Supplementary material

- Equations of motion

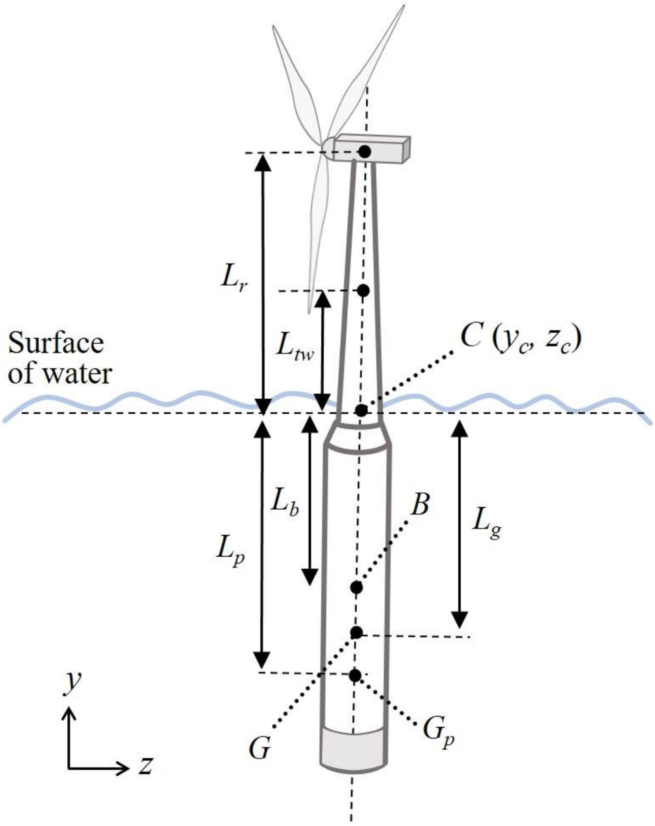

Fig. S1 Schematic diagram of the FOWT.

To solve the equations of motion using Lagrange's equations in Eq. (1), kinetic energy ,potential energy and other non-conservative forces (i.e. thrust force F_a; M_a, hydrodynamic loads F_h; M_h, and mooring force F_m; M_m, and damping force F_{dmp}) should be defined. The kinetic energy T of the system can be obtained as

$$
\begin{aligned}
& T = T_s + T_t, \\
& \text{where} \quad T_s = \tfrac{1}{2}m_{\text{tot}}(\dot{y}_c^2 + \dot{z}_c^2 + L_g^2\dot{\theta}^2 - 2\dot{y}_c\dot{\theta}L_g\cos\theta + 2\dot{z}_c\dot{\theta}L_g\sin\theta) + \tfrac{1}{2}I\dot{\theta}^2, \\
& \qquad\quad T_t = \frac{1}{2}m_t(\dot{y}_c^2 + \dot{z}_c^2 + u^2\dot{\theta}^2 + \dot{u}^2 - 2\dot{u}\dot{\theta}L_t + L_t^2\dot{\theta}^2 - 2\dot{y}_cL_t\cos\theta\dot{\theta} + 2\dot{z}_cL_t\sin\theta\dot{\theta} \\
& \qquad\qquad + 2\dot{y}_c\dot{u}\cos\theta - \dot{z}_c\dot{u}\sin\theta - 2\dot{y}_cu\sin\theta\dot{\theta} - 2\dot{z}_cu\cos\theta\dot{\theta}),
\end{aligned}
\tag{S1}
$$

Where m_{tot} is total mass of SFWT without the TMD; I is the total moment of inertia of the system about z-axis without the TMD; L_g is the distance from point C to point G; L_t is the distance from point C to the location where the TMD is installed. m_t is the mass of the TMD.

Next, the potential energy V of the system can be obtained as,

$$
\begin{aligned}
&V = V_s + V_t, \\
&\text{where} \quad V_s = \; m_{tw}g(z_c + L_{tw}\cos\theta) + m_r g(z_c + L_r\cos\theta) \\
&\qquad\qquad + m_p g(z_c - L_p\cos\theta) - \rho_w g V_s(z_c - L_b\cos\theta), \\
&\qquad V_t = m_t g(z_c - L_t\cos\theta - u\sin\theta) + \tfrac{1}{2}ku^2,
\end{aligned} \tag{S2}
$$

where m_{tw} is the mass of the tower; m_r is the mass of RNA; m_p is the mass of the platform; g is the gravitational acceleration; ρ_w is the density of seawater ; V_s is the volume of the submerged structure into the water; L_{tw} is the distance from point *C* to point G_{tw}; L_r is the distance from point *C* to point G_R; L_p is the distance from point *C* to point G_P; L_b is the distance from point *C* to point *B*; *k* is the spring coefficient of the TMD.

Because of the structure limitation, range of TMD distance u, should be restricted into the platform diameter (9.4m with reference to [1]). The range of the *u* is defined as, from -4 m to 4 m in this work. When the *u* reached to the constraints, virtual spring and damping coefficients are added to the spring and damping coefficients to prevent the TMD from oscillating out of the range.

When the wind blowing, thrust force is generated by the rotating motion of the SFWT's blades. The direction of the thrust force F_a is parallel to the *x*-axis. The magnitude of thrust force corresponding to blowing wind velocity is obtained by wind-thrust force curve [2] of the NREL 5-MW wind turbine.

The hydrodynamic loads acting on the SFWT were considered with Morison's equation and additional damping. The Morison's equation is able to capture the hydrodynamic loads when flow separation occurs around the structure. To improve accuracy of the model for SFWT, additional damping forces are incorporated into the model. Specifically, Jonkman et al. [1, 3] showed that the error of their model was decreased by adding extra viscous damping forces. Then, the hydrodynamic load can be obtained as

$$
\begin{aligned}
&F_{h,y} = \int f_{\mathrm{mrs}} dz + H_y \dot{y}_c, \\
&F_{h,z} = H_z \dot{z}_c, \\
&M_{h,\theta} = \int z f_{\mathrm{mrs}} dz + H_\theta \dot{\theta}, \\
&\text{where} \quad f_{\mathrm{mrs}} = \rho_w C_a A(\dot{v_w} - \dot{v_s}) + \tfrac{1}{2}\rho_w C_d D(v_w - v_s) \mid v_w - v_s \mid,
\end{aligned} \tag{S3}
$$

where f_{mrs} represents the force per unit length calculated from the Morison's equation; v_s is the local velocity of the submerged locations in SFWT; v_w is the velocity of water; *A* is the cross section area of platform; *D* is the diameter of the platform; C_a is the added mass coefficient; C_d is the viscous drag coefficient; H_x, H_y, and H_θ are included to consider the additional damping. The values of Morison's equation coefficients and additional damping forces are provided in [1].

Previous studies [1, 4] have shown that the loads applied to a structure by the mooring lines can be modeled as two different forces: 1) the combined gravitational and buoyant forces of the mooring lines and 2) the linear elastic forces in the mooring lines. These equivalent forces that are used to exert the mooring effect on the SFWT are obtained as

$$F_{m,y} = F_{m0,y} + k_{m,yy}y_c + k_{m,yz}z_c + k_{m,y\theta}\theta,$$

$$F_{m,z} = F_{m0,z} + k_{m,zy}y_c + k_{m,zz}z_c + k_{m,z\theta}\theta, \tag{S4}$$

$$F_{m,\theta} = F_{m0,\theta} + k_{m,\theta y}y_c + k_{m,\theta z}z_c + k_{m,\theta\theta}\theta,$$

where F_{m0} is the constant force corresponding to the gravitational and buoyant forces of the mooring lines, and k_m is the stiffness of the mooring lines. The values of the coefficients are provided in a previous work [1].

To check the stability of the platform, $S_{b,0}$ and $S_{b,T}$ are defined as

$$\begin{aligned}
&S_{b,0} = F_{b,0}l_{BG,0},\\
&S_{b,T} = F_{b,T}l_{BG,T},\\
&\text{where } F_{b,0} = \rho_w gAl_s,\\
&\qquad l_{BG,0} = l_{B,0} - l_{G,0} = \frac{l_s}{2} - l_{G,0}\\
&\qquad F_{b,T} = \rho_w gA(l_s + \triangle l_s),\\
&\qquad l_{BG,T} = l_{B,T} - l_{G,T} = \frac{l_s + \Delta l_s}{2} - l_{G,T}.
\end{aligned} \tag{S5}$$

Here, L_s is the submerged length of the platform in the absence of the TMD, and ΔL_s is the incremental of L_s due to the TMD. The lengths are shown in Fig S2. A is the cross section area of the FWOT platform. Because the submerged length is determined such that the weight is the same with the buoyancy force, the following relations should be satisfied:

$$\begin{aligned}
&F_{b,0} = \rho_w gAl_s \overset{!}{=} m_{\text{tot}}g,\\
&F_{b,T} = \rho_w gA(l_s + \triangle l_s) \overset{!}{=} (m_{\text{tot}} + m_t)g = (1+\gamma)m_{\text{tot}}g.
\end{aligned} \tag{S6}$$

In addition, the distances can be calculated as

$$\begin{aligned}
&l_{G,T} = \frac{m_{\text{tot}}l_{G,0} + m_t(l_s - x_3)}{m_{\text{tot}} + m_t},\\
&l_s = \frac{m_{\text{tot}}}{\rho_w A},\\
&\triangle\, l_s = \gamma l_s.
\end{aligned} \tag{S6}$$

Using Eqs. (S4–S6), the values of $S_{b,0}$ and $S_{b,T}$ can be determined for various mass ratios and TMD locations, as shown in Fig 3(b).

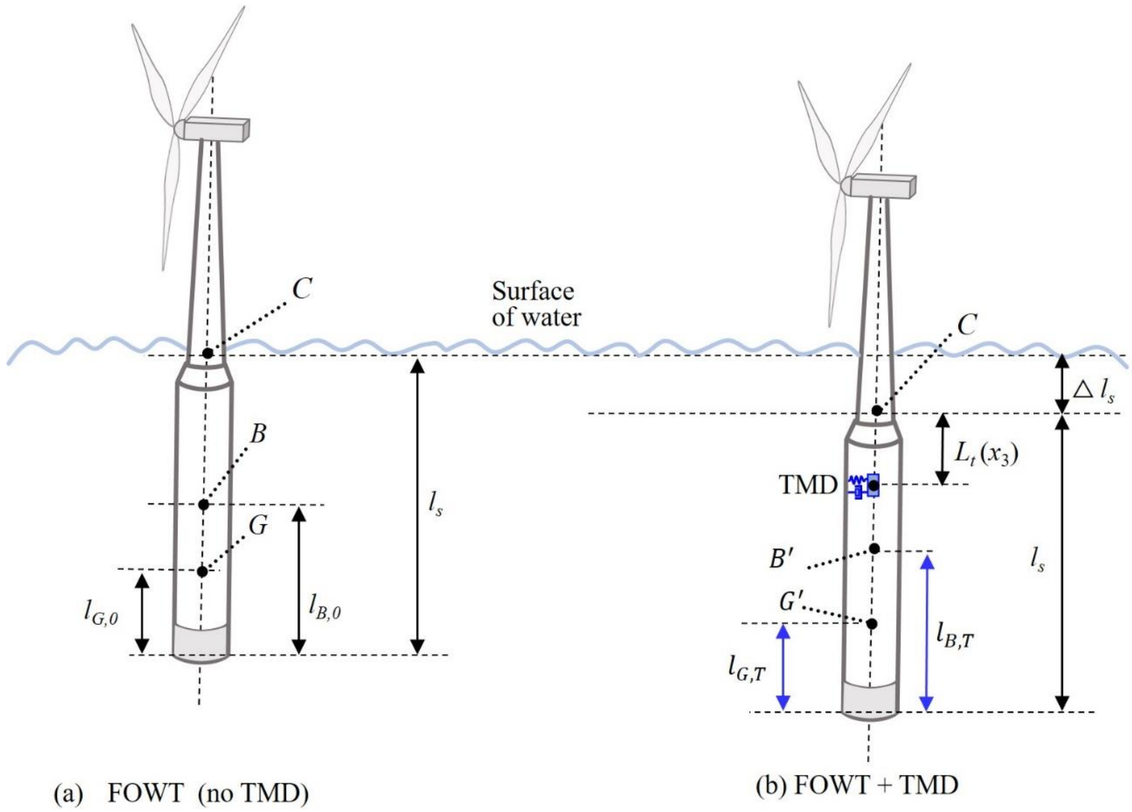

Fig. S2 Dimensions used to consider stability.

- Stochastic load conditions

To apply the condition of the marine environment, stochastic wind conditions are proposed for thrust force and hydrodynamic force. The stochastic wind speed V_h is obtained by using the Kaimal spectrum model recommended by the IEC [5]. The instantaneous wind speed can be obtained as,

$$V_h = V_{\text{avg}} + \sum_i \left[\sqrt{2\text{PSD}_\text{v}(f_i)\Delta f}\,\sin(2\pi f_i + \varphi_i)\right] \quad \text{(S7)}$$

where f_i is the frequency components of the wind speed and φ_i is the random phase of the random numbers in 0 to 2π. PSD_v is PSD of instantaneous wind speed. The kaimal spectrum model recommended by the IEC [5] for wind turbine class 2 and turbulence category 2 was used to create the PSD_v. Obtained stochastic wind velocity and corresponding thrust force is shown is Fig. S2 (a) and (b).

The stochastic winds create waves that induced a hydrodynamic load on the submerged platform of the SFWT. The velocity and acceleration of the water caused by wind was predicted with linear wave theory [6]. The velocity of the water was obtained by

$$v_w = \sum_i [2\pi f_i e^{k_i z_c} \sqrt{2\text{PSD}_w(f_i)\Delta f} \sin(2\pi f_i t - k_i y)], \quad \text{(S8)}$$

where f_i is the frequency of the wave and k_i is the wavenumber, which can be obtained as $k_i = (2\pi f_i)^2/g$. An empirical equation on the PSD$_w$ of the wave elevation proposed by Pierson and Moskowitz [7] is used. This equation can be obtained as

$$\mathrm{PSD}_w(f) = \frac{\alpha g^2}{32\pi^5 f^5}\exp[-\beta(\frac{g}{2\pi f V_{19.4}})^4], \tag{S9}$$

where α is 0.008, β is 0.74, g is the gravitational acceleration; $V_{19.4}$ is the wind speed measured at 19.4m height; and f is frequency of the waves. ; $V_{19.4}$ is calculated by using wind velocity gradient (power law with exponent of 0.12). Wave velocity and acceleration obtained by linear theory at 10 m depth from still water level is shown in Fig. S2 (c) and (d).

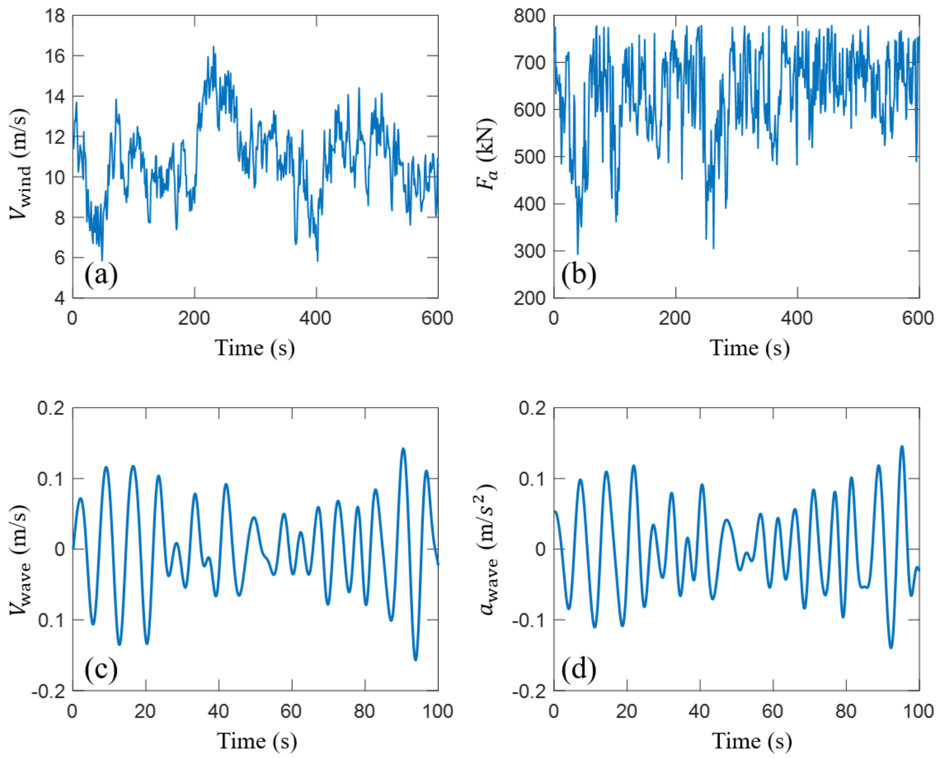

Fig. S2 Stochastic wind and wave. (a) and (b) show the stochastic wind velocity when $V_{avg} = 11$ m/s and corresponding thrust force acting on the hub of the SFWT. (c) and (d) show the wave velocity and acceleration caused by stochastic wind speed at 10 m depth form mean sea water level.

- Extension of the optimization approach

The proposed approach can also be used when four or more parameters have to be considered for optimization. For example, when four parameters x_{1-4} are used, the cost function can be represented as $h(x_1, x_2, x_3, x_4)$. Then, the optimization can be realized with the calculation structure shown in Fig. S1. Note again that the mathematical properties of the cost function (described in Section 3.3) should be satisfied to use this approach.

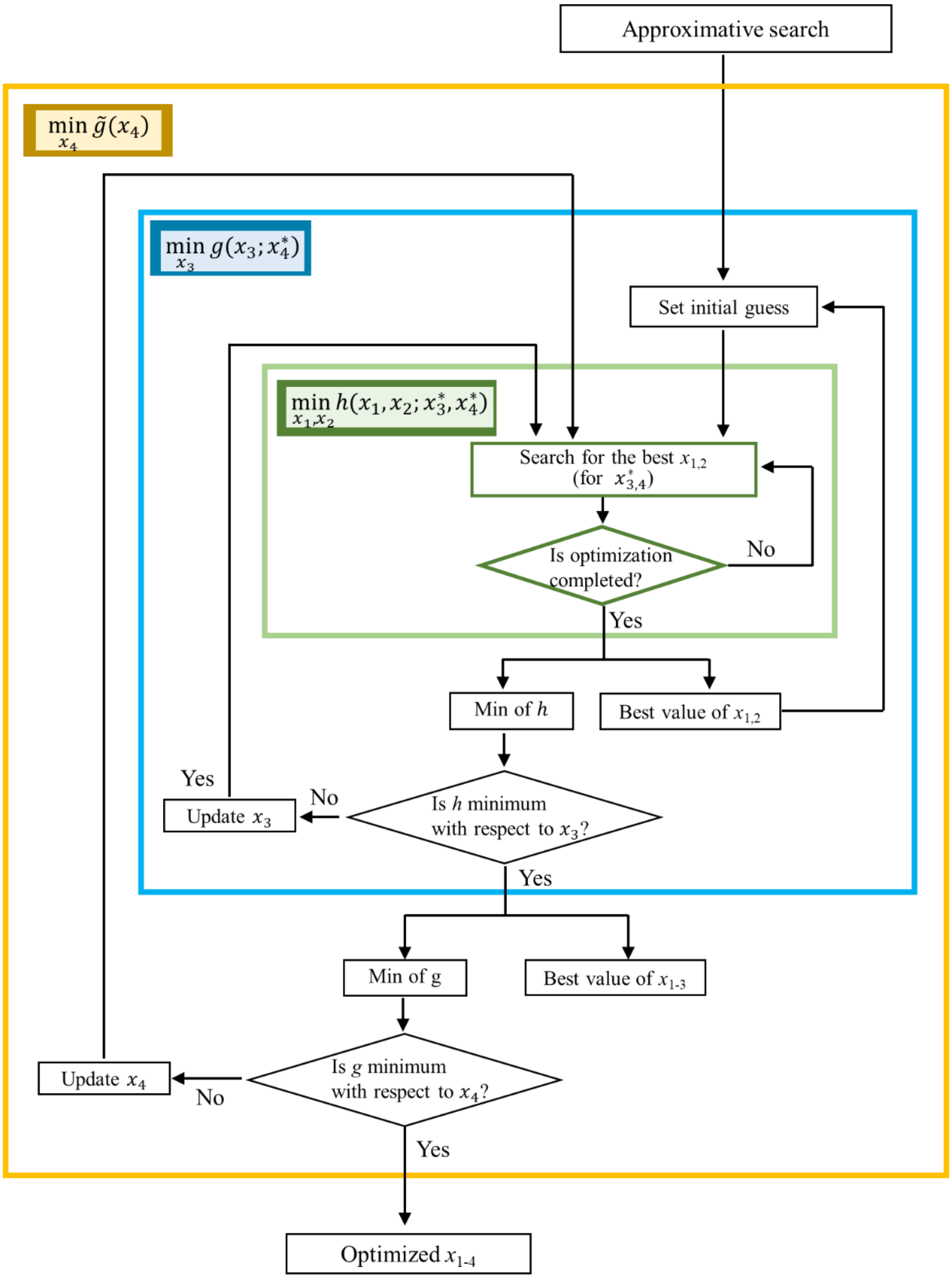

Figure S3. Optimization flowchart with four parameters. Here, $g(x_3, x_4) = \min_{x_1,x_2} h(x_1, x_2; x_3, x_4)$ and $\tilde{g}(x_4) = \min_{x_1,x_2,x_3} h(x_1, x_2, x_3; x_4)$

References

1. Jonkman, J., *Definition of the floating system of phase IV of OC3*. 2010: NREL.

2. Wan, L., et al., *Comparative experimental study of the survivability of a combined wind and wave energy converter in two testing facilities.* Ocean Engineering, 2016. **111**: p. 82-94.
3. J.Jonkman, *Offshore code comparison collaboration within IEA wind task 23: phase lV Results regarding floating wind turbine modeling*, in *European Wind Energy Conference*. 2010: Warsaw, Poland.
4. Jonkman, J., et al., *Definition of a 5-MW reference wind turbine for offshore system development*. 2009: NREL.
5. IEC61400-1, *Wind turbines –Part 1: Design requirements.3rd ed*. 2005, IEC.
6. Krogstad, H.E. and Ø.A. Arntsen, *Linear wave theory : PART A -Regular waves* 2000, Norwegian University of Science and Technology.
7. Jr., W.J. Pierson and L. Moskowitz, *A proposed spectral form for fully developed wind seas based on the similarity theory of S. A. Kitaigorodskii.* Journal of Geophysical Research, 1964. **69**(24): p. 5181-5190.