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Horizontal Detection Range Estimation of Deep-Sea Acoustic Shadow Area Based on Vertical Phased Emission Array

Naibin Chen ^{1,2,*} , Dayong Peng ^{1,*}, Li Ma ¹, Tao Hu ¹, Haijun Liu ¹ and Haifeng Li ¹

¹ Key Laboratory of Underwater Acoustic Environment, Institute of Acoustics, Chinese Academy of Sciences, Beijing 100190, China; mali@mail.ioa.ac.cn (L.M.); hutao@mail.ioa.ac.cn (T.H.); liuhaijun@mail.ioa.ac.cn (H.L.); lihaifeng@mail.ioa.ac.cn (H.L.)

² College of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China

* Correspondence: cnb1009369176@163.com (N.C.); pengdayong@mail.ioa.ac.cn (D.P.)

Abstract: To scan the horizontal range of the deep-sea shadow zone with high accuracy and efficiency, enabling active target detection, a horizontal detection range estimation method based on phased sound focusing technology is presented. The model of the phased emission sound field is derived, along with the theoretical estimation formula for the horizontal detection range, by combining the half-power beamwidth of the beam curve at the phased-emitter end with the ray propagation law of deep-sea acoustics. The corresponding relationships among each discrete phased angle, the horizontal distance, and the detection range of the focused beam boundary are detailed. The estimation formula for the horizontal detection range is then improved to account for the sea surface reflection effect, and angle optimization is applied to the transmitted beam curve to counteract the boundary error accumulation effect of the focused sound beam as the sound ray propagates. To address the issue of reduced scanning efficiency at large phased angles, the grating lobe-focused sound beam generated by phased emission array parameters is used to achieve angle scanning within the close deep-sea sound shadow zone, increasing the phased angle scanning efficiency. Finally, the deep-sea phased emission active target detection experiments were carried out, and the echo signal generated by the phased focused sound beam and received by the vertical receiving array was recorded. Numerical simulation and echo signal processing show that the high-efficiency angle scanning of the entire deep-sea sound shadow zone is achieved by the proposed estimation method.

Keywords: phased emission; acoustic shadow zone; target detection; active sonar; acoustic focusing; half-power beamwidth



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1. Introduction

Active sonar generates an echo signal by emitting a specific acoustic waveform towards an underwater target, subsequently processing, identifying, and displaying the received echo signal to facilitate target detection and tracking. In active detection, spatial directivity can be achieved using array-based emission, enabling the concentration of acoustic energy in a specified direction or spatial region. This approach enhances spatial gain and serves as an effective means to increase detection range. Traditional vertical emission arrays typically employ uniform excitation, where all array elements transmit signals with identical amplitude and phase. This configuration maximizes the radiated power of the transmitting array, thereby extending the effective range. In contrast, phased emission allows for precise control over the directivity of array emissions. By adjusting the amplitude and phase of individual array elements, sound field focusing can be achieved

at specified angles, enabling the scanning of distances and depths through focus position adjustments. This technique significantly improves active detection performance by leveraging the flexibility of phased arrays.

Phased emission technology employs multiple transmitting elements to form a controllable beam with specific directionality and shape by modulating their phase relationships and amplitudes [1]. This technology is widely utilized in underwater acoustics for applications such as Doppler current profilers, which measure the velocity and direction of seawater flow, and multi-beam echo sounders, which map bottom topography [2,3]. These systems typically operate at frequencies above 10 kHz. However, high-frequency sound waves experience significant attenuation in water, making high-frequency phased arrays unsuitable for long-range target detection. In contrast, low-frequency vertical phased arrays offer longer wavelengths, reduced attenuation, and greater detection range, making them increasingly relevant for underwater target detection. Among the established technologies applied to shallow-water low-frequency vertical phased arrays, single-mode excitation and time-reversal mirrors are the most prominent [4–8]. Golubeva derived an expression for the acoustic power emitted by a vertical phased array in a Pekeris waveguide, where the acoustic field is expressed as the sum of discrete and continuous spectra. Their work also examined the dependence of emitted power on the number of array elements and the array compensation angle [9]. Gao was the first to study the “single-mode excitation technology” of underwater low-frequency phased arrays [10]. Similarly, Yi conducted research on shallow-water low-frequency vertical phased arrays, demonstrating their potential for underwater target detection and active sonar reverberation suppression [11,12].

Existing research on low-frequency phased emission technology has primarily focused on applications in shallow-water environments, with limited exploration of its use in deep-sea target detection. In deep-sea environments, the low sound energy region between convergence zones, defined as the acoustic shadow zone, predominantly contains sound energy reflected from the bottom [13]. Within this zone, sound propagation is primarily characterized by bottom bounce modes. These reflections result in significant attenuation of sound energy, causing transmission loss within the acoustic shadow zone to exceed that of spherical spreading loss [14]. The bottom bounce mode plays a critical role in target detection within the acoustic shadow zone, offering advantages such as simpler calculation of spreading loss and reduced sensitivity to ocean fluctuations [15]. Diana observed and grouped the sound energy distribution of the bottom bounce modes [16]. Udovydchenkov emphasized the critical role of bottom reflection paths in target detection through observations made using a 1400 m vertical receiving array at a distance of 50 km [17]. Liu proposed a sound source location method using the time delay difference of different arrival paths in the bottom bounce modes [18]. Han established a quantitative relationship between the sonar-detectable areas in deep bottom reflection zones and the sound rays emitted at various angles. Furthermore, Han proposed a pulse train signal design method capable of targeting multiple discrete sonar-detectable regions [19]. Phased emission technology, with its capability to focus acoustic energy at specified grazing angles, offers compatibility with the bottom bounce mode. By leveraging the bottom reflection path, it is possible to “illuminate” deep-sea acoustic shadow zones within a certain horizontal range, stimulating targets to generate echo signals and enabling active target detection within these shadow zones. While the theoretical foundation for enhancing acoustic energy in shadow zones via the bottom bounce mode has existed for a long time, practical implementation remains challenging. Active sonar systems designed for such applications are complex, and experimental validation is difficult. Consequently, most studies in this area rely on simulations, with limited reports addressing active target detection using phased emission technology to focus acoustic energy. Additionally, during angle scanning of deep-sea shadow zones

using bottom reflection, the absence of a robust basis for angle selection often results in low scanning efficiency and limited accuracy.

This paper derives the theoretical formula for the phased emission sound field using the Gaussian beam tracking method and active beamforming principles. To quantitatively describe the detection range of the acoustic shadow zone corresponding to each phased emission angle, a horizontal detection range estimation method based on half-power beam width is proposed. This method first determines the half-power angle of the beam pattern for each phased emission angle. Then, using ray acoustics theory, it calculates the horizontal span of the half-power emission angle, defining the boundaries of the phased-focused acoustic beam. The method is further refined by incorporating the principles of deep-sea sound propagation and phased emission parameters, establishing a relationship between the grazing angle and the corresponding horizontal detection range and distance. This approach addresses the challenges of low scanning accuracy and efficiency in deep-sea acoustic shadow zones. Based on the estimation results, specific emission angles can be selected to scan the entire horizontal range of the acoustic shadow zone, significantly improving the precision and efficiency of active target detection using phased emission technology. Additionally, this method provides a theoretical basis for phased angle selection. Finally, the application effect of this method in the actual deep-sea environment was verified using the phased emission active target detection test data in a deep-sea area in 2020.

2. Phased Emission Acoustic Field Modeling

References [20,21] give theoretical expressions of ray acoustic fields based on the Gaussian beam tracing method. Under the condition of point source acoustic fields, the specific expressions are as follows:

$$P(r, z) = \sum_{m=1}^M \frac{\Delta\theta}{c_0} \exp\left(\frac{i\pi}{4}\right) \sqrt{\frac{q(0)\omega \cos \theta_m}{2\pi}} \sqrt{\frac{c(s)}{rq(s)}} \exp\left\{-i\omega \left[\tau(s) + \frac{p(s)}{2q(s)}\eta^2\right]\right\} \quad (1)$$

where M is the number of eigenrays, s is the arc length of the sound ray introduced in the ray coordinates, $\Delta\theta$ is the angle of the sound beam, c_0 is the sound speed at the source point, $c(s)$ is the speed of sound at the arc length s of the ray, ω is the angular frequency of the source, θ_m represents the grazing angle of the sound ray, p and q are arc lengths and relative changes in adjacent rays at the arc length s [22], η is the vertical distance from the center sound ray, and $\tau(s)$ is the phase delay along the sound ray.

Consider a vertical transmitting array with N elements and an array element spacing of d . Assume that the weight of each array element is $W_n(\theta_m; \theta_B)$; the center depth of the vertical array is taken as the reference depth. Combining Equation (1),

$$P_N(r, z) = \sum_{n=1}^N W_n^*(\theta_m; \theta_B) P(r, z) \quad (2)$$

$$W_n(\theta_m; \theta_B) = \exp\left(i\left(n-1-\frac{N-1}{2}\right)\frac{\omega}{c_0}d(\sin \theta_m - \sin \theta_B)\right) \quad (3)$$

where θ_B is the phased emission angle. Combined with Equations (2) and (3), the ray theoretical expression of the phased emission sound field can be obtained as follows:

$$P_N(r, z) = \sum_{m=1}^M \frac{\Delta\theta}{c_0} \frac{\sin\left(\frac{N}{2}\frac{\omega}{c_0}d(\sin \theta_m - \sin \theta_B)\right)}{\sin\left(\frac{\omega}{c_0}d(\sin \theta_m - \sin \theta_B)\right)} \exp\left(i\frac{\pi}{4}\right) \times \sqrt{\frac{q(0)\omega \cos \theta_m}{2\pi}} \sqrt{\frac{c(s)}{rq(s)}} \exp\left\{-i\omega \left[\tau(s) + \frac{p(s)}{2q(s)}\eta^2\right]\right\} \quad (4)$$

The essence of Equation (4) is to perform amplitude weighting on the sound rays of different grazing angles of the point source, that is, active beamforming at a specified angle. The corresponding weighted value of the sound ray is as follows:

$$W(\theta_m; \theta_B) = \frac{\sin\left(\frac{N}{2} \frac{\omega}{c_0} d(\sin \theta_m - \sin \theta_B)\right)}{\sin\left(\frac{\omega}{c_0} d(\sin \theta_m - \sin \theta_B)\right)} \quad (5)$$

3. The Phased Emission Horizontal Detection Range Estimation Method and Simulation

3.1. Estimation Method

In active beamforming, acoustic energy focusing forms a beam within a specific angular range, where the energy exhibits significant gain. As illustrated in Figure 1, the beamwidth between the two angles at which the normalized main lobe amplitude drops to $1/\sqrt{2}$ on either side of the peak is defined as the -3 dB bandwidth, commonly referred to as the half-power beam width [23]. This parameter is a critical measure of the attenuation characteristics of focused energy and can be determined through the following calculation:

$$\Delta\theta_{1/2} \approx \frac{51\lambda}{\cos \theta_B N d} (^\circ) \quad (6)$$

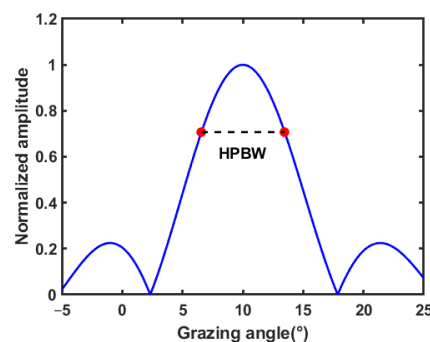


Figure 1. HPBW schematic diagram.

Therefore, for the phased emission grazing angle θ_B , the angles corresponding to the half-power points on the left and right sides are as follows:

$$\theta_{BL} = \theta_B - \frac{1}{2} \Delta\theta_{1/2} = \theta_B - \frac{25.5\lambda}{\cos \theta_B N d} \quad (7)$$

$$\theta_{BH} = \theta_B + \frac{1}{2} \Delta\theta_{1/2} = \theta_B + \frac{25.5\lambda}{\cos \theta_B N d} \quad (8)$$

Unless otherwise specified, the positive angle in this paper refers to the angle between the normal direction of the vertical phased array and the bottom direction, and the negative angle refers to the angle between the normal direction and the surface direction. According to relevant research in ray acoustics, the horizontal distance that the sound ray emitted by the sound source propagates can be described by the following equation [24]:

$$x = \cos \alpha_1 \int_{sd}^z \frac{dz}{\sqrt{n^2(z) - \cos^2 \alpha_1}} \quad (9)$$

where α_1 is the grazing angle of the sound ray emitted from the sound source, $n(z) = c_0/c(z)$ is the refractive index of the sound ray, $c(z)$ is the sound speed at a certain depth z , and sd is the depth of the sound source. To estimate the horizontal range of the focused sound beam

“illuminating” the acoustic shadow zone, it is necessary to calculate the horizontal span of the sound ray emitted at two angles corresponding to the half-power beam width. The focused sound beam requires bottom reflection to “illuminate” the shadow zone. Therefore, the calculation of the horizontal span consists of two components: the downward path of the sound ray to the bottom and the upward path to the sea surface. By sound field reciprocity, the upward path can be considered the inverse of the sound ray emitted from a point source at the receiving point. According to the “virtual source” theory, the receiving point can be mirrored to create a virtual source relative to the bottom. Consequently, the upward path can be treated as a downward path directed toward the virtual source. It is important to note that when mirroring the receiving point, the sound speed profile (SSP, referring to the curve of sound speed changing with sea depth) must also be mirrored relative to the bottom. A schematic representation of this process is shown in Figure 2. By substituting Equations (7) and (8) into Equation (9), the horizontal span of the sound ray at the receiving depth can be calculated as follows:

$$X_{BL} = \cos \theta_{BL} \int_{sd}^{2H-rd} \frac{dz}{\sqrt{n^2(z) - \cos^2 \theta_{BL}}} \quad (10)$$

$$X_{BH} = \cos \theta_{BH} \int_{sd}^{2H-rd} \frac{dz}{\sqrt{n^2(z) - \cos^2 \theta_{BH}}} \quad (11)$$

where H represents the sea depth, rd is the depth of the receiver, and X_{BH} and X_{BL} represent the left and right boundaries of the upward focused acoustic beam, respectively. The horizontal detection range can then be expressed as follows:

$$\Delta range = X_{BL} - X_{BH} = \cos \theta_{BL} \int_{sd}^{2H-rd} \frac{dz}{\sqrt{n^2(z) - \cos^2 \theta_{BL}}} - \cos \theta_{BH} \int_{sd}^{2H-rd} \frac{dz}{\sqrt{n^2(z) - \cos^2 \theta_{BH}}} \quad (12)$$

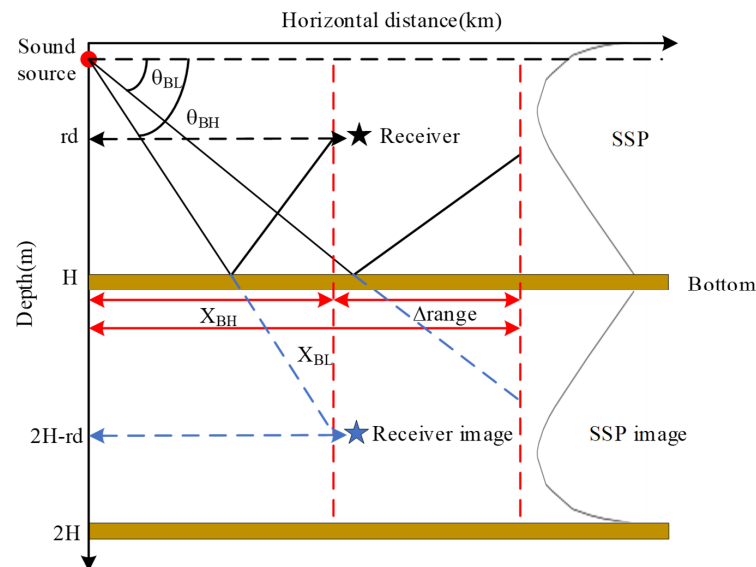


Figure 2. The estimation of the boundary of the upward focused acoustic beam.

From Equation (12), it can be inferred that the horizontal detection range and the horizontal detection position are greatly affected by the phased emission angle. Based on the half-power beam width, the horizontal detection range of active targets at any receiving depth can be estimated under known horizontal layered sound speed distribution, and then a certain number of relatively discrete emission grazing angles can be selected for

angle scanning, so that the sum of their respective horizontal detection ranges can cover the entire deep-sea acoustic shadow area, avoiding problems such as repeated detection at the same horizontal distance, and greatly improving the efficiency and accuracy of deep-sea phased emission active target detection.

3.2. Simulation Verification of Horizontal Detection Range Estimation

In order to verify the applicability of the method in the deep-sea environment, we used Bellhop to simulate the acoustic field. Figure 3 presents the full-depth sound speed profile measured during the sea trial. The black circles indicate the discrete sound speed measurements at various depths, while the red solid line represents the interpolated profile derived from these discrete measurements. The selection of the depth infinitesimal dz in Formula (12) is governed by the interpolation accuracy. Once a certain level of accuracy was achieved, further refinement had a negligible impact on the calculation results. The sound channel axis was approximately located at a depth of 1270 m, the sound speed was 1484.6 m/s, and the sea surface sound speed was 1546.3 m/s, which was greater than the 1529.4 m/s near the bottom. It was a typical deep-sea incomplete sound channel. The depth of the first element of the 20-element phased array was set to 31.5 m below the sea surface, and the other elements were evenly arranged in the direction of the bottom with a spacing of 1.6 m. Regarding the selection of transmission frequency, the lower the transmission frequency, the larger the size of the required transmission transducer, and the more difficult the engineering implementation. In addition, the absorption loss of seawater is proportional to the transmission frequency, and the energy of high-frequency sound waves will decay rapidly as the propagation distance increases [14]. Therefore, a low-to-medium frequency range, around 1 kHz, is typically chosen to balance engineering feasibility and control absorption loss. In this study, the transmission frequency for simulations was set at 720 Hz, which fell within the low-to-medium frequency band. The simulated bottom parameters were based on measured data. The bottom sound speed was 1580 m/s, the density was 1.583 g/cm³, and the bottom attenuation was 0.178 dB/λ.

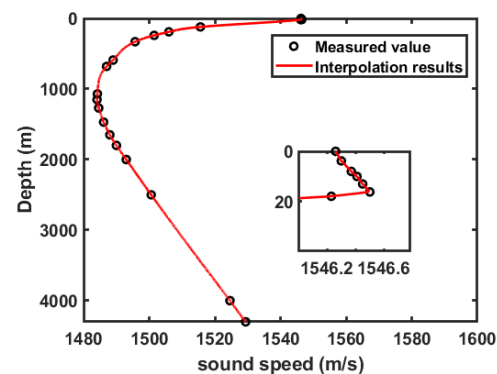


Figure 3. Measured sound speed profile.

Figure 4 shows the estimated results of Equations (10)–(12) and compares them with the Bellhop simulation results. The red solid line is the result of the focused beam boundary calculation, also known as the HPBW (half-power beam width) boundary. From the comparison results, it can be seen that the estimated results of the horizontal detection range are highly consistent with the simulation results of the simulated focused beam. Figure 5 shows the curve of the horizontal detection range changing with the phased grazing angle. It can be seen that the detection range increases within a small angle range, and the maximum value of the horizontal detection range appears at around 10°. The detection range shows a decreasing trend until around 55°, and the active detection angle resolution increases.

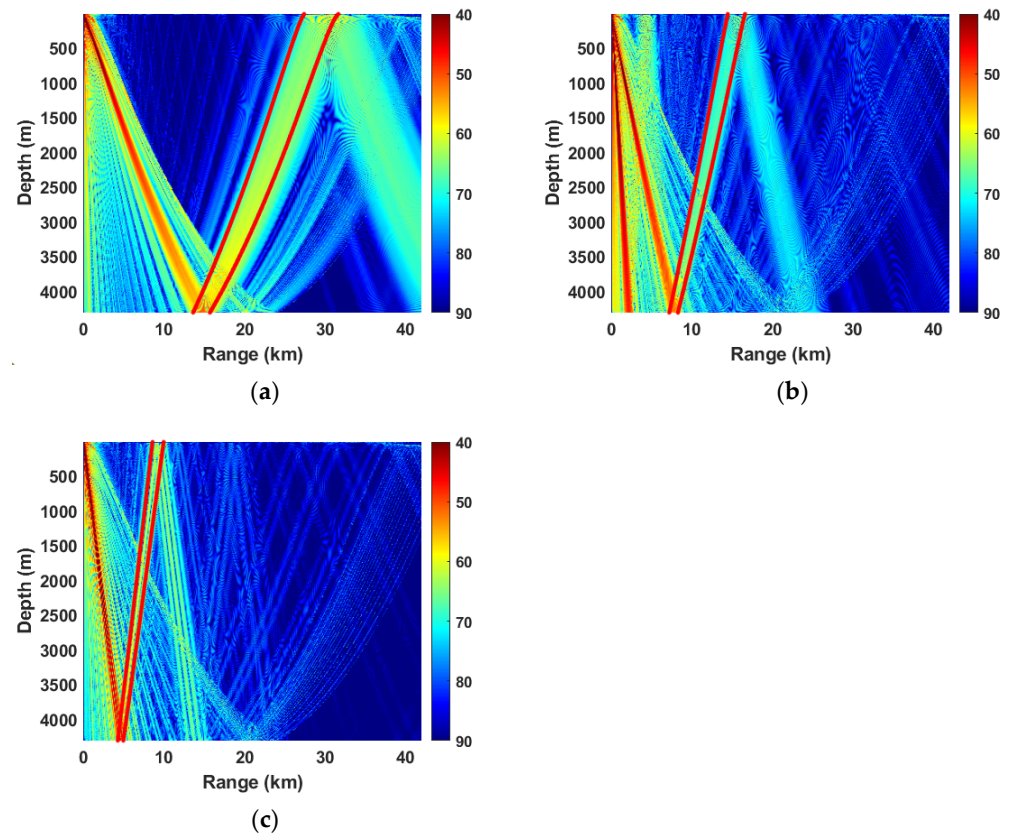


Figure 4. Comparison of theoretical estimation and simulation results of focused acoustic beam: (a) 10° phased emission; (b) 26° phased emission; (c) 41° phased emission.

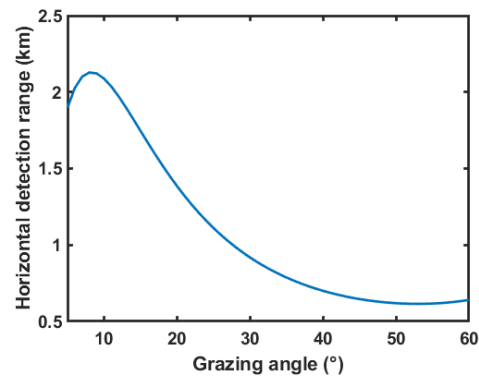


Figure 5. The curve of horizontal detection range with grazing angle.

Figure 6 presents the simulated sound transmission loss curves for two cases: one considering the sound speed layered structure and the other assuming a constant sound speed structure. The comparison reveals that, for small grazing angles, the horizontal position of the sound focus differs significantly between the two cases. However, as the phased grazing angle increases, the discrepancies in the horizontal position and the horizontal range of the focused sound beam gradually diminish, basically the same at 42°. Notably, large phased angles correspond to a small horizontal detection range. This indicates that in the wide horizontal range of the near-surface acoustic shadow zone, the horizontal detection range predicted without accounting for the sound speed layered structure deviates substantially from the actual scenario. These results confirm that the proposed estimation method provides improved accuracy for small-angle phased emissions when the deep-sea sound speed layered structure is taken into account.

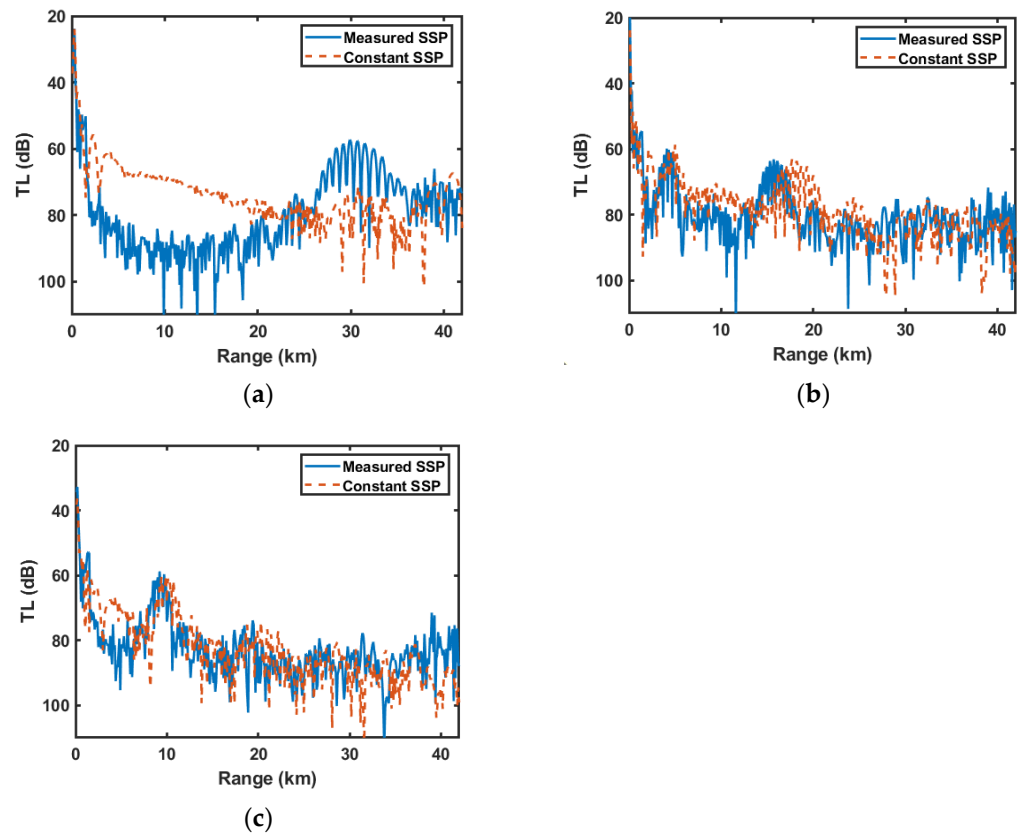


Figure 6. Comparison curve of horizontal transmission loss of two sound speed distributions: (a) simulation results of 10° phased emission; (b) simulation results of 26° phased emission; (c) simulation results of 41° phased emission.

4. Optimization of the Estimation Method

4.1. Optimization of Horizontal Detection Range Estimation

As shown in Figure 4, the transmission of the focused sound beam toward the bottom after reflecting off the sea surface forms the surface bounce area illustrated in Figure 7. This phenomenon leads to an expansion of the horizontal detection area. Below the surface bounce area, the focused sound beams in two directions decoupled in distance, and the energy of the beam reflected by the sea surface remains high. As a result, the horizontal detection range at the same depth is further expanded, with the expansion rate significantly exceeding that within the surface bounce area. To account for this, Equation (12) requires further refinement. Here, the limit depth of the surface bounce area is defined as Δd , and the horizontal detection range is analyzed separately for regions above and below Δd .

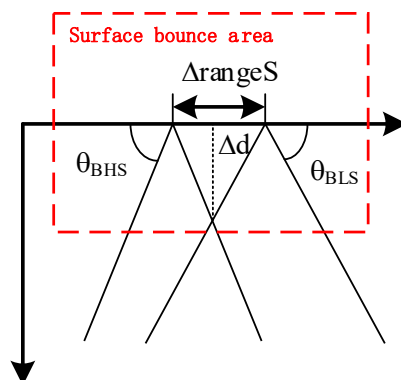


Figure 7. Horizontal detection range expansion.

In Figure 7, θ_{BHS} and θ_{BLS} represent the grazing angles of the sound rays emitted from θ_{BH} and θ_{BL} when they transmission to the sea surface, which can be calculated using Snell's law [24]:

$$\theta_{BLS} = \cos^{-1}\left(\frac{c_s}{c_0} \cos \theta_{BL}\right) \theta_{BHS} = \cos^{-1}\left(\frac{c_s}{c_0} \cos \theta_{BH}\right) \quad (13)$$

In Figure 7, $\Delta rangeS$ represents the horizontal detection range at the sea surface. For simplicity of calculation, the region where the upward and downward focused acoustic beams overlap horizontally is approximated as a triangular area. This approximation allows the region to be determined through the following calculation:

$$\Delta d = \frac{\sin(\theta_{BHS}) \sin(\theta_{BLS})}{\sin(\theta_{BHS} + \theta_{BLS})} \Delta rangeS \quad (14)$$

Assuming that the horizontal spans of the left and right boundaries of the downward focused sound beam reflected by the sea surface are X'_{BH} and X'_{BL} , respectively, as shown in Figure 8, it is necessary to perform mirror processing based on the interface with a depth of $2H$ on the basis of Figure 2, and the following can be obtained:

$$X'_{BL} = \cos \theta_{BL} \int_{sd}^{2H+rd} \frac{dz}{\sqrt{n^2(z) - \cos^2 \theta_{BL}}} \quad (15)$$

$$X'_{BH} = \cos \theta_{BH} \int_{sd}^{2H+rd} \frac{dz}{\sqrt{n^2(z) - \cos^2 \theta_{BH}}} \quad (16)$$

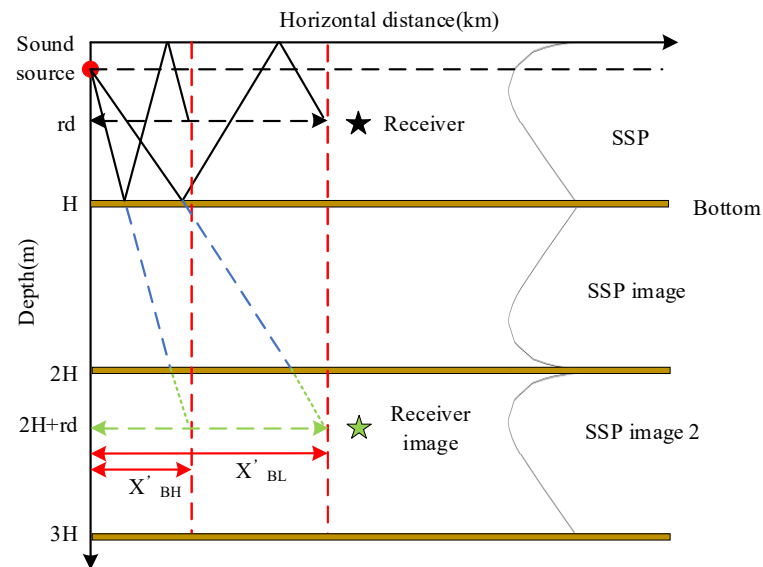


Figure 8. Downward focused beam boundary estimation.

If $rd < \Delta d$, then

$$\Delta Range = \Delta range + 2 \cos \theta_{BLS} \int_0^{rd} \frac{dz}{\sqrt{n_1^2(z) - \cos^2 \theta_{BLS}}} \quad (17)$$

where $n_1(z) = c_s/c(z)$ is the refractive index of the sound ray at each depth relative to the sea surface, and $\Delta Range$ is the estimated result of the improved horizontal detection range.

If $rd > \Delta d$, then

$$\Delta Range = \Delta range + \Delta range S + \cos \theta_{BLS} \int_0^{rd} \frac{dz}{\sqrt{n_1^2(z) - \cos^2 \theta_{BLS}}} - \cos \theta_{BHS} \int_0^{rd} \frac{dz}{\sqrt{n_1^2(z) - \cos^2 \theta_{BHS}}} \quad (18)$$

4.2. Optimization of Grazing Angle

Figure 9 illustrates the comparative results after incorporating the HPBW boundary of the downward focused beam. It is evident that the consistency is proportional to the phased grazing angle. In Figure 9a, the sound ray emitted with θ_{BL} exhibits a deviation of approximately 1.1 km from the simulated right boundary after the first bottom reflection, increasing to 2.4 km upon reaching the sea surface. This deviation accumulates during transmission, leading to underestimated horizontal detection ranges at various depths. Consequently, this affects the design of the phased emission angle and reduces the efficiency of phased angle scanning. To mitigate these issues, the phased emission angle can be optimized based on the simulation results of the deep-sea waveguide environment.

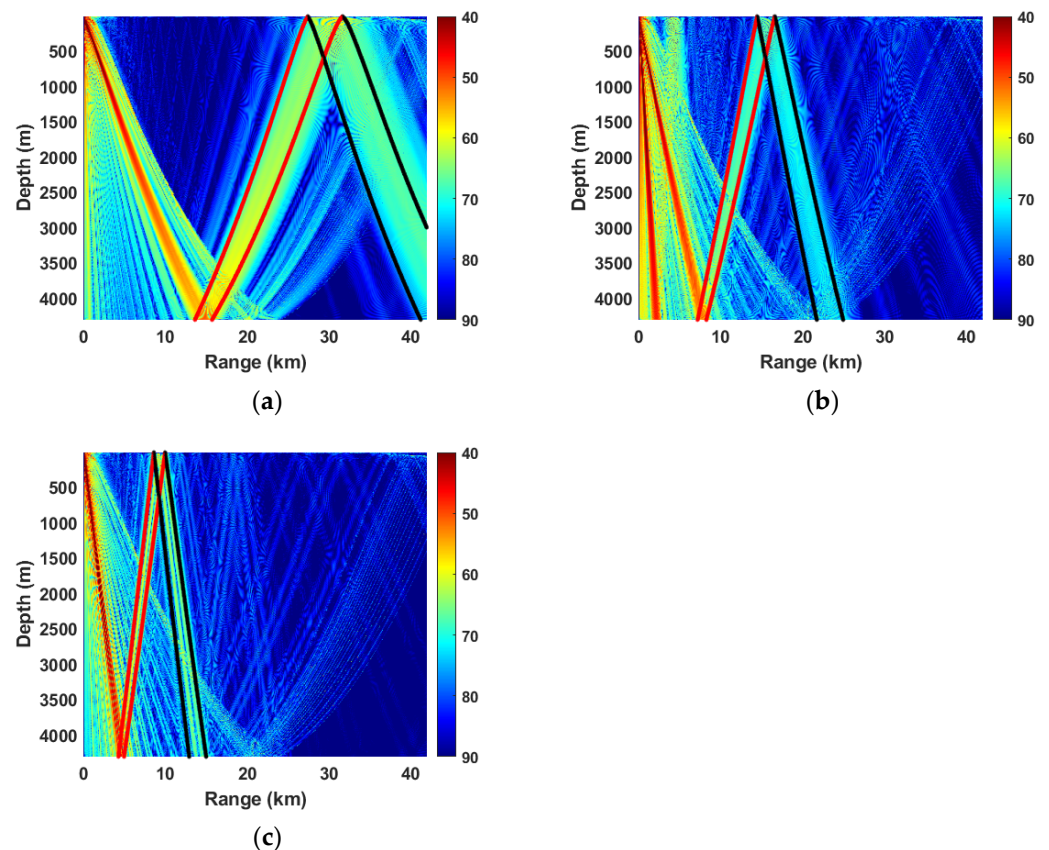


Figure 9. Downward focused beam HPBW boundary estimation results: (a) the 10° phased emission; (b) the 26° phased emission; (c) the 41° phased emission.

Similar to Figure 1, the -6 dB bandwidth is the angle range corresponding to the normalized amplitude of the beam curve dropping to 0.5. When $N\pi d(\sin \theta_m - \sin \theta_B)/\lambda = 1.895$, $W(\theta_m; \theta_B)/N = 0.5$, the -6 dB bandwidth can be calculated as follows:

$$\Delta\theta_{1/4} \approx \frac{69\lambda}{\cos\theta_B Nd} (^\circ) \quad (19)$$

θ_{BL} can be improved as follows:

$$\theta_{BL} = \theta_B - \frac{1}{2}\Delta\theta_{1/4} = \theta_B - \frac{34.5\lambda}{\cos\theta_B Nd} \quad (20)$$

Figure 10 presents the calculation results of the horizontal span following angle optimization. Compared with Figure 9, it is evident that the right boundary of the focused beam has been significantly expanded. Table 1 summarizes the estimated results for X_{BH} , X_{BL} , X'_{BH} , X'_{BL} , and $\Delta Range$ at a receiving depth of 60 m. The horizontal range of the acoustic shadow area is approximately 3 km to 40 km. From the results in Table 1, it can be observed that scanning the horizontal distance using 12 phased angles enables active detection to cover the entire acoustic shadow area, with minimal overlap between the horizontal detection ranges corresponding to each phased angle. Furthermore, the combined horizontal detection ranges of the first four angles account for 56.51% of the total horizontal span of the acoustic shadow area. This indicates that while increasing the phased angle reduces the horizontal detection range, it enhances the detection angle resolution. However, this comes at the expense of reduced scanning efficiency across the entire acoustic shadow area.

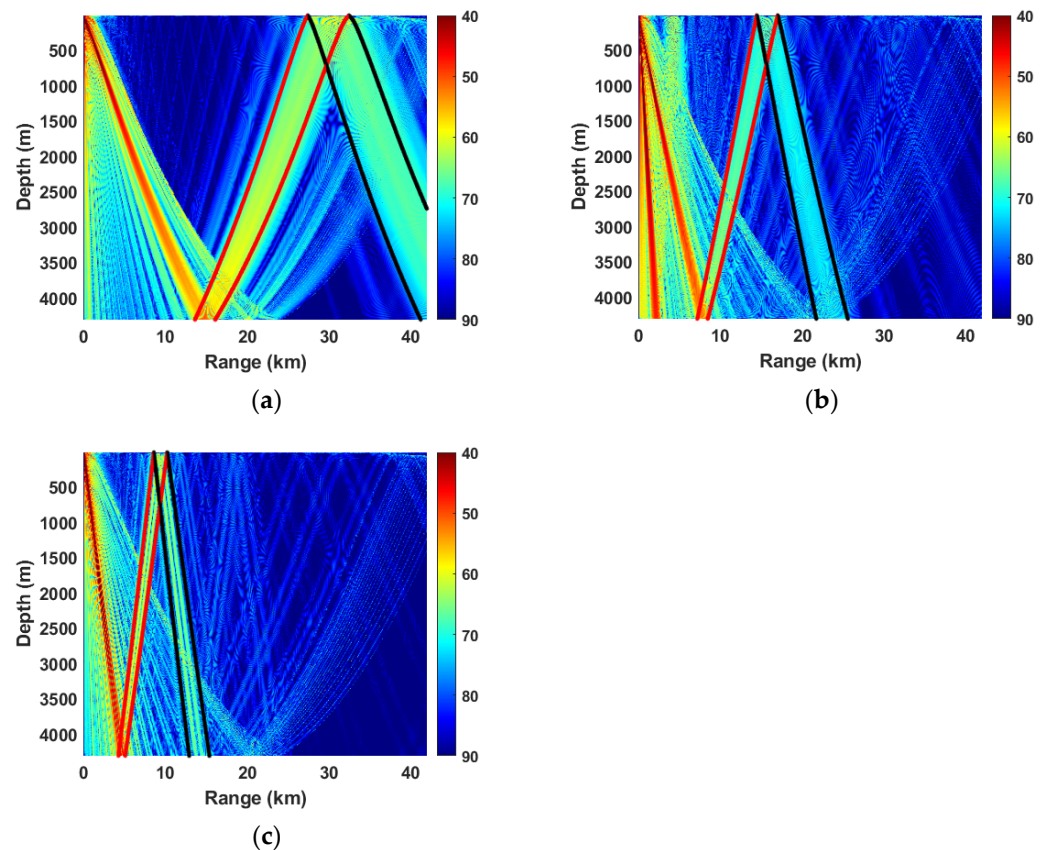


Figure 10. Angle optimized focused beam boundary estimation results: (a) the 10° phased emission; (b) the 26° phased emission; (c) the 41° phased emission.

Table 1. Estimation results of the main lobe-focused beam parameters.

θ_B (°)	X_{BH} (km)	X_{BL} (km)	X'_{BH} (km)	X'_{BL} (km)	$\Delta Range$ (km)
6	31.98	36.53	32.88	40.39	8.41
10	27.15	32.03	27.73	32.93	5.78
14	22.94	27.26	23.36	27.84	4.90
18	19.48	23.09	19.81	23.52	4.04
22	16.67	19.68	16.94	20.02	3.35
26	14.37	16.91	14.60	17.19	2.82
31	12.04	14.15	12.22	14.37	2.33
36	10.14	11.96	10.29	12.15	2.01
41	8.55	10.19	8.68	10.34	1.79
47	6.94	8.45	7.05	8.57	1.63
53	5.56	7.02	5.64	7.12	1.56
60	4.12	5.64	4.18	5.72	1.6

4.3. Optimization of Angle Scanning Efficiency

It can be seen from Figure 10b that in addition to the main lobe-focused beam, there is also a focused beam with an abnormal direction. This is caused by the appearance of grating lobes in the beam curve due to the transmit array parameters. The condition for no grating lobes in the transmit beam is as follows [1]:

$$d < \frac{\lambda}{1 + |\sin \theta_B|} \quad (21)$$

The angle of the grating lobes is as follows:

$$\theta_G = \sin^{-1}\left(-\frac{\lambda}{d} + \sin \theta_B\right) \quad (22)$$

Figure 11a illustrates the transmitting beam pattern when the phased angle is 26°. According to Equation (22), the grating lobe is calculated to appear at −63.2°. The half-power beam width corresponding to the grating lobe-focused beam, indicated by the black dot in Figure 11, can also be determined. The grating lobe angle is negative, meaning that the phased emission is directed toward the sea surface, resulting in the scenario shown in Figure 10b, where two focused beams coexist. Equation (22) further demonstrates that the grating lobe angle increases with the phased angle. Given the negative value of the grating lobe angle, this increase manifests as a reduced slope for the grating lobe-focused beam. The corresponding horizontal detection position moves relative to the main lobe-focused beam. As shown in Figure 11b, when the phased angle reaches 41°, the grating lobe angle becomes nearly symmetrical to the phased angle about 0°, therefore in Figure 10c, the main lobe and grating lobe-focused beams overlap spatially.

To address the reduced efficiency of the phased emission horizontal detection range estimation method when scanning at larger grazing angles, the characteristics of the grating lobe can be leveraged during large phased angle scanning. By utilizing the grating lobe-focused acoustic beam to perform angular scanning from near to far distances, the number of required angles for detection can be streamlined. This approach enhances the efficiency of angular scanning while maintaining comprehensive coverage.

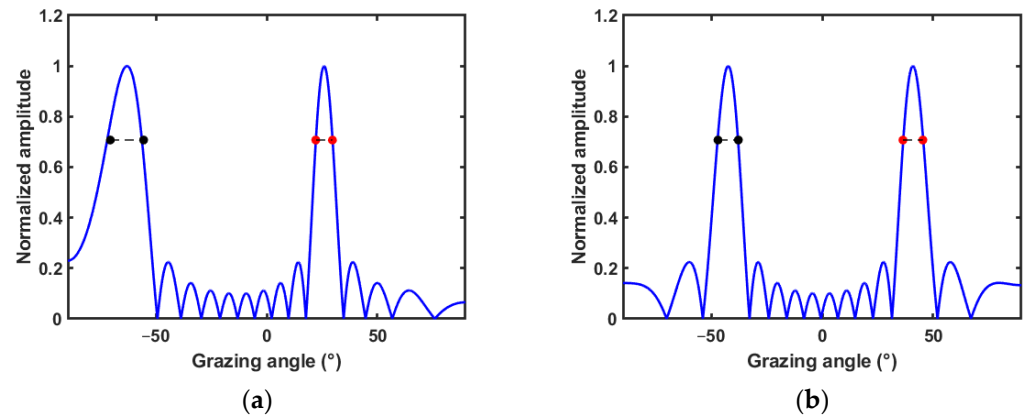


Figure 11. Grating lobe-focused beam: (a) 26° phased emission; (b) 41° phased emission.

The calculation process of calculating the horizontal detection distance of the grating lobe-focused acoustic beam at each receiving depth is similar to the previous article, combined θ_{BL} with θ_{BH} the selection θ_{GL} and θ_{GH} :

$$\theta_{GL} = \theta_G + \frac{1}{2}\Delta\theta_{1/4} = \theta_G + \frac{34.5\lambda}{\cos\theta_G Nd} \quad (23)$$

$$\theta_{GH} = \theta_G - \frac{1}{2}\Delta\theta_{1/2} = \theta_G - \frac{25.5\lambda}{\cos\theta_G Nd} \quad (24)$$

It can be seen from Figure 12 that when calculating the horizontal detection range corresponding to the grating lobe-focused acoustic beam, the sound source can be processed as a virtual source image relative to the sea surface [25]. At this time, the depth of the sound source is $-sd$, which can be calculated as:

$$\Delta range_G = X_{GL} - X_{GH} = \cos\theta_{GL} \int_{-sd}^{2H-rd} \frac{dz}{\sqrt{n^2(z) - \cos^2\theta_{GL}}} - \cos\theta_{GH} \int_{-sd}^{2H-rd} \frac{dz}{\sqrt{n^2(z) - \cos^2\theta_{GH}}} \quad (25)$$

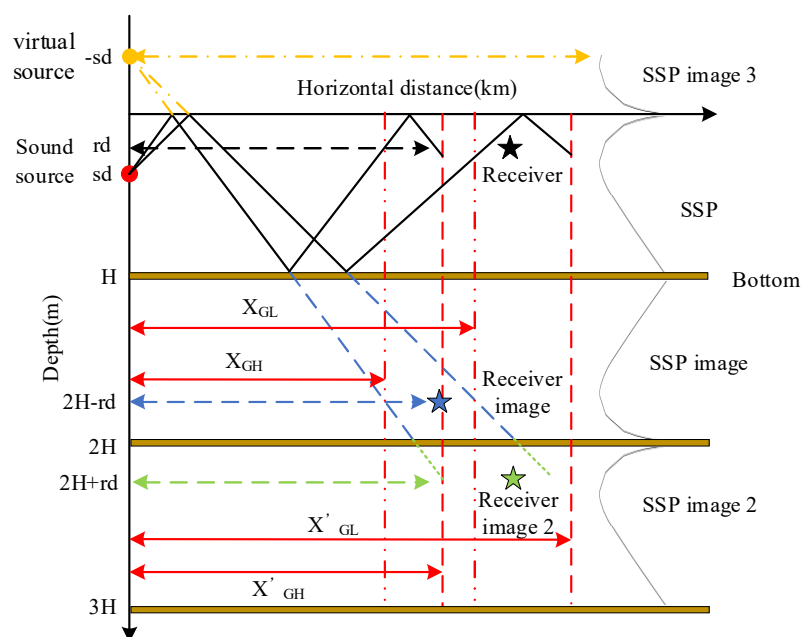


Figure 12. Grating lobe-focused beam boundary estimation.

The limit depth of the surface bounce area of the grating lobe-focused beam ΔdG is:

$$\Delta dG = \frac{\sin(\theta_{GHS}) \sin(\theta_{GLS})}{\sin(\theta_{GHS} + \theta_{GLS})} \Delta rangeSG \quad (26)$$

The G in the symbol represents the grating lobe. The horizontal detection range corresponding to the grating lobe-focused beam $\Delta RangeG$ can be deduced by analogy with Equations (17) and (18) through the relationship between rd and ΔdG , which will not be repeated here.

Figure 13 presents the boundary estimation results for the grating lobe-focused beam at phased angles of 26° , 31° , 36° , and 41° . The pink and blue solid lines represent the boundaries of the upward and downward grating lobe-focused beams, respectively. The results indicate a high degree of consistency in the proposed method's estimation of the grating lobe-focused beam within the given angle range. Table 2 summarizes the estimation results for points X_{GH} , X'_{GL} , and $\Delta RangeG$ at five phased angles. As the phased angle increases, the grating lobe-focused beam performs angular scanning from near to far distances. The horizontal detection ranges at the specified angles effectively cover the close-range acoustic shadow zone. Compared with relying solely on the main lobe-focused beam, this approach improves angular scanning efficiency by 33.3%.

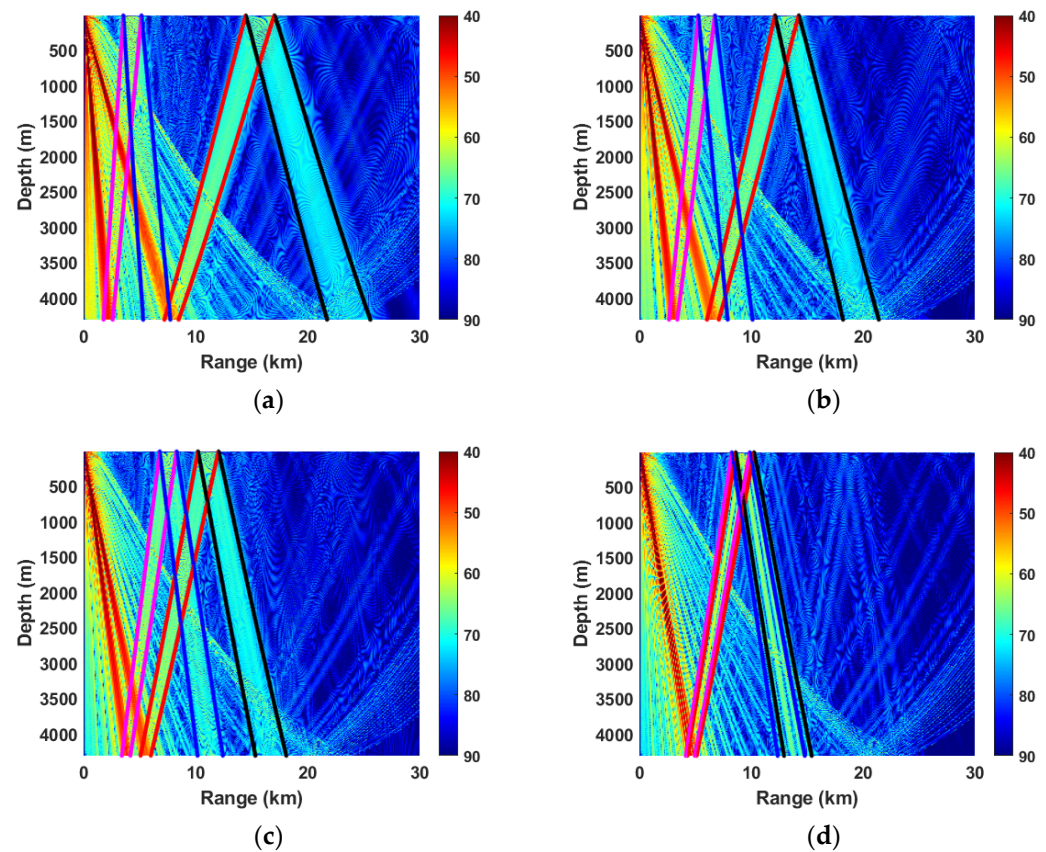


Figure 13. Main lobe and grating lobe-focused beam boundary estimation results: (a) the 26° phased emission; (b) the 31° phased emission; (c) the 36° phased emission; (d) the 41° phased emission.

Table 2. Estimation results of grating lobe-focused beam parameters.

θ_B (°)	θ_G (°)	X_{GH} (km)	X'_{GL} (km)	$\Delta RangeG$ (km)
22	−73.1	1.64	3.89	2.26
26	−63.2	3.52	5.21	1.69
31	−54.7	5.23	6.81	1.57
36	−48.0	6.75	8.37	1.62
41	−42.5	8.20	9.95	1.75

4.4. Estimation Method Optimization Results

According to the relationship between the receiver depth and the limit depth of the sea surface reflection area of the main lobe-focused beam and the grating lobe-focused acoustic beam, the expression of the total horizontal detection range ΔR when the grating lobe-focused acoustic beam exists is directly given as follows:

$$(1) \quad rd < \Delta d \text{ and } rd < \Delta dG$$

$$\begin{aligned} \Delta R = & \Delta range + 2 \cos \theta_{BLS} \int_0^{rd} \frac{dz}{\sqrt{n_1^2(z) - \cos^2 \theta_{BLS}}} \\ & + \Delta rangeG + 2 \cos \theta_{GLS} \int_0^{rd} \frac{dz}{\sqrt{n_1^2(z) - \cos^2 \theta_{GLS}}} \end{aligned} \quad (27)$$

$$(2) \quad \Delta dG < rd < \Delta d$$

$$\begin{aligned} \Delta R = & \Delta range + 2 \cos \theta_{BLS} \int_0^{rd} \frac{dz}{\sqrt{n_1^2(z) - \cos^2 \theta_{BLS}}} \\ & + \Delta rangeG + \Delta rangeSG + \cos \theta_{GLS} \int_0^{rd} \frac{dz}{\sqrt{n_1^2(z) - \cos^2 \theta_{GLS}}} \\ & - \cos \theta_{GHS} \int_0^{rd} \frac{dz}{\sqrt{n_1^2(z) - \cos^2 \theta_{GHS}}} \end{aligned} \quad (28)$$

$$(3) \quad \Delta d < rd < \Delta dG$$

$$\begin{aligned} \Delta R = & \Delta range + \Delta rangeS + \cos \theta_{BLS} \int_0^{rd} \frac{dz}{\sqrt{n_1^2(z) - \cos^2 \theta_{BLS}}} \\ & - \cos \theta_{BHS} \int_0^{rd} \frac{dz}{\sqrt{n_1^2(z) - \cos^2 \theta_{BHS}}} + \Delta rangeG \\ & + 2 \cos \theta_{GLS} \int_0^{rd} \frac{dz}{\sqrt{n_1^2(z) - \cos^2 \theta_{GLS}}} \end{aligned} \quad (29)$$

$$(4) \quad rd > \Delta dG \text{ and } rd > \Delta d$$

$$\begin{aligned} \Delta R = & \Delta range + \Delta rangeS + \cos \theta_{BLS} \int_0^{rd} \frac{dz}{\sqrt{n_1^2(z) - \cos^2 \theta_{BLS}}} \\ & - \cos \theta_{BHS} \int_0^{rd} \frac{dz}{\sqrt{n_1^2(z) - \cos^2 \theta_{BHS}}} + \\ & \Delta rangeG + \Delta rangeSG + \cos \theta_{GLS} \int_0^{rd} \frac{dz}{\sqrt{n_1^2(z) - \cos^2 \theta_{GLS}}} \\ & - \cos \theta_{GHS} \int_0^{rd} \frac{dz}{\sqrt{n_1^2(z) - \cos^2 \theta_{GHS}}} \end{aligned} \quad (30)$$

Table 3 provides the estimation results of X_{BH} , X'_{BL} , X_{GH} , X'_{GL} and under nine phased angles. It can be seen that after the estimation method is improved, the nine-angle phased emission at the transmitting end can achieve the angle scanning of the horizontal range of the entire acoustic shadow zone at a depth of 60 m, and the overlap rate of the horizontal detection range is low; thus, the angle scanning efficiency is improved.

Table 3. Estimation results of optimization estimation method.

θ_B (°)	X_{BH} (km)	X'_{BL} (km)	X_{GH} (km)	X'_{GL} (km)	ΔR (km)
6	31.98	40.39	-	-	8.41
10	27.15	32.93	-	-	5.78
14	22.94	27.84	-	-	4.90
18	19.48	23.52	-	-	4.04
22	16.67	20.02	1.64	3.89	5.61
26	14.37	17.19	3.52	5.21	4.51
31	12.04	14.37	5.23	6.81	3.9
36	10.14	12.15	6.75	8.37	3.63
41	8.55	10.34	8.20	9.95	3.54

4.5. Discussion of Uneven Seabed

The estimation method proposed in this paper mainly uses the bottom reflection. When transmitting at a certain angle, if the seabed is not flat enough, the grazing angle of the sound ray after the bottom reflection will change, which will affect the actual horizontal detection range. In this section, the proposed estimation method is optimized for a seafloor that gently undulates in depth with horizontal distance.

When the phased angle is fixed, the exit angle and propagation trajectory of the boundary rays of the focused acoustic beam can be calculated. Based on the propagation trajectory and seabed depth information, the reflection positions of the boundary rays on the seabed can be estimated. The accuracy of this estimation depends on the precision of the sound speed profile interpolation. Figure 14 presents a schematic diagram of the focused acoustic beam boundary reflected by the seabed. The corresponding grazing angles of the boundary rays at the seabed reflection points are denoted as θ_{BHB} and θ_{BLB} . Due to the gentle undulations of the seabed, the angles between the seabed tangents and the horizontal plane at the reflection points are denoted as α_H and α_L , respectively. Through geometric analysis, the grazing angles of the rays after reflection can be determined as θ'_{BHB} and θ'_{BLB} :

$$\theta'_{BHB} = \theta_{BHB} \pm 2\alpha_H, \theta'_{BLB} = \theta_{BLB} \pm 2\alpha_L \quad (31)$$

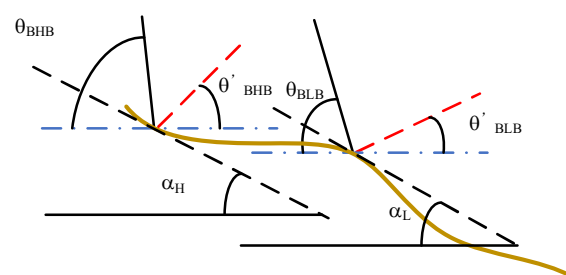


Figure 14. Schematic diagram of the acoustic beam boundary reflecting off the seabed.

The positive and negative operations correspond to the upward and downward tilts of the seabed at the reflection points, respectively.

Taking θ_{BHB} as an example, it is assumed that the acoustic ray reflects at a depth of z_1 and propagates upward to the receiving depth rd . In this case, the left boundary X_{BH} corresponding to the left edge of the focused acoustic beam is expressed as follows:

$$X_{BH} = \cos \theta_{BH} \int_{sd}^{z_1} \frac{dz}{\sqrt{n^2(z) - \cos^2 \theta_{BH}}} + \frac{c(rd)}{c(z_1)} \cos \theta'_{BHB} \int_{rd}^{z_1} \frac{dz}{\sqrt{n_2^2(z) - \frac{c^2(rd)}{c^2(z_1)} \cos^2 \theta'_{BHB}}} \quad (32)$$

where $n_2(z) = c(rd)/c(z)$. Similarly, X_{BL} can be determined as follows:

$$X_{BL} = \cos \theta_{BL} \int_{sd}^{z_2} \frac{dz}{\sqrt{n^2(z) - \cos^2 \theta_{BL}}} + \frac{c(rd)}{c(z_2)} \cos \theta'_{BLB} \int_{rd}^{z_2} \frac{dz}{\sqrt{n_2^2(z) - \frac{c^2(rd)}{c^2(z_2)} \cos^2 \theta'_{BLB}}} \quad (33)$$

where z_2 is the reflection depth corresponding to the right beam boundary ray.

Based on this, the phased emission horizontal detection distance and range under the conditions of gentle terrain undulation can be further derived using the previous content. The impact of changes in seabed topography is essentially the change in ray propagation trajectories caused by the topography. Given the diverse and complex nature of these variations, a detailed discussion is beyond the scope of this study.

5. Sea Trial Data Analysis

The equipment deployment and relative position are shown in Figure 15. The seabed in the detection area was relatively flat, with an average depth of 4300 m. The sound speed profile during the test is shown in Figure 3. The transmitting platform, designated as Ship No. 1, was equipped with a 20-element low-frequency vertical phased array suspended from its stern. The top element of the array was positioned at a depth of approximately 31.5 m, with an element spacing of about 1.6 m. The phased array transmitted a linear frequency modulation (LFM, signal with an instantaneous frequency that varies linearly over time) signal with a pulse width of 2 s, a center frequency of 720 Hz, and a bandwidth of 160 Hz at a specified phased angle. Additionally, Ship No. 2 was equipped with a transponder to simulate the echo signal. The transponder is capable of both receiving and transmitting acoustic signals. Upon receiving an incident acoustic signal, the hardware circuit within the transponder measures the signal strength. If the received signal strength exceeds a predefined threshold, the signal is amplified by a fixed gain and retransmitted. Consequently, the retransmitted signal inherently carries the intensity information of the incident signal. During the sea trial, the transponder was maintained at an average depth of 60 m.

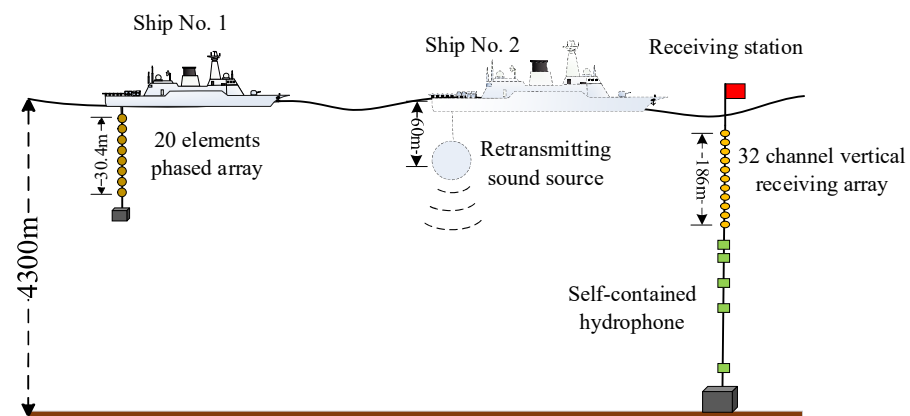


Figure 15. Sea trial equipment deployment.

A 32-element acoustic vertical receiving array was deployed at point O to receive acoustic signals. The sampling rate of each receiving channel was 12,800 Hz, and the hydrophone covered a depth from 120 m to 306 m underwater. In addition, self-contained hydrophones were installed at unequal intervals, ranging from 514 m to 4114 m, and the sampling rate of the self-contained hydrophone was 64,000 Hz.

5.1. Simulation Analysis

During the sea trial, the horizontal distances between the transmitting array, the transponder, and the receiving station were 31 km and 39 km, with the transponder positioned approximately 8 km from the receiving station. Referring to Table 3 and Figure 10a, it can be observed that when the phased angle is 10° , the horizontal detection range estimation is 31 km. Figure 16a,b show the simulation results of the eigenrays of the transmitting array and the transponder reaching the receiver. In the figures, the pink star indicates the position of the transponder, while the red dot represents the average depth of the receiving array. The eigenray types are categorized based on the number of bottom reflections. The blue solid lines denote the eigenrays with a single bottom reflection, whereas the black dashed and dotted lines represent eigenrays with double and triple bottom reflections, respectively. The two simulation results are consistent, both revealing three main arrival structures corresponding to the first, second, and third bottom reflections. Figure 16c presents the arrival structure results after applying the weighting defined by Equation (5), while Figure 16d depicts the arrival structure of the echo signal. The line patterns corresponding to the arrival structures in the two figures align one-to-one with the simulation results of the eigenrays. It can be seen that the bottom reflection has a significant attenuation effect on the sound energy.

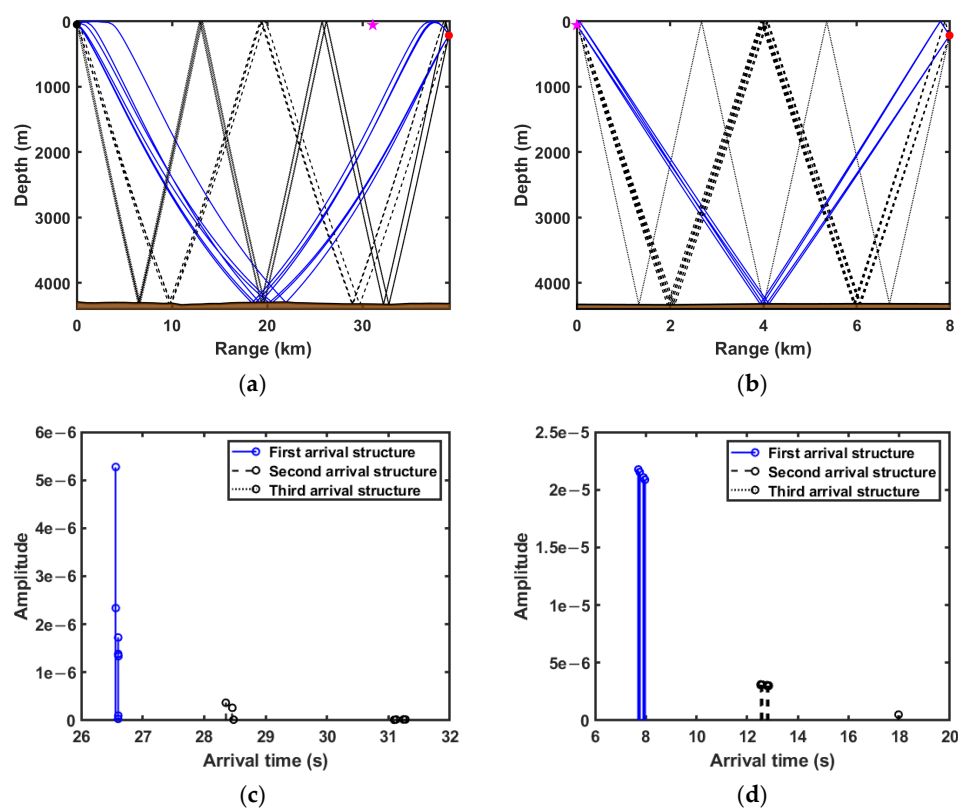


Figure 16. Simulation results of sea trial: (a) eigenray of the transmitting array; (b) eigenray of the transponder; (c) arrival structure of the transmitting array; (d) arrival structure of the transponder.

The receiving beam pattern was generated using the simulated grazing angles of the eigenrays corresponding to the six arrival structures as the transmitting array and transponder signals reached the receiver, as shown in Figure 16, along with the parameters of the vertical receiving array. Figure 17 presents the calculated results for each of the six arrival structures. The curve amplitude was normalized relative to the maximum amplitude of arrival structures. The results reveal the presence of multiple grating lobe structures in the beam pattern. The angles at which these grating lobe structures appear can serve as a basis for extracting the corresponding echo structures in the beam domain.

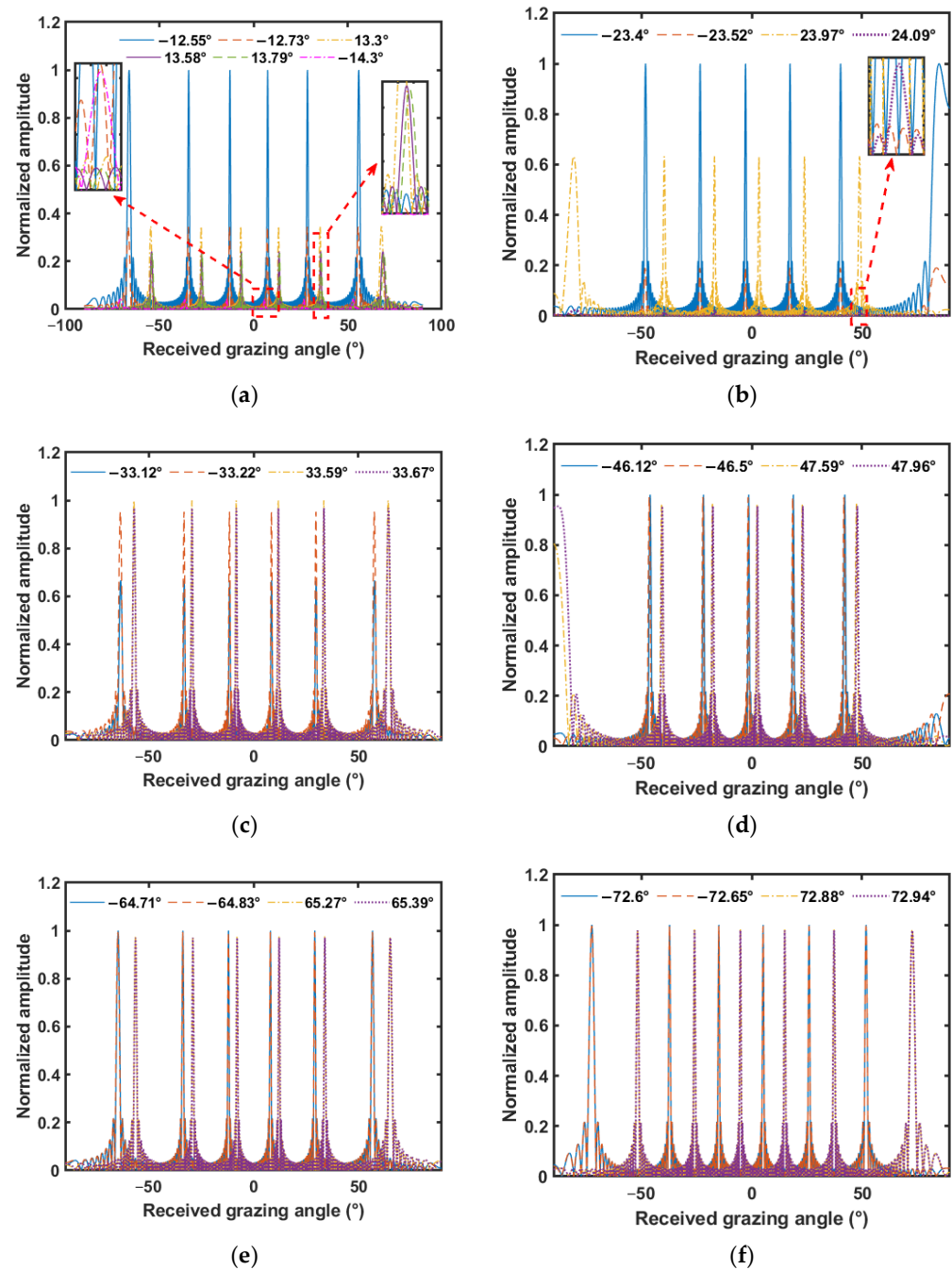


Figure 17. The receiving beam pattern: (a) the first phased arrival structure; (b) the second phased arrival structure; (c) the third phased arrival structure; (d) the first echo arrival structure; (e) the second echo arrival structure; (f) the third echo arrival structure.

5.2. Sea Trial Data Verification

The sea trial data were processed by conventional beamforming (CBF) and matched filtering (MF) in sequence, beamforming functions as a spatial filter, converting the array-received signals into the delay-beam domain and enabling the analysis of the processing results in both the delay structure and angle domain. Matched filtering further suppressed interference and enhanced signal detection capability. The delay-beam domain results obtained are called CBF-MF results.

Figure 18 presents the CBF-MF results for each arrival structure of the 10° phased emission signal. Comparing Figure 18a with Figure 17a, it is evident that the grating lobe angles align well with the bright-spot angles in the beam pattern. Additionally, in the same arrival structure, the arrival delay is proportional to the absolute value of the received grazing angle. This relationship is reflected in the delay-beam domain results as a delay in the bright spots. From Figure 18a, the bright spots with absolute angle values less than 20° appear in pairs, where the right-side spot in each pair exhibits a slight delay. For bright-spot pairs with absolute angle values greater than 20° , the spots overlap; however, the right-side spot consistently shows a delay relative to the left. This result corresponds well with the eigenray arrival angles shown in Figure 17a. According to Figure 16c, the average delay difference between the second and first arrival structures is 1.82 s, while the delay difference between the third and first structures is 4.57 s. In the CBF-MF results, the first arrival structure shows a delay of approximately 0.8 s, with prominent bright spots appearing at 2.68 s and 5.43 s. The respective delay differences of 1.88 s and 4.63 s align closely with the simulated values, indicating good agreement between the measured and simulated results in terms of delay. Comparing Figure 18b with Figures 17b and 18c with Figure 17c, it is evident that the measured data are highly consistent with the beam curve calculation results. This consistency is observed in the angular and delay relationships between the bright-spot pairs.

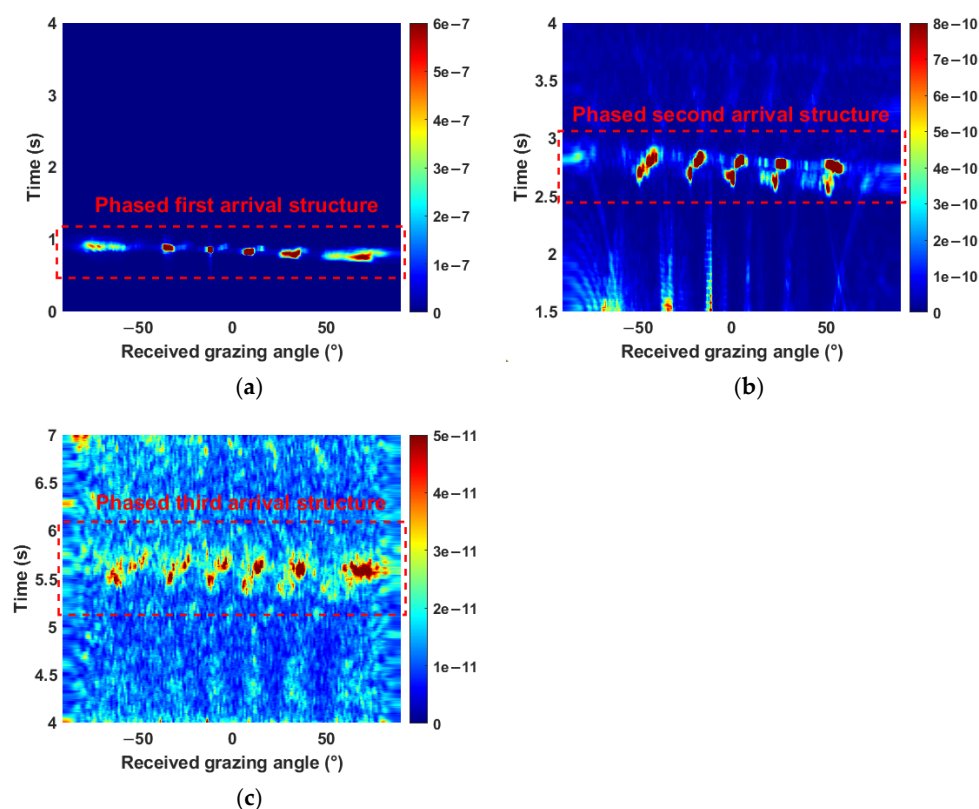


Figure 18. The 10° phased receive signal CBF-MF results: (a) 0–4 s; (b) 1.5–4 s; (c) 4–7 s.

Figure 19 illustrates the CBF-MF results for each arrival structure of the echo signal, with each subfigure representing an enlarged view of the processing results for the 10° phased emission signal. In Figure 19a, the delay range of 2–6 s is shown in detail. The bright-spot structure observed at 3.11 s is symmetrical about the 0° axis and aligns well with the beam pattern grating lobe angles in Figure 17d. This bright spot represents the first arrival structure of the echo. The second arrival structure of the phased emission has a delay of 2.68 s, which is close to the delay of the echo's first arrival structure. In the measured data, a bright spot caused by a low-energy arrival structure is present between these two arrival structures. The overlap of this bright spot with the first arrival structure of the echo in the time domain is the primary cause of the observed axial symmetry.

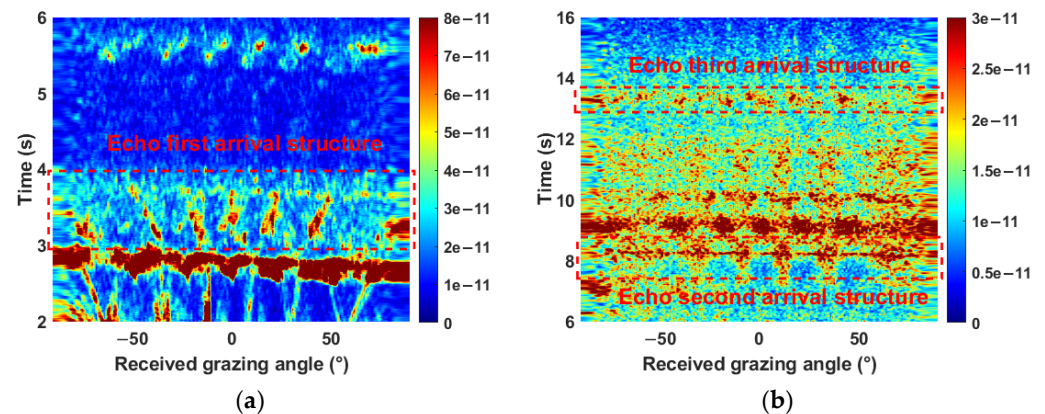


Figure 19. The echo signal CBF-MF results: (a) 2–6 s; (b) 6–16 s.

In Figure 16d, the average delay differences between the second and third arrival structures of the echo and the first structure are 4.81 s and 10.16 s, respectively. In Figure 19b, bright-spot structures are observed at delays of approximately 8 s and 13.24 s, which are consistent in angle with the results shown in Figure 17e,f. The corresponding delay differences from the first arrival structure are 4.89 s and 10.13 s, respectively. These results confirm the consistency between the measured data and the simulation results for echo data verification.

However, due to the low energy levels of the second and third arrival structures in the echo signal and the significant time-domain aliasing with the phased transmission signal, the CBF-MF results become more complex. As a result, details such as the time delay differences between the second and third arrival structure bright-spot pairs of the echo signal are not sufficiently distinct.

To further validate the proposed estimation method, Figure 20 presents the CBF-MF processing results for the received signals at four phased angles. The color bar range is consistently set across all subfigures for comparison. The results for the three arrivals corresponding to the phased emission in Figure 20b–d exhibit higher clarity and demonstrate good alignment with Figure 20a in both the time and beam domains. According to the horizontal detection range estimates provided in Table 3, only the 10° phased transmission encompasses the 31 km position of the transponder. It is difficult to observe a distinct echo signal arrival structure in Figure 20b–d.

Figure 21a,b illustrate the CBF-MF processing results of the received signal for the 31° and 53° phased transmissions, respectively, with the color bar range consistent between the two figures. The transponder was located 12.4 km from the phased array. According to the estimation results in Table 3, the horizontal detection range of the main lobe-focused beam for the 31° spanned 12.04–14.37 km, encompassing the transponder's location. In contrast, the range for the 53° was 5.56–7.12 km, which excluded the transponder's position. Using Equation (22), the corresponding grating lobe-focused beam angle was calculated

as -32.18° , with a horizontal detection range of 11.83–13.92 km, which included the transponder's location. As shown in Figure 21, when the phased emission occurred at large angles, the grating lobe-focused acoustic beams could still effectively detect targets within the acoustic shadow zone, enhancing the efficiency of angular scanning across the entire acoustic shadow zone.

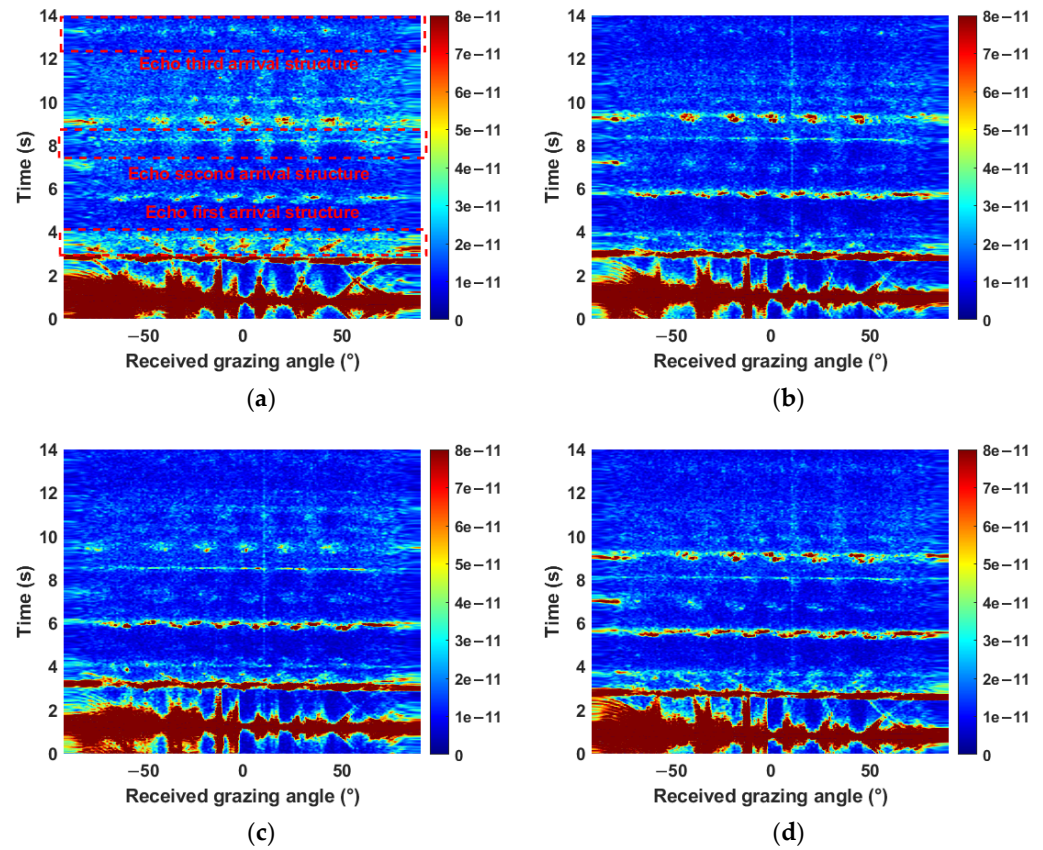


Figure 20. Received signal CBF-MF results: (a) the 10° phased emission; (b) the 22° phased emission; (c) the 26° phased emission; (d) the 30° phased emission.

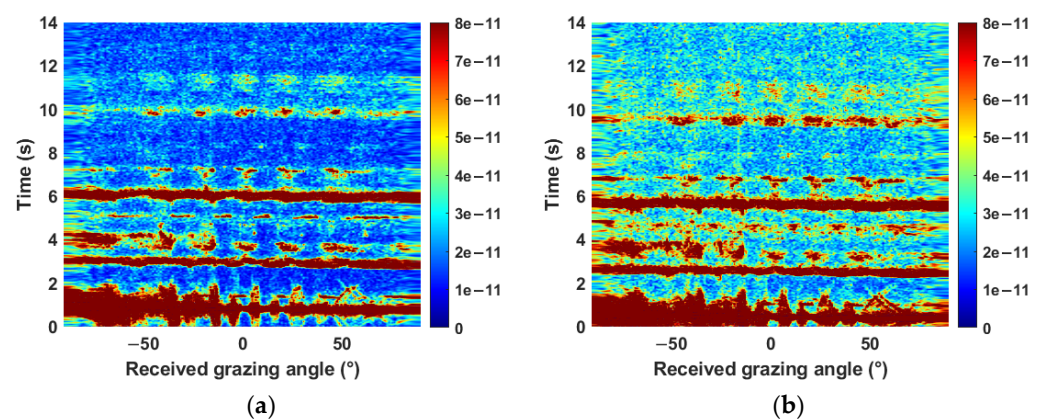


Figure 21. Grating lobe-focused beam detection verification results: (a) the 31° phased emission; (b) the 53° phased emission.

6. Conclusions

This paper presents a method for estimating the horizontal detection range of the shadow zone by combining phased emission technology and transmitting beam HPBW. The quantitative relationship between the phased angle and the horizontal detection distance

and range was determined, and several discrete phased angles were designed to achieve the angle scanning of the entire horizontal range of the acoustic shadow zone at a certain receiving depth.

(1) This method must take into account the hydrological information of the detected sea area; otherwise, the calculation result of the horizontal span of the focused beam will be biased, and the degree of deviation will be inversely proportional to the phased angle.

(2) This method takes into account the expansion of the horizontal detection range caused by the surface bounce area and adapts to the law of sound ray propagation in the deep sea environment. Based on the -6 dB beam width, the boundary grazing angle of the focused beam is improved, and the horizontal detection range of the specified grazing angle can be accurately estimated.

(3) This method is adapted to the phased array parameters, and the grating lobe-focused beam caused by the transmitting array parameters is used to improve the angle scanning efficiency. In the case of close-range and large-angle phased emission, the horizontal detection range can be significantly expanded.

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