

Article

Flow-Induced Vibrations of a Square Cylinder in the Combined Steady and Oscillatory Flow

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Abstract

The paper presents a two-dimensional RANS–SST k – ω investigation of vortex-induced vibration of a square cylinder with two degrees of freedom under combined steady and oscillatory flow at the Reynolds number of 5000, Keulegan–Carpenter number of 10, mass ratio of 2.5, and zero structural damping. Flow ratio a (steady-to-total velocity) is varied from 0 to 1.0, and the reduced velocity U_r from 2 to 25 to map lock-in regimes, response amplitudes, frequency content, hydrodynamic loads, trajectories, and wake patterns. At low $a \leq 0.4$, in-line vibrations dominate at $U_r > 5$, with double-frequency transverse lock-in peaking near $U_r = 5$. As $a \rightarrow 1.0$, in-line motion diminishes, and single-frequency transverse oscillation prevails, with the maximum transverse displacement up to $0.54D$. The mean drag coefficient increases with increasing flow ratio; the fluctuating drag coefficient decreases with increasing a ; while the lift coefficient peaks at $a = 1$, $U_r = 2$. Wake topology transitions from a mixed vortex shedding towards a 2S pattern, as $a \rightarrow 1$.

Keywords: vortex-induced vibration (VIV); square cylinder; combined steady–oscillatory flow; lock-in; RANS–SST k – ω simulation

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1. Introduction

Subsea components, supports, and platform legs in offshore systems are frequently subjected to combined flows induced by both waves and tides [1,2]. Many of these components have square or rectangular cross-sections, including long and slender structures. Unlike circular geometries, square cross-sections introduce sharp corners that define flow separation points and leading to distinct wake dynamics, significantly affecting hydrodynamic loading and vibration characteristics. A combined steady and oscillatory flow results in unsteady fluid–structure interactions and, together with complex cross-sectional shapes, can amplify fatigue accumulation and impact structural integrity. Despite the prevalence of combined flows in real-world marine environments, the dynamics of square structures under such conditions remains insufficiently characterized. A deeper understanding of these interactions is critical for the continuous reliability of offshore systems.

Circular structures, due to their interaction with the incoming flow, experience vortex-induced vibration (VIV) lock-in [3–5], which occurs when the vortex shedding frequency becomes close to the structural natural frequency. The lock-in state manifests as a locking of the vibration frequency with increased vibration amplitudes. For 1-degree-of-freedom (1DOF) structures, the lock-in peak in terms of vibration amplitudes in the reduced velocity range presents a combination of initial, upper and lower branches. Two-degrees-of-freedom (2DOF) structures can experience substantially larger displacement amplitudes, with the

highest amplitudes forming what is referred to as a super-upper branch. The development of lock-in depends on multiple factors, including structural parameters such as mass [6,7], damping [8] and aspect [9] ratios, as well as flow characteristics, like Reynolds number Re [8,10], sheared flow parameter [10], oscillations in the flow velocity [2,11], etc.

A fundamental comparison of VIV lock-in in circular and square structures is undertaken in [12], which considers varying angles of attack for a square cylinder. The study focuses on 1DOF structures and finds that the diamond configuration of the square cylinder leads to vibration amplitudes and frequencies similar to those of circular cylinders. It also observes that galloping instability in square structure can occur at incidence angles between 0° and 10° , depending on the selected mass-damping ratio. The research shows that square cylinders can experience both vortex-induced vibration and galloping, and that an asymmetric configuration can produce a new branch in the lock-in response. This new branch features overall higher vibration amplitudes and a frequency approximately half of that of the upper branch in the circular cylinder lock-in peak. The impact of flow incidence angle on forces acting on a stationary square cylinder is further discussed in [13–15], and its effect on lock-in behavior in oscillating square cylinders under uniform flow is explored in [16–18]. In [15], a minimum drag coefficient is observed at an incidence angle of 22.5° .

The influence of the Reynolds number on hydrodynamic forces, vortex dynamics, and lock-in behavior of a square cylinder is studied in [19,20] for low Reynolds numbers ($Re \leq 170$), in [21] for moderate Reynolds numbers ($Re = 150\text{--}500$), in [14,22] for high Reynolds numbers ($Re \geq 4000$), and comprehensively in [23] for $Re \leq 10^7$. At low Reynolds numbers [20], the drag force on a square cylinder is found to exceed that of a circular cylinder. The work in [21] indicates a transition from 2D to 3D vortex shedding in the range $Re = 150\text{--}200$, and a pulsating nature of fluid forces in the range $Re = 200\text{--}300$. The study also finds an increasing lift force with increasing Reynolds number from 300 to 500. In [23], the shear layer transition I regime is observed in the range $220 \leq Re \leq 1000$, and the shear layer transition II regime appears at $Re > 1000$, where the mean drag coefficient for a stationary square structure becomes nearly constant, with no observed drag crisis regime.

The impact of mass and damping ratios on VIV and galloping of a square structure is investigated in [24]. The study reports that the displacement amplitude of a square cylinder decreases with increasing mass ratio. The effect of the damping ratio on vibration and hydrodynamic forcing is observed to differ between VIV and galloping. An increase in the damping ratio leads to a reduction in the displacement amplitude of VIV and a decrease in the mean drag coefficient. The impact of mass ratio is also examined in [19] at a Reynolds number of 170 for both VIV and galloping regimes. This work finds that galloping begins at a lower reduced velocity and exhibits higher displacement amplitudes at larger mass ratios, whereas in VIV, a larger mass ratio results in reduced vibration amplitudes.

To address the gap between modeling approximations and real-world unsteady, non-uniform flows, square cylinders have been studied under turbulent [25], sheared [26,27], accelerating [28–30], oscillatory [31–33] and combined [1,34,35] flows, particularly in recent years. In [28], Gaussian-type accelerating flows in the range of $Re = 1.72 \times 10^4\text{--}6.536 \times 10^4$ are simulated in 3D using the LES method to obtain drag and lift coefficients, Strouhal number, and pressure density functions. The study reveals discontinuities in the vortex shedding process, with a vortex street restarting at a higher frequency and reduced cross-flow force fluctuations.

A fixed square cylinder in oscillatory flow is simulated in [31] using a 3D RANS method for the Keulegan–Carpenter number KC values ranging from 3 to 13 and blockage ratios from 0.05 to 0.5. The level set method (LSM) is used in this work to simulate the free surface in a numerical wave tank. The study finds that, at a fixed blockage ratio, lift and

drag coefficients increase with increasing KC number. The results also show that, at a fixed KC number, these coefficients increase with increasing blockage ratio.

A combined steady and oscillatory flow over a square cylinder is modeled in 2D in [34] for the KC values ranging from 10 to 40 and a Reynolds number of 100. In this work, the square structure is rigid with 2DOFs, and the considered mass ratio is 2. The results indicate changes in the lock-in peak characteristics associated with varying KC numbers. Specifically, the initial branch is found to be the widest in the reduced velocity range at $KC = 10$. The widest lower branch occurs at $KC = 20$, while the shortest lower branch at $KC = 40$. This work also reports the occurrence of a beating phenomenon at $KC = 20$, where the structural vibration frequency aligns with the frequency of the inflow velocity.

A review of the published research reveals a significant lack of data on the dynamics of square structures in oscillatory and combined flows under turbulent conditions. The present study addresses this gap by focusing on a Reynolds number of 5000 and $KC = 10$, with the square structure modeled as having a mass ratio of 2.5 and 2DOFs. The aim of this research is to investigate the influence of the flow ratio in combined steady and oscillatory flow on characteristics of the lock-in peak. The objectives of the study include developing a 2D CFD simulation model in ANSYS Fluent 2025 R1, verification and validation of the model, calculations and analysis of the results for response amplitudes, frequency content, hydrodynamic loads, XY trajectories, and wake patterns.

The paper is organized as follows. Section 1 reviews the published research on flow over a single square cylinder and its complex dynamics. Section 2 describes the 2D numerical model. Section 3 presents the simulation results for vibration characteristics and observed vortex shedding patterns. Section 4 concludes the study with a summary of the key findings.

2. Mathematical Formulation

2.1. Computational Model

Offshore structures must endure the combined effects of currents and waves, which can be represented in numerical studies as a combination of steady and oscillatory flows. The current study investigates vortex-induced vibration of a square 2DOF cylinder subjected to combined steady and oscillatory flow, as illustrated in Figure 1a, at a low mass ratio of 2.5. The influence of the flow ratio a , defined as the relationship of the steady flow component to the overall fluid velocity, on the cylinder's response is examined. Simulations are conducted with a constant Keulegan–Carpenter number of 10. Flow ratios range from 0 to 1, and for each flow ratio, the reduced velocity varies between 2 and 25 to ensure full coverage of the lock-in regime.

The lock-in state, which is responsible for vibration amplification and potential fatigue damage in structures, is the primary focus of the current work. The range of reduced velocities is selected to enable the observation of multiple dynamic regimes, including the initiation of vortex shedding, peak amplitudes, and behavior after the lock-in. A mass ratio of 2.5 [2], typical of offshore pipelines, is selected. A damping coefficient of zero is used to specifically capture the structure's maximum possible vibration. Considering such worst-case scenarios early in the design process is key to defining adequate safety margins, ensuring that vibration amplitudes remain within the levels reported in this work.

The size of the computational domain is set at $60D \times 40D$, consistent with the work of [2]. The domain dimensions are defined relative to the side length of the square cylinder, denoted as D , which is taken to be 0.02 m. The cylinder is positioned $20D$ from the inlet to ensure the inflow has sufficient space to develop before interacting with the structure.

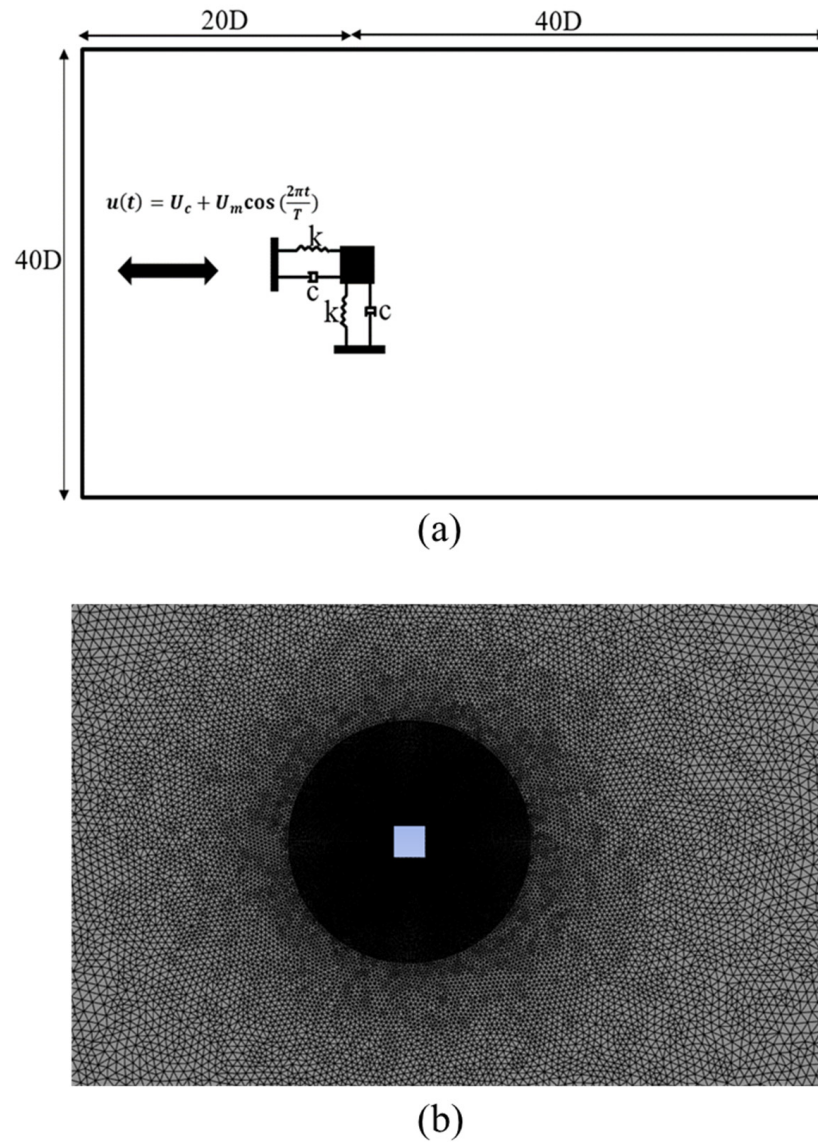


Figure 1. General schematic of the system: (a) schematic of the computational domain; (b) mesh in proximity to the square cylinder.

The boundary to the left of the rectangular computational domain is defined as a velocity-inlet with combined steady and oscillatory flow velocity U :

$$u(x = -20D, y, t) = U, v(x = -20D, y, t) = 0. \quad (1)$$

At the outlet boundary ($x = 40D$) a zero-gauge pressure condition and convective (zero velocity gradient) are imposed:

$$p(x = 40D, y, t) = 0, \frac{\partial u}{\partial x}(x = 40D, y, t) = 0, \frac{\partial v}{\partial x}(x = 40D, y, t) = 0 \quad (2)$$

Symmetry top and bottom boundaries are assumed for simulations:

$$\begin{aligned} v(x, y = 20D, t) &= 0, \frac{\partial u}{\partial y}(x, y = 20D, t) = 0, \\ v(x, y = -20D, t) &= 0, \frac{\partial u}{\partial y}(x, y = -20D, t) = 0. \end{aligned} \quad (3)$$

For the surface of the cylinder, no-slip conditions are used.

2.2. Flow Model

The flow velocity $u(t)$ in the combined steady and oscillatory flow is defined as

$$u(t) = U_c + U_m \cos\left(\frac{2\pi t}{T}\right) \quad (4)$$

where time is presented by t , U_m is the amplitude of the oscillatory flow velocity, U_c is the steady flow velocity, and T is the period of the oscillatory flow velocity. The study examines the influence of flow ratio on the dynamics of the square cylinder. In the current work, the flow ratio a is given by

$$a = \frac{U_c}{U_c + U_m}. \quad (5)$$

A constant Reynolds number of 5000 is utilized in simulations, and this is expressed as

$$Re = \frac{\rho(U_c + U_m)D}{\mu}. \quad (6)$$

The KC number is expressed as

$$KC = \frac{U_m T}{D}. \quad (7)$$

A transient incompressible two-dimensional Reynolds-Averaged Navier–Stokes (RANS) equation [36] is employed to model the flow around the square cylinder, and the continuity and momentum equations are as follows:

$$\frac{\partial(\rho \bar{u}_i)}{\partial x_i} = 0, \quad (8)$$

$$\frac{\partial(\rho \bar{u}_i)}{\partial t} + \frac{\partial}{\partial x_j} (\rho \bar{u}_i \bar{u}_j + \rho \overline{u'_i u'_j}) = \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial \bar{\tau}_{ij}}{\partial x_j}, \quad (9)$$

where the mean pressure is denoted by p , $\bar{u}_i$ represent the mean components of the velocity vector, $\rho \overline{u'_i u'_j}$ mean the Reynolds stresses, ρ represents the fluid density and $\bar{\tau}_{ij}$ is the average viscous stress vector components, defined as:

$$\bar{\tau}_{ij} = \mu \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right), \quad (10)$$

where μ denotes the dynamic viscosity. The study uses the shear stress transport (SST) k – ω turbulence model [37], and it has been shown to accurately predict flows under adverse pressure gradients. The PISO Algorithm is employed for calculations, enabling robust, time-accurate pressure-velocity coupling and divergence-free velocity solutions. Discretization is performed using the Green–Gauss Cell-Based method for gradients, second-order pressure interpolation, and second-order upwind momentum scheme to mitigate numerical diffusion, which is key for capturing wake dynamics and shear layers.

Cylinder motion is modeled using a User-Defined Function (UDF) controlled dynamic mesh. The dynamic mesh cells are allowed to undergo smoothing and remeshing, with the cylinder initially stationary at its equilibrium position having initial conditions for the streamwise and transverse displacement and velocity given as $X_1(0) = 0$, $\dot{X}_1(0) = 0$, $Y_1(0) = 0$, and $\dot{Y}_1(0) = 0$, where $X_1(0)$ represents the streamwise displacement at time $t = 0$, $\dot{X}_1(0)$ is the streamwise velocity at $t = 0$, $Y_1(0)$ is the transverse displacement at $t = 0$, and $\dot{Y}_1(0)$ is the transverse velocity at $t = 0$.

2.3. Structural Model

The square cylinder with elastic support is represented as a spring–mass system. The streamwise displacement X and transverse displacement Y are governed by the equations:

$$m_s \ddot{X} + c \dot{X} + kX = F_X, \quad (11)$$

$$m_s \ddot{Y} + c \dot{Y} + kY = F_Y, \quad (12)$$

where the mass of the structure is denoted as m_s , the damping coefficient is represented as c , k denotes the elastic support's stiffness coefficient, F_X , F_Y denote forces of hydrodynamic nature acting in the streamwise and transverse directions.

The moving square cylinder is characterized by the mass ratio m^* :

$$m^* = \frac{m_s}{m_{fd}}, \quad (13)$$

where m_{fd} denotes the mass of the displaced fluid, or

$$m_{fd} = \rho_f D^2. \quad (14)$$

The reduced velocity U_r of the combined flow is dependent on the velocity components and the natural frequency f_n :

$$U_r = \frac{U_c + U_m}{f_n D}, \quad (15)$$

where f_n is given by

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m_e}}, \quad (16)$$

where m_e denotes the effective mass, or

$$m_e = \rho_f D^2 C_A + m_s, \quad (17)$$

where $C_A = 1$ represents the coefficient of the fluid added mass.

2.4. Mesh Tests

To ensure the accuracy of the numerical results, a grid independence test was conducted for a 1DOF circular cylinder subjected to steady flow conditions for five different unstructured meshes using the parameters of [2]. A body-fitted O-mesh was applied around the cylinder surface. The mesh configuration close to the surface of the cylinder is displayed in Figure 2. A mesh of higher resolution is implemented around the cylinder to resolve the vortex formation with great fidelity and also to meet $y^+ \leq 1$ requirements. To achieve that, the first layer cell height of 0.00003 m with a growth ratio of 1.2 is used.

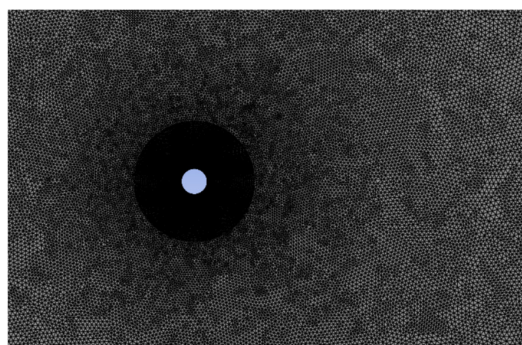


Figure 2. Sample of Mesh 4 for the area around the cylinder.

Table 1 presents the complete details of the mesh specifications and the corresponding results. A damping ratio of 0 and a time step size of 0.005 s are employed for the main simulation. The main simulation uses mesh settings optimized through this test, as illustrated in Figure 1b. Also, a time convergence study is conducted using Mesh 4 for three different time steps, and the results are presented in Table 2.

Table 1. Mesh independence test results for the Reynolds number of 5000 and mass ratio of 2.5.

Mesh	Element Number Used	Node Number Used	Re	U_r	$\frac{A_y}{D}$
Current study					
Mesh 1	49,607	25,124	5000	7	0.623
Mesh 2	63,218	31,954			0.650
Mesh 3	71,740	36,228			0.668
Mesh 4	82,585	41,667			0.667
Mesh 5	105,696	53,253			0.640
Published research					
[2]	-	-	5000	7	0.68

Table 2. Time convergence study.

Mesh	Element Number Used	Time Step	Re	U_r	$\frac{A_y}{D}$
Current study					
Mesh 4	82,585	0.01	5000	7	0.654
Mesh 4	82,585	0.005			0.667
Mesh 4	82,585	0.003			0.671
Published research					
[2]	-	-	5000	7	0.68

Validation of the numerical model is conducted against the numerical work of [38,39] for a square structure. In the work of [38], VIV of a 2DOF square cylinder at Reynolds number of 100 for a mass ratio of 3 and damping coefficient of 0 is modeled using the SIMPLEC algorithm. Table 3 presents the validation results, which demonstrate strong agreement with the numerical findings reported in [38,39]. Oscillation amplitudes are defined as $A_x = \frac{X_{max} - X_{min}}{2}$ and $A_y = \frac{Y_{max} - Y_{min}}{2}$, where A_x is the streamwise displacement amplitude, A_y is the transverse displacement amplitude, X_{max} is the maximum streamwise displacement, X_{min} is the minimum in-line displacement, Y_{max} is the maximum transverse displacement, and Y_{min} is the minimum transverse displacement.

Table 3. Validation with the numerical work of [38,39] for the Reynolds number of 100 and mass ratio of 3.

U_r	$\frac{A_y}{D}$ (Current Work)	$\frac{A_y}{D}$ (Numerical) [38]	$\frac{A_y}{D}$ (Numerical) [39]
4	0.045	0.043	0.043
5	0.275	0.300	0.280
6	0.200	0.200	0.150
7	0.088	0.101	0.076
8	0.059	0.065	0.065

3. Results and Discussion

The results of numerical simulations of a square cylinder with two degrees of freedom subjected to combined steady and oscillatory flow at a Reynolds number of 5000 are presented below to examine the effects of the flow ratio on the cylinder's dynamic response. The flow ratios analyzed include 0, 0.2, 0.4, 0.6, 0.8, and 1, with a fixed Keulegan–Carpenter number of 10 and reduced velocities ranging from 2 to 25. The findings cover response amplitudes, transverse vibration frequencies, mean drag and root mean square (RMS) values of the hydrodynamic coefficients, XY trajectories, and vorticity contours of the square cylinder.

3.1. Vibration Characteristics

Figure 3 illustrates the response amplitudes of the cylinder dependent on the reduced velocity for the specified flow ratios. For a flow ratio $a \leq 0.8$, the vibration is dominated by streamwise motion, while at $a = 1$, it is dominated by transverse motion, consistent with the findings in [2]. At $a < 0.8$, a shift in the peak streamwise displacement amplitude is observed: as the flow ratio increases, the maximum peak occurs at a higher reduced velocity. The streamwise vibration amplitude of the cylinder at $a = 0.8$ differs notably from other flow ratios [2], increasing with reduced velocity. However, at $a = 1$, the streamwise amplitude remains small compared to the transverse amplitude, except at $U_r = 3$.

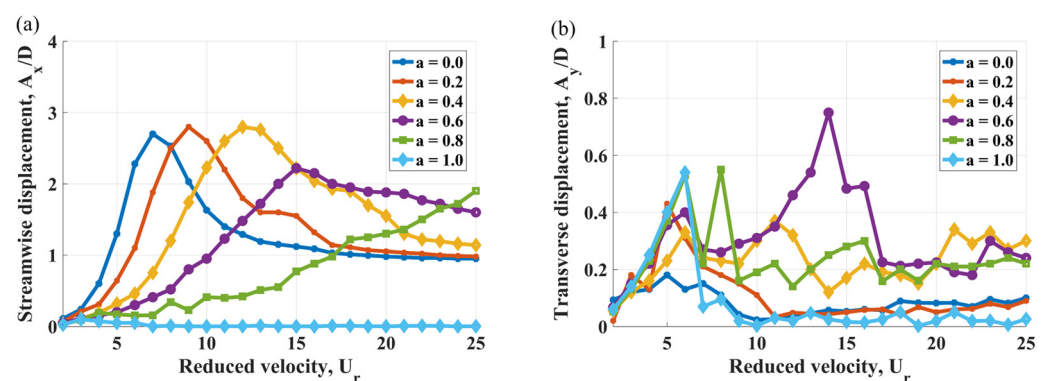


Figure 3. Displacement amplitudes for different flow ratios, the Reynolds number of 5000 and mass ratio of 2.5: (a) in streamwise direction; (b) in transverse direction.

At flow ratios $0 \leq a \leq 0.4$, and reduced velocities $U_r \leq 5$, the transverse vibration amplitude is nearly equivalent to the streamwise vibration amplitude. As the reduced velocity increases beyond 5, the transverse amplitudes become smaller than the streamwise amplitudes, and the difference between them grows with increasing reduced velocity. The dominance of in-line vibrations at $a \leq 0.4$ and $U_r > 5$ stems from the oscillatory flow's acceleration phase. Unlike transverse VIV driven by vortex shedding asymmetry, in-line motion responds directly to drag fluctuations. At low a , the strong oscillatory component (U_m) induces large pressure gradients during the flow reversal, exciting streamwise resonance. This aligns with the 'drag crisis' phenomenon observed in oscillatory flows [11,40], where rapid acceleration amplifies inertial forces.

For $a = 1$, the maximum transverse vibration amplitude of 0.54 occurs at $U_r = 6$. The maximum transverse amplitude ($A_y/D = 0.54$ at $a = 1$ is 32% lower than values reported for circular cylinders under similar conditions [2,4]. This attenuation results from fixed flow separation at sharp corners of the square cylinder, which suppresses shear layer coherence, reducing lift force magnitude. This contrasts with circular cylinders, where separation point variability enables stronger vortex coherence. At $a = 0$, the maximum transverse amplitude of 0.18 occurs at $U_r = 5$ where the transverse vibration frequency is twice the oscillatory

flow frequency, as shown in Figure 4. At $a = 0.2$, a maximum transverse amplitude of 0.43 is observed at a reduced velocity of 5, again corresponding to a vibration frequency twice that of the oscillatory flow.

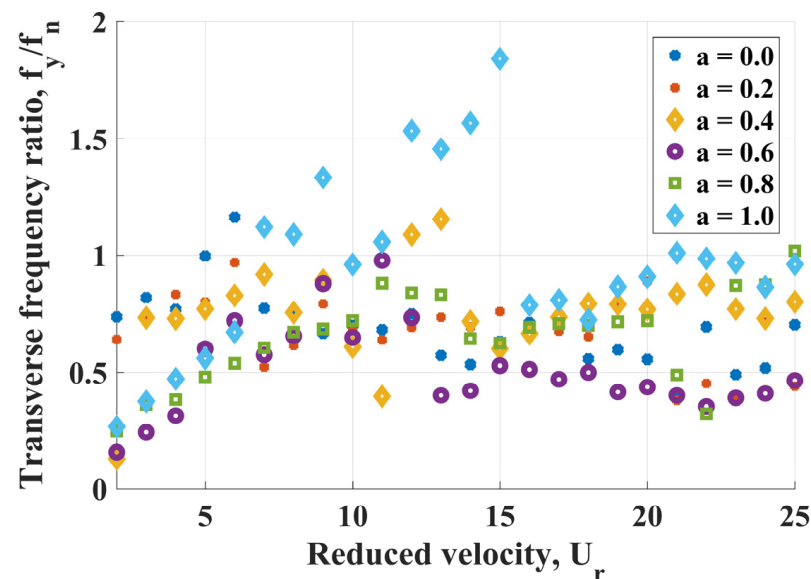


Figure 4. Transverse frequency ratio for different flow ratios, the Reynolds number of 5000 and mass ratio of 2.5.

Figure 4 illustrates the transverse vibration frequencies of the cylinder as a function of reduced velocity for the specified flow ratios. The oscillatory flow frequency dominates the vibration frequency in the streamwise direction for flow ratios $a \leq 0.8$. However, the transverse vibration exhibits a different frequency behavior. A Fast Fourier Transform (FFT) is applied to analyze the frequency content of the transverse vibration amplitude. For oscillatory flow, the oscillatory frequency is defined as $f_w = \frac{1}{T}$ where T is the period of the oscillatory flow. The transverse vibration frequency ratio is defined as the ratio of the vibration frequency to the structural natural frequency.

References [2,40] reported that the cross-flow frequency decreases with increasing reduced velocity at a constant KC value, which is consistent with the present findings. For flow ratio $a = 0$ (pure oscillatory flow), the frequency of the transverse vibration at reduced velocities in the range of $4 \leq U_r < 6$ is twice the frequency of the oscillatory flow, in agreement with experimental work results of [40,41]. At $U_r = 7$ in the current study, the transverse vibration frequency matches the oscillatory flow frequency.

Studies conducted by [41,42], and ref. [11] have shown that a circular cylinder in a pure oscillatory flow can exhibit transverse vibrations at frequencies that are integer multiples of the oscillatory flow frequency. In [11], the vibration mode is characterized based on the dominant vibration frequency. When a cylinder vibrates primarily at a frequency equal to the oscillatory flow frequency, it is referred to as the single-frequency mode; when the vibration frequency is twice the oscillatory flow frequency, it is referred to as the double-frequency mode.

At a flow ratio $a = 0$, the vibration of the cylinder transitions from a double-frequency mode in the reduced velocity range $4 \leq U_r \leq 6$ to a single-frequency mode at $U_r = 7$. At a flow ratio of $a = 0.2$, a similar transition is observed: the cylinder exhibits a double-frequency mode for $5 \leq U_r \leq 6$, and a single-frequency mode at $7 \leq U_r \leq 9$. The double-frequency transverse lock-in at $a \leq 0.2$ and $U_r = 5-6$ arises from harmonic synchronization between vortex shedding and structural motion. During one oscillatory flow cycle (T), the cylinder experiences two vortex shedding events (2S mode) with each triggering a transverse force

peak. This forces the structure to vibrate at $2f_w$, matching the ‘wake capture’ mechanism described by Sumer and Fredsøe [42] for oscillatory flows.

For flow ratios of $a = 0.4$ and $a = 0.6$, the transverse vibration frequency corresponds to a multiple of the oscillatory flow frequency only at specific reduced velocities. At $a = 0.6$, the double-frequency mode is observed at $U_r = 3, 7$, and 8 . At $a = 0.8$, neither the single-, nor double-frequency mode is observed, which is consistent with the finding reported in [2].

3.2. Hydrodynamic Coefficients

Figure 5 shows the mean drag coefficients of the cylinder as a function of reduced velocity at the considered flow ratios. Overall, the mean drag coefficient increases with increasing flow ratio, with $a = 0$ (pure oscillatory flow) yielding the lowest values and $a = 1$ (steady flow) yielding the highest. For flow ratios $a \leq 8$, the mean drag coefficient remains approximately constant with increasing reduced velocity. A maximum mean drag coefficient value of 2.14 is recorded for $a = 1$ and $U_r = 3$, which decreases at $U_r = 5$ and remains within the range of 1.35–1.54 at $7 \leq U_r \leq 15$. The rise in mean drag coefficient (C_{D0}) with the increasing flow ratio reflects the growing dominance of a steady momentum transfer. Conversely, RMS fluctuating drag peaks at $a = 0$ due to oscillatory flow reversals inducing large pressure transients.

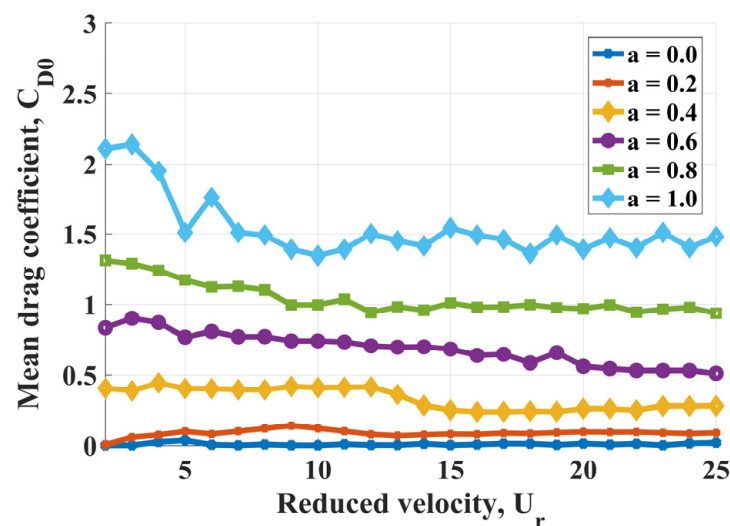


Figure 5. Mean drag coefficient for different flow ratios, the Reynolds number of 5000 and mass ratio of 2.5.

Figure 6 illustrates the RMS fluctuating drag C_D^{fl} and lift C_L coefficients as functions of reduced velocity at the specified flow ratios. For the RMS fluctuating drag coefficient, a similar trend is observed for $a \leq 4$, with $a = 0$ consistently showing the highest RMS fluctuating drag coefficient across all cases. For flow ratios $0.6 \leq a \leq 1$, the RMS drag coefficient remains nearly constant, with only slight variation at $a = 1$. The RMS lift coefficient decreases to a constant value with increasing reduced velocity for all examined flow ratios. The maximum value of the RMS lift coefficient is observed for $a = 1$ at a reduced velocity of 2. Within the range $2 \leq U_r \leq 15$, the RMS lift coefficient increases with increasing flow ratio from $a = 0.6$ to $a = 1$.

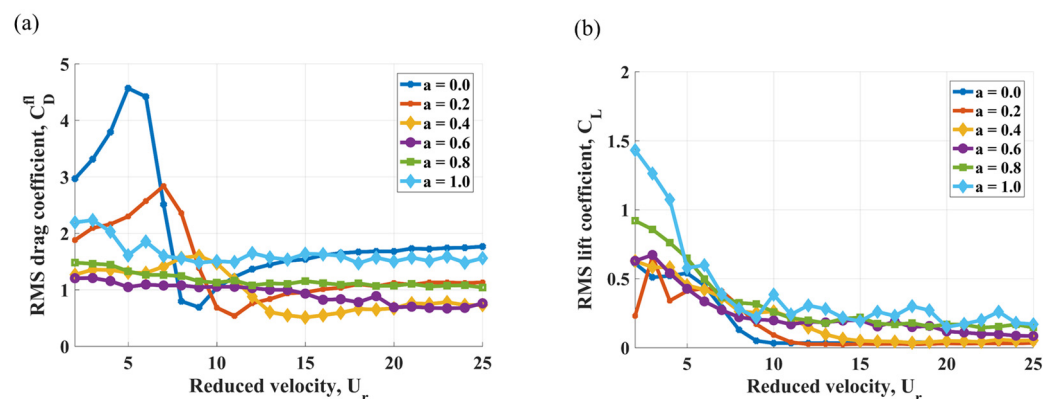


Figure 6. RMS hydrodynamic coefficients for different flow ratios, the Reynolds number of 5000 and mass ratio of 2.5: (a) fluctuating drag coefficient; (b) fluctuating lift coefficient.

3.3. XY Trajectories

Figure 7 presents the XY trajectories of the cylinder for various flow ratios and selected reduced velocities within the lock-in regime. The infinity “ ∞ ” shaped trajectory at $a = 0$ and $U_r = 4$ and 5 are consistent with the patterns previously reported in [2,40]. As shown in Figure 8, for $a < 1$, the motion of the cylinder is primarily dominated by the in-line vibration. At $a = 1$, the vibration is predominantly transverse, with negligible in-line motion. The figure-of-eight trajectory pattern observed at $U_r = 4$ and 5 also agrees with the findings of [2].

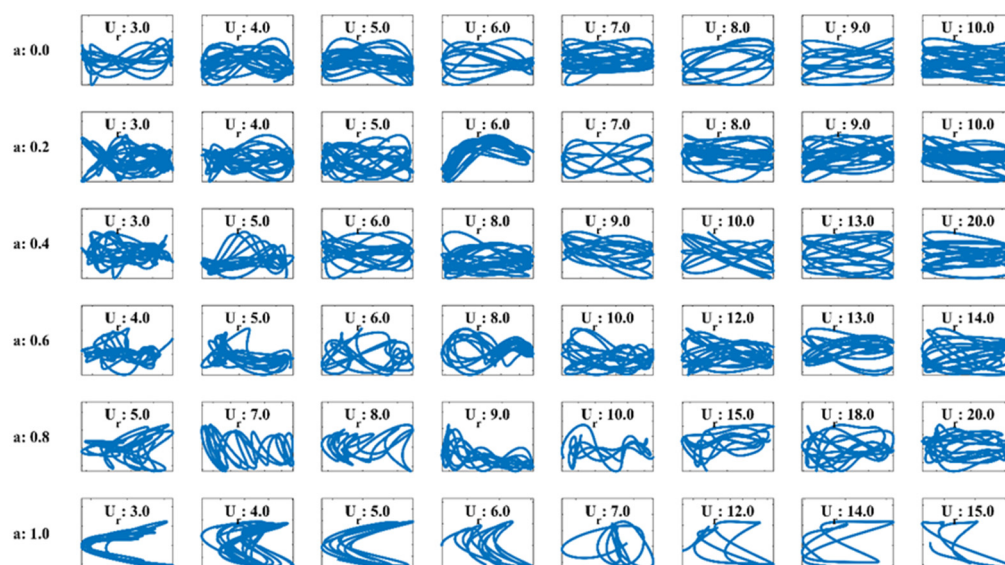


Figure 7. XY trajectories at selected reduced velocities and different flow ratios, the Reynolds number of 5000 and mass ratio of 2.5.

3.4. Vorticity Contours

Figures 8–11 depict the vorticity magnitude for the specified flow ratios at selected reduced velocities. According to [43], a stationary circular cylinder in a pure oscillatory flow with $KC = 10$ sheds one pair of vortices per cycle. In the current study, for flow ratios $a = 0$ and $a = 0.2$, shown in Figures 8 and 9, a defined pair of vortices is shed from the cylinder within the first 5 s. However, these vortices do not develop fully before flow reversal, and those not shed dissipate after the reversal. About ten vortices are formed during one oscillation cycle at these flow ratios, as depicted in Figures 8 and 9 with black circles. For higher flow ratios $0.8 \leq a \leq 1$, illustrated in Figures 10 and 11, a well-defined

pair of vortices is observed, corresponding to the 2S vortex shedding mode, with vortices growing larger and more stable as the steady-flow component increases.

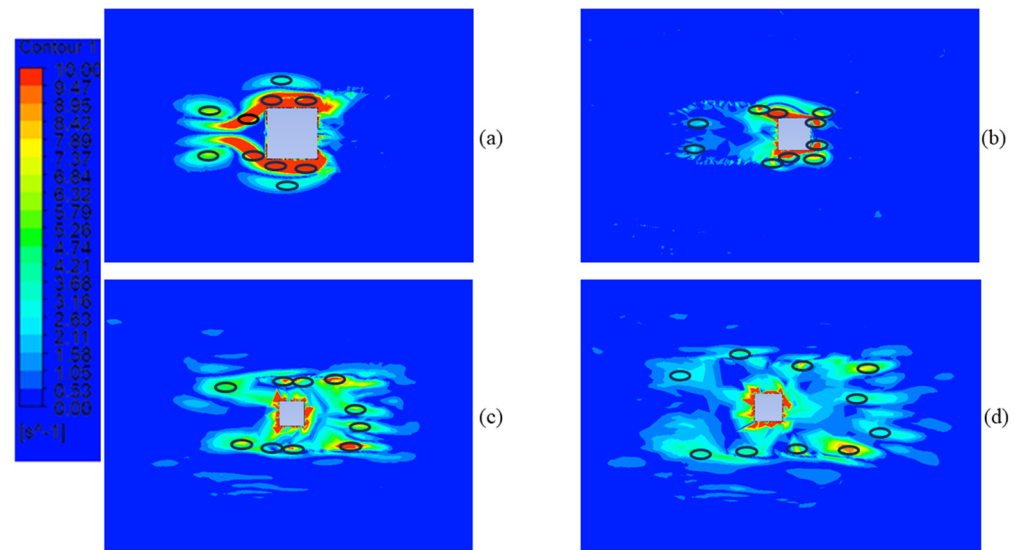


Figure 8. Vorticity magnitude for $a = 0$ at $U_r = 9$ at time: (a) 0.5 s; (b) 1.0 s; (c) 20 s; (d) 40 s.

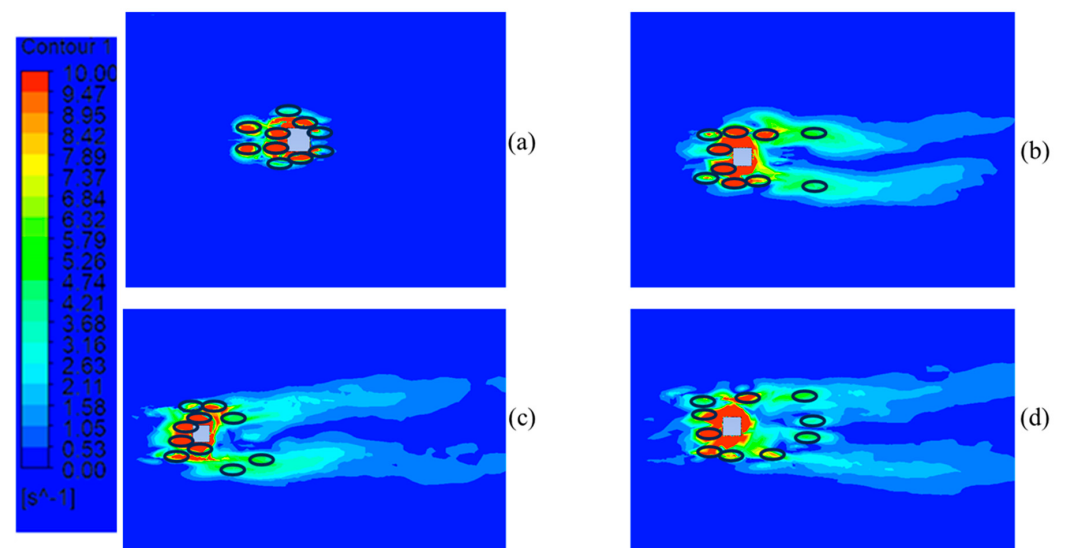


Figure 9. Vorticity magnitude for $a = 0.2$ at $U_r = 10$ at time: (a) 1 s; (b) 10 s; (c) 20 s; (d) 30 s.

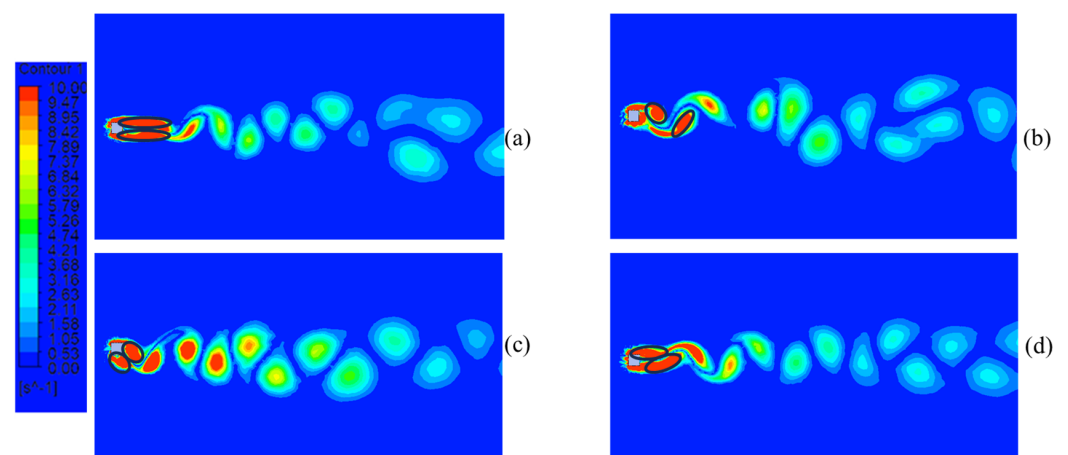


Figure 10. Vorticity magnitude for $a = 0.8$ at $U_r = 8$ at time: (a) 8 s; (b) 15 s; (c) 30 s; (d) 40 s.

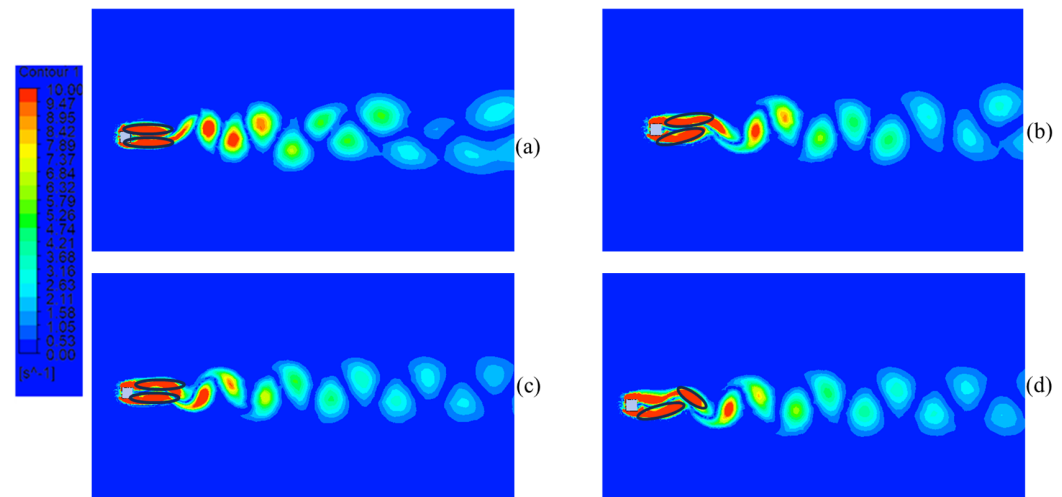


Figure 11. Vorticity magnitude for $a = 1.0$ at $U_r = 9$ at time: (a) 5 s; (b) 15 s; (c) 26.29 s; (d) 40 s.

4. Conclusions

This work presents a numerical investigation of the vortex-induced vibration of an elastically mounted square cylinder with two degrees of freedom subjected to combined steady and oscillatory flow. Simulations are performed at $Re = 5000$, $KC = 10$, $m^* = 2.5$, and zero structural damping. The effects of varying flow ratio ($a = 0, 0.2, 0.4, 0.6, 0.8, 1.0$) and reduced velocity ($U_r = 2\text{--}25$) on the dynamic response of the cylinder are systematically examined. Key aspects analyzed include vibration amplitudes, frequency content, hydrodynamic forces, motion trajectories, and vortex shedding patterns.

The results show that at low flow ratios ($a \leq 0.2$), in-line vibrations dominate when the reduced velocity exceeds 6. The streamwise amplitude remains within $3D$ for all flow ratios. The transverse vibration amplitudes peak within the lock-in region and do not exceed $0.54D$, observed at $a = 1.0$.

The frequency analysis reveals that transverse vibration can exhibit both single- and double-frequency lock-in behaviors. For flow ratios $a \leq 0.2$, double-frequency lock-in occurs within the range $U_r = 5\text{--}6$, transitioning to single-frequency modes as the reduced velocity increases. As the steady component becomes dominant, the vibration response becomes primarily single-frequency in the transverse direction, with minimal in-line motion.

Hydrodynamic loading was also found sensitive to the flow ratio. The mean drag coefficient increases, as the steady component becomes stronger. The RMS fluctuating drag coefficient is highest for pure oscillatory flow and decreases gradually with increasing flow ratio, becoming nearly constant at $a = 1$. The RMS fluctuating lift coefficient declines with increasing reduced velocity across all flow ratios, reaching its maximum value at $a = 1$, $U_r = 2$.

The motion trajectories of the cylinder evolve with the flow conditions. At flow ratios less than 1, the cylinder exhibits trajectories resembling infinity shape, indicating coupled in-line and transverse motion. At $a = 1$, the motion becomes predominantly transverse, with a clear figure-of-eight trajectory observed, reflecting the dominance of cross-flow oscillations.

Vortex streets also evolve as a function of flow ratio. For low values of a ($a \leq 0.2$), a vortex shedding in the 2S mode is observed. As the flow ratio increases toward unity, the wake structure becomes more complex, transitioning to a combination of 2S and P + S vortex shedding modes, indicative of increased interaction between steady and oscillatory flow components.

Overall, the study demonstrates how varying the balance between steady and oscillatory flow significantly influences the vibration amplitude, frequency behavior, and wake of the square cylinder. These insights provide a useful foundation for developing vortex-induced vibration control strategies in offshore engineering application and for designing flow-driven energy harvesting devices.

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