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Singularity-Free Fixed-Time Cooperative Tracking Control of Unmanned Surface Vehicles with Model Uncertainties

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Abstract

This article addresses the problem of singularity-free fixed-time tracking control for multiple unmanned surface vehicles (USVs) with model uncertainties. To compensate for the uncertain nonlinearities in the multi-USV systems, fuzzy logic approximators are employed to estimate unknown hydrodynamic parameters. By integrating adaptive fixed-time control theory with backstepping methodology, a novel singularity-free fixed-time consensus control scheme is developed, incorporating a error switching mechanism to prevent singularities arising from the differentiation of speed control laws. Through rigorous analysis via fixed-time stability theory, the proposed control scheme guarantees that consensus tracking errors reach a small region around zero within fixed-time. Numerical simulations demonstrate the efficacy of the presented method.

Keywords: unmanned surface vehicles (USVs); fixed-time control; consensus control; backstepping technology

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1. Introduction

Distributed cooperative control of unmanned surface vehicles (USVs) has attracted considerable research attention in recent years, owing to its diverse applications in marine engineering [1–3]. These include marine environmental monitoring, maritime patrol, and hydrographic surveys. Formation control serves as a foundational framework for enabling coordinated motion in multi-USV systems, where the relative positions and orientations of the vehicle group are regulated through localized inter-agent communication [4]. Formation control of USVs has been extensively studied, yielding diverse methodological frameworks. The leader-follower framework operates on a fundamental principle wherein the followers synchronize their states with the given leader's motion trajectory [5]. This architecture effectively transforms the formation problem into a consensus-based coordination task. Owing to its critical role in distributed cooperative control, numerous consensus algorithms have been devised for multi-USV systems. For example, for USVs with disturbances, the work in [6] develops an event-triggered cooperative consensus control method. The authors in [7] present an obstacle avoidance-based leader-follower consensus control for multi-USV systems. A formation collision avoidance control algorithm is proposed for leader-follower USV systems [8]. A distributed consensus sliding mode control algorithm is proposed for heterogeneous USVs in [9]. The authors in [10] give the secure leader-following consensus tracking control protocol for USVs subject to actuator faults and cyber attacks.

Despite considerable improvements have been realized in consensus control of multi-USV systems, it is crucial to emphasize that existing approaches [6–10] addressing distributed consensus problems rely heavily on precise knowledge of dynamic parameters. However, obtaining accurate USV model parameters remains challenging due to unpredictable disturbances induced by ocean currents and wave forces. Consequently, known model-dependent control schemes become impractical when the system contains uncertain hydrodynamic nonlinearities. Notably, approximation-based methods, including neural networks [11] and fuzzy logic systems [12], offer the robust adaptive control framework to address diverse uncertainties such as unknown nonlinearities and external disturbances. The work in [13] proposes a leader–follower neural network control strategy of USVs considering performance constraint and collision avoidance. A distributed cooperative intelligent tracking control protocol is designed for underactuated USVs with input quantization [14]. The authors in [15] propose a fuzzy anti-saturation formation controller of multiple USVs with model uncertainty. A neural networks-based guaranteed performance formation tracking control scheme is developed for USVs in the presence of unknown dynamic nonlinearities in [16]. The authors in [17] design a fuzzy logic system-based formation sliding mode controller for leader-follower USV systems. Nevertheless, the abovementioned works primarily achieve asymptotic stability of closed-loop systems, where stabilization is achieved only as time approaches infinity.

In practice, the rate of convergence represents a critical control metric in distributed cooperative control. The fixed-time control strategy ensures not only rapid convergence but also eliminates dependence on initial conditions [18]. The concept of fixed-time stability is originally introduced to address stabilization problems in uncertain linear systems [19]. The authors in [20,21] pioneer two innovative fixed-time consensus protocols for first-order multi-agent systems. However, immediate extension to double-integrating multi-agent systems introduced a singularity issue in the control law. To resolve this limitation, the work [22] subsequently develops a fixed-time cooperative control algorithm for a class of multi-agent systems. The authors in [23] subsequently propose a fixed-time consensus scheme that achieves optimal convergence within a specified time interval, irrespective of initial conditions. A fixed-time formation sliding mode control problem is solved for multi-USV systems successfully [24]. The authors in [25] propose an event-triggered fixed-time cooperative scheme for USVs with faults using a disturbance observer. Backstepping-based control is a reliable method, which can decompose a complex nonlinear USV system into kinematic and kinetic subsystems. It then begins the design from the kinematic level and progressively works inward towards the dynamic level. As noted in [26], the inherent singularity problem arising from exponential power terms in fixed-time control schemes remains a fundamental challenge. For USV systems, the singularity problem may be produced when the fixed-time speed control laws with the exponential power errors is differentiated. A critical examination of current research reveals that the development of a backstepping-based fixed-time consensus protocol with singularity avoidance capabilities for multi-USV systems subject to model uncertainties remains an open challenge with significant theoretical and practical implications.

Building upon these research insights, this paper investigates the backstepping-based fuzzy adaptive fixed-time cooperative control problem for USV systems considering dynamic uncertainties. The principal contributions of this study are summarized as follows.

(1) Unlike existing cooperative tracking control methods for USV systems [13–17] that only guarantee slow convergence, the proposed distributed fixed-time controller constructed via the backstepping framework, enables the USV systems' outputs to rapidly converge to the desired leader signal within a fixed time.

(2) A novel error switching mechanism is designed for multi-USV controller design, enabling automatic transition of tracking errors with exponential terms between two operational intervals, which directly circumvents singularity issues within the backstepping framework.

(3) Fuzzy logic systems are utilized to handle the unknown hydrodynamic nonlinearities of the USV systems. Moreover, in order to broaden the practical applicability of the proposed scheme, the collision avoidance requirement is considered by designing the consensus tracking errors with the safe distance.

The remainder of the paper is organized as follows. Section 2 proposes the system model and problem statement. The controller design and stability analysis are elaborated in Section 3. Simulation results are shown in Section 4. Conclusions are given in Section 5.

2. Modeling and Problem Statement

2.1. Communication Network

The communication topology among K USVs is modeled by a directed graph $G_j = (H_j, B_j, A_j)$ for $(j = x, y, \psi)$, where $H_j = \{1, 2 \ldots, K\}$ represents the set of nodes, $B_j \subseteq H_j \times H_j$ denotes the edge set defining inter-agent communication links, and $A_j = [a_{ijc}] \in \mathbb{R}^{K \times K}$ is the weighted adjacency matrix with $a_{ijc} > 0$ if node i receives information from node c and $a_{ijc} = 0$ otherwise [27]. The neighborhood of node i is defined as $N_{ij} = \{c | (c, i) \in B_j, c \neq i\}$. The degree matrix $D_j = diag(d_{1j}, \ldots, d_{Kj})$ is constructed with $d_{ij} = \sum_{c=1}^{K} a_{ijc}$, and the graph Laplacian is given by $L_j = D_j - A_j$, which encodes the connectivity structure of the network.

2.2. System Formulation

For each USV ($i \in H_l$), the dynamic model follows [28]:

$$\begin{aligned} \dot{\mu}_i &= R^{(i)}(\psi_i)\nu_i \\ M_i\dot{\nu}_i &= -C_i(\nu_i)\nu_i - D_i(\nu_i)\nu_i + T_i \end{aligned} \tag{1}$$

in which $\mu_i = [x_i, y_i, \psi_i]^T$ is the i-th USV's position vector; $\nu_i = [u_i, v_i, r_i]^T$ is the speed vector; The rotation matrix of each USV is

$$R^{(i)}(\psi_i) = \begin{bmatrix} R_{11}^{(i)} & R_{12}^{(i)} & R_{13}^{(i)} \\ R_{21}^{(i)} & R_{22}^{(i)} & R_{23}^{(i)} \\ R_{31}^{(i)} & R_{32}^{(i)} & R_{33}^{(i)} \end{bmatrix} = \begin{bmatrix} \cos\psi_i & -\sin\psi_i & 0 \\ \sin\psi_i & \cos\psi_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $M_i = diag(M_{ix}, M_{iy}, M_{i\psi})$, $C_i(\nu_i) = diag(C_{ix}, C_{iy}, C_{i\psi})$, and $D_i(\nu_i) = diag(D_{ix}, D_{iy}, D_{i\psi})$ are the inertia matrix, Coriolis matrix, and damping matrix, respectively. $T_i = [T_{ix}, T_{iy}, T_{i\psi}]^T$ is the i-th USV's control input vector.

Control Objective: This paper focuses on developing a backstepping-based fixed-time cooperative control scheme for USVs, ensuring that their position vectors $\mu_i = [x_i, y_i, \psi_i]^T$ tracks the given leader signal vector $y_0 = [x_0, y_0, \psi_0]^T$. The proposed controllers aim to achieve consensus tracking in multi-USV systems, as shown in Figure 1.

The consensus error of the kinematic level is given by $s_{i1} = [s_{ix1}, s_{iy1}, s_{i\psi1}]^T$, and the speed tracking error of the kinetic level $s_{i2} = [s_{ix2}, s_{iy2}, s_{i\psi2}]^T = \nu_i - \alpha_i = \left[u_i - \alpha_{ix}, v_i - \alpha_{iy}, r_i - \alpha_{i\psi}\right]^T$, where α_{ij} $(i \in H_l, j = x, y, \psi)$ denote the virtual speed controls in surge, sway and yaw, respectively, and

$$
\begin{aligned}
s_{ix1} &= \sum_{c=1}^{K} a_{ixc}(x_i + \xi_{ix} - x_c - \xi_{cx}) + a_{ix0}(x_i + \xi_{ix} - x_0) \\
s_{iy1} &= \sum_{c=1}^{K} a_{iyc}(y_i + \xi_{iy} - y_c - \xi_{cy}) + a_{iy0}(y_i + \xi_{iy} - y_0) \\
s_{i\psi 1} &= \sum_{c=1}^{K} a_{i\psi c}(\psi_i - \psi_c) + a_{i\psi 0}(\psi_i - \psi_0)
\end{aligned} \tag{2}
$$

where $\xi_{ix} > 0$, $\xi_{iy} > 0$, $\xi_{cx} > 0$, and $\xi_{cy} > 0$ represent the demanding distances.

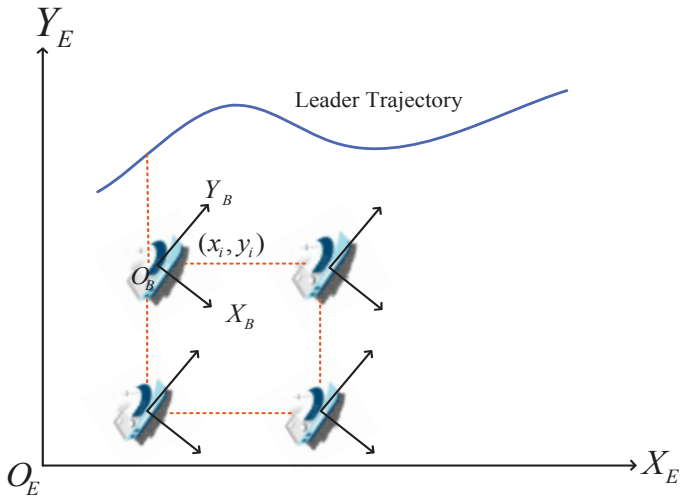


Figure 1. Geometrical illustration of cooperative tracking (note: $X_E O_E Y_E$ and $X_B O_B Y_B$ denote the earth and body frames, respectively).

Lemma 1 ([29]). *A directed graph G_j admits a spanning tree rooted at node 0 if and only if there exists a directed path from node 0 to every other node in the graph. Let $F_j = diag(a_{1j0}, \ldots, a_{Kj0})$ be a diagonal matrix where the entries a_{ij0} represent the connection weights from node 0 to each node i. Under this condition, the composite matrix $L_j + F_j$ formed by summing the weighted Laplacian L_j of G_j and the root-coupling matrix F_j is guaranteed to be nonsingular.*

Lemma 2 ([30]). *For the i-th USV (1), if there exists a positive definite function which is also radially unbounded $E(z) : \Re^s \to \Re_+$, such that*

$$
\begin{aligned}
Q_1(\|z\|) \le E(z) \le Q_2(\|z\|) \\
\dot{E}(z) \le -\chi_2 E^p(z) - \chi_3 E^q(z) + \sigma_2,\ t \ge 0
\end{aligned} \tag{3}
$$

in which $Q_1(\|z\|)$ and $Q_2(\|z\|)$ denote $\kappa_\infty-$ functions: $\chi_2 > 0$, $\chi_3 > 0$, $0 < p < 1$ and $q > 1$ represent adjustable constants meeting $\sigma_2 < \min\{\chi_2(1-\omega), \chi_3\ (1-\omega)\}$ $(0 < \omega < 1)$, thus the system (1) is practically fixed-time stable and the convergence time S is calculated as

$$
S \le S_{\max} = \frac{1}{\chi_2 \omega (1-p)} + \frac{1}{\chi_3 \omega (q-1)}. \tag{4}
$$

Lemma 3 ([31,32]). *Letting a continuous function $f(Z)$ defined on a compact set Ω there exists the fuzzy logic system $\Theta^T \pi(Z)$ such that*

$$
\sup_{Z \in \Omega} \left| f(Z) - \Theta^T \pi(Z) \right| \le \phi^* \tag{5}
$$

with $\forall \phi^* > 0$.

Remark 1. *The fuzzy logic system is a nonlinear framework capable of inferring unknown nonlinear relationships between input and output variables. Mathematically, it can be expressed as a linear combination of fuzzy basis functions. Using empirical data collected from the USV system, these fuzzy basis functions can be constructed and subsequently transformed into IF-THEN rule representations. In the backstepping control strategy developed in this study, an fuzzy logic system-based approximator is employed to estimate the inherently uncertain hydrodynamic damping components within the kinetics of the USV under control.*

To illustrate the construction of fuzzy rules, consider a simple example of approximating an unknown nonlinear function $f(Z)$, where Z is the input variable. First, we define linguistic labels for Z (e.g., "Negative," "Zero," "Positive") using Gaussian membership functions. A basic fuzzy rule base can then be constructed as follows: Rule 1: IF Z is Negative, THEN $\hat{f}(Z)$ is Θ_1; Rule 2: IF Z is Zero, THEN $\hat{f}(Z)$ is Θ_2; Θ_1; Rule 2: IF Z is Positive, THEN $\hat{f}(Z)$ is Θ_3; The output of the fuzzy logic system, $\hat{f}(Z)$, is computed as a weighted average of the consequent parameters $(\Theta_1, \Theta_2, \Theta_3)$, where the weights are the normalized firing strengths of each rule.

3. Control Design

This section proposes a backstepping-based fixed-time cooperative control scheme for USVs, with stability guaranteed via Lyapunov analysis. The control block diagram is described in Figure 2.

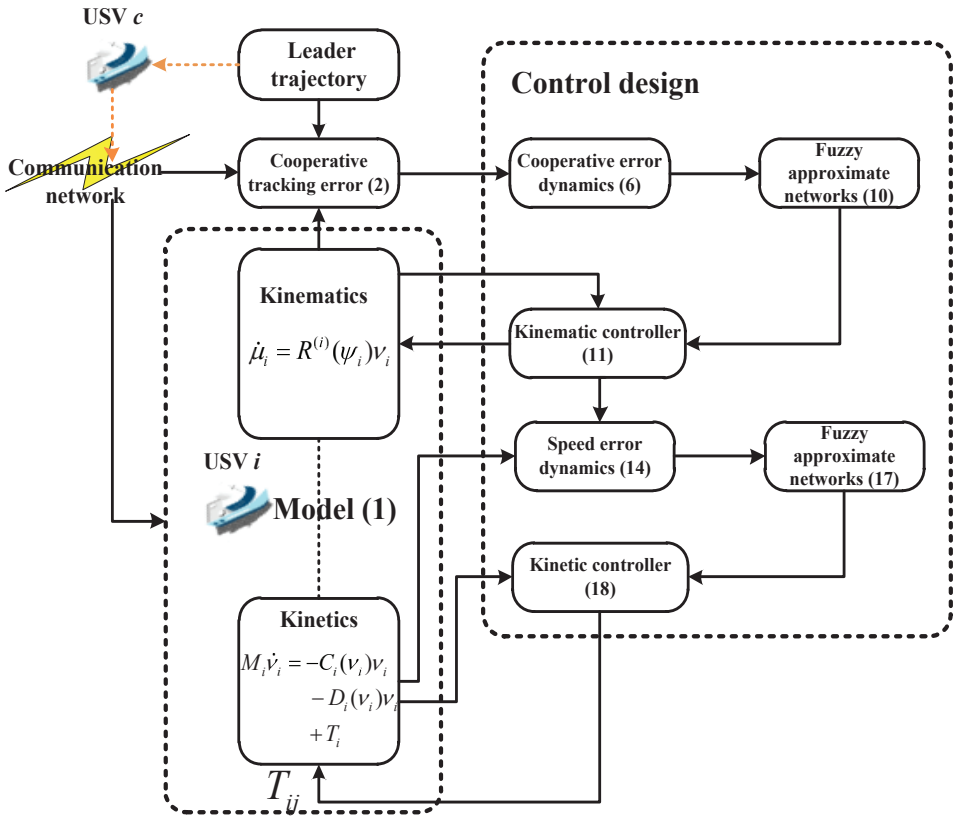


Figure 2. Block diagram of the developed control algorithm.

3.1. Distributed Fixed-Time Kinematics and Kinetics Controller Design

Initially, a distributed kinematic controller generates virtual speed signals to drive the consensus position and orientation error to a small region around zero within a fixed time.

Based on (1), we have

$$\begin{aligned}\dot{s}_{ix1} =&(d_{ix}+a_{ix0})(u_i\cos\psi_i - v_i\sin\psi_i) - \sum_{c=1}^{K} a_{ixc}(u_c\cos\psi_c - v_c\sin\psi_c) - a_{ix0}\dot{x}_0\\ \dot{s}_{iy1} =&(d_{iy}+a_{iy0})(u_i\sin\psi_i + v_i\cos\psi_i) - \sum_{c=1}^{K} a_{iyc}(u_c\sin\psi_c + v_c\cos\psi_c) - a_{iy0}\dot{y}_0\\ \dot{s}_{i\psi 1} =&(d_{i\psi}+a_{i\psi 0})r_i - \sum_{c=1}^{K} a_{i\psi c}r_c - a_{i\psi 0}\dot{\psi}_0.\end{aligned} \tag{6}$$

Chose the Lyapunov function as

$$E_1 = \sum_{i=1}^{K}\frac{s_{ix1}^2}{2} + \frac{s_{iy1}^2}{2} + \frac{s_{i\psi 1}^2}{2} + \frac{\tilde{\theta}_{ix1}^2}{2a_{ix1}} + \frac{\tilde{\theta}_{iy1}^2}{2a_{iy1}} + \frac{\tilde{\theta}_{i\psi 1}^2}{2a_{i\psi 1}} \tag{7}$$

in which $a_{il1} > 0$ are the design parameters; $\tilde{\theta}_{ij1} = \theta_{ij1} - \hat{\theta}_{ij1}(i = 1,...K, j = x, y, \psi)$; $\theta_{ij1} = ||\Theta_{ij1}||^2$ is the unknown constant, where $\Theta_{ij1} = [\Theta_{ij11},...,\Theta_{ij1n}]^T$ is the unknown weight vector with the number of fuzzy rules n; The unknown constant θ_{ij1} can be estimated as $\hat{\theta}_{ij1}$.

From (6), the derivative of E_1 is obtained as

$$\begin{aligned}\dot{E}_1 =& \sum_{i=1}^{K} s_{ix1}(d_{ix}+a_{ix0})\Big[\cos\psi_i(s_{ix2}+\alpha_{ix}+f_{ix1}(Q_{ix1})) - \sin\psi_i(s_{iy2}+\alpha_{iy}+f_{iy1}(Q_{iy1}))\Big]\\ &+ s_{iy1}(d_{iy}+a_{iy0})\Big[\sin\psi_i(s_{iy2}+\alpha_{ix}+f_{ix1}(Q_{ix1})) + \cos\psi_i(s_{iy2}+\alpha_{iy}+f_{iy1}(Q_{iy1}))\Big]\\ &+ s_{i\psi 1}(d_{i\psi}+a_{i\psi 0})\Big[s_{i\psi 2}+\alpha_{i\psi}+f_{i\psi 1}(Q_{i\psi 1})\Big] + g_{ix1}\kappa_{ix1}s_{ix1}^4 + g_{iy1}\kappa_{iy1}s_{iy1}^4 + g_{i\psi 1}\kappa_{i\psi 1}s_{i\psi 1}^4\\ &- s_{ix1}^2(d_{ix}+a_{ix0})(\cos\psi_i + \sin\psi)/2 - s_{iy1}^2(d_{iy}+a_{iy0})(\sin\psi_i + \cos\psi)/2\\ &- s_{i\psi 1}^2(d_{i\psi}+a_{i\psi 0})/2 - \frac{\tilde{\theta}_{ix1}\dot{\hat{\theta}}_{ix1}}{a_{ix1}} - \frac{\tilde{\theta}_{iy1}\dot{\hat{\theta}}_{iy1}}{a_{iy1}} - \frac{\tilde{\theta}_{i\psi 1}\dot{\hat{\theta}}_{i\psi 1}}{a_{i\psi 1}}\end{aligned} \tag{8}$$

where $g_{ij1} > 0(j = x, y, \psi)$ are design parameters; $\kappa_{ij1} = (p-1)\varepsilon_{ij1}^{2p-4}$ with $\varepsilon_{ij1} > 0$ and $p = [(2n-1)/(2n+1)]$ $(n \geq 2, n \in N)$; $\hat{\theta}_{ij1}$ is the estimate of the unknown constant θ_{ij1};

And

$$\begin{aligned}f_{ix1}(Q_{ix1}) =& -\left(\frac{\cos\psi_i}{d_{ix}+a_{ix0}}\right)\Big[a_{ix0}\dot{x}_0 + \sum_{c=1}^{K} a_{ixc}\Big(\cos\psi_c u_c - \sin\psi_c v_c\Big)\\ &- (d_{ix}+a_{ix0})\left(\frac{\cos\psi_i + \sin\psi_i}{2}\right)s_{ix1} + g_{ix1}\kappa_{ix1}s_{ix1}^3\Big]\\ &-\left(\frac{\sin\psi_i}{d_{iy}+a_{iy0}}\right)\Big[a_{iy0}\dot{y}_0 + \sum_{c=1}^{K} a_{iyc}\Big(\sin\psi_c u_c + \cos\psi_c v_c\Big)\\ &- (d_{iy}+a_{iy0})\left(\frac{\cos\psi_i + \sin\psi_i}{2}\right)s_{iy1} + g_{iy1}\kappa_{iy1}s_{iy1}^3\Big]\\ f_{iy1}(Q_{iy1}) =& -\left(\frac{-\sin\psi_i}{d_{ix}+a_{ix0}}\right)\Big[a_{ix0}\dot{x}_0 + \sum_{c=1}^{K} a_{ixc}\Big(\cos\psi_c u_c - \sin\psi_c v_c\Big)\\ &- (d_{ix}+a_{ix0})\left(\frac{\cos\psi_i + \sin\psi_i}{2}\right)s_{ix1} + g_{ix1}\kappa_{ix1}s_{ix1}^3\Big]\\ &-\left(\frac{\cos\psi_i}{d_{iy}+a_{iy0}}\right)\Big[a_{iy0}\dot{y}_0 + \sum_{c=1}^{K} a_{iyc}\Big(\sin\psi_c u_c + \cos\psi_c v_c\Big)\\ &- (d_{iy}+a_{iy0})\left(\frac{\cos\psi_i + \sin\psi_i}{2}\right)s_{iy1} + g_{iy1}\kappa_{iy1}s_{iy1}^3\Big]\\ f_{i\psi 1}(Q_{i\psi 1}) =& -\left(\frac{1}{d_{i\psi}+a_{i\psi 0}}\right)\Big[a_{i\psi 0}\dot{\psi}_0 + \sum_{c=1}^{K} a_{i\psi c}r - (d_{i\psi}+a_{i\psi 0})\left(\frac{1}{2}\right)s_{i\psi 1} + g_{i\psi 1}\kappa_{i\psi 1}s_{i\psi 1}^3\Big].\end{aligned} \tag{9}$$

Utilizing the fuzzy logic systems in Lemma 3, for $\phi_{il1} > 0$ and $b_{il1} > 0$, we have

$$\begin{aligned}
&s_{ix1}(d_{ix} + a_{ix0}) \cos\psi_i f_{ix1}(Q_{ix1}) \\
\leq &(d_{ix} + a_{ix0}) \cos\psi_i \left(\frac{s_{ix1}^2 \theta_{ix1} \pi_{ix1}^T \pi_{ix1}}{2b_{ix1}^2} + \frac{b_{ix1}^2}{2} + \frac{s_{ix1}^2}{2} + \frac{\phi_{ix1}^2}{2} \right) \\
&- s_{ix1}(d_{ix} + a_{ix0}) \sin\psi_i f_{iy1}(Q_{iy1}) \\
\leq &(d_{ix} + a_{ix0}) \sin\psi_i \left(\frac{s_{ix1}^2 \theta_{iy1} \pi_{iy1}^T \pi_{iy1}}{2b_{iy1}^2} + \frac{b_{iy1}^2}{2} + \frac{s_{ix1}^2}{2} + \frac{\phi_{iy1}^2}{2} \right) \\
&s_{iy1}(d_{iy} + a_{iy0}) \sin\psi_i f_{ix1}(Q_{ix1}) \\
\leq &(d_{iy} + a_{iy0}) \sin\psi_i \left(\frac{s_{iy1}^2 \theta_{ix1} \pi_{ix1}^T \pi_{ix1}}{2b_{ix1}^2} + \frac{b_{ix1}^2}{2} + \frac{s_{iy1}^2}{2} + \frac{\phi_{ix1}^2}{2} \right) \\
&s_{iy1}(d_{iy} + a_{iy0}) \cos\psi_i f_{iy1}(Q_{iy1}) \\
\leq &(d_{iy} + a_{iy0}) \cos\psi_i \left(\frac{s_{iy1}^2 \theta_{iy1} \pi_{iy1}^T \pi_{iy1}}{2b_{iy1}^2} + \frac{b_{iy1}^2}{2} + \frac{s_{iy1}^2}{2} + \frac{\phi_{iy1}^2}{2} \right) \\
&s_{i\psi1}(d_{i\psi} + a_{i\psi0}) f_{i\psi1}(Q_{i\psi1}) \\
\leq &(d_{i\psi} + a_{i\psi0}) \left(\frac{s_{i\psi1}^2 \theta_{i\psi1} \pi_{i\psi1}^T \pi_{i\psi1}}{2b_{i\psi1}^2} + \frac{b_{i\psi1}^2}{2} + \frac{s_{i\psi1}^2}{2} + \frac{\phi_{i\psi1}^2}{2} \right).
\end{aligned} \tag{10}$$

Establish the distributed kinematic controller as

$$\begin{aligned}
\alpha_{ix} = &\left(\frac{\cos\psi_i}{d_{ix} + a_{ix0}} \right) \Bigg[- g_{ix1} \beta_{ix1}(s_{ix1}) - g'_{ix1} s_{ix1}^{2q-1} - (d_{ix} + a_{ix0}) s_{ix1} \Bigg(\frac{\cos\psi_i \hat{\theta}_{ix1} \pi_{ix1}^T \pi_{ix1}}{2b_{ix1}^2} \\
&+ \frac{-\sin\psi_i \hat{\theta}_{iy1} \pi_{iy1}^T \pi_{iy1}}{2b_{iy1}^2} \Bigg) \Bigg] + \left(\frac{\sin\psi_i}{d_{iy} + a_{iy0}} \right) \Bigg[- g_{iy1} \beta_{iy1}(s_{iy1}) - g'_{iy1} s_{iy1}^{2q-1} \\
&- (d_{iy} + a_{iy0}) s_{iy1} \left(\frac{\sin\psi_i \hat{\theta}_{ix1} \pi_{ix1}^T \pi_{ix1}}{2b_{ix1}^2} + \frac{\cos\psi_i \hat{\theta}_{iy1} \pi_{iy1}^T \pi_{iy1}}{2b_{iy1}^2} \right) \Bigg] \\
\alpha_{iy} = &\left(\frac{-\sin\psi_i}{d_{ix} + a_{ix0}} \right) \Bigg[- g_{ix1} \beta_{ix1}(s_{ix1}) - g'_{ix1} s_{ix1}^{2q-1} - (d_{ix} + a_{ix0}) s_{ix} \Bigg(\frac{\cos\psi_i \hat{\theta}_{ix1} \pi_{ix1}^T \pi_{ix1}}{2b_{ix1}^2} \\
&+ \frac{-\sin\psi_i \hat{\theta}_{iy1} \pi_{iy1}^T \pi_{iy1}}{2b_{iy1}^2} \Bigg) \Bigg] + \left(\frac{\cos\psi_i}{d_{iy} + a_{iy0}} \right) \Bigg[- g_{iy1} \beta_{iy1}(s_{iy1}) - g'_{iy1} s_{iy1}^{2q-1} \\
&- (d_{iy} + a_{iy0}) s_{iy1} \left(\frac{\sin\psi_i \hat{\theta}_{ix1} \pi_{ix1}^T \pi_{ix1}}{2b_{ix1}^2} + \frac{\cos\psi_i \hat{\theta}_{iy1} \pi_{iy1}^T \pi_{iy1}}{2b_{iy1}^2} \right) \Bigg] \\
\alpha_{i\psi1} = &\left(\frac{1}{d_{i\psi} + a_{i\psi0}} \right) \Bigg[- g_{i\psi1} \beta_{i\psi1}(s_{i\psi1}) - g'_{i\psi1} s_{i\psi1}^{2q-1} - (d_{i\psi} + a_{i\psi0}) s_{i\psi1} \left(\frac{\hat{\theta}_{i\psi1} \pi_{i\psi1}^T \pi_{i\psi1}}{2b_{i\psi1}^2} \right) \Bigg]
\end{aligned} \tag{11}$$

where $q > 1$, $g_{ix1} > 0$, $g_{iy1} > 0$, $g_{i\psi1} > 0$, $g'_{ix1} > 0$, $g'_{iy1} > 0$, and $g'_{i\psi1} > 0$ are design parameters , and

$$\begin{aligned}
\beta_{ix1}(s_{ix1}) = &\begin{cases} s_{ix1}^{2p-1}, & |s_{ix1}| \geq \varepsilon_{ix1}, \\ \varsigma_{ix1} s_{ix1} + \kappa_{ix1} s_{ix1}^3, & |s_{ix1}| < \varepsilon_{ix1}, \end{cases} \\
\beta_{iy1}(s_{iy1}) = &\begin{cases} s_{iy1}^{2p-1}, & |s_{iy1}| \geq \varepsilon_{iy1}, \\ \varsigma_{iy1} s_{iy1} + \kappa_{iy1} s_{iy1}^3, & |s_{iy1}| < \varepsilon_{iy1}, \end{cases} \\
\beta_{i\psi1}(s_{i\psi1}) = &\begin{cases} s_{i\psi1}^{2p-1}, & |s_{i\psi1}| \geq \varepsilon_{i\psi1}, \\ \varsigma_{i\psi1} s_{i\psi1} + \kappa_{i\psi1} s_{i\psi1}^3, & |s_{i\psi1}| < \varepsilon_{i\psi1}, \end{cases}
\end{aligned} \tag{12}$$

where $p = [(2n-1)/(2n+1)]$ $(n \geq 2, n \in N)$, $\varsigma_{ij1} = (2-p)\varepsilon_{ij1}^{2p-2}$ and $\kappa_{ij1} = (p-1)\varepsilon_{ij1}^{2p-4}$ with $\varepsilon_{ij1} > 0$.

Invoking (10)–(12) into (8), it follows that

$$\begin{aligned}\dot{E}_1 \leq & \sum_{i=1}^{K}\sum_{j=x}^{\psi} -g_{ij1}\beta_{ij1}s_{ij1} - g_{ij1}^{'}s_{ij1}^{2q} + \frac{\tilde{\theta}_{ij1}}{a_{ij1}}\left[\frac{a_{ij1}(d_{ix}+a_{ix0})R_{1j}^{(i)}s_{ix1}^2\pi_{ij1}^T\pi_{ij1}}{2b_{ij1}^2}\right.\\ & \left. + \frac{a_{ij1}(d_{iy}+a_{iy0})R_{2j}^{(i)}s_{iy1}^2\pi_{ij1}^T\pi_{ij1}}{2b_{ij1}^2} + \frac{a_{ij1}(d_{i\psi}+a_{i\psi0})R_{3j}^{(i)}s_{i\psi1}^2\pi_{ij1}^T\pi_{ij1}}{2b_{ij1}^2} - \dot{\hat{\theta}}_{ij1}\right]\\ & + (d_{ij}+a_{ij0})\left[\frac{R_{j1}^{(i)}(b_{ix1}^2+\phi_{ix1}^2)}{2} + \frac{R_{j2}^{(i)}(b_{iy1}^2+\phi_{iy1}^2)}{2} + \frac{R_{j3}^{(i)}(b_{i\psi1}^2+\phi_{i\psi1}^2)}{2}\right]\\ & + (d_{ij}+a_{ij0})s_{ij1}\left(R_{j1}^{(i)}s_{ix2} + R_{j2}^{(i)}s_{iy2} + R_{j3}^{(i)}s_{i\psi2}\right) + g_{ij1}c_{ij1}s_{ij1}^4. \end{aligned} \tag{13}$$

Remark 2. *Based on (12), we have that if* $s_{ij1} > 0$, $\beta_{ij1}(\varepsilon_{ij1}^{+}) = \lim_{s_{ij1}\to\varepsilon_{ij1}^{+}} s_{ij1}^{2p-1} = \varepsilon_{ij1}^{2p-1}$ *and* $\beta_{ij1}(\varepsilon_{ij1}^{-}) = \lim_{s_{ij1}\to\varepsilon_{ij1}^{-}} \varsigma_{ij1}s_{ij1} + \kappa_{ij1}s_{ij1}^3 = \varepsilon_{ij1}^{2p-1}$, *then we have* $\beta_{ij1}(\varepsilon_{ij1}^{+}) = \beta_{ij1}(\varepsilon_{ij1}^{-})$. *Conversely,* $\beta_{ij1}(-\varepsilon_{ij1}^{+}) = \beta_{ij1}(-\varepsilon_{ij1}^{-})$ *when* $s_{ij1} < 0$. *Likewise,* $\dot{\beta}_{ij1}(\varepsilon_{ij1}^{+}) = \lim_{s_{ij1}\to\varepsilon_{ij1}^{+}} (2p-1)s_{ij1}^{2p-2} = (2p-1)\varepsilon_{ij1}^{2p-2}$ *and* $\dot{\beta}_{ij1}(\varepsilon_{ij1}^{-}) = \lim_{s_{ij1}\to\varepsilon_{ij1}^{-}} \varsigma_{ij1} + 3\kappa_{ij1}s_{ij1}^2 = (2p-1)\varepsilon_{ij1}^{2p-2}$ *when* $s_{ij1} > 0$, *and* $\dot{\beta}_{ij1}(-\varepsilon_{ij1}^{+}) = \dot{\beta}_{ij1}(-\varepsilon_{ij1}^{-})$ *when* $s_{ij1} < 0$. *Then, the proposing function* $\beta_{ij1}(s_{ij1})$ *can be continuous differentiable, which ensures that the control laws (11) can also be continuous differentiable.*

Remark 3. *Notice that* s_{ij1}^{2p-1} *and its differentiation* $(2p-1)s_{ij1}^{2p-2}$ *may lead to the control singularity issue, that means* $2p-1 < 0$ *and* $2p-2 < 0$. *For avoiding the singularity problem, a novel function* $\beta_{ij1}(s_{ij1})$ *is designed. Clearly, when* $|s_{ij1}| < \varepsilon_{ij1}$, $\beta_{ij1}(s_{ij1}) = \varsigma_{ij1}s_{ij1} + \kappa_{ij1}s_{ij1}^3$, *the singularity issue is settled.*

Next, a distributed kinetics controller leverages the virtual speed signals to compute the surge force, sway force, and yaw moment, ensuring fixed-time stability of the closed-loop system.

Using $s_{i2} = [s_{ix2}, s_{iy2}, s_{i\psi2}]^T = [u_i - \alpha_{ix}, v_i - \alpha_{iy}, r_i - \alpha_{i\psi}]^T$, the derivatives of s_{ij2} are

$$\begin{aligned}\dot{s}_{ix2} &= [T_{iu} + f_{ix2}(Q_{ix2})]/M_{ix}\\ \dot{s}_{iy2} &= [T_{iv} + f_{iy2}(Q_{iy2})]/M_{iy}\\ \dot{s}_{i\psi2} &= [T_{ir} + f_{i\psi2}(Q_{i\psi2})]/M_{i\psi}\end{aligned} \tag{14}$$

where

$$\begin{aligned}f_{ix2}(Q_{ix2}) =& -C_{ix}u - D_{ix}u - M_{ix}\dot{\alpha}_{ix} + \frac{s_{ix2}}{2} + (d_{ix}+a_{ix0})R_{1j}^{(i)}s_{ix1} + (d_{iy}+a_{iy0})R_{2j}^{(i)}s_{iy1}\\ & + (d_{i\psi}+a_{i\psi0})R_{3j}^{(i)}s_{i\psi1} - g_{ix2}\kappa_{ix2}s_{ix2}^3\\ f_{iy2}(Q_{iy2}) =& -C_{iy}v - D_{iy}v - M_{iy}\dot{\alpha}_{iy} + \frac{z_{iy2}}{2} + (d_{ix}+a_{ix0})R_{1j}^{(i)}s_{ix1} + (d_{iy}+a_{iy0})R_{2j}^{(i)}s_{iy1}\\ & + (d_{i\psi}+a_{i\psi0})R_{3j}^{(i)}s_{i\psi1} - g_{iy2}\kappa_{iy2}s_{iy2}^3\\ f_{i\psi2}(Q_{i\psi2}) =& -C_{i\psi}r - D_{i\psi}r - M_{i\psi}\dot{\alpha}_{i\psi} + \frac{s_{i\psi2}}{2} + (d_{ix}+a_{ix0})R_{1j}^{(i)}s_{ix1} + (d_{iy}+a_{iy0})R_{2j}^{(i)}s_{iy1}\\ & + (d_{i\psi}+a_{i\psi0})R_{3j}^{(i)}s_{i\psi1} - g_{i\psi2}\kappa_{i\psi2}s_{i\psi2}^3.\end{aligned}$$

Select the Lyapunov function as

$$E_2 = E_1 + \sum_{i=1}^{K}\sum_{j=x}^{\psi}\frac{M_{ij}s_{ij2}^2}{2} + \frac{\tilde{\theta}_{ij2}^2}{2a_{ij2}}. \tag{15}$$

where $\tilde{\theta}_{ij2} = \theta_{ij2} - \hat{\theta}_{ij2}(i = 1,...K, j = x, y, \psi)$; $\theta_{ij2} = ||\Theta_{ij2}||^2$ is the unknown constant, where $\Theta_{ij2} = [\Theta_{ij21}, ..., \Theta_{ij2n}]$ is the unknown weight vector; The unknown constant θ_{ij2} can be estimated as $\hat{\theta}_{ij2}$.

Invoking (14) into (15), one obtains that

$$\begin{aligned}\dot{E}_2 =& \dot{E}_1 + \sum_{i=1}^{K}\sum_{j=x}^{\psi} s_{ij2}\Big(T_{ij} + f_{ij2}(Q_{ij2})\Big) - \frac{s_{ij2}^2}{2} - (d_{ij} + a_{ij0})s_{ij1}\Big(R_{j1}^{(i)} \\ &\times s_{ix2} + R_{j2}^{(i)}s_{iy2} + R_{j3}^{(i)}s_{i\psi 2}\Big) + g_{ij2}\kappa_{ij2}s_{ij2}^4 - \frac{\tilde{\theta}_{ij2}\dot{\hat{\theta}}_{ij2}}{a_{ij2}}.\end{aligned} \tag{16}$$

Akin to (10), we have

$$s_{ij2}f_{ij2}(Q_{ij2}) \leq \frac{s_{ij2}^2\theta_{ij2}\pi_{ij2}^T\pi_{ij2}}{2b_{ij2}^2} + \frac{b_{ij2}^2}{2} + \frac{s_{ij2}^2}{2} + \frac{\phi_{ij2}^2}{2}. \tag{17}$$

Propose the kinetic controller as

$$T_{ij} = -g_{ij2}\beta_{ij2} - g'_{ij2}s_{ij2}^{2q-1} - \frac{s_{ij2}\hat{\theta}_{ij2}\pi_{ij2}^T\pi_{ij2}}{2b_{ij2}^2} \tag{18}$$

where $g_{ij2} > 0$, $g'_{ij2} > 0$ and $b_{ij2} > 0$ are design parameters, and

$$\beta_{ij2}(s_{ij2}) = \begin{cases} s_{ij2}^{2p-1}, & |s_{ij2}| \geq \varepsilon_{ij2}, \\ \varsigma_{ij2}s_{ij2} + \kappa_{ij2}s_{ij2}^3, & |s_{ij2}| < \varepsilon_{ij2}, \end{cases} \tag{19}$$

where $\varsigma_{ij2} = (2-p)\varepsilon_{ij2}^{2p-2}$ and $\kappa_{ij2} = (p-1)\varepsilon_{ij2}^{2p-4}$ with $\varepsilon_{ij2} > 0$.

Invoking (17) and (18) into (16) results in

$$\begin{aligned}\dot{E}_2 \leq& \sum_{i=1}^{K}\sum_{j=x}^{\psi} + \Big(\sum_{o=1}^{2} -g_{ijo}\beta_{ijo}s_{ijo} - g_{ijo}^{'}s_{ijo}^{2q}\Big) + (d_{ij} + a_{ij0}) \\ &\times \Big[\frac{R_{jx}^{(i)}(b_{ix1}^2 + \phi_{ix1}^2)}{2} + \frac{R_{j2}^{(i)}(b_{iy1}^2 + \phi_{iy1}^2)}{2} + \frac{R_{j3}^{(i)}(b_{i\psi 1}^2 + \phi_{i\psi 1}^2)}{2}\Big] \\ &+ \frac{b_{ij2}^2}{2} + \frac{\phi_{ij2}^2}{2} + \frac{\tilde{\theta}_{ij1}}{a_{ij1}}\Big[\frac{a_{ij1}(d_{ix} + a_{ix0})R_{1j}^{(i)}s_{ix1}^2\pi_{ij1}^T\pi_{ij1}}{2b_{ij1}^2} \\ &+ \frac{a_{ij1}(d_{iy} + a_{iy0})R_{2j}^{(i)}s_{iy1}^2\pi_{ij1}^T pi_{ij1}}{2b_{ij1}^2} + \frac{s_{ij1}(d_{i\psi} + a_{i\psi 0})R_{3j}^{(i)}s_{i\psi 1}^2\pi_{ij1}^T\pi_{ij1}}{2b_{ij1}^2} \\ &- \dot{\hat{\theta}}_{ij1}\Big] + \frac{\tilde{\theta}_{ij2}}{a_{ij2}}\Big(\frac{a_{ij2}s_{ij2}^2 pi_{ij2}^T\pi_{ij2}}{2b_{ij2}^2} - \dot{\hat{\theta}}_{ij2}\Big) + \sum_{o=1}^{2} g_{ijo}\kappa_{ijo}s_{ijo}^4.\end{aligned} \tag{20}$$

Design the updating laws as

$$\begin{aligned}\dot{\hat{\theta}}_{ij1} =& \frac{a_{ij1}R_{1j}^{(i)}(d_{ix}+a_{ix0})s_{ix1}^2\pi_{ij1}^T\pi_{ij1}}{2b_{ij1}^2} + \frac{a_{ij1}R_{2j}^{(i)}(d_{iy}+a_{iy0})s_{iy1}^2\pi_{ij1}^T\pi_{ij1}}{2b_{ij1}^2} \\ &+ \frac{a_{ij1}R_{3j}^{(i)}(d_{i\psi}+a_{i\psi0})s_{i\psi1}^2\pi_{ij1}^T\pi_{ij1}}{2b_{ij1}^2} - \lambda_{ij1}\hat{\theta}_{ij1} - \lambda'_{ij1}\hat{\theta}_{ij1}^{2q-1} \\ \dot{\hat{\theta}}_{ij2} =& \frac{a_{ij2}s_{ij2}^2\pi_{ij2}^T\pi_{ij2}}{2b_{ij2}^2} - \lambda_{ij2}\hat{\theta}_{ij2} - \lambda'_{ij2}\hat{\theta}_{ij2}^{2q-1}.\end{aligned} \tag{21}$$

Remark 4. *In the conventional adaptive control, the parameter update law is commonly developed as* $\dot{\hat{\theta}}_{ij1} = \frac{a_{ij1}R_{1j}^{(i)}(d_{ix}+a_{ix0})s_{ix1}^2\pi_{ij1}^T\pi_{ij1}}{2b_{ij1}^2} + \frac{a_{ij1}R_{2j}^{(i)}(d_{iy}+a_{iy0})s_{iy1}^2\pi_{ij1}^T\pi_{ij1}}{2b_{ij1}^2} + \frac{a_{ij1}R_{3j}^{(i)}(d_{i\psi}+a_{i\psi0})s_{i\psi1}^2\pi_{ij1}^T\pi_{ij1}}{2b_{ij1}^2} - \lambda_{ij1}\hat{\theta}_{ij1}$ *and* $\dot{\hat{\theta}}_{ij2} = \frac{a_{ij2}s_{ij2}^2\pi_{ij2}^T\pi_{ij2}}{2b_{ij2}^2} - \lambda_{ij2}\hat{\theta}_{ij2}$. *The conventional approach, which relies on the linear differential equation method, fails to meet the fixed-time stability criterion stipulated in Lemma 2. Consequently, this limitation necessitates the development of a novel parameter adaptation law, given by (21). The incorporation of the nonlinear terms* $-\lambda'_{ij1}\hat{\theta}_{ij1}^{2q-1}$ *and* $-\lambda'_{ij2}\hat{\theta}_{ij2}^{2q-1}$ *in (21) guarantees that the fixed-time system performance will be established in the subsequent stability analysis.*

3.2. Stability Analysis

Theorem 1. *Based on the kinematic controller (11), the kinetic controller (18), and the updating laws (21), the following results hold: (1) the designed strategy guarantees that consensus tracking errors reach a small region around zero within fixed-time, and the convergence time is limited as* $S \le S_{\max} = \frac{1}{\chi_2\omega(1-p)} + \frac{1}{\chi_3(18K)^{1-q}\omega(q-1)}(0<\omega<1)$; *and (2) the closed-loop system ensures boundedness of all its signals.*

Proof. Invoking (21) into (20) gives

$$\begin{aligned}\dot{E}_2 \le& \sum_{i=1}^{K}\sum_{j=x}^{\psi}\left(\sum_{o=1}^{2} -g_{ijo}\beta_{ijo}s_{ijo} - g'_{ijo}s_{ijo}^{2p} + \frac{\lambda_{ijo}\tilde{\theta}_{ijo}\hat{\theta}_{ijo}}{a_{ijo}} + \frac{\lambda'_{ijo}\tilde{\theta}_{ijr}\hat{\theta}_{ijo}^{2q-1}}{a_{ijo}}\right) \\ &+ \frac{a_{ij2}^2}{2} + (d_{ij}+a_{ij0})\left[\frac{R_{j1}^{(i)}(a_{ix1}^2+\phi_{ix1}^2)}{2} + \frac{R_{j2}^{(i)}(a_{iy1}^2+\phi_{iy1}^2)}{2} + \frac{R_{j3}^{(i)}(a_{i\psi1}^2+\phi_{i\psi1}^2)}{2}\right] \\ &+ \frac{\phi_{ij2}^2}{2} + \frac{\lambda_{ijo}\tilde{\theta}_{ijo}\hat{\theta}_{ijo}}{a_{ijo}} + \frac{\lambda'_{ijo}\tilde{\theta}_{ijo}\hat{\theta}_{ijo}^{2q-1}}{a_{ijo}} + \sum_{o=1}^{2} g_{ijo}c_{ijo}s_{ijo}^4.\end{aligned} \tag{22}$$

From the Young's inequality [33–35], one has

$$\frac{\lambda_{ijo}\tilde{\theta}_{ijo}\hat{\theta}_{ijr}}{s_{ijo}} \le -\frac{\lambda_{ijo}}{2a_{ijo}}\tilde{\theta}_{ijo}^2 + \frac{\lambda_{ijo}}{2a_{ijo}}\theta_{ijo}^2.$$

From Lemmas 3–4 in [36], one has

$$\frac{\lambda'_{ijo}\tilde{\theta}_{ijo}\hat{\theta}_{ijo}^{2q-1}}{a_{ijo}} \le \frac{\lambda'_{ijo}}{qa_{ijo}}\theta_{ijo}^{2q} - \frac{\lambda'_{ijo}}{2qa_{ijo}}\tilde{\theta}_{ijo}^{2q}. \tag{23}$$

Case 1: If $|s_{ijo}| < \varepsilon_{ijo}$, it yields that

$$\begin{aligned}\dot{E}_2 \leq & \sum_{i=1}^{K}\sum_{j=x}^{\psi} -2g_{ij1}\varsigma_{ij1}\frac{s_{ij1}^2}{2} - \frac{2g_{ij2}\varsigma_{ij2}}{M_{ij}}\frac{M_{ij}s_{ij2}^2}{2} + \Bigg(\sum_{o=1}^{2} -\lambda_{ijo}\frac{\tilde{\theta}_{ijo}^2}{2a_{ijo}} \\ & + \frac{\lambda_{ijo}^{'}}{qa_{ijo}}\theta_{ijo}^{2q} + \frac{\lambda_{ijo}}{2a_{ijo}}\theta_{ijo}^2\Bigg) + \frac{b_{ij2}^2}{2} + \frac{\phi_{ij2}^2}{2} + (d_{ij}+a_{ij0})\Bigg[\frac{R_{j1}^{(i)}(a_{ix1}^2+\phi_{ix1}^2)}{2} \\ & + \frac{R_{j2}^{(i)}(a_{iy1}^2+\phi_{iy1}^2)}{2} + \frac{R_{j3}^{(i)}(a_{i\psi1}^2+\phi_{i\psi1}^2)}{2}\Bigg] \\ \leq & -\chi_1 V_2 + \sigma_1 \end{aligned} \tag{24}$$

where $\chi_1 = \min\{\ 2g_{ij1}\varsigma_{ij1}, \frac{2g_{ij2}\varsigma_{ij2}}{M_{ij}}, \lambda_{ijo}$, and $\sigma_1 = \sum_{i=1}^{K}\sum_{j=x}^{\psi}\frac{\lambda_{ijo}^{'}}{qa_{ijr}}\theta_{ijo}^{2q} + \frac{\lambda_{ijo}}{2a_{ijo}}\theta_{ijo}^2 + (d_{ij}+a_{ij0})\Big[\frac{R_{j1}^{(i)}(b_{ix1}^2+\phi_{ix1}^2)+R_{j2}^{(i)}(b_{iy1}^2+\phi_{iy1}^2)+R_{j3}^{(i)}(a_{i\psi1}^2+\pi_{i\psi1}^2)}{2}\Big] + \frac{a_{ij2}^2}{2} + \frac{\phi_{ij2}^2}{2}$.

Applying the integrating factor $\exp^{\chi_1 t}$ to (24) results in

$$d(\exp^{\chi_1 t}E_2)/dt \leq \exp^{\chi_1 t}\sigma_1. \tag{25}$$

Figuring (25) gives

$$E_2(t) \leq E_2(0) + \frac{\sigma_1}{\chi_1}. \tag{26}$$

Hence, for $|s_{ijo}| < \varepsilon_{ijr}$, the closed-loop signals achieve semi-global uniform ultimate bounded.

Case 2: If $|s_{ijo}| \geq \varepsilon_{ijo}$. According to Lemma 2 in [37], we have

$$-\frac{\lambda_{ijo}}{2a_{ijo}}\tilde{\theta}_{ijo}^2 \leq \quad -\Big(\frac{\lambda_{ijo}}{2a_{ijo}}\tilde{\theta}_{ijo}^2\Big)^p + (1-p)p^{\frac{p}{1-p}}. \tag{27}$$

Invoking (27) into (22), one has

$$\begin{aligned}\dot{E}_2 \leq & \sum_{i=1}^{K}\sum_{j=x}^{\psi} -2^p g_{ij1}\Big(\frac{s_{ij1}^2}{2}\Big)^p - \Big(\frac{2}{M_{ij}}\Big)^p g_{ij2}\Big(\frac{M_{ij}s_{ij2}^2}{2}\Big)^p - \sum_{o=1}^{2}\sigma_{ijo}^p\Big(\frac{\tilde{\theta}_{ijr}^2}{2a_{ijo}}\Big)^p \\ & - 2^q g_{ij1}^{'}\Big(\frac{s_{ij1}^2}{2}\Big)^p - \Big(\frac{2}{M_{ij}}\Big)^q g_{ij2}^{'}\Big(\frac{M_{ij}s_{ij2}^2}{2}\Big)^q - \sum_{o=1}^{2}\frac{2^q a_{ijo}^{q-1}\lambda_{ijo}^{'}}{2q}\Big(\frac{\tilde{\theta}_{ijo}^2}{2a_{ijo}}\Big)^q + \sigma_2 \\ \leq & -\chi_2 E_2^p - \chi_3(18K)^{1-q}E_2^q + \sigma_2, \end{aligned} \tag{28}$$

where $\sigma_2 = \sigma_1 + 3(1-p)p^{\frac{p}{1-p}}$, $\chi_2 = \min\Big\{\ 2^p g_{ij1}, \Big(\frac{2}{M_{ij}}\Big)^p g_{ij2}, \lambda_{ijo}^p\Big\}$, and $\chi_3 = \min\Big\{\ 2^q g_{ij1}^{'}, \Big(\frac{2}{M_{ij}}\Big)^q g_{ij2}^{'}, \frac{2^q a_{ijo}^{q-1}\lambda_{ijo}^{'}}{2q}\Big\}$.

Based on the Lemma 2, we get that

$$S \leq S_{\max} = \frac{1}{\chi_2\omega(1-p)} + \frac{1}{\chi_3(18K)^{1-q}\omega(q-1)}. \tag{29}$$

Furthermore, all error signals remain bounded within the range

$$
\begin{aligned}
|s_{ij1}| &\leq \sqrt{2}\Big(\frac{\sigma_2}{(1-\omega)\chi_2}\Big)^{\frac{1}{2p}} \\
|s_{ij2}| &\leq \sqrt{\frac{2}{M_{ij}}}\Big(\frac{\sigma_2}{(1-\omega)\chi_2}\Big)^{\frac{1}{2p}} \\
|\tilde{\theta}_{ij1}| &\leq \sqrt{2a_{ij1}}\Big(\frac{\sigma_2}{(1-\omega)\chi_2}\Big)^{\frac{1}{2p}} \\
|\tilde{\theta}_{ij2}| &\leq \sqrt{2a_{ij2}}\Big(\frac{\sigma_2}{(1-\omega)\chi_2}\Big)^{\frac{1}{2p}}.
\end{aligned} \tag{30}
$$

According to (30), the boundedness of $\hat{\theta}_{ijo}$ is ensured, thus we have that α_{ij} and T_{ij} are bounded. From the definition of E_2 and Lemma 3 in [38], for $t \geqslant S$, one obtains

$$
\begin{aligned}
||x + \xi_x - \overline{x}_0|| &\leq \frac{||s_{x1}||}{\lambda_{\min x}\{L_x + F_x\}} \\
||y + \xi_y - \overline{y}_0|| &\leq \frac{||s_{y1}||}{\lambda_{\min y}\{L_y + F_y\}} \\
||\psi - \overline{\psi}_0|| &\leq \frac{||s_{\psi 1}||}{\lambda_{\min \psi}\{L_\psi + F_\psi\}}
\end{aligned} \tag{31}
$$

where $\overline{x}_0 = [x_0, ..., x_0]^T$, $\overline{y}_0 = [y_0, ..., y_0]^T$, $\overline{\psi}_0 = [\psi_0, ..., \psi_0]^T$, $s_{x1} = [s_{1x1}, ..., s_{Kx1}]^T$, $s_{y1} = [s_{1y1}, ..., s_{Ky1}]^T$, $s_{\psi 1} = [s_{1\psi 1}, ..., s_{K\psi 1}]^T$, $x = [x_1, ..., x_K]^T$, $y = [y_1, ..., y_K]^T$, $\psi = [\psi_1, ..., \psi_K]^T$, $\xi_x = [\xi_{1x}, ..., \xi_{Kx}]^T$, and $\xi_y = [\xi_{1y}, ..., \xi_{Ky}]^T$; $\lambda_{\min j}\{L_j + F_j\}$ is the minimum singular value of $L_j + F_j$. That is, the consensus error remains in a small neighborhood of origin after the fixed time. □

4. Simulation Validation

The multi-USV systems operate under the directed communication topology illustrated in Figure 3, with five USVs maintaining bidirectional information exchange. The leader signal is given as

$$
\begin{bmatrix} x_0 \\ y_0 \\ \psi_0 \end{bmatrix} = \begin{bmatrix} 10\cos(0.1t) \\ 10\sin(0.1t + 10) \\ \text{atan2}(\frac{\dot{y}_0}{\dot{x}_0}) \end{bmatrix}.
$$

The control parameters are selected in Table 1. The initial states are selected in Table 2. The fuzzy sets of fuzzy logic systems $\Theta_{ijl}^T \pi_{ijl}(Q_{ijl})(l = 1, 2)$ are selected over the interval $[-\frac{11}{2}, \frac{11}{2}]$. For $n = 1, ..., 12$, define $Q_{ij1} = [x_i, y_i, \psi_i]^T$ and $Q_{ij2} = [u_i, v_i, r_i]^T$,

$$
\begin{aligned}
Q_{ij1}^0 &= \underbrace{\Big[[-13/2 + n, -13/2 + n]^T, ..., [-13/2 + n, -13/2 + n]^T\Big]}_{3}^T \\
Q_{ij2}^0 &= \underbrace{\Big[[-13/2 + n, -13/2 + n]^T, ..., [-13/2 + n, -13/2 + n]^T\Big]}_{3}^T.
\end{aligned}
$$

Hence, the fuzzy membership functions of fuzzy logic systems are selected as $\mu_{ijl^n}(Q_{ijl}) = \exp(-(Q_{ijl} - Q_{ijl}^0)^T(Q_{ijl} - Q_{ijl}^0)/2)$. Then, the fuzzy basis function vectors are given as $\pi_{ijl}(Q_{ijl}) = [\pi_{ijl,1}(Q_{ijl}), \pi_{ijl,2}(Q_{ijl}), ..., \pi_{ijl,12}(Q_{ijl})]$ with $\pi_{ijl,n}(Q_{ijl}) = \frac{\mu_{ijl^n}}{\sum_{n=1}^{12} \mu_{ijl}^n}$.

Figure 4 shows that the actual position of four USVs agrees on the leader position with obstacle avoidance strategy. Figure 5 exhibits the yaw angles of four USVs can track

the given reference yaw angle to realize the orientation consensus. Figure 6 describes the bounded speed curves in surge, sway, and yaw. The actual surge force, sway force, and yaw moment of four USVs are shown in Figure 7, which are acceptable.

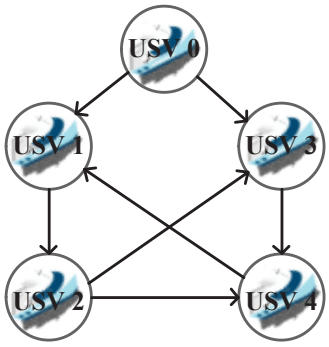


Figure 3. Communication network of multi-USV systems.

Table 1. Control parameters.

Symbol	Interpretation
$g_{i11}, g_{i12}, g_{i21}, g_{i22}, g_{i31}, g_{i32}$	2, 2.5, 15, 13, 5, 34
$g'_{i11}, g'_{i12}, g'_{i21}, g'_{i22}, g'_{i31}, g'_{i32}$	0.5, 0.3, 1.3, 4, 0.15, 3.4
q, p	197/101, 97/99
$\lambda_{ij1}, \lambda_{ij2}$	0.4, 0.4
b_{ij1}, b_{ij2}	3, 3

Table 2. Initial states.

Symbol	Interpretation
$x_1(0), x_2(0), x_3(0), x_4(0)$	$-2, -3, 0, -1$
$y_1(0), y_2(0), y_3(0), y_4(0)$	$0, -2, 0.1, 0.1$
$\psi_1(0), \psi_2(0), \psi_3(0), \psi_4(0)$	0.1, 0.1, 0.1, 0.1
$u_i(0), v_i(0), r_i(0)$	0, 0, 0

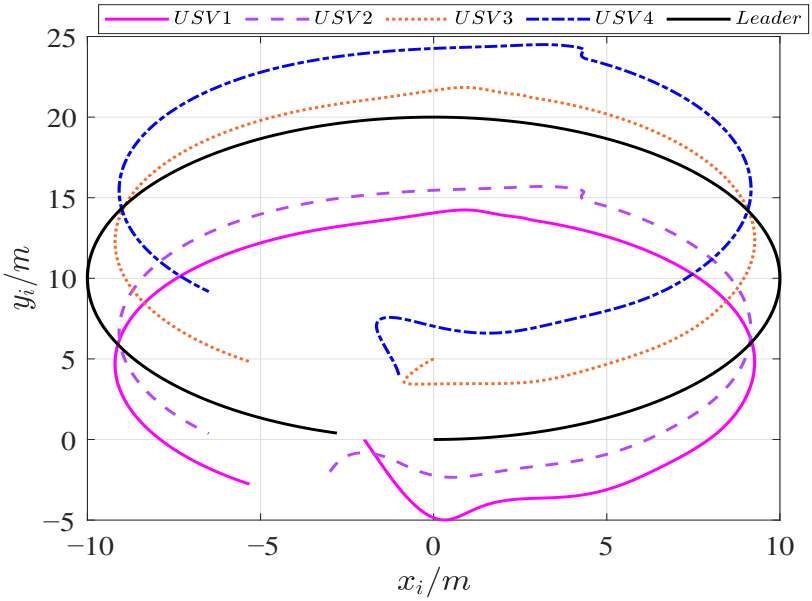


Figure 4. Position consensus control performance of multiple USVs.

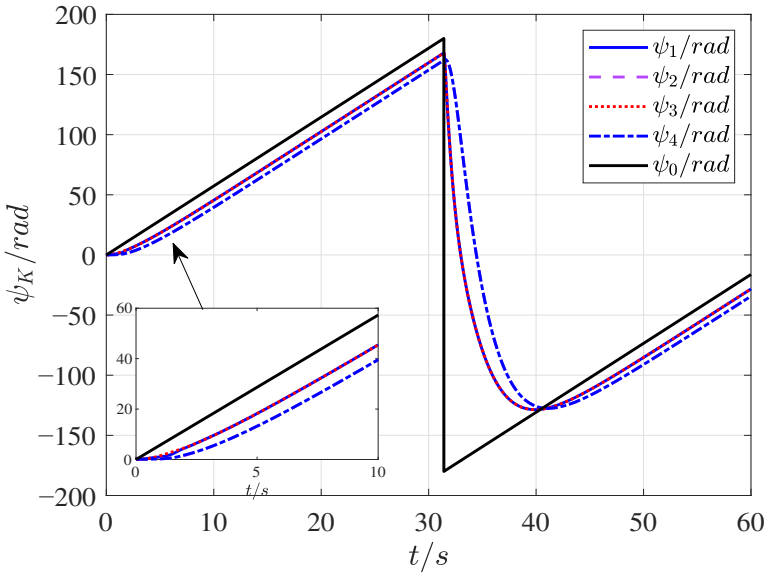


Figure 5. Orientation consensus control performance multiple USVs.

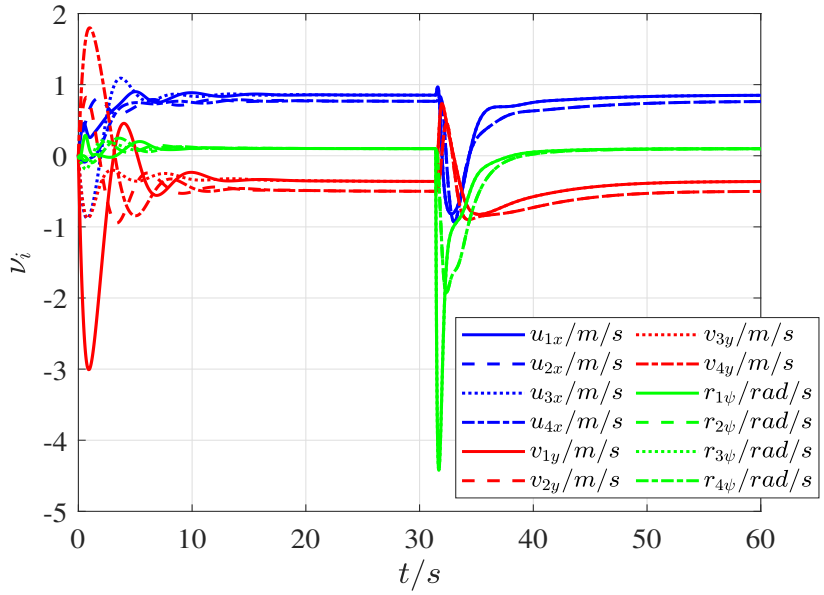


Figure 6. Linear speed and yaw rate of follower USVs.

To evaluate the resilience of the proposed control method under practical marine conditions, a robustness testing is developed, where the influence of waves, wind, and currents is emulated. In the simulations, environmental disturbances are represented as Gaussian random noise. Following the approach in [39], high-frequency wave-induced motions are generated via a second-order bandstop filter, whereas low-frequency perturbations resulting from wave drift, currents, and wind in the yaw direction are modeled using a first-order transfer function. Thus, we consider the disturbances as $\tau_u^D = \sin(\psi)\overline{P}(s)$, $\tau_v^D = \cos(\psi)\overline{P}(s)$, and $\tau_r^D = P'(s)$, in which $\overline{P}(s)$ and $P'(s)$ denote the high-frequency wave motion and the slow-varying environmental disturbances, respectively. Chose reference leader as $[x_0, y_0, \psi_0]^T = [10\sin(0.1t + 10), t, \text{atan2}(\frac{\dot{y}_0}{\dot{x}_0})]^T$. The configuration parameters remain consistent with those used in the earlier simulation. The cooperative tracking performance can be realized under real-world marine disturbances using the strategy of this paper, showing Figures 8 and 9. To demonstrate the superiority of the proposed fixed-time control method, the comparisons of consensus error convergence between our control strategy and the traditional consensus tracking strategy in [15] are shown in Figure 10 and

Table 3. It is evident that the proposed fixed-time method in this paper can achieve faster and lower convergence of the consensus errors.

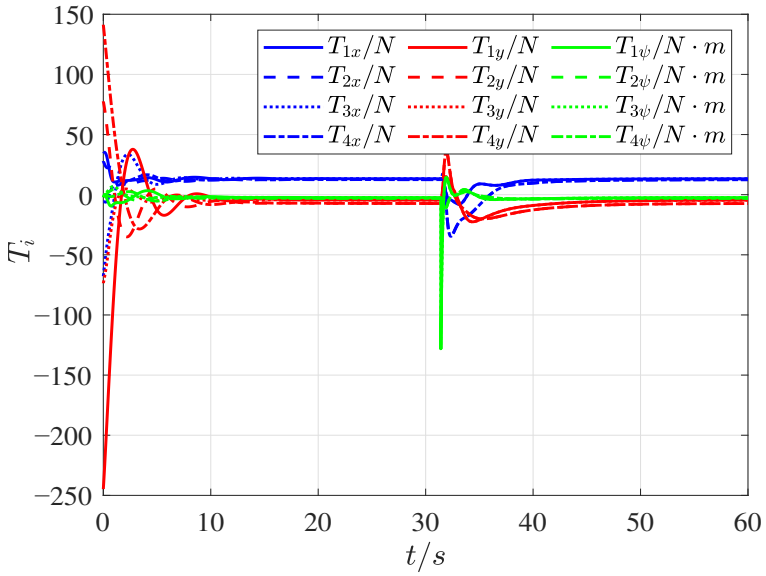


Figure 7. Control inputs.

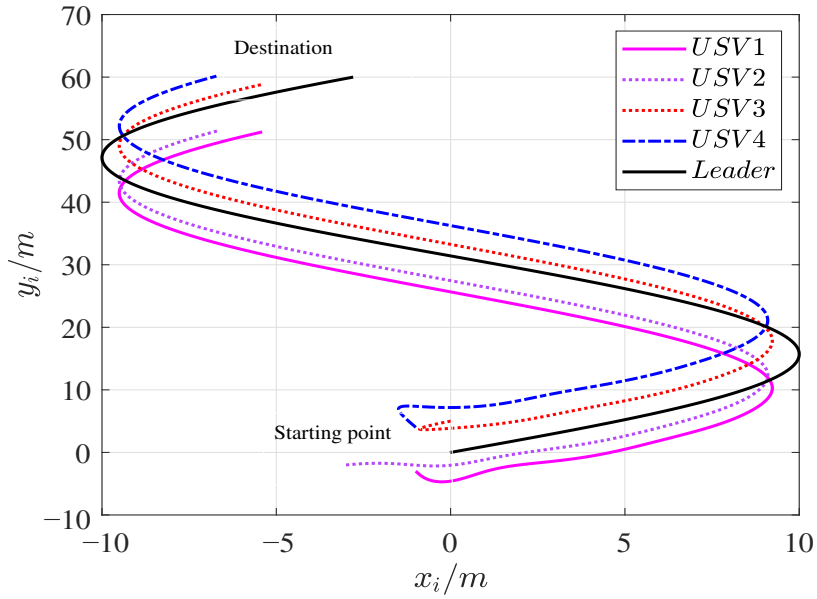


Figure 8. Position consensus control performance under real-world marine disturbances.

Table 3. Comparation of consensus errors.

Metric	Under Our Control Method	Under the Control Method [15]
Average consensus error of USV 1 in surge s_{1x1}	$-0.0124/\mathrm{m}$	$-0.0925/\mathrm{m}$
Average consensus error of USV 1 in sway s_{1y1}	$-0.3206/\mathrm{m}$	$-1.0781/\mathrm{m}$
Average consensus error of USV 1 in yaw $s_{1\psi1}$	$-0.320/\mathrm{rad}$	$-3.8166/\mathrm{rad}$

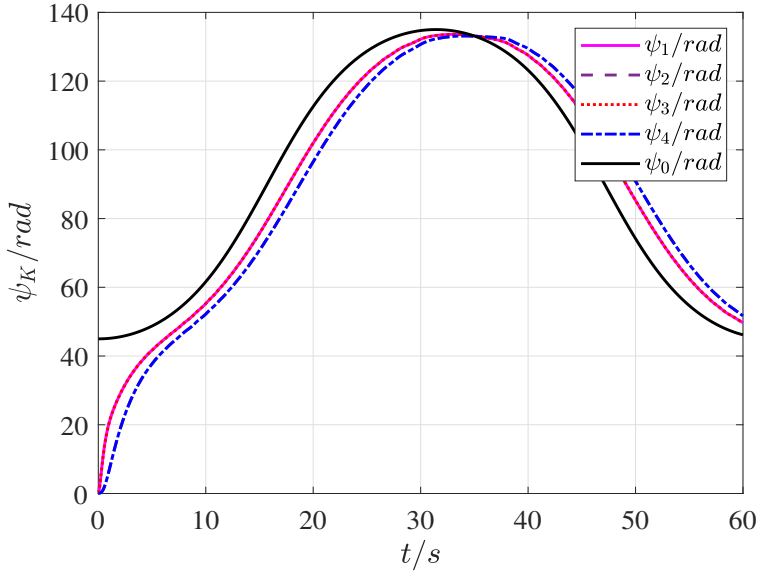


Figure 9. Orientation consensus control performance under real-world marine disturbances.

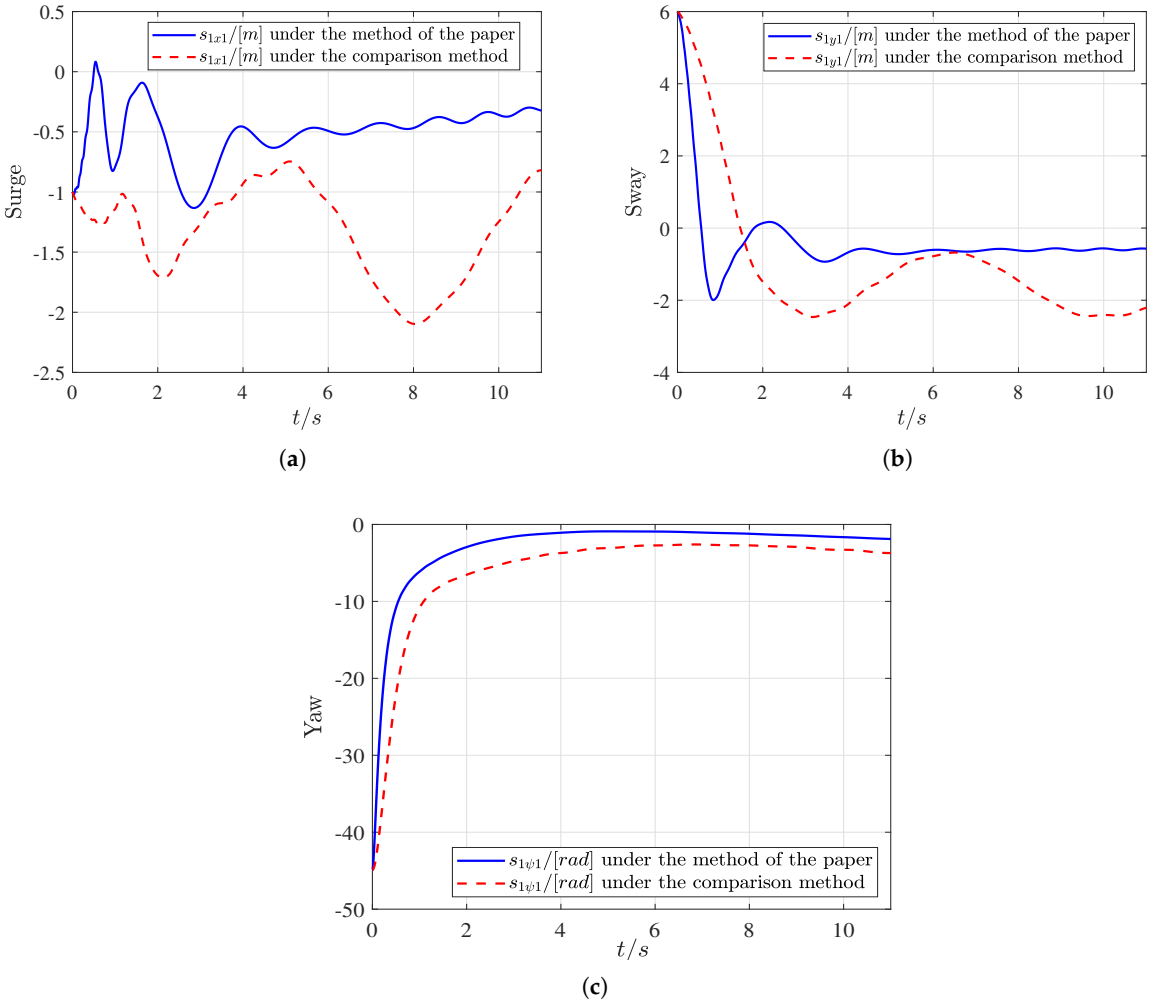


Figure 10. Comparisons of consensus errors of the USV 1. (**a**) In surge. (**b**) In sway. (**c**) In yaw.

5. Conclusions

This article has addressed the fixed-time cooperative tracking control problem for USVs with model uncertainties. By integrating the approaches of fixed-time control and

fuzzy approximation-based adaptive backstepping control under a directed topology, we have developed a distributed fuzzy adaptive fixed-time control algorithm. A novel switching function has been designed in the distributed fixed-time controllers to avoid the singularity problem in the backstepping design framework. Based on the fixed-time stability theory, the proposed control scheme guarantees that consensus tracking errors reach a small region around zero within a fixed time. Finally, simulation results have demonstrated the effectiveness of the developed approach. Future work will focus on addressing actuator thrust/torque saturation in practical applications. We will explore integrating anti-windup compensation mechanisms such as auxiliary dynamic systems or disturbance observer-based techniques into the proposed fixed-time control framework to ensure both stability and consensus performance under input constraints.

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