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A Probabilistic Method for Estimating the Percentage of Corrosion Depth on the Inner Bottom Plates of Aging Bulk Carriers

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Abstract: This paper presents an approach for the model estimating the probabilistic percent corrosion depth for inner bottom plates of fuel oil tanks located in the double bottom of aging bulk carriers. Assuming that corrosion begins after four years of exploitation, a statistical approach to investigations on the ratio of the corrosion rate and the average initial inner bottom plate's thickness of considered bulk carriers is given. We consider this ratio to be a random variable since it is included in the usual linear corrosion model. By applying adequate statistical tests to the available empirical dataset, three best fitted three-parameter distributions for estimating the cumulative density function and the probability density function of the random variable were obtained. These three distributions were further used to estimate the studied percentage of corrosion depth. Lastly, we present the corresponding numerical and graphical results concerning the obtained statistical and empirical results and give concluding remarks.

Keywords: bulk carrier; fuel oil tanks; inner bottom plates; corrosion rate; percentage of corrosion depth; probabilistic method; three-parameter distribution

1. Introduction

Corrosion is one of the most frequent degradation mechanisms that affects ship hulls as they age. Although there are numerous types of corrosion, such as galvanic corrosion, fretting corrosion, bio-corrosion, and pitting corrosion, the most widespread type that damages ship hull structures is general corrosion. The long periods of exploitation include operations with cargo. Additionally, the influential factors of the environment and maintenance all affect the corrosion rate over time. A high intensity of corrosion causes a faster wear of structural parts and requires a significant investment in maintenance. The maintenance and repairs delay the usage of ships, increase the expenses, and shorten the life expectancy of ships.

Previous studies have mainly focused on tankers, bulk carriers, and ships considered to be the most susceptible to the rapid decay caused by corrosion. Moreover, previously conducted research has been based on the analysis of corrosion mechanisms [1–3], influential factors of the environment [2,4–6], and the decay of particular structural parts of bulk carriers [4,7–14].

The findings reported so far have multiple benefits for shipbuilders, ship repair yards, ship owners, ship management companies, insurance companies, and classification societies. The quality standards and allowable wear limits are important for all stakeholders since each of the them has a specific role in the process of shipbuilding and exploitation.

Ship designers aim to create optimal ships with a flawless performance, and shipbuilders attempt to ensure a high quality of built-in materials, devices, and the services of suppliers and other stakeholders. The research conducted so far has investigated corrosion mechanisms over time in order to predict the corrosion margin that should be considered during the construction of steel plates. Furthermore, ship owners want to use their ships for a long period of time and postpone potential repairs and replacements of corroded surfaces. This can be achieved by choosing an appropriate maintenance method, which can significantly contribute to slowing down the corrosion process and reducing the number of repairs. Moreover, higher quality steels and adequate painting surface protection delay initial corrosion, which significantly extends the life cycle of steel ship structures. In this sense, the rules of classification societies regarding further exploitation and the usage of corroded surfaces represent a limiting factor. The rules clearly define the types of wastage of structural parts or entire structural areas.

This paper is organized in the following way. Section 2 explains the motivation for this study and relies on a survey investigating the analytic and probabilistic corrosion rate estimation models in relation to ship hull structures. The primary focus is the investigation of corrosion wastage of the inner bottom plates of bulk carriers. Additionally, the same section presents the data collection methods, a brief description of the input data set, and the probabilistic method developed in this study. The purposefully developed probabilistic model is a modified version of a frequently used linear probabilistic model that has been employed in previous studies and in our recent works [14–16]. In relation to this model, Section 3 describes the application of the statistical tests that introduced three best fitted three-parameter continuous distributions for the random variable $\frac{c_1}{d_0}$ into the considered class consisting of numerous three-parameter continuous distributions. Section 4 presents concluding remarks.

2. Materials and Methods

Currently, all stakeholders in the maritime industry are primarily concerned about the sustainability of ships in exploitation and the protection of the environment. However, the attitudes regarding ship sustainability vary among stakeholders. For example, ship owners and managers require the maximum exploitation of ships, while classification societies and flag states require compliance with the defined standards aimed at protecting people, material goods, and the environment. Likewise, in order to assess the need for repair and partial or complete replacement of the parts that are damaged by corrosion, classification societies analyze the allowed wear expressed as a percentage in relation to the original thickness of steel plates. A less significant wear percentage indicates minor corrosion and a longer period of exploitation, while excessive wear indicates notable corrosion and requires the replacement of a greater number of corroded surfaces. Classification societies prescribe allowable wear limits that ensure optimal security and protection of the environment, while ship owners want to maximally exploit their ships, up to the limits defined, with an intention to reduce the cost of maintenance and repairs.

Previous studies have examined the different types of corrosion such as general, pitting, cavitation, and galvanic corrosion. General corrosion was researched in relation to a reduction in the thickness of steel plates and expressed as the millimeters of wear [4,6,7,14,16]. Additionally, the thickness reduction caused by the pitting corrosion of steel plates has been analyzed in terms of the thickness wear percentage. Most authors have employed an analytic and probabilistic method to determine the millimeters of wear [12–17]. However, this paper established a corresponding methodology that suggested the best distributions for assessing the wear percentage of steel plates from a probabilistic and statistical perspective.

Monitoring the condition of ships in exploitation detected the beginning of corrosion. Based on previous research and the observation of an extensive database, it was assumed that the corrosion of susceptible areas starts after several years of exploitation [6,14,16] while, in less sensitive areas, the beginning of corrosion is postponed. For instance, some authors have assumed that corrosion starts after more than five years after construction (see References [10,11] where deck plates and ballast tanks are considered).

Previous research [12–14,16] has confirmed that inner bottom plates are significantly influenced by various factors that affect the rapid decay of plates. Regarding this, one side of the inner bottom plates was exposed to ballast water, dry space, and fuel oil tanks, while the other side was affected by cargo. More precisely, inner bottom plates were in constant contact with cargo in cargo holds and under the influence of handling equipment and maintenance processes such as cleaning before and after cargo operations. Therefore, this paper focuses on the plates constituting the parts of fuel oil tanks. Since fuel oil tanks become warm over time, the emergence of corrosion is naturally faster and requires early maintenance and repairs. This motivated research on corrosion rates expressed as the percentage of metal thickness reduction in relation to the original thickness.

In this regard, assuming that corrosion starts after four years of ship exploitation, this paper proposed a probabilistic method for estimating the corrosion depth percentage. The point at which the total wear reached 10% of the entire structural area of the fuel oil tank sheets was determined based on the estimations made and corresponded to the regulations. According to the estimations made, the structural area examined cannot be used throughout the 25 years of the projected life expectancy of ships.

2.1. Data Collecting Methodology

This research examined bulk carriers whose age varied between five and twenty-five years. Since previous research has proven that inner bottom plates and the bottom of cargo holds are the most susceptible to decay caused by corrosion, this study focused on the steel plates of fuel oil tanks. The steel plates are the sheets of fuel oil tanks and subsidiary cargo holds. Consequently, steel plates are exposed to different influences—ower sides are affected by warm fuel, while upper sides are under the influence of the atmosphere (if cargo holds are empty) or cargo (if cargo holds are full).

This research encompasses both sides of fuel oil tanks—the portside (P) and starboard (S) side. The data were collected by means of special surveys and regular measurements of each bulk carrier. The data obtained indicated the general corrosive decay expressed as the percentage of the reduction in the original steel thickness. The database does not include the damage caused by damages, cracks, or stress.

Since the fuel is located in the oil tanks (which are below inner bottom plates) in all bulk carriers, examined corrosion emerges from cargo holds, i.e., affects the top of sheets. The measuring of the thickness reduction was performed for cargo holds, but not for oil tanks.

Measuring ship hull structures required compliance with special procedures and standards with the aim of estimating a real condition of all structural parts. Although each special survey required the measuring of all structural elements of bulk carrier construction that are in contact with the sea, atmosphere, and cargo, this paper focused on the measured data regarding the wear of the sheets of fuel oil tanks located in double bottom areas. These steel plates are the inner bottom plates of cargo holds. Since the amount of measured data is precisely defined by classification societies, we selected the data in accordance with a predetermined order. Measuring the examined area only represents part of the extensive measuring of ships in exploitation. The selection included the data that objectively presented the condition of the structural area. In this sense, with a view toward ensuring a proper data analysis, this paper examined each sheet of fuel oil tanks as a separately measured surface. Each surface was transversally and longitudinally intersected by tin extending along the entire sheets of the double bottom of cargo holds.

The length of each fuel oil tank corresponded to the length of a cargo hold, except in three cases, when the lengths of fuel oil tanks equaled double the length of cargo holds. For that reason, only three tanks were in the way of two cargo holds and had 10 cross sections considered in this research. Each fuel oil tank in the way of a cargo hold was divided into five sections: two sections for after and before ends, and three sections at equal mutual distances in the middle, between the ends of tanks (Figure 1).

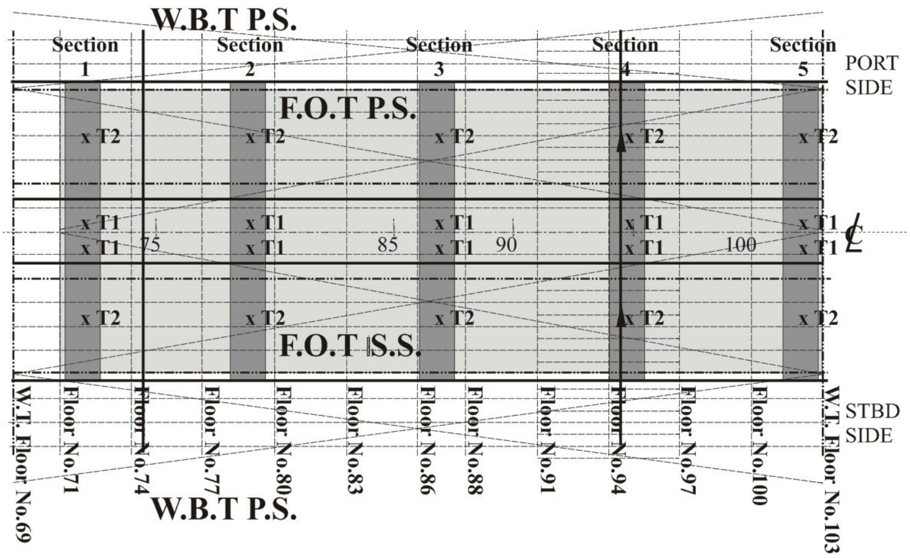

Figure 1. Collecting data scheme for inner bottom plates in the way of fuel oil tanks.

Each cross section included one measured point on each tin transversal on the fuel oil tanks that were included in the section, and is represented by an average value of the section. In this way, the average value of the section wear percentage represents several measured points. This established a database containing a total of 570 sections, which indicated the wear percentage of each section, i.e., the wear of inner bottom plates in relation to the original thickness of fuel oil tanks on the bulk carriers examined.

Considering that corrosion begins after four years of exploitation and in accordance with the statistical, graphical, and obtained numerical results, it can be assumed that three three-parameter continuous distributions are suitable for a probabilistic model for estimating the corrosion depth percentage of inner bottom plates of bulk carriers. The estimation concerned the percentage of thickness reduction of built-in structure plates of fuel oil tanks. Statistical tests confirmed this assumption with a high precision.

2.2. A Brief Description of the Input Data Set

As previously indicated, this paper only analyzed bulk carriers whose decay is, over time, caused by general corrosion and expressed as the percentage of the wear of steel plates. Based on previous research conducted by the authors of this paper [14,16,17], the examined database included 25 bulk carriers in exploitation whose age varied between five and twenty-five years. The measured data were gathered between 2005 and 2017, where certain bulk carriers were measured three times and others only once in special surveys. The analysis focused on four ships whose age was between five and ten years, seven ships that were 15 years old, 13 ships that were 20 years old, and 10 ships that were 25 years old. The measurements were conducted through 38 different special surveys, which established an input database for further analysis. Table 1 exhibits the database analyzed, including the number of ships in relation to their age, and the number of special surveys, fuel oil tanks, measured points, and sections examined. The last column of Table 1 presents the average values of wear, depending on the age intervals of the ships. A total of 110 fuel oil tanks were examined through 2810 measured points and 570 sections.

2.3. The Proposed Problem and Related Methodology

For a good survey of investigations of the analytic and probabilistic corrosion rate estimation model for different hull structural elements of bulk carriers, see the survey paper by Qin and Cui [18].

It is known that the corrosion wastage, $d(t)$ (at moment t), may be generally expressed as a power function of time (usually expressed in years) after the corrosion starts [18,19], i.e.,

$$d(t) = c_1(t - T_{cl})^{c_2}. \tag{1}$$

where $d(t)$ is the corrosion wastage, t is the elapsed time after the plate is used, T_{cl} is the life of the coating, and c_1 and c_2 are positive real coefficients. As noticed in Reference [18], in most of the related studies on the time-dependent reliability of ship structures [20–24], the effect of corrosion was represented by an uncertain but constant corrosion rate, which resulted in a linear decrease of plate thickness with time. Accordingly, for the subject of research of this paper, we use expression (1) with $c_2 = 1$ proposed by Paik, Kim, and Lee [4], and a related linear corrosion model is defined as

$$d(t) = c_1(t - T_{cl}), \tag{2}$$

where c_1 is the corrosion rate, usually expressed in mm/year.

Table 1. A database of the percentage of thickness reduction of inner bottom plates.

The Age of Ships (Years)	The Number of Ship Surveys	The Number of Tanks	The Number of Measured Points	The Number of Sections	The Average Values of Plate Thickness Reduction Caused by Corrosion (%)
0–5	4	9	230	45	0.5
5–10	4	10	266	55	2.8
10–15	7	19	500	100	9.8
15–20	13	43	998	220	11.7
20–25	10	29	816	150	17.7
SUM:	38	110	2810	570	

In the case of the investigation of the corrosion wastage of an arbitrary structural area of a group of bulk carriers, it can be of interest to estimate a percentage, $p(t)$ of corrosion wastage with respect to the average initial bulk carriers' structural area thickness. More precisely, if we denote the average initial structural area of the considered group of bulk carriers by $\overline{d_0}$, then it is natural to define a related value of $p(t)$ as:

$$p(t) = \frac{d(t)}{\overline{d_0}} (\text{with } t > T_{cl}). \tag{3}$$

Dividing Equation (2) by $\overline{d_0}$, we can obtain the following.

$$p(t) = \frac{d(t)}{\overline{d_0}} = \frac{c_1}{\overline{d_0}}(t - T_{cl})\ (\text{with } t > T_{cl}). \tag{4}$$

Since the effect of corrosion is generally of an uncertain nature, the coefficient c_1 can be considered as a continuous random variable. Since $\overline{d_0}$ is a real constant, this is also true for the ratio $\frac{c_1}{\overline{d_0}}$. Hence, the determination of the values $p(t)$ is closely related to the probabilistic estimation of a continuous random variable $\frac{c_1}{\overline{d_0}}$ for determining the best fitted three-parameter continuous distributions for the probability density function (PDF) and the cumulative density function (CDF) for $\frac{c_1}{\overline{d_0}}$. To the best of our knowledge, three-parameter continuous distributions have very rarely been considered in related literature. Very recently, in Reference [25], the suggested theoretical distribution for a related initial distribution approach is the three-parameter-Weibull distribution. Applying a Kolmogorov-Smirnov test, the probabilistic estimation model for the random variable $\frac{c_1}{\overline{\overline{d_0}}}$ and related applications are given in the next section.

3. Results

Assuming that corrosion starts after four years of exploitation and, based on the application of Equation (2) from Section 2.3, this section proposes a probabilistic method for estimating the random variable $\frac{c_1}{\overline{\overline{d_0}}}$ that was defined in Section 2.3 for the inner bottom plates of the investigated fuel oil tanks on 38 aging bulk carriers. Namely, the section suggests a statistical approach to approximating the CDF of the variable $\frac{c_1}{\overline{\overline{d_0}}}$. The relevant estimations were only detected among three-parameter continuous distributions, which defined the three best-fitted distributions for the random variable $\frac{c_1}{\overline{\overline{d_0}}}$. Expression (4) where $T_{cl} = 4$, i.e.,

$$\frac{c_1}{\overline{\overline{d_0}}} = \frac{d(t)}{(t-4)\overline{\overline{d_0}}} \tag{5}$$

was used for these purposes.

Figure 2 presents a total of 570 measurements of plate thickness expressed as the percentage of the reduction of the original thickness due to corrosion on the inner bottom plates of 38 ships surveyed. The data set includes all measured points from each plate in corresponding transversal sections. The following text presents different multi-parameter functions that were validated by corresponding tests. Based on the tests considered, the paper analyzes the average values of the obtained parameters in order to determine the linear dependence of the values examined and the point at which the average values exceeded the allowable limits defined by classification societies.

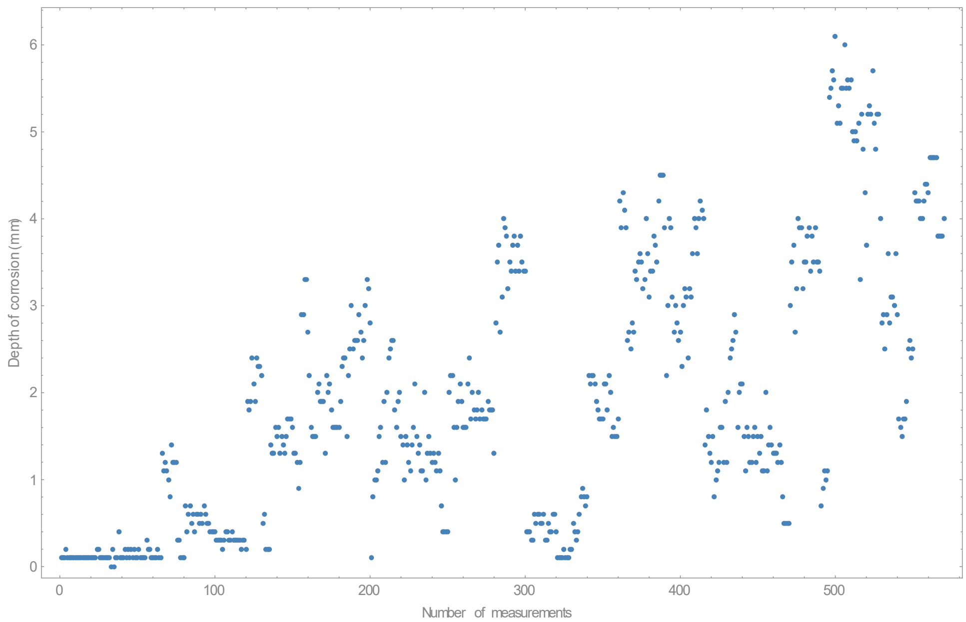

Figure 2. The data of 570 thickness measurements of inner bottom plates obtained through 38 ship surveys.

3.1. Appropriate Statistical Analysis Related to Measurements of Inner Bottom Plates

Table 2 presents the numerical characteristics of 465 empirical data that are expressed as the percentage of corrosion wastage and examined in this paper. Table 2 contains the standard statistical parameters that describe the empirical data (first and third quartiles are denoted as Q1 and Q3 respectively). These parameters describe the shape of the data as well as some of the related percentiles.

Table 2. The descriptive statistics of the empirical data for $T_{cl} = 4$ years.

Statistic	Value	Percentile	Value
Sample Size	465	Min	0.00129
Range	0.02041	5%	0.00197
Mean	0.00788	10%	0.00284
Variance	1.5617×10^{-5}	25% (Q1)	0.00448
Standard Deviation	0.00395	50% (Median)	0.00746
Coefficient of Variation	0.5017	75% (Q3)	0.01111
Standard Error	1.8326×10^{-4}	90%	0.01336
Skewness	0.22245	95%	0.01424
Excess Kurtosis	−0.83477	Max	0.0217

As previously indicated, a total of 570 measurements of corrosion wear were performed in the structural area of inner bottom plates (IBP) of bulk carriers and all obtained values are expressed as millimeters. The papers by Ivošević et al. [14] and Ivošević et al. [16] examined the same dataset consisting of 570 measured data on plate thickness reduction due to the corrosion of IBP (i.e., related values $d(t)$ on the depth of corrosion, expressed as millimeters), which were measurements carried out on 38 bulk carriers. It is known that, for practical reasons, a difference between the measured steal thickness and initial thickness of less than 0.3 mm can be considered negligibly small (no corrosion due to steel under the paint), and, hence, those measurements were eliminated from our further statistical analysis. In this sense, the database relevant for the paper consists of 465 measurements on the corrosive wear of the IBP area. This section statistically analyzes the corrosion wear of steel plates expressed as the wear percentage in relation to the average values of the original thickness of inner bottom plates. Therefore, the 465 sets of data expressed as millimeters were first converted into equivalent percentages. Since the initially measured steel wear caused by corrosion was measured in millimeters, the examined data set had to be expressed as a percentage, i.e., as the ratio ($\frac{d(t)}{\overline{d_0}}$) of corrosion wear at a particular moment t ($d(t)$) to the original thickness of inner bottom plates ($\overline{d_0}$) (see Section 2.3). The IBP area was constructed of steel plates whose thickness varied. For the purposes of further analysis, the steel plates were grouped into the following categories, depending on the original thickness: [16,17], [17,18], [18,19], [19,20], [20,21], [21,22], [22,23] (where all values are expressed as mm). The ordinal numbers of intervals are denoted by i, where $i \in \{1, 2, \ldots, 7\}$, and the average value of an interval is denoted by d_i, where $d_i \in \{16.5, 17.5, 18.5, 19.5, 20.5, 21.5, 22.5\}$. Since the original thickness of the steel examined was considered when measuring, the entire data set (i.e., all measured points) can be related to one of the previously formed intervals. In this way, it is possible to determine the total number of points that belong to the formed interval i ($i \in \{1, 2, \ldots, 7\}$). The average value of the original thickness of inner bottom plates ($\overline{d_0}$) was calculated in accordance with the grouped data in the following way.

$$\overline{d_0} = \frac{1}{N}\sum_{i=1}^{7} d_i n(d_i) \tag{6}$$

where $n(d_i)$ denotes the frequency of d_i ($i \in \{1, 2, \ldots, 7\}$).

Since N is the total number of samples (measured points) based on the dataset used for the statistical analysis, the calculated value of the original average thickness is $\overline{d_0} = 18.43455$ mm. Taking into account the average value obtained for the original thickness of inner bottom plates ($\overline{d_0}$), the data expressed as mm were easily converted into a percentage. A probabilistic model that can describe the wear percentage caused by corrosion of the steel plates of the IBP was developed based on the percentage data listed in Table 2. An initial assumption in the development of this model was that $T_{cl} = 4$ years and that the ratio $\frac{c_1}{\overline{d_0}}$ is a continuous random variable (see Section 2.3).

The fitting of theoretical distributions based on a set of empirical data includes three basic phases.

1. The choice of an adequate model, i.e., the choice of optimal theoretical distributions,
2. The determination of the optimal values for the parameters that characterize theoretical distributions,
3. The determination of the significance level and the quality of the distributions fitted.

Each theoretical distribution depends on the set of initial parameters that describe its characteristics. The most frequent parameters of theoretical distributions are the location, scale, shape, and threshold parameters. The location parameter describes the position of the graph of the theoretical distribution in relation to the horizontal axis (x-axis). The scale parameter defines the degree of dispersion. The shape parameter describes the tendency of a distribution to skew to the left, while the threshold parameter describes the lowest value that a distribution reaches in relation to the x-axis. A distribution does not necessarily have all the parameters listed. There are two-parameter and multi-parameter distributions.

This paper focuses on the theoretical distributions that are characterized by three parameters, i.e., three-parameter distributions. The optimal set of parameters that characterizes relevant distributions should be determined during the distribution fitting process in order to determine the best distributions that adequately describe the set of empirical percentage data. For this purpose, this paper relies on the maximum likelihood estimation method (e.g., see Hays, [26]), which is one of the most commonly used methods for determining the unknown parameters of theoretical distributions. This method is based on a procedure that detects the parameters that maximize the likelihood of the observed data that occur with respect to a theoretical model. In order to determine the goodness of fit (e.g., see Stephens, [27]), the last phase applies the Kolmogorov–Smirnov test. This test compares empirical and theoretical models by computing the maximum absolute difference between empirical and theoretical distribution functions. The first column of Table 3 exhibits an overview of the most prominent three-parameter distributions that successfully fit into the set of the 465 empirical data previously described. The second column of Table 3 presents the parameters that describe each theoretical distribution, along with the values obtained by means of the maximum likelihood estimation method. The theoretical distributions in Table 3 are not sorted in accordance with the goodness of fit, but in alphabetical order.

Table 3. Fitted three-parameter distributions of $\frac{c_1}{d_0}$ for IBP where $T_{cl} = 4$ years.

Distribution	Parameters
Burr	$k = 4.2273E + 13$ $\alpha = 0.93196$ $\beta = 3.4010x10^{12}$
Dagum	$k = 0.08817$ $\alpha = 15.238$ $\beta = 0.01399$
Erlang	$m = 7$ $\beta = 0.0016$ $\gamma = -0.00262$
Error	$k = 4.1549$ $\sigma = 0.00395$ $\mu = 0.00788$
Fatigue Life	$\alpha = 0.21019$ $\beta = 0.01867$ $\gamma = -0.0112$
Frechet	$\alpha = 1.0402E + 8$ $\beta = 3.4816x10^5$ $\gamma = -3.4816x10^5$
Gamma	$\alpha = 6.5552$ $\beta = 0.0016$ $\gamma = -0.00262$
Gen. Extreme Value	$k = -0.1979$ $\sigma = 0.00381$ $\mu = 0.00631$
Gen. Gamma	$k = 0.94793$ $\alpha = 3.6606$ $\beta = 0.00198$
Gen. Logistic	$k = 0.04888$ $\sigma = 0.00226$ $\mu = 0.00769$
Gen. Pareto	$k = -0.81361$ $\sigma = 0.01158$ $\mu = 0.00149$
Inv. Gaussian	$\lambda = 0.45761$ $\mu = 0.0195$ $\gamma = -0.01163$
Log-Logistic	$\alpha = 7.6305$ $\beta = 0.01799$ $\gamma = -0.01047$
Log-Pearson3	$\alpha = 6.1211$ $\beta = -0.2504$ $\gamma = -3.4722$
Lognormal	$\sigma = 0.18192$ $\mu = -3.8403$ $\gamma = -0.01397$
Pearson 5	$\alpha = 57.607$ $\beta = 1.6874$ $\gamma = -0.02194$
Pearson 6	$\alpha\,1 = 3.4686$ $\alpha\,2 = 6330.4$ $\beta = 14.369$
Pert	$m = 0.00571$ $a = 7.0282x10^{-4}$ $b = 0.0232$
Power Function	$\alpha = 0.57172$ $a = 0.00129$ $b = 0.0217$
Triangular	$m = 0.00129$ $a = 0.00128$ $b = 0.02175$
Weibull	$\alpha = 1.9729$ $\beta = 0.00845$ $\gamma = 3.8091x10^{-4}$

All three-parameter distributions were rated based on the Kolmogorov–Smirnov test, which identified the three best fitted three-parameter distributions, i.e., the three distributions that best described the empirical data of the percentage of corrosion wear. Based on the distribution of the random variable $\frac{c_1}{d_0}$ for IBP where $T_{cl} = 4$ years, the Kolmogorov–Smirnov test indicated that the three best fitted three-parameter distributions were Generalized Pareto, Error, and Log-Pearson 3. These three distributions and corresponding fitted parameters are presented in Table 4.

Table 4. The three best fitted three-parameter distributions of $\frac{c_1}{d_0}$ for IBP with $T_{cl} = 4$ years.

Distribution	Parameters
Gen. Pareto	k = −0.8 σ = 0.01 μ = 0.00149
Error	k = 7.0 σ = 0.08 μ = 0.16
Log-Pearson 3	α = 5.7 β = −0.26 γ = −0.54

The second column of Table 4 shows the parameters and corresponding values that characterize the best fitted three-parameter distributions. These parameters are defined as follows.

(1) For the Generalized Pareto distribution:

k—continuous shape parameter $(-\infty < \sigma < +\infty)$;

σ—continuous scale parameter $(0 < \sigma < +\infty)$;

μ—continuous location parameter $(-\infty < \mu < +\infty)$.

The PDF, $f_{GP}(x)$, of the Generalized Pareto distribution with its domain $-\infty < x \leq \mu$ when $k \geq \mu$ and $\mu \leq x \leq \mu - \rho/k$ when $k < 0$ is defined as:

$$f_{GP}(x) = \frac{1}{\sigma}\left(1 + \frac{k(x-\mu)}{\rho}\right)^{-\left(\frac{1}{\xi}-1\right)} \tag{7}$$

The CDF of the Generalized Pareto distribution, $F_{GP}(x)$, is defined as:

$$F_{GP}(x) = \begin{cases} 1 - \left(1 + k\frac{(x-\mu)}{\sigma}\right)^{-1/k} & k \neq 0 \\ 1 - exp\left(-\frac{(x-\mu)}{\sigma}\right) & k = 0 \end{cases} \tag{8}$$

(2) For the Error distribution:

k—continuous shape parameter,

σ—continuous scale parameter (σ > 0),

μ—continuous location parameter.

The PDF, $f_E(x)$, of the Error Distribution for its domain $-\infty < x < +\infty$ is defined as:

$$f_E(z) = \frac{c_1}{\sigma} exp(-|c_0 z|^k) \tag{9}$$

where $c_0 = \left(\frac{\Gamma\left(\frac{3}{k}\right)}{\Gamma\left(\frac{1}{k}\right)}\right)^{1/2}$, $c_1 = \frac{kc_0}{2\Gamma\left(\frac{1}{k}\right)}$, and $z = \frac{x-\mu}{\sigma}$.

The CDF, $F_E(x)$, of the Error Distribution for its domain $-\infty < x < +\infty$ is defined as:

$$F_E(x) = 0.5\left(1 + \frac{\Gamma_{|c_0 x^k|}(1/k)}{\Gamma(1/k)}\right) \text{when } x \geq \mu, \text{and} \tag{10}$$

$$F_E(x) = 0.5\left(1 - \frac{\Gamma_{|c_0 x^k|}(1/k)}{\Gamma(1/k)}\right) \text{when } x < \mu. \tag{11}$$

(3) For the Log-Pearson 3 Distribution:

α—continuous shape parameter ($0 < \alpha < +\infty$),
β—continuous scale parameter ($\beta \neq 0$),
γ—continuous location parameter.

The PDF, $f_{LP}(x)$, of the Log-Pearson 3 Distribution for its domain $e \leq x < +\infty$ when $\beta < 0$ and $e \leq x < +\infty$ when $\beta \geq 0$, is defined as:

$$f_{LP}(x) = \frac{1}{\left(x\middle|\beta\middle|\Gamma(\alpha)\right)\left(\frac{lnx-\gamma}{\beta}\right)^{\alpha-1}} e^{-\left(\frac{lnx-\gamma}{\beta}\right)} \quad (12)$$

The CDF, $F_{LP}(x)$, of the Log-Pearson 3 Distribution is defined as:

$$F_{LP}(x) = \frac{\Gamma_{(\ln(x)-\gamma)/\beta}(\alpha)}{\Gamma(\alpha)} \quad (13)$$

The data from Table 4, together with the expressions (1)–(4), imply the following statistical results, given in Tables 5 and 6. In these tables, $f_{GP}(x)$, $f_E(x)$, and $f_{LP}(x)$, ($F_{GP}(x)$, $F_E(x)$, and $F_{LP}(x)$) denote the probability density functions of the best fitted Generalized Pareto, Error, and Log-Pearson 3 distributions (the cumulative distribution functions) for $\frac{c_1}{d_0}$, respectively.

Table 5. The empirical values and the values of PDF for the three best fitted three-parameter distributions for $\frac{c_1}{d_0}$ with respect to IBP.

IBP with T_{cl} = 4 Years					
Lower Bound	**Upper Bound**	**Empirical PDF of $\frac{c_1}{d_0}$**	$f_{GP}(x)$	$f_E(x)$	$f_{LP}(x)$
0	0.00227	0.073	0.06681	0.08237	0.05704
0.00227	0.00454	0.187	0.18958	0.15123	0.18019
0.00454	0.00681	0.191	0.18092	0.18039	0.21897
0.00681	0.009079	0.155	0.17050	0.18290	0.19497
0.009079	0.011349	0.157	0.15752	0.17981	0.14648
0.011349	0.013619	0.161	0.13914	0.14778	0.09680
0.013619	0.015889	0.067	0.09552	0.06668	0.05684
0.015889	0.018159	0.006	0.00001	0.00870	0.02941
0.018159	0.020429	0.000	0	0.00014	0.01306
0.020429	0.022699	0.002	0	0	0.00474

Table 6. The empirical values and the values of cumulative density functions (CDF) for the three best fitted distributions for $\frac{c_1}{d_0}$ with respect to IBP.

IBP T_{cl} = 4 Years					
Lower Bound	**Upper Bound**	**Empirical CDF of $\frac{c_1}{d_0}$**	$F_{GP}(x)$	$F_E(x)$	$F_{LP}(x)$
0	0.00227	0.073	0.06681	0.08237	0.05704
0.00227	0.00454	0.260	0.25639	0.23360	0.23723
0.00454	0.00681	0.451	0.43731	0.41399	0.45620
0.00681	0.009079	0.606	0.60781	0.59689	0.65117
0.009079	0.011349	0.763	0.76533	0.77670	0.79765
0.011349	0.013619	0.924	0.90447	0.92448	0.89445
0.013619	0.015889	0.991	0.99999	0.99116	0.95129
0.015889	0.018159	0.997	1	0.99986	0.98070
0.018159	0.020429	0.999	1	1	0.99376
0.020429	0.022699	0.999	1	1	0.99850

Figures 3 and 4 present the graphs of PDF and CDF of the three best fitted three-parameter distributions of $\frac{c_1}{d_0}$ for IBP where $T_{cl} = 4$ years, as given in Table 4. Figure 5 shows the probability difference graph of the three best fitted three-parameter distributions for $\frac{c_1}{d_0}$ for IBP where $T_{cl} = 4$ years. The probability difference graph was formed in order to show the quality of the fitted distributions by means of a comparison of the corresponding theoretical CDF and empirical CDF (see Figure 5). Additionally, the probability difference graph enables a comparison of the goodness- of-fit of the three best fitted distributions presented in Table 4. The graph in Figure 5 represents the difference between the empirical CDF and theoretical CDF and illustrates the quality of the fitted distributions, bearing in mind that the difference does not exceed the interval (−0.06,0.06).

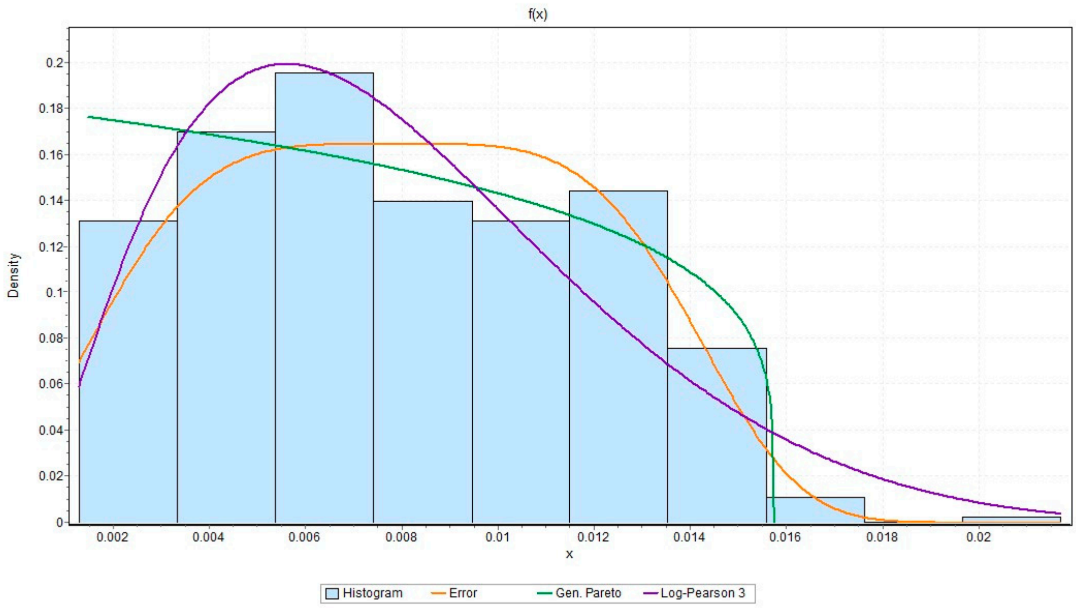

Figure 3. The empirical PDF and the PDF of the three best fitted three-parameter distributions of $\frac{c_1}{d_0}$ for IBP with $T_{cl} = 4$ years.

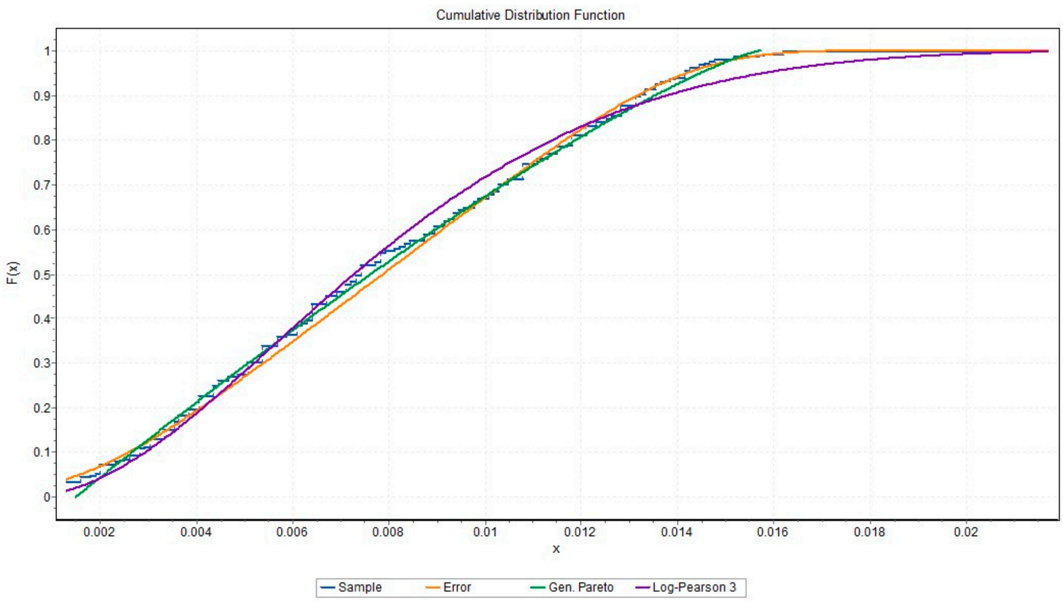

Figure 4. The empirical CDF and the CDF of the three best fitted three-parameter distributions of $\frac{c_1}{d_0}$ for IBP with $T_{cl} = 4$ years.

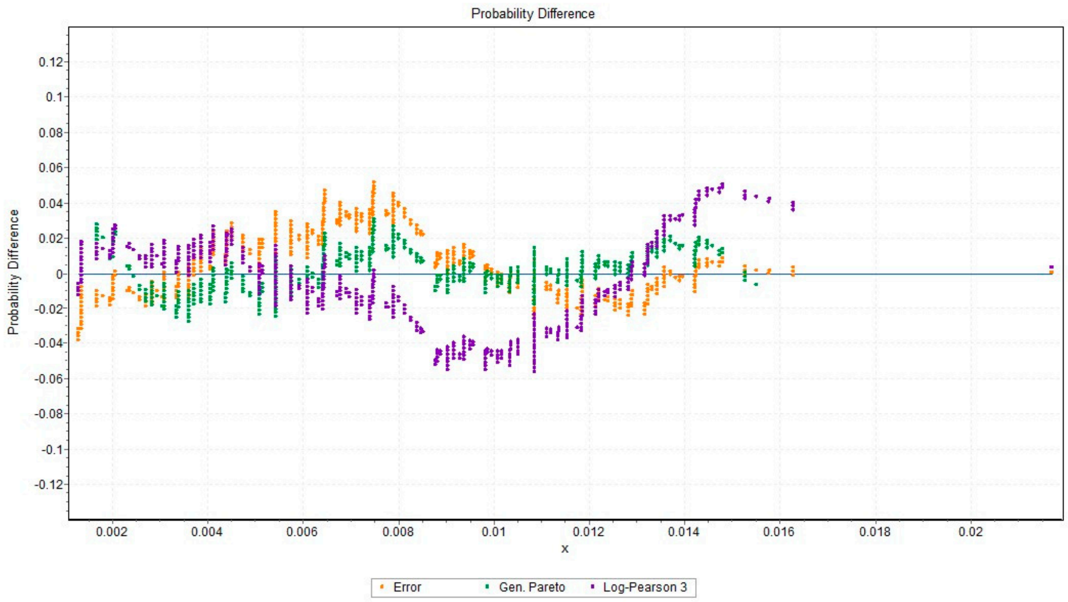

Figure 5. The probability difference graph of the three best fitted three-parameter distributions of $\frac{c_1}{d_0}$ for IBP with $T_{cl} = 4$ years.

Furthermore, Figure 6 presents probability-probability (P-P) plots of the three best fitted three-parameter distributions of $\frac{c_1}{d_0}$ for IBP where $T_{cl} = 4$ years given in Table 4. According to Figure 6, it is evident that the Generalized Pareto, Error, and Log-Pearson 3 distributions properly describe the tendencies of the empirical data since the deviations from the line shown in the graph are negligible. On the basis of the linearity observed in Figure 6, it can be concluded that the three theoretical distributions adequately shape all aspects of the empirical data including the location and scope.

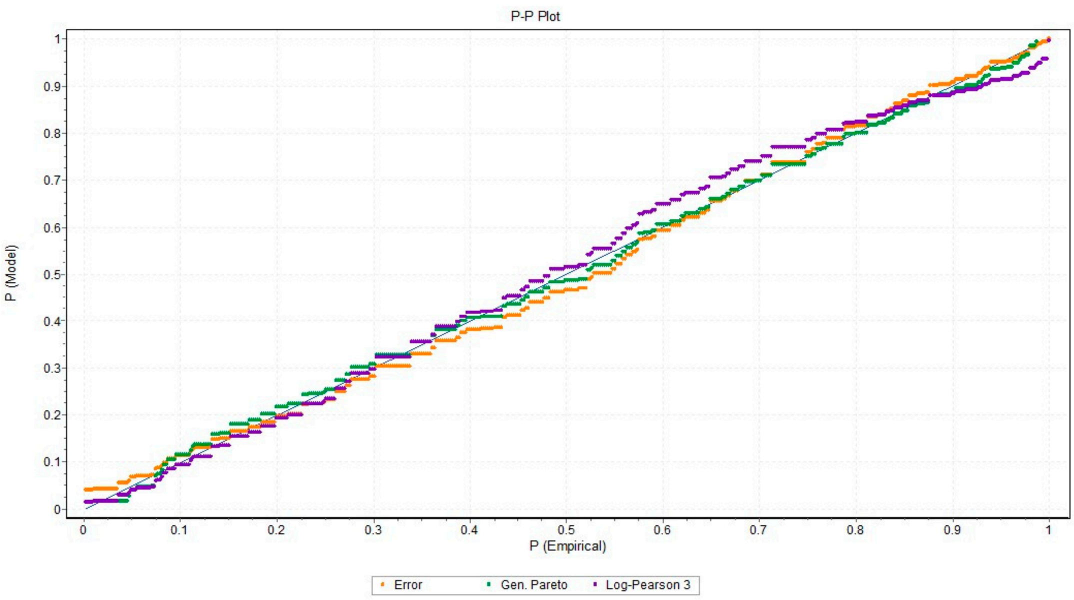

Figure 6. Probability-probability (P-P) plots of the three best fitted three-parameter distributions of $\frac{c_1}{d_0}$ for IBP with $T_{cl} = 4$ years.

Table 7 shows the statistical values and corresponding p-values in the case of the three best fitted three-parameter distributions that were previously presented in Table 4. Since the three calculated p-values are below the value chosen for the significance level (α), it is evident that the three theoretical distributions properly follow the empirical data. The same conclusion is still relevant for a lower significance level (e.g., 0.02 or 0.01).

Table 7. Test statistical values for the best fitted three-parameter distributions.

Distribution	Statistic	*p*-Value
Gen. Pareto	0.03075	0.75958
Error	0.05153	0.16339
Log-Pearson 3	0.05805	0.08375

The second column of Table 8 shows the average values of the wear percentage of steel, which were obtained on the basis of empirical data on the structural area of the IBP on the examined bulk carriers that were 5, 10, 15, 20, and 25 years old. The equation $\frac{d(t)}{\mathrm{d}_0} = 0.00793523\,(\mathrm{t}-4)$ was obtained upon fitting a line based on these data. More precisely, the solution of the equation $\frac{d(t)}{\mathrm{d}_0} = 0.10$ is $t = 16.602$ years, while the solution of the equation $\frac{d(t)}{\mathrm{d}_0} = 0.15$ is $t = 22.903$ years. These facts are presented in Figure 7.

Table 8. Average values of $\frac{d(t)}{\mathrm{d}_0}$ for IBP.

Years	$\frac{d(t)}{\mathrm{d}_0}$
5	0.0216986
10	0.0399617
15	0.104289
20	0.116896
25	0.166935

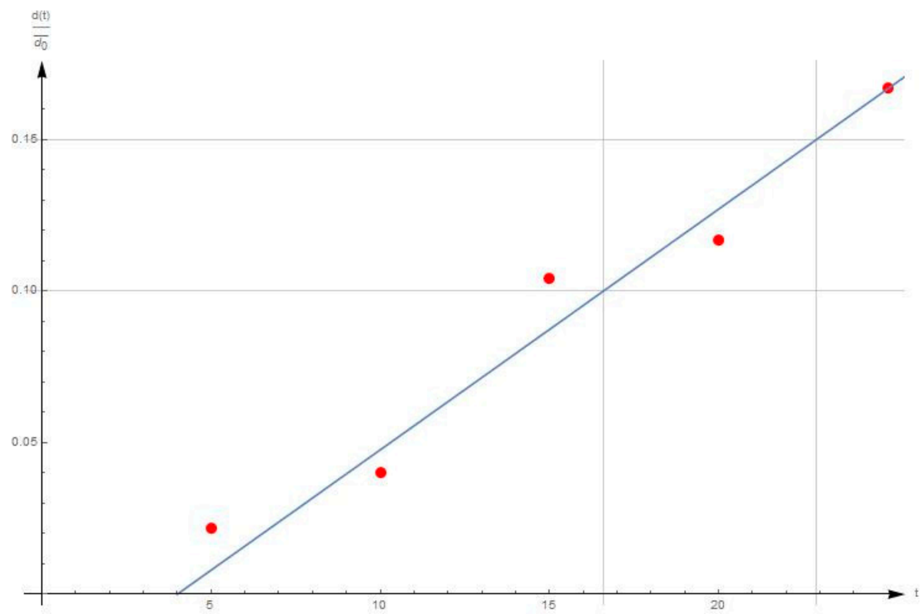

Figure 7. Estimates of $\frac{\mathrm{d(t)}}{\mathrm{d}_0}$ for IBP as a function of t.

The key parameter of the estimation of the further exploitation of the structural area of bulk carriers (bottom and side shell plating, deck plating, and inner bottom plating) is based on the percentage of the allowable wear of the entire structural area. Although particular cases of allowable wear of inner bottom plates vary between 15% and 30% (depending on the ship size and classification society), classification societies usually allow the wear of 10% for the entire structural area. In that regard, the data presented in Figure 7 are in accordance with the values of wear percentage obtained for the entire inner bottom area.

We conducted an additional analysis in order to verify whether (in statistical terms) it can be confirmed that the proposed methodology follows the empirical data. It is well-known that the expected value of the Generalized Pareto distribution is equal to (see Reference [28]):

$$E(GP) = \mu + \frac{\sigma}{1-k} \tag{14}$$

Bearing in mind the results presented in Table 4 and the previous formula, it can be concluded that the mean of the best fitted three-parameter distribution of variable $\frac{c_1}{d_0}$ for IBP with $T_{cl} = 4$ years is equal to $E(GP) = 0.00704556$.

In order to estimate the percentage values of corrosion $p(t)$ (with $t > 4$ years) given by Equation (4), we applied the usual statistical assumption that the mean value of $\frac{c_1}{d_0}$ is equal to its expected value, i.e., for the General Pareto distribution of $\frac{c_1}{d_0}$, its expected value is $E(GP) = 0.00704556$/year Substituting this "probability approximation" into Equation (5), we found that:

$$p(t) = \frac{d(t)}{\overline{d_0}} = \frac{c_1}{\overline{d_0}}(t-4) \approx E(GP)(t-4) = 0.00704556(t-4) \tag{15}$$

and it follows that:

$$p(t) = \frac{d(t)}{\overline{d_0}} = \frac{c_1}{\overline{d_0}}(t-4) \approx E(GP)(t-4) = 0.00704556(t-4) = 0.7046(t-4)\%. \tag{16}$$

The numerical results given in Table 9 present the estimated values of $p(t)$. These estimations were obtained by applying Equation (16) and varying variable T_{cl} from 5 to 24 years.

Table 9. Estimated values of $p(t) = \frac{d(t)}{\overline{d_0}}$ for IBP with $T_{cl} > 4$ years (expressed as a percentage) in accordance with Equation (16).

T_{cl}	$p(t)$	T_{cl}	$p(t)$	T_{cl}	$p(t)$	T_{cl}	$p(t)$
5	0.704556	10	4.22734	15	7.75012	20	11.2729
6	1.40911	11	4.93189	16	8.45467	21	11.9775
7	2.11367	12	5.63645	17	9.15923	22	12.682
8	2.81822	13	6.341	18	9.86378	23	13.3866
9	3.52278	14	7.04556	19	10.5683	24	14.0911

From Table 9, it can be seen that the calculated numerical values are very close to the related empirical data presented in Figure 7. The correctness of the proposed methodology is confirmed by this analysis and the fact that the numerical values follow the empirical data set well.

As can be seen from Figure 7, a linear curve intersects the value of 10% of the wear of the structural area after 16.602 years while the value of 15% of the total wear is achieved after 22.903 years. This proves that the corrosion processes on the inner bottom plates of fuel oil tanks are so intensive that the entire structural area should be replaced after 16.602 years. Considering the fact that a projected life expectancy of ships is between 20 and 25 years, the structural area examined in this paper and presented in Figure 7 cannot withstand the projected life expectancy due to excessive corrosive processes.

4. Conclusions

This paper is an extension of the authors' investigations on the probabilistic corrosion rate estimation model for inner bottom plates of bulk carriers. It has presented a probabilistic method for estimating the percentage of corrosion depth on the inner bottom plates of a considered group of aging bulk carriers. By applying the usual linear corrosion model, we have focused our attention on a statistical approach toward estimating the ratio of the corrosion rate and the average initial inner bottom plates thickness of considered bulk carriers. Motivated by several earlier investigations and the fact that many factors influence the corrosion wastage of ship hull structures and are of an uncertain nature, the previously defined ratio was considered as a continuous random variable.

Applying the Kolmogorov-Smirnov test at a significance level of 0.05, it was confirmed that, of the three-parameter continuous distributions, the Generalized Pareto distribution, Error distribution, and Log-Pearson 3 distribution were the best fitted distributions for the defined ratio. We have also given related statistical, numerical, and graphical results. These numerical and graphical results show that, in a statistical sense, the three best-fitted distributions for the defined ratio closely match the corresponding empirical data and follow all of their important characteristics well. In particular, the numerical results related to the Generalized Pareto distribution confirm the fact that the values calculated for estimating the studied percentage of corrosion depth are very close to the corresponding empirical data.

The proposed methodology clearly shows the dependence of the corrosion depth as a function of time. This methodology based on statistical analysis allows us to determine the final limits of utilizing the inner bottom plating of fuel oil tanks and to compare them with those defined by the rules of classification societies.

The proposed statistical methodology could be applied for related investigations involving other ship hull structure members of bulk carriers. Clearly, related statistical analysis can be performed for a large class of two-parameter continuous distributions.

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Conflicts of Interest: The authors declare no conflicts of interest

References

1. Paik, J.F.; Brennan, F.; Carlsen, C.A.; Daley, C.; Garbatov, Y.; Ivanov, L.; Rizzo, C.; Simonsen, B.C.; Yamamoto, N.; Zhuang, H.Z. Report of Committee V.6 Condition Assessment of Aging Ships. In Proceedings of the 16th International Ship and Offshore Structures Congress, Southampton, UK, 20–25 August 2006.
2. Roberts, S.E.; Marlow, P.B. Casualties in dry bulk shipping (1963–1996). *Mar. Policy* **2002**, *26*, 437–450. [CrossRef]
3. Bulk carrier casualty report, IMO, MSC 83/INF.6, 3 July 2007. Available online: http://www.safedor.org/resources/MSC_83-INF-8.pdf (accessed on 16 June 2020).
4. Paik, J.K.; Kim, S.K.; Lee, S.K. A probabilistic corrosion rate estimation model for longitudinal strength members of bulk carriers. *Ocean Eng.* **1998**, *25*, 837–860. [CrossRef]
5. Guedes Soares, C.; Garbatov, Y.; Zayed, A. Effect of environmental factors on steel plate corrosion under marine immersion conditions. *Corros. Eng. Sci. Technol.* **2011**, *46*, 524–541. [CrossRef]

6. Paik, J.K.; Thayamballi, A.K.; Park, Y.I.; Hwang, J.S. A time-dependent corrosion wastage model for seawater ballast tank structures of ships. *Corros. Sci.* **2004**, *46*, 471–486. [CrossRef]
7. Yamamoto, N.; Ikagaki, K. A Study on the Degradation of Coating and Corrosion on Ship's Hull Based on the Probabilistic Approach. *J. Offshore Mech. Arct. Eng.* **1998**, *120*, 121–128. [CrossRef]
8. Guo, J.; Wang, G.; Ivanov, L.; Perakis, A.N. Time-varying ultimate strength of aging tanker deck plate considering corrosion effect. *Mar. Struct.* **2008**, *21*, 402–419. [CrossRef]
9. Garbatov, Y.; Guedes Soares, C.; Wang, G. Non-linear time dependent corrosion wastage of deck plates of ballast and cargo tanks of tankers. In Proceedings of the 22nd International Conference on Offshore Mechanics and Arctic Engineering, OMAE 2005–67579, Halkidiki, Greece, 12–17 June 2005.
10. Jurišić, P.; Parunov, J.; Garbatov, Y. Aging effects on Ship Structural integrity. *Brodogradnja/Shipbuilding* **2017**, *68*, 15–28. [CrossRef]
11. Jurišić, P.; Parunov, J.; Garbatov, Y. Comparative analysis based on two nonlinear corrosion models commonly used for prediction of structural degradation of oil tankers. *Trans. FAMENA* **2014**, *38*, 21–30.
12. Paik, J.K.; Lee, J.M.; Park, Y.I.; Hwang, J.S.; Kim, C.W. Time–variant ultimate longitudinal strength of corroded bulk carriers. *Mar. Struct.* **2003**, *16*, 567–600. [CrossRef]
13. Paik, J.K.; Thayamballi, A.K.; Park, Y.I.; Hwang, J.S. A time-dependent corrosion wastage model for bulk carrier structures. *Int. J. Marit. Eng.* **2003**, *145 Pt A2*, 61–87.
14. Ivošević, Š.; Meštrović, R.; Kovač, N. Probabilistic estimates of corrosion rate of fuel tank structures of aging bulk carriers. *Int. J. Naval Arch. Ocean Eng.* **2019**, *11*, 165–177. [CrossRef]
15. Ivošević, Š. Analysis of Ships Hull Structural Degradation. Ph.D. Thesis, University of Montenegro, Maritime Faculty Kotor, Kotor, Montenegro, 2012.
16. Ivošević, Š.; Meštrović, R.; Kovač, N. An approach to the probabilistic corrosion rate estimation model for inner bottom plates of bulk carriers. *Brodogradnja/Shipbuilding* **2017**, *68*, 57–70. [CrossRef]
17. Ivošević, Š.; Meštrović, R.; Kovač, N. A comparison of some multi-parameter distributions related to estimation of corrosion rate of aging bulk carriers. In Proceedings of the 7th International Conference on Marine Structures, Dubrovnik, Croatia, 6–8 May 2019; CRC Press: Boca Raton, FL, USA, 2019; pp. 403–410.
18. Qin, S.; Cui, W. Effect of corrosion models on the time-dependent reliability of steel plated elements. *Mar. Struct.* **2003**, *16*, 15–34. [CrossRef]
19. Paik, J.K.; Thayamballi, A.K. Ultimate strength of aging ships. *J. Eng. Marit. Environ.* **2002**, *1*, 57–77.
20. Guedes Soares, C.; Garbatov, Y. Reliability of maintained ship hulls subjected to corrosion. *J. Ship Res.* **1996**, *40*, 235–243.
21. Guedes Soares, C.; Garbatov, Y. Reliability of maintained ship hull girders subjected to corrosion and fatigue. *Struct. Saf.* **1998**, *20*, 201–219. [CrossRef]
22. Guedes Soares, C.; Garbatov, Y. Reliability of plate elements subjected to compressive loads and accounting for corrosion and repair. In *Structural Safety and Reliability*; Shiraishi, N., Shinozuka, M., Wen, Y.K., Eds.; Balkema: Rotterdam, The Netherlands, 1998; Volume 3, pp. 2013–2020.
23. Guedes Soares, C.; Garbatov, Y. Reliability of corrosion protected and maintained ship hulls subjected to corrosion and fatigue. *J. Ship Res.* **1998**, *43*, 65–78.
24. Guedes Soares, C.; Garbatov, Y. Reliability of maintained ship hulls subjected to corrosion and fatigue under combined loading. *J. Constr. Steel Res.* **1999**, *52*, 93–115. [CrossRef]
25. Katalinić, M.; Parunov, J. Uncertainties of Estimating Extreme SignificantWave Height for Engineering Applications Depending on the Approach and Fitting Technique—Adriatic Sea Case Study. *J. Mar. Sci. Eng.* **2019**, *8*, 259. [CrossRef]
26. Hays, W.L. *Statistics for the Social Sciences*, 2nd ed.; Rinehart & Winston: New York, NY, USA, 1973.
27. Stephens, M.A. Introduction to Kolmogorov (1033) on the empirical determination of a distribution. In *Breakthroughs in Statistics*; Springer: New York, NY, USA, 1992.
28. De Smith, M.J. *Statistical Analysis Handbook a Comprehensive Handbook of Statistical Concepts, Techniques and Software Tools*; The Winchelsea Press: London, UK, 2018.