Article

A Model for the Spread of Infectious Diseases Supplemental Material

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Given the amount of data processed and associated graphs for each of the ten countries selected for the study of presumed deaths COVID-19 related, these have been separated into this supplemental material section that is associated with the paper. Table S1 shows the accumulated number of presumed deaths due to COVID-19 per day for the selected ten countries.

Table S1: accumulated number of presumed deaths due to COVID-19 taken from [1].
Abbreviation Key: China (**CN**), South Korea (**SK**), Italy (**IT**), Iran (**IR**), USA (**US**), Spain (**SP**), Germany (**DE**), United Kingdom (**UK**), Netherlands (**NE**).

Date	CN	SK	IT	IR	US	SP	FR	DE	UK	NE
01/21	9	0	0	0	0	0	0	0	0	0
01/22	17	0	0	0	0	0	0	0	0	0
01/23	25	0	0	0	0	0	0	0	0	0
01/24	41	0	0	0	0	0	0	0	0	0
01/25	56	0	0	0	0	0	0	0	0	0
01/26	80	0	0	0	0	0	0	0	0	0
01/27	106	0	0	0	0	0	0	0	0	0
01/28	132	0	0	0	0	0	0	0	0	0
01/29	170	0	0	0	0	0	0	0	0	0
01/30	213	0	0	0	0	0	0	0	0	0
01/31	259	0	0	0	0	0	0	0	0	0
02/01	304	0	0	0	0	0	0	0	0	0
02/02	361	0	0	0	0	0	0	0	0	0
02/03	425	0	0	0	0	0	0	0	0	0
02/04	490	0	0	0	0	0	0	0	0	0
02/05	563	0	0	0	0	0	0	0	0	0
02/06	636	0	0	0	0	0	0	0	0	0
02/07	722	0	0	0	0	0	0	0	0	0
02/08	811	0	0	0	0	0	0	0	0	0
02/09	908	0	0	0	0	0	0	0	0	0
02/10	1016	0	0	0	0	0	0	0	0	0
02/11	1113	0	0	0	0	0	0	0	0	0
02/12	1259	0	0	0	0	0	0	0	0	0
02/13	1380	0	0	0	0	0	0	0	0	0
02/14	1523	0	0	0	0	0	0	0	0	0
02/15	1665	0	0	0	0	0	1	0	0	0

02/16	1770	0	0	0	0	0	1	0	0	0
02/17	1868	0	0	0	0	0	1	0	0	0
02/18	2004	0	0	0	0	0	1	0	0	0
02/19	2118	0	0	2	0	0	1	0	0	0
02/20	2236	1	0	2	0	0	1	0	0	0
02/21	2345	2	1	4	0	0	1	0	0	0
02/22	2442	2	2	6	0	0	1	0	0	0
02/23	2592	6	3	8	0	0	1	0	0	0
02/24	2663	8	7	12	0	0	1	0	0	0
02/25	2715	11	11	16	0	0	1	0	0	0
02/26	2744	12	12	19	0	0	2	0	0	0
02/27	2788	13	17	26	0	0	2	0	0	0
02/28	2835	16	21	34	0	0	2	0	0	0
02/29	2870	17	29	43	1	0	2	0	0	0
03/01	2912	21	41	54	1	0	2	0	0	0
03/02	2943	28	52	66	6	0	3	0	0	0
03/03	2981	32	79	77	9	1	4	0	0	0
03/04	3012	35	107	92	11	2	4	0	0	0
03/05	3042	42	148	108	12	3	7	0	1	0
03/06	3070	43	197	124	15	8	9	0	1	1
03/07	3097	48	233	145	19	10	16	0	1	1
03/08	3119	50	366	194	22	17	19	0	2	3
03/09	3136	53	463	237	26	30	30	2	3	4
03/10	3158	60	631	291	30	36	33	2	7	4
03/11	3169	60	827	354	38	55	48	3	7	5
03/12	3176	66	1016	429	41	86	61	6	9	5
03/13	3189	67	1266	514	49	133	79	8	10	10
03/14	3199	72	1441	611	57	196	91	9	28	12
03/15	3213	75	1809	724	68	294	127	13	43	20
03/16	3226	75	2158	853	86	342	148	17	65	24
03/17	3237	81	2503	988	109	533	175	26	81	43
03/18	3245	84	2978	1135	150	638	264	28	115	58
03/19	3248	91	3405	1284	207	831	372	44	158	76
03/20	3255	94	4032	1433	256	1093	450	68	194	106
03/21	3261	102	4825	1556	302	1381	562	84	250	136
03/22	3270	104	5476	1685	414	1772	674	94	285	179
03/23	3277	111	6077	1812	555	2311	860	123	359	213
03/24	3281	120	6820	1934	780	2991	1100	159	508	276
03/25	3287	126	7503	2077	1027	3647	1331	206	649	356
03,26	3292	131	8215	2234	1295	4365	1696	267	877	434
03/27	3295	139	9134	2378	1696	5138	1995	351	1161	546
03/28	3300	144	10023	2517	2221	5982	2314	433	1459	639
03/29	3300	152	10779	2640	2583	6803	2606	541	1669	771
03/30	3305	158	11591	2757	3141	7716	3024	645	2043	864
03/31	3312	162	12482	2898	4053	8464	3523	775	2425	1039
04/01	3318	165	13155	3036	5102	9387	4032	931	3095	1173
04/02	3326	169	13915	3160	6076	10348	5387	1107	3747	1339
04/03	3329	174	14681	3294	7121	11198	6507	1275	4461	1487

04/04	3331	177	15362	3452	8452	11947	7560	1444	5221	1651
04/05	3331	183	15887	3603	9616	12641	8078	1584	5865	1766
04/06	3331	186	16523	3739	10941	13341	8911	1810	6433	1867
04/07	3333	192	17127	3872	12848	14045	10328	2016	7471	2101
04/08	3335	200	17669	3993	14788	14729	10869	2349	8505	2248
04/09	3336	204	18279	4110	16691	15447	12210	2607	9608	2396
04/10	3339	208	18849	4232	18747	16081	13197	2736	10760	2511
04/11	3339	211	19468	4357	20577	16606	13832	2871	11599	2643
04/12	3341	214	19899	4474	22105	17209	14393	3022	12285	2737
04/13	3341	217	20465	4585	23640	17756	14967	3194	13029	2823
04/14	3342	222	21067	4683	29825	18255	15729	3495	14073	2945
04/15	3342	225	21645	4777	32443	18812	17167	3804	14915	3134
04/16	3342	229	22170	4869	34619	19130	17920	4052	15944	3315
04/17	4632	230	22745	4958	37154	19478	18681	4352	16879	3459
04/18	4632	232	23227	5031	39014	20043	19323	4538	17994	3601
04/19	4632	234	23660	5118	40575	20453	19718	4642	18492	3668
04/20	4632	236	24114	5209	42514	20852	20265	4862	19051	3751
04/21	4632	238	24648	5297	45318	21282	20796	5086	20223	3916
04/22	4632	240	25085	5391	47681	21717	21340	5315	21060	4055
04/23	4632	240	25549	5481	50243	22157	21856	5575	21787	4177
04/24	4632	240	25669	5574	52193	22524	22245	5760	22792	4289
04/25	4632	242	26384	5650	54265	22902	22614	5877	23635	4409
04/26	4633	243	26644	5710	55415	23190	22856	5976	24055	4475
04/27	4633	244	26977	5806	56803	23521	23293	6126	24393	4518
04/28	4633	244	27359	5877	59266	23822	23660	6314	25302	4566
04/29	4633	246	27682	5957	61656	24275	24087	6467	26097	4711
04/30	4633	247	27967	6028	63861	24543	24376	6623	26771	4795
05/01	4633	248	28236	6091	65753	24824	24594	6736	27510	4893
05/02	4633	250	28710	6156	67444	25100	24760	6812	28131	4897
05/03	4633	250	28884	6203	68597	25264	24895	6866	28446	5056
05/04	4633	252	29079	6277	69921	25428	25201	6993	28734	5082
05/05	4633	254	29315	6340	72271	25613	25531	6993	29427	5168
05/06	4633	255	29684	6418	74799	25875	25809	7275	30076	5204
05/07	4633	256	29958	6486	76928	26070	25987	7392	30615	5288
05/08	4633	256	30201	6541	78615	26299	26230	7510	31241	5359
05/09	4633	256	30395	6589	80037	26478	26310	7549	31587	5422
05/10	4633	256	30560	6640	80787	26621	26380	7569	31855	5440
05/11	4633	256	30739	6685	81847	26744	26643	7661	32065	5456
05/12	4633	258	30911	6733	83718	26920	26991	7738	32692	5510
05/13	4633	259	31106	6783	85540	27104	27074	7861	33186	5562
05/14	4633	260	31368	6854	87293	27321	27425	7928	33614	5590
05/15	4633	260	31610	6902	88895	27459	27529	8001	33988	5643
05/16	4633	262	31763	6937	90113	27563	27625	8027	34466	5670
05/17	4634	262	31908	6988	90978	27650	28108	8049	34636	5680
05/18	4634	263	32007	7057	91981	27709	28239	8123	34796	5694
05/19	4634	263	32169	7119	93533	27778	28022	8193	35341	5715
05/20	4634	263	32330	7183	94936	27888	28132	8270	35704	5748
05/21	4634	264	32486	7249	96347	27940	28215	8309	36042	5775

05/22	4634	264	32616	7300	97645	28628	28289	8352	36393	5788
05/23	4634	266	32735	7359	98678	28678	28332	8366	36675	5810
05/24	4634	266	32785	7417	99293	28752	28367	8371	37116	5822
05/25	4634	267	32877	7451	99798	26837	28432	8428	37237	5830
05/26	4634	269	32955	7508	100572	27117	28530	8498	37373	5856
05/27	4634	269	33072	7564	102107	27118	28596	8533	37807	5871
05/28	4634	269	33142	7627	103330	27119	28662	8570	38220	5903
05/29	4634	269	33229	7677	104542	27121	28714	8594	38593	5931
05/30	4634	269	33340	7734	105557	27125	28771	8600	38819	5951
05/31	4634	270	33415	7797	106195	27127	28802	8605	38934	5956
Date	**CN**	**SK**	**IT**	**IR**	**US**	**SP**	**FR**	**DE**	**UK**	**NE**

The graphs that were generated for several of the countries listed in **Table-1** include the following curves:

(1) Actual data $y_a(t)$ and the actual data model estimates curve $\hat{y}_a(t)$.

(2) The natural logarithm of the actual data $y_1(t)$ and the model estimate curve $\hat{y}_1(t)$.

(3) The linearization of the actual data logarithm $y_2(t)$ and the linear fit $\hat{y}_2(t)$.

(4) The derivative of the actual data model $d\hat{y}_a(t)/dt$.

(5) Some additional graphs of interest, like fit residuals are in some cases provided.

Included also with the figures are the horizontal asymptote value y_∞ for the natural logarithm of the actual data and the process P_1 time constant $\tau = 1/\alpha$ [days], both parameters corresponding to our model, the curve $\hat{y}_1(t)$.

Some additional information is provided when applicable, such as changes in the data recorded.

Data extrapolations are indicated via asterisks, following the model estimate curves.

[1] Modeling Applied to China (CN)

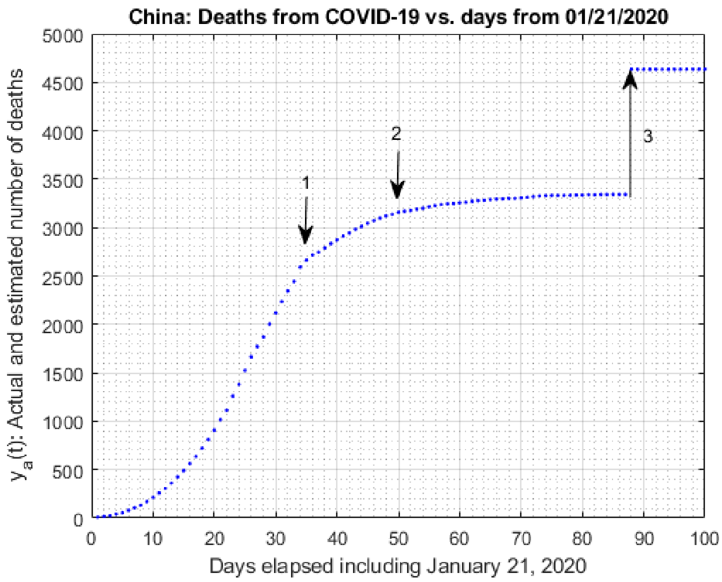

Figure S1 (CN): Actual Reported Data. There is a markedly step in the first time-derivative in point indicated by (1) that occurred around February 23, and another minor step in the first derivative indicated by (2) that occurred around March 10, 2020, and a major correction between April 16 and 17, 2020, from 3342 accumulated presumed deaths to 4632, a jump of 1290 presumed deaths. Because of these critical points in the actual curve above, the authors modeled (and extrapolated) the pandemic presumed deaths using only the data recorded at the beginning up to break point (1) in this graph. The best fit provided extrapolation estimates that joined the correction after the jump (3) very closely. This is shown in the following FigureS2.

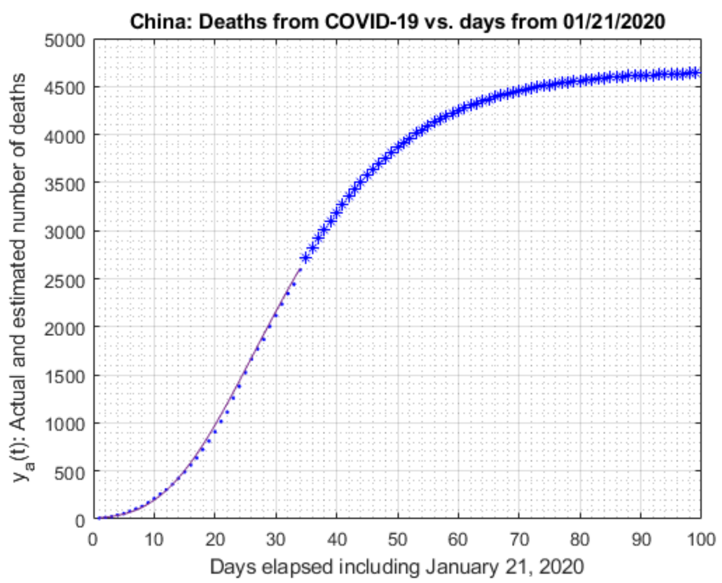

Figure S2 (CN): Actual and Extrapolated number of presumed deaths using only the consistent data from January 21 to February 23.

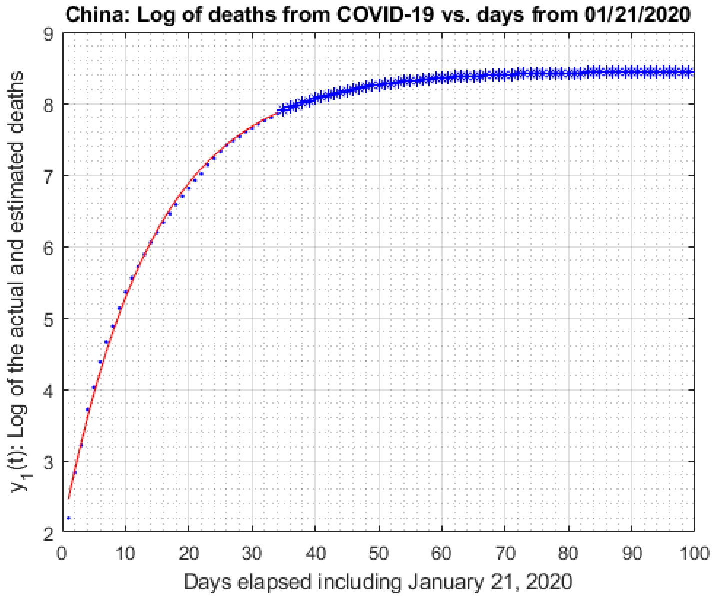

Figure S3 (CN): Natural Logarithm of the Actual (and extrapolated) Data in Figure S2.
Note: In this figure the plateau or saturation value is $y_\infty = 8.448$ and the time-constant is $\tau = 14.1493$ days. These numbers indicate that the number of presumed deaths might reach a value of $y_a(t \to \infty) = \exp(8.448) \simeq 4666$ people. The sum of squares of the residuals (i.e., $r_i(t) = \hat{y}_1(t) - y_1(t)$ for estimated minus actual values) for this data is $RSS = (\mathbf{r}^T\mathbf{r})^{1/2} = 0.4399$

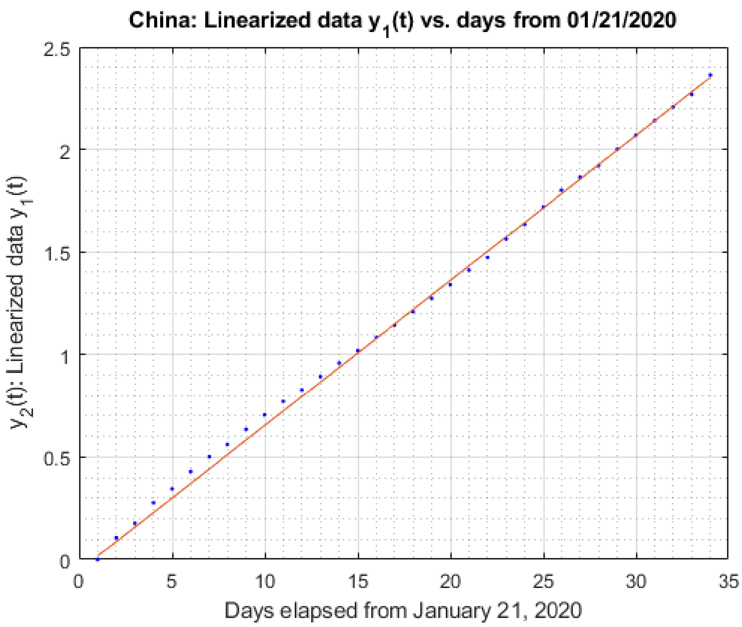

Figure S4 (CN): Linearization of Data in Figure S3 (dots) and Weighted Least Squares Linear Fit (continuous curve).

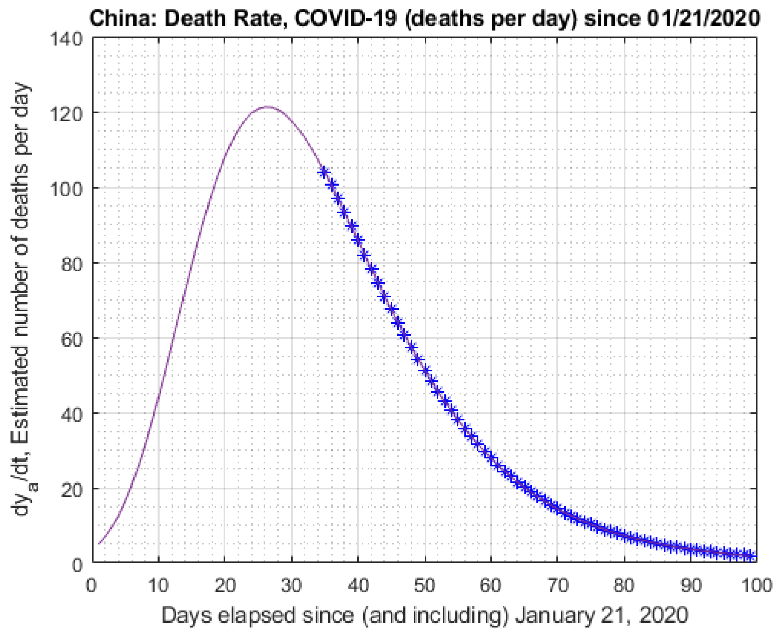

Figure S5 (CN): Date Rate corresponding to the Derivative of the Estimated Data in Figure S2. The peak occurred in February 15, 2020.

[2] Modeling Applied to South Korea (SK)

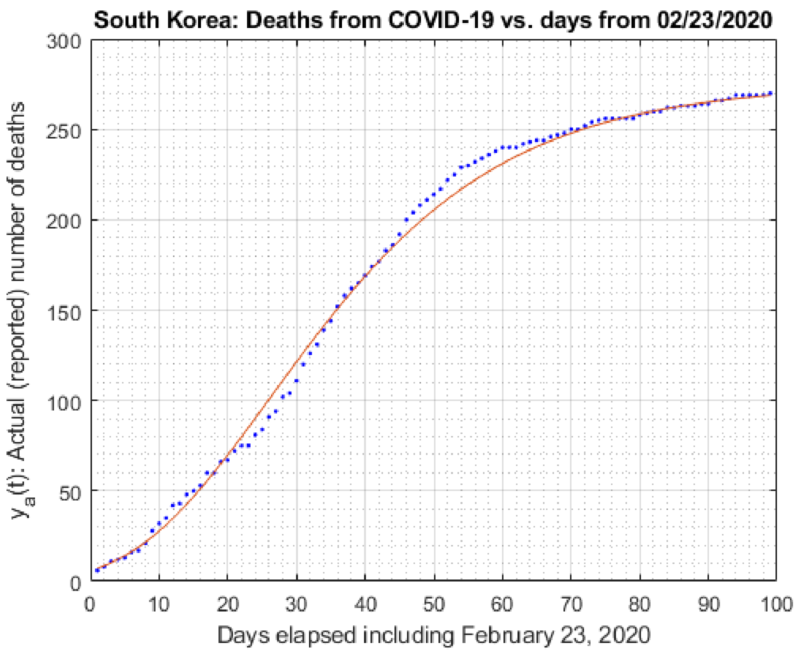

Figure S6 (SK): Actual and modeled number of presumed deaths using only data from February 23, 2020.

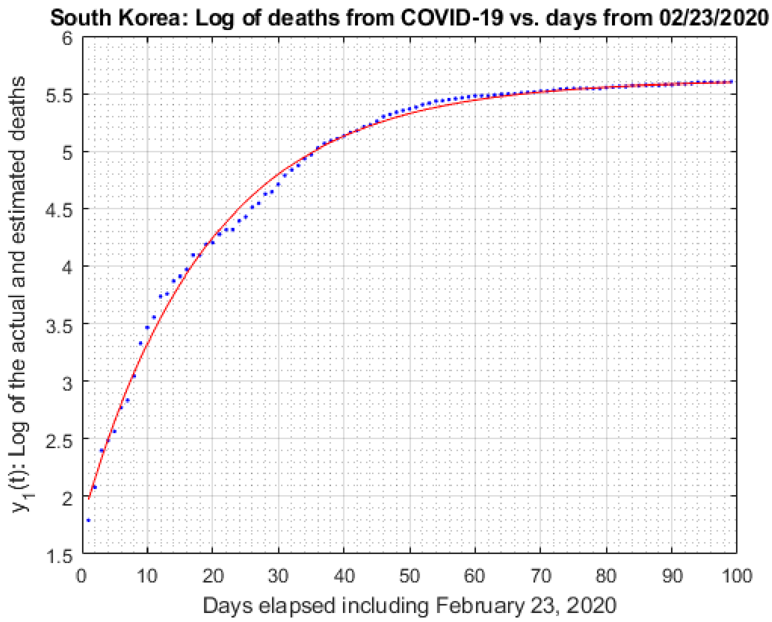

Figure S7 (SK): Natural Logarithm of the Actual (and extrapolated) Data in Figure S6**.**
Note: In this figure the plateau or saturation value is $y_\infty = 5.6173$ and the time-constant is $\tau = 19.415$ days. These numbers indicate that the number of presumed deaths might reach a value of $y_a(t \to \infty) = \exp(5.6173) \simeq 275$ people. The sum of squares of the residuals (i.e., $r_i(t) = \hat{y}_1(t) - y_1(t)$ for estimated minus actual values) for this data is $RSS = (\mathbf{r}^T\mathbf{r})^{1/2} = 0.5745$.

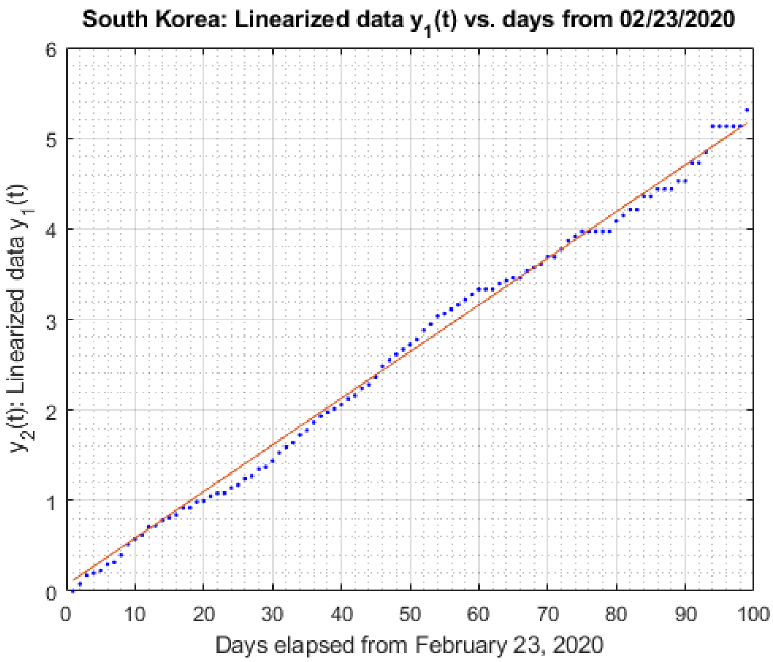

Figure S8 (SK): Linearization of Data in Figure S7 (dots) and Weighted Least Squares Linear Fit (continuous curve).

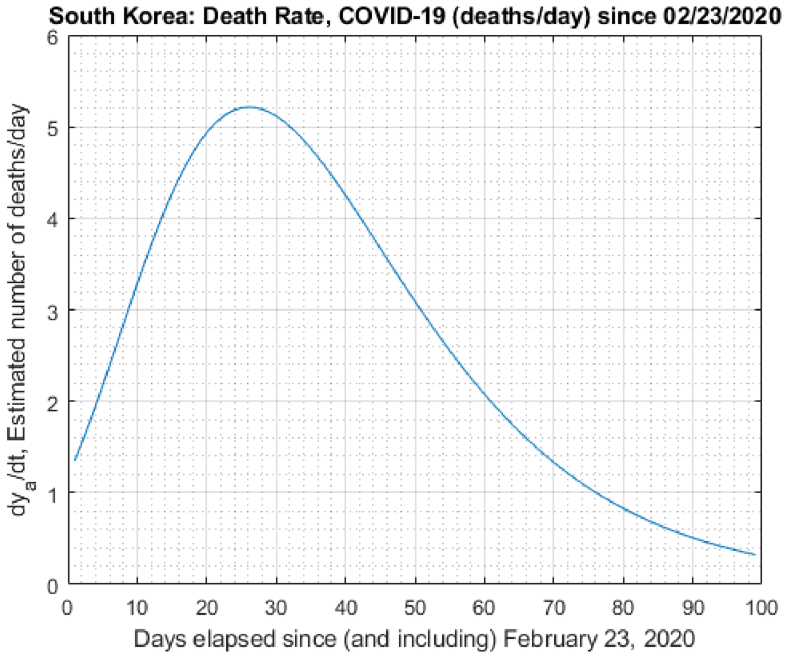

Figure S9 (SK): Date Rate corresponding to the Derivative of the Estimated Data in **Figure S6**. The peak occurred in March 19, 2020 according to the model.

[3] Modeling Applied to Italy (IT)

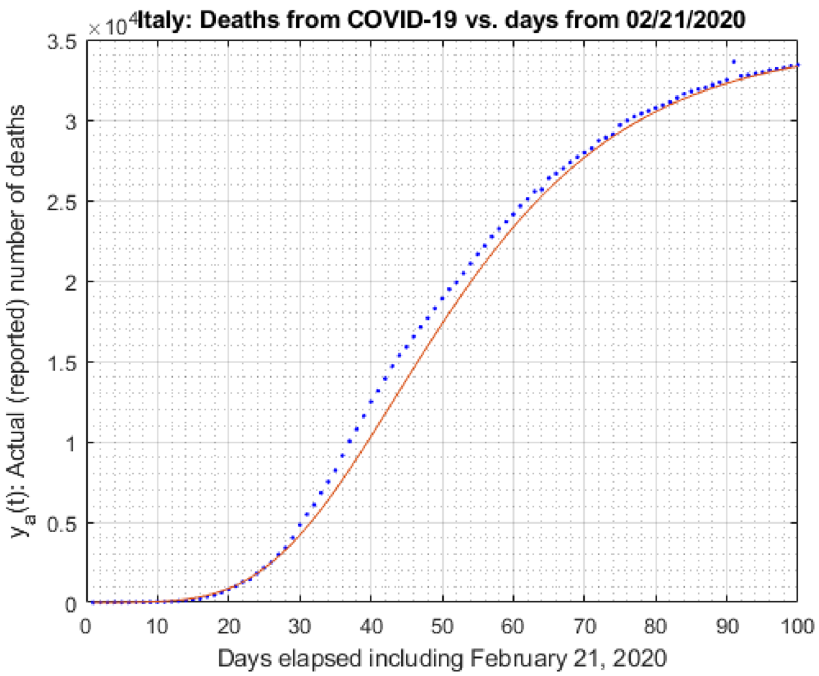

Figure S10 (IT): Actual and modeled number of presumed deaths using only data from February 21, 2020

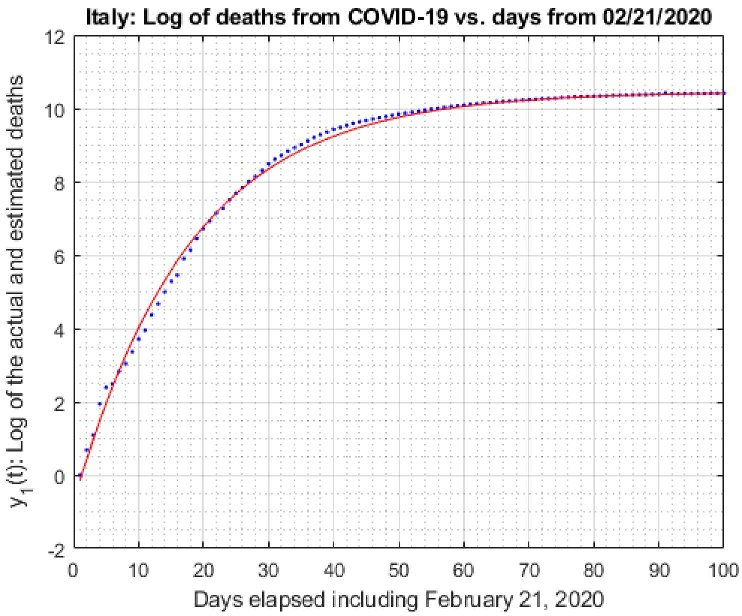

Figure S11 (IT): Natural Logarithm of the Actual (and extrapolated) Data in Figure S10. **Note:** In this figure the plateau or saturation value is $y_\infty = 10.4574$ and the time-constant is $\tau = 18.0002$ days. These numbers indicate that the number of presumed deaths might reach a value of $y_a(t \to \infty) = \exp(10.4574) \simeq 34801$ people. The sum of squares of the residuals (i.e., $r_i(t) = \hat{y}_1(t) - y_1(t)$ for estimated minus actual values) for this data *is* $RSS = (\mathbf{r}^T\mathbf{r})^{1/2} = 1.4644$.

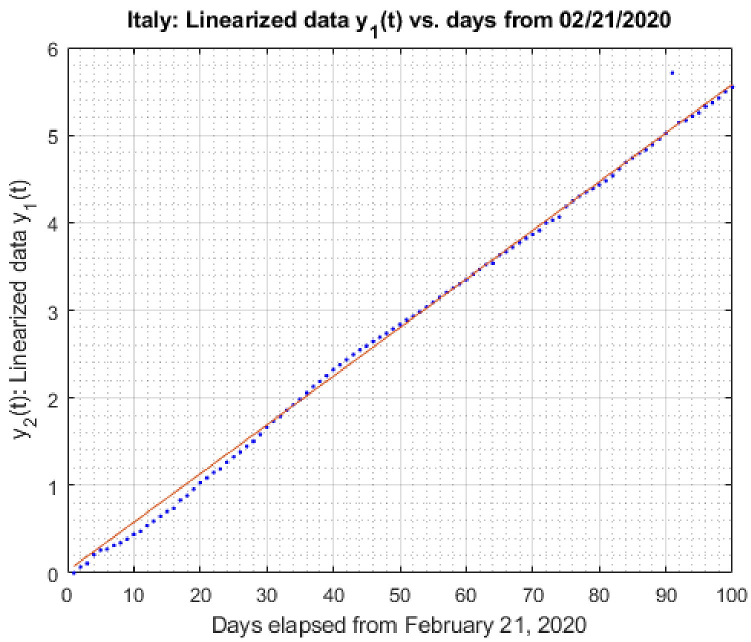

Figure S12 (IT): Linearization of Data in Figure S11 (dots) and Weighted Least Squares Linear Fit (continuous curve).

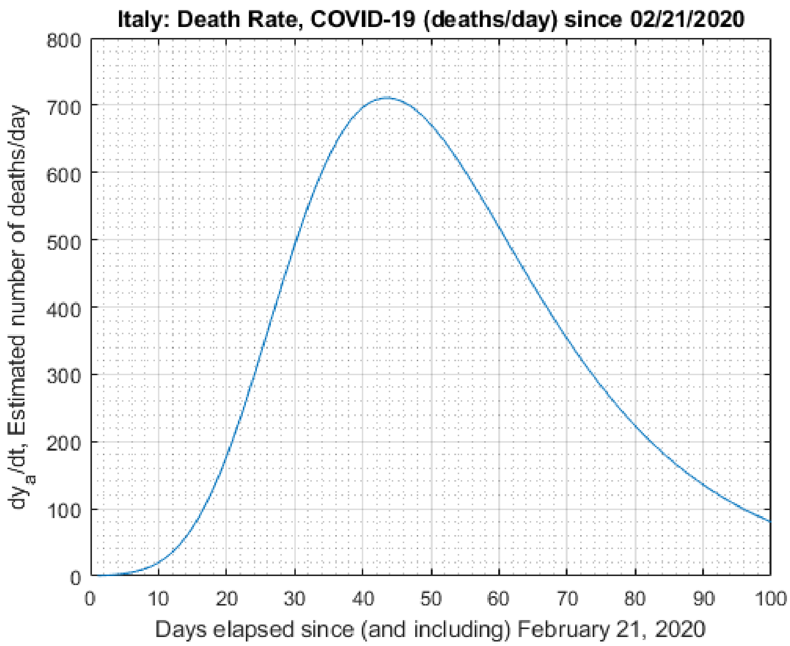

Figure S13 (IT): Date Rate corresponding to the Derivative of the Estimated Data in **Figure S10.** The peak occurred in April 04, 2020 according to the model.

[4] Modeling Applied to Iran (IR)

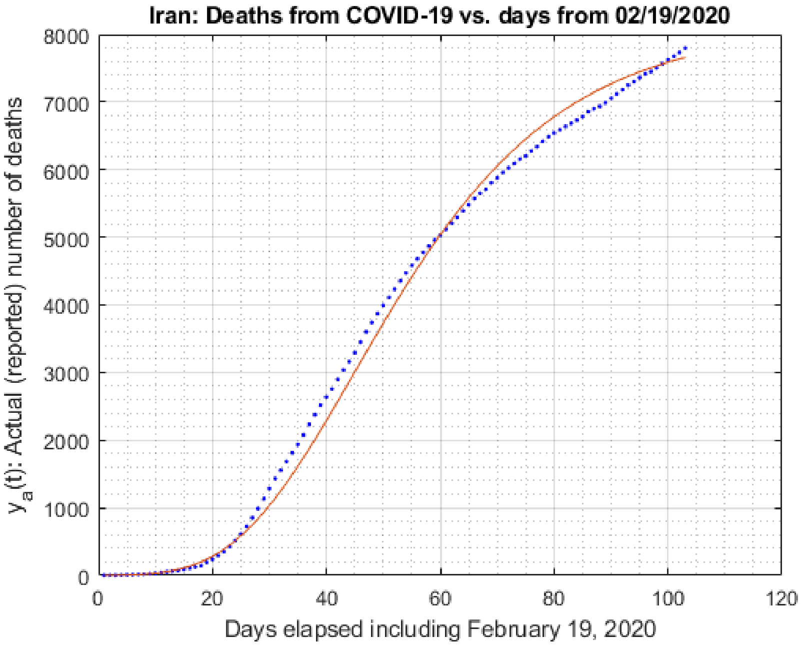

Figure S14 (IR): Actual and modeled number of presumed deaths using only data from February 19, 2020

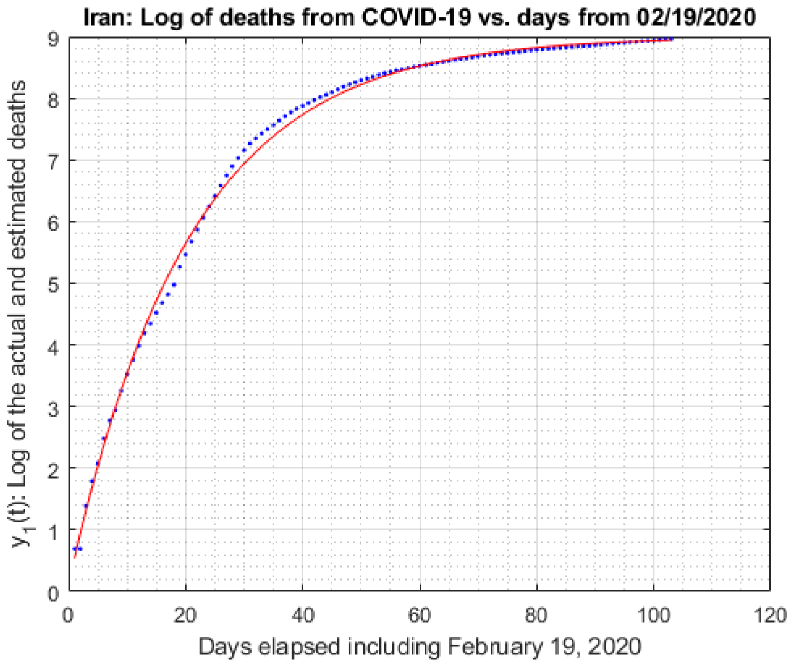

Figure S15 (IR): Natural Logarithm of the Actual Data in Figure S14**.**

Note: In this figure the plateau or saturation value is $y_\infty = 9.00253$ and the time-constant is $\tau = 20.5296$ days. These numbers indicate that the number of presumed deaths might reach a value of $y_a(t \to \infty) = \exp(9.00253) \simeq 8124$ people. The sum of squares of the residuals (i.e., $r_i(t) = \hat{y}_1(t) - y_1(t)$ for estimated minus actual values) for this data is $RSS = (\mathbf{r}^T\mathbf{r})^{1/2} = 1.0933$.

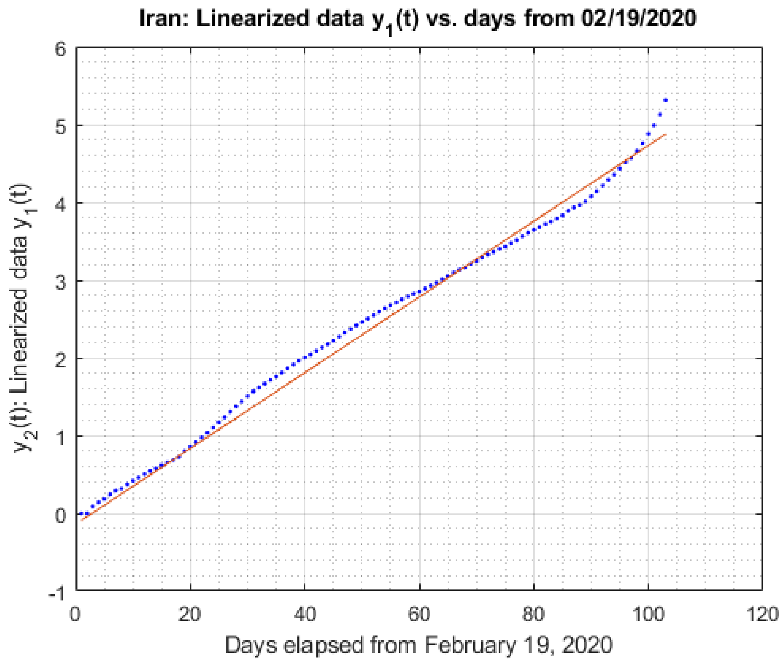

Figure S16 (IR): Linearization of Data in Figure S15 (dots) and Weighted Least Squares Linear Fit (continuous curve).

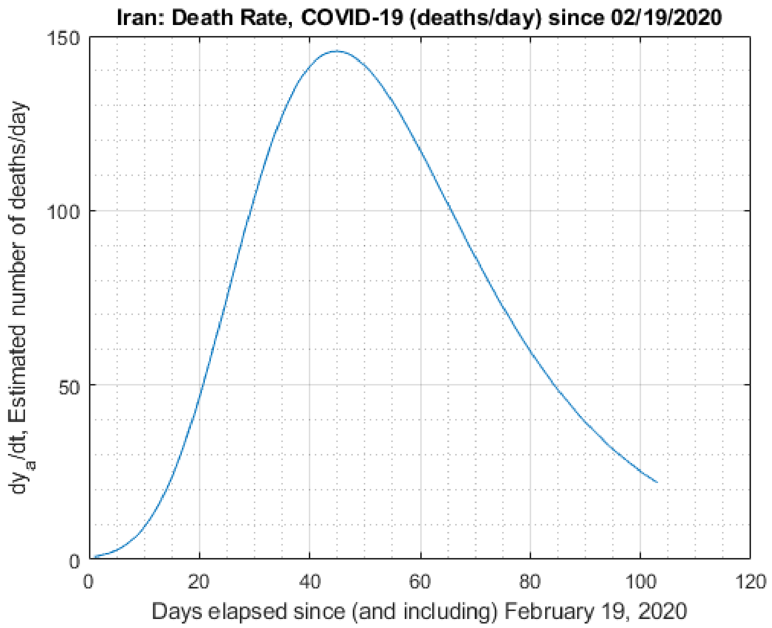

Figure S17 (IR): Date Rate corresponding to the Derivative of the Estimated Data in Figure S14. The peak occurred in April 03, 2020 according to the model.

[5] Modeling Applied to USA (US)

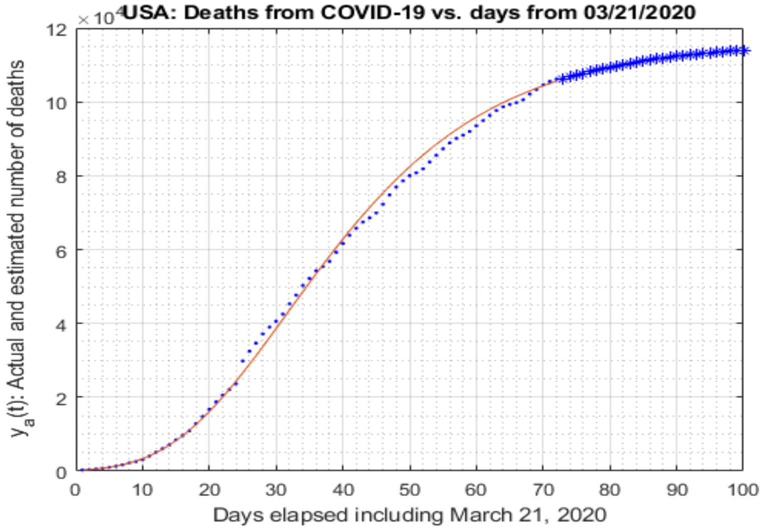

Figure S18 (US): Actual Data (dots) and Model Estimate (continuous curve)

Note: In the following figure (**Figure A-2**), the plateau or saturation value is $y_\infty = 11.6622$ and the time-constant $\tau = 17.1261$ days. These numbers indicate that the number of presumed deaths might reach a value of $y_a(t \to \infty) = \exp(11.6622) \simeq 116{,}099$ people. The sum of squares of the residuals (i.e., $r_i(t) = \hat{y}_1(t) - y_1(t)$ for estimated minus actual values) for this data is $RSS = (\mathbf{r}^T\mathbf{r})^{1/2} = 0.3230$.

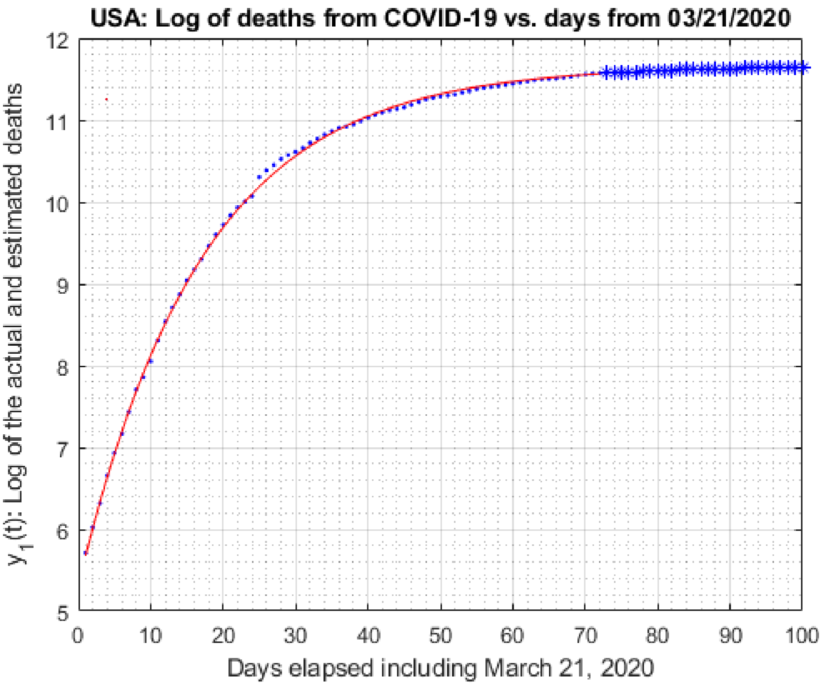

Figure S19 (US): Natural Logarithm of Actual Data in Figure S18 (dots) and Model Estimate (continuous curve). Here $y_\infty = 11.6622$ and $\tau = 17.126$ days.

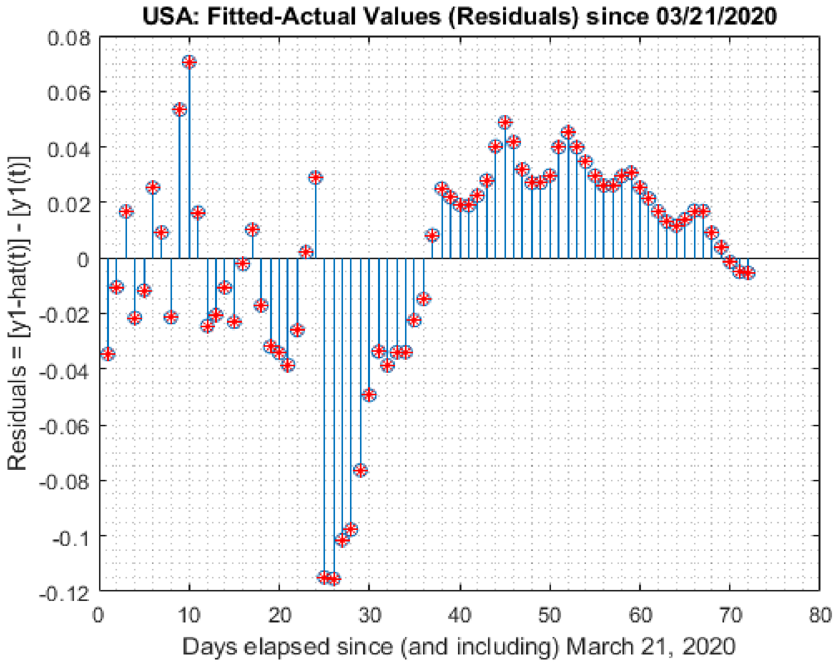

Figure S20 (US): Residuals $r_i(t)=\hat{y}_1(t)-y_1(t)$ of fitted minus actual data shown in Figure S19**.** An unexplained weekly periodicity can be observed in the residuals throughout.

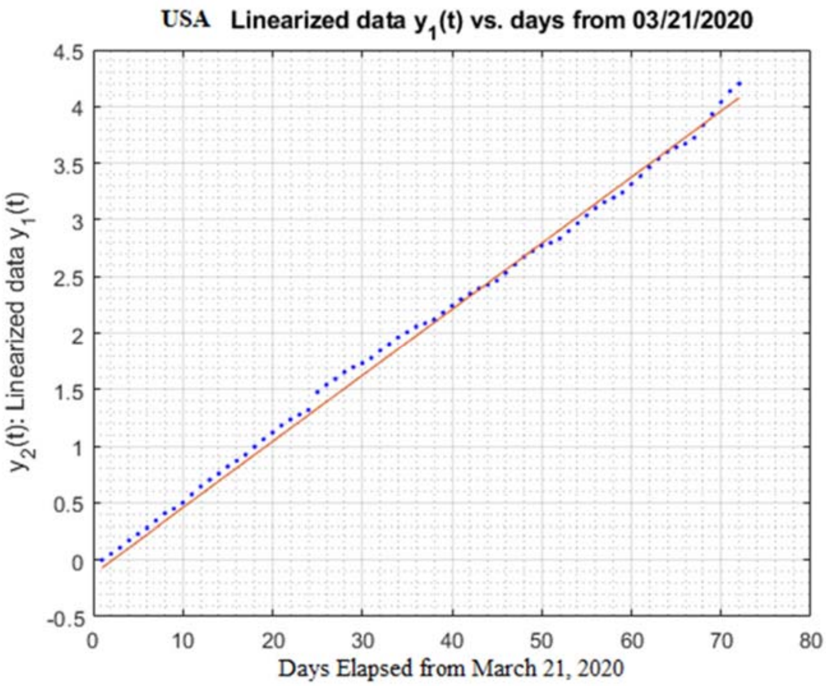

Figure S21 (US): Linearization of Data in Figure S19 (dots) and Weighted Least Squares Linear Fit (continuous curve).

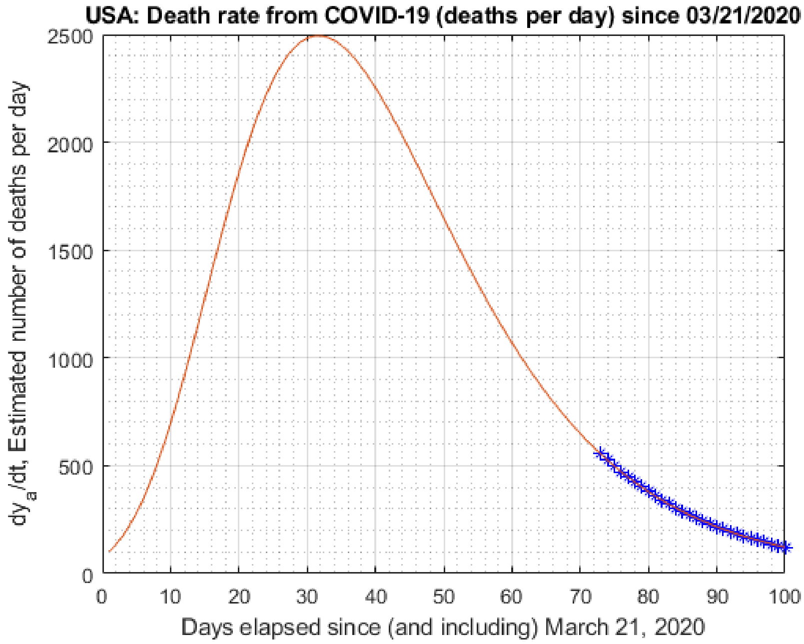

Figure S22 (US): Date Rate corresponding to the Derivative of the Estimated Data in Figure S18. The peak occurred in April 22, 2020, according to the model.

[6] Modeling Applied to Spain (SP)

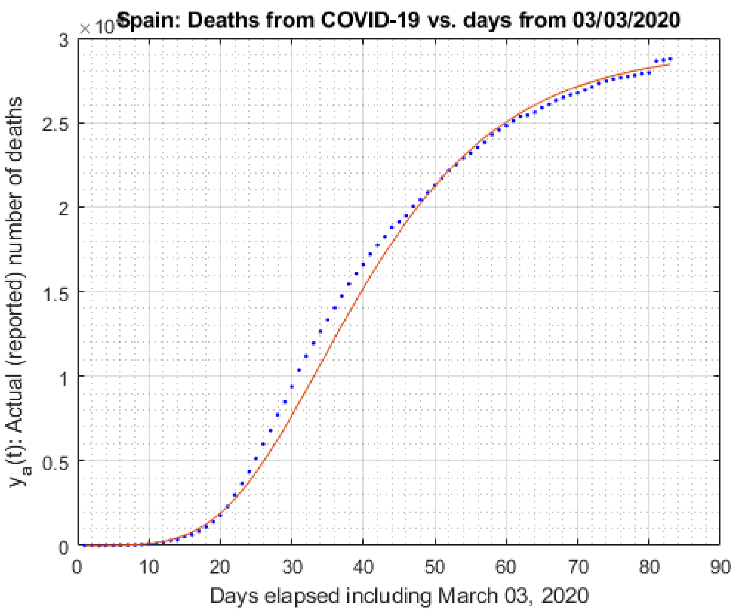

Figure S23 (SP): Actual and modeled number of presumed deaths using only data from March 03, 2020.

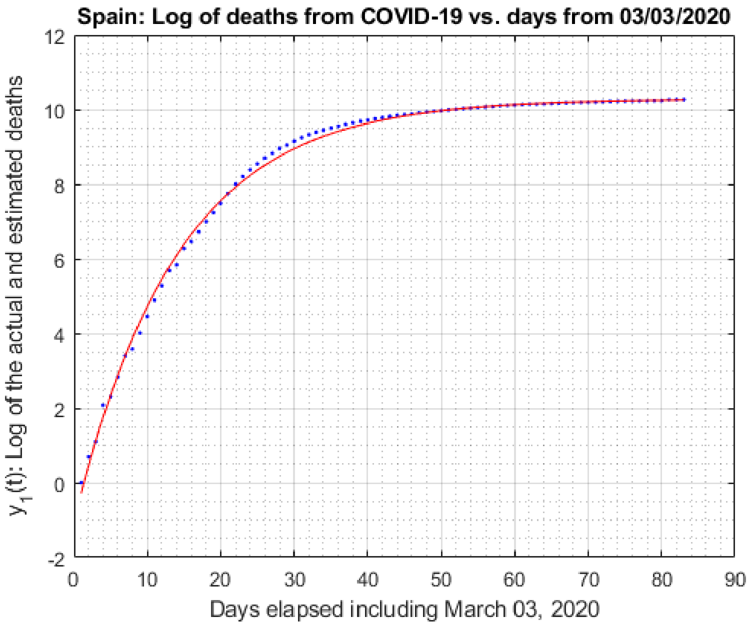

Figure S24 (SP): Natural Logarithm of the Actual (and extrapolated) Data in Figure S23. **Note:** In this figure the plateau or saturation value is $y_\infty = 10.28584$ and the time-constant is $\tau = 14.0376$ days. These numbers indicate that the number of presumed deaths might reach a value of $y_a(t \to \infty) = \exp(10.28584) \simeq 29315$ people. The sum of squares of the residuals (i.e., $r_i(t) = \hat{y}_1(t) - y_1(t)$ for estimated minus actual values) for this data is $RSS = (\mathbf{r}^T\mathbf{r})^{1/2} = 1.376$

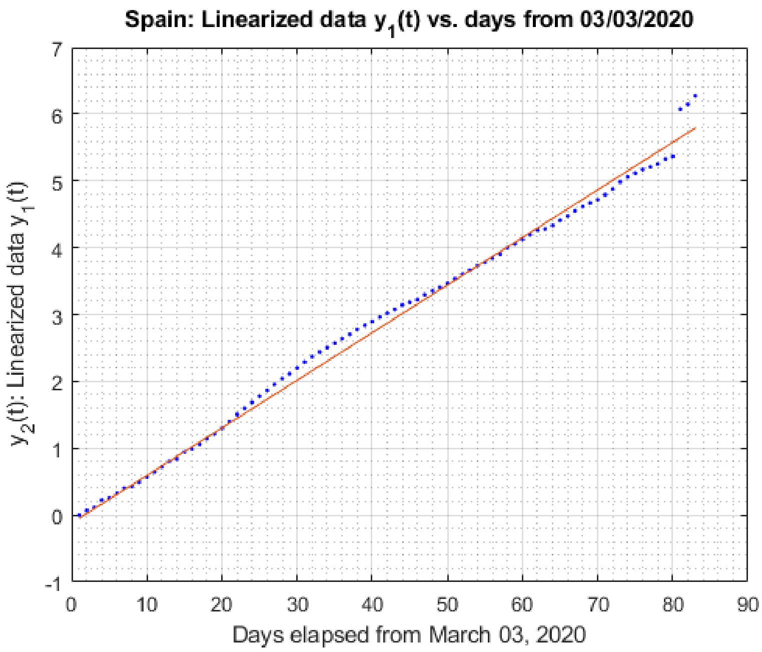

Figure S25 (SP): Linearization of Data in Figure S24 (dots) and Weighted Least Squares Linear Fit (continuous curve). The plot clearly shows a jump discontinuity between May 20 and May 21.

The data (not shown here but is included in Table 1) had a down-step from 28752 on 05/25/2020 to 26837 on 05/25, not a correct step for cumulative data, and then another step-up discontinuity on 05/27/2020.

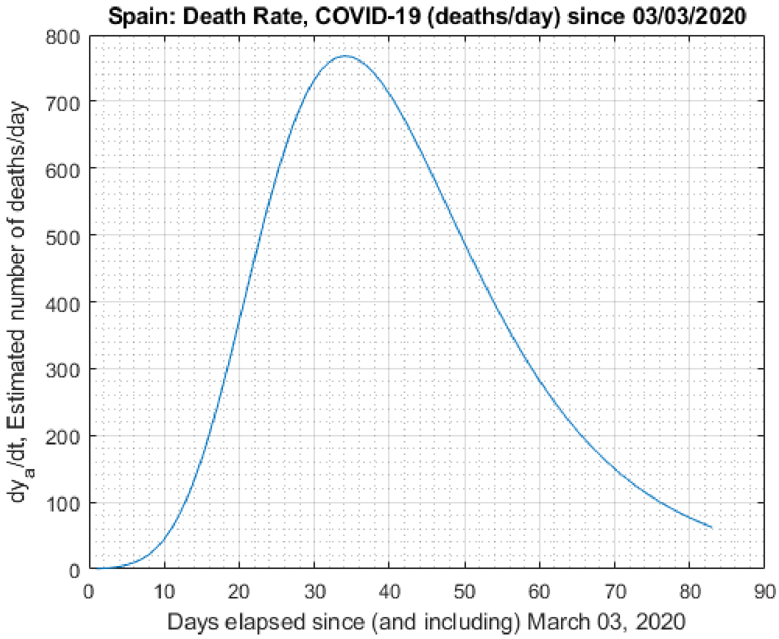

Figure S26 (SP): Date Rate corresponding to the Derivative of Estimated Data in Figure S23. The peak occurred in April 05, 2020, according to the model.

[7] Modeling Applied to France (FR)

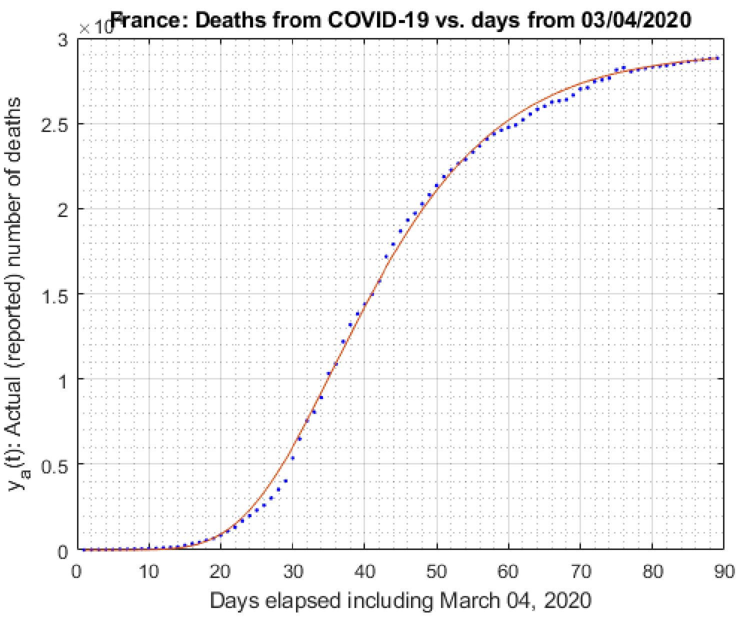

Figure S27 (FR): Actual and modeled number of presumed deaths using only data from March 04, 2020.

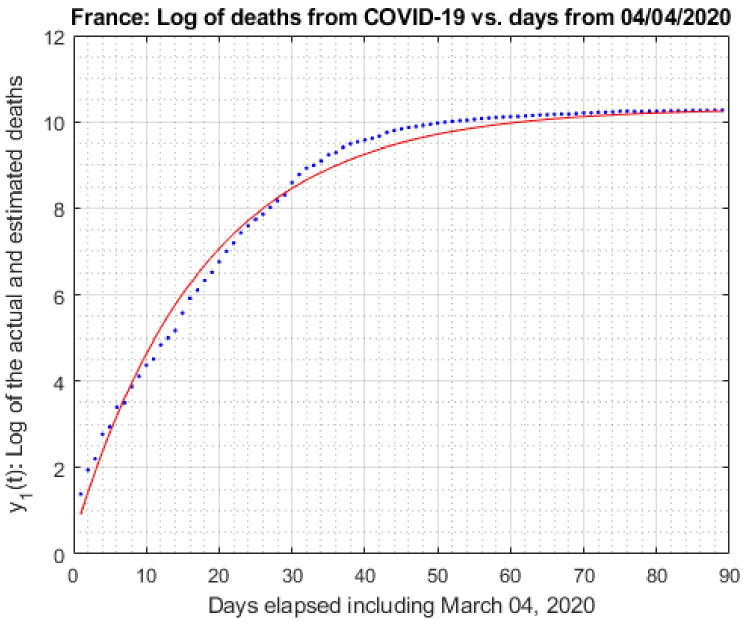

Figure S28 (FR): Natural Logarithm of the Actual (and extrapolated) Data in Figure S27. **Note:** In this figure the plateau or saturation value is $y_{\infty} = 10.3182$ and the time-constant is $\tau = 17.8885$ days. These numbers indicate that the number of presumed deaths might reach a value of $y_a(t \to \infty) = \exp(10.3182) \simeq 30279$ people. The sum of squares of the residuals (i.e., $r_i(t) = \hat{y}_1(t) - y_1(t)$ for estimated minus actual values) for this data is $RSS = (\mathbf{r}^T\mathbf{r})^{1/2} = 2.2950$.

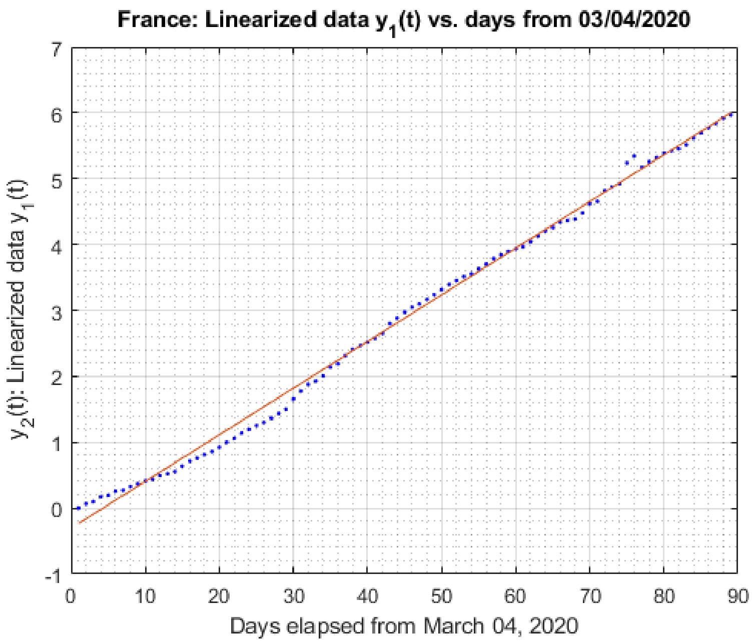

Figure S29 (FR): Linearization of Data in Figure S28 (dots) and Weighted Least Squares Linear Fit (continuous curve). The fit improves for later data due to the weighted Lest Squares putting higher weights on the later data. A couple of outliers can be observed for elapsed days 75 and 76.

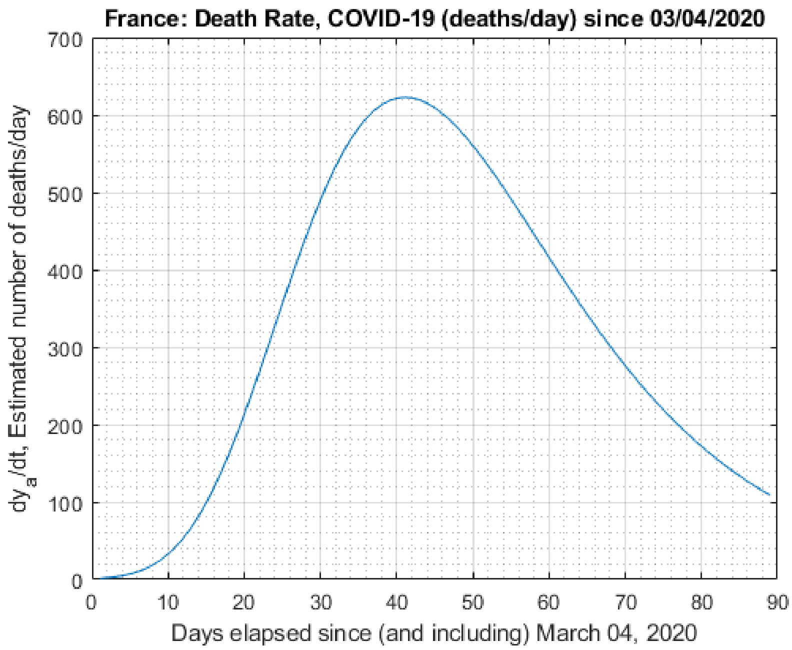

Figure S30 (FR): Date Rate corresponding to the Derivative of Estimated Data in Figure S27. The peak occurred in April 13, 2020, according to the model.

[8] Modeling Applied to Germany (DE)

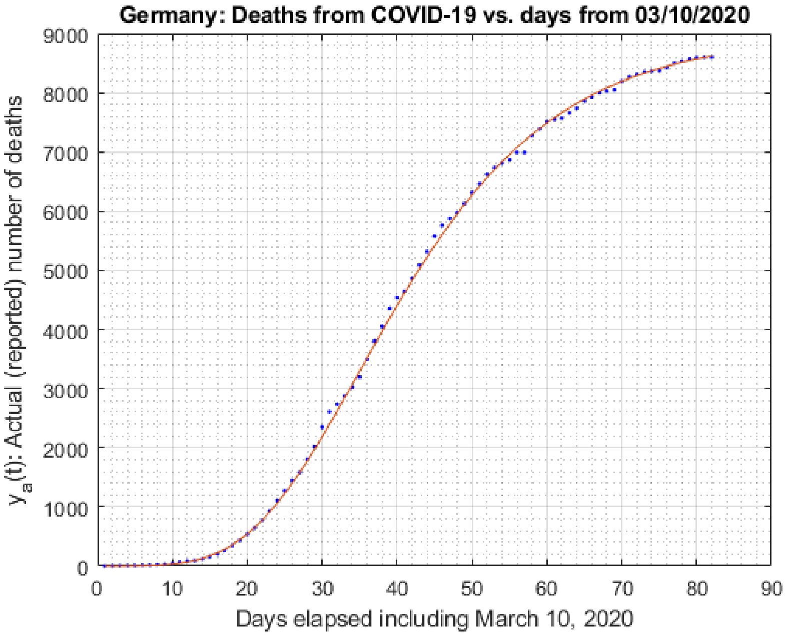

Figure S31 (DE): Actual and modeled number of presumed deaths using only data from March 10, 2020.

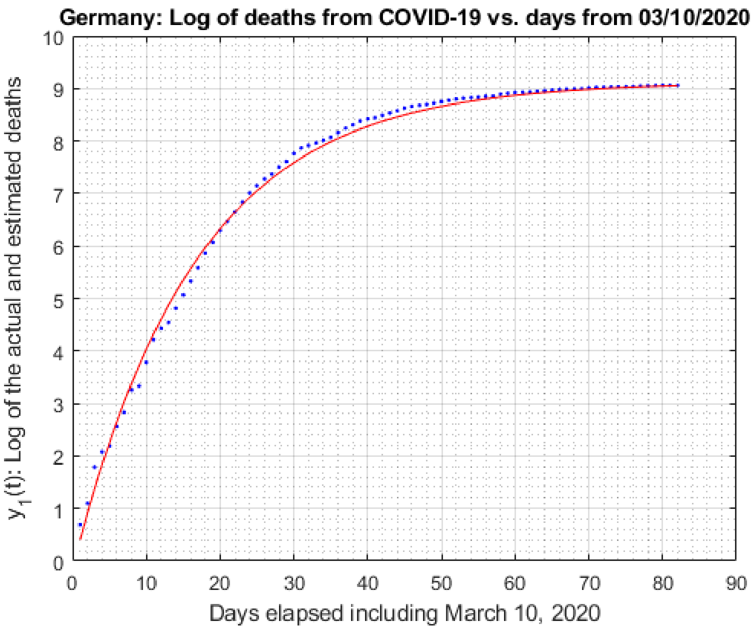

Figure S32 (DE): Natural Logarithm of the Actual (and extrapolated) Data in Figure S31**.**

Note: In this figure the plateau or saturation value is $y_\infty = 9.1235$ and the time-constant is $\tau = 16.7038$ days. These numbers indicate that the number of presumed deaths might reach a value of $y_a(t \to \infty) = \exp(9.1235) \simeq 9168$ people. The sum of squares of the residuals (i.e., $r_i(t) = \hat{y}_1(t) - y_1(t)$ for estimated minus actual values) for this data is $RSS = (\mathbf{r}^T\mathbf{r})^{1/2} = 1.2656$.

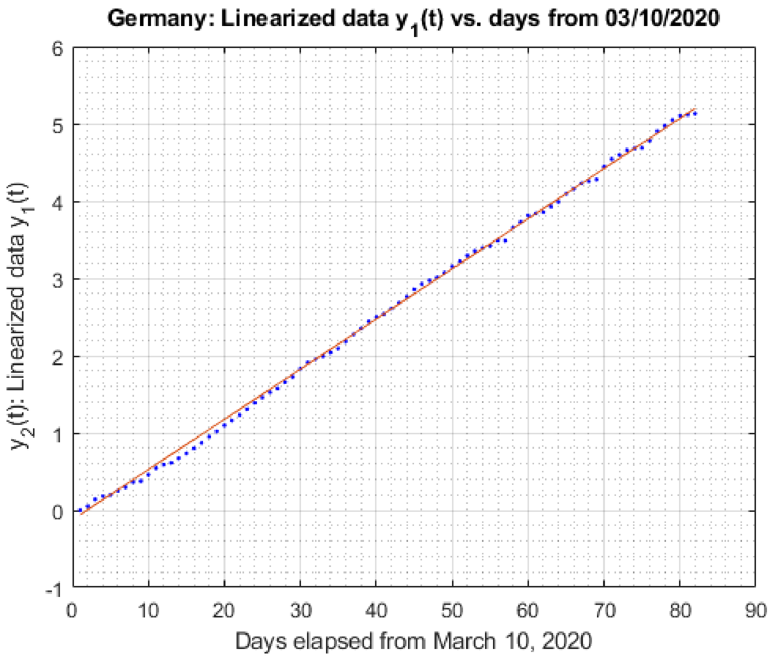

Figure S33 (DE): Linearization of Data in Figure S32 (dots) and Weighted Least Squares Linear Fit (continuous curve).

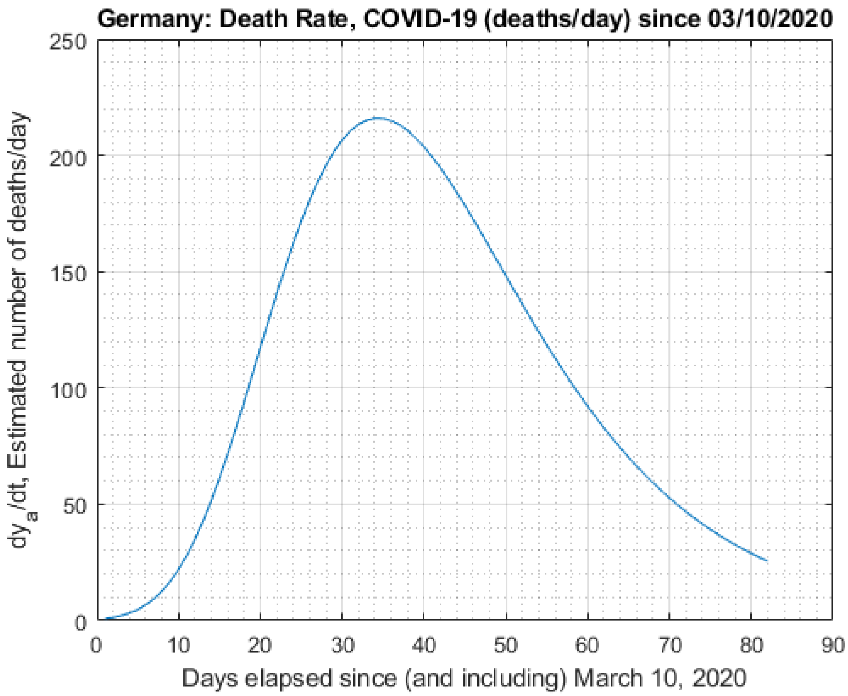

Figure S34 (DE): Date Rate corresponding to the Derivative of Estimated Data in Figure S31. The peak occurred in April 13, 2020, according to the model.

[9] Modeling Applied to United Kingdom (UK)

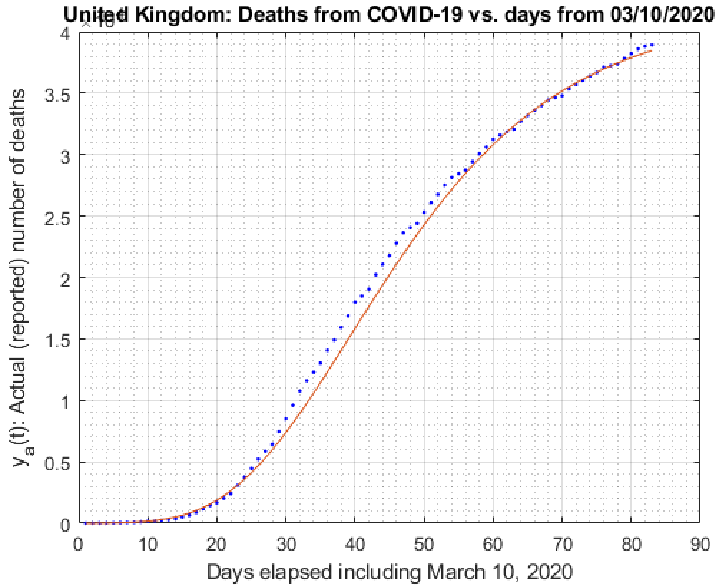

Figure S35 (UK): Actual and modeled number of presumed deaths using only data from March 10, 2020.

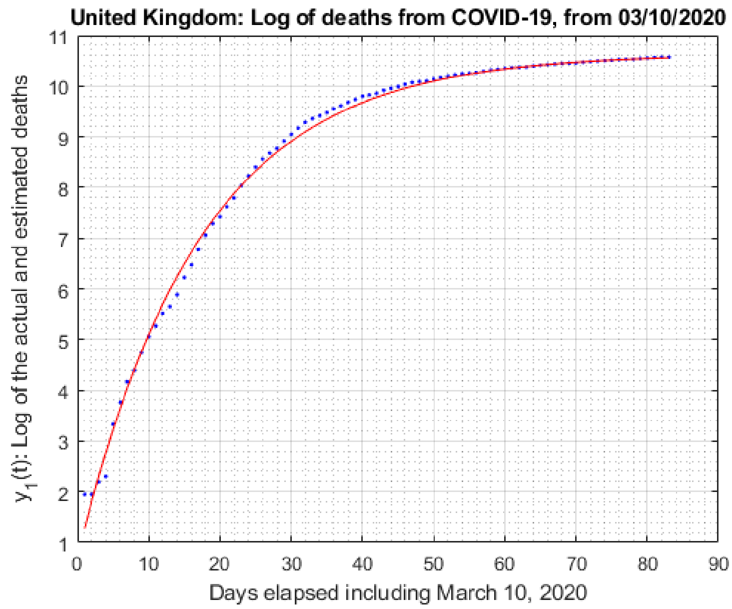

Figure S36 (UK): Natural Logarithm of the Actual (and extrapolated) Data in Figure S35.
Note: In this figure the plateau or saturation value is $y_\infty = 10.636$ and the time-constant is $\tau = 17.1554$ days. These numbers indicate that the number of presumed deaths might reach a value of $y_a(t \to \infty) = \exp(10.636) \simeq 41606$ people. The sum of squares of the residuals (i.e., $r_i(t) = \hat{y}_1(t) - y_1(t)$ for estimated minus actual values) for this data is $RSS = (\mathbf{r}^{\mathrm{T}}\mathbf{r})^{1/2} = 1.2653$.

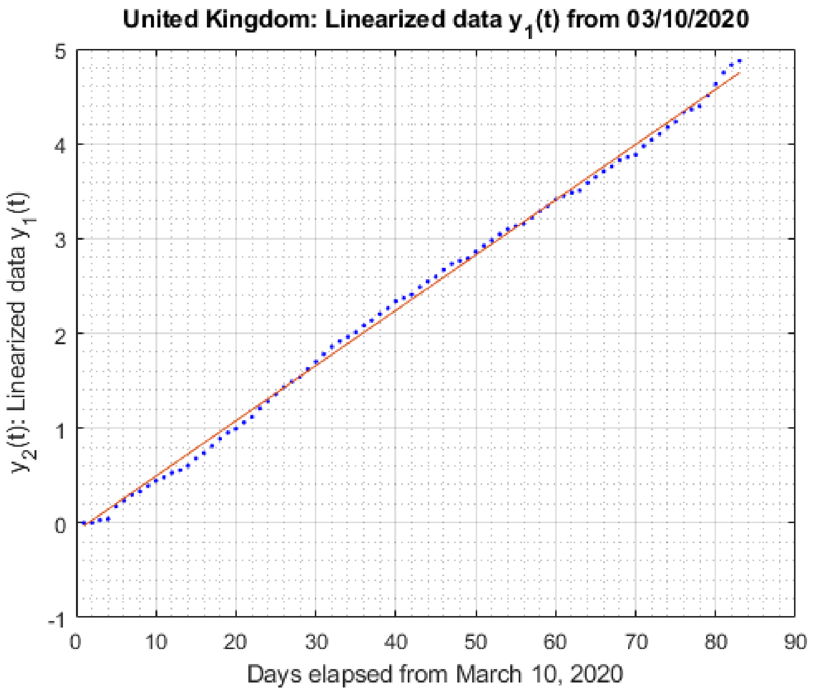

Figure S37 (UK): Linearization of Data in Figure S36 (dots) and Weighted Least Squares Linear Fit (continuous curve).

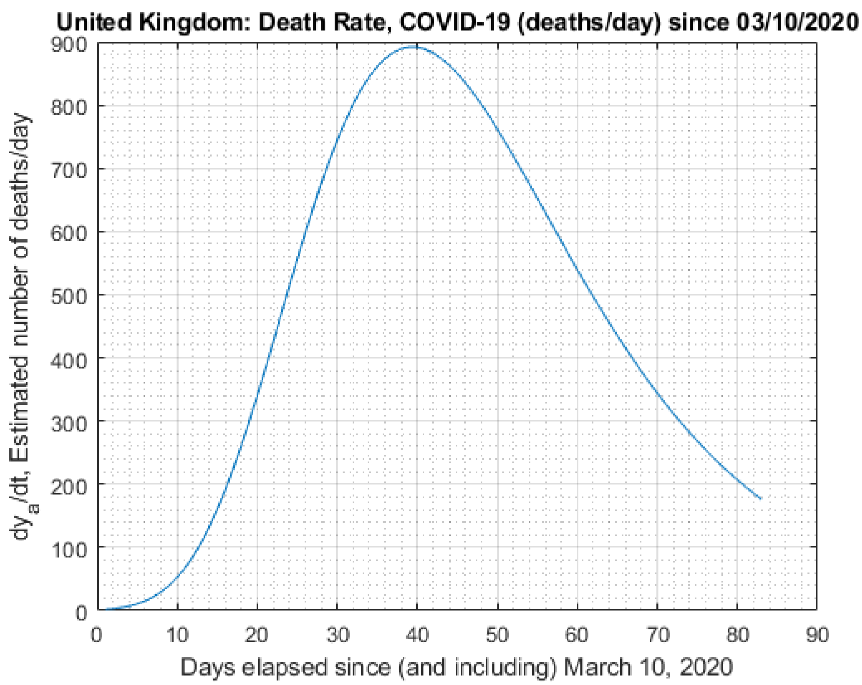

Figure S38 (UK): Date Rate corresponding to the Derivative of Estimated Data in Figure S35. The peak occurred in April 17, 2020, according to the model.

[10] Modeling Applied to Netherlands (NE)

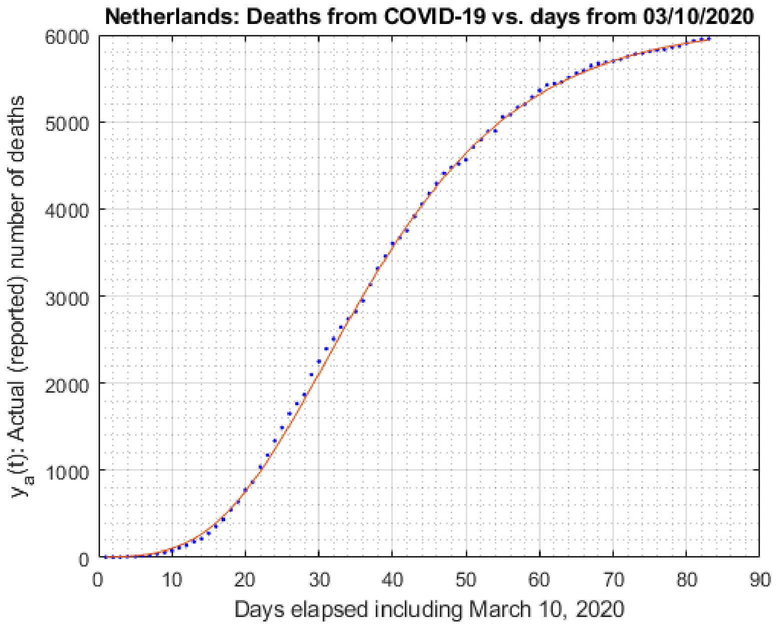

Figure S39 (NE): Actual and modeled number of presumed deaths using only data from March 10, 2020.

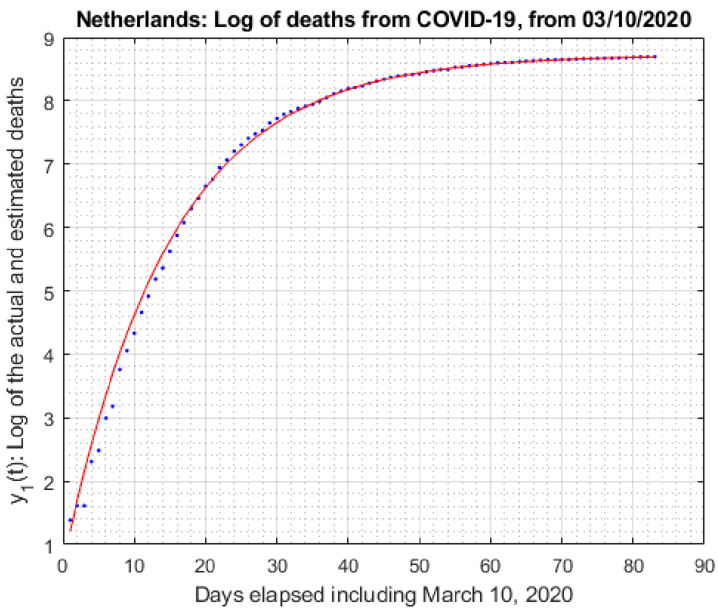

Figure S40 (NE): Natural Logarithm of the Actual (and extrapolated) Data in **Figure S39. Note:** In this figure the plateau or saturation value is $y_{\infty} = 8.720$ and the time-constant is $\tau = 14.8473$ days. These numbers indicate that the number of presumed deaths might reach a value of $y_a(t \rightarrow \infty) = \exp(8.720) \simeq 6124$ people. The sum of squares of the residuals (i.e., $r_i(t) = \hat{y}_1(t) - y_1(t)$ for estimated minus actual values) for this data is $RSS = (\mathbf{r}^T\mathbf{r})^{1/2} = 1.2483$.

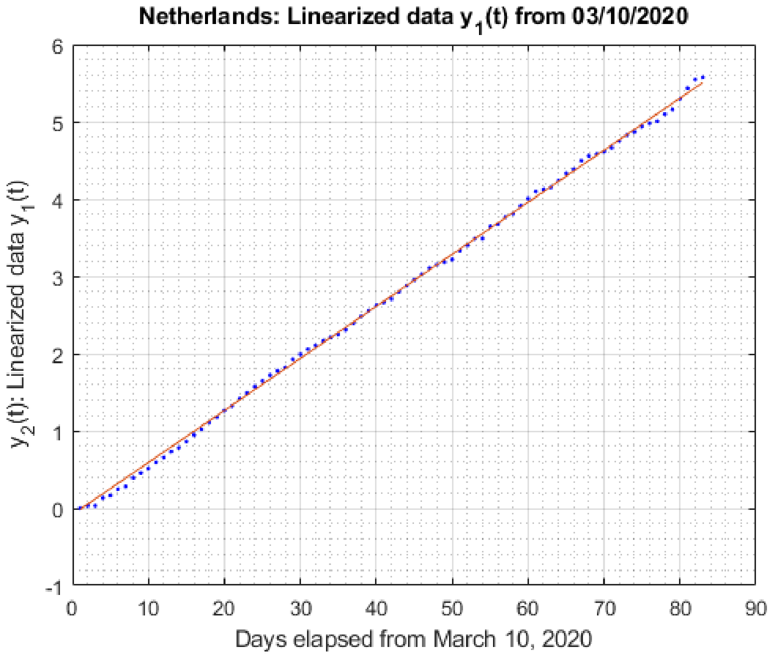

Figure S41 (NE): Linearization of Data in Figure S40 (dots) and Weighted Least Squares Linear Fit (continuous curve).

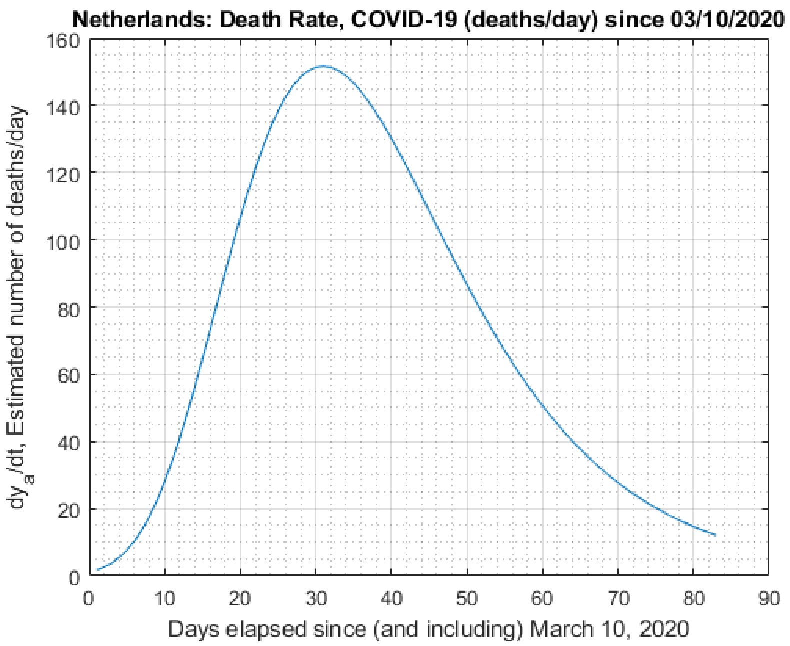

Figure S42 (NE): Date Rate corresponding to the Derivative of Estimated Data in Figure S39. The peak occurred in April 09, 2020, according to the model.

Figure S42 is the last graph in the set of graphs in this **Supplemental Materials** section.

Mathematics of first-order differential equation

This section is included in this Supplemental Materials as it contains background material that may be useful for readers that would benefit from a brief description of the solution to a first-order differential equation for a Heaviside or step-input function *u(t).*

The first-order model here proposed consists of the response of a first-order system or first-order, linear, constant coefficients differential equation to a Heaviside [or unit step *u(t)*] input (or disturbance) function (scaled by a given amplitude A). The response is given is given in Eq. (5) as

$$y_1(t \geq t_d) = y_\infty - \left(y_\infty - y_i\right) e^{-\alpha(t-t_d)}, \quad t \geq t_d$$

This response function is fitted to the data that is computed as the natural logarithm of the actual data, in our COVID-19 case, as the number of deceased people versus days elapsed since some given data-recording start time, and this original (recorded) data generally describes a sigmoidal behavior as a function of time, such as the logistic (or similar) functions.

A first-order Differential equation:

$$\frac{dy}{dt} + ay = f(t) \tag{A.1}$$

Here *f(t)* denotes a forcing function, interpreted as the input to a first-order system. Described by the differential equation (A.1). Here we look for the solution to an input $f(t) = Au(t)$, a step-function of amplitude A applied at time $t = 0$. So (A.1) is rewritten as the initial-value problem (IVP):

$$\begin{gathered} \frac{dy}{dt} + ay = Au(t) \\ y(0) = y_0 \end{gathered} \tag{A.2}$$

Equation (A.2) can be solved via several methods. Using the Laplace transform method, equation (A.2) can be written in the complex-frequency domain as

$$sY(s) - y(0) + aY(s) = \frac{A}{s} \tag{A.3}$$

With some algebra, one obtains

$$Y(s) = \frac{A + sy_0}{s(s+a)} = \frac{k_1}{s} + \frac{k_2}{s+a} \tag{A.4}$$

The partial fraction decomposition coefficients on the RHS of (A.4) are given by:

$$k_1 = \frac{A}{a}, \quad k_2 = -\left(\frac{A}{a} - y_0\right) \tag{A.5}$$

Substituting these coefficients into (A.4) yield

$$Y(s)=\frac{A}{a}\frac{1}{s}-\left(\frac{A}{a}-y_0\right)\frac{1}{s+a} \tag{A.6}$$

Taking the inverse Laplace transform of (A.6) yields the time-domain solution (or system response)

$$y(t)=\frac{A}{a}-\left(\frac{A}{a}-y_0\right)\exp(-at) \tag{A.7}$$

It can be shown that (A.7) with $y_\infty = A/a$ is algebraically the same equation as Eq.(5)

References:

1. COVID-19 Coronavirus Pandemic | Worldometer Available online: https://www.worldometers.info/coronavirus/ (accessed on Oct 27, 2020).