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Effect of Mooring Lines on the Hydroelastic Response of a Floating Flexible Plate Using the BIEM Approach

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Abstract: A boundary integral equation method (BIEM) model for the problem of surface wave interaction with a moored finite floating flexible plate is presented. The BIEM solution is obtained by employing the free surface Greens function and Green's theorem, and the expressions for the plate deflection, reflection, and transmission coefficients are derived from the integro-differential equation. Furthermore, the shallow water approximation model and its solution is obtained based on the matching technique in a direct manner. The accuracy of the present BIEM code is checked by comparing the results of deflection amplitude, reflection, and transmission coefficients with existing published results and experimental datasets as well as the shallow water approximation model. The hydroelastic response of the moored floating flexible plate is studied by analyzing the effects of the mooring stiffness, incidence angle, and flexural rigidity on the deflection amplitude, plate deformations, reflection, and transmission coefficients. The present analysis may be helpful in understanding the different physical parameters to model a wave energy conversion device with mooring systems over BIEM formulations.



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Keywords: floating flexible plate; BIEM; SWA; spring moorings; deflection amplitude; plate displacement

1. Introduction

In recent years, researchers have been increasingly interested in studying the hydroelastic response of floating or submerged structures connected with mooring lines application to coastal or marine engineering problems in order to model floating breakwaters and wave energy converters (see [1–4]). These floating flexible structures need to survive the failure or any kind of damage in waves, wind, and currents during storm events. To model a floating flexible structure, the hydroelastic response and its analysis in different design parameters ensuring the reliability and safety of the flexible structures are of great importance as the floating structures can be located in unprotected seas and exposed to rigorous environmental conditions.

Due to the flexible nature of the horizontal structure, the computational burden becomes too bulky. To overcome these difficulties, researchers often use numerical methods and solutions. The wave response analysis of a flexible floating structure by combining the two numerical methods boundary element method and finite element method (BEM-FEM) is shown in [5]. A B-spline Galerkin scheme was applied to calculate the hydroelastic response of a very large floating structure in waves in [6]. The hydroelastic response of a flexible floating structure was analyzed by applying a numerical method FEM in [7]. The numerical methods BEM and FEM have been applied to the hydrodynamic analysis of floating elastic surfaces in [8] using FEM; in [9] using BEM and the direct method in [10]. The hydroelastic response of a compliant floating raft modelled as sea ice flow using the boundary-integral method in [11]. The hydroelastic waves in a channel covered by cracked ice with arbitrary edge conditions at the walls expanded the solution into a series of cosine functions with unknown coefficients was studied in [12].

It may be noted that the numerical computations of the present solution based on integro-differential equation were carried out in a desktop machine with Intel® core i7-4790 CPU with a 3.60 GHz processor and 16 GB of RAM. On average, each case was run roughly 8–10 min (simulation time). The present BIEM would be one of the alternative methods to compare and analyze the hydroelastic response of this problem by incorporating the structural deformations and flexible modes into the formulation. This method is computationally efficient and it also involves less computational cost than the different numerical methods used in [13–15].

On the other hand, the interaction of waves with floating or submerged flexible structures of finite depth connected with mooring lines can hardly be analyzed analytically. Some of the most relevant previous work relating to moored finite floating and or submerged flexible horizontal plate problems are discussed below.

There has been considerable progress in the study of real physical boundary value problems associated with moored finite horizontal floating structures in different water depths based on an analytical approach. Surface wave interaction with a moored finite floating elastic plate was formulated using linear water wave theory and the effect of different design parameters on the floating elastic plate were analyzed using the eigenfunction expansion method in [16]. The effect of mooring lines on the floating elastic plate in the presence of submerged flexible horizontal membrane under eigenfunction expansion method and application of orthogonal mode coupling relation was studied in [17]. Under the velocity decomposition method, the effect of a fixed submerged flexible membrane on a moored floating elastic plate in three dimensions was analyzed in [18]. Furthermore, the effect of a moored submerged flexible porous plate on the moored finite floating flexible plate was analyzed in two dimensions by studying different design parameters in [19]. Recently, a review on the numerical approaches in the hydroelastic response of horizontal elastic structures was presented in [20].

From the above literature, it is confirmed that until now there is no study reported to the public related to wave interaction with a moored finite floating flexible plate using BIEM and its hydroelastic response via BIEM. Therefore, this paper presents a BIEM model for linear water wave interaction with a moored floating flexible plate in finite water depth to analyze the effect of mooring lines on the floating flexible structure on various design parameters.

The derivation of the boundary integral equation along with the integrodifferential equation in terms of plate deflection for the moored finite floating flexible plates are presented by using three-dimensional free surface Green's function and applying Green's theorem. A BIEM code is developed to present numerical results of deflection amplitude, plate displacements, reflection, and transmission coefficients to study the hydroelastic response of a moored floating flexible plate. To show the level of accuracy of the BIEM code, the present results of deflection amplitude, reflection, and transmission coefficients are compared with existing published results and experimental datasets as well as the shallow water approximation (SWA) model. Furthermore, the hydroelastic response of moored finite floating flexible plate is studied by analyzing the effects of mooring stiffness, incident angle, and flexural rigidity of the moored floating flexible plate on the deflection amplitude, plate displacements, reflection, and transmission coefficients.

2. Model Formulation

Under linear wave theory, the mathematical formulation is considered in a three-dimensions Cartesian Coordinate system and BVP is solved by applying BIEM in finite water depth. It is assumed that a floating flexible plate is placed at $y = 0$, $0 < x < b$ connected with mooring lines with stiffness s_j , and $z < \infty$ over the impermeable bottom bed $y = h$. Hence, the whole fluid domain is divided into three fluid domains defined by open water domain, plate covered domain, and interface domain are referred as $\mathcal{D}_O$, $\mathcal{D}_P$, and $\partial\mathcal{D}$, respectively (see Figure 1).

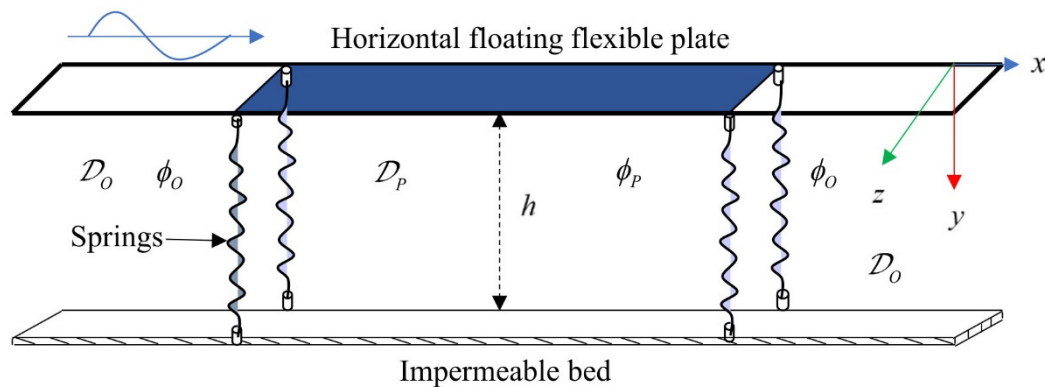


Figure 1. Geometrical representation of the model.

It is considered that surface gravity wave interacts with floating flexible plate making an oblique angle β with the positive x -axis and the fluid is inviscid, incompressible, motion is irrotational and simple harmonic in time with angular frequency ω . Therefore, there exists velocity potential $\Phi(x, y, z; t)$ that can be defined as $\Phi(x, y, z, t) = \text{Re}\{\phi(x, y, z)e^{-i\omega t}\}$ where $\phi(x, y, z)$ is the complex wave potential with $\phi(x, y, z) = \phi_O(x, y, z)$ for $\mathcal{D}_O$ and $\phi(x, y, z) = \phi_P(x, y, z)$ for $\mathcal{D}_P$ which satisfies the Laplace equation in the whole fluid domain as

$$\nabla_{xyz}^2 \Phi = 0 \quad (1)$$

and the impermeable sea bed condition,

$$\partial_y \Phi = 0 \text{ at } y = h \text{ for } -\infty < x < \infty. \quad (2)$$

The linearized free surface condition in $\mathcal{D}_1$ is obtained by the combination of kinematic and dynamic boundary conditions yields

$$g\partial_y \Phi + \omega^2 \Phi = 0 \text{ on } y = 0, \quad -\infty < x < 0, \quad b < x < \infty, \quad (3)$$

where g is the gravitational constant. The linearized kinematic condition at $y = 0$ in plate covered domain is given by

$$\partial_y \Phi = \partial_t \eta \quad (4)$$

where $\eta(x, z; t)$ is the deflection of the floating flexible plate.

The floating plate condition is obtained by satisfying plate deflection, equating the hydrodynamic and dynamic pressure along with kinematic condition (4) at $y = 0$ as:

$$\left\{ \mathcal{X} \nabla_{xz}^4 + \ell(\partial_x^2 + \partial_z^2) + \rho_p d \partial_t^2 \right\} \partial_y \Phi = \rho(\partial_t^2 \Phi - g \partial_y \Phi) \quad (5)$$

where $\mathcal{X}$ and ℓ are the flexural rigidity and the uniform compressive force of the plate, respectively.

At the moored edges $x = 0, b; y = 0$, the bending moment and shear force yield the conditions as

$$\mathcal{X}(\partial_x^2 \eta + \nu \partial_z^2 \eta) = 0 \quad (6)$$

$$\mathcal{X} \partial_x (\partial_x^2 \eta + (2 - \nu) \partial_z^2 \eta) + \ell \partial_x \eta = s_j \quad (7)$$

The continuity of pressure and velocity at the interface of the plate covered and the water surface are given by

$$\phi_O = \phi_P \quad (8)$$

$$\partial_x \phi_O = \partial_x \phi_P \quad (9)$$

Finally, the complex wave potential $\phi(x, y, z)$ also satisfies the Sommerfeld radiation condition in the far-field.

2.1. Integro-Differential Equation Based on BIEM

In this Section, the solution of wave interaction with a moored floating flexible plate via integral equation and plate deflection in terms of integro-differential equation will be presented. The complex velocity potential $\phi_O(x, y, z)$ is decomposed into the incident wave potential and the diffracted wave potential, which are denoted by ϕ_I and ϕ_S , respectively. So, the total potential in the domain $\mathcal{D}_O$ can be expressed as

$$\phi_O(x, y, z) = \phi_I(x, y, z) + \phi_S(x, y, z) \quad (10)$$

where

$$\phi_I(x, y, z) = \frac{-igI_0}{2\omega} \varepsilon(y) e^{i\alpha_0(x \cos \beta + z \sin \beta)} \quad (11)$$

with $\varepsilon(y) = \cosh k(h - y) / \cosh kh$, I_0 is the incident wave amplitude, ω is the frequency and k_0 satisfies the gravity wave dispersion relation. Let $\mathcal{G}(x, y, z; x_0, y_0, z_0)$ refer to the three-dimensional Green's function with (x_0, y_0, z_0) and (x, y, z) are the source point and any point in the fluid domain, respectively. Therefore, the Green's function $\mathcal{G}(x, y, z; x_0, y_0, z_0)$ at the free surface $y = y_0 = 0$ is obtained as

$$\mathcal{G}(x, z; x_0, z_0) = 2 \int_C \frac{\alpha J_0(\alpha \varphi)}{(K - \alpha \tanh \alpha h)} d\alpha \quad (12)$$

where C is the contour of integration, $J_0(\alpha \varphi)$ is the Bessel's function with $\varphi = \{(x - x_0)^2 + (z - z_0)^2\}^{1/2}$ and satisfies the governing equation

$$\nabla^2 \mathcal{G} = 4\pi \delta(x - x_0) \delta(z - z_0) \quad (13)$$

with Dirac delta function δ along with free surface boundary condition (3) and bottom boundary condition (2). Now, applying Green's theorem to the velocity potentials ϕ_O and ϕ_P , for the domain $(x, z) \in \mathcal{D}_O$, one can get

$$0 = \int_{\partial \mathcal{D} \cup \mathcal{D}_P} (\phi_P \partial_n \mathcal{G} - \mathcal{G} \partial_n \phi_P) d\Omega, 4\pi \phi_S = - \int_{\partial \mathcal{D} \cup \mathcal{D}_O} (\phi_S \partial_n \mathcal{G} - \mathcal{G} \partial_n \phi_S) d\Omega \quad (14)$$

and for $(x, z) \in \mathcal{D}_P$, result in

$$4\pi \phi_P = \int_{\partial \mathcal{D} \cup \mathcal{D}_P} (\phi_P \partial_n \mathcal{G} - \mathcal{G} \partial_n \phi_P) d\Omega, 0 = - \int_{\partial \mathcal{D} \cup \mathcal{D}_O} (\phi_S \partial_n \mathcal{G} - \mathcal{G} \partial_n \phi_S) d\Omega, \quad (15)$$

where n denotes the outward normal to Ω (Ω is a smooth (regular) surface).

By adding the two Equations (15) and using boundary condition (5) along with Equation (3) for the Green's function, one can derive as:

$$4\pi \phi_P = \int_{\mathcal{D}_P} (\phi_I \partial_n \mathcal{G} - \mathcal{G} \partial_n \phi_I) d\Omega + \int_{\mathcal{D}_P} \{(\mathcal{X} \nabla_{x_0 z_0}^4 + \ell \nabla_{x_0 z_0}^2 - m\omega^2)(\partial_y \phi_P)\} \mathcal{G} d\Omega \quad (16)$$

The integro-differential equation can be derived by applying Green's formula, free surface condition for Green's function and Laplace equation to incident potential as

$$\begin{aligned} & \{(\mathcal{X} \nabla_{xz}^4 + \ell \nabla_{xz}^2 + (\rho g - m\omega^2)) \partial_y \phi_P\} \\ & + \frac{\theta}{4\pi} \int_{\mathcal{D}_P} \{(\mathcal{X} \nabla_{x_0 z_0}^4 + \ell \nabla_{x_0 z_0}^2 - m\omega^2)(\partial_y \phi_P)\} \mathcal{G} d\Omega = \rho g \partial_y \phi_I, \end{aligned} \quad (17)$$

where $\vartheta = \omega^2/g$. Using the kinematic and free surface boundary conditions as in Equations (2) and (3) and substituting incident potential from Equation (11) into Equation (17), an integro-differential equation in terms of plate deflection $\eta(x, z)$ is obtained as:

$$\left\{ \mathcal{X} \nabla_{xz}^4 + \ell \nabla_{xz}^2 + (\rho g - m\omega^2) \right\} + \frac{\vartheta}{4\pi} \int_{\mathcal{D}_p} (\mathcal{X} \nabla_{x_0 z_0}^4 + \ell \nabla_{x_0 z_0}^2 - m\omega^2) \eta(x_0, z_0) \mathcal{G}(x, z; x_0, z_0) dx_0 dz_0 = I_0 e^{ik_0(x \cos \beta + z \sin \beta)}. \quad (18)$$

The Greens function (12) satisfy the free surface boundary condition at $y = y_0 = 0$, bottom boundary condition, and radiation condition where the contour C defined over the integral is the complex α - plane from 0 to $+\infty$ beneath the singularity $\alpha = \alpha_0$ (see Figure 2) chosen for satisfying the radiation condition and Bessel's function $J_0(\alpha\varphi)$. To overcome the singularity under the integrals of the integro-differential Equation (18) associated with the Green's function by using residue theorem based on the complex function theory. Furthermore, we applied the Sonine- Gegenbauer expression for $J_0(\alpha\varphi)$ to evaluate the integration with respect to z_0 (see [21]).

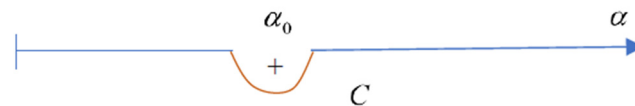


Figure 2. Contour of integration of singularity α_0 .

In the next subsection, the displacement of the moored floating flexible plate, reflection and transmission coefficients will be determined in terms of series form.

2.2. Plate Displacement, Reflection and Transmission Coefficients

In this Section, the plate deflection can be expressed as a linear superposition of the eigenfunctions associated with the flexural gravity waves in the following form

$$\eta(x, z) = \sum_{n=0, I, II, III, IV, 1}^M \{A_n \exp(i\gamma_n x) + B_n \exp(-i\gamma_n x)\} e^{iz\kappa}, \quad 0 < x < b, \quad 0 < \beta < 90^\circ \quad (19)$$

where A_n and B_n are the unknown amplitudes are to be determined with $\kappa = \alpha_0 \sin \beta$. The wavenumber γ_n satisfies the relation $\gamma_n = \sqrt{\lambda_n^2 - \kappa^2}$ where λ_n satisfies the flexural gravity dispersion relation.

$$\mathcal{S}(\lambda_n) \equiv \left\{ \mathcal{X} \lambda_n^4 + \ell \lambda_n^2 + (\rho g - m\omega^2) \right\} \lambda_n \tanh \lambda_n h - \vartheta \quad (20)$$

In Equation (20), $\mathcal{S}(\lambda_n) \equiv 0$ is the dispersion relation associated with a floating horizontal flexible plate in finite water depth.

Substituting the Green's function as in Equation (12) and plate deflection as in Equation (19) into Equation (18), a system of two linear equations is obtained and another four linear equations are obtained using the moored edge conditions (6) and (7), the bending moment and shear force at $x = 0, b$ gives

$$\sum_{n=0}^M \mathcal{X}(\gamma_n^2 + v\kappa^2) \{A_n \exp(-i\gamma_n b) + B_n \exp(i\gamma_n b)\} = 0 \quad (21)$$

$$\sum_{n=0}^M \mathcal{X}(\gamma_n^2 + v\kappa^2) \{A_n \exp(i\gamma_n b) + B_n \exp(-i\gamma_n b)\} = 0 \quad (22)$$

$$\sum_{n=0}^M \left\{ \mathcal{X} \left\{ \gamma_n^3 + (2 - v)\gamma_n \kappa^2 \right\} - \ell \gamma_n \right\} \{A_n \exp(-i\gamma_n b) - B_n \exp(i\gamma_n b)\} - s_1 = 0 \quad (23)$$

$$\sum_{n=0}^M \left\{ \mathcal{X} \left\{ \gamma_n^3 + (2 - v)\gamma_n \kappa^2 \right\} - \ell \gamma_n \right\} \{A_n \exp(i\gamma_n b) - B_n \exp(-i\gamma_n b)\} - s_2 = 0 \quad (24)$$

Thus, there are six equations for the determination of the unknowns A_n and B_n . Using the relations in Equation (14), the amplitudes of reflected and transmitted waves R and T in terms of A_n and B_n are obtained as:

$$C_r = v(\alpha_0, h) \sum_{n=0}^M \exp(i\alpha_0 b) (\mathcal{X}\lambda_n^4 - \ell\lambda_n^2 - m\omega^2) \left(\frac{A_n \exp(-i\gamma_n b)}{u(\alpha_0, \beta)} - \frac{B_n \exp(i\gamma_n b)}{u(-\alpha_0, \beta)} \right) \quad (25)$$

$$C_t = 1 - v(\alpha_0, h) \sum_{n=0}^M \exp(-i\alpha_0 b) (\mathcal{X}\lambda_n^4 - \ell\lambda_n^2 - m\omega^2) \left(\frac{A_n \exp(-i\gamma_n b)}{u(-\alpha_0, \beta)} - \frac{B_n \exp(i\gamma_n b)}{u(\alpha_0, \beta)} \right) \quad (26)$$

where $u(\alpha_0, \beta) = (\gamma_n + \alpha_0 \cos \theta) \cos \theta$ and $v(\alpha_0, h) = \alpha_0 \vartheta / (\vartheta^2 - \vartheta^2 h + \alpha_0^2)$. In the next section, the referred problem will be modelled in SWA in a direct manner.

3. Shallow Water Approximation Model

Proceeding similarly in the case of SWA as in [22,23], using the kinematic condition (4) and plate covered condition (5) in terms of plate deflection, one can derive

$$\phi_P(x, z) = i\omega h \sum_{n=0, I, II, III, IV, 1}^M \{A_n \exp(i\gamma_n x) + B_n \exp(-i\gamma_n x)\} e^{iz\kappa} \quad (27)$$

Solving the Laplace equation using separation of variables along with free surface boundary condition, the velocity potential in $\mathcal{D}_0$ associated with the reflection and transmission wave amplitudes R and T can be derived as:

$$\phi_O(x, z) = \begin{cases} e^{i\alpha_0(x \cos \theta + z \sin \theta)} + R e^{i\alpha_0(-x \cos \theta + z \sin \theta)} & \text{for } x < 0, \\ T e^{i\alpha_0(x \cos \theta + z \sin \theta)} & \text{for } x > b. \end{cases} \quad (28)$$

Using the conditions of pressure and velocity (8–9) at $x = 0, b$ into the Equations (27) and (28), we obtain four linear equations and, also, we have another four linear equations from moored edge conditions (21)–(24). A MATLAB code is developed by using the system of eight linear equations to obtain the unknowns A_n , B_n , R , and T associated with Equations (27) and (28). Once the reflection and transmission wave amplitudes are determined, then the reflection and transmission coefficients can be computed by the following formulae $C_r = |R|$ and $C_t = |T|$.

4. Numerical Results and Discussions

In numerical computation, the roots of the dispersion relation (20) in $\mathcal{D}_P$ are in the upper complex plane. The dispersion relation (20) has two real complex roots, four complex roots, and many infinitely imaginary roots. In this work, for developing a numerical code, 10 different roots are utilized which give realistic physical solutions for λ and are positioned in the upper complex half-plane: one real positive root λ_0 , two complex roots λ_1 and λ_2 which are of the form $\pm a + ib$, and seven imaginary roots of the form $+i\lambda_n$ for $n = 3, 4, \dots, 9$.

Initially, to verify the accuracy of the present BIEM code, the obtained results of deflection amplitude, reflection and transmission coefficients are compared with existing published results of other calculations, experimental datasets, and SWA models. Then, the hydroelastic response of moored finite floating plate is analyzed by studying the effects of mooring stiffness, incidence angle, and flexural rigidity on the deflection amplitudes and plate displacements as well as reflection and transmission coefficients in detail.

4.1. Comparison of Results with Existing Published Results, Experimental Datasets, and SWA Model

Figure 3 compares the present deflection amplitude with existing published results in [24] and experimental dataset in [25] with $EI = 10^5$, $\ell = 1.5\sqrt{EI\rho g}$, and $b/h = 20$ versus plate length. The comparison results show that the trend of the deflection amplitude

between the present and [24] are similar. Slightly higher values of mooring stiffness $s = 10^{3.5} \text{Nm}^{-1}$ and lower compressive force $\ell = 0.02 \sqrt{EI\rho g}$ were chosen which led to a similar trend between the results from the model [24] and experimental data [25] and the present BIEM model.

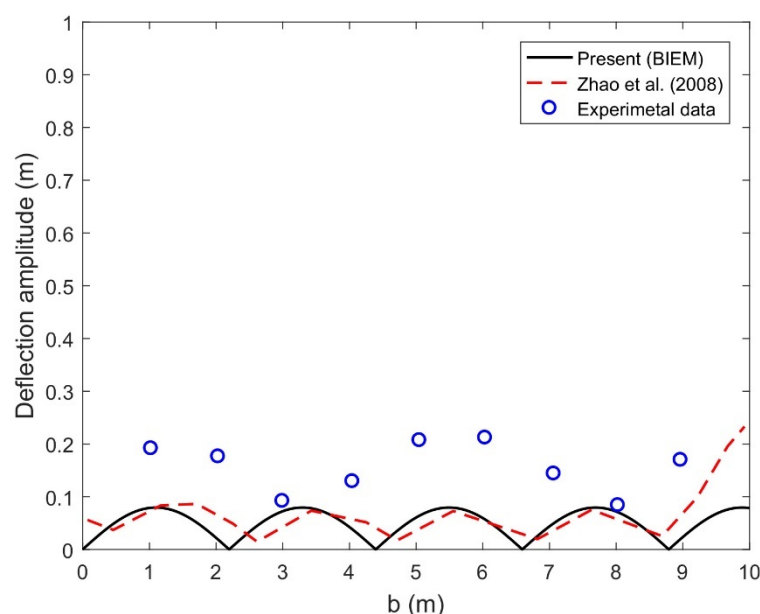
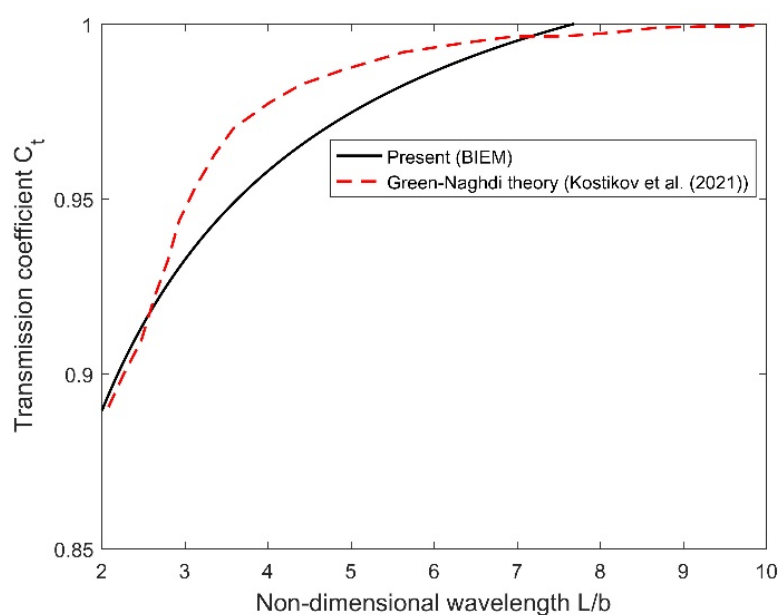


Figure 3. Comparison of the deflection amplitude with the model [24] and experimental data in [25].

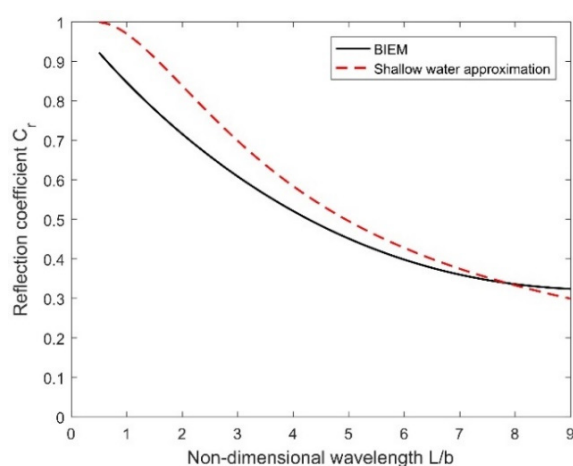
In Figure 3, comparison results indicated that few of the experimental data points with BIEM are in agreement and not all data points but the trends are almost similar. These differences can be explained by the present BIEM model associated with the mooring lines connected at the edges of the flexible plate under compressive force, while these parameters were not included in the model [25]. Therefore, the present BIEM model reproduces a lower amplitude crest of the floating flexible plate that the model [25] cannot reproduce.

Furthermore, in Figure 3, the deflection amplitude of the horizontal floating flexible plate from the [24] model is 0.09 m, while the BIEM model is 0.08 m, which is 11% larger. On the other hand, the deflection amplitude from the Experimental data [25] model is 0.2 m, while the present BIEM model is 0.8 m, which is 66% larger. However, the other two points from the model [25] are in almost agreement with the present BIEM model.

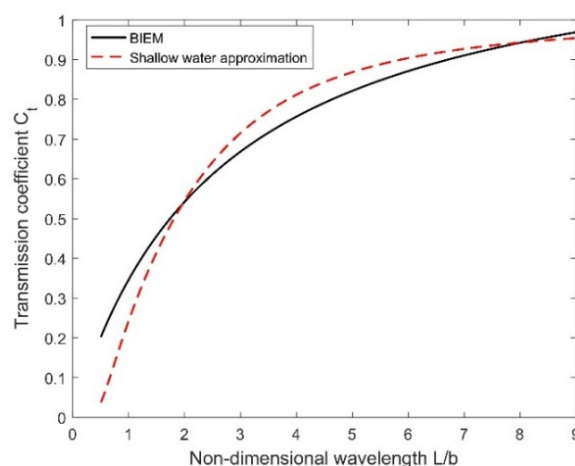
Figure 4 shows the comparisons of C_t between the present BIEM with (a) Green-Naghdi model ([26], Figure 13), SWA of the (b) C_r and (c) C_t versus non-dimensional wavelength with $EI = 10^5$, $\ell = 1.5 \sqrt{EI\rho g}$, and $b/h = 20$. In Figure 4a, the comparison shows that the trend of C_t is in a similar pattern however, there are some deviations between the two models. This is because of the nonlinearity in [26] and the linearity in the present BIEM model. From Figure 4b,c, it is found that the trend of the C_r and C_t are similar and also the data between the BIEM and SWA are very close to each other. It is also seen that the SWA results are over or under predicted which is because of the neglect of evanescent modes in the case of SWA.



(a) Comparison of the C_t : present (BIEM) and Green-Naghdi model [26].



(b) Comparison of C_r : BIEM and SWA



(c) Comparison of C_t : BIEM and SWA

Figure 4. Comparison of present BIEM with Green-Naghdi model [26] and SWA of the reflection and the transmission coefficients C_r and C_t .

Furthermore, in Figure 4a, the transmission coefficient from the Green-Naghdi theory model is 0.956 m, while the present BIEM model is 1 m, which is 4.37% larger. In Figure 4b: The reflection coefficient from the shallow water approximation model is 1 m, while the BIEM model is 0.9 m, which is 9% larger. In Figure 4c: The transmission coefficient from the Shallow water approximation model is 0.943 m, while the BIEM model is 0.84 m, which is 10.25% larger.

As a result of Figures 3 and 4, it is ensured that the present solution based on BIEM, is supported by the existing published results, experimental datasets and as well as SWA model.

4.2. Hydroelastic Response of the Moored Floating Flexible Plate Based on BIEM

In Figure 5, the variations of the deflection amplitude of the floating flexible plate for different (a) mooring stiffness versus plate length, (b) mooring stiffness versus non-dimensional wavenumber, (c) incidence angle with a certain value of mooring stiffness $s = 10^{3.5} \text{Nm}^{-1}$, and (d) flexural rigidity for mooring stiffness $s = 10^{3.5} \text{Nm}^{-1}$ versus non-

dimensional wavenumber kb with $EI = 10^5$, $\ell = 1.5\sqrt{EI\rho g}$, and $b/h = 20$ are plotted. From Figure 5a,b, we can observe that the deflection amplitude decreases with an increase in the values of mooring stiffness whilst, the deflection amplitude becomes higher for higher incidence angle (shown in Figure 5c) which is because, when the incident wave becoming perpendicular to the floating plate, it propagates along the floating horizontal plate in the z -direction. This is because, as the rigidity parameter of the floating increases, the plate behaves as a rigid structure which leads to less deflection or no deflection. From Figure 5d, it is observed that with a certain value of mooring stiffness $s = 10^{3.5}\text{Nm}^{-1}$, the deflection amplitude of the floating flexible plate decreases with an increase in flexural rigidity.

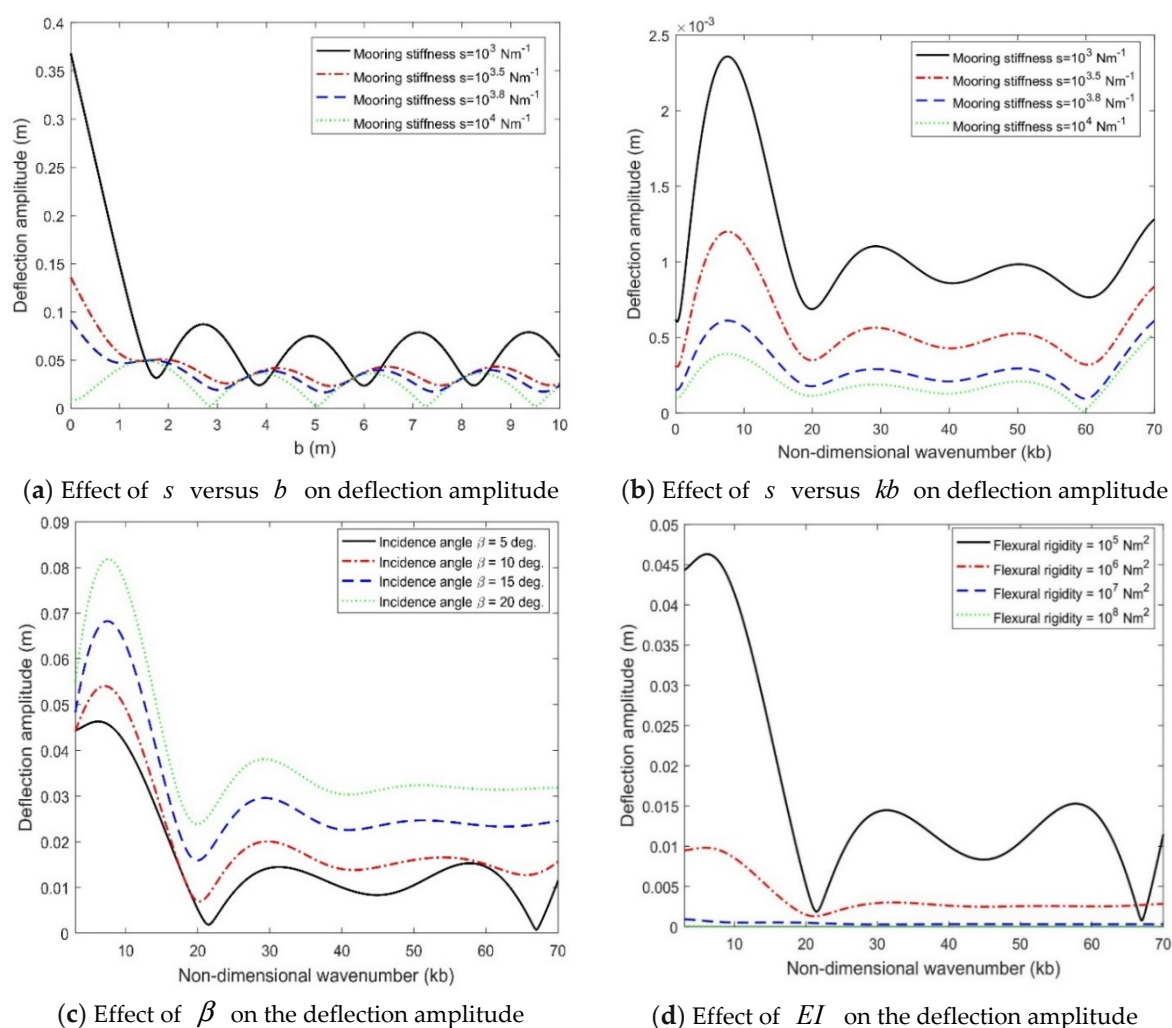
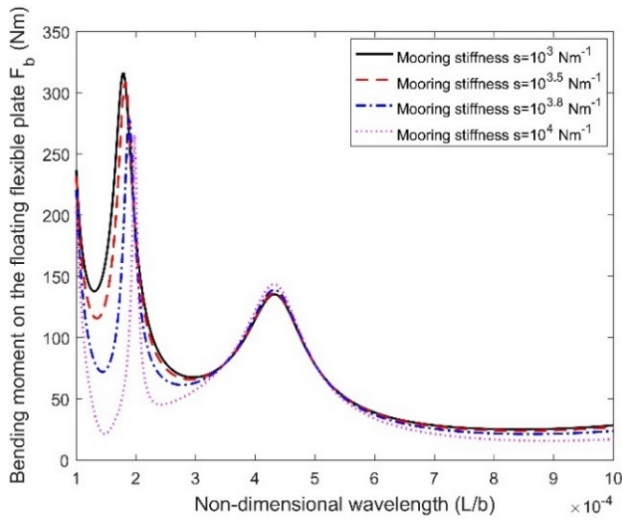


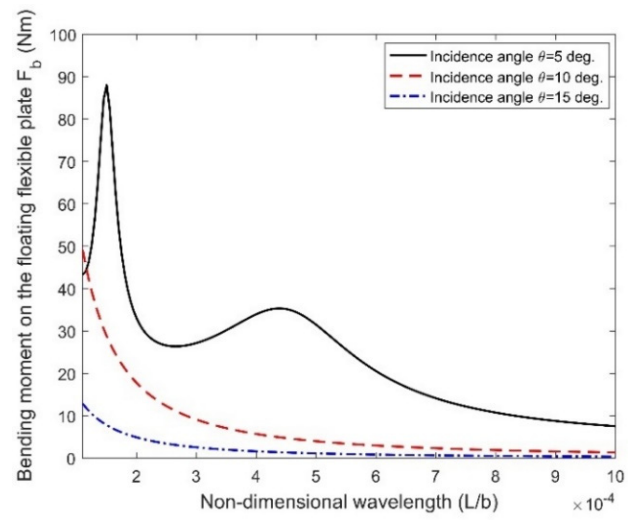
Figure 5. Effects of (a,b) mooring stiffness, (c) incidence angle β with $s = 10^{3.5}\text{Nm}^{-1}$, and (d) flexural rigidity with $s = 10^{3.5}\text{Nm}^{-1}$ on the deflection amplitude with $EI = 10^5$, $\ell = 1.5\sqrt{EI\rho g}$, and $b/h = 20$.

Figures 6 and 7 show the effects of different mooring stiffness and incident angle on the bending moment and shear force on the moored floating flexible plate versus non-dimensional wavelength L/b with $EI = 10^5$, $\ell = 1.5\sqrt{EI\rho g}$, and $b/h = 20$. From Figure 6a, it is observed that the bending moment acting on the floating flexible plate decreases with an increase in the values of mooring stiffness. However, for larger wavelengths, the resonating pattern of the shear force vanishes. In Figure 7a,b, it is found that the observations are similar as in Figure 6a,b. Furthermore, it was observed that the bending moment attends higher values than those of shear force whilst, this trend is the reverse in case of the effect

of incident angle as seen from Figures 6b and 7b with a certain value of mooring stiffness $s = 10^{3.5} \text{Nm}^{-1}$.

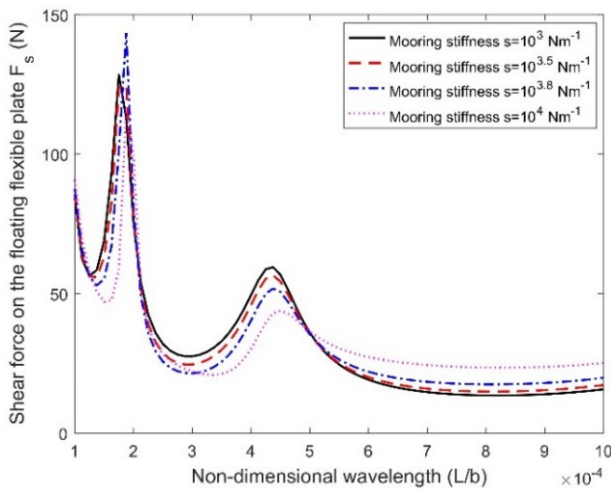


(a) Effect of S on the bending moment

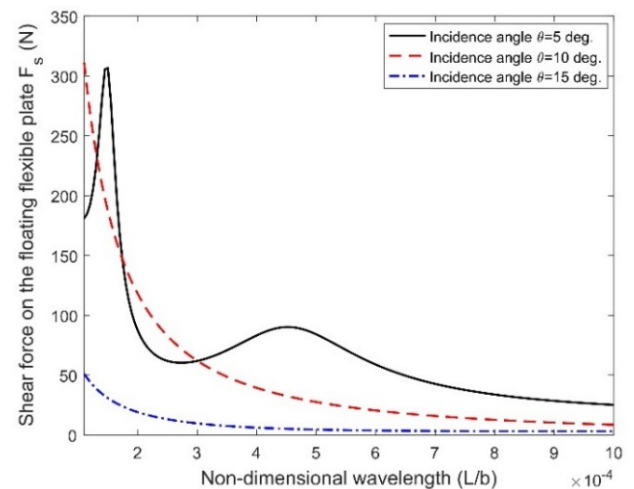


(b) Effect of β on the bending moment

Figure 6. Effects of (a) mooring stiffness and (b) incidence angle for mooring stiffness $s = 10^{3.5} \text{Nm}^{-1}$ on the bending moment with $EI = 10^5$, $\ell = 1.5\sqrt{EI\rho g}$, and $b/h = 20$.



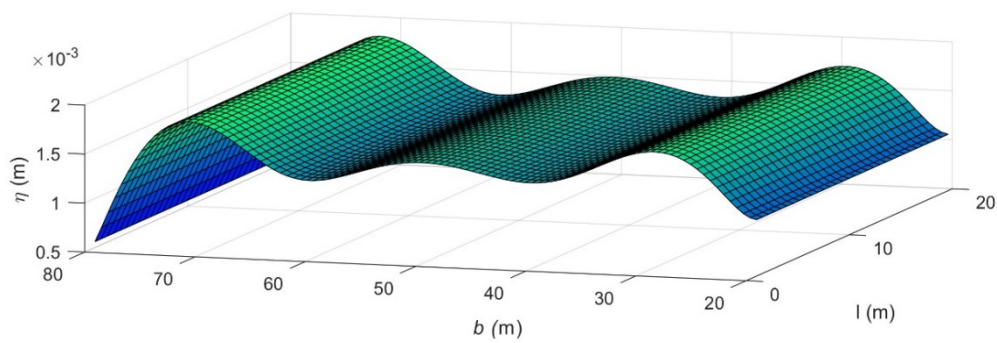
(a) Effect of s on the shear force



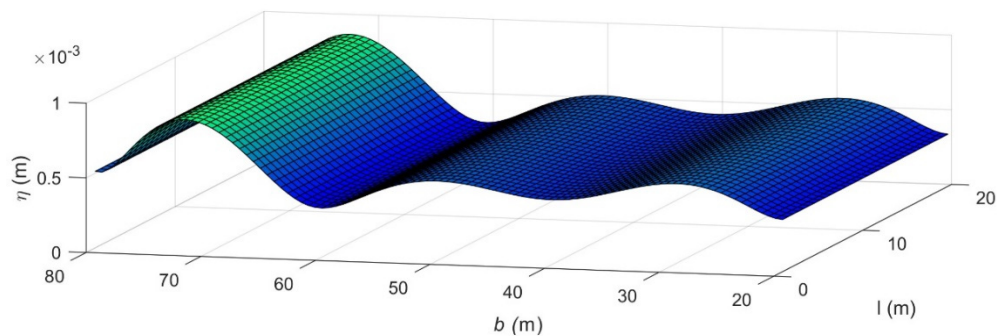
(b) Effect of β on the shear force

Figure 7. Effects of (a) mooring stiffness and (b) incident angle for mooring stiffness $s = 10^{3.5} \text{Nm}^{-1}$ on the shear force for $EI = 10^5$, $\ell = 1.5\sqrt{EI\rho g}$, and $b/h = 20$.

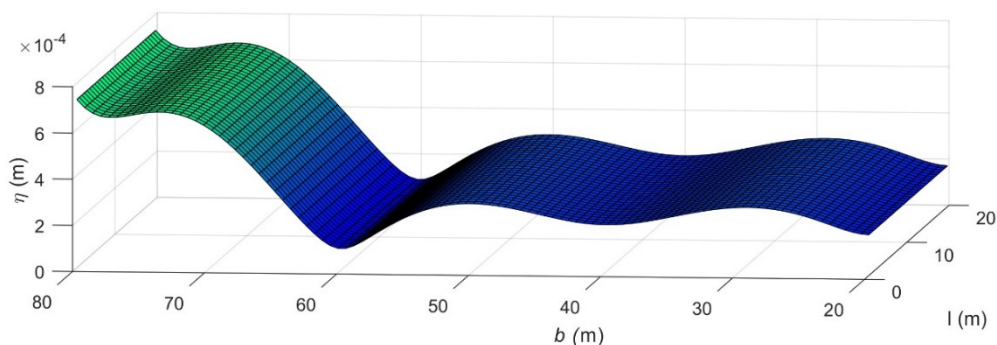
Figure 8 simulates the displacements of the moored floating flexible plate for different values of mooring stiffness versus length and width with $\mathcal{X} = 10^5 \text{Nm}^{-1}$, $\ell = 1.5\sqrt{EI\rho g}$, $h = 8 \text{m}$. It is observed that the plate deformation decreases with an increase in the values of mooring stiffness. This is because as the mooring stiffness of the plate increases, the plate becomes stiffer which leads to less deformation of the plate.



(a) Plate deformation for $s = 10^3 \text{ Nm}^{-1}$



(b) Plate deformation for $s = 10^{3.5} \text{ Nm}^{-1}$



(c) Plate deformation for $s = 10^{3.8} \text{ Nm}^{-1}$

Figure 8. Effect of mooring stiffness on the deformation of the moored floating flexible plate.

Figure 9 demonstrates the effect of the mooring stiffness and incident angle on the reflection coefficient C_r and the transmission coefficient C_t versus (a) non-dimensional wavelength L/b and (b) wave period $T(\text{s})$ with $EI = 10^5$, $\ell = 1.5\sqrt{EI\rho g}$, and $b/h = 20$. In Figure 9a, it is seen that for higher values of mooring stiffness, the reflection coefficient C_r increases for $L/b > 8$ which is because as the stiffness becomes higher the flexible plate becomes stiffer which results in higher reflection (see [18]). On the other hand, in the case of C_t the observations are similar to C_r throughout the non-dimensional wavelength L/b . In Figure 9b, it is found that the values of the C_r becomes lower as the values of incident angle increase whilst, the transmission coefficients decrease for higher values of incident angle. This is because as the stiffness becomes higher, the flexible plate becomes stiffer which results in a higher reflection (see [18]). Furthermore, the observations in Figure 9c are similar to Figure 9a. From Figure 9d, it is observed that the reflection coefficient increases with an increase in flexural rigidity whilst, the trend of transmission coefficient becomes the reverse to that of reflection coefficients.

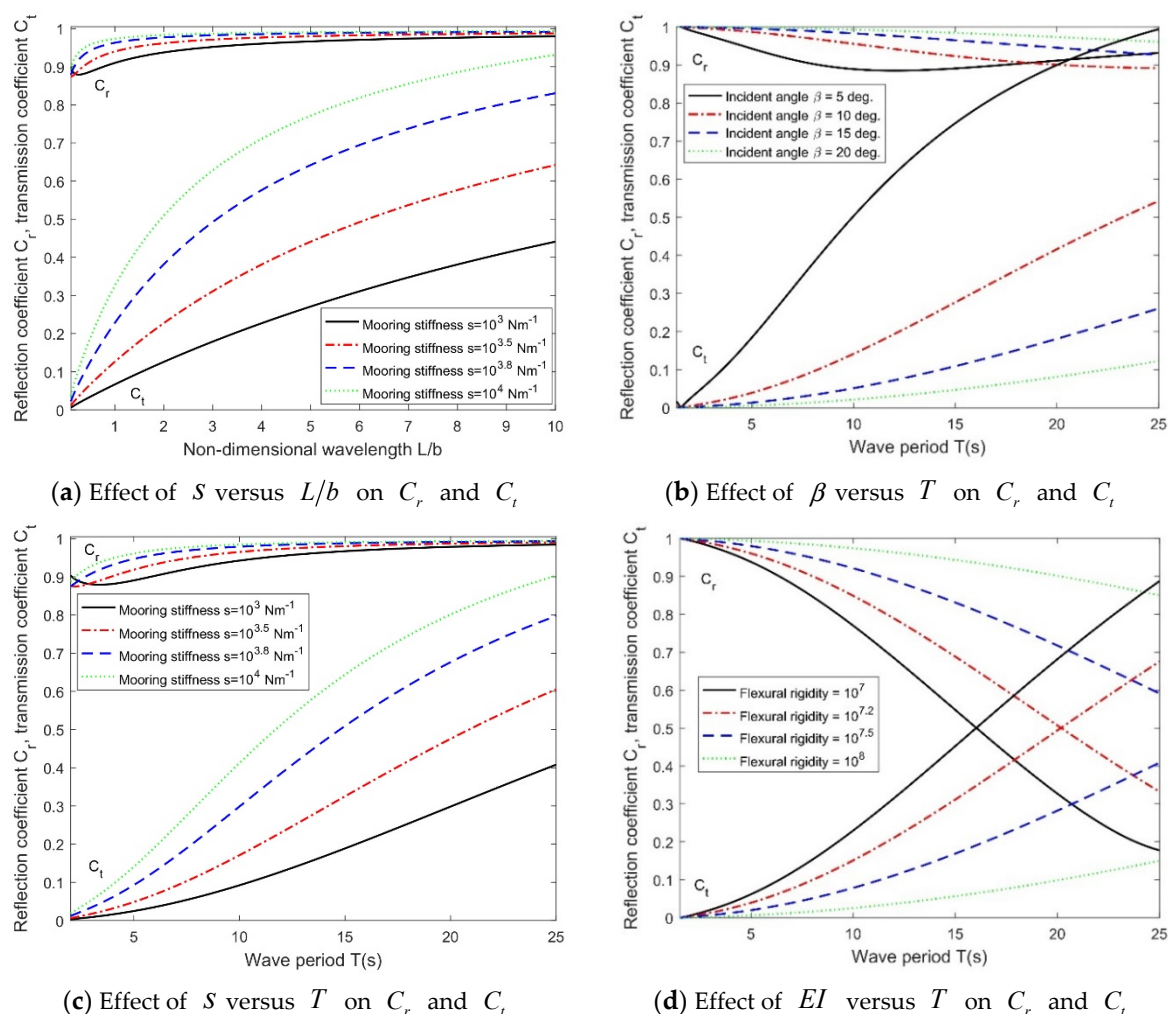


Figure 9. Effects of (a) mooring stiffness versus non-dimensional wavelength, (b) incidence angle, (c) mooring stiffness, and (d) flexural rigidity versus wave period on the C_r and C_t for $EI = 10^5$, $\ell = 1.5\sqrt{EI\rho g}$, and $b/h = 20$.

5. Conclusions

In the present paper, the new contribution is the addition of mooring edge boundary conditions to the BIEM formulation and the effect of mooring stiffness on the hydroelastic response of the floating plate. The BIEM code is compared with existing published results and experimental datasets from the other calculations as well as the SWA model. Furthermore, the hydroelastic response of moored finite floating flexible plate is studied by analyzing the effect of mooring stiffness, incident angle, and flexural rigidity versus different physical parameters on the deflection amplitude, the plate displacement, and the reflection and the transmission coefficients. It has been concluded that:

- The mathematical model of the moored floating flexible plate using the BIEM technique and the analysis of its hydroelastic response is reported for the first time here.
- Comparison of the results reveal that the present BIEM model results are supported by existing published results from other calculations and experimental datasets as well as the SWA model.
- The deflection amplitudes of the moored floating flexible plate decrease for higher values of mooring stiffness and flexural rigidity which is because as these two parameters go on increasing the flexible plate becomes stiffer that leads to less deflection.
- The shear force and bending moment are decreased in nature when there are higher values of mooring stiffness and incident angle. Furthermore, the reflection coefficient

cients increasing in nature as the mooring stiffness increases whilst the transmission coefficients decrease.

- The hydroelastic response analysis of different physical parameters suggested that, for higher spring stiffness, the deflection pattern becomes more regular whilst rigidity in the flexible plate creates fewer deformations for higher values of flexural rigidity. This concludes that a suitable choice of springs and flexural rigidity play a vital role in modelling a wave energy conversion device.

As a future step, the present formulation and the integro-differential equation can be generalized for problems arising in civil engineering application to breakwater and WECs.

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