

Article

Influence of Bragg Resonance on the Hydrodynamic Performance of a Fixed-Detached Asymmetric Oscillating Water Column Device

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Abstract: The present study analyzes the hydrodynamic performance of an asymmetric offshore Oscillating Water Column device positioned in close proximity to multiple bottom standing and fully submerged breakwaters and trenches. The breakwaters and trenches are located on the leeward side of the Oscillating Water Column device. The structures are investigated in combination with a shore-fixed vertical wall. The analysis is carried out using the Boundary Element Method based on the linear potential flow theory. The results are compared with the existing analytical, numerical, and experiment results available in the literature. The effects of the various shape parameters of the submerged breakwaters/trenches and the shape parameters of the Oscillating Water Column device are investigated. The results show that the resonance effects on the efficiency performance increase as the number of breakwaters/trenches increases. The undulating bottom trench shape is effective in improving the efficiency of the Oscillating Water Column device compared to the breakwater. The efficiency bandwidth is greater in the case of a rectangular trench than in the case of a parabolic- or triangular-shaped trench. In addition, the first peak value in the efficiency curve for a lower frequency is higher in the case of a larger-draft Oscillating Water Column device front wall compared to that of the rear wall. This study demonstrates that in the long wave-length regime, a zero efficiency point is observed between two consecutive resonant peaks, whereas in the intermediate and short wave-length regimes, a trough and a zero efficiency point alternately occur between two consecutive resonance peaks. Various parameters relevant to the behavior of the Oscillating Water Column Wave Energy Converter, such as radiation susceptance, radiation conductance, hydrodynamic efficiency, and volume flux due to a scatter potential, are addressed.

Keywords: Oscillating Water Column; 2D BEM method; hydrodynamic efficiency; Bragg resonance

Academic Editors: Eugen Rusu,
Kostas Belibassakis and George
Lavidas

Received: 28 April 2025

Revised: 31 May 2025

Accepted: 31 May 2025

Published: 2 June 2025

Citation: Kar, P.; Mayon, R.; Ning, D. Influence of Bragg Resonance on the Hydrodynamic Performance of a Fixed-Detached Asymmetric Oscillating Water Column Device. *J. Mar. Sci. Eng.* **2025**, *13*, 1115. <https://doi.org/10.3390/jmse13061115>

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1. Introduction

Ocean wave energy harvesting technology is one of the most promising renewable energy sectors due to a vast but untapped energy potential worldwide. China has approximately 249.7 Terawatt-hours per year (TWh/year) of extractable nearshore ocean wave energy resource, and ten provinces have an ocean wave energy potential greater than 850 Gigawatt-hours per year (GWh/year) (Xie and Zuo [1]). Many countries, such as China, Spain, Portugal, the United Kingdom, etc., developed many leading ocean wave energy technologies in the past century. With rational exploitation and effective utilization, wave energy could provide a reliable alternative renewable energy resource to mitigate concerns

about environmental issues. Investigations on the extraction of wave power from water waves have been published since the 1970s. The intensification of the energy crisis in the previous decades has motivated many researchers (Evans and Porter [2]; Mustapa et al. [3]; Ning and Ding [4] etc.) to work toward the development of ocean wave energy harvesters. Various types of Wave Energy Converters (WECs) were patented, fabricated, and installed in appropriate locations to extract ocean wave energy. WECs can be classified into (i) Oscillating Water Columns (OWCs), (ii) over-topping devices, (iii) wave attenuators, and (iv) point absorbers. The operating principle of an OWC device is well-known, and the device provides many advantages over other ocean energy sources. The OWC-WEC consists of an air turbine that is attached to a semi-submerged chamber through an orifice on either the top side or the shore side of the device. The bottom side of the OWC chamber is open to the sea. By harnessing the rise and fall of the water column within the chamber driven by the incident wave action, the pneumatic air inside the chamber is cyclically compressed and decompressed to actuate an air-driven turbine and generate electricity. A comprehensive review of the OWC device was reported by Heath et al. [5].

In recent decades, the hydrodynamic performance of OWC devices has been extensively investigated by many researchers by means of theoretical analysis (Yang et al. [6], Ding et al. [7], Zhao et al. [8]) and experimentally (Boccotti [9], Altomare et al. [10], Zhao and Ning [11]). Evans [12] analytically studied the hydrodynamic efficiency of a two-dimensional OWC device with an internal water free surface in the frequency domain using the Galerkin approximation method. The propagation of water waves in the presence of bottom undulation is important in several circumstances, including wave transformations over continental shelves and wave scattering by submerged breakwaters and trenches of various designs. The bottom effects have a substantial influence on the incident wave energy. This is significant, as the waves propagate toward coastal locations. Many physical processes, such as wave breaking, refraction, and shoaling occur because of bottom friction effects. Therefore, the role of bottom undulations is an important characteristic for analyzing the hydrodynamic efficiency of an OWC device in coastal locations. The influence of stepped sea bottom topography on the efficiency of a nearshore OWC device was carried out using both theoretical and numerical approaches by Rezanejad et al. [13]. Deng et al. [14] reported a theoretical analysis of an asymmetric offshore stationary OWC device with a bottom plate by means of the Eigenfunction Expansion Method (EEM). They observed that the bottom plate can provide an additional resonance mechanism that can help to increase the OWC efficiency bandwidth.

The aforementioned studies predominantly employ a theoretical-based approach, and more recently, experimental as well as numerical research has been conducted by several investigators. Experimental and numerical investigations on the hydrodynamic performance of an OWC-WEC on a stepped bottom using a 2D model were reported by Rezanejad et al. [15]. Over the past few decades, numerical methods, such as the Boundary Element Method (BEM), have been developed and extensively used by many researchers. The advantages of the BEM can be attributed primarily to the fact that the boundary of the physical domain is discretized instead of the complete domain. Moreover, the method can handle physical problems of varied bed profiles, which include submerged structures, undulation in the bottom bed profile and trenches of varied configurations (Kar et al. [16,17]). The role of dual breakwaters and trenches on the efficiency of an OWC was studied by Naik et al. [18]. Furthermore, the study was extended to wave power extraction by a dual-chamber OWC in the presence of a bottom undulation by Naik et al. [19]. Recently, Naik et al. [20] investigated the impact of a sloping porous seabed on the efficiency of an OWC in oblique waves. Narendran and Vijay [21] studied the three different types of OWC lip walls to evaluate their effect on the performance

of OWC using the Dual Boundary Element Method (DBEM) with CFD. A 3D BEM for analyzing the hydrodynamic performance of a land-fixed OWC device was investigated by Rodríguez et al. [22]. A hydrodynamic analysis of an OWC array in the presence of variable bathymetry was performed by Zhao et al. [23].

The performance of an OWC can be improved significantly by leveraging the phenomenon of fluid resonance. Larger-amplitude trapped waves can be controlled and manipulated with the introduction of Bragg resonance instigated by a designed bottom topography. Bragg reflection by multiple bottom standing breakwaters/trenches and seabed undulations causes significant changes in the wave scattering and can influence the OWC performance due to larger fluid oscillations. Gao et al. [24] investigated the effects of Bragg reflection on harbor oscillations. The study was further extended by Gao et al. [25] to understand the mitigation mechanism of harbor oscillations in the presence of arc-shaped sinusoidal bars. The study showed that under the non-resonance condition, the wave amplitude inside the harbor can be reduced up to 80–89% due to the presence of the sinusoidal bars. A similar phenomenon was observed by Gao et al. [26] while analyzing the gap resonance between closely spaced boxes in a numerical wave flume. Kar et al. [27] studied the influences of Bragg resonance due to the occurrence of fluid resonances on the hydrodynamic performance of a trapezoidal-shape fixed-detached OWC using the BEM. A theoretical study of the hydrodynamic performance of an asymmetric fixed-detached OWC device was conducted by Rodríguez et al. [28] by means of the Eigenfunction Expansion Method and BEM. Although there are some studies on the effects of breakwaters and trenches on the performance of an OWC, to the authors' knowledge, there is an absence of research on the effects of breakwater/trench-induced Bragg resonance on a fixed-detached OWC.

The main objectives of the present study is to examine modifications in bandwidth and Bragg resonance in the presence of trenches/breakwaters on the efficiency curves and the shifting of the peak resonant frequency due to different rear and front wall drafts/widths, chamber breadths, and gap distances between the OWC chamber and the reflecting vertical wall. Section 2 defines the mathematical model of the physical problem corresponding to the water wave radiation and scattering by a fixed-detached OWC device, based on the linear potential flow theory in the presence of the trenches/breakwaters. In Section 3, the numerical methodology based on the Boundary Element Method (BEM) for solving the Boundary Value Problem is presented. Section 4 describes the procedure for obtaining the energy relations. In Section 5, convergence and truncation analyses are carried out, respectively, and a comparison is made with the analytic numerical results of [2,13,29]. Section 6 shows the results for various modeled cases. Finally, in Section 7, the main conclusions of this study are drawn.

2. Mathematical Model

The present Boundary Value Problem (BVP) numerically analyzes the performance of an OWC with different bottom variations using the BEM under the assumptions that the fluid is inviscid and incompressible and that fluid motion is irrotational. The computational domain employs the Cartesian coordinate frame $O\text{-}xz$, with the x axis aligned in the direction of the incident wave and the z -axis oriented in the vertically upwards direction. h_1 represents the still water depth. As depicted in Figure 1, a_1 and a_2 denote the drafts of an OWC front wall and rear wall, respectively, whilst D signifies the breadth of an OWC chamber, and b_1 and b_2 correspond to the thickness of the front and rear walls of the OWC. The OWC is positioned towards the incident wave side, away from the multiple bottom standing breakwaters/trenches. The breakwater is submerged with a uniform depth h_2 and w corresponds to the width of the trench/breakwater, while d represents the gap distance

between the two consecutive breakwaters/trenches. The gap distance between the OWC and the leftmost breakwater/trench corresponds to l , while L represents the gap distance between the rightmost breakwater/trench and the shore-fixed vertical wall, and N denotes the number of trenches/breakwaters.

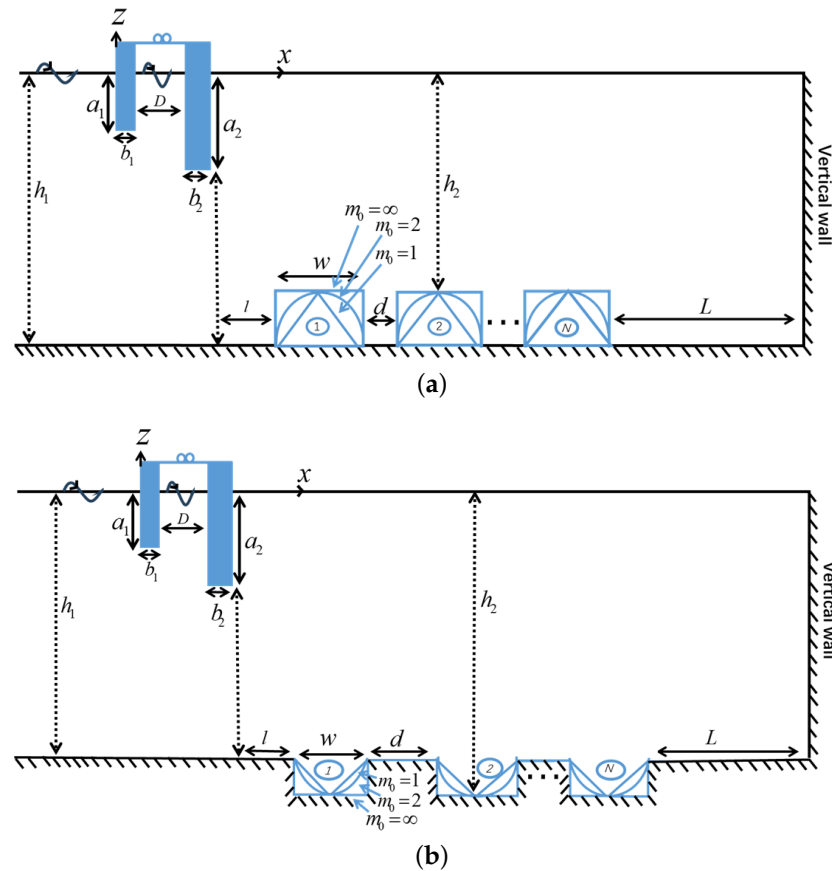


Figure 1. Schematic of a fixed-detached OWC and undulating bottom of (a) breakwater and (b) trench types with shape parameter m_0 where $m_0 = 1, 2$ correspond to the triangular and parabolic shapes, respectively, whilst $m_0 = \infty$ corresponds to the rectangular shape. The bottom profile is similar to that considered in Kar et al. [16] and Xie et al. [30].

The instantaneous pressure distribution within the chamber is given by $P(t) = \mathcal{R}\{pe^{-i\omega t}\}$, where p is the complex amplitude of the air pressure above the water within the chamber. Since the flow is irrotational, the velocity potential in the computational domain is given by $\Phi(x, z, t) = \mathcal{R}\{\phi(x, z)e^{-i\omega t}\}$, where ϕ satisfies the Laplace Equation as

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0, \text{ in the fluid region.} \quad (1)$$

The linearized boundary condition on the mean free surface is

$$\frac{\partial \phi}{\partial z} - K\phi = \begin{cases} \frac{i\omega p}{\rho g}, & \text{inside the chamber,} \\ 0, & \text{outside the chamber,} \end{cases} \quad (2)$$

where $K = \omega^2/g$ and ω is the angular frequency ($\omega = 2\pi/T$, T being the wave period). g is the gravity acceleration taken as 9.81 m/s^2 , and ρ is the water density taken as 10^3 kg/m^3 .

The potential ϕ decomposes into the radiation and scattering potential (for details, see Evans & Porter [2]) as

$$\phi = \phi^S + \frac{i\omega p}{\rho g} \phi^R, \quad (3)$$

where ϕ^S , $\frac{i\omega p}{\rho g} \phi^R$ represent the scattering and radiation potential, respectively.

Applying Equation (3) in Equation (2), the boundary condition on the mean free surface at $z = 0$ transforms into

$$\frac{\partial \phi^{S,R}}{\partial z} - K\phi^{S,R} = 0, \quad \text{outside the chamber}, \quad (4)$$

$$\frac{\partial \phi^R}{\partial z} - K\phi^R = 1, \quad \text{inside the chamber}, \quad (5)$$

$$\frac{\partial \phi^S}{\partial z} - K\phi^S = 0, \quad \text{inside the chamber}. \quad (6)$$

The no-flow condition is imposed on the impermeable bottom bed, $z = -h_1 \cup -h_2$, and on the vertical shore-fixed wall and on the surfaces of OWC front and rear walls, yielding

$$\frac{\partial \phi^{S,R}}{\partial n} = 0, \quad (7)$$

where $\partial/\partial n$ denotes the normal derivative.

The far-field boundary condition at the incident wave side is

$$\phi^S(x, z) = \psi_0(k_0; z)e^{ik_0x} + A_S\psi_0(k_0; z)e^{-ik_0x}, \quad \text{as } x \rightarrow -\infty, \quad (8)$$

$$\phi^R(x, z) = A_R\psi_0(k_0; z)e^{ik_0x}, \quad \text{as } x \rightarrow -\infty, \quad (9)$$

where A_S , A_R are the unknown complex coefficients related to amplitudes of the reflected and radiated waves where

$$\psi_0(k_0; z) = \frac{\cosh k_0(z + h_1)}{\cosh k_0 h_1},$$

with k_0 being the real positive root of the dispersion relation $\omega^2 = gk \tanh kh_1$.

The alternative form of the far-field boundary condition at the incident wave side as given in Equations (8) and (9) can be expressed as

$$\frac{\partial(\phi^S - \psi_0(k_0; z)e^{ik_0x})}{\partial x} = -ik_0(\phi^S - \psi_0(k_0; z)e^{ik_0x}) \quad \text{as } x \rightarrow -\infty, \quad (10)$$

$$\frac{\partial \phi^R}{\partial x} = ik_0\phi^R \quad \text{as } x \rightarrow -\infty. \quad (11)$$

The bottom profile $h(x)$ as depicted in Figure 1 is taken to be

$$h(x) = \begin{cases} -h_1 & \text{for } (-\infty < x < \tilde{s}) \cup (\tilde{s} + w < x < r), \\ -h_2 - \left(\frac{x + \tilde{s}}{w/2}\right)^{m_0} \times (h_1 - h_2) & \text{for } \tilde{s} < x < \tilde{s} + w/2, \\ -h_2 - \left(\frac{x - \tilde{s}}{w/2}\right)^{m_0} \times (h_1 - h_2) & \text{for } \tilde{s} + w/2 < x < \tilde{s} + w, \end{cases} \quad (12)$$

where $\tilde{s} = (b_1 + D + b_2 + l)$, $r = \tilde{s} + w + L$.

3. Solution Methodology: Boundary Elementary Method (BEM)

Here, the derivation of the solution procedure for the radiation problem is discussed using the constant BEM technique. The two-dimensional free-space Green function $G(x, z; x_0, z_0)$ is given as

$$G(x, z; x_0, z_0) = \frac{1}{2\pi} \ln(r), \quad r = \sqrt{(x - x_0)^2 + (z - z_0)^2}, \quad (13)$$

where $\ln(r)$ is the logarithmic function and (x, z) , (x_0, z_0) are the field and source points in the computational domain, respectively. The following integral equation is obtained when Green's integral theorem, Equation (1), and Green's function G as defined in Equation (13) are applied:

$$\int_{\Gamma} \left(\phi^R(x, z) \frac{\partial G(x, z; x_0, z_0)}{\partial n} - G(x, z; x_0, z_0) \frac{\partial \phi^R(x, z)}{\partial n} \right) d\Gamma = \begin{cases} -\frac{\phi^R(x, z)}{2} & \text{if } (x, z) \notin \Gamma \\ -\phi^R(x, z) & \text{if } (x, z) \in \Gamma \end{cases}. \quad (14)$$

Substituting Equations (3)–(7) into the integral Equation (14),

$$\begin{aligned} c\phi^R + \int_{\Gamma_{f_1}} \left(\frac{\partial G}{\partial n} - KG \right) \phi^R d\Gamma + \int_{\Gamma_l} \left(\frac{\partial G}{\partial n} - ik_0 G \right) \phi^R d\Gamma + \int_{\Gamma_b} \frac{\partial G}{\partial n} \phi^R d\Gamma + \int_{\Gamma_r} \frac{\partial G}{\partial n} \phi^R d\Gamma \\ + \int_{\Gamma_{f_2}} \left(\frac{\partial G}{\partial n} - KG \right) \phi^R d\Gamma + \int_{\Gamma_s} \frac{\partial G}{\partial n} \phi^R d\Gamma + \int_{\Gamma_{f_3}} \left(\frac{\partial G}{\partial n} - KG \right) \phi^R d\Gamma = 0, \end{aligned} \quad (15)$$

and

$$\begin{aligned} c\phi^S + \int_{\Gamma_{f_1}} \left(\frac{\partial G}{\partial n} - KG \right) \phi^S d\Gamma + \int_{\Gamma_l} \left(\frac{\partial G}{\partial n} - ik_0 G \right) \phi^S d\Gamma + \int_{\Gamma_b} \frac{\partial G}{\partial n} \phi^S d\Gamma + \int_{\Gamma_r} \frac{\partial G}{\partial n} \phi^S d\Gamma \\ + \int_{\Gamma_{f_2}} \left(\frac{\partial G}{\partial n} - KG \right) \phi^S d\Gamma + \int_{\Gamma_s} \frac{\partial G}{\partial n} \phi^S d\Gamma + \int_{\Gamma_{f_3}} \left(\frac{\partial G}{\partial n} - KG \right) \phi^S d\Gamma = 0, \end{aligned} \quad (16)$$

where $c = 1/2$.

ϕ^R , ϕ^S , $\partial\phi^R/\partial n$, $\partial\phi^S/\partial n$ are assumed to be constant on each panel in the boundary of the domain Γ . Hence, using Equations (10) and (11), Equations (15) and (16) transform into

$$\begin{aligned} \sum_i (H^{i,j} - KF^{i,j}) \phi^R|_{\Gamma_{f_1}} + \sum_i (H^{i,j} - ik_0 F^{i,j}) \phi^R|_{\Gamma_l} + \sum_i H^{i,j} \phi^R|_{\Gamma_b} + \sum_i H^{i,j} \phi^R|_{\Gamma_r} \\ + \sum_i (H^{i,j} - KF^{i,j}) \phi^R|_{\Gamma_{f_2}} + \sum_i H^{i,j} \phi^R|_{\Gamma_s} + \sum_i (H^{i,j} - KF^{i,j}) \phi^R|_{\Gamma_{f_3}} = \sum_i F^{i,j}, \end{aligned} \quad (17)$$

and

$$\begin{aligned} \sum_i (H^{i,j} - KF^{i,j}) \phi^S|_{\Gamma_{f_1}} + \sum_i (H^{i,j} - ik_0 F^{i,j}) \phi^S|_{\Gamma_l} + \sum_i H^{i,j} \phi^S|_{\Gamma_b} + \sum_i H^{i,j} \phi^S|_{\Gamma_r} \\ + \sum_i (H^{i,j} - KF^{i,j}) \phi^S|_{\Gamma_{f_2}} + \sum_i H^{i,j} \phi^S|_{\Gamma_s} + \sum_i (H^{i,j} - KF^{i,j}) \phi^S|_{\Gamma_{f_3}} = \sum_i F^{i,j}, \end{aligned} \quad (18)$$

where the influence coefficients $F^{i,j}$, $H^{i,j}$ are defined as follows:

$$F^{i,j} = \int_{\Gamma_j} G d\Gamma, \quad H^{i,j} = \frac{\delta_{ij}}{2} + \int_{\Gamma_j} \frac{\partial G}{\partial n} d\Gamma. \quad (19)$$

By solving Equations (17) and (18), a matrix equation is obtained, which is used to find the values of the unknown coefficients of interest. The BEM was studied previously for the scattering of surface gravity waves over an undulating bed by Kar et al. [16,17,31], Figure 2.

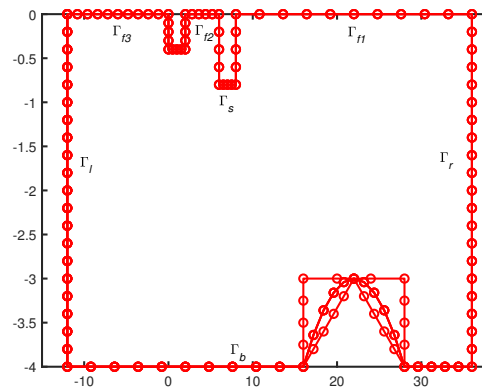


Figure 2. BEM computational domain.

4. Power Takeoff (PTO) Model

Here, the expressions for the various physical parameters associated with the performance of an OWC device are provided. The time harmonic volume flow rate $Q(t) = \Re\{qe^{-i\omega t}\}$ where

$$q = \int_{S_i} \frac{\partial \phi}{\partial z} dx = q^S + \frac{i\omega p}{\rho g} q^R, \quad (20)$$

where $q^S = \int_{S_i} \frac{\partial \phi^S}{\partial z} dx$ and $q^R = \int_{S_i} \frac{\partial \phi^R}{\partial z} dx$ represent the volume flux rate across the chamber free surface of the scattering and radiation problem, respectively.

The radiation volume flux q^R across the chamber free surface as defined in Equation (20) can be separated into real and imaginary parts as

$$\frac{i\omega p}{\rho g} q^R = -(\tilde{B} - i\tilde{A})p = -Zp, \quad (21)$$

where $Z = \tilde{B} - i\tilde{A}$ is the complex admittance, while $\tilde{A}$ (transfer of energy) and $\tilde{B}$ (uncaptured energy) are analogous to the coefficients of added mass and radiation-damping in rigid body systems. The parameters $\tilde{A}$ and $\tilde{B}$ in Equation (21), termed the radiation susceptance and conductance of an OWC, can be expressed as

$$\tilde{A} = \frac{\omega}{\rho g} \Re\{q^R\}, \quad \tilde{B} = \frac{\omega}{\rho g} \Im\{q^R\}. \quad (22)$$

Using the far-field behavior (detailed derivation given in Koley and Trivedi [32]), the empirical relation for the radiation conductance parameter $\tilde{B}$ can be expressed as

$$\tilde{B} = \frac{\omega}{\rho g} K k_0 |A_R|^2 \mathcal{A}, \quad (23)$$

and a relation between the radiated amplitude and the induced volume flux due to a scattered potential yields

$$q^S = -2iKk_0 A_R \mathcal{A}. \quad (24)$$

A linear relation between the volume flux q through the turbine and the pressure drop p across the OWC chamber yields

$$q = \Lambda p, \quad (25)$$

where Λ is a real control parameter associated with the linear turbine damping applied on the airflow.

After combining Equations (20), (21), and (25), the imposed internal pressure gives

$$p = \frac{q^S}{\Lambda + Z}. \quad (26)$$

The total rate of working pressure forces inside the OWC is $Q(t) \times P(t)$. The mean rate of work performed by the pressure produced in the OWC for one wave period (see Rodriguez et al. [29]) is

$$W = \frac{1}{2} \text{Re}\{\bar{p}q\}, \quad (27)$$

where bar (–) denotes the complex conjugate. After inserting Equations (25) and (26) into Equation (27) and simplifying, we get

$$W = \frac{1}{2} \text{Re}\left\{\bar{p}\left(q^S - Zp\right)\right\} = \frac{|q^S|^2}{8\tilde{B}} - \frac{\tilde{B}}{2} \left|p - \frac{q^S}{2\tilde{B}}\right|^2. \quad (28)$$

If $\tilde{B}$ exists, from Equation (28), the maximum work W_{max} yields

$$W_{max} = \frac{|q^S|^2}{8\tilde{B}}, \text{ for } p = \frac{q^S}{2\tilde{B}}, \quad (29)$$

where $\Lambda = \tilde{Z}$ for maximum power.

After inserting Equation (26) into Equation (28), we get

$$W = \frac{|q^S|^2}{8\tilde{B}} \left[1 - \underbrace{\left(\frac{|\Lambda - Z|}{|\Lambda + Z|} \right)^2}_f \right]. \quad (30)$$

In order to optimize the power conversion efficiency, the last term in the square brackets must be minimized. Therefore,

$$\frac{df}{d\Lambda} = 0. \quad (31)$$

After algebraic manipulation,

$$\Lambda = \Lambda_{opt} = \sqrt{\tilde{A}^2 + \tilde{B}^2}. \quad (32)$$

Using Equations (30) and (32), the maximum value of extracted power becomes

$$W_{opt} = \frac{|q^S|^2}{8\tilde{B}} \left\{ 1 - \frac{\Lambda_{opt} - \tilde{B}}{\Lambda_{opt} + \tilde{B}} \right\}. \quad (33)$$

Moreover, since $\tilde{A}(\omega)$ and $\tilde{B}(\omega)$, then $\Lambda(\omega)$, which means that for each wave frequency, the turbine parameter must be altered appropriately to satisfy Equation (32). Therefore, the method is an optimization for all frequencies.

Hence, the maximum hydrodynamic efficiency of this power extraction is given as

$$\eta_{max} = \frac{W_{opt}}{W_{max}} = \frac{2\tilde{B}}{\Lambda_{opt} + \tilde{B}}. \quad (34)$$

The dimensionless form of the radiation susceptance $\tilde{A}$ and conductance $\tilde{B}$ of an OWC as defined in Equation (22) can be expressed as

$$\mu = \frac{\rho g}{\omega D} \tilde{A}, \quad \nu = \frac{\rho g}{\omega D} \tilde{B}. \quad (35)$$

Using Equation (35), the maximum hydrodynamic efficiency of the OWC device η_{max} as defined in Equation (34) can be expressed in non-dimensional form as (see Evans and Porter [2])

$$\eta_{max} = \frac{2}{1 + \sqrt{1 + \left(\frac{\mu}{\nu}\right)^2}}. \quad (36)$$

Moreover, it is noted that the absolute value of the dimensionless volume flux due to scattered potential is represented as $|q^S/q^I|$, where q^I is the incident volume flux as

$$|q^I| = k_0 D \tanh k_0 h_1. \quad (37)$$

5. Numerical Convergence and Model Validation

5.1. Convergence of Solution: Panel Size

The convergence of the numerical solution depends on the panel size p_s used to discretize the physical boundaries. The number of panels n_{p_s} are

$$n_{p_s} = \left\lfloor \frac{\text{computational domain boundary length}}{p_s} \right\rfloor,$$

where $\lfloor \cdot \rfloor$ is the floor function.

In Table 1, the truncation analysis is carried out to nullify the effect of local disturbance of the far-field boundary in the case of a single rectangular breakwater. It can be seen that at a distance of 4 times the water depth between the left face of the OWC and the left far-field boundary, the results converge up to four decimal places. Tables 2–4 show the convergence of the solution in the case of a single trench with three different shapes positioned near the fixed-detached OWC. Tables 2–4 show that the values of the radiation conductance, radiation susceptance, and hydrodynamic efficiency converge up to two decimal places for $p_s \geq 0.05$. Therefore, $p_s = 0.05$ is assumed in subsequent computations.

Table 1. The values of μ, ν, η_{max} for different distances (dist.) at which the left far-field boundary is truncated in the case of a single rectangular breakwater with $a_1/h_1 = 3/4, a_2/h_1 = 1/2, b_1/h_1 = b_2/h_1 = 1/8, D/h_1 = 1/2, L/h_1 = 1/2, l/h_1 = 1/2, w/h_1 = 1/2$.

Dist.	$Kh_1 = 0.5$			$Kh_1 = 1.5$			$Kh_1 = 2.5$		
	μ	ν	η_{max}	μ	ν	η_{max}	μ	ν	η_{max}
$2h_1$	1.7239	0.0380	0.0432	−1.2060	0.0777	0.1208	−0.3649	0.0040	0.0219
$3h_1$	1.7239	0.0380	0.0432	−1.2060	0.0777	0.1208	−0.3649	0.0040	0.0218
$4h_1$	1.7239	0.0380	0.0432	−1.2060	0.0777	0.1208	−0.3649	0.0040	0.0218
$5h_1$	1.7239	0.0380	0.0432	−1.2060	0.0777	0.1208	−0.3649	0.0040	0.0218

Table 2. The values of μ, ν, η_{max} for different values of p_s for a single triangular trench with $a_1/h_1 = a_2/h_1 = 1/2, b_1/h_1 = b_2/h_1 = 1/8, D/h_1 = 1, l/h_1 = 2, w/h_1 = 1, L/h_1 = 1$.

p_s	$Kh_1 = 0.5$			$Kh_1 = 1.5$			$Kh_1 = 2.5$		
	μ	ν	η_{max}	μ	ν	η_{max}	μ	ν	η_{max}
0.1	0.7784	1.8116	0.9576	0.6327	0.0496	0.1451	−0.4344	0.0141	0.0631
0.05	0.7749	1.8540	0.9597	0.6368	0.0503	0.1462	−0.4728	0.0142	0.0585
0.01	0.7748	1.8544	0.9584	0.6347	0.0511	0.1487	−0.4736	0.0143	0.0555

Table 3. The values of μ, ν, η_{max} for different values of p_s for a single parabolic trench with $a_1/h_1 = a_2/h_1 = 1/2, b_1/h_1 = b_2/h_1 = 1/8, D/h_1 = 1, l/h_1 = 2, w/h_1 = 1, L/h_1 = 1$.

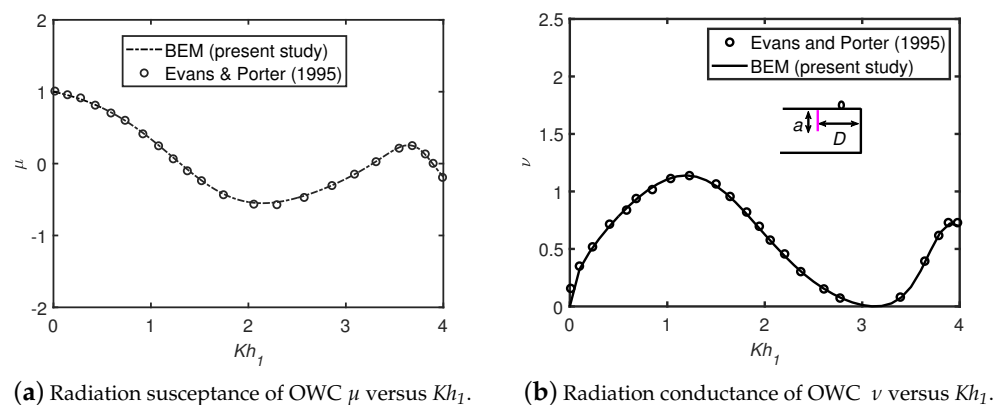
p_s	$Kh_1 = 0.5$			$Kh_1 = 1.5$			$Kh_1 = 2.5$		
	μ	ν	η_{max}	μ	ν	η_{max}	μ	ν	η_{max}
0.1	0.8025	1.8242	0.9557	0.5581	0.0543	0.1766	−0.4344	0.0141	0.0630
0.05	0.8001	1.8672	0.9578	0.5574	0.0553	0.1797	−0.4729	0.0142	0.0585
0.01	0.8037	1.8660	0.9595	0.5523	0.0562	0.1781	−0.4736	0.0143	0.0555

Table 4. The values of μ, ν, η_{max} for different values of p_s for a single rectangular trench with $a_1/h_1 = a_2/h_1 = 1/2, b_1/h_1 = b_2/h_1 = 1/8, D/h_1 = 1, l/h_1 = 2, w/h_1 = 1, L/h_1 = 1$.

p_s	$Kh_1 = 0.5$			$Kh_1 = 1.5$			$Kh_1 = 2.5$		
	μ	ν	η_{max}	μ	ν	η_{max}	μ	ν	η_{max}
0.1	0.8287	1.8370	0.9537	0.4823	0.0592	0.2174	−0.4345	0.0141	0.0630
0.05	0.8270	1.8802	0.9558	0.4789	0.0604	0.2224	−0.4729	0.0142	0.0585
0.01	0.8251	1.8853	0.9564	0.4720	0.0615	0.2288	−0.4737	0.0143	0.0555

5.2. Comparison with Existing Analytical Results of an OWC Positioned on a Flat Bottom Topography (Evans & Porter [2])

In Figure 3, the radiation susceptance μ , radiation conductance ν , and hydrodynamic efficiency η_{max} as defined in Equations (35) and (36) and the magnitude of the induced volume flux due to the scattering potential $|q^S|/|q^I|$ are plotted against non-dimensional wave frequency Kh_1 . The results from the current method are compared with those from the earlier study of Evans & Porter [2]. Figure 3 shows the verification of the numerical method in the case of a shore-fixed OWC device with a flat rigid bottom. The parameters, such as OWC front wall draft $a_1/h = 0.125$, OWC chamber breadth $D/h = 1$, and still water depth $h (= h_1)$, are the same as those of Evans & Porter [2]. The results from the present study and the previous studies are in very good agreement.

**Figure 3.** Cont.

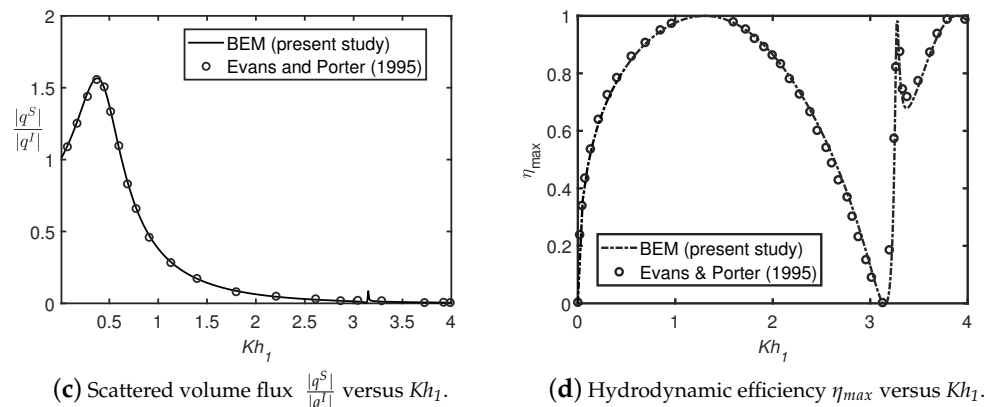


Figure 3. The variation in (a) radiation susceptibility μ , (b) radiation conductance ν , (c) the magnitude of the induced volume flux due to the scattering potential $|q^S|/|q^I|$, and (d) hydrodynamic efficiency of an OWC η_{max} versus Kh_1 in the case of a flat sea bed. Evans & Porter [2].

5.3. Comparison with Existing Experimental Result (Thomas et al. [33]) and Analytic Result (Rezanejad et al. [13])

Figure 4 shows the variation in hydrodynamic efficiency η_{max} of a shore-fixed OWC in the presence of (a) flat bottom and (b) over a stepped bottom. Figure 4a shows a comparison between the present numerical result with the experimental data of Thomas et al. [33]. The similar trends between the present numerical result and experimental result are observed. Moreover, a small deviation between two results is due to the assumption that the fluid is inviscid in the present linearized model. In Figure 4b, the hydrodynamic efficiency matches well with the existing analytical result of Rezanejad et al. [13] in the case of a stepped seabed.

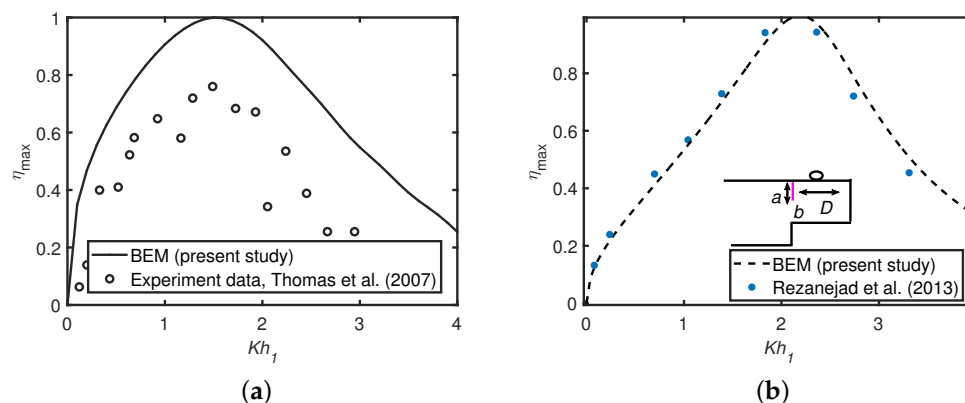


Figure 4. The variation in hydrodynamic efficiency η_{max} versus Kh_1 for a shore-fixed OWC (a) in the presence of a flat bottom, and (b) over a stepped bottom bed with a thick OWC front wall. Rezanejad et al. [13] and Thomas et al. [33].

5.4. Comparison with Existing Results of a Fixed-Detached OWC Device in the Presence and Absence of a Vertical Wall (Rodríguez et al. [28])

Figure 5 illustrates the effect of seabed undulation on hydrodynamic efficiency η_{max} in the case of a fixed-detached OWC with (Figure 5a) and without (Figure 5b) a shore-fixed reflecting wall with $D/h_1 = 1, a_1/h_1 (= a_2/h_1) = 1/2, b_1/h_1 (= b_2/h_1) = 1/8, l/h_1 = 2, w/h_1 = 1, L/h_1 = 1$. Figure 5a reveals that when the distance between the top of a parabolic breakwater/trench and the mean free surface h_2/h_1 decreases, the peak efficiency value shifts to the lower frequencies and the efficiency bandwidth is reduced. The result in the case of a flat seabed agrees well with the numerical result of Rodríguez et al. [28] with $h_2/h_1 = 1$. In Figure 5b, the hydrodynamic efficiency matches well with the existing

analytical result of Rodriguez et al. [28] in the absence of a reflecting wall with $h_2/h_1 = 1$. A comparison between Figure 5a,b shows that resonance in the efficiency curves occurs in the presence of a reflecting wall. Hence, the validations as shown in Figures 3–5 confirm the accuracy of the present numerical model.

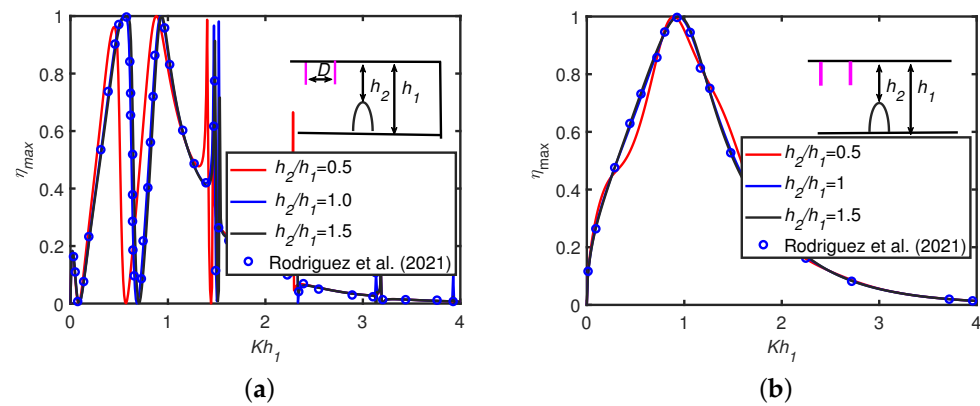


Figure 5. The variation in hydrodynamic efficiency η_{max} versus Kh_1 for a fixed-detached OWC in the presence of a parabolic bottom (a) with a vertical reflecting wall and (b) without a vertical reflecting wall. Rodriguez et al. [28].

6. Results and Discussion

In order to analyze the effects of different wave and structural parameters on the hydrodynamic performance of an OWC in the presence of different seabed configurations, numerical codes are written using MATLAB R2023a software. Various non-dimensional structural parameters associated with OWC and breakwaters/trenches are summarized in Tables 5 and 6. The values of the selected parameters listed in Tables 5 and 6 are kept fixed unless it is highlighted in the appropriate figure's caption. The OWC chamber walls are considered to have constant thickness, i.e., b/h_1 ($=b_1/h_1 = b_2/h_1$), unless otherwise mentioned.

Table 5. Numerical data employed for computation in the case of a single trench/breakwater.

Parameters	Values	Parameters	Values
h_1 water depth	4m	D/h_1 OWC chamber length	1/2
a_1/h_1 OWC front wall draft	3/4	w/h_1 trench/breakwater width	1/2
a_2/h_1 OWC rear wall draft	1/2	l/h_1 gap distance between trench and OWC	1/2
b_1/h_1 OWC front wall thickness	1/8	L/h_1 gap distance between trench and vertical wall	1/2
b_2/h_1 OWC rear wall thickness	1/8		

Table 6. Numerical data employed for computation in the case of multiple trenches/breakwaters.

Parameters	Values	Parameters	Values
h_1 water depth	4 m	D/h_1 OWC chamber length	1/2
a_1/h_1 OWC front wall draft	2/10	w/h_1 trench/breakwater width	1
a_2/h_1 OWC rear wall draft	1/10	l/h_1 gap distance between trench and OWC	2
b_1/h_1 OWC front wall thickness	1/10	L/h_1 gap distance between trench and vertical wall	1
b_2/h_1 OWC rear wall thickness	1/10	d/h_1 gap distance between two consecutive trenches/breakwaters	1/2

6.1. Effects of a Single Breakwater/Trench: Shapes, Depth, and Width

Figure 6 demonstrates the influence of different distances from the top of the breakwater to the mean free surface h_2/h_1 on (a) radiation susceptance μ , and (b) radiation conductance ν in the case of a single parabolic breakwater/trench with $a_1/h_1 = a_2/h_1 = 1/2$, $b_1/h_1 = b_2/h_1 = 1/8$, $D/h_1 = 1$, $l/h_1 = 2$, $w/h_1 = 1$, $L/h_1 = 1$. The results in Figure 6 in the case of a flat bottom agree well with the existing result of Rodriguez et al. [29]. Figure 6b shows that the area under the curves decreases as h_2/h_1 decreases, which is similar to the trend observed in efficiency η_{max} in Figure 5a. A comparison between Figures 6a and 5a reveals that the minimal values in radiation susceptance μ correspond to the higher values in the efficiency η_{max} in the case of a single parabolic breakwater/trench, which is obvious as it follows from Equation (36).

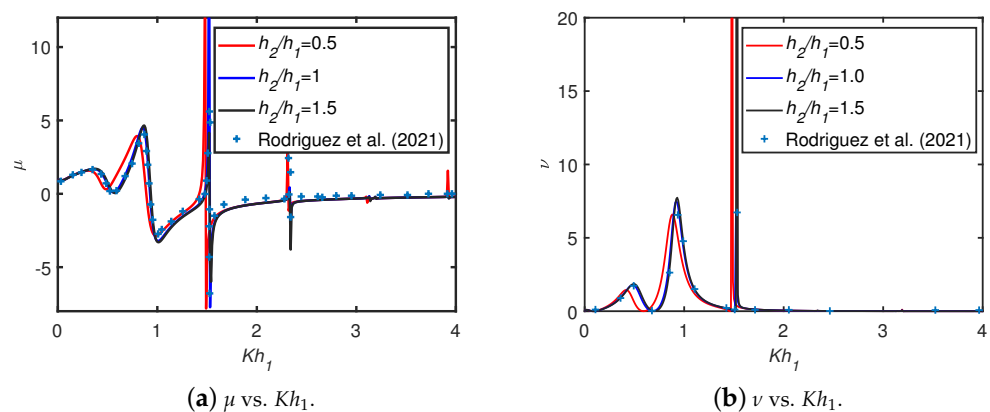


Figure 6. The influence of water depth h_2/h_1 on (a) radiation susceptance μ , and (b) radiation conductance ν in the case of a single parabolic breakwater/trench. Rodriguez et al. [29].

Figure 7 demonstrates the variation in hydrodynamic efficiency of an OWC against non-dimensional frequency Kh_1 for three different bottom configurations, namely, triangular, parabolic, and rectangular, in the case of unequal (Figure 7a,b) and equal (Figure 7c,d) front walls drafts of an OWC. Figure 7 shows that the first peak efficiency value that occurs at lower frequencies is greater in the case of a larger front wall draft compared to a larger rear wall draft. Figure 7a,b reveal that the first hydrodynamic peak efficiency value that occurs for a lower frequency, particularly $Kh_1 < 0.5$, is higher in the case of a rectangular ($m_0 = \infty$) configuration compared to that of a triangular ($m_0 = 1$) and a parabolic ($m_0 = 2$) bottom shape. Figure 7a,b show that the second bandwidth in the case of a rectangular breakwater ($0.2 < Kh_1 < 3.2$) is larger compared to that of a parabolic and a triangular shaped breakwater ($0.2 < Kh_1 < 1.8$). The area under the largest resonance peak efficiency is reduced in the case of a smaller front wall draft compared to a large front wall draft, as shown in Figure 7a,b. This scenario occurs because more wave energy is captured in the case of a smaller draft front wall OWC in the presence of the breakwater. The reflected wave energy is greater in the case of a smaller draft OWC front wall. In Figure 7c,d, it is observed that the first peak efficiency value is higher in the case of a trench compared to a breakwater. It may be noted that the theoretical value of $\eta_{max} = 1$ means that the OWC device successfully captures the entirety of the wave energy.

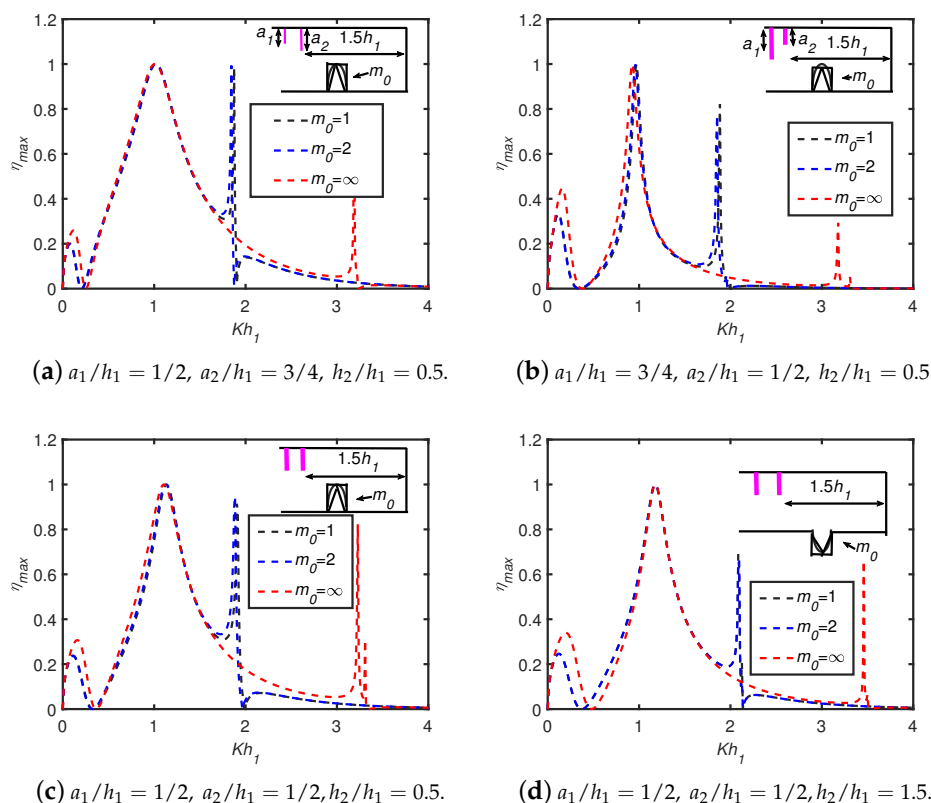


Figure 7. Hydrodynamic efficiency η_{max} versus Kh_1 for different shapes of a breakwater/trench in the case of (a,b) unequal and (c,d) equal front wall drafts of an OWC.

The variation in hydrodynamic efficiency η_{max} versus Kh_1 is plotted for different (a) submerged depths of the front and rear walls, (b) submerged depths of the front wall, and (c) submerged depths of the rear wall in the case of a single triangular trench as shown in Figure 8. Figure 8a reveals that the maximum peak efficiency value that occurs for $Kh_1 < 2$ shifts to lower frequencies with an increase in the front and rear wall drafts. In Figure 8b,c, it is observed that when the submergence of one of the walls increases, the values of the frequency at which the maximum resonance efficiency $\eta_{max} = 1$ occurs decrease. Figure 8b reveals that with an increase in draft of the OWC front wall, the efficiency bandwidth is reduced. The peaks at low-frequency ($Kh_1 \approx 0.2$) decrease with an increase in the front wall draft of an OWC. For the smaller frequencies, particularly $Kh_1 < 0.5$ in terms of the peak efficiency value, Figure 8c shows an opposite trend to that observed in Figure 8b.

Figure 9 shows the variation in hydrodynamic efficiency η_{max} plotted against non-dimensional frequency Kh_1 for equal (Figure 9a) and unequal (Figure 9b,c) thicknesses of the front and rear walls in the case of a single triangular trench. The bandwidth of maximum-height peak efficiency decrease with an increase in the wall thickness of the OWC. The frequency at which maximum efficiency occurs decreases with an increase in wall thickness of an OWC. Figure 9 reveals that when the thicknesses of the front and rear walls increase, the peak efficiency value at $Kh_1 \approx 0.2$ increases. Figure 9a,b show that the zero efficiency point shifts toward the lower frequencies with an increase in wall thickness. A comparison of Figure 9c with Figure 9a,b shows that the first peak efficiency value is greater when the thickness of an OWC front wall increases. The bandwidth of the efficiency curves is significantly decreased when the front wall thickness increases as shown in Figure 9c. The zero efficiency occurs at a similar frequency irrespective of the change in front wall thickness of an OWC.

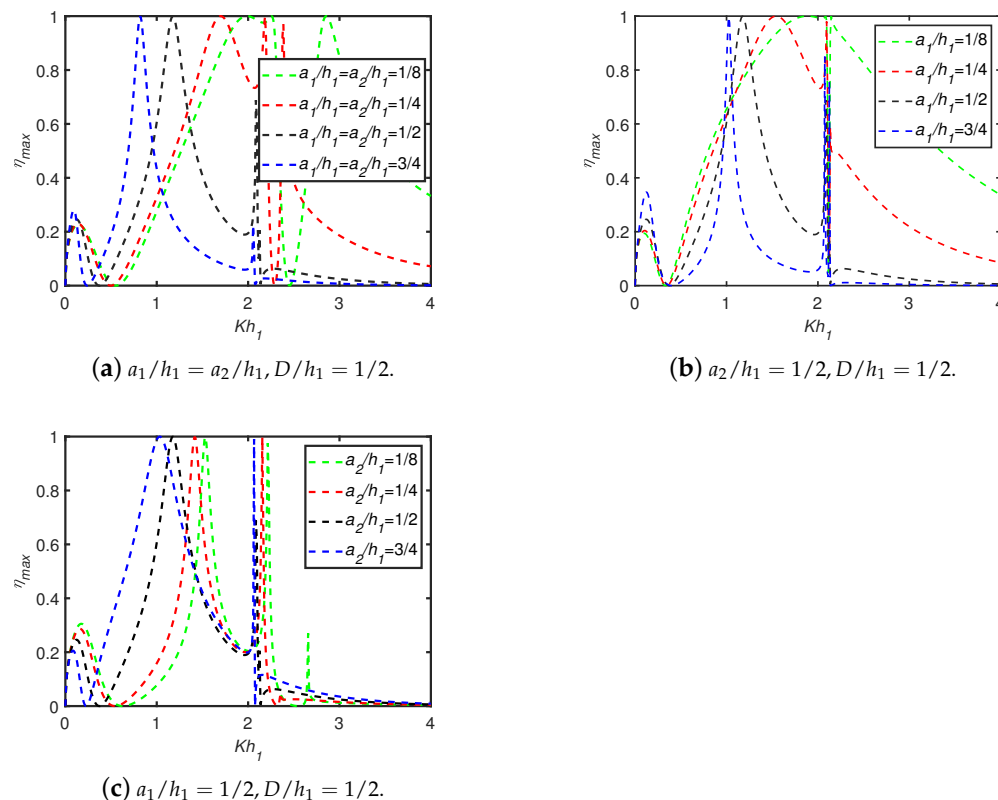


Figure 8. Hydrodynamic efficiency η_{max} versus Kh_1 for different (a) submerged depths of front and rear walls, (b) submerged depths of front wall, and (c) submerged depths of rear wall in the case of a single triangular trench.

In Figure 10, the results of hydrodynamic efficiency η_{max} are plotted against non-dimensional frequency Kh_1 for different values of the chamber length-to-water depth ratio $D/h_1 = 1/4, 1/2, 3/4, 1$ in the presence of a single triangular trench. Figure 10 reveals that the first peak efficiency value increases with increasing OWC chamber length D/h_1 irrespective of the different values of the front and rear wall drafts of an OWC. Moreover, the frequency at which maximum-height peak value occurs is shifted towards the lower frequencies when the chamber length D/h_1 increases. The first peak efficiency value at $Kh_1 \approx 0.2$ is greater in the case of a larger front wall draft as shown in Figure 10a compared to Figure 10c,b as the chamber length D/h_1 increases. Figure 10c demonstrates the maximum-height peak efficiency bandwidth increases with an increase in the chamber length of an OWC. It is seen that the bandwidth for maximum-height peak efficiency is highly reduced in the case of a larger front wall draft of an OWC. Figure 10 reveals that the variation in frequency points at which the maximum-height peak efficiency occurs is more compared with Figure 9. Moreover, in Figure 10 for $Kh_1 \approx 0.5$, all the efficiency curves coincide irrespective of the chamber length D/h_1 . The reason may be the occurrence of destructive interference.

Figure 11 shows the variation in hydrodynamic efficiency η_{max} versus Kh_1 for different values of (a) trench width w/h_1 , and (b) gap length between the trench and OWC l/h_1 (maintaining a fixed gap length between the OWC and vertical wall) in the case of a single triangular trench. Figure 11a reveals that the first peak efficiency value remains unchanged with an increase in the trench width w/h_1 . The bandwidth of peak efficiency at $Kh_1 = 1$ changes slightly, while for a larger trench width, the bandwidth decreases. In Figure 11b, a variation in the efficiency curves is observed with an increase in the gap distance between the trench and the OWC. A greater number of resonance peaks are observed compared

with Figure 11a. Figure 11b reveals that when the gap distance increases, the first peak efficiency value occurs for the lower frequency; particularly, $Kh_1 < 0.5$ decreases. Troughs between two consecutive resonance peaks are observed in the case of short and intermediate wave-length regimes. The efficiency bandwidth at the lower frequency decreases with an increase in the gap distance between the trench and OWC as shown in Figure 11b. The peak frequency value is shifted to the lower frequencies. This is because increasing the gap distance increases the horizontal distance a fluid particle must travel during a period of motion, resulting in a reduction in the value at which the resonance occurs.

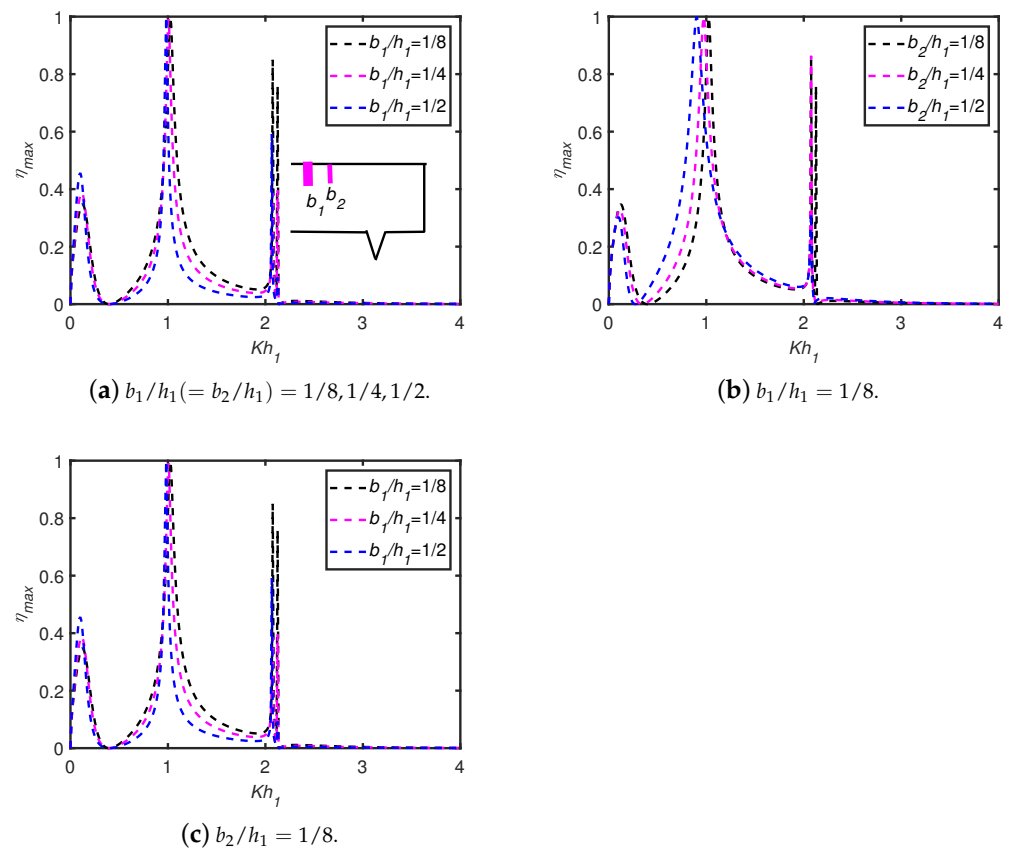


Figure 9. Hydrodynamic efficiency η_{max} versus Kh_1 for different (a) thicknesses of front and rear walls, (b) thicknesses of rear wall, and (c) thicknesses of front wall in the case of a single triangular trench.

Figure 12 demonstrates the variation in hydrodynamic efficiency η_{max} against dimensionless wave frequency Kh_1 for different values of (a) the distance from the top of the breakwater to the mean free surface h_2/h_1 , (b) the width of a breakwater w/h_1 with OWC front wall draft $a_1/h_1 = 2/10$, and OWC rear wall draft $a_2/h_1 = 1/10$, chamber length $D/h_1 = 1/2$, and thickness of OWC wall $b_1/h_1 = b_2/h_1 = 1/10$. Figure 12a shows that in the case of the short wave-length regime, there is minimal variation in the efficiency curves with an increase in distance from the top of a breakwater to the mean free surface h_2/h_1 . Figure 12a shows that the variation in the OWC efficiency in the case of a flat bottom $h_2/h_1 = 1$ agrees very well in the case of the breakwater ($h_2 < h_1$). The variation in efficiency improves in the case of the trench ($h_2 > h_1$) compared to the breakwater ($h_2 < h_1$) in the long wave-length regime. This phenomenon is due to the occurrence of additional trapped wave energy in the case of trench-type bottom undulation compared to a breakwater undulation. In short and intermediate wave-length regimes, an efficiency phase shift is observed with an increase in wave frequency Kh_1 . Figure 12b demonstrates the variation in efficiency η_{max} against dimensionless wave frequency Kh_1 with a change in trench width w/h_1 in the case of a single triangular shape trench ($m_0 = 1$). Figure 12b reveals

that the efficiency increases with an increase in Kh_1 in the case of long and intermediate wave-length regimes, while it decreases in the short wave-length regime. Furthermore, the amplitude of the resonance efficiency peaks decreases with an increasing width of the trench. The occurrence of resonance peaks and the number of zero efficiency points increases as the width of the trench increases, which is due to the formation of nodes and anti-nodes inside the trench and gap region.

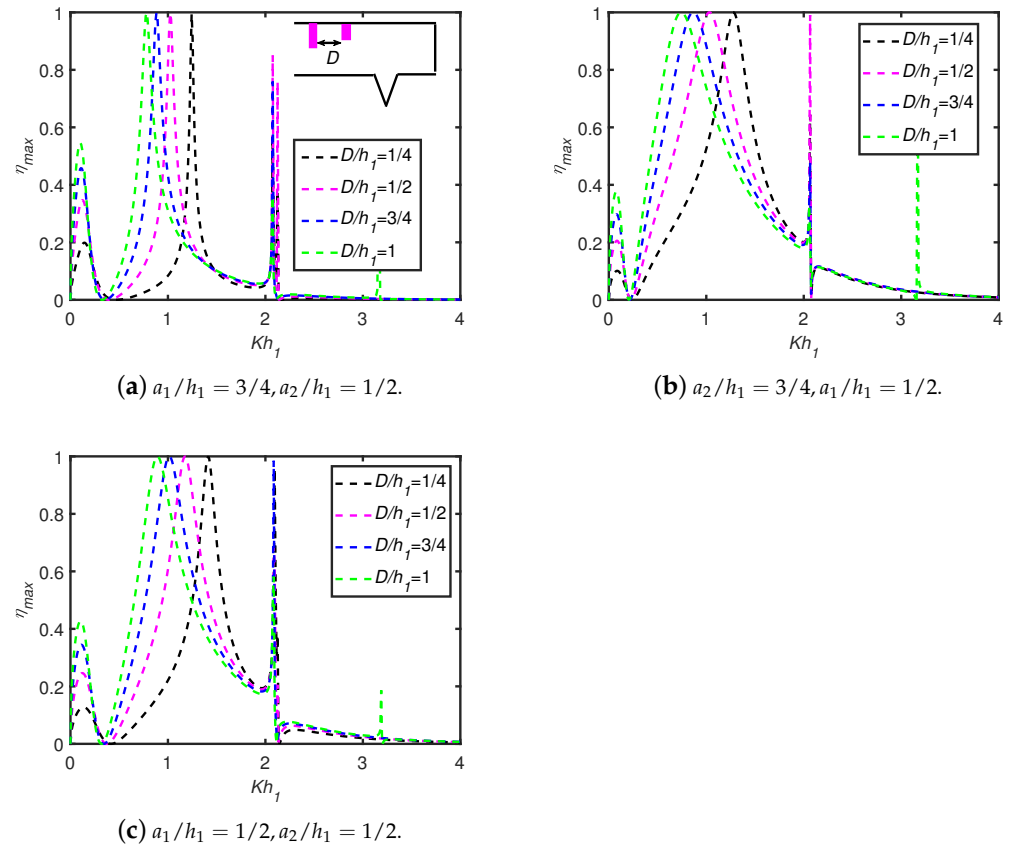


Figure 10. Hydrodynamic efficiency η_{max} versus Kh_1 for different chamber lengths of an OWC in the case of (a,b) unequal front walls drafts, and (c) equal draft front wall in the case of a single triangular trench.

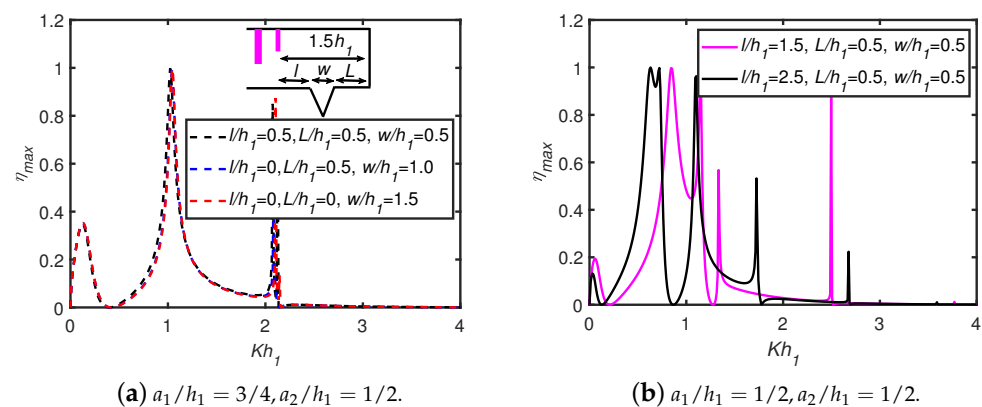


Figure 11. Hydrodynamic efficiency η_{max} versus Kh_1 for different values of (a) trench width, and (b) gap length between a single trench and an OWC.

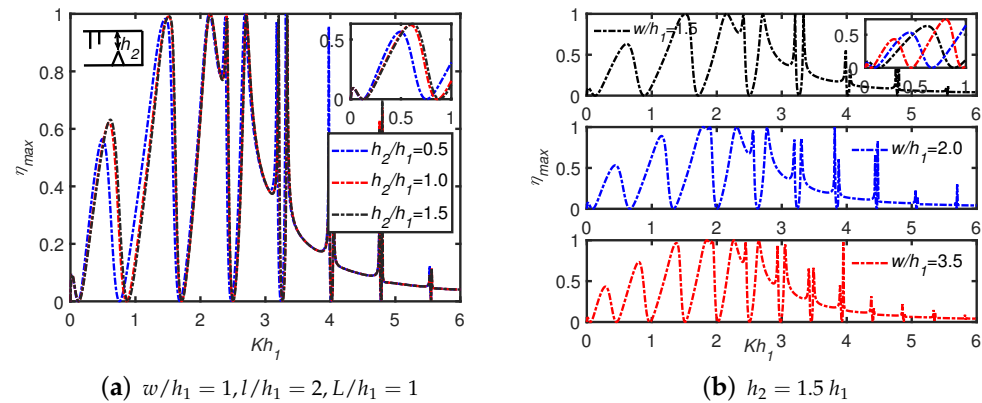


Figure 12. Hydrodynamic efficiency η_{max} versus Kh_1 for different values of (a) distance from top of the breakwater to the mean free surface h_2/h_1 , and (b) width w/h_1 with $h_1 = 4$ m.

6.2. Effects of Multiple Trenches/Breakwaters

Figure 13 demonstrates the variation in the hydrodynamic efficiency η_{max} against dimensionless wave frequency Kh_1 in the case of (a) double and (b) triple triangular breakwaters–trenches in combination. Figure 13a reveals that the efficiency is higher in the case of multiple trenches as compared to the trench–breakwater combination. A similar scenario is observed in the case of a single trench as shown in Figure 12a. The amplitude of the resonance peaks increases with an increase in Kh_1 for the long wave-length regime, whilst efficiency decreases for intermediate and short wave-length regimes. A comparison between Figures 13a and 7a reveals that the number of resonance peaks for the long wave-length regime is greater as the number of breakwaters/trenches increases. The total number of peaks is equal to $N + 2$ for N number of breakwaters/trenches in the long wave-length regime. The number of resonance peaks and troughs increases with an increase in the number of breakwaters. Figure 13b illustrates that there is a shift in the resonant efficiency peak positions with a change in depth h_3/h_1 where h_3 is the gap distance between the mean free surface to the top of the second breakwater. A similar phenomenon is observed in Figure 13a. Moreover, the occurrence of a shift in resonance peak points happens due to the alternating depth from the top of the breakwater to the mean free surface. In Figure 13b, an increase in the number of resonance peaks and troughs is observed, which is due to additional trapped wave energy.

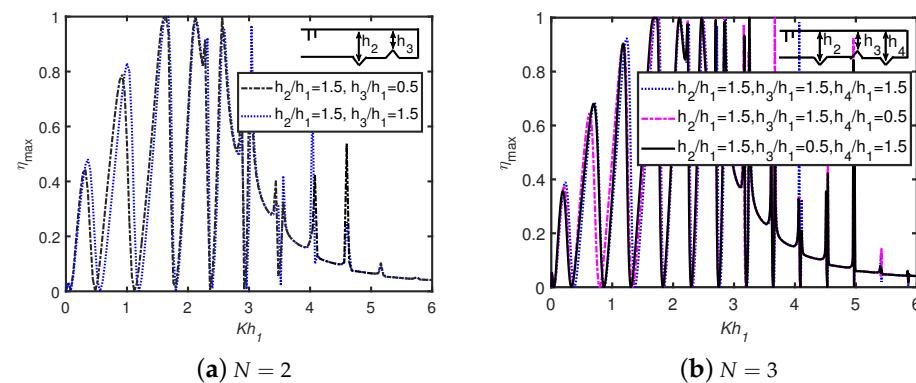


Figure 13. Hydrodynamic efficiency η_{max} versus Kh_1 for the case of (a) double ($N = 2$) and (b) triple ($N = 3$) breakwaters/trenches in combination.

Figure 14 demonstrates the variation in the hydrodynamic efficiency η_{max} against the dimensionless wave frequency Kh_1 for different values of trench width, (a) $w/h_1 = 1.5$,

(b) $w/h_1 = 2.5$, in the case of triple trenches. Figure 14a shows that the peak amplitude in efficiency η_{max} increases with an increase in Kh_1 for $Kh_1 < 2$, whilst the reverse trend is observed for $Kh_1 > 2$. Therefore, $0 < Kh_1 < 6$ can be defined as one full cycle resonance bandwidth in terms of the efficiency. Two consecutive resonant peaks are separated by a zero minimum for $Kh_1 < 2$. However, for $Kh_1 > 2$, a trough and a zero minimum point alternately occur between two successive peaks. Figure 14a demonstrates that the number of resonance peaks and troughs between two consecutive resonance peaks increases with an increase in wave frequency Kh_1 in the case of multiple trenches as compared to a single trench. The peaks occur due to the formation of anti-nodes in the standing wave pattern, and the minimum values occur due to nodes over the trenches and gap distance between the trenches. A similar result was demonstrated by Kar et al. [31]. A comparison between Figure 14a,b reveals that the amplitude of the resonance peaks decreases for a larger trench width due to resonance.

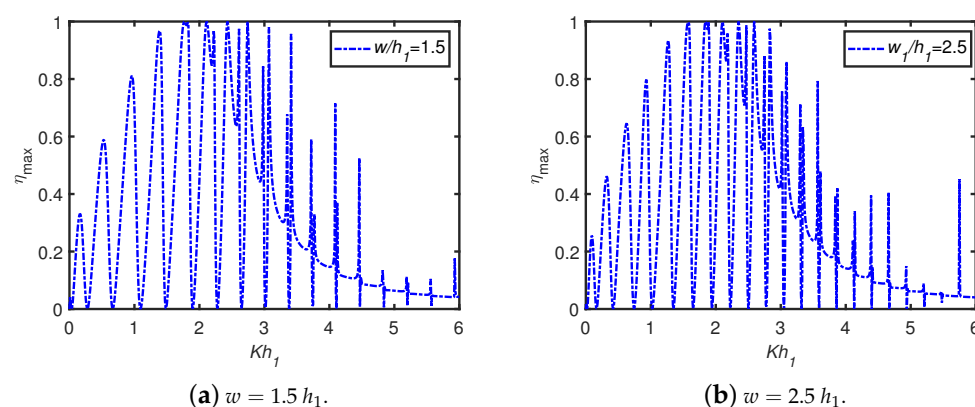


Figure 14. Hydrodynamic efficiency η_{max} versus Kh_1 for different values of trench width: (a) $w/h_1 = 1.5$ and (b) $w/h_1 = 2.5$.

6.3. Effects of OWC Parameters: Front Wall Draft, Chamber Breadth, Wall Thickness

Figure 15 shows the variation in efficiency η_{max} against the dimensionless wave frequency Kh_1 for different values of (a) OWC front wall drafts a_1/h_1 , and (b) chamber breadths D/h_1 in the case of triple trenches. The number of resonance peaks is higher in the presence of triple trenches compared to single and double trenches as observed in Figures 12a and 13a. Figure 15a shows that the amplitude of the resonance peak efficiency η_{max} increases with an increase in wave frequency Kh_1 for $Kh_1 < 1$ in the case of $a_1/h_1 = 0.8$, while it decreases and tends to zero for higher values of Kh_1 , i.e., $Kh_1 > 3$ in the case of $a_1/h_1 = 0.4, 0.6$, respectively. Figure 15a shows that zero efficiency is observed for the higher wave frequencies Kh_1 , i.e., $Kh_1 > 2$ in the case of $a_1/h_1 = 0.8$. The bandwidth of a complete cycle decreases with an increase in the OWC front wall draft a_1/h_1 . Figure 15a shows that the number of resonance peaks decreases with an increase in wave frequency Kh_1 for $Kh_1 < 2$ with the deeper OWC front wall draft a_1/h_1 . Troughs occur between the consecutive resonance peaks, which is a similar observation as seen in Figure 14. The number of troughs decreases with an increase in the OWC front wall draft a_1/h_1 . Figure 15b reveals that the peak amplitude efficiency and bandwidth for $Kh_1 < 1$ increase when chamber length D/h_1 increases. A similar observation is seen in Figure 10 in the case of a single trench. Therefore, this study shows that suitable values of an OWC wall draft are important to achieve a higher efficiency from the OWC.

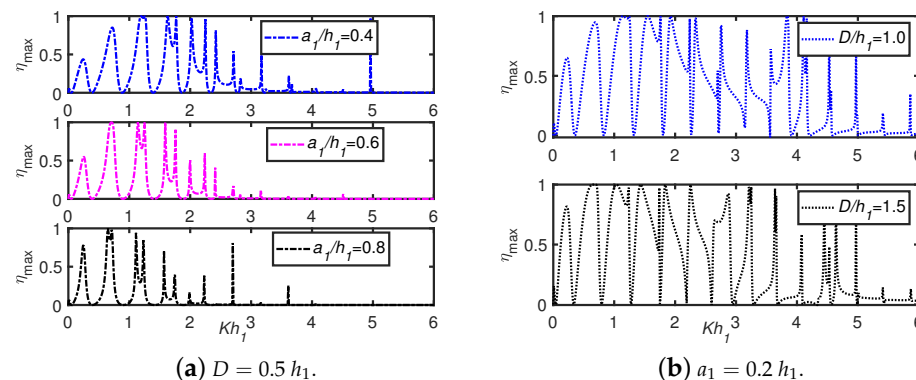


Figure 15. Hydrodynamic efficiency η_{max} versus Kh_1 for different values of (a) front wall draft, (b) a chamber breadth.

Figure 16 reveals the variation in hydrodynamic efficiency η_{max} versus Kh_1 in the case of different values of OWC wall thickness (in Figures 16a,b, the front and back walls have equal thickness, and in Figure 16c,d the front and back wall thicknesses are unequal) in the case of triple triangular trenches. Figure 16a,b reveal that in the case of the long wave-length regime, particularly for $Kh_1 < 0.5$, the peak amplitude value in efficiency η_{max} increases with an increase in the OWC wall thickness b/h_1 and zero efficiency occurs between two consecutive resonance peaks. A similar observation was discussed in Rodriguez et al. [29]. Moreover, it is observed that the pattern of the trough region in the efficiency curve between two consecutive resonance peaks for $3 < Kh_1 < 4$ follows a reverse trend as that seen for $1 < Kh_1 < 3$. This phenomenon occurs because of the change in phase of wave motion inside the chamber. Figure 16a,b also show that the trough amplitude between two consecutive resonance peaks decreases with an increase in OWC wall thickness b/h_1 . Additionally, the number of zero efficiency points in Figure 16a,b is the same irrespective of different values of wall thickness b/h_1 . Figure 16c shows that resonance peak efficiency amplitude increases with an increase in front wall thickness in the long wave-length region, whilst the trough between two consecutive resonance peaks decreases in magnitude and tends to zero for intermediate and short wave-length regimes. In Figure 16d, the variation in efficiency is negligible in the case of a smaller wave frequency for change in OWC rear wall thickness compared to a change in thickness of OWC front wall. Figures 15 and 16 show that chamber breadth D/h_1 and OWC wall thickness b/h_1 play an important role in enhancing the efficiency of the OWC.

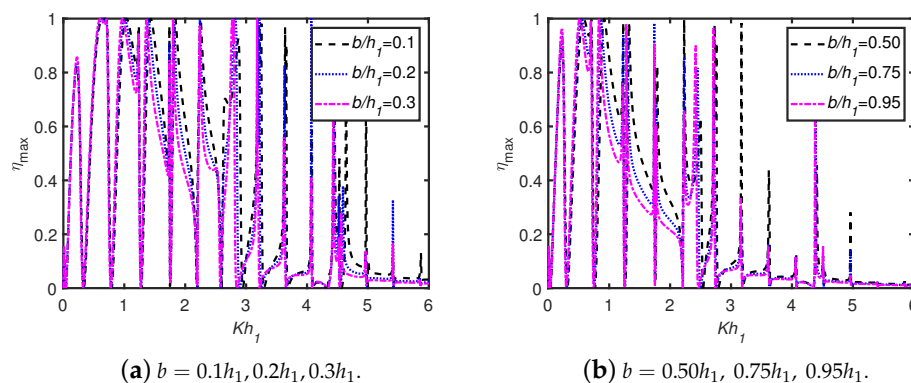


Figure 16. Cont.

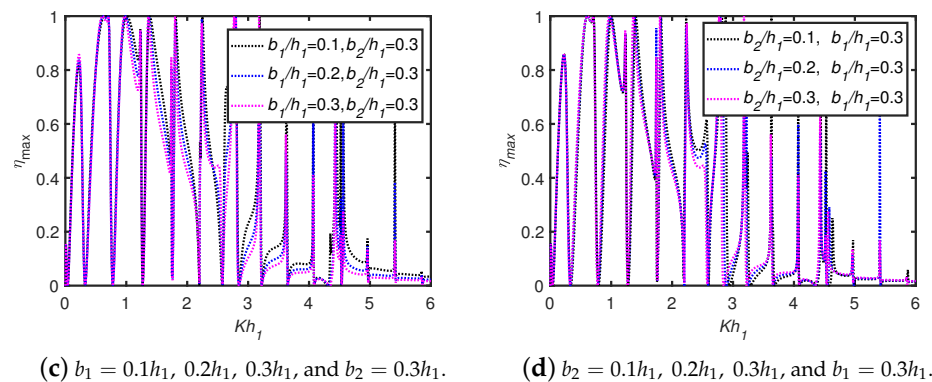


Figure 16. Hydrodynamic efficiency η_{max} versus Kh_1 in the case of different values of (a,b) equal OWC wall thickness and (c,d) unequal OWC wall thickness with OWC chamber breadth $D = 1.5h_1$.

Figure 17 demonstrates the variation in hydrodynamic efficiency η_{max} versus non-dimensional frequency Kh_1 in the case of different values of unequal OWC wall thickness in the case of triple triangular trenches with OWC chamber breadth $D/h_1 = 1/2$. Figure 17a reveals that the peak amplitude efficiency for smaller frequencies increases with an increase in the front wall thickness, while the trough between consecutive resonance peaks decreases, which is similar to the trend observed in Figure 16c. Figure 17b shows the minimal variation in efficiency when the OWC rear wall thickness increases. Figure 17 shows that the number of zero efficiency points is the same irrespective of the change in the front or rear wall thickness. A comparison between Figures 16c,d and 17 shows that with an increase in OWC chamber breadth, the resonance peak amplitude increases in the long wave-length regime. Moreover the troughs between two consecutive resonance peaks decreases uniformly, whilst in Figure 16c,d, the change in pattern of trough is observed in the case of intermediate and short wave-length regimes. This happens due to wave resonance, which is increased in the case of a larger chamber breadth.

The radiation susceptance μ (Figure 18), radiation conductance ν (Figure 19), and volume flux due to a scattered potential $|q^S|/|q^I|$ (Figure 20) versus non-dimensional wave frequency Kh_1 are plotted for different numbers of trenches N and different values of the chamber breadths of an OWC D/h_1 with $a_1/h_1 = 0.2, a_2/h_1 = 0.1, b_1/h_1 = b_2/h_1 = 0.1, l/h_1 = 2, w/h_1 = 0.5, d/h_1 = 0.5, L/h_1 = 1$. Figure 18a shows the radiation susceptance μ decreases with an increase in chamber breadth D/h_1 . The phase shift occurs with an increase in Kh_1 for a change in different values of D/h_1 . In Figure 18b, the number of resonance peaks in the radiation susceptance increases with an increase in the number of trenches N , which is a similar observation to that of Figure 13. Figure 19 shows that the peak amplitude is greater in the long and intermediate wave-length regimes, while the peak amplitude in efficiency equals zero in the short wave-length regime. In Figure 19b, a common maximum occurs for $Kh_1 = 2.2$, while a common zero minimum is observed for $Kh_1 = 1.9$ irrespective of the even and odd number of trenches. Figure 20a demonstrates the peak amplitude of volume flux due to a scattered potential $|q^S|/|q^I|$ increases with an increase in chamber breadth for long wave-length regimes whilst peak amplitude decreases with an increase in chamber breadth in the case of intermediate and short wave-length regimes. Figure 20b shows that the occurrence of resonance peaks increases with an increase in the number of trenches. Moreover, the common minima and maxima occur around $Kh_1 = 2$, which is a similar observation as that seen in Figure 19b. The occurrence of maximum and minimum values is due to the occurrence of nodes and anti-nodes inside the chamber. Therefore, a comparison between Figures 19 and 20b shows that the common maxima and minima in the radiation conductance of an OWC correspond to the com-

mon maxima and minima of the volume flux $|q^S|/|q^I|$. The reason for this follows from Equations (23) and (24).

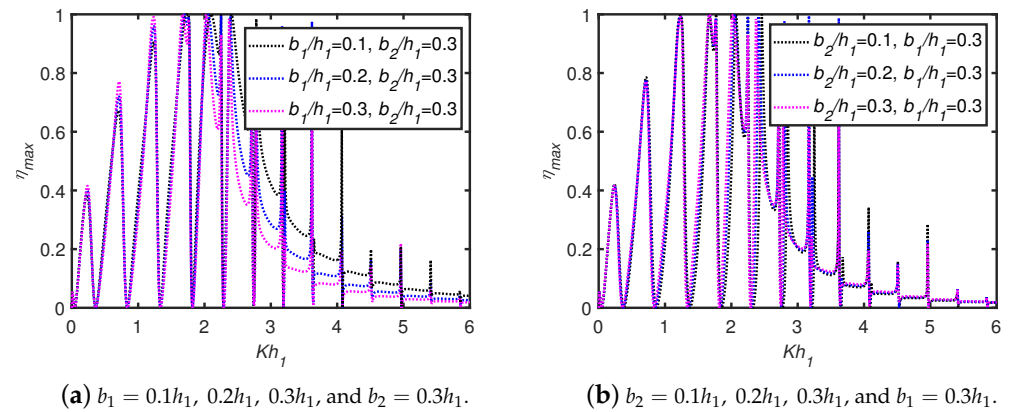


Figure 17. Hydrodynamic efficiency η_{max} versus Kh_1 in the case of different values of (a,b) unequal OWC wall thickness in the case of triple triangular trenches with OWC chamber breadth $D = 0.5h_1$.

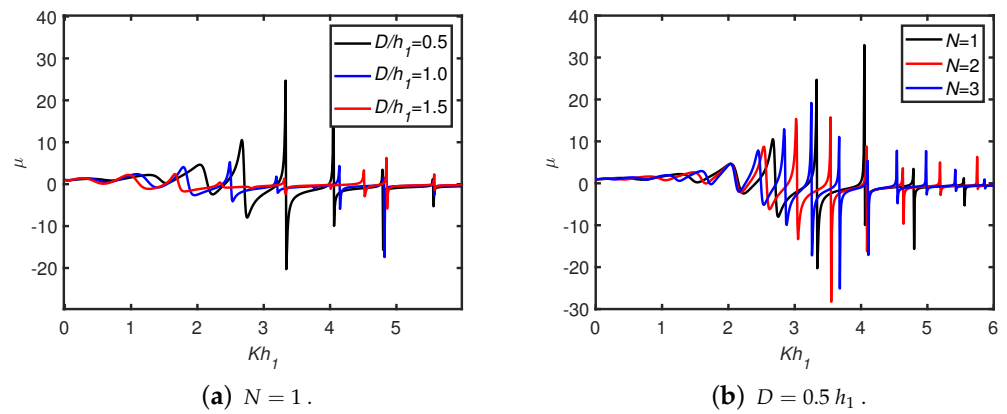


Figure 18. Radiation susceptance of an OWC against Kh_1 for different values of (a) chamber breadth, and (b) the number of trenches.

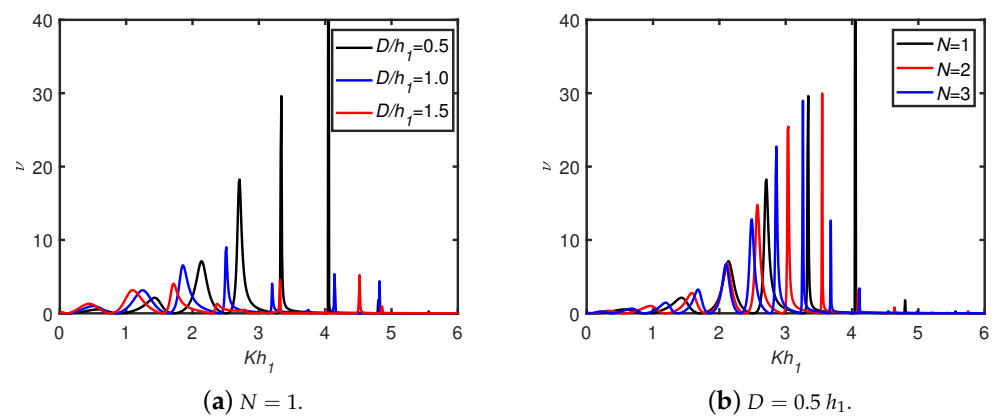


Figure 19. Radiation conductance of an OWC against Kh_1 for different values of (a) chamber breadth, and (b) the number of trenches.

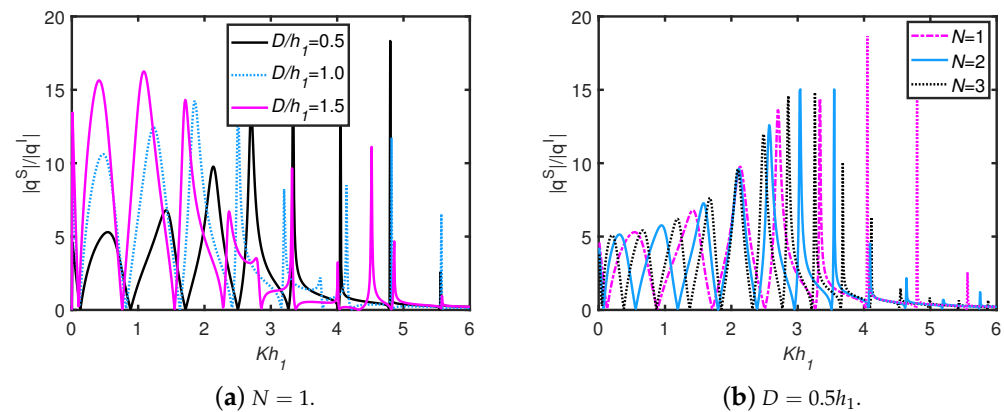


Figure 20. The volume flux due to a scattered potential $|q^S|/|q^I|$ against Kh_1 for different values of (a) chamber breadth, and (b) the number of trenches.

7. Conclusions

In the present study, the influence of submerged bottom standing trenches/breakwaters on the hydrodynamic performance of an OWC is investigated numerically using the BEM, based on the principles of linear potential flow theory. Moreover, fluid is considered as inviscid and incompressible, and the fluid motion is irrotational. The experimental comparison suggests that the results may not necessarily behave the same way in reality, where viscous effects could play a significant role. Various important physical parameters, such as radiation conductance, radiation susceptance, volume flux due to scatter potential, and hydrodynamic efficiency of an OWC are plotted and analyzed for different values of wave length and OWC chamber structural parameters. The following conclusions can be made:

- The waves scattered from the trenches/breakwaters and the OWC device undergo periodic oscillations resulting in multiple sharp peaks and dips in the efficiency curves. As the number of breakwaters as well as width of breakwaters/trenches increases, the number of resonant efficiency peaks increases, whilst the peak amplitudes decrease in the case of a long wave-length regime.
- For an OWC device, the wall draft and chamber length are the two critical geometrical factors that improve wave energy absorption.
- The peak efficiency value for lower-frequency wave conditions is observed in the efficiency curves due to trapped waves between the OWC and the vertical wall. When the width of multiple trenches increases, then the number of peak resonance values in the efficiency curves for lower frequencies increases. The total number of peaks is equal to $N + 2$ for N number of breakwaters/trenches in the long wave-length regime. In higher-frequency wave conditions, when the gap between the OWC and vertical wall is increased, the occurrence of number of troughs increases.
- Trenches are effective in increasing the efficiency of an OWC compared to breakwaters. In addition, the efficiency bandwidth is greater in the case of rectangular trenches compared to the trenches of parabolic and triangular shapes.
- For multiple resonances, two consecutive resonant peaks are separated by a zero minimum for smaller frequencies, particularly $Kh_1 < 2$. However, for $Kh_1 > 2$, a trough and a zero minimum point alternately occur between two successive peaks.

The present study is based on linear wave–structure interaction theory without considering non-linear effects, which could be included in future works.

Author Contributions: Validation, P.K.; Software, P.K.; Methodology, P.K., R.M. and D.N.; Writing—Original Draft Preparation, P.K.; Writing—review & editing, P.K., R.M. and D.N.; Investigation, P.K., R.M. and D.N.; Formal Analysis, P.K., R.M. and D.N.; Funding acquisition, R.M. and D.N. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding. P. Kar and D. Ning acknowledge the National Natural Science Foundation of China for their support through Grant No. U22A20242, and R. Mayon thanks the National Natural Science Foundation of China for the support through Grant No. 52250410362.

Data Availability Statement: The data that support the findings of this study are available upon reasonable request.

Conflicts of Interest: The authors declare no conflicts of interest.

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