Journal of *Marine Science and Engineering*

MDPI

Technical Note

A New Expression for a Goda Type Breaker Index Formula in Spectral Wave Models

Zereng Chen [1,2], Qinghe Zhang [2,*] and Yang Nie [2]

[1] College of Harbour, Coastal and Offshore Engineering, Hohai University, Nanjing 210024, China; zrchen@hhu.edu.cn
[2] State Key Laboratory of Hydraulic Engineering Intelligent Construction and Operation, Tianjin University, Tianjin 300350, China; nieyang2000@tju.edu.cn
* Correspondence: qhzhang@tju.edu.cn

Abstract: The breaker index in contemporary spectral wave models is a vital parameter in studying depth-induced wave breaking, which is an important mechanism in coastal engineering. In the previous study, the authors modified a Goda type breaker index formula and parameterized it in a spectral wave model. Although this parameterization model can accurately simulate nearshore wave height variations under various laboratory and field conditions, the computation of deep-water wave conditions in the formula necessitates reverse calculation based on the incident waves, thus limiting its applicability. To solve this problem, a new expression for this Goda type breaker index is rederived, enabling its applicability to simulate breaking waves caused by variable wave conditions on open boundaries and wind field inputs in the whole computational region. The model's capability to simulate wave heights in laboratory and field experiments is comparable to that of the original formula.

Keywords: SWAN model; Goda type breaker index; depth-induced wave breaking; irregular waves

check for updates

Academic Editor: Angelo Rubino

Received: 7 January 2025
Revised: 3 February 2025
Accepted: 6 February 2025
Published: 7 February 2025

Citation: Chen, Z.; Zhang, Q.; Nie, Y. A New Expression for a Goda Type Breaker Index Formula in Spectral Wave Models. *J. Mar. Sci. Eng.* **2025**, *13*, 313. https://doi.org/10.3390/jmse13020313

Correction Statement: This article has been republished with a minor change. The change does not affect the scientific content of the article and further details are available within the backmatter of the website version of this article.

Copyright: © 2025 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

1. Introduction

Gravity waves in the ocean are generated by wind, propagating towards the coast and finally breaking in the surf zone [1]. In coastal regions, the process of depth-induced wave breaking is ubiquitous. Accurately predicting variations in wave height within the surf zone is of paramount importance for coastal engineering research and design [2]. Presently, phase-averaged parameterized spectral wave models, such as the SWAN (Simulating WAves Nearshore) model, have become convenient approaches to simulate the propagation and breaking of irregular waves near the shore [3,4].

In these spectral wave models, although the dissipation modules have differences [5–7], the breaker index (γ, the ratio of the maximum or breaking wave height to the local water depth, d) serves as a key parameter, fundamentally influencing the accuracy of the model's simulation of breaking wave heights. Battjes and Janssen [5] (hereafter BJ78) considered that γ is a constant with a value of 0.8, and the value in current third-generation wave models is set to 0.73, as determined from extensive datasets. Based on comprehensive datasets with laboratory and field observations, some researchers found that the breaker index is related to the deep-water wave steepness [8,9], the beach slope [10], or the normalized water depth kd [11], where k is the local wavenumber. A new parameterization for depth-induced wave breaking was presented by Salmon et al. [12] (hereafter SA15), where the breaker index is calculated by the beach slope and kd. Lin and Sheng [13] (hereafter LS17) further

justified nonlinear slope dependency in SA15, and the modified formula improved the performance in simulating significant wave heights in shallow water. Using the inverse modeling approach, Zhang et al. [4] (hereafter ZL21) also developed a formula on the basis of Ruessink et al. [11], which revealed the composite effects of the offshore wave steepness and kd on γ.

In general, the development of the breaker index formula has evolved from a constant value to being influenced by a single factor to subsequently being influenced by dual factors [14]. Furthermore, it seems that breaker index formulas considering multiple factors have better performance in simulating wave heights than those considering single factors [4,12,13,15]. For formulas involving multiple factors, the classic one is Goda's formula, which has been widely used in coastal engineering [16]. Goda [17] developed a breaker index formula, which is a function of the beach slope, m, and relative water depth (the radio of d to the deep-water wavelength, L_0), based on various laboratory data, where the formula can be written as follows:

$$\gamma = A\left\{1 - \exp\left[-1.5\pi\frac{d}{L_0}\left(1 + 15m^{4/3}\right)\right]\right\} \Big/ \frac{d}{L_0} \tag{1}$$

By assigning the values of an empirical coefficient A, which ranges from 0.12 to 0.18 with a recommended value of 0.17 for random waves, the breaker index has good agreement with laboratory and field data. Since then, Goda's formula has been extensively employed for the simulation of waves breaking on sloping beaches [18], identification of the breaking point [19], prediction of overtopping and wave forces [20], exploration of wave breaking over sandy beaches [21], and wave height estimation [22,23].

Additionally, some improvement and modification suggestions have been proposed for the Goda type formula. Rattanapitikon and Shibayama [15] modified the slope effect term of $(1 + 15m^{4/3})$ into $(1.033 + 4.71m - 10.46m^2)$ by using diverse datasets from published laboratory experiments. By the reanalysis of various sets of field and laboratory data, Goda [24] (hereafter G10) weakened the impact of the slope effect term to $(1 + 11m^{4/3})$, and this updated formula has better agreement with laboratory data. Other researchers focus on the empirical coefficient A in Equation (1). By utilizing data from laboratory experiments, Kriebel [25] observed a variation in the coefficient A ranging from 0.090 to 0.180, with a mean of 0.142 and a standard deviation of 0.017. Li et al. [26] also reported A as 0.150, along with a standard deviation of 0.031. Wood et al. [27] suggested that the value of A varies with the critical wave steepness, defined as the ratio of breaking wave heights to the deep-water wavelength. Shen et al. [28] identified a correlation between A and deep-water wave steepness, which is the ratio of the deep-water wave height to deep-water wavelength. Chen et al. [29] (hereafter CZ22) also found that the proportionality coefficient A of G10 is highly related to the deep-water wave steepness and modified G10, in which CZ22 is a function of the deep-water wave steepness, the beach slopes, and the relative water depth. This formula is applicable over a wide range of beach slope values, requires no empirical adjustment of parameters, and demonstrates good performance when compared to the previous formulas. In the spectral wave model, the calculation of the Goda type breaker index requires the predetermination of deep-water wave conditions. When utilizing the CZ22 formula, the initial step involves calculating and inputting the deep-water wave height. In laboratory-scale tank experiments, the deep-water wave height can be inversely determined through wave shoaling calculations based on the incident wave height. However, in more general situations, particularly when addressing issues at field scales, the incident wave height at the computational boundary may not be readily known. Additionally, when changes occur in the incident wave characteristics during the calculation process, the model is unable to update the deep-water wave steepness in real

time based on the current wave conditions. These limitations hinder the formula's practical applicability in field contexts.

Therefore, this study aims to refine the formulation of CZ22, making it no longer dependent on incident wave conditions and more seamlessly integrated into spectral wave models, all the while maintaining its accuracy in simulating wave heights. The arrangement of this paper is as follows: Section 2 describes the model and formula optimization, and the simulation results and discussions are shown in Section 3. The conclusion is presented in Section 4.

2. Model and Formula Optimization

As a third-generation wave model, SWAN (version 41.31), originally developed by Booij et al. [30], simulates wave propagation in nearshore waters by solving the wave action density in wave spectrum balance equation. The introduction to the model has been omitted to avoid redundancy, where details can be found in SA15, LS17, and CZ22. According to CZ22, the breaker index formula is reproduced as

$$\begin{cases} \gamma = 0.234\left(\frac{H_{s0}}{L_0}\right)^{0.105}\left\{1-\exp\left[-1.5\pi\frac{d}{L_0}\left(1+11m^{4/3}\right)\right]\right\}/\frac{d}{L_0}, \text{ if } 0.002 \le m \le 0.1 \\ \gamma = 0.50, \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } m < 0.002 \end{cases} \tag{2}$$

in which H_{s0} is the significant wave height in deep water. In CZ22, the calculation of H_{s0} relies on linear wave theory, which entails deriving it from the incident significant wave height and subsequently inputting it into the model as a known condition, while L_0 is calculated by

$$L_0 = gT_p^2/(2\pi) \tag{3}$$

where T_p is the peak period, which also needs to be input based on the incident wave conditions. Hence, before executing the aforementioned model, users must specify the initial wave conditions. While the model can simulate nearshore wave height variations well, its practical utility is substantially curtailed by the absence of known deepwater wave conditions in fields. Therefore, considering the effect of shoaling and the linear wave dispersion relation, H_{s0} can be represented as

$$H_{s0} = H_s/\sqrt{1/(2n\tanh kd)}, \tag{4}$$

where $n = \frac{1}{2}\left(1+\frac{2kd}{\sinh 2kd}\right)$ is the ratio of group velocity to phase speed and H_s is the local significant wave height. After substituting Equation (4) to Equation (2), merging the H_s, d, and L_0 at both ends of the equation, and removing the power exponent on the left end, we have

$$\frac{H_s}{L_0} = 0.2055(n\tanh kd)^{0.0587}\left\{1-\exp\left[-1.5\pi\frac{d}{L_0}\left(1+11m^{4/3}\right)\right]\right\}^{1.1173}. \tag{5}$$

Finally, the formula can be restored to the Goda type, and the optimized formula is written as

$$\begin{cases} \gamma = 0.2055(n\tanh kd)^{0.0587}\left\{1-\exp\left[-1.5\pi\frac{d}{L_0}\left(1+11m^{4/3}\right)\right]\right\}^{1.1173}/\frac{d}{L_0}, \text{ if } 0.002 \le m \le 0.1 \\ \gamma = 0.50, \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } m < 0.002 \end{cases} \tag{6}$$

In the present formula, some details need to be illustrated: L_0 is also calculated following Equation (3), but T_p is now provided by the local spectrum in the wave model. The value of the beach slope m in this formula is determined by the gradient of the beach slope in the computational grid along wave directions. Although the linear dispersion equation

is used as a preliminary step in calculating the value of the breaker index, the nearshore wave breaking process is still calculated as a strong nonlinear physical mechanism in the SWAN model. In other words, the linear dispersion equation in the breaking index formula is solely aimed at eliminating the variable H_{s0}, where the unknown or variable condition of H_{s0} will cause CZ22 to be unusable, rather than the linear theory itself having advantages. Following SA15 and LS17, m also has the limitation of the maximal beach slope being 0.1 to prevent unrealistic simulation results on very steep slopes. Moreover, this formula shows that the breaker index decreases as the beach slope decreases. When the water depth is relatively large, the influence of m on the waves will gradually decrease, and the beach slope will not greatly affect the value of the breaking index. To weaken the sensitivity of m on this condition, the normalized water depth, kd, is applied in this model. It spans a wide range from 0 to 1.5, with a lower limit of 1.1 observed in laboratory settings, according to Salmon et al. [12]. When kd is high, indicating minimal influence from water depth, Equation (6) is substituted with a constant value of 0.78, as proposed by McCowan [31], to avoid generating unreasonable breaker index values in deep water.

3. Verification and Comparison

Several cases, including laboratory and field observations [27,29,32–37], with beach slopes ranging from 1/10 to 0 are used to assess the performance of the present model, and the simulated wave heights found by CZ22, BJ78, SA15, LS17 are also shown to compare the accuracy of the model. In addition, three parameters are used to quantitatively evaluate the comparison. The first is *RMSE*, a widely used and intuitive parameter in error analysis, which is defined as

$$RMSE = \left[\frac{1}{M} \sum_{i=1}^{M} (H_{m,i} - H_{o,i})^2 \right]^{0.5} \tag{7}$$

where M is the total number of measured data in each case and $H_{m,i}$ and $H_{o,i}$ are the modeled and observed wave heights, respectively. The second and the third are called the scatter index (*SI*) and the relative bias (*RB*), respectively, which can be written as

$$SI = RMSE / \overline{H_o} \tag{8}$$

$$RB = \frac{1}{M} \sum_{i=1}^{M} (H_{m,i} - H_{o,i}) \Big/ \overline{H_o} \tag{9}$$

where $\overline{H}_0$ represents the mean value of measured data. The values of *RMSE* and *SI* are always positive, and a smaller value indicates a higher level of accuracy between the simulated and observed results. Considering that *RMSE* may exhibit significant differences between laboratory and field cases, these disparities can be normalized using *SI* for error comparison [13]. Meanwhile, the value of *RB* can reflect whether the simulated wave height is higher or lower than the measured wave height, so the *RB* value is not always positive.

3.1. Cases Verification

The results of the error analysis are summarized in Table 1. Overall, both models perform well in simulating wave height variations under different slopes of laboratory cases. The *RMSE* errors of CZ22 and Equation (6) are both on the order of millimeters, with minimal differences between each other. The average *SI* values for the two formulas are 4.45% and 4.69%, respectively, both within 5%.

For field observations, Figure 1a shows the nearshore bathymetry during 2–6 February in 1980 at Leadbetter Beach, Santa Barbara [36]. The water depth in the computational domain ranges from 0 m to 8.5 m and the mesh size is 5 m. Waves propagate from the northern boundary to the nearshore in the south, with a small angle between the

wave direction and the normal direction to the shore. At x = 0 m, a 150 m long profile was extracted for analyzing the variation in wave heights from February 2 to 6. As the incident wave parameters vary daily, the variable incident wave conditions of the five days can calculate five different H_{s0}, which means the CZ22 formula cannot provide continuous simulations. Therefore, the CZ22 simulations were calculated separately for each day, whereas the present model provides continuous simulations for the entire five days (Figure 1b). Over the course of five days, the range of wave incident angles is between 15.2° and 19.1°, while the incident wave heights range from 0.37 m to 0.52 m, with peak wave periods ranging from 11.6 s to 16.0 s. As the waves propagate to approximately y = 60–80 m, corresponding to water depths of around 1.5 m, waves reached their maximum deformation due to shoaling, followed by wave breaking. Both formulas are able to adequately simulate the variations in waves over the five-day period (Figure 1b). The breaker index values of Equation (6) are similar to CZ22 on 3 February and smaller than CZ22 on the other four days, leading to lower modeled wave heights, especially near the shore. Compared to the above field cases, the *RMSE* errors of both formulas in simulating wave height range from 0.02 to 0.05 m, with errors on the order of centimeters (Table 1). The *RMSE* of CZ22 is slightly smaller than that of Equation (6). Meanwhile, the average *SI* errors of CZ22 and Equation (6) are 7.30% and 7.94%, respectively. Although the mean *SI* errors between the simulated wave heights and the measured value has slightly increased by 0.64% compared to CZ22, for wave height simulations in the field, it is more attractive to continuously simulate under different incident wave conditions than to rerun the model with multiple external inputs.

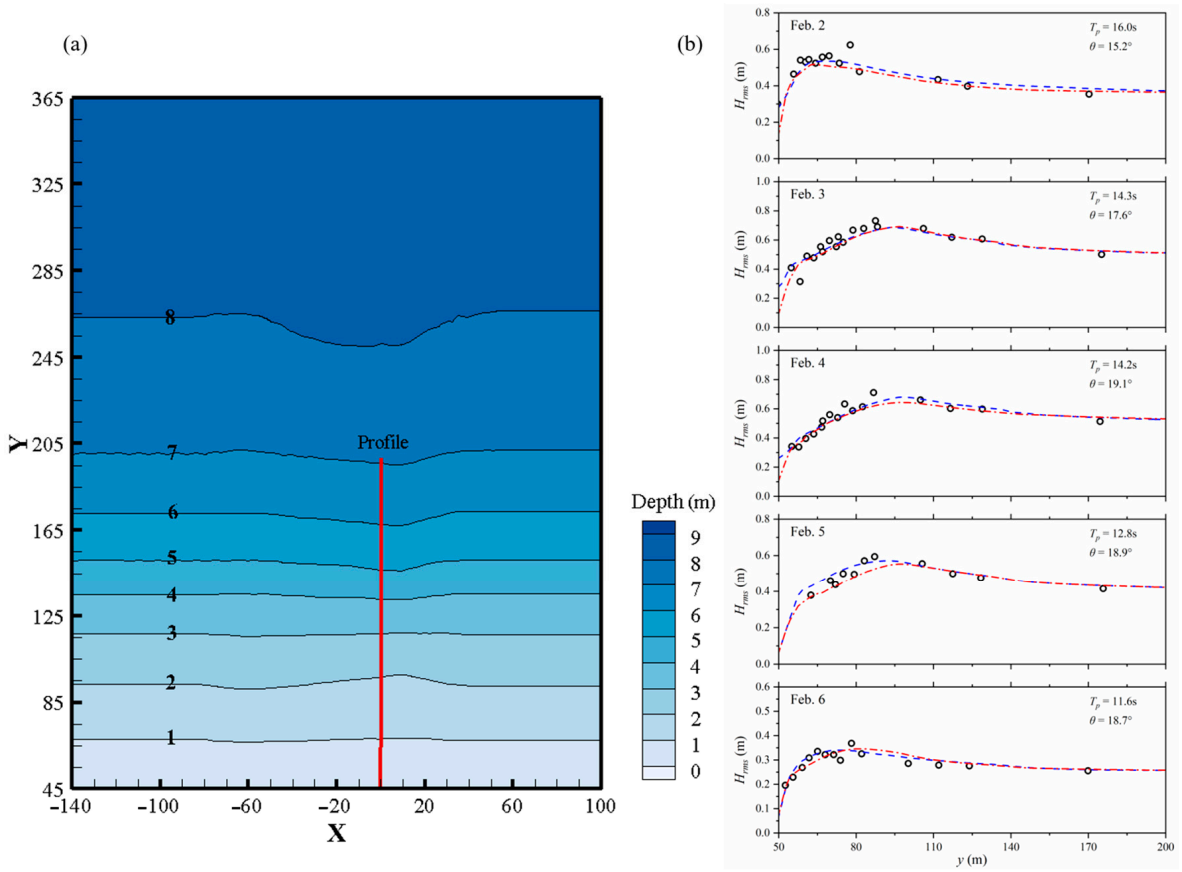

Figure 1. The nearshore bathymetry of the field case at the Leadbetter beach (**a**). The incident waves propagate from the northern boundary to south areas. The red solid line is the location of a profile with a slope of about 1/22. (**b**) shows simulation results profile on 2 February to 6 February in 1980. Blue dashed lines and red dash dot lines are the wave heights simulated by CZ22 and Equation (6), respectively. Black circles are the observation data of wave heights in the field.

Table 1. Summary of *RMSE* and *SI* values of CZ22 and Equation (6).

Case		Sources	Incident Wave Conditions			CZ22		Equation (6)	
			H_s (m)	T_p (s)	d (m)	*RMSE* (m)	*SI* (%)	*RMSE* (m)	*SI* (%)
a	Small-scale	[32]	0.105	1.67	0.800	0.0053	7.68	0.0050	7.30
b	Small-scale	[33]	0.070	1.00	0.450	0.003	5.35	0.0031	5.53
c	Large-scale	[34]	0.100	1.57	0.850	0.0031	3.69	0.0036	4.25
d	Small-scale	[29]	0.040	1.10	0.120	0.001	3.69	0.001	3.65
e	Large-scale	[27]	0.170	1.67	0.700	0.0045	3.07	0.0042	2.88
f	Small-scale	[35]	0.160	2.20	0.655	0.0031	3.24	0.0044	4.54
2 February			0.523	16.0	9.41	0.0384	7.86	0.0519	9.64
3 February			0.707	14.3	9.26	0.0438	7.66	0.0472	8.26
4 February	Field	[36]	0.735	14.2	9.25	0.0361	6.79	0.0369	6.94
5 February			0.566	12.8	9.07	0.0345	7.04	0.0364	7.44
6 February			0.368	11.6	8.92	0.0208	7.16	0.0216	7.44
DALILAH	Field	[37]	——	——	7.8	0.0552	8.69	0.0472	7.43
Average							5.99		6.28

Another essential consideration is the case of wind-generated waves, where there are no incident wave parameters at the open boundary. Waves within the computational domain are generated by input wind files. In this scenario, the CZ22 formula would be unable to calculate the breaker index, because there is no input incident wave conditions at the open boundary to calculate a certain H_{s0}. To illustrate the ability of the optimized formula to simulate wind wave breaking, we use the field case from Birkemeier et al. [37], whose wave height observations were carried out at Duck Beach, North Carolina (Figure 2a). The computational domain of the model spans from 80.14° W to 67.96° W longitudinally and from 28.05° N to 40.72° N latitudinally, with the deepest water depth reaching 5500 m. The grid scale ranges from tens of meters to tens of thousands of meters (Figure 2b). Wind field data are sourced from ERA5 reanalysis data, with a spatial resolution of 0.25 degrees and a temporal resolution of 1 h. We first run the model by using Equation (6) to simulate the wave propagation in the period of 4–16 October 1990. At the Field Research Facility (FRF) instrument, where the location is shown in Figure 2c, wave heights remained below 0.8 m until 10 October, and the simulated values closely matched the measured data. Between 10 and 14 October, significant fluctuations in wave height occurred twice due to strong winds, with maximum wave heights reaching 1.74 m. The model is able to accurately simulate the peak wave heights. However, around the 12th, simulated wave heights are higher compared to the measured data, which may be attributed to the lower water level during that period. The model in this paper does not account for the influence of tidal currents on wave height variations. After the 14th, wave heights gradually decreased, and the simulated results matched well with the measured values (Figure 2d). Due to the requirement of providing wave conditions at the incident open boundary using the CZ22 formula, the incident wave conditions at the 8 m isobath are extracted from the above simulation results, a computational grid (Figure 2c) is drawn to simulate the wave-breaking process by CZ22, and Equation (6) is run again using this mesh to compare the simulating wave heights. In addition, the wave heights measured at the Delilah array instrument locations [38,39] are utilized for comparison in this study (Figure 2c). On 10 October, for hour 7, both the two formulas successfully simulate the variation in wave heights before and after breaking. The *RMSE* of CZ22 is slightly larger than that of Equation (6), and the *SI* errors of CZ22 and Equation (6) are 8.69% and 7.43%, respectively (Table 1).

Overall, for the breaker index of Equation (6) after the optimization of CZ22, the simulated wave height variations in both laboratory and field cases align well with the observed values, with an average *SI* error of 6.28%, comparable to the performance of CZ22.

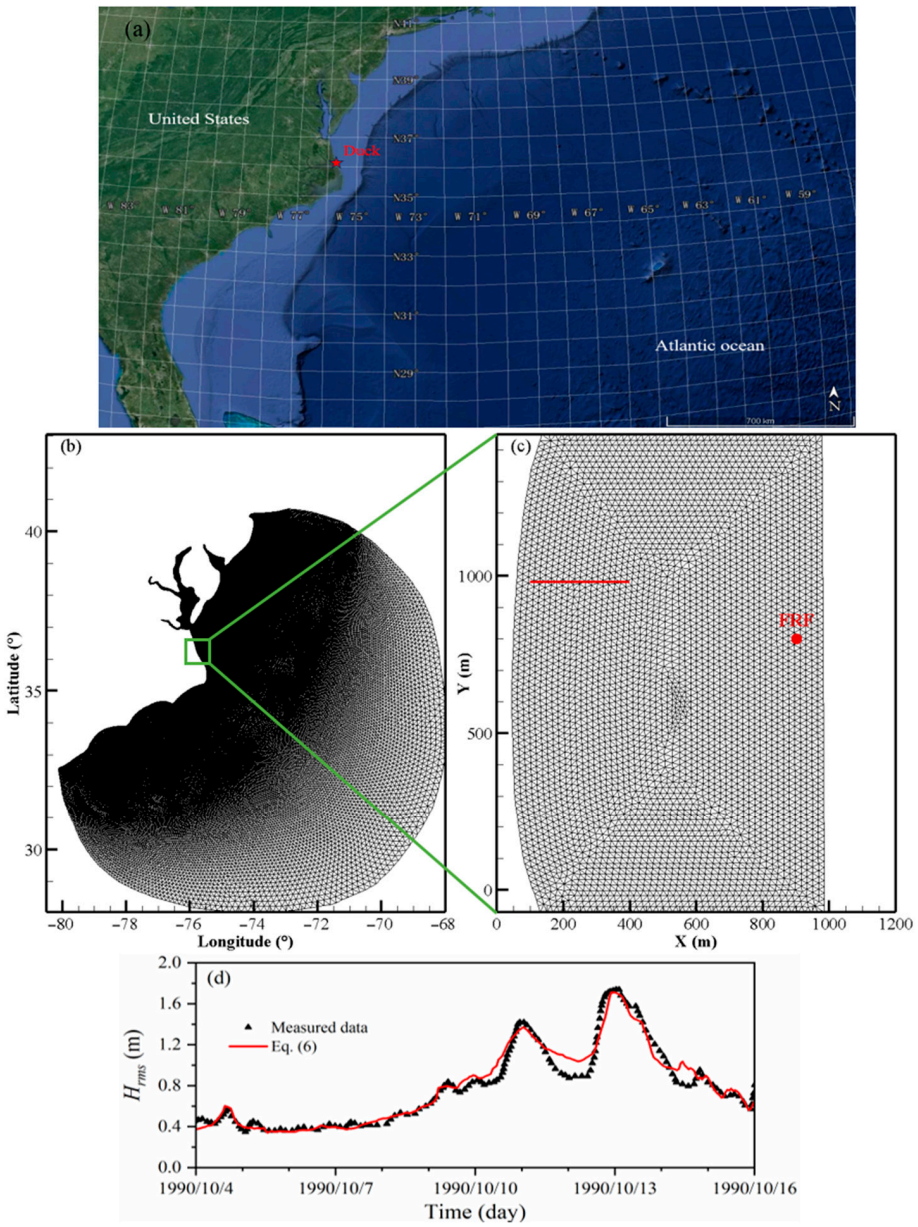

Figure 2. The field case of the Duck Beach. (**a**) The location. (figure source from Google Earth). (**b**) The computational domain in the latitude and longitude coordinate system and its grid partition diagram. (**c**) A small grid partition diagram in Cartesian coordinate system, in which the red dot is the FRF instrument location and the red line contains the Delilah array instrument locations. (**d**) The simulated wave heights of Equation (6) compared to field observations at the FRF point from 4 to 16 October in 1990.

3.2. Comparison with Other Formulas

For the field cases in Section 3.1, we compare the values of *SI* and *RB* for Equation (6), BJ78, SA15, and LS17, where the last three formulas can be written as

$$\gamma_{\mathrm{BJ78}} = 0.73 \tag{10}$$

$$\begin{cases} \gamma_{\mathrm{SA15}} = \gamma_1 / \tanh(\gamma_1 / \gamma_2) \\ \gamma_1 = 0.54 + 7.59m \\ \gamma_2 = -8.06 + 8.09kh \end{cases} \tag{11}$$

$$\begin{cases} \gamma_{LS17} = \gamma_1 / \tanh(\gamma_1 / \gamma_2) \\ \gamma_1 = 0.54 + 0.47 \exp(-0.018/m) \\ \gamma_2 = -8.06 + 8.09kh \end{cases} \quad (12)$$

In Figure 3, it can be observed that the *SI*-*RB* point of Equation (6) is generally closer to the origin compared to the other three formulas, indicating that the overall error of that is smaller. Equation (6) can still maintain high accuracy in predicting wave heights at the field scale.

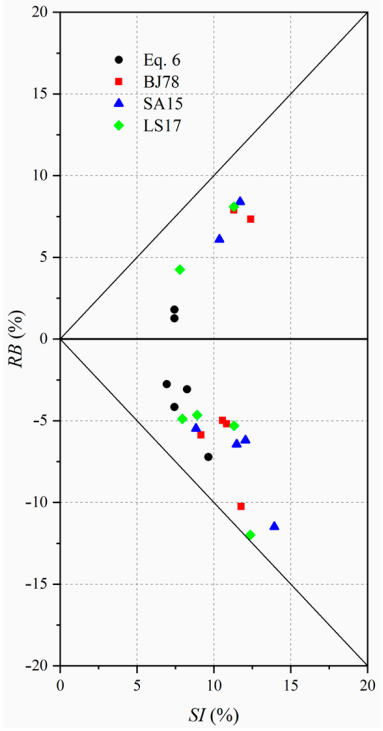

Figure 3. The scatter plots of the relative bias versus the scatter index from the simulated and observed wave heights for the six field cases.

4. Conclusions

In this study, a new expression for the Goda type breaker index in the spectral wave model is rederived, enabling its applicability to simulate breaking waves caused by variable wave conditions on open boundaries and wind field inputs in the whole computational region. This study optimizes the formulation of the Goda type breaker index proposed by Chen et al. (2022) [29], allowing the model to calculate the breaker index independently of the incident wave characteristics in deep water. The new expression of the breaker index formula has been embedded in the SWAN model (see https://github.com/ZerengChen/BreakerIndex_SWAN (accessed on 12 April 2024)). Six laboratory cases with beach slopes ranging from 1/10 to 0, five-day wave height observations near the Leadbetter beach, and the DALILAH field case are employed to assess the present formula. The model exhibits an average SI error of 6.28%, comparable to the performance of the original formula of Chen et al. (2022) [29], and it can also maintain high accuracy in these field cases compared with other breaking index formulas.

Author Contributions: Software, Z.C.; validation, Z.C. and Y.N.; formal analysis, Z.C.; writing—original draft preparation, Z.C.; writing—review and editing, Z.C. and Q.Z.; supervision, Q.Z.; funding acquisition, Q.Z. All authors have read and agreed to the published version of the manuscript.

Funding: The Joint Funds of the National Natural Science Foundation of China (Grant no. U1906231).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Holthuijsen, L.H. *Waves in Oceanic and Coastal Waters*; Cambridge University Press: New York, NY, USA, 2007.
2. Tran, K.Q.; Duong, N.T.; Luu, L.X.; Tran, L.H.; Rattanapitikon, W. Development of novel parametric wave model for irregular wave height transformation. *Ocean. Eng.* **2023**, *278*, 114493. [CrossRef]
3. Apotsos, A.; Raubenheimer, B.; Elgar, S.; Guza, R.T. Testing and calibrating parametric wave transformation models on natural beaches. *Coast. Eng.* **2008**, *55*, 224–235. [CrossRef]
4. Zhang, C.; Li, Y.; Cai, Y.; Shi, J.; Zheng, J.H.; Cai, F.; Qi, H. Parameterization of nearshore wave breaker index. *Coast. Eng.* **2021**, *168*, 10394. [CrossRef]
5. Battjes, J.A.; Janssen, J.P.F.M. Energy loss and set-up due to breaking of random waves. In Proceedings of the 16th International Conference on Coastal Engineering ASCE, Hamburg, Germany, 27 August–3 September 1978; pp. 569–587. [CrossRef]
6. Thornton, E.B.; Guza, R.T. Transformation of wave height distribution. *J. Geophys. Res. Oceans* **1983**, *88*, 5925–5938. [CrossRef]
7. Baldock, T.E.; Holmes, P.; Bunker, S.; Weert, P.V. Cross-shore hydrodynamics within an unsaturated surf zone. *Coast. Eng.* **1998**, *34*, 173–196. [CrossRef]
8. Battjes, J.A.; Stive, M.J.F. Calibration and verification of a dissipation model for random breaking waves. *J. Geophys. Res.* **1985**, *90*, 9159–9167. [CrossRef]
9. Southgate, H.N.; Nairn, R.B. Deterministic profile modelling of nearshore processes. Part 1. Waves and currents. *Coast. Eng.* **1993**, *19*, 27–56. [CrossRef]
10. Nelson, R.C. Design wave heights on very mild slopes—An experimental study. *Trans. Inst. Eng. Aust. Civ. Eng.* **1987**, *29*, 157–161.
11. Ruessink, B.G.; Walstra, D.J.R.; Southgate, H.N. Calibration and verification of a parametric wave model on barred beaches. *Coast. Eng.* **2003**, *48*, 139–149. [CrossRef]
12. Salmon, J.E.; Holthuijsen, L.H.; Zijlema, M.; van Vledder, G.P.; Pietrzak, J.D. Scaling depth-induced wave-breaking in two-dimensional spectral wave models. *Ocean. Model.* **2015**, *87*, 30–47. [CrossRef]
13. Lin, S.F.; Sheng, J.Y. Assessing the performance of wave breaking parameterizations in shallow waters in spectral wave models. *Ocean Model.* **2017**, *120*, 41–59. [CrossRef]
14. Robertson, B.; Hall, K.; Zytner, R.; Nistor, I. Breaking Waves: Review of Characteristic Relationships. *Coast. Eng. J.* **2013**, *55*, 1350002. [CrossRef]
15. Rattanapitikon, W.; Shibayama, T. Verification and modification of breaker height formulas. *Coast. Eng. J.* **2000**, *42*, 389–406. [CrossRef]
16. Duong, N.T.; Tran, K.Q.; Luu, L.X.; Tran, L.H. Prediction of breaking wave height by using artificial neural network-based approach. *Ocean. Model.* **2023**, *182*, 102177. [CrossRef]
17. Goda, Y. Irregular wave deformation in the surf zone. *Coast. Eng. J.* **1975**, *18*, 13–26. [CrossRef]
18. Izumiya, T.; Isobe, M. Breaking criterion on non-uniformly sloping beach. In Proceedings of the 20th International Conference on Coastal Engineering ASCE, Taipei, Taiwan, 9–14 November 1986; pp. 318–327. [CrossRef]
19. Sakai, S.; Hirayama, K.; Saeki, H. A new parameter for wave breaking with opposing current on sloping sea bed. In Proceedings of the 21st International Conference on Coastal Engineering ASCE, Costa Del Sol-Malaga, Spain, 20–25 June 1988; pp. 1035–1044. [CrossRef]
20. Allsop, N.; Durand, N.; Hurdle, D. Influence of steep seabed slopes on breaking waves for structure design. In Proceedings of the 26th International Conference on Coastal Engineering ASCE, Copenhagen, Denmark, 22–26 June 1998; pp. 906–919. [CrossRef]
21. Udo, K.; Mano, A. Backshore coarsening processes triggered by wave-induced sand transport: The critical role of storm events. *Earth. Surf. Proc. Land.* **2010**, *35*, 1269–1280. [CrossRef]
22. Goda, Y. A performance test of nearshore wave height prediction with clash datasets. *Coast. Eng.* **2009**, *56*, 220–229. [CrossRef]
23. Kang, H.; Chun, I.; Oh, B. New procedure for determining equivalent deep-water wave height and design wave heights under irregular wave conditions. *Int. J. Nav. Arch. Ocean.* **2020**, *12*, 168–177. [CrossRef]
24. Goda, Y. Reanalysis of regular and random breaking wave statistics. *Coast. Eng. J.* **2010**, *52*, 71–106. [CrossRef]
25. Kriebel, D. Breaking waves in intermediate-depths with and without current. In Proceedings of the 27th International Conference on Coastal Engineering ASCE, Sydney, Australia, 16–21 July 2000; pp. 203–215. [CrossRef]

26. Li, Y.; Tu, Y.; Cui, L.; Dong, G. Transformation and breaking of irregular waves on very gentle slope. *China Ocean Eng.* **2000**, *14*, 261–278. [CrossRef]
27. Wood, D.J.; Muttray, M.; Oumeraci, H. The SWAN model used to study wave evolution in a flume. *Ocean. Eng.* **2001**, *28*, 805–823. [CrossRef]
28. Shen, Y.; Zhang, Q.H.; Chen, C.; Ji, C. Application of modified breaking index of Goda in SWAN model. *Port. Eng. Technol.* **2017**, *54*, 1–6. (In Chinese) [CrossRef]
29. Chen, Z.R.; Zhang, Q.H.; Wu, Y.S.; Ji, C. A modified breaker index formula for depth-induced wave breaking in spectral wave models. *Ocean Eng.* **2022**, *264*, 112527. [CrossRef]
30. Booij, N.; Ris, R.C.; Holthuijsen, L.H. A third-generation wave model for coastal regions 1. Model description and validation. *J. Geophys. Res.* **1999**, *104*, 7649–7666. [CrossRef]
31. McCowan, J., VII. On the solitary wave. *Lond. Edinb. Dublin Philos. Mag. J. Sci.* **1891**, *32*, 45–58. [CrossRef]
32. Baldock, T.E.; Huntley, D.A. Long-wave forcing by the breaking of random gravity waves on a beach. *Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci.* **2002**, *458*, 2177–2201. [CrossRef]
33. Zhang, Z.; Zou, Z. Vertical distribution of longshore currents over plane and barred beaches. *J. Hydrodyn.* **2012**, *24*, 718–728. [CrossRef]
34. Ruessink, B.G.; Michallet, H.; Bonneton, P.; Mouazé, D.; Lara, J.; Silva, P.A.; Wellens, P. GLOBEX: Wave dynamics on a gently sloping laboratory beach. In Proceedings of the 7th International Conference on Coastal Dynamics, Arcachon, France, 24–28 June 2013; pp. 1351–1362.
35. Jensen, M.S. Breaking of Waves Over a Steep Bottom Slope. Ph.D. Thesis, Hydraulics & Coastal Engineering Laboratory, Department of Civil Engineering, Aalborg University, Aalborg Øst, Denmark, 2004.
36. Wu, C.; Thornton, E.B.; Guza, R.T. Waves and longshore currents: Comparison of a numerical model with field data. *J. Geophys. Res.* **1985**, *90*, 4951–4958. [CrossRef]
37. Birkemeier, W.A.; Donoghue, C.; Long, C.E.; Hathaway, K.H.; Baron, C.F. *1990 DELILAH Nearshore Experiment: Summary Report*; Technical Report CHL-97-4-24; US Army Engineer Waterways Experiment Station: Vicksburg, MS, USA, 1997.
38. Reniers, A.J.H.M.; Van Dongeren, A.R.; Battjes, J.A.; Thornton, E.B. Linear modeling of infragravity waves during Delilah. *J. Geophys. Res. Oceans* **2002**, *107*, 1–18. [CrossRef]
39. Reniers, A.J.H.M.; Zijlema, M. SWAN SurfBeat-1D. *Coast. Eng.* **2022**, *172*, 104068. [CrossRef]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.