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Effects of Oceanic Turbulence on Orbital Angular Momenta of Optical Communications

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Abstract: The propagation properties of Laguerre-Gaussian beams in oceanic turbulence are investigated for both single-photon and biphoton cases. For single-photon communication, the channel capacity and trace distance are employed, both of which effectively reveal the communication performance via different viewpoints. For the biphoton case, we consider distributions of quantum resources including entanglement and quantum coherence. Turbulence conditions with a larger inner-scale and anisotropic factors, higher dissipation rate of kinetic energy, lower dissipation rate of the mean-squared temperature, and lower temperature-salinity contribution ratio combined with longer wavelength and an appropriate range of optimal beam width are beneficial to communication performances. Our results provide theoretical significance to improve the orbital-angular-momentum communication via oceanic turbulence.

Keywords: oceanic turbulence; orbital angular momentum; optical communication; quantum information

1. Introduction

Optical vortex beams, carrying orbital angular momenta (OAM), may find potential applications in wireless communication systems [1,2]. In addition to polarization of photons, OAM provides a new degree of freedom for encoding non-classical information. This encoding manner may possess more security [3] and higher capacity [4]. These years, wireless OAM communication has received intensive attention and the propagation media gradually extends from atmosphere [5,6] to ocean [7–9], so as to meet the growing demand of underwater submarine communication or ocean exploration [10–13]. However, the disturbance of quantumness will be unavoidable when the encoded photons with OAM transmit through oceanic turbulence [14].

Investigations on oceanic turbulence properties and their influences on quantum states propagation are useful to improve oceanic communication performances. For vortex wave propagation in turbulent ocean, the OAM mode is susceptible to wave-front distortions caused by the random fluctuations of the refractive index of turbulent seawater [15]. Baghdady et al. [16] explored the effects of turbulence on the propagation of vortex beams with optical vortices and gained considerable interest due to the possibility of implementing quantum optical communication links with OAM modes. Zhang et al. [17] studied the influence of anisotropic turbulence on the OAM modes of vortex beams in the ocean. Li et al. [18] found the effects of oceanic turbulence on the evolution of channel capacity performance by studying quantitatively in a series of numerical simulations.

In quantum information science, it is critical to understand the behaviors of nonclassically correlated photons traveling in the turbulent media since the quantumness contained in the encoded states are usually fragile and can be easily destroyed. Quantum entanglement [19], a fundamental quantum resource in quantum information science, is a typical kind of quantumness which is usually

considered. For instance, the decay of entanglement for photonic OAM qubit states in the turbulent atmosphere has been reported [20] via concurrence [21]. However, it has been proved that entanglement is not the unique resource which can be exploited. Quantum coherence [22] has been considered as a new resource responsible for certain quantum communication protocols. On the other hand, oceanic turbulence is mainly induced by the temperature and salinity fluctuations, distinguishing from the case of atmospheric turbulence, which is mainly caused by temperature or humidity fluctuations. Therefore, the understanding of propagation properties for quantum resources (such as entanglement and quantum coherence) of OAM photons in turbulent ocean is crucial. To the best of our knowledge, this issue has not been addressed yet. In this work, we consider the influences of oceanic turbulence on the channel capacity and trace distance in the single-photon communication case and entanglement and quantum coherence in the biphoton communication case.

The paper is organized as follows. In Section 2, we give a typical model as Laguerre-Gaussian (LG) beams propagating through turbulent ocean. Both the single-photon communication and biphoton quantum communication are considered. In Section 3, we numerically discuss the effects of turbulence on the channel capacity and trace distance for the single-photon case as well as entanglement and quantum coherence for the biphoton case. Conclusions are presented in Section 4.

2. Laguerre-Gaussian Beams Propagating through Turbulent Ocean

The OAM module is related to the spatial distribution of wave function. There are some kinds of vortex beams, such as LG beams [15,18], Bessel-Gaussian Schell beams [6], Hermite-Gaussian beam [17] and Lommel-Gaussian beam [7], having been used for OAM modes in recent years. In the following, we choose LG beams [23] which are good candidates for encoding and division-multiplexing of OAM modes. The parameter values used for simulation are selected according to Ref. [24], where some typical values of oceanic turbulence were given. Unless otherwise mentioned, the adopted parameters are $\chi = 10^{-7} \text{ K}^2/\text{s}$ (K is the unit of thermodynamic temperature, kelvin), $\varepsilon = 10^{-6} \text{ m}^2/\text{s}^3$, $\eta = 1 \text{ mm}$, $\gamma = -2$, $\zeta = 2$, $z = 10 \text{ m}$, $p_0 = 0$, $l_0 = 1$, $\kappa = 1$, $\vartheta = \pi/2$, $\omega_0 = 0.01 \text{ m}$ and $\lambda = 532 \text{ nm}$.

2.1. Single-Photon Communication Case

The oceanic turbulence on the transmission of single photons has been modeled as random refractive index inhomogeneities, the effect of which is a phase aberration on the wave function. As is shown in Figure 1, we consider that a LG vortex beam with well-defined OAM, emitted by a source, propagates through the turbulent ocean before being received by a detector on the optical axis. The complex amplitude (wave function in the cylindrical coordinates) of a LG beam with its initial OAM (azimuthal) quantum number l_0 and radial quantum number p_0 can be expressed as [23]

$$U_{p_0}^{|l_0|}(r, \phi, z) = R_{p_0}^{|l_0|}(r, z) \frac{\exp(i l_0 \phi)}{\sqrt{2\pi}}, \quad (1)$$

where the radial part of the LG beam can be expressed as

$$R_{p_0}^{|l_0|}(r, z) = \frac{2}{\omega(z)} \sqrt{\frac{p_0!}{(p_0 + |l_0|)!}} \left[\frac{\sqrt{2}r}{\omega(z)} \right]^{|l_0|} \exp \left[-\frac{r^2}{\omega(z)^2} \right] L_{p_0}^{|l_0|} \left[\frac{2r^2}{\omega(z)^2} \right] \times \exp \left[\frac{ikr^2}{2R(z)} \right] \exp \left[-i(2p_0 + |l_0| + 1) \tan^{-1} \left(\frac{z}{z_R} \right) \right], \quad (2)$$

where $\omega(z) = \omega_0[1 + (z/z_R)^2]^{1/2}$ is the spot size with ω_0 the beam waist at $z = 0$, $z_R = \frac{1}{2}k\omega_0^2$ the Rayleigh range and $k = 2\pi/\lambda$ the wavenumber. Besides, the parameter $R(z) = z[1 + (z/z_R)^2]$ is the radius of wavefront curvature, and $L_{p_0}^{|l_0|}(x)$ is the generalized Laguerre polynomial given by

$$L_{p_0}^{|l_0|}(x) = \sum_{m=0}^{p_0} (-1)^m \frac{(|l_0| + p_0)!}{(p_0 - m)! (|l_0| + m)! m!} x^m. \quad (3)$$

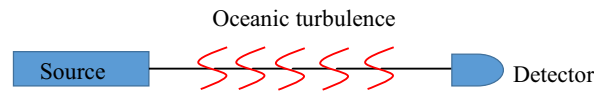


Figure 1. Sketch of single-photon communication: a LG beam, produced by a source, propagates via an oceanic channel with turbulence and is received by a detector at certain distance.

When the LG beam propagates in turbulent ocean, its complex amplitude inevitably suffers from phase perturbation caused by the turbulence. With the help of the extended Huygens–Fresnel integral [25], the cross spectral density function $W(r, \phi, \phi', z)$ at z plane is expressed as [26]

$$W(r, \phi, \phi', z) = U_{p_0}^{|l_0|}(r, \phi, z) U_{p_0}^{|l_0|}(r, \phi', z) \exp\left[-\frac{1}{2} D_S(r, r', z)\right], \quad (4)$$

where $D_S(r, r', z)$ denotes the wave structure function in anisotropic oceanic turbulence. In the case of spherical wave, $D_S(r, r', z)$ is given by [27]

$$D_S(r, r', z) = \frac{2|r - r'|^2}{\rho_c^2}, \quad (5)$$

where ρ_c is the spatial coherence length which can be generally expressed as follows [28]

$$\rho_c^{-2} = \frac{1}{3} k^2 \pi^2 z \int_0^\infty \kappa^3 \Phi(\kappa) d\kappa. \quad (6)$$

Here $\kappa = \sqrt{\kappa_x^2 + \kappa_y^2 + \kappa_z^2} = \sqrt{\kappa_r^2 + \kappa_z^2}$ is the magnitude of the spatial frequency, and $\Phi(\kappa)$ denotes the spatial power spectrum of the refractive-index fluctuations of oceanic turbulence [29]:

$$\begin{aligned} \Phi(\kappa) = & 0.388 \times 10^{-8} C_n^2 \zeta^2 \kappa_\zeta^{-11/3} [1 + 2.35(\kappa_\zeta \eta)^{2/3}] \\ & \times (e^{-A_1 \delta} - 2\tau^{-1} e^{-A_2 \delta} + \tau^{-2} e^{-A_3 \delta}), \end{aligned} \quad (7)$$

where $\kappa_\zeta = \sqrt{\zeta^2 \kappa_r^2 + \kappa_z^2}$, η and ζ are inner-scale factor and anisotropic factor of oceanic turbulence respectively, and τ is the balance parameter that determines the relative strength of temperature and salinity varying from 0 to -5 , with $\tau \rightarrow 0$ corresponding to the salinity-driven turbulence and $\tau \rightarrow -5$ corresponding to the temperature-driven turbulence. The constants A_1 , A_2 and A_3 are 1.863×10^{-2} , 9.41×10^{-3} and 1.9×10^{-4} in turn, and $\delta = 8.284(\kappa_\zeta \eta)^{4/3} + 12.978(\kappa_\zeta \eta)^2$. The equivalent temperature structure parameter C_n^2 , in units of $K^2 m^{-2/3}$ [27], is a measure of the strength of the temperature fluctuation in turbulent ocean. Over short time intervals at a fixed propagation distance and constant depth, it may be reasonable to assume that C_n^2 is essentially constant. The structure parameter C_n^2 is related to the rate of dissipation of mean-squared temperature χ and the rate of dissipation of turbulent kinetic energy ε by

$$C_n^2 = 10^{-8} \chi \varepsilon^{-1/3}. \quad (8)$$

In clean ocean water, χ is in the range of $10^{-4} K^2/s$ to $10^{-10} K^2/s$ (surface water and deep water, respectively); ε ranges from $10^{-1} m^2/s^3$ at the ocean surface to $10^{-10} m^2/s^3$ in the deep water. According to the Markov approximation in the anisotropic random media, κ and κ_ζ can be further simplified as $\kappa = \kappa_r$ and $\kappa_\zeta = \zeta \kappa_r$ since circular symmetry is maintained in the plane orthogonal to

the propagation direction z . Substituting Equation (7) into Equation (6) yields the spatial coherence length of the spherical wave propagation in turbulent ocean:

$$\rho_c^{-2} = 8.659 \times 10^{-8} k^2 (\epsilon \eta)^{-1/3} \zeta^{-2} \chi z (1 - 2.605 \tau^{-1} + 7.013 \tau^{-2}). \quad (9)$$

Then the detecting probability of azimuthal quantum number l with respect to initial l_0 is given by

$$P(l|l_0) = \frac{1}{2\pi} \int_0^\infty |R_{p_0}^{l_0}(r, z)|^2 r dr \int_0^{2\pi} e^{-i(l-l_0)\Delta\phi} \exp\left[-\frac{4r^2 \sin^2 \frac{\Delta\phi}{2}}{\rho_c^2}\right] d\Delta\phi. \quad (10)$$

2.2. Biphoton Communication Case

In this subsection, we turn to consider the case of quantum communication with biphoton entangled states. The coherence of OAM entanglement can be destroyed by the beam wandering, wavefront distortions, and scintillation when the OAM biphoton propagate across turbulence [30,31]. This is one of the biggest challenge for realizing quantum communication based on OAM. Therefore, it is more imperative to describe the OAM entanglement state in turbulence so the OAM modes can be successfully applied to the quantum communication.

As is shown in Figure 2, two LG beams, carrying pairs of entangled photons, are distributing to two detectors at opposite directions, in which case the two channels are independent. The entangled LG beams have the same beam waist ω_0 and radial quantum number $p_0 = 0$ but possess opposite azimuthal quantum numbers l_0 and $-l_0$, respectively. The initial state of the photon pair is prepared in an Werner-like state as usually considered in quantum information processing:

$$\hat{\rho}^{(0)} = \frac{1-\gamma}{4} I + \gamma |\Psi_0\rangle \langle \Psi_0|, \quad (11)$$

where $0 \leq \gamma \leq 1$ characterizes the purity of the initial state, I is the identity matrix and the Bell-like state $|\Psi_0\rangle$ reads as

$$|\Psi_0\rangle = \cos\left(\frac{\vartheta}{2}\right) |l_0, -l_0\rangle + e^{i\phi} \sin\left(\frac{\vartheta}{2}\right) |-l_0, l_0\rangle, \quad (12)$$

with $0 \leq \vartheta \leq \pi$ and $0 \leq \phi < 2\pi$. In realistic experiments, quantum states are usually mixed with an ensemble of pure states, not definitely prepared. The purity parameter γ is crucial for preparation procedure, i.e., it means perfect preparation for $\gamma = 1$ (pure Bell state) and failure preparation for $0 \leq \gamma \leq 1/(1 + \sin(\vartheta))$ since no entanglement is present. Recent researches suggest that there is still quantum coherence in this non-entangled states as long as $\gamma \neq 0$ and $\vartheta \neq 0, \pi$. Quantum coherence has been considered as another quantum resource as compared to entanglement. For detail, please refer to the recent review paper Ref. [22] and reference therein.



Figure 2. Sketch of biphoton communication: a pair of OAM-entangled photons, produced by the source, is sent via independent oceanic channels with turbulence, and finally received by two detectors at certain distance.

The influence of the oceanic turbulence on the entangled biphoton can be treated as a linear map $\hat{M}_i$ with $i = 1, 2$, in terms of which the received state at the detectors can be expressed as

$$\hat{\rho} = (\hat{M}_1 \otimes \hat{M}_2) \hat{\rho}^{(0)}. \quad (13)$$

For short distance, one has $M_1 = M_2 = M$ and elements of M given by [20,32]

$$M_{\tilde{l},\tilde{l}'}^{l,l'} = \frac{\delta_{l-l',\tilde{l}-\tilde{l}'}}{2\pi} \int_0^\infty |R_0^{l_0}|(r,z)|^2 r dr \int_0^{2\pi} e^{i(l-\tilde{l})\Delta\phi} \exp\left[-\frac{Ds(\mathbf{r},\mathbf{r}',z)}{2}\right] d\Delta\phi. \quad (14)$$

According to Equation (14), the post-selected state in the truncated basis $\{|l_0, l_0\rangle, |l_0, -l_0\rangle, |-l_0, l_0\rangle, |-l_0, -l_0\rangle\}$ can be written in an X form as

$$\rho = \begin{pmatrix} \rho_{11} & & & \rho_{14} \\ & \rho_{22} & \rho_{23} & \\ & \rho_{32} & \rho_{33} & \\ \rho_{41} & & & \rho_{44} \end{pmatrix}, \quad (15)$$

with the normalized density matrix elements

$$\begin{aligned} \rho_{11} &= \left(\mu^2 \rho_{11}^{(0)} + \mu\nu \rho_{22}^{(0)} + \mu\nu \rho_{33}^{(0)} + \nu^2 \rho_{44}^{(0)} \right) / (\mu + \nu)^2, \\ \rho_{22} &= \left(\mu\nu \rho_{11}^{(0)} + \mu^2 \rho_{22}^{(0)} + \nu^2 \rho_{33}^{(0)} + \mu\nu \rho_{44}^{(0)} \right) / (\mu + \nu)^2, \\ \rho_{33} &= \left(\mu\nu \rho_{11}^{(0)} + \nu^2 \rho_{22}^{(0)} + \mu^2 \rho_{33}^{(0)} + \mu\nu \rho_{44}^{(0)} \right) / (\mu + \nu)^2, \\ \rho_{44} &= \left(\nu^2 \rho_{11}^{(0)} + \mu\nu \rho_{22}^{(0)} + \mu\nu \rho_{33}^{(0)} + \mu^2 \rho_{44}^{(0)} \right) / (\mu + \nu)^2, \\ \rho_{14} &= \mu^2 \rho_{14}^{(0)} / (\mu + \nu)^2, \rho_{41} = \mu^2 \rho_{41}^{(0)} / (\mu + \nu)^2, \\ \rho_{23} &= \mu^2 \rho_{23}^{(0)} / (\mu + \nu)^2, \rho_{32} = \mu^2 \rho_{32}^{(0)} / (\mu + \nu)^2. \end{aligned} \quad (16)$$

and

$$\begin{aligned} \mu &= M_{l_0, l_0}^{l_0, l_0} = M_{-l_0, -l_0}^{-l_0, -l_0} = M_{-l_0, l_0}^{-l_0, l_0} = M_{l_0, -l_0}^{l_0, -l_0}, \\ \nu &= M_{l_0, l_0}^{-l_0, -l_0} = M_{-l_0, -l_0}^{l_0, l_0}. \end{aligned} \quad (17)$$

3. Results and Discussions

3.1. Channel Capacity and Trace Distance in Single-Photon Communication

The channel capacity [33] is related to the maximal reliable information transfer through a (noisy) communication channel. First, we briefly outline the basic concept of channel capacity defined by

$$C = \max\{H(x) - H(x|y)\}, \quad (18)$$

with

$$H(x) = - \sum_{x_i} P(x_i) \log_2 P(x_i), \quad (19)$$

the Shannon entropy of the source, and

$$H(x|y) = - \sum_{y_j} \sum_{x_i} P(x_i, y_j) \log_2 P(x_i|y_j), \quad (20)$$

the conditional entropy, where $P(x_i)$ is the probability of the transmitted signals $\{x_i\}$, $P(x_i, y_j)$ is the joint probability of $\{x_i\}$ and received signals $\{y_j\}$, and $P(x_i|y_j)$ donates the conditional probability of $\{x_i\}$ given $\{y_j\}$.

We consider the input LG mode with an initial azimuthal quantum number in the range $l_0 = -L, \dots, L$. The maximum in Equation (18) is reached when the probability for each mode is $P(x_i) = 1/(2L+1)$, in terms of which one has the entropy $H(x) = \log_2(2L+1)$ and the conditional entropy $H(x|y) = - \sum_{ij} P(x_i)P(y_j|x_i)[\log_2 P(y_j|x_i) - \log_2 \sum_i P(y_j|x_i)]$. The different values of the

OAM quantum number can be distinguished by rotations and sorting schemes [34]. In the presence of turbulence, the OAM of photons received by the detector will not remain in the initial range $-L, \dots, L$ due to crosstalk. Detected photons with y_j can then be labeled as two types: the “reliable” photons with OAM eigenvalues remaining in the initial range $|l| \leq L$, and a “lost” photon with OAM eigenvalues $|l| > L$. According to Ref. [5], the conditional probability of detecting a “reliable” photon is simply Equation (20) and that of a “lost” photon is given by $1 - \sum_{|l| \leq L} P(l|l_0)$.

To illustrate the effects of crosstalk, an alternative tool termed as trace distance [35,36] can be employed, which precisely measures the distinguishability of states [36]. Trace distance is a natural metric on the space of physical states and gives the achievable upper bound on the distinguishability between probability distributions arising from measurements performed on the two states. The trace distance of two distributions $\{P(x_i)\}$ and $\{P(y_i)\}$ is defined by

$$D = \frac{1}{2} \sum_i |P(x_i) - P(y_i)|. \quad (21)$$

In Figure 3a, we plot the channel capacity as a function of the propagation distance z with different L . The channel capacity decays fast in a short range and then the decay slows down. Comparing different curves in the figure, we find that channel capacity is always higher in the propagation process for larger L . This is clear that for larger L the initial value of channel capacity at $z = 0$ is enhanced since quantum states are encoded in higher dimensions $(2L + 1)$. Figure 3b gives a similar plot of the trace distance. The trace distance is increasing from its initial value (zero) at $z = 0$ where there is no crosstalk before propagation. As the propagation distance z increases, the trace distance grows fast in a short propagation distance. The two measures both effectively illustrate that state transfers with higher OAM numbers are more robust against to the oceanic turbulence, but rather from different points of view.

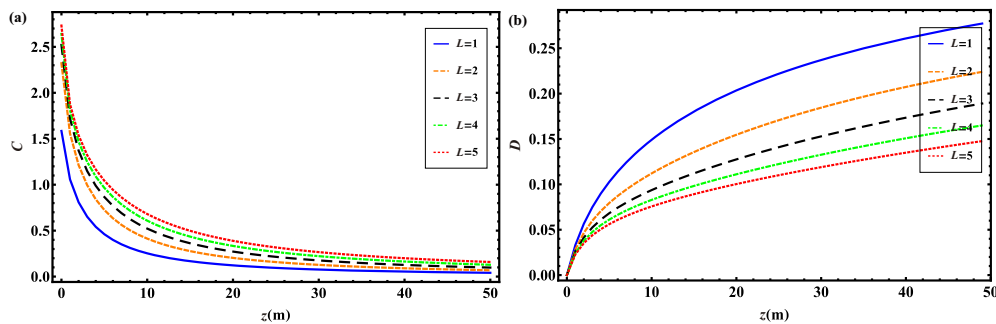


Figure 3. (a) Channel capacity and (b) trace distance as functions of propagation distance z with different L .

Since oceanic turbulence is closely related to dissipation rates χ of the mean-squared temperature and dissipation rates ε of kinetic energy per unit mass of fluid, we plot channel capacity and trace distance as functions of χ with different ε in Figure 4. The total OAM channel capacity of LG beams reduces if χ decreases or ε increases and the trend of the trace distance is opposite. The higher values of χ , expressed the strength of the small scale temperature gradient, means a stronger turbulent ocean. ε is closely associated with the turbulence scale. A smaller ε means a large turbulence scale, and at the same time the spreading of the LG beams will be wider [37]. So when the water has a larger χ and a smaller ε , it means that both of them contribute significantly to the stronger turbulence and larger beam wander. This finding is beneficial for some applications operating in relatively deep ocean, such as submarine communication and ocean detection.

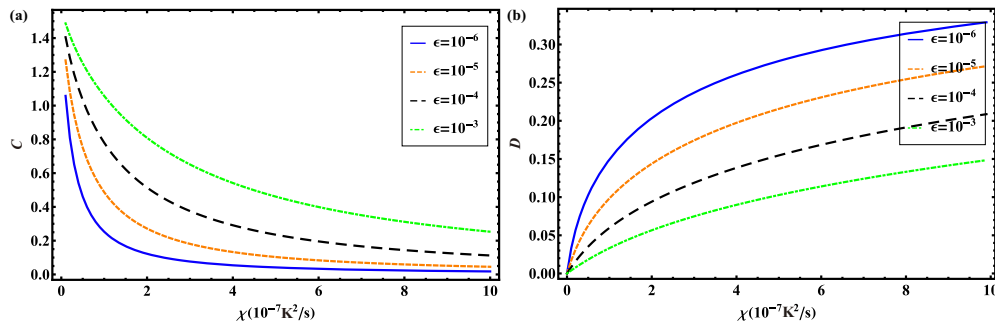


Figure 4. (a) Channel capacity and (b) trace distance as functions of dissipation rate of mean-squared temperature χ with different dissipation rate of kinetic energy per unit mass of fluid ϵ .

Since both temperature and salinity variations contribute to refraction fluctuations of oceanic turbulence, the contribution ratio τ of temperature to salinity is an important factor determining the turbulence effect. Figure 5 displays the effect of τ with different inner-scale factor η . The channel capacity decreases with the increment of τ and the variation trend with η is contrary to that with τ . From Figure 5a, with η uniformly increased, the enhancement of channel capacity becomes less and less apparent. The trace distance increases with the increment of τ and the reduction of η . As for a small τ approaching to -5 , the temperature induced changes dominates, while as for a large τ approaching to 0, the salinity induced changes dominates. The results reveal that LG beams have better immunity to the adverse effects of temperature variation than that of salinity variation.

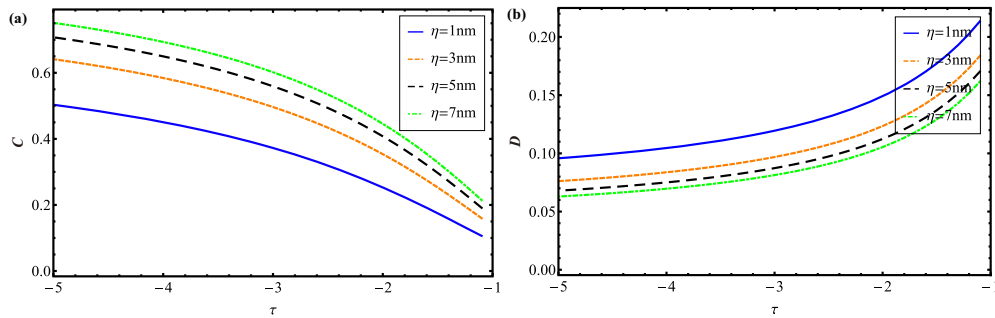


Figure 5. (a) Channel capacity and (b) trace distance as functions of contribution ratio of temperature and salinity to refractive index fluctuation τ with different inner-scale factor η .

3.2. Quantum Entanglement and Quantum Coherence in Biphoton Communication

In this subsection, we turn to study the behaviors of nonclassical information remained in the output state. The first measure of nonclassical information we employ is entanglement. To quantify the degree of entanglement, a measure termed as concurrence *Con* [21] has been put forward, which varies from *Con* = 0 for a separable state to *Con* = 1 for a maximally entangled state. For a two-qubit X state ρ in Equation (15), the concurrence has an explicit expression as

$$Con = 2 \max\{0, |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}, |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}\}. \quad (22)$$

We plot the concurrence *Con* as a function of the propagation distance z and the initial state parameter ϑ in Figure 6a. It is seen that the concurrence decays in a non-asymptotical manner with the increase of the propagation distance z to vanish. This phenomenon is termed as entangled sudden death [38]. As shown by the red dotted curves, the vanishing distance increases rapidly as ϑ increases to $\pi/2$, and then it decreases fast as ϑ increases from $\pi/2$ to π , exhibiting a symmetric distribution as well as a peak at $\vartheta = \pi/2$. Besides, the influence of purity γ is demonstrated in Figure 6b. The vanishing distance also decreases non-asymptotically as the initial state purity γ decreases from 1 to $1/3$ for $\vartheta = \pi/2$. This is due to the fact that the initial state is non-entangled for $0 \leq \gamma \leq 1/3$ and $\vartheta = \pi/2$.

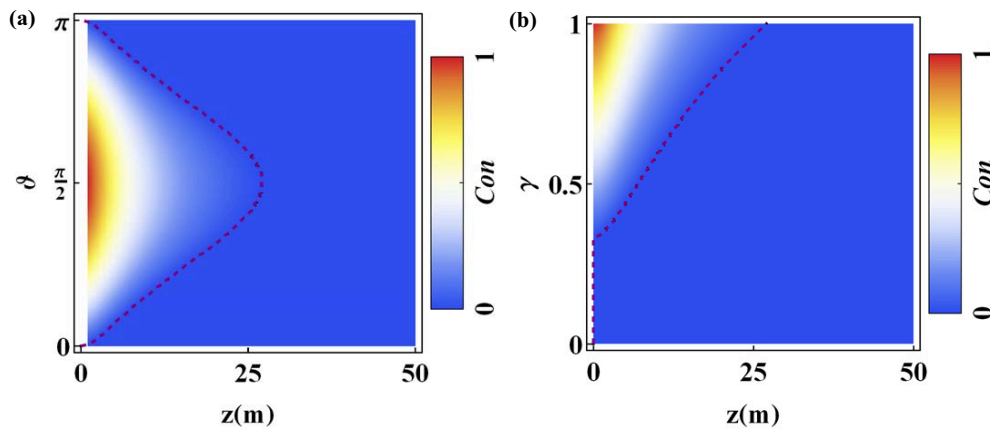


Figure 6. Concurrence as functions of (a) z and ϑ and of (b) z and γ . The red dotted curves are vanishing distance as functions of (a) ϑ and of (b) γ .

The second quantity of quantum information is the quantum coherence. The concept of coherence is familiar to us, since it plays crucial roles in various phenomena, such as quantum optics [39], quantum information [35] and biological systems [40]. However, coherence has not been considered as a physical resource until very recently [22,41]. To quantify the degree of coherence, a measure defined as trace-distance coherence Coh is used. Trace distance measure of coherence is a strong monotone for the X state ρ in Equation (15), which is equivalent to the l_1 norm of coherence [41] with

$$Coh = 2(|\rho_{23}| + |\rho_{14}|). \quad (23)$$

We also plot the behaviors of coherence Coh in Figure 7. As clearly shown for ϑ near $\pi/2$, coherence decreases more fast at the distance between 0 to 20 m than that at the longer propagation distance. Except $\vartheta = 0, \pi$, the coherence is always non-vanishing, which means the quantum coherence is more robust than entanglement which exhibits sudden vanishing. To further illustrate this, in Figure 8, we compare the propagations of concurrence and coherence for different OAM modes l_0 . It is clear shown that entanglement is always less than coherence and will expedience sudden vanishing (at 27 m for $l_0 = 1$), although the vanishing distance can be prolonged with increasing OAM number. In general, quantum resources encoded by larger OAM numbers can be better preserved via turbulent channels.

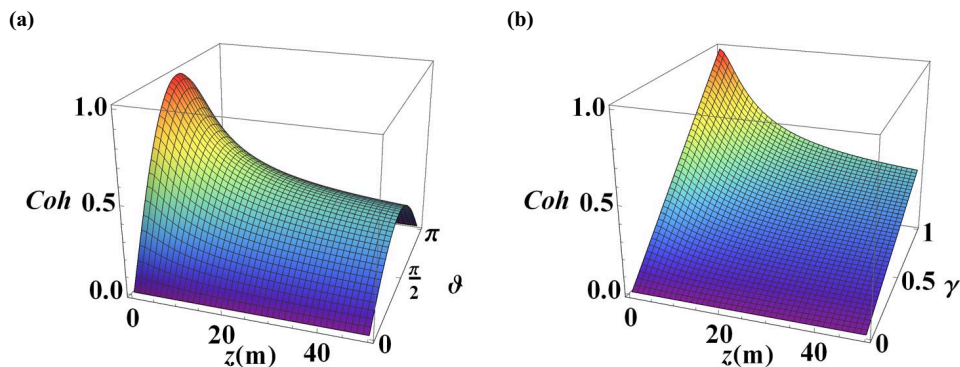


Figure 7. Coherence as a function of (a) z and ϑ and of (b) z and γ .

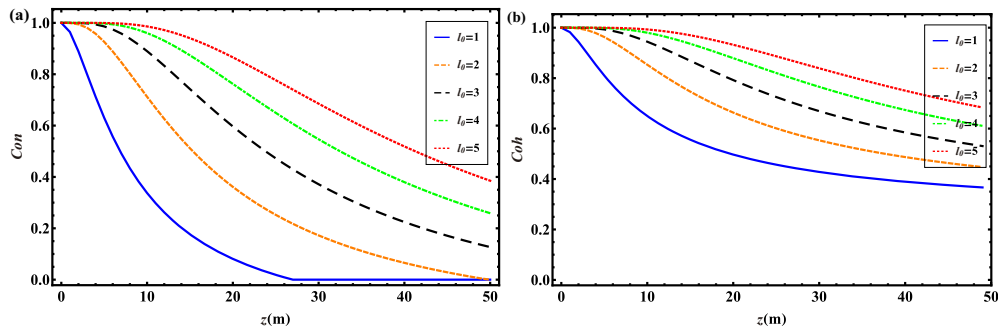


Figure 8. (a) Concurrence and (b) coherence as functions of propagation distance z with different initial OAM azimuthal quantum number l_0 .

At last, the effects of temperature structure parameter C_n^2 and wavelength on the concurrence and coherence are displayed in Figures 9 and 10, respectively. For small C_n^2 such as reaching the order of 10^{-15} , the behaviors of concurrence and coherence are similar. As C_n^2 grows to the order of 10^{-13} , the dynamics of entanglement exhibits faster decay than that of coherence. Since C_n^2 is determined by the rate of dissipation of mean-squared temperature χ and the rate of dissipation of turbulent kinetic energy ε given in Equation (8), we may conclude that quantum resources are significantly influenced by the ocean-turbulent changes rates as well. From Figure 10, we can see that the choice of longer wavelength in transmission window is superior, especially when the beam waist is comparable to or larger than the optimum waist width. It is interesting to observe that the amount of two different resources can be maximized in a short range of optimal beam width which is almost independent of the wavelength.

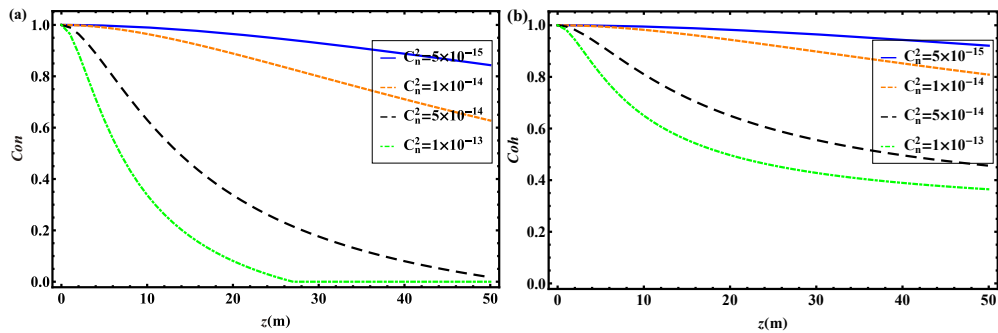


Figure 9. (a) Concurrence and (b) coherence as functions of propagation distance z with different equivalent temperature structure parameter C_n^2 .

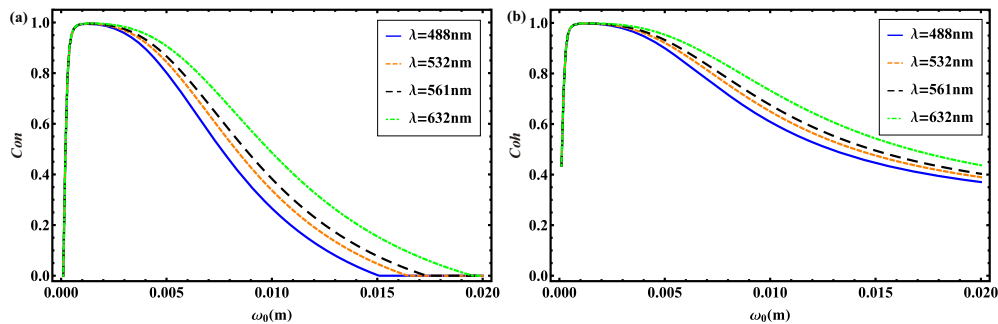


Figure 10. (a) Concurrence and (b) coherence as functions of waist width ω_0 with different wavelength λ .

4. Conclusions

In conclusion, we have quantitatively described the effects of the oceanic turbulence on channel capacity, trace distance, concurrence and trace-distance coherence of LG beams. The influences of

the source parameters of LG beams, the parameters of oceanic turbulence and propagation distance are considered.

First, for single-photon communication, we consider channel capacity and trace distance as measures of the communication performance. The channel capacity decreases by the longer propagation distance z , lower OAM numbers, higher dissipation rate of mean-squared temperature χ and contribution ratio of temperature and salinity τ , lower dissipation rate of kinetic energy per unit mass of fluid ε and inner-scale factor η . By contrast, the trace distance displays converse behaviors versus the variations of these parameters, which provides an alternative tool for the research of underwater optical communication.

Besides, for biphoton communication, we explore the distributions of quantum concurrence and trace-distance coherence. The results show that both quantum resources decreases as the farther propagation distance z , smaller purity γ and azimuthal quantum number l_0 , larger equivalent temperature structure parameter C_n^2 . Comparing to entanglement which exhibits sudden vanishing, quantum coherence can be better preserved during the distribution which exhibits only asymptotic decay. It is also interesting to there exists a short range of optimal beam width, maximizing both resources and almost independent of the wavelength.

Before ending, we remark that it is unclear whether other beams (such as Hermite-Gaussian beams, Bessel-Gaussian beams and Airy beams) possess these similar properties, which is beyond the scope of this work. Our results contribute to the underwater OAM communication, especially these using LG beams.

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