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A Method for Reducing Cogging Torque of Integrated Propulsion Motor

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Received: 27 June 2019; Accepted: 18 July 2019; Published: 21 July 2019

Abstract: How to reduce the cogging torque of the integrated propeller motor is an important means to improve its noise performance because cogging torque is one of the key factors causing torque ripple. We proposed a method to reduce the cogging torque by optimizing the size of the Halbach array's auxiliary pole. First, an analytical model for the airgap magnetic field of Halbach array based on different dimensions (including the circumference ratio and the radial thickness) of the auxiliary pole is given. Then the finite element method is used to verify the analytical model. On the basis, we calculated the cogging torque of different size of auxiliary poles as sample data by combining different circumference ratio and radial thickness. Furthermore, using the two-variable single-objective neural network genetic optimization algorithm based on Backpropagation (BP), we obtain the optimal size of the auxiliary pole. Finally, comparing the motor cogging torque and torque ripple before and after optimization indicated that the cogging torque and torque ripple are effectively reduced after optimizing the size of the auxiliary pole.

Keywords: Halbach array; auxiliary poles; magnetic field; cogging torque; BP neural network; genetic algorithm

1. Introduction

Compared with the traditional form of propulsion motor, the Integrated Motor Propulsor (IMP) is popular because of its compact structure, modular design and assembly and disassembly, no need for dynamic sealing, and low noise [1,2]. However, for conventional propeller motors of the same power, the IMP has a much larger diameter. According to the calculation formula of the moment of inertia: $J = \frac{1}{2}m(r_1^2 + r_2^2)$, where r_1 and r_2 are the inner and outer diameters of the rotor, respectively. We can know that IMP has a larger moment of inertia. In order to ensure the starting characteristics, the IMP imposes more stringent restrictions on the cogging torque. From the knowledge of the motor, it can be known that cogging torque is one of the important reasons for motor torque ripple. Large cogging torque can eventually cause very large output torque ripple that affect the efficiency and even availability of the IMP. Therefore, it is necessary to reduce the cogging torque for the IMP.

The Halbach array was proposed by physicist Dr. K. Halbach of the U.S. in 1979 [3]. Compared with the previous surface-mount motor magnetic structure, the Halbach array enhances the utilization of permanent magnet materials by the directional magnetic focusing of the auxiliary magnetic poles, which increases the magnetic field on one side of the magnetic steel and weakens the other side. For the inner rotor motor, the Halbach array can reduce the magnetic flux density of the rotor yoke, which reduces the thickness of the rotor yoke and improves the overall power density of the motor. The most important point is that the Halbach array approximates the air gap flux density distribution

to a sine wave, helping to reduce cogging torque and torque ripple [4,5]. Therefore, the Halbach array is widely used in the design of permanent magnet motors.

Due to the many advantages of the Halbach array, it is widely used in the IMP of underwater vehicles, such as paper [6,7]. However, comparing with a same-power traditional underwater vehicle propulsion motor, IMP has more stringent restrictions on the cogging torque due to its larger diameter. Therefore, it is necessary to further reduce the cogging torque on the basis of the Halbach array magnetic structure, so that the output torque is more stable and the performance of the IMP is improved.

For surface-mounted internal PM (SPM) machines, many scholars have proposed various methods for reducing cogging torque by optimizing the permanent magnet structure. Li et al. proposed three different shapes of permanent magnet and effectively reduced the cogging torque by optimizing the dimensions of different shapes [8]. Ik-Hyun Jo and co-workers showed a method that uses the skew for reducing cogging torque by more than 80% [9]. Daohan Wang and co-workers showed us that using different permanent magnet widths can also reduce cogging torque [10]. Furthermore, the cogging torque can be effectively reduced by the asymmetrical distribution of the permanent magnets in the circumferential direction [11,12]. However, these methods of reducing the cogging torque by optimizing the permanent magnets make the permanent magnet structure more complicated and less technical. Compared to these proposed methods, a relatively simple method of reducing cogging torque is proposed in the paper. On the other hand, in order to supplement a new method of reducing the cogging torque of the SPM machines by optimizing the permanent magnet structure, we propose a relatively simple optimization method.

In this paper, we started with the size of the auxiliary pole of the Halbach array and reduced the cogging torque by optimizing the circumferential ratio and the radial thickness of the auxiliary pole. After preliminary analysis using the finite element method, this method can effectively reduce the cogging torque. However, when using the finite element method to optimize the size of the auxiliary poles of the Halbach array, it is very time-consuming to calculate various operating conditions. Therefore, we optimized the size of the auxiliary pole by establishing an analytical model and using the BP neural network and genetic algorithm.

2. Motor Model Description

In order to verify the cogging torque analytical model for different sizes of auxiliary poles, we use the finite element solution of cogging torque as a comparison. Nowadays, the finite element calculation software is mature and the calculation accuracy is very high. Therefore, we use Ansys Maxwell software to obtain the finite element solution.

According to the requirements of the technical indicators, we used the traditional magnetic circuit method to design the magnetic circuit structure and obtained the structural parameters of the magnetic circuit structure. Table 1 shows the main structural parameters of the original surface-mount brushless direct current (BLDC) Motors based on Halbach array. We first established a two-dimensional model of the motor in the Maxwell software according to the parameters in Table 1. Figure 1 shows the 1/4 model of the motor. It can be seen from the partial enlargement that for the model to be studied, the thickness of the auxiliary pole of the Halbach array is generally not equal to that of the main magnetic pole to achieve the effect of reducing the cogging torque.

Table 1. Main structural parameters of the motor.

Structural Parameters	Value	Structural Parameters	Value
Number of pole pairs	10	Stator tooth height (mm)	14
Number of slots	60	Rotor outer diameter (mm)	319
OD of stator (mm)	380	Rotor yoke thickness (mm)	6
ID of stator (mm)	324	Main pole thickness (mm)	3.5
Notch width (mm)	2.8	Pole embrace	0.7
Stator tooth width (mm)	6.6

Figure 1. 1/4 model of surface-mount BLDC motors.

3. Cogging Torque Analysis Model

3.1. The Magnetic Field of Slotless Model

An accurate magnetic field of air gap based on different auxiliary pole sizes is a prerequisite for calculating the analytical solution of cogging torque. Considering the periodicity of the motor's magnetic field, in order to simplify the calculation, we only need to calculate the magnetic field in one pair of poles. Then, the periodic arrangement is performed to obtain the air gap magnetic field distribution of the entire motor model. Figure 2 shows the geometric model of a magnetic field period before slotting. As shown in the figure, the reference point is set at the middle of the N pole, then a magnetic field period can be expressed as $(-\frac{\pi}{p}, \frac{\pi}{p})$. R_m is the radius of the main pole of the Halbach array. R_{m1} is the auxiliary pole radius. R_r is the outer diameter of the rotor and R_s is the inner diameter of the stator. If the pole embrace is α_p , the angle of the N pole as shown in Figure 2 can be expressed as $(-\alpha_p \frac{\pi}{2p}, \alpha_p \frac{\pi}{2p})$. The boundary between the S pole and the auxiliary pole is expressed as $\pm(2 - \alpha_p) \frac{\pi}{2p}$. The airgap region is defined as region 1 and the permanent magnet region (including the main pole and the auxiliary pole) is defined as region 2. In addition, it can be seen from the model that the pole embrace reflects the circumferential length of the auxiliary pole, so we use α_p as the symbolic value of the circumferential length of the auxiliary pole.

Figure 2. Motor model in one period.

In order to simplify the calculation, we first calculate the magnetic field generated by the main pole and the auxiliary pole of the Halbach array separately in the air gap, and then linearly superimpose it to obtain the airgap magnetic field distribution of the slotless model. The error of the method of this separating calculation and linear superposition is mainly due to the mutual influence of the main and auxiliary poles. We assume that this effect is negligible.

First, we calculated the magnetic field of the main magnetic pole. The geometric model is shown in Figure 3. If the main pole is radially magnetized, the radial and tangential distribution of magnetization in a pair of poles can be expressed in the form shown in Figure 4.

Figure 3. The main pole of the Halbach array.

Figure 4. Magnetization distribution of the main pole: (a) the radial component of magnetization and (b) the tangential component of magnetization.

Then the radial and tangential components of the magnetization of the main pole are given by

$$\begin{cases} M_r = -\frac{B_r}{\mu_0} \\ M_\theta = 0 \end{cases}, -\frac{\pi}{p} \leq \theta < (\alpha_p - 2)\frac{\pi}{2p} \text{ \& } (2 - \alpha_p)\frac{\pi}{2p} \leq \theta < \frac{\pi}{p} \quad (1)$$

$$\begin{cases} M_r = 0 \\ M_\theta = 0 \end{cases}, (\alpha_p - 2)\frac{\pi}{2p} \leq \theta < -\alpha_p\frac{\pi}{2p} \text{ \& } \alpha_p\frac{\pi}{2p} \leq \theta < (2 - \alpha_p)\frac{\pi}{2p} \quad (2)$$

$$\begin{cases} M_r = \frac{B_r}{\mu_0} \\ M_\theta = 0 \end{cases}, -\alpha_p\frac{\pi}{2p} \leq \theta < \alpha_p\frac{\pi}{2p} \quad (3)$$

Obviously, both the radial and tangential components of the magnetization are periodic, so Fourier decomposition can be performed. Fourier series expansion is performed on the radial and tangential components of the magnetization of the main pole, by

$$\begin{cases} M_r(\theta) = \sum_{n=1,3,5,\dots}^{\infty} M_{rn} \cos(np\theta) \\ M_{\theta}(\theta) = 0 \end{cases} \quad (4)$$

where

$$M_{rn} = 2\alpha_p \frac{B_r}{\mu_0} \frac{\sin(\frac{1}{2}n\pi\alpha_p)}{\frac{1}{2}n\pi\alpha_p}$$

On the other hand, airgap region 1 is a passive field and scalar magnetic position φ_1 satisfies the Laplace equation; the main pole permanent magnet region 2 is an active field and the scalar magnetic position φ_2 is governed by the Poisson equation [13], i.e.,

$$\frac{\partial^2 \varphi_1}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi_1}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \varphi_1}{\partial \theta^2} = 0 \quad (5)$$

$$\frac{\partial^2 \varphi_2}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi_2}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \varphi_2}{\partial \theta^2} = \frac{\text{div } \mathbf{M}}{\mu_r} = \frac{1}{\mu_r} \left(\frac{M_r}{r} + \frac{\partial M_r}{\partial r} + \frac{1}{r} \frac{\partial M_{\theta}}{\partial \theta} \right) = \frac{1}{\mu_r} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{r} M_{rn} \cos(np\theta) \quad (6)$$

Through the simultaneous Laplace equation and the Poisson equation, the general solution is obtained as

$$\varphi_1(r, \theta) = \sum_{n=1,3,5,\dots}^{\infty} (A_{n1} r^{np} + B_{n1} r^{-np}) \cos(np\theta) \quad (7)$$

$$\varphi_2(r, \theta) = \sum_{n=1,3,5,\dots}^{\infty} (A_{n2} r^{np} + B_{n2} r^{-np}) \cos(np\theta) + \sum_{n=1,3,5,\dots}^{\infty} \frac{M_{rn}}{\mu_r [1 - (np)^2]} \cos(np\theta) \quad (8)$$

where A_{n1} , B_{n1} , A_{n2} , and B_{n2} are undetermined coefficients.

Assuming that the magnetic permeability of the core is infinite, according to the knowledge of electromagnetics, the magnetic field at the interface between the air and the magnetically permeable material is perpendicular to the tangent of the interface, that is, the tangential component of the magnetic field strength at $r = R_s$ and $r = R_r$ is 0. And the magnetic field strength at the interface between the permanent magnet and the air gap is equal everywhere [14]. Thus, boundary conditions are expressed as

$$\begin{cases} H_{\theta 1}(r, \theta) \Big|_{r=R_s} = 0 \\ H_{\theta 2}(r, \theta) \Big|_{r=R_r} = 0 \\ B_{r1}(r, \theta) \Big|_{r=R_m} = B_{r2}(r, \theta) \Big|_{r=R_m} \\ H_{\theta 1}(r, \theta) \Big|_{r=R_m} = H_{\theta 2}(r, \theta) \Big|_{r=R_m} \end{cases} \quad (9)$$

In polar coordinates, the relationship between the scalar magnetic position φ and the magnetic field strength H can be given by

$$H_r = - \frac{\partial \varphi}{\partial r} \quad (10)$$

$$H_{\theta} = - \frac{1}{r} \frac{\partial \varphi}{\partial \theta} \quad (11)$$

Then, the special solution of the partial differential equation can be obtained by combining the general solution and the boundary conditions. According to the relationship between the scalar

magnetic position and the magnetic field strength, the airgap magnetic field distribution can be expressed as

$$B_{r1}(r, \theta) = \sum_{n=1,3,5,\dots}^{\infty} K_B(n) f_{B_r}(r) \cos(np\theta) \quad (12)$$

$$B_{\theta 1}(r, \theta) = \sum_{n=1,3,5,\dots}^{\infty} K_B(n) f_{B_\theta}(r) \sin(np\theta) \quad (13)$$

where

$$K_B(n) = \frac{\mu_0 M_{rn}}{\mu_r} \frac{np}{(np)^2 - 1} \left\{ \frac{(A_{3n} - 1) + 2\left(\frac{R_r}{R_m}\right)^{np+1} - (A_{3n} + 1)\left(\frac{R_r}{R_m}\right)^{2np}}{\frac{\mu_r + 1}{\mu_r} - \left[1 - \left(\frac{R_r}{R_s}\right)^{2np}\right] - \frac{\mu_r - 1}{\mu_r} \left[\left(\frac{R_m}{R_s}\right)^{2np} - \left(\frac{R_r}{R_m}\right)^{2np}\right]} \right\}$$

$$f_{B_r}(r) = \left(\frac{r}{R_s}\right)^{np-1} \left(\frac{R_m}{R_s}\right)^{np+1} + \left(\frac{R_m}{r}\right)^{np+1}$$

$$f_{B_\theta}(r) = -\left(\frac{r}{R_s}\right)^{np-1} \left(\frac{R_m}{R_s}\right)^{np+1} + \left(\frac{R_m}{r}\right)^{np+1}$$

$$A_{3n} = np$$

So far, we have obtained the radial component and the tangential component of the air gap flux density of the main pole. On the other hand, if only the auxiliary pole is considered, the model is as shown in Figure 5.

Figure 5. Auxiliary pole of Halbach array.

If the main pole is tangentially magnetized, the geometric relationship of magnetization and its radial and tangential components can be expressed as Figure 6. Different from the R_m is the radius of the main pole R_{m1} is the auxiliary pole radius. It can be seen that the radial and tangential components can be mathematically expressed as

$$M_r = M \sin(\theta) \quad (14)$$

$$M_\theta = M \cos(\theta) \quad (15)$$

Then, the radial and tangential components of the magnetization of the pair of poles can be expressed as shown in Figure 7.

Figure 6. Geometric relationship of magnetization components.

Figure 7. Magnetization of auxiliary pole: (a) radial component and (b) tangential component.

Mathematically expressed as

$$\begin{cases} M'_r = 0 \\ M'_\theta = 0 \end{cases}, -\frac{\pi}{p} \leq \theta < (\alpha_p - 2)\frac{\pi}{2p} \text{ \& } (2 - \alpha_p)\frac{\pi}{2p} \leq \theta < \frac{\pi}{p} \text{ \& } -\alpha_p\frac{\pi}{2p} \leq \theta < \alpha_p\frac{\pi}{2p} \quad (16)$$

$$\begin{cases} M'_r = \frac{B_r}{\mu_0} \sin(\theta + \frac{\pi}{2p}) \\ M'_\theta = \frac{B_r}{\mu_0} \cos(\theta + \frac{\pi}{2p}) \end{cases}, (\alpha_p - 2)\frac{\pi}{2p} \leq \theta < -\alpha_p\frac{\pi}{2p} \quad (17)$$

$$\begin{cases} M'_r = -\frac{B_r}{\mu_0} \sin(\theta - \frac{\pi}{2p}) \\ M'_\theta = -\frac{B_r}{\mu_0} \cos(\theta - \frac{\pi}{2p}) \end{cases}, \alpha_p\frac{\pi}{2p} \leq \theta < (2 - \alpha_p)\frac{\pi}{2p} \quad (18)$$

Similarly, it is expanded by Fourier series, i.e.,

$$\begin{cases} M'_r(\theta) = \sum_{n=1}^{\infty} M'_{rn} \cos(np\theta) \\ M'_\theta(\theta) = \sum_{n=1}^{\infty} M'_{\theta n} \sin(np\theta) \end{cases} \quad (19)$$

where

$$M'_{rn} = \frac{2p}{[1 - (np)^2]\pi} \frac{B_r}{\mu_0} \left\{ (np) \sin\left[(1 - \alpha_p)\frac{\pi}{2p}\right] \left[\sin\left(\frac{np\pi}{2}(2 - \alpha_p)\right) + \sin\left(\frac{np\pi}{2}\alpha_p\right) \right] + \cos\left[(1 - \alpha_p)\frac{\pi}{2p}\right] \left[\cos\left(\frac{np\pi}{2}(2 - \alpha_p)\right) - \cos\left(\frac{np\pi}{2}\alpha_p\right) \right] \right\}$$

$$M'_{\theta n} = \frac{2p}{[(np)^2 - 1]\pi} \frac{B_r}{\mu_0} \left\{ (np) \cos\left[(1 - \alpha_p)\frac{\pi}{2p}\right] \left[\cos\left(\frac{np\pi}{2}(2 - \alpha_p)\right) - \cos\left(\frac{np\pi}{2}\alpha_p\right) \right] + \sin\left[(1 - \alpha_p)\frac{\pi}{2p}\right] \left[\sin\left(\frac{np\pi}{2}(2 - \alpha_p)\right) + \sin\left(\frac{np\pi}{2}\alpha_p\right) \right] \right\}$$

Similarly, in Regions 1 and 2, the Laplace equation and the Poisson equation are also satisfied, respectively. Combined with the boundary conditions, the radial and tangential components of the air gap flux density of the auxiliary pole can also be obtained as

$$B'_{r1}(r, \theta) = - \sum_{n=1}^{\infty} G \cos(np\theta) \left[\left(\frac{r}{R_s}\right)^{np-1} \left(\frac{R_{m1}}{R_s}\right)^{np+1} + \left(\frac{R_{m1}}{r}\right)^{np+1} \right] \quad (20)$$

$$B'_{\theta 1}(r, \theta) = \sum_{n=1}^{\infty} G \sin(np\theta) \left[\left(\frac{r}{R_s}\right)^{np-1} \left(\frac{R_{m1}}{R_s}\right)^{np+1} - \left(\frac{R_{m1}}{r}\right)^{np+1} \right] \quad (21)$$

where

$$G = \frac{np\mu_0 \left\{ \frac{M'_{rn} - M'_{\theta n}}{np+1} - \frac{M'_{rn} + M'_{\theta n}}{np-1} \left(\frac{R_r}{R_{m1}}\right)^{2np} + \frac{2(M'_{rn} + npM'_{\theta n})}{(np)^2 - 1} \left(\frac{R_r}{R_{m1}}\right)^{np+1} \right\}}{\left[\left(\mu_r + 1\right) \left(\frac{R_r}{R_s}\right)^{2np} - \left(\mu_r - 1\right) \left(\frac{R_r}{R_{m1}}\right)^{2np} \right] - \left[\left(\mu_r + 1\right) - \left(\mu_r - 1\right) \left(\frac{R_{m1}}{R_s}\right)^{2np} \right]}$$

By linearly superimposing the radial and tangential components of the air gap flux density of the main and auxiliary poles, the total air gap magnetic field distribution of the Halbach array based on different auxiliary pole sizes can be obtained, i.e.,

$$B_r(r, \theta) = B_{r1}(r, \theta) + B'_{r1}(r, \theta) \quad (22)$$

$$B_\theta(r, \theta) = B_{\theta1}(r, \theta) + B'_{\theta1}(r, \theta) \quad (23)$$

In order to verify the analytical model of the air gap flux density of the slotless model, the results calculated using the finite element method were used as a comparison. The resulting comparison of the finite element solution and the analytical solution is shown in Figures 8 and 9. It can be seen from Figures 8 and 9 that the radial and tangential components of the air gap magnetic flux density obtained by the analytical method agree very well with the results of the finite element method. Their differences mainly occur on higher harmonics, which may be caused by the analytic solution's neglect of higher harmonics. However, the frequency and amplitude of their main harmonic components are basically the same, so the analytical model can be considered correct.

Figure 8. Comparison of the radial component of magnetic field.

Figure 9. Comparison of the tangential component of magnetic field.

3.2. The Magnetic Field of the Slotted Model

Conformal mapping is an important tool for transforming complex fields into simple fields. The airgap magnetic density of the slotless model has been obtained above, which can be converted into the airgap magnetic field of the slotted model by four conformal mappings. In order to simplify the calculation, the stator slot is assumed to be infinitely deep before the conformal mapping. Since the magnetic field of the stator slot mainly acts near the slot, it is assumed that the infinite depth of the slot has little effect on the final result. The process of the conformal mapping is shown in Figure 10 [15–17]. The model of plane S represents the model within one pitch of the slotted motor, while the model of plane K represents the model within one pitch of the slotless motor. They can be transformed into each other by a four-step conformal mapping as shown in Figure 10. In the above we have obtained the magnetic field distribution of the air gap in the slotless motor, then the air gap magnetic field distribution of the slotted motor can be obtained by transforming into the slotted model.

Figure 10. The process of conformal mapping.

The mapping expressions are

$$\left\{ \begin{array}{l} S \leftrightarrow Z : z = \ln(s) \\ Z \leftrightarrow W : z(w) = j \frac{g'}{\pi} \left[\ln \left(\frac{1+p}{1-p} \right) - \ln \left(\frac{b+p}{b-p} \right) - \frac{2(b-1)}{\sqrt{b}} \tan^{-1} \frac{p}{\sqrt{b}} \right] + c \\ W \leftrightarrow T : t = j \frac{g'}{\pi} \ln w + \ln R_s + j \frac{\theta_s}{2} \\ T \leftrightarrow K : k = e^t \end{array} \right. \quad (24)$$

where $g' = \ln \left(\frac{R_s}{R_r} \right)$ is the length of the air gap in the plane Z and $b'_0 = \theta_2 - \theta_1$ is the slot width in the plane Z. s , z , w , t , and k are complex expressions of coordinates of one point in the corresponding plane. The coefficients in the formula are

$$b = \left[\frac{b'_0}{2g'} + \sqrt{\left(\frac{b'_0}{2g'} \right)^2 + 1} \right]^2 \quad (25)$$

$$a = \frac{1}{b} \quad (26)$$

$$c = \ln R_s + j \theta_2 \quad (27)$$

$$p = \sqrt{\frac{w-b}{w-a}} \quad (28)$$

Then, within one slot distance, the effect of the slots on the magnetic field can be represented by a complex form of relative airgap permeability, i.e.,

$$\lambda = \frac{k}{s} \frac{w-1}{(w-a)^{\frac{1}{2}}(w-b)^{\frac{1}{2}}} = \lambda_a + j\lambda_b \quad (29)$$

where k , s , and w are the complex forms of the coordinates of one point in the planes k , s , and w , respectively.

In the analytical model, there are six slots under a pair of poles, so to calculate the relative permeability under a pair of poles, it is necessary to periodically extend the relative permeance six times within one slot. Finally, the real part and the imaginary part and the position of the pair of poles relative magnetic flux can be obtained as shown in Figure 11.

Figure 11. The real and imaginary parts of the complex relative magnetic conductance.

Obviously, the relative magnetic permeability $\lambda = \lambda_a + j\lambda_b$ of slotless model satisfies $\lambda_a = 1$, $\lambda_b = 0$. Due to the influence of the slot, the real and imaginary parts of the relative magnetic flux change at the corresponding positions of the slots.

Therefore, the air gap magnetic field distribution of the slotted model can be calculated by

$$B_s = B_k \lambda^* = (B_{kr} + jB_{k\theta}) \quad (30)$$

where $B_s = B_{sr} + jB_{s\theta}$ is the magnetic flux density of the slotted model and the magnetic flux density of the slotless model is $B_k = B_{kr} + jB_{k\theta}$. λ^* is the conjugate complex number of λ . By substituting λ into the above Equation (30), the radial and tangential components of the slotted model air gap flux density are

$$B_{sr} = \text{Re}(B_k \lambda^*) = B_{kr} \lambda_a + B_{k\theta} \lambda_b \quad (31)$$

$$B_{s\theta} = \text{Im}(B_k \lambda^*) = B_{k\theta} \lambda_a - B_{kr} \lambda_b \quad (32)$$

It can be seen that when the model is slotless ($\lambda_a = 1$, $\lambda_b = 0$) the above formula becomes $B_{sr} = B_{kr}$, $B_{s\theta} = B_{k\theta}$. It further verified that the conformal mapping can accurately describe the effect of slotting on the airgap magnetic field. So far, the airgap magnetic field distribution of the slotted model is obtained.

To verify the analytical model, the finite element method was used to calculate the radial and tangential components of the air gap flux density of the slotted model, respectively. Figures 12 and 13 shows the comparison of the results of the analytical and finite element method calculations. Because the analytical method does not consider the influence of magnetic flux leakage and slotting depth,

the analytical solution and the finite element solution have certain errors. The error is mainly reflected in the amplitude of some harmonics. However, the results obtained by the two methods are not very inaccurate. In engineering, the analytical solution can be considered to accurately describe the slotted model based on different auxiliary pole sizes.

Figure 12. Comparison of the radial component of magnetic field of slotted model.

Figure 13. Comparison of the tangential component of magnetic field of slotted model.

Maxwell tensor method is to convert the volumetric force in the magnetic field into the tension on the surface of the body. Here we can use the Maxwell tensor method to calculate the cogging torque [18], the expression is as

$$T_c = \frac{L_a}{\mu_0} r^2 \int_0^{2\pi} B_{sr} B_{s\theta} d\theta \quad (33)$$

where L_a is the effective axial length of the stator and r is a certain measurement radius in the range of air gap. By substituting the above-mentioned airgap magnetic flux density radial component B_{sr} (31) and tangential component $B_{s\theta}$ (32) into the cogging torque calculation Formula (33), the cogging torque at the initial position can be obtained.

To calculate the cogging torque at different positions, it is necessary to offset the air gap magnetic field of the slotless model, which symbolizes the rotor position, from the equivalent magnetic flux that symbolizes the stator position by a certain angle. substituting (31), (32) obtains the airgap magnetic fields B_{sr} and $B_{s\theta}$ of the rotor at this position. Then substitute (33) to get the cogging torque at this position. Figure 14 shows a comparison of slot torque and finite element method for one cogging torque period. The analytical solution has some discrepancies with the finite element method due to factors such as magnetic flux leakage and slot depth, but the error is not large. It can be considered that the analytical model can describe the cogging torque of the model based on the thickness of the unequal auxiliary magnetic pole.

Figure 14. Comparison of analytical solutions and finite element solutions.

4. Optimization of Halbach Auxiliary Pole Size

4.1. Calculate Sample Data

The auxiliary pole mainly includes two factors: the circumferential length (i.e., the pole embrace α_p) and the radial thickness $h_{m1} = R_{m1} - R_r$. We mainly adjust the two factors to obtain the optimal size of the Halbach array's auxiliary pole, which makes the cogging torque is minimal.

We take the adjustment range of the pole embrace α_p , which symbolizes the circumferential length of the auxiliary pole as 0.6–0.8 mm and the step is 0.5 mm; the radial thickness h_{m1} has an adjustment range of 2.7 to 3.9 mm and a step size of 0.2 mm. Through the combination of the different parameters of these two factors, the amplitude of cogging torque based on different auxiliary pole sizes is shown in Table 2.

Table 2. Cogging torque of different auxiliary pole sizes.

Cogging Torque (Nm)		Radial Thickness of Auxiliary Pole h_{m1} (mm)						
		2.7	2.9	3.1	3.3	3.5	3.7	3.9
Pole Embrace α_p	0.60	5.354	5.281	5.196	5.054	4.527	4.268	4.493
	0.65	1.867	1.424	0.945	0.546	1.122	1.932	3.080
	0.70	2.936	3.531	4.156	5.593	6.585	7.258	8.236
	0.75	5.182	5.545	5.917	6.366	6.842	7.412	8.475
	0.80	5.852	6.210	6.620	7.120	7.689	8.450	9.297

It can be seen from the data in the table that under a certain pole embrace α_p , the cogging torque has a large change of amplitude when the radial thickness h_{m1} is changed; when the pole embrace

is different, the variation range is different. Especially, the variation is greatest near the optimally estimated pole embrace. Conversely, under a certain radial thickness h_{m1} , the cogging torque of the motor is also different when the pole embrace α_p changed. It indicates that the two main dimensions of the auxiliary pole of the Halbach array have a large influence on the magnitude of the cogging torque. However, because of the step of the calculation, we only get the optimized size of the auxiliary pole approximately. For engineering, the size is necessarily more accurate. Therefore, we need to obtain the optimized size using optimization methods.

4.2. Size Optimization

The BP (Backpropagation) neural network and genetic algorithm is a network learning optimization algorithm [19,20]. The main process is as follows; Input the learning sample, that is, the training data, and use the backpropagation algorithm to repeatedly train the weights and deviations of the network, so that the data of the training output and the expected data, which is the predicted data, are as close as possible. When the sum of squared errors is smaller than the specified training error, the training learning is completed, and the model that needs to be optimized is obtained. Then, the minimum value of the network model is obtained, and the corresponding independent variable is the optimal auxiliary pole size.

Our optimization model has two independent variables: the pole embrace α_p , representing the circumferential length of the auxiliary pole, and the radial thickness h_{m1} . One optimization target is the magnitude of the cogging torque. The sample data in Table 2 is divided into two categories: the first four rows of data in the table are used as model training data and the 5th-row data is used as verification data, i.e., the expected data.

After repeated learning with the training samples, the comparison of training results between the predicted data and the expected data are shown in Figure 15. As can be seen from Figure 15, the predicted curve is substantially consistent with the expected curve; Figure 16 shows the performance of the neural network. This figure shows that the Validation Performance is 0.0021278 at epoch 12. It has been very close to our target error of the network. Figure 17 shows the regression analysis of the BP neural network. It can be seen from the figure that the validation set and the test set of complex correlation number (R) are very close to 1. In summary, it can be considered that the neural network model is effective.

Figure 15. Backpropagation (BP) neural network prediction results.

Figure 16. The performance of the neural network.

Figure 17. The regression analysis of the BP neural network.

By the fitting of the neural network based on BP, the mathematical model of the different auxiliary polar dimensions of the Halbach array shown in Figure 18 is obtained. The two independent variables represent the pole embrace and the radial thickness of the permanent magnet, respectively, and the dependent variable represents the magnitude of the cogging torque. As can be seen from the model shown in Figure 18, the auxiliary magnetic pole of the Halbach array of the motor has an optimum size, which minimizes the cogging torque of the motor.

Figure 18. Model of influence of different auxiliary pole sizes on cogging torque.

Through the further iterative process of the genetic algorithm, the size of the optimal Halbach array auxiliary pole is, the main magnetic pole ratio (pole embrace) of the Halbach array is 0.6674, and the radial thickness of the permanent magnet is 2.8831 mm.

We take these two cases as a comparison: the pole embrace is the optimal pole embrace, the radial thickness is the normal thickness, i.e., 3.5 mm; the radial thickness is the optimal thickness, and the pole embrace is the optimally estimated pole embrace of this motor, i.e., 0.65. Calculated their cogging torque curves separately, as shown in Figure 19. It can be seen that when $\alpha_p = 0.6674$, $h_m = 3.5$ mm, the maximum cogging torque is 2.7–2.8 Nm, and the maximum cogging torque is 1.7–1.8 Nm when $\alpha_p = 0.65$, $h_m = 2.8831$ mm. After the auxiliary pole size is optimized, the maximum cogging torque is 0.3 Nm. When the pole embraces are both 0.6674, by comparing the two conditions of $h_m = 3.5$ mm and optimized $h_m = 2.8831$ mm, it can be known that the radial thickness of the auxiliary pole has a significant influence on the cogging torque. On the other hand, when the radial thickness is 2.8831 mm, by comparing the two conditions of $\alpha_p = 0.65$ and the optimized $\alpha_p = 0.6674$, it is also known that the circumferential length of the auxiliary pole is significant for reducing the cogging torque. In summary, optimizing the size of the auxiliary pole has a significant effect on reducing the cogging torque.

Figure 19. Comparison of cogging torque based on different auxiliary pole sizes.

In order to compare the performance of the motor before and after optimization, we compare the electromagnetic torque (output torque) at steady-state in the three operating conditions mentioned in Figure 19. As shown in Figure 20, we can see that when $\alpha_p = 0.6674$, $h_m = 3.5$ mm, the average value of the output torque is about 33 Nm, the torque fluctuation is about 5 Nm. When $\alpha_p = 0.65$,

$h_m = 2.8831$ mm, the torque ripple is not reduced, and the average torque is slightly reduced. By using the optimal Halbach array auxiliary pole size, it can be seen that not only the average electromagnetic torque is not reduced, but also the torque ripple is effectively reduced. In addition, according to the knowledge of electrical engineering, we can know that the slight change in the radial thickness of the permanent magnet has a negligible effect on the maximum value of the air gap flux density. From Figure 20 we can see that the pole arc coefficient has a greater influence on the average output torque than the radial thickness. Therefore, we believe that optimizing the size of the auxiliary pole does have a great effect on improving the performance of the motor. Obviously, there are many factors that affect the torque ripple. For example, current commutation and armature reaction can also cause large torque ripple. Here, we effectively suppress the cogging torque by the method described above without destroying the output performance of the motor. In this respect, the method of reducing the cogging torque has engineering reference value. Of course, as can be seen from Figure 20, it is very necessary to further study to suppress the torque ripple so that the output torque performance of the motor meets the requirements of the IMP.

Figure 20. Comparison of electromagnetic torque.

In summary, we obtained the cogging torque, the torque ripple and the average electromagnetic torque of the motor based on the auxiliary magnetic pole before and after optimization. By comparison, we obtained the effect of optimizing the auxiliary pole size as shown in Table 3. It can be seen in Table 3, after optimizing the size of the auxiliary pole, the cogging torque is reduced by approximately 88%, and the output torque ripple is reduced by approximately 45% without affecting the average value of the electromagnetic torque.

Table 3. Comparison of torque before and after optimization.

Comparison Items	The Reduction Percentages
Cogging Torque	88%
Torque Ripple	45%
Average Electromagnetic Torque	Not changes

5. Conclusions

This paper proposes a method to reduce the cogging torque by optimizing the size of the auxiliary pole of the Halbach array. In this paper, we first used the superposition of the magnetic field to obtain the analytical model based on different Halbach array's auxiliary pole dimensions and compared it

with the finite element. The results show that the analytical model can accurately describe the magnetic field of this motor. Then, we optimized the two size factors of the auxiliary pole by BP-based neural network algorithm. By comparing the cogging torque and the electromagnetic torque curve before and after optimization, we proposed that the circumferential length and radial thickness of the auxiliary pole of the Halbach array have a significant influence on the cogging torque, which can be optimized to reduce the cogging torque of the motor.

However, the research in this paper is limited to the two-dimensional (2-D) level. For the three-dimensional (3-D) model, we need further research to obtain a more accurate model of the auxiliary pole size prediction. In addition, in order to more accurately predict the performance of the motor, we also need to study the optimization algorithm to obtain two optimization goals of cogging torque and torque ripple.

Author Contributions: Conceptualization, H.O. and Y.H.; methodology, Y.H. and H.O.; software, H.O. and Y.L.; validation, Y.H. and Z.M.; formal analysis, H.O. and Y.L.; writing—original draft preparation, H.O. and Z.M.; writing—review and editing, Y.H.; supervision, Y.L. and Y.H.; project administration, Z.M. and Y.H.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

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