

Article

Effect of Nonlinear Factors on Typhoon-Induced Storm Surges

Fanjun Chen ¹, Kaixuan Ding ² and Zhilin Sun ^{1,*}¹ College of Civil Engineering and Architecture, Zhejiang University, Hangzhou 310058, China; 11912116@zju.edu.cn² Water Affairs Bureau of Jiujiang District, Jiujiang District, Wuhu 241001, China; 21934101@zju.edu.cn

* Correspondence: oceanzls@163.com

Abstract: This study employs the Delft3D numerical model to elucidate nonlinear interactions in velocity dynamics across four key marine regions during typhoon-induced storm surges (Typhoon In-fa, No. 2106). To address gaps in understanding how typhoon winds drive storm surges, this study aims to analyze the relative contributions of bottom friction, wind stress, and convective terms to storm surge dynamics, providing insights for predictive modeling and marine hazard mitigation. Introducing a novel metric, the “Flow Velocity Nonlinear Coupling Proportion” (“FVNCP”, abbreviated as “NCP”), this research quantifies the interactive effects of storm surge flow velocity by dissecting the contributions of bottom friction, wind stress, and the convective term. Through decision tree modeling, wind stress emerges as the primary driver of *NCP* in open sea and sheltered areas, with peak values reaching 1.50×10^{-4} and 2.14×10^{-4} m/s², respectively. In contrast, the convective term dominates the strait and bypassing regions, exhibiting maximum impacts of 3.21×10^{-4} and 2.94×10^{-4} m/s², while bottom friction’s influence is consistently minor across all regions. Wind stress contributes the most to *NCP* in open waters, at an average of 48.28%, while the convective term exerts a comparable 38.85% effect. In confined areas like the strait and bypassing regions, the convective term accounts for 40–44% of the *NCP*, with wind stress contributing 32–39%. The role of bottom friction is the least among the three factors though its impact intensifies in shallower zones. These findings offer critical insights for advancing predictive models and informing strategies to mitigate typhoon-driven marine hazards.

Keywords: typhoon storm tide; nonlinear coupling; convective term; shear stress; numerical simulation; decision tree model



Received: 2 November 2024

Revised: 12 December 2024

Accepted: 19 December 2024

Published: 24 December 2024

Citation: Chen, F.; Ding, K.; Sun, Z. Effect of Nonlinear Factors on Typhoon-Induced Storm Surges. *J. Mar. Sci. Eng.* **2025**, *13*, 5. <https://doi.org/10.3390/jmse13010005>

Copyright: © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Typhoon-induced storm surges pose severe threats to coastal ecosystems and infrastructure, particularly in regions like the Zhoushan sea area where unique geographic and climatic conditions amplify surge impacts. The challenges of observing critical flow dynamics, such as velocity and direction shifts during typhoon events, are compounded by the complex marine environment where nonlinear interactions between forces like wind stress and tides intensify flow variability. Advances in ocean dynamics modeling have enabled researchers to simulate these events with increasing precision. While many studies have primarily focused on tidal levels, fewer have investigated the nuanced transformations in flow velocity—a gap this study addresses by modeling velocity changes during typhoon storm surges in the Zhoushan area.

In the early 1970s, operational numerical model forecasts for storm surges on the northwest European shelf, such as those developed by R.A. Flather, successfully integrated

surge–tide interaction effects, marking a significant advancement in storm surge modeling [1]. By the 1980s, B. Johns and M. Anwar Ali extended these concepts using nonlinear numerical models for typhoon-induced storm surges in the Bay of Bengal, demonstrating that such interactions could dramatically elevate coastal water levels due to the coupling of surge dynamics [2]. Building on this foundation, J.A. As Salek and T. Yasuda introduced a more comprehensive model in 1995, incorporating critical factors such as island geography, sea level rise, and variable water depths. Their work underscored how these elements intensify coupling effects, driving significant increases in coastal water levels [3]. Building on Flather's early work, researchers such as Kim and Yasuda (2008) refined storm surge modeling through the SuWAT model, which captures interactions between depth-averaged nonlinear shallow water equations and wave effects using the SWAN model, enhancing water level prediction accuracy by 40% during Typhoon Ewiniar [4]. Subsequently, Zhou and Tan (2018) employed coupled models based on the Takahashi pressure distribution method, achieving high precision in surge level predictions along the Ningbo coast [5]. Recently, Xu and Ma incorporated a linear wind drag coefficient C_d in storm surge models for the Bohai, Yellow, and East Seas, coupling it with astronomical tides. Their approach yielded superior performance over traditional models, especially during Typhoons 7203 and 7303, highlighting the added precision achieved by integrating nonlinear dynamics into storm surge predictions [6].

The development of numerical models for typhoon storm surges has predominantly focused on simulating total water levels, which result from the coupling of tidal forces, wind stress, and pressure gradients. However, as early as 1978, Prandle identified that the nonlinear interaction between tides and storm surges introduces significant errors if treated additively, underscoring the necessity of nonlinear consideration for improved accuracy [7]. Qin et al. demonstrated that incorporating nonlinear effects into storm surge models could substantially enhance forecast precision, particularly in reducing water level prediction errors [8]. Yang et al. (2018) employed the finite element ocean model ADCIRC to investigate Typhoon Rammasun (No. 1409), delving into critical nonlinear interactions between tides and surges. Their study highlighted that nonlinear terms such as wind stress, bottom friction, and convective forces significantly influence surge dynamics, advancing the predictive capability of numerical models under extreme weather conditions [9].

With advancements in algorithms, research on nonlinear coupling within typhoon storm surges has progressed from early linear regression approaches to sophisticated nonlinear and machine learning models. Initial studies often relied on linear regression to predict storm surge impacts on flow velocity, examining relationships among basic variables such as wind speed, air pressure, and tides [10]. As more robust models, including polynomial regression and support vector regression, emerged, they enabled more accurate simulations of complex relationships between storm surges and flow dynamics. These nonlinear models incorporate the interactive effects of wind fields, pressure gradients, and topography [11], allowing researchers to better capture surge–current interactions, particularly under extreme weather conditions.

This study addresses submarine cable protection by examining extreme flow velocities induced by typhoons, which can scour the seabed and expose or suspend cables. While much research on seabed scouring has focused on pipelines—drawing on observational analysis, physical modeling, and numerical simulations [12–14]—submarine cable studies have primarily relied on sensor monitoring and physical simulations to assess burial depth changes and damage severity [15–17]. Few studies, however, approach cable protection from the perspective of flow velocity dynamics. This study provides a novel framework for analyzing nonlinear flow velocity dynamics during typhoon-induced storm surges, introducing the 'Flow Velocity Nonlinear Coupling Proportion (*NCP*)' as a metric to quantify the

interplay of key factors such as wind stress, convection, and bottom friction. Unlike prior studies, this research integrates qualitative and quantitative analyses, leveraging regression models to elucidate the spatial heterogeneity of nonlinear interactions across different marine regions. These findings not only advance our understanding of typhoon-driven surge dynamics but also have practical implications for protecting critical infrastructure like submarine cables.

2. Typhoon Description and Storm Surge Model

2.1. Typhoon 2106 Description

This section first describes the characteristics of Typhoon In-fa (No. 2106) (for consistency, this study will hereafter refer to this event as Typhoon 2106), followed by a comprehensive overview of the storm surge model used, Delft3D. Detailed explanations of experimental design and data sources are also provided. Typhoon 2106 originated in the western Pacific on 18 July 2021, at 2:00 AM (Beijing Time, UTC + 8) at 132.5° E, 22.2° N. (All subsequent times are reported in (Beijing Time, UTC + 8) unless otherwise stated.) By 5:00 PM on 21 July, the typhoon had crossed the 24-hour warning line, approaching the Zhoushan Islands. Typhoon 2106 ultimately made landfall in Zhoushan City, Zhejiang Province, China at approximately 9:50 AM on July 26 (as shown in Figure 1), before moving northward and dissipating in the Bohai Bay [18]. According to the China Meteorological Administration's final path report, Typhoon 2106 struck Zhoushan with a wind speed of 38 m/s and an atmospheric pressure of 968 hPa, marking it as one of the most intense typhoons in the region since 1949. This study selected typhoon 2106 as a critical case for storm surge modeling due to its extreme intensity, with numerical simulations conducted to capture surge dynamics under the specific wind field conditions of this landmark event.

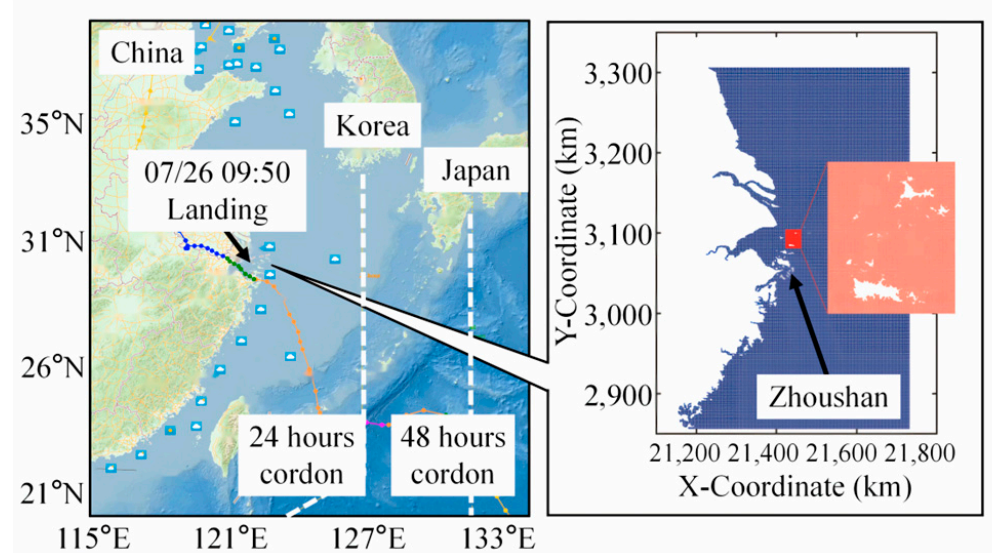


Figure 1. Typhoon 2106 track and storm surge model area.

In September 2019, the Yellow River Hydrological Survey and Mapping Bureau conducted detailed hydrological surveys for the Zhoushan Daqu-Shengsi 110 kV transmission line project, encompassing seven core parameters: tidal levels, tidal currents (velocity and direction), sediment concentration, suspended sediment, seabed composition, basic meteorological conditions, and sea state. Based on these measurements, a comprehensive statistical analysis of hydrological data was performed (Table 1; Figure 2). Subsequent observations in July 2021 spanned approximately 12 observation periods, conducted mainly on Sundays, focusing on tidal levels at two fixed stations on Qushan and Sijiao Islands.

Each observation period lasted for 24 h, starting from midnight, to capture a full tidal cycle and ensure consistency in data collection. The observations were conducted weekly, providing a regular and systematic dataset for tidal dynamics analysis. Between July 14 and 24, additional measurements during spring and neap tides were recorded at four stations (S1, S2, S3, and S4), providing key insights into tidal dynamics in the region.

Table 1. List of hydrographic stations in the test area.

Station	Physical Observatory (WSG-84 Coordinates)		Depth
	B	L	
S1	30°40′07.76″ N	122°19′51.87″ E	18.7 m
S2	30°38′48.60″ N	122°26′21.40″ E	58.3 m
S3	30°33′48.23″ N	122°18′16.29″ E	19.5 m
S4	30°31′59.36″ N	122°24′45.12″ E	29.2 m
Temporary tide stations	30°36′30.32″ N	122°23′19.17″ E	/

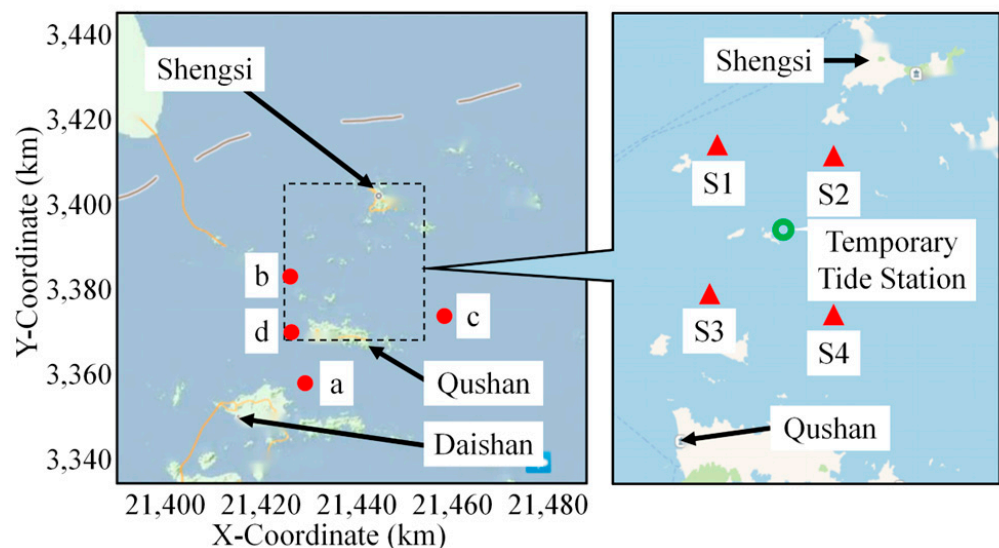


Figure 2. A schematic diagram of the location of the characterization area and the deployment of the actual station.

The cable scour protection requires special attention, so four characteristic sea areas near the Daishan Island are selected: (a) strait, (b) bypassing area, (c) open sea area, and (d) sheltered area, as shown in Figure 2.

2.2. Storm Surge Model

Delft3D is widely recognized as a robust tool for storm surge investigations, particularly in complex coastal regions. Its application in simulating Typhoon 2106 conditions leverages its strength in resolving flow dynamics through depth-averaged equations and nested grid systems [19,20]. By capturing the distribution and variation in flow velocities with precision, Delft3D enables a clearer understanding of the interactions between typhoon-driven wind fields and storm surge coupling.

This study implemented a two-level nested grid approach: a large-scale offshore grid (26.28°–34.41° N, 119.44°–125.52° E) with a 1500 m resolution and 166,069 grid points, and a high-resolution small-scale grid (30.38°–30.80° N, 122.20°–122.60° E) with a 100 m resolution and 170,564 grid points (Figure 1). Using a cold start method, initial water levels and flow velocities were set to zero. At the open ocean boundary, 13 primary tidal components were modeled with harmonic constants for tidal accuracy. Eddy viscosity

coefficients were set to $80 \text{ m}^2/\text{s}$ for the offshore model and $10 \text{ m}^2/\text{s}$ for nearshore conditions, while Manning roughness coefficients varied between 0.012 and $0.020 \text{ mm}^{-1}/^3\cdot\text{s}$ to reflect topographical differences.

For bathymetry, the GEBCO global dataset provided $30'' \times 30''$ resolution depth data ($\sim 200 \text{ m}$), interpolated onto a detailed nautical chart. Nearshore, dynamic boundary dry-wet grid techniques with a 0.02 m threshold were applied, omitting grid points with total depths below this threshold from calculations as dry grid points.

2.2.1. Governing Differential Equation

The mathematical model for typhoon-induced storm surges is based on the Boussinesq assumption and the incompressible fluid continuity equations under the hydrostatic pressure assumption for the Navier–Stokes (N-S) equations [21]:

$$\frac{\partial \zeta}{\partial t} + \frac{1}{\sqrt{G_{yy}}\sqrt{G_{xx}}} \frac{\partial((d + \zeta)U\sqrt{G_{yy}})}{\partial x} + \frac{1}{\sqrt{G_{yy}}\sqrt{G_{xx}}} \frac{\partial((d + \zeta)V\sqrt{G_{xx}})}{\partial y} = 0 \quad (1)$$

The momentum equations in the x - and y -directions are as follows:

$$\begin{aligned} \frac{\partial u}{\partial t} + \frac{u}{\sqrt{G_{xx}}} \frac{\partial u}{\partial x} + \frac{v}{\sqrt{G_{yy}}} \frac{\partial u}{\partial y} + \frac{w}{d + \zeta} \frac{\partial u}{\partial \sigma} = & \frac{v^2}{\sqrt{G_{yy}}\sqrt{G_{xx}}} \frac{\partial \sqrt{G_{yy}}}{\partial x} - \frac{uv}{\sqrt{G_{yy}}\sqrt{G_{xx}}} \frac{\partial \sqrt{G_{xx}}}{\partial y} \\ & + fv - \frac{1}{\rho_0 \sqrt{G_{xx}}} P_x + F_x + \frac{1}{(d + \zeta)^2} \frac{\partial}{\partial \sigma} \left(\nu_V \frac{\partial v}{\partial \sigma} \right) + M_x \end{aligned} \quad (2)$$

σ represents the vertical coordinate transformation factor, which is dimensionless and ranges between 0 and 1, commonly used in sigma-coordinate systems for numerical ocean modeling. Specifically, σ is defined as $\sigma = \frac{z - \eta}{H + \eta}$, where z is the vertical position (positive upward), To clarify, η represents the free surface elevation relative to the reference water level (mean sea level), and H is the still water depth (total water depth below the reference level). This nondimensionalization ensures that $\sigma = 0$ at the reference water level, $\sigma = -1$ at the seabed, and $\sigma = 1$ at the free surface. The use of this transformation facilitates accurate representation of the free surface and bottom boundary layers under varying bathymetric conditions, as noted in previous studies on sigma-coordinate systems [22,23].

This approach ensures accurate representation of free surface and bottom boundary layers under varying water depths [20,24,25].

$$\begin{aligned} \frac{\partial v}{\partial t} + \frac{u}{\sqrt{G_{xx}}} \frac{\partial v}{\partial x} + \frac{v}{\sqrt{G_{yy}}} \frac{\partial v}{\partial y} + \frac{w}{d + \zeta} \frac{\partial v}{\partial \sigma} = & \frac{uv}{\sqrt{G_{yy}}\sqrt{G_{xx}}} \frac{\partial \sqrt{G_{yy}}}{\partial x} - \frac{u^2}{\sqrt{G_{yy}}\sqrt{G_{xx}}} \frac{\partial \sqrt{G_{xx}}}{\partial y} \\ & - fu - \frac{1}{\rho_0 \sqrt{G_{yy}}} P_y + F_y + \frac{1}{(d + \zeta)^2} \frac{\partial}{\partial \sigma} \left(\nu_V \frac{\partial v}{\partial \sigma} \right) + M_y \end{aligned} \quad (3)$$

where ζ on the other hand, represents the total water depth, which includes the reference depth h and the free surface elevation η . The term ζ in Equation (3) explicitly includes both the reference water depth h and the dynamic free surface elevation η , as $\zeta = h + \eta$. This distinction allows us to decouple the surface elevation dynamics (driven by atmospheric and tidal forces) from the total depth variation. d is the reference water depth; $\sqrt{G_{xx}}$ and $\sqrt{G_{yy}}$ are the coordinate transformation factors; u , v , and w represent the velocities in the x , y , and z directions, respectively; ρ_0 is the density of seawater; g is the gravitational acceleration; U and V are the vertically averaged velocities in the x - and y -directions, respectively; ν_V is the vertical eddy viscosity coefficient; P_ζ and P_η represent the atmospheric pressure gradients in the x - and y -directions; f is the Coriolis force coefficient; F_x and F_y are the Reynolds stress terms; M_x and M_y are momentum source/sink terms. Equations (1)–(3) provide the foundational framework based on the Boussinesq and incompressible flow assumptions, elucidating the physical mechanisms underlying nonlinear coupling effects.

Although these equations are not directly solved in the numerical model, they are essential for interpreting the nonlinear interactions in storm surge dynamics.

Given that most existing research focuses on the surge effect of typhoons, this study uses the surge effect as a reference to analyze the nonlinear interaction of flow velocity. In the Cartesian coordinate system, the Delft3D model employs depth-integrated two-dimensional continuity and momentum equations, expressed as follows. To account for irregular bathymetry and the curvature of the Earth in coastal regions, coordinate transformation factors (a_x and a_y) are introduced to adapt the equations for curved grid systems, enabling the model to simulate hydrodynamic behavior more accurately in complex geometries [19,26]. These factors are particularly important in sigma-coordinate systems where the vertical coordinate is scaled relative to the free surface and bottom boundaries. By transforming the vertical coordinate into a dimensionless scale (σ) ranging from 0 (at the bottom) to 1 (at the free surface), the model ensures consistent resolution across varying depths. This transformation allows the governing equations to better capture flow characteristics in regions with complex bathymetry, such as steep slopes or shallow coastal zones [24,25]. Equations (4)–(6) are the governing equations implemented in the Delft3D model, which underpin the numerical simulations conducted in this study.

$$\frac{\partial H}{\partial t} + \frac{\partial}{\partial x}(UH) + \frac{\partial}{\partial y}(VH) = 0 \quad (4)$$

$$\frac{\partial U}{\partial t} + U \frac{\partial}{\partial x} U + V \frac{\partial}{\partial y} U - fv = -g \frac{\partial \left[\zeta + \frac{P_s}{g\rho_0} \right]}{\partial x} + \frac{\tau_{sx} - \tau_{bx}}{H\rho_0} - \frac{D_x}{H} \quad (5)$$

$$\frac{\partial U}{\partial t} + U \frac{\partial}{\partial x} V + V \frac{\partial}{\partial y} V - fU = -g \frac{\partial \left[\zeta + \frac{P_s}{g\rho_0} \right]}{\partial y} + \frac{\tau_{sy} - \tau_{by}}{H\rho_0} - \frac{D_y}{H} \quad (6)$$

Here, $H = h + \zeta$ is the total water depth where h is the reference water depth. τ_{sx} and τ_{sy} are the surface wind stress components in the x - and y -directions, while τ_{bx} and τ_{by} are the bottom friction forces in the x - and y -directions. D represents the horizontal momentum dissipation term. The wind stress is given by:

$$\vec{\tau}_s = \rho_a C_d \vec{W} \sqrt{W_x^2 + W_y^2} \quad (7)$$

where ρ_a is the air density, taken as 1.205 kg/m^3 at 20°C . W_x and W_y are the wind velocity components in the x and y directions, respectively, and C_d is the drag coefficient of the wind stress. Typically, it can be expressed as [26]:

$$C_d = 7.5 \times 10^{-4} + 6.7 \times 10^{-4} \times \sqrt{W_x^2 + W_y^2} \quad (8)$$

C_d generally depends on the Reynolds number and surface roughness. For relatively smooth surfaces, C_d is usually taken as 0.002, while under typhoon conditions with rougher surfaces, it may be as high as 0.003. The bottom friction force is given by [27]:

$$\tau_{bx} = \rho_0 C_f U \frac{\sqrt{U^2 + V^2}}{H}, \quad \tau_{by} = \rho_0 C_f V \frac{\sqrt{U^2 + V^2}}{H} \quad (9)$$

The bottom friction coefficient C_f is calculated as $C_f = \frac{g}{C^2}$, where C is the Chezy coefficient. In this study, C is selected between 55 and 65, with 60 being used for the calculations.

Whether it is bottom friction, wind stress, or the convective term, all are calculated as vector quantities based on their x - and y -direction components. As outlined in Equations (7) and (9) and “Section 4.1.3”, these represent the calculation methods for the magnitudes

of the vector sums of the three nonlinear terms. The overall vector direction is determined by the distribution of the original x -direction components. If the x -direction component is positive, the resultant vector sum is positive, indicating an overall vector direction toward the east; conversely, if the x -direction component is negative, the resultant vector sum is negative, indicating an overall vector direction toward the west.

2.2.2. Parameterized Typhoon

In typhoon simulations, the choice of typhoon model plays a critical role in determining the accuracy of the simulation results [28]. Different wind field models exhibit significant variations in their precision when generating wind speed and pressure fields [29]. This study employs the optimized Sun Zhilin-Zhong Sanhong typhoon model, which introduces a sea surface resistance term into the gradient wind equation for calculating wind fields. Compared with the Holland model and the Fujita-Takahashi model, this approach demonstrates a clear advantage in wind speed calculation accuracy [30]. The pressure Equation (10), wind speed Equation (11), and the maximum wind speed Equation (12) are presented as follows:

$$P(r) = P_c + (P_\infty - P_c) \exp\left(-\frac{R}{r}\right) \quad (10)$$

$$V_g(r) = \sqrt{\frac{(P_\infty - P_c)R}{\rho_a r} \exp\left[-\left(\frac{R}{r}\right)\right] + \frac{(f + 3\nu/4\delta_a^2)^2 r^2}{4}} - \frac{(f + 3\nu/4\delta_a^2)r}{2} \quad (11)$$

$$V_{max} = \sqrt{\frac{(P_\infty - P_c)}{\rho_a e} + \frac{(f + 3\nu/4\delta_a^2)^2 R^2}{4}} \frac{(f + 3\nu/4\delta_a^2)R}{2} \quad (12)$$

where $P(r)$ represents the pressure at a distance r from the typhoon center; P_c is the central pressure of the typhoon, which can be obtained from the China Meteorological Administration (CMA) best-track dataset; P_∞ is the ambient pressure at the outer edge of the typhoon, typically set to 1013 hPa; R denotes the radius of maximum wind speed (RMW) of the typhoon; $V_g(r)$ represents the gradient wind speed at a distance r from the typhoon center, and f is the Coriolis force, calculated as $f = 2\omega \sin\varphi$, where ω is the Earth's angular velocity of rotation, with a value of $7.292 \times 10^{-5} \text{ s}^{-1}$, and φ is the latitude. The dynamic viscosity of air, ν , is given as $15.8 \times 10^{-6} \text{ m}^2/\text{s}$. By substituting $r = R$ into Equation (11), the maximum gradient wind speed, V_{max} , can be derived, representing the maximum wind speed of the typhoon. δ_a is the thickness of the sea surface boundary layer, typically on the order of 10^{-1} m , and can be adjusted as needed during calculations.

2.2.3. Astronomical Tide Verification

To minimize the impact of open boundary conditions on the simulation results when using Delft3D, a $400 \times 600 \text{ km}$ open boundary area was selected. Only tidal boundary conditions and constants were applied at the open boundary to obtain the simulation results for pure astronomical tide levels. As mentioned earlier, in September 2019, tidal level data from a temporary tidal station and flow velocity and direction data from four current measurement points (S1, S2, S3, and S4) were collected. The measured data were used to validate the accuracy of the astronomical tide model. The validation period for spring tide was from 10:00 PM on the 14th to 11:00 PM on the 15th, and for neap tide, it was from 2:00 PM on the 23rd to 3:00 PM on the 24th. As shown in Figure 3, 91.44% of the simulated tidal level data points had an error within 10 cm compared with the measured data. The validation results for flow velocity and direction, as seen in Figures 3 and 4, show that 80.3% of the flow velocity data points had a relative error of less than 10%, indicating a high degree of accuracy in the astronomical tide simulation.

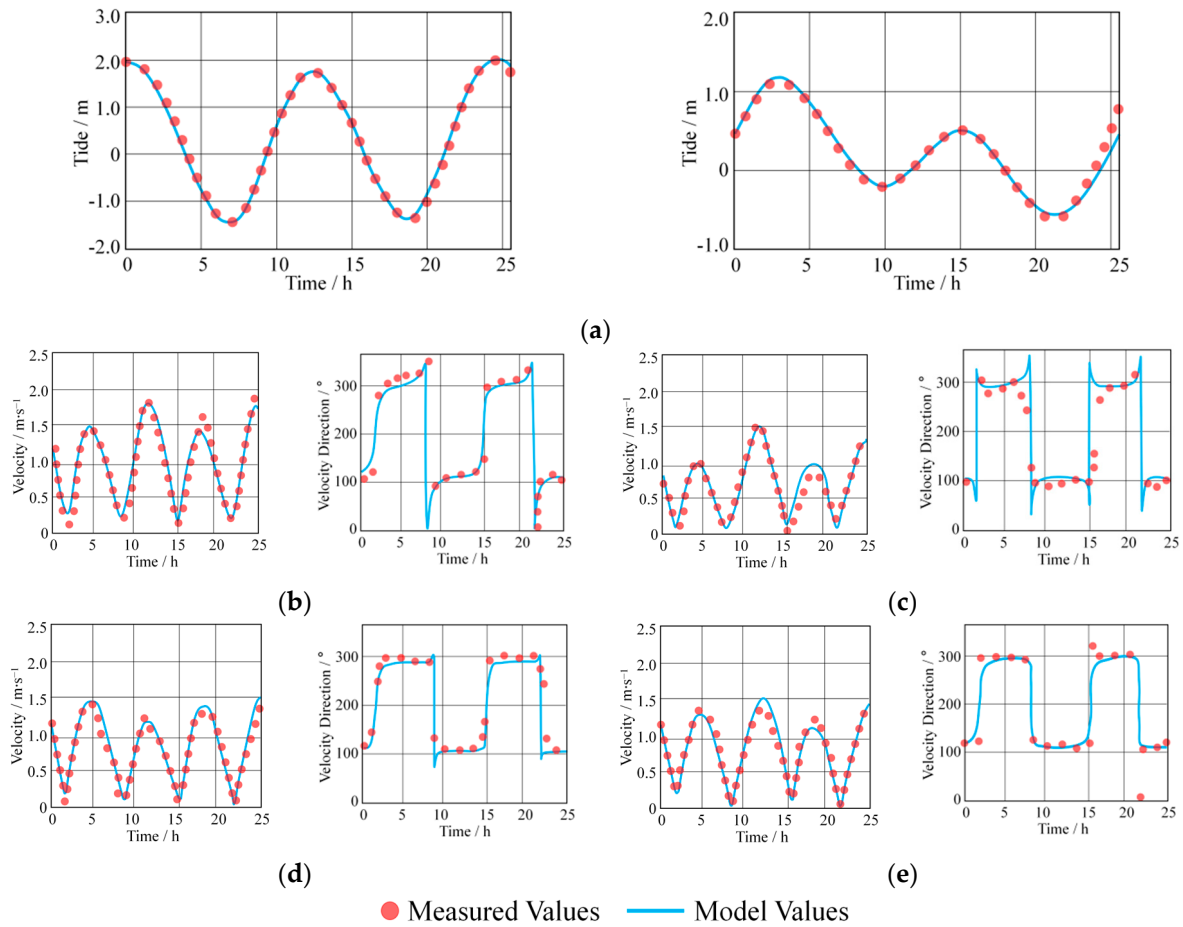


Figure 3. Verification of flow velocity and direction during the high tide period. (a) Temporary sea-surface station astronomical tide verification; (b) S1 site; (c) S2 site; (d) S3 site; (e) S4 site.

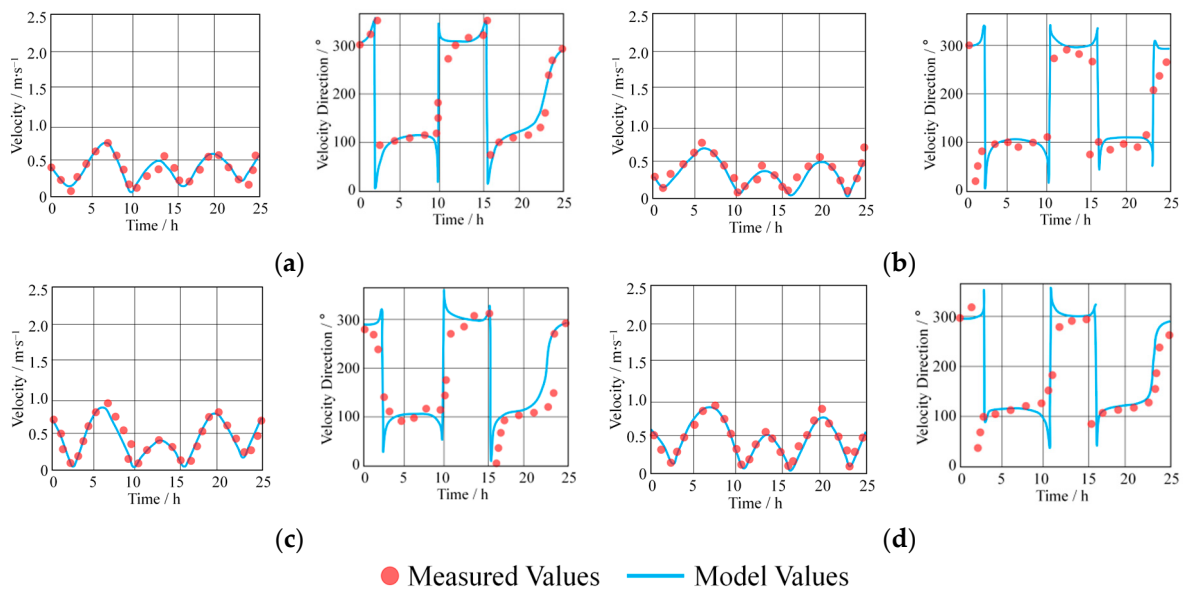


Figure 4. Verification of flow velocity and direction during the neap tide period. (a) S1 site; (b) S2 site; (c) S3 site; (d) S4 site.

2.2.4. Typhoon Storm Surge Verification

Following a similar validation approach as used for astronomical tides, storm surge validation in Delft3D employed the same large open boundary area. Using path data

from Typhoon 2106, boundary conditions for air pressure and wind fields were established to isolate sea-surface levels driven solely by storm surge effects. Numerical simulations incorporating both astronomical tides and Typhoon 2106 as driving forces spanned from 01:00 on 21 July to 00:00 on 27 July 2021. The model's predictive accuracy was assessed using the technical score index S , ranging from 0 to 1, with values closer to 1 indicating greater accuracy. Additionally, the coefficient of determination (R^2) quantified the correlation between predicted and observed data, with values closer to 1 indicating stronger predictive alignment. Both S and R^2 metrics underpinned the model's validation, ensuring consistency between simulated and actual storm surge-induced water levels [26,31].

$$S = 1 - \frac{\sum_{i=1}^N (s_i - o_i)^2}{\sum_{i=1}^N (|s_i - \bar{o}| + |o_i - \bar{o}|)^2} \quad (13)$$

$$R^2 = 1 - \frac{\sum_{i=1}^N (s_i - o_i)^2}{\sum_{i=1}^N (s_i - \bar{s}_i)^2} \quad (14)$$

where o_i and s_i represent the observed and simulated values at time point i , respectively, while $\bar{o}_i$ and $\bar{s}_i$ denote the averages of the observed and simulated values. During the typhoon, this study validated the storm surge tidal levels using measured sea-surface data from two fixed hydrological stations, Qushan and Sijiao. The validation results are shown in Figure 5a,b. At the Qushan station, 80.2% of the data points had an absolute error of less than 30 cm, with an average error of 17 cm. During the simulations, oscillations with a periodicity of approximately 12 h were observed, which might correspond to the M2 tidal component or twice the inertial period. To ensure these oscillations were not artifacts of insufficient spin-up time, we extended the spin-up period to 72 h, as recommended by Prandle [7] and Gopalakrishnan [32]. Additional sensitivity tests confirmed that these oscillations are inherent to the coupled tide-typhoon forcing and do not arise from transient effects or numerical instabilities. Specifically, the M2 tidal component's influence was validated using the harmonic analysis method described in Pawlowicz [33]. The technical score S was 0.991, and the coefficient of determination R^2 was 0.96. At the Sijiao station, 86.3% of the data points had an absolute error of less than 30 cm, with an average error of 13 cm. The technical score S was 0.99, and the R^2 value was 0.97. Overall, the Sun Zhilin-Zhong Sanhong storm surge model performed well in simulating storm surge sea-surface levels.

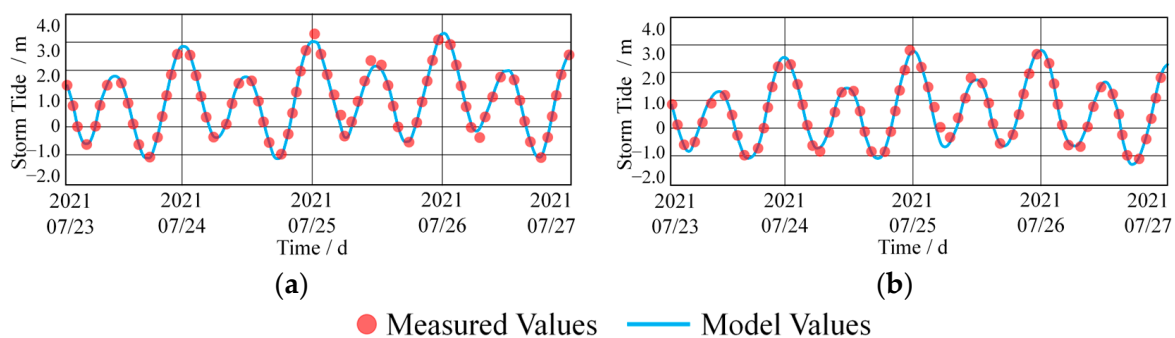


Figure 5. Verification of sea surface elevation during Typhoon 2106. (a) Qushan Station; (b) Sijiao Station.

2.3. Regression Model

The methods employed in this study include multiple regression models to analyze nonlinear interaction effects of storm surge flow velocities. These methods focus on identifying the influence of bottom friction, wind stress, and the convective term on the nonlinear coupling proportion (NCP) ($NCR = \left(\frac{U_w + U_A - U_c}{U_c} \right) \times 100\%$, NCP quantifies the degree of

nonlinear interaction in flow velocity where values closer to 0 indicate higher linearity and larger values reflect stronger nonlinear coupling effects.) The analysis framework includes linear regression, ridge regression, lasso regression, elastic net regression, decision tree regression, random forest regression, and support vector regression (SVR), with each method applied to four characteristic regions.

Building on existing studies of surge-related hazards, this research investigates how three nonlinear factors—bottom friction, wind stress, and the convective term—affect the interaction of flow velocity in typhoon storm surges. While previous studies have largely focused on qualitative analysis, the quantitative relationship between these nonlinear factors and their contributions to flow velocity dynamics remains underexplored. Regression analysis provides a robust framework for examining interdependencies among multidimensional physical factors and their collective impact on response variables [34], with options ranging from linear regression [35] to multicollinearity and regularization techniques, as well as extended nonlinear models.

Although various methods, such as clustering, neural networks, principal component analysis (PCA), and factor analysis, exist for analyzing multidimensional relationships [36], regression analysis offers particular advantages for storm surge studies. Its explanatory strength and scalability make it well-suited to examining the complex coupling effects of typhoon surge flow velocities.

In this study, seven regression models were developed: linear regression, ridge regression [37], lasso regression [38], elastic net regression [39], decision tree regression [40], random forest regression [41], and support vector regression (SVR) [42]. Each model was applied across four regions to determine the most effective approach for quantifying relationships between the three nonlinear factors—bottom friction, wind stress, and the convective term—and the degree of flow velocity nonlinear coupling. The regression analyses quantitatively attribute the observed nonlinear coupling proportions (*NCP*) to the contributions of wind stress, convection, and bottom friction. Wind stress emerged as the dominant forcing factor in the open sea and sheltered regions, contributing up to 48.28% and 39% of *NCP*, respectively, consistent with emphasizing its role in driving flow dynamics during extreme weather events. In contrast, the convective term was the primary driver in strait and bypassing regions, contributing 40–44% of *NCP*. The observed spatial heterogeneity underscores the importance of considering regional characteristics in attributing nonlinear forcing effects. Although the regression models provide robust insights into the relative contributions of these factors, their quantitative predictions should be interpreted within the context of the dataset and model assumptions.

2.3.1. Linear Regression

Linear regression assumes a direct relationship between independent variables (bottom friction, wind stress, and the convective term) and the dependent variable *NCP* (*NCP* quantifies flow velocity coupling; values near 0 indicate linearity and larger values reflect nonlinearity). Ridge regression and lasso regression introduce regularization to handle multicollinearity, while elastic net regression combines both L1 and L2 penalties to enhance model performance. This method minimizes the residual sum of squares between the observed and predicted values. However, it does not inherently handle multicollinearity among predictors, which can lead to unstable estimates and inflated variances of coefficients. In storm surge studies, linear regression serves as a baseline method to capture straightforward relationships among bottom friction, wind stress, and convective terms.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \epsilon \quad (15)$$

where y is the dependent variable representing the degree of flow velocity nonlinearity coupling, and x_1, x_2, x_3 are the independent variables representing bottom friction, wind stress, and the convective term, respectively. β_0 is the intercept, and $\beta_1, \beta_2, \beta_3$ are the coefficients for each independent variable, indicating the extent to which x influences y . ϵ is the error term accounting for any discrepancies not explained by the model.

2.3.2. Ridge Regression

Ridge regression improves upon standard ordinary least squares regression by adding an $L2$ regularization term to the loss function, which enhances model stability, especially when multicollinearity exists among predictors. The goal is to minimize both the residual sum of squares and the squared magnitude of the regression coefficients, thereby preventing overfitting, particularly in cases where predictors are highly correlated. Ridge regression uses $L2$ regularization to constrain the magnitude of regression coefficients but does not shrink any coefficients to zero. Therefore, unlike lasso regression, ridge regression cannot simplify the model by sparsifying the coefficients; instead, it retains all features [43]. Nonetheless, ridge regression has significant advantages in addressing multicollinearity and enhancing predictive stability, particularly in cases where there are many features but their correlations with the response variable are low.

This can be further illustrated through the following mathematical expression: When minimizing the loss function in ridge regression,

$$L(\beta) = \|y - X\beta\|^2 + \lambda\|\beta\|^2 \quad (16)$$

the regularization term $\lambda\|\beta\|^2$ constrains the absolute size of the coefficients but does not drive any β to zero. This contrasts with the $L1$ regularization used in lasso regression:

$$L(\beta) = \|y - X\beta\|^2 + \lambda\|\beta\|_1 \quad (17)$$

where $L1$ regularization simplifies the model by sparsifying the coefficients (shrinking some coefficients to exactly zero). Consequently, ridge regression is more advantageous when the emphasis is on predictive accuracy rather than model simplification [40].

Ridge regression “shrinks” the regression coefficients β by adding a penalty to their magnitude, especially in the presence of highly correlated predictors. By carefully selecting λ , we can balance model complexity and predictive performance.

In ridge regression, “model complexity” primarily refers to the magnitude of the coefficients β , which reflects the model’s capacity to fit the data. Unlike lasso regression, ridge regression ($L2$ regularization) does not set coefficients β to zero but, instead, shrinks them toward smaller values. This reduces the model’s sensitivity to noise and collinearity without explicitly eliminating predictors, leading to a more stable solution [40]. While the flexibility of the model decreases as the regularization parameter λ increases, the total number of non-zero coefficients remains unchanged, implying that ridge regression controls complexity by regularizing the size of the coefficients rather than performing variable selection. Thus, the balance between model variance and bias is achieved through λ as it reduces overfitting without directly reducing the dimensionality of the model.

2.3.3. Lasso Regression

Lasso regression adds an $L1$ regularization term, addressing multicollinearity while also performing feature selection. By penalizing the absolute values of the coefficients, lasso can shrink the coefficients of less important features toward zero, but never exactly to zero, thus simplifying the model. By adjusting the value of λ , the model’s sparsity and

complexity can be controlled. Unlike ridge regression, lasso regression adds a $L1$ -norm penalty to the loss function:

$$L1 = \underset{\beta}{\operatorname{argmin}} \|y - X\beta\|^2 + \lambda \|\beta\|^2 \quad (18)$$

The $L1$ -norm penalty enables feature selection by shrinking some coefficients exactly to zero, thus simplifying the model. This is particularly useful when dealing with high-dimensional data or when only a subset of predictors significantly affects the dependent variable.

Lasso regression is particularly advantageous in identifying key factors, such as the dominant influence of wind stress or convective terms on NCP in different regions. However, it may struggle in cases where predictors exhibit high multicollinearity as the selection of one variable may exclude others with similar effects.

2.3.4. Elastic Net Regression

Elastic net regression combines the strengths of both lasso and ridge regression by incorporating both $L1$ and $L2$ regularization terms into the loss function. This approach allows for effective feature selection, like lasso, while also handling multicollinearity, like ridge regression.

$$\hat{\beta} = \underset{\beta}{\operatorname{argmin}} \|y - X\beta\|^2 + a\lambda \|\beta\|_1 + (1 - a)\lambda \|\beta\|^2 \quad (19)$$

$\|\beta\|_1 = \sum_{j=1}^p |\beta_j|$ represents the $L1$ norm, $\|\beta\|_2^2 = \sum_{j=1}^p |\beta_j|^2$ represents the squared $L2$ norm, and $\lambda_1, \lambda_2 \geq 0$ act as hyperparameters controlling the strength of the $L1$ (lasso) and $L2$ (ridge) regularization, respectively. a determines the relative weight of $L1$ and $L2$ penalties. When $a = 0.5$, elastic net balances the strengths of ridge and lasso, making it ideal for datasets with correlated predictors and sparse signals.

In elastic net regression, the parameters λ_1 and λ_2 represent the relative weights of the $L1$ and $L2$ regularization terms, respectively. These parameters are typically chosen through cross-validation to optimize the model's predictive performance while balancing sparsity and multicollinearity. Specifically, λ_1 controls the extent of feature selection, shrinking some coefficients to exactly zero, while λ_2 adjusts the degree of coefficient regularization to mitigate multicollinearity. The interplay between these parameters enables elastic net to handle datasets with high-dimensional features and correlated predictors effectively. This flexibility is particularly beneficial for disentangling the combined effects of wind stress and convective terms in this study where predictors exhibit moderate collinearity. Common approaches for determining λ_1 and λ_2 include grid search or randomized search strategies [43].

Elastic net is particularly effective in this study for disentangling the combined effects of wind stress and convective terms, which often exhibit moderate multicollinearity. Its flexibility allows it to simultaneously handle sparsity and model complexity.

2.3.5. Decision Tree Regression

Decision tree regression is a nonparametric approach capable of capturing complex nonlinear patterns in data by iteratively partitioning the dataset into smaller subsets. This process is guided by minimizing the variance of the target variable within each subset, thereby identifying patterns that are difficult to capture using linear models. Random forest regression builds upon this by using ensemble learning to improve stability and prediction accuracy, while support vector regression (SVR), employing a Gaussian radial

basis function kernel, provides robust performance for small sample sizes by balancing model complexity and fitting accuracy.

In decision tree regression, the relationship between input variables (X) and the output variable (Y) is established by recursively splitting the data into subsets, with each split determined by a specific feature and threshold value that minimizes a predefined criterion, such as the mean squared error (MSE). For a given node m , which represents a subset of the dataset, the MSE is defined as:

$$MSE_m = \frac{1}{N_m} \sum_{i \in N_m} (y_i - \bar{y}_m)^2 \quad (20)$$

Here, N_m denotes the number of samples in node; m, y_i represents the target value for sample; and i , and $\bar{y}_m$ is the average value of the target variable within node m . However, Equation (20) only quantifies the variance within a single node. The regression relationship between the input variables and the target variable is defined by minimizing the weighted MSE of the child nodes resulting from each split. The splitting criterion can be expressed as $\min_{j,t} [\frac{n_{left}}{n_m} \cdot MSE_{left(t)} + \frac{n_{right}}{n_m} \cdot MSE_{right(t)}]$. Here, j is index of the feature used for splitting; t is threshold value for splitting; n_{left} and n_{right} are the number of samples in the left and right child nodes, respectively; MSE_{left} and MSE_{right} are mean squared errors for the left and right child nodes.

The splitting process continues recursively until one of the stopping criteria is met: (1) the tree reaches a predefined maximum depth; (2) the number of samples in a node falls below a minimum threshold; or (3) further splitting does not significantly reduce the MSE.

Each “node” in a decision tree serves a specific purpose: intermediate nodes are decision points where the dataset is split based on a feature and threshold; leaf nodes are terminal nodes that represent the final predictions of the model, and the prediction for a leaf node is the average of the target values ($\bar{y}_m$) for all samples within the node.

The depth of a node refers to its hierarchical level in the tree. Deeper nodes typically correspond to smaller, more specific subsets of the data, leading to more refined predictions. For a new observation X_{new} , the tree traverses from the root node through the intermediate nodes based on the splitting rules until it reaches a leaf node. The output prediction is the mean target value of the leaf node: $\hat{y} = \bar{y}_m$, where $X_{new} \in leaf\ m$.

Decision tree regression provides a flexible and interpretable approach for modeling nonlinear relationships between input variables and a target variable. The splitting process ensures that each resulting subset minimizes the internal variance, thereby capturing the underlying structure of the data.

2.3.6. Random Forest Regression

Random forest regression generally improves prediction performance and stability by constructing multiple decision trees and averaging their results. However, as observed in Figure 13, its performance in regions such as “Strait” and “Sheltered Area” exhibits limitations, likely due to a combination of feature sparsity and high noise in these specific datasets. This behavior highlights the sensitivity of the model to data quality and distribution in localized regions, which warrants further investigation. Given B trees, the prediction result for a new observation is computed as:

$$\hat{y} = \frac{1}{B} \sum_{b=1}^B \hat{y}^{(b)} \quad (21)$$

where $\hat{y}^{(b)}$ is the prediction from the b -th decision tree.

2.3.7. Support Vector Regression

Support vector regression (SVR) is a method for analyzing nonlinear relationships in datasets, particularly when the sample size is small. It seeks to find an optimal balance between model complexity and prediction accuracy. The approach involves minimizing a loss function while maintaining the model's capacity to generalize beyond the training data.

Loss Function and Optimization: The loss function in SVR is designed to minimize the prediction error while incorporating a margin of tolerance ϵ . For a given dataset $\{(x_i, y_i)\}_{i=1}^N$, the optimization objective is expressed as:

$$L(w, b) = \frac{1}{2} \|w\|^2 + C \sum_{i=1}^n (\xi_i + \xi_i^*) \quad (22)$$

where w is the weight vector, b is the bias term, ξ_i and ξ_i^* are slack variables accounting for deviations beyond the ϵ -margin, and C is a regularization parameter controlling the trade-off between the model complexity and tolerance to errors.

The constraints for the optimization problem are given as:

$$\begin{cases} y_i - (w \cdot \phi(x_i) + b) \leq \epsilon + \xi_i \\ (w \cdot \phi(x_i) + b) - y_i \leq \epsilon + \xi_i^* \\ \xi_i, \xi_i^* \geq 0, \forall i \end{cases} \quad (23)$$

where $\phi(x_i)$ is a nonlinear mapping of the input vector x_i into a higher-dimensional feature space.

Dual Formulation and Solution: Using the Lagrangian dual formulation, the problem can be re-expressed in terms of Lagrange multipliers a_i and a_i^* . The dual objective function becomes:

$$f(a_i, a_i^*) = \sum_{i=1}^I (a_i - a_i^*) y_i - \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N (a_i - a_i^*) (a_j - a_j^*) k(x_i, x_j) \quad (24)$$

where $k(x_i, x_j) = \phi(x_i) \cdot \phi(x_j)$ is the kernel function. The Lagrange multipliers must satisfy the following constraints: $\sum_{i=1}^I (a_i - a_i^*) = 0, 0 \leq a_i, a_i^* \leq C, \forall i$.

Prediction and Kernel Function: After solving for a_i and a_i^* , the predicted output for a new input x is calculated as:

$$k(x_i, x_j) = \exp\left(-\frac{\|x_i - x_j\|^2}{2\sigma^2}\right) \quad (25)$$

where σ is the kernel width parameter.

Relationship Between Equations (24) and Optimization: Equation (24) represents the dual form of the optimization problem where the solution relies on maximizing the function $f(a_i, a_i^*)$ with respect to the Lagrange multipliers. This formulation directly influences the coefficients w and b in Equation (25). Thus, minimizing the primal loss function (22) and solving the dual problem (24) are mathematically equivalent processes, ensuring consistency between the optimization objectives.

By integrating these formulations, SVR effectively balances prediction accuracy and model robustness, making it well-suited for analyzing nonlinear coupling effects in storm surge flow velocities.

3. Methods

3.1. Flow Velocity Simulation Under Different Conditions

This section simulates the flow velocity fields resulting purely from astronomical tides and storm surges, exploring the variations in vertically averaged flow velocity data in the

study area during Typhoon 2106. These simulations provide the preliminary conditions for analyzing the nonlinear interactions between astronomical tides and storm surges. Typhoon 2106 made landfall on Zhoushan Island at approximately 9:50 a.m. on July 26, having reached the boundary of the study area around 4:00 PM on July 25. Four characteristic locations near Daishan Island were selected for the study: the strait (a), bypassing area (b), open sea area (c), and sheltered area (d), as shown in Figure 2.

The simulation covers a 72 h period from 00:00 on July 24 to 00:00 on July 27. Since the model uses a cold start, the simulation data begin at 00:00 on July 24. The Delft3D model outputs primarily include ocean currents (flow velocity) represented by x - and y -directional components. Additionally, atmospheric pressure and wind speed are critical input parameters that influence the simulation. These are provided in the Supplementary Materials (Figure S1a,b). To obtain the true simulation values, the vector sum of these components is calculated, as shown in Figure 6. The details of x - and y -directional flow velocity components can be found in the Supplementary Materials (Figure S1a,b).

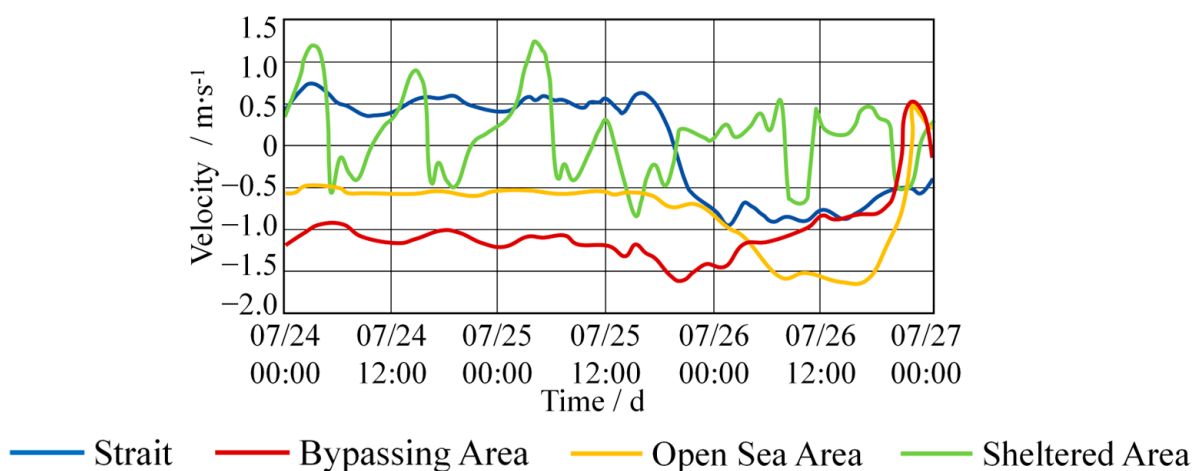


Figure 6. The distribution of flow velocity components in four characteristic regions during pure storm surge conditions. Note: Figure 6 represents the magnitude of flow velocity, derived from the vector sum of x and y components at each location, averaged over the specified time intervals.

After vector summation of the flow velocities, under the influence of the pure storm surge, the flow velocity in the strait region remained relatively stable between 0 and 0.7 m/s before 5:00 a.m. on July 25. Following the typhoon's landfall, the velocity reached its peak at -0.96 m/s around 5:00 a.m. on July 27. The term "error" refers to the difference between the simulated and measured velocities at locations (a) to (d), which are derived from hydrodynamic observations during Typhoon 2106. While the specific measurements are not graphically displayed here, they were used to validate the model outputs. This change can be attributed to the local acceleration effect caused by the strait's topography.

In the bypassing area, the flow velocity primarily moved westward, exhibiting the widest fluctuation range, from -1.62 to 0.45 m/s. This indicates that the region was significantly affected by the typhoon. The relatively open waters and shallow depth of this area, with an average depth of 14.7 m, result in trends similar to those observed in the open sea region. In the open sea area, the flow velocity predominantly moved westward, but with smaller fluctuations. The root-mean-square error (RMSE) was 0.90 , and the mean absolute error (MAE) was 0.80 , indicating that the typhoon had a relatively balanced impact on this region, leading to more stable changes in flow velocity. The sheltered area displayed the most distinctive vector flow velocity characteristics. Before July 25, the flow velocity exhibited a quasi-periodic pattern, which may be associated with tidal effects due to its alignment with the diurnal tidal cycle in the region. This periodicity is approximately

twice the inertial frequency, potentially generated by the superposition of tidal forces and localized wind-driven oscillations, as suggested by observations in shallow, sheltered regions during typhoon events [44]. After July 25, the peak velocity ranged between 0.5 and 1.0 m/s, while still maintaining some quasi-periodic features.

Under the influence of purely astronomical tides, the flow velocity characteristics in the four regions (Figure 7) exhibited distinct differences. The strait, acting as a “corridor” of seawater with an average depth of 15.7 m, experienced significant fluctuations in flow velocity, ranging from -2.02 to 1.99 m/s. This was the largest amplitude observed among the four regions, reflecting the amplifying effect of the strait’s topography. In contrast, the bypassing area and the open sea region displayed relatively steady flow velocity changes, with extremes only ranging from -1.67 to 0.81 m/s. The lower fluctuation in flow velocity in these areas indicates more stable tidal conditions. The presence of islands in the bypassing area likely dampened the tidal energy, while in the open sea, the dispersal of tidal energy contributed to the more moderate flow velocity variations. The sheltered area also exhibited notable flow velocities, possibly due to the protective effect of the surrounding topography, which could induce local recirculation and surges in tidal currents.

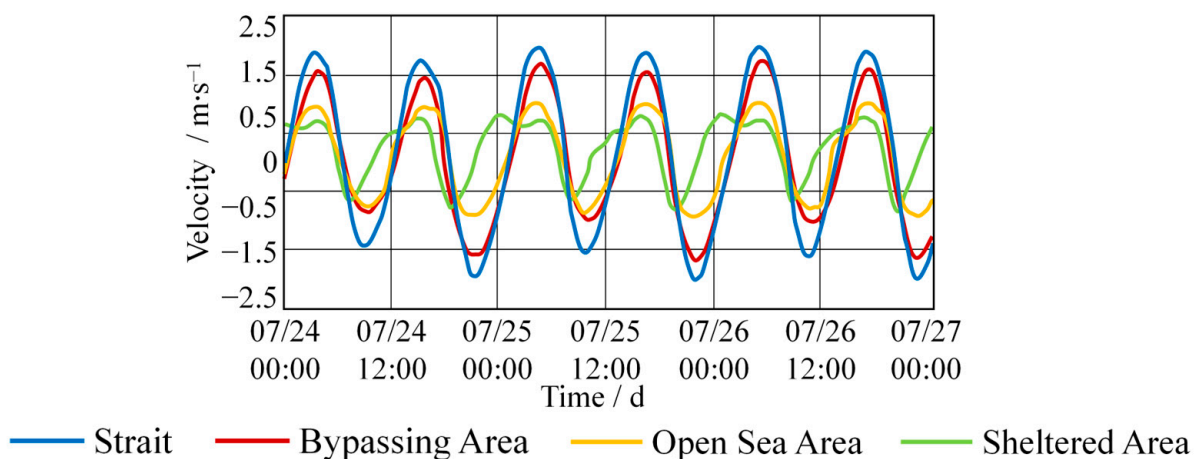


Figure 7. The distribution of vector velocities in four characteristic regions driven solely by astronomical tide forces.

3.2. Flow Velocity Field Under Coupled Effects

The previous sections analyzed the flow velocity under pure storm surge and pure astronomical tide conditions across the four characteristic regions. It was observed that the impact of the typhoon on the flow velocity field is more complex and intense, while the astronomical tide exhibited periodic, oscillatory changes. For Typhoon No. 2106, it was found that the sum of the flow velocities from the pure storm surge and pure astronomical tide does not equal the flow velocity under the combined influence of the storm surge and astronomical tide, revealing the nonlinear nature of these interactions. In this section, the flow velocity from the pure storm surge U_w is added to that from the pure astronomical tide U_A , and the result is subtracted from the flow velocity under coupled conditions U_c , yielding the nonlinear flow velocity difference ΔU , as shown in Figure 8. If the flow velocity difference ΔU approaches zero, it indicates that the storm surge and astronomical tide velocities exhibit near-linear coupling where the flow velocity system behaves as a simple additive effect of each component. However, the further ΔU deviates from zero, the more pronounced the nonlinear coupling becomes, indicating complex interactions occurring between the various flow velocities within the fluid system.

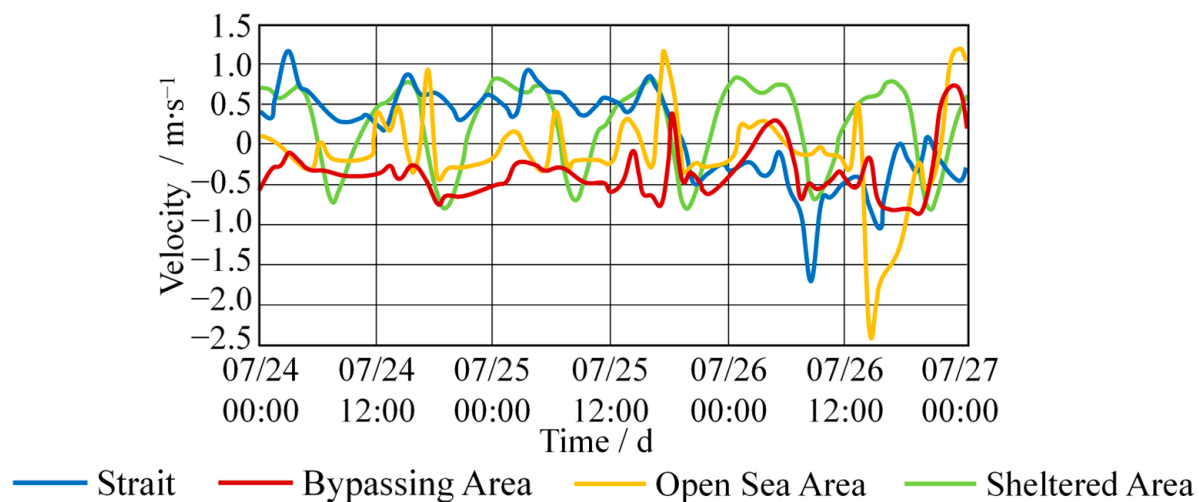


Figure 8. Nonlinear flow velocity differences (ΔU) across four characteristic regions during Typhoon 2106.

The data presented in Figures 7 and 8 depict the magnitude of velocity (speed), calculated as the vector sum of the x - and y -directional components of flow velocity. Directional information, while essential in vector representation, is omitted from the plots for clarity and to focus on the magnitude variations across the studied regions. This approach aligns with prior studies that emphasize the scalar characteristics of flow velocities in regional analysis under complex dynamic conditions. The methodology for calculating the magnitude is consistent with Equation (6) where the total velocity $U = \sqrt{u_x^2 + u_y^2}$ and where U_x and U_y represent the velocity components in the x - and y -directions, respectively.

The nonlinear flow velocity difference ΔU across the four regions exhibits significant variations. In the strait, the nonlinear flow velocity difference is the most pronounced among the four regions. Before 19:00 on July 25 (prior to the typhoon's landfall), the flow velocity difference remained relatively stable at around 0.69 m/s. When compared with the coupled flow velocity, this period can be described as the "nonlinear weakening phase" where the sum of the pure storm surge and pure astronomical tide velocities is greater than the velocity under the coupled conditions. In contrast, after 19:00 on July 25, the period can be termed the "nonlinear increasing phase" as the narrow channel causes more intense coupling between the tidal forces and wind stress. The more concentrated the wind stress, the more likely a "nonlinear increase effect" will occur. Similarly, the bypassing area shows smaller fluctuations in the flow velocity difference compared with the strait. Its wider expanse makes it more likely to exhibit the "nonlinear increasing effect" but with less intensity than in the strait. This is because a broader sea surface can accommodate larger water masses and facilitate the lateral dispersion of flow energy. The Ekman spiral theory supports this as wind stress over an expansive sea surface induces Ekman transport, creating stronger surface currents that amplify the interaction with subsurface currents. As a result, the convective term becomes more pronounced, intensifying the nonlinear coupling effect [45]. The interaction between wind-driven currents and topographical resistance in broader sea areas also contributes to the amplification of nonlinear effects [46]. In the open sea area, the flow velocity difference ΔU exhibits a negative peak of approximately -1.37 m/s around 14:00 on July 26, representing the largest deviation observed during the typhoon event. This peak reflects a pronounced nonlinear interaction likely driven by the combined effects of wind stress and the convective term, which amplifies flow divergence under specific conditions. Beyond this peak, the ΔU fluctuates within a range of 0 to 1 m/s, indicating moderate variability rather than true stability, as the flow field

responds to dynamic forcing mechanisms such as the periodic alternation of storm surge and tidal contributions. This region is the most likely to exhibit “linear coupling” among the four regions, possibly because the storm surge’s driving force is insufficient to generate a widespread nonlinear increase effect in the open sea. The sheltered area displays a distinct 12 h cycle, with the average positive and negative peak values of the flow velocity difference at 0.80 m/s and -0.77 m/s, respectively. This periodicity may be related to the region’s uniquely shallow topography, which amplifies tidal resonance and enhances periodic variations in flow velocity due to the interaction of incoming and reflected tidal waves. Studies [47,48] have shown that shallow water regions often exhibit strong resonance effects when the local water depth and topographical constraints align with the natural period of tidal forcing, leading to quasi-periodic flow velocity patterns. Overall, the sheltered area is characterized by a predominant “nonlinear weakening” effect.

4. Analysis and Discussion of Nonlinear Terms

4.1. Nonlinear Terms

The interactions driving typhoon storm surges are inherently nonlinear. A simple linear summation of the components would lead to significant errors in the simulation results. This occurs primarily because the flow directions of astronomical tides and storm surges can influence the additive effect. When the phases of the two align, the maximum flow velocity effect may be observed; conversely, when their flow directions oppose each other, they may cancel each other out. Wind fields, as the driving force behind storm surges, directly affect the development of storm surge systems. According to the depth-integrated Equations (5) and (6), bottom friction is particularly significant in shallow water regions. As the flow velocity increases, bottom friction increases quadratically, following the well-established drag laws for turbulent flows in shallow water systems [9]. Additionally, the convective term can alter both the direction and the speed of the water flow, further amplifying the system’s nonlinear coupling effects. Drawing on common studies of the surge-related hazards caused by typhoon storm surges, this paper analyzes the nonlinear coupling effects of astronomical tides and typhoons based on three major coupling terms in the equations: wind stress, bottom friction, and the convective term.

4.1.1. Bottom Friction

The interaction between wind stress and bottom friction determines the propagation speed and attenuation of storm surges. The calculation of bottom friction typically uses a modified quadratic drag law, given by the formulas as $\tau_{bx} = \rho_0 C_f U \frac{\sqrt{U^2 + V^2}}{H}$, $\tau_{by} = \rho_0 C_f V \frac{\sqrt{U^2 + V^2}}{H}$. The flow velocities can be computed using Delft3D, and the bottom friction coefficient C_f can be obtained from Equation (9). The vector bottom friction forces for the four regions are calculated under the conditions of pure storm surge, pure astronomical tide, and the coupled effects of typhoon storm surges, as shown in Figure 9.

As observed in Figure 9, the strait region clearly reflects the dominant influence of astronomical tides on bottom friction. Before the landfall of the typhoon on July 25, the bottom friction under pure astronomical tide conditions can be used as a “baseline.” During this period, the bottom friction due to astronomical tides is only slightly lower than the coupled bottom friction at extreme values, with an average difference of just 0.09×10^{-4} m/s². The bottom friction is hardly affected by the pure typhoon field. After July 25, the average difference between the extreme values of the pure astronomical tide and coupled bottom friction increases to 0.47×10^{-4} m/s². This indicates that the bottom friction in the strait is more sensitive to the influence of astronomical tides than to the typhoon field, with the narrow topography amplifying the interaction between wind stress and tidal forces.

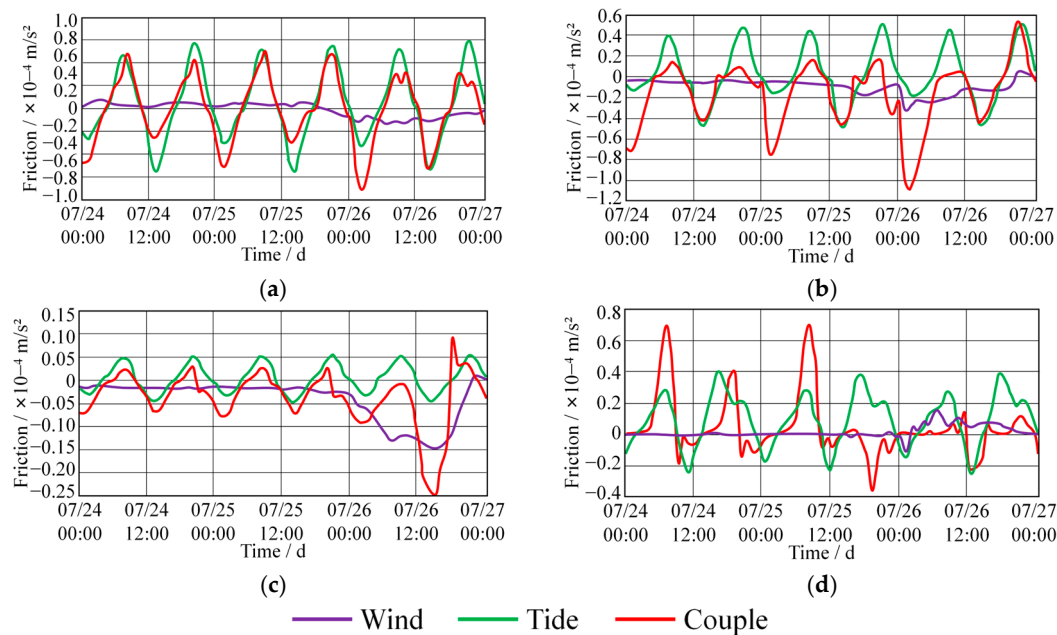


Figure 9. Temporal variations of bottom friction forces in four characteristic regions during Typhoon 2106. (a) Strait; (b) Bypassing area; (c) Open sea area; (d) Sheltered area.

In the bypassing area, the coupled bottom friction exhibits significant changes compared with the bottom friction under astronomical tides. After the typhoon's landfall, the westward extreme value difference reaches $0.69 \times 10^{-4} \text{ m/s}^2$. Although the water depth in the bypassing area is similar to that of the strait, the water surface is much wider, indicating that the sea surface width is more sensitive to the effects of the typhoon field. "Width" refers to the unobstructed surface area of water above sea level, which varies depending on the presence of islands, channels, and other topographical features.

In the open sea area, which is much broader and more than three times deeper than the other regions, all three types of bottom friction fall within a very narrow range of -0.25 to $0.09 \times 10^{-4} \text{ m/s}^2$. This represents the smallest bottom friction among the four regions. It can be inferred that compared with sea surface width, water depth has a more dominant influence on bottom friction—the deeper the water, the smaller the bottom friction.

In the sheltered area, before the typhoon's landfall on July 25, the extreme values of bottom friction under astronomical tides and the coupled conditions differed by over 34.6%. After the typhoon made landfall, the coupled bottom friction tended to stabilize. Due to the shallow depth of only 5.8 m in this area, the influence of the typhoon field on bottom friction was more pronounced, making the region more sensitive to the effects of the typhoon.

The weaker influence of bottom friction in deeper waters, as observed in this study, echoes the conclusions of Pugh [27] who noted that bottom friction diminishes with increasing water depth.

4.1.2. Wind Stress

In the coupling of typhoon storm surges, the size and extent of the storm surge are directly related to the intensity of the wind field, its duration, and the typhoon's path. The typhoon wind field exhibits uneven spatial distribution, with wind direction serving as the core driving force for storm surge propagation, while wind speed affects the height and energy of the storm surge. The region near the typhoon's center, where the maximum wind speed occurs, typically generates the largest storm surge. According

to Equation (7), the wind stress components are expressed as: $\tau_{sx} = \rho_a C_d W_x \sqrt{W_x^2 + W_y^2}$, $\tau_{sy} = \rho_a C_d W_y \sqrt{W_x^2 + W_y^2}$.

Since the pure storm surge data in this study are entirely derived from Typhoon 2106 (No. 2106) and the pure astronomical tide model does not incorporate any wind calculations while the wind component under coupled conditions still comes from the storm surge data of Typhoon 2106, this section directly compares the wind stress magnitudes across the four regions. The results are shown in the wind stress time series in Figure 10.

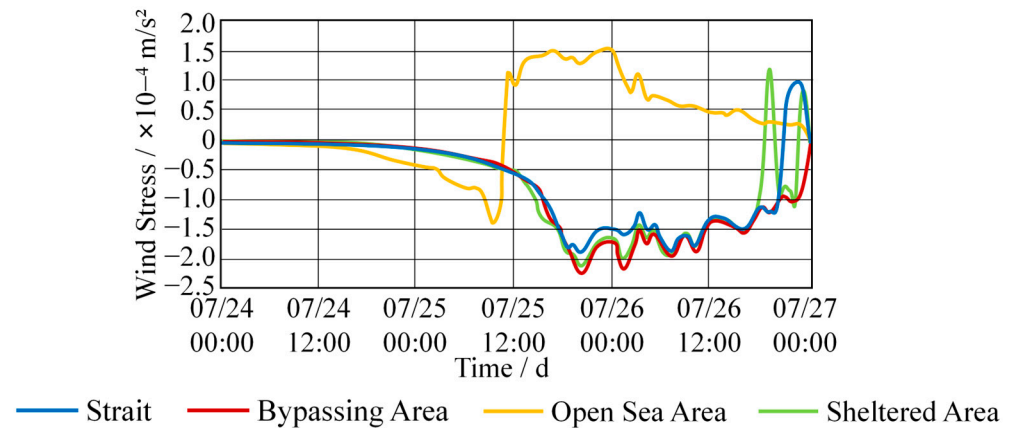


Figure 10. Temporal evolution of wind stress in four characteristic regions during Typhoon 2106.

Among the four characteristic regions, the open sea area, with no obstructions and relatively greater average water depth, is the only region where the wind stress began to shift westward around 20:00 on July 25, just before the typhoon made landfall. Throughout the event, the wind stress generally exhibited a trend opposite to the typhoon's wind direction, blowing eastward. This could be due to the strong winds directly impacting the sea surface without being blocked or guided by coastal topography. Additionally, the phenomenon of a leading front may have occurred, where the wind direction shifts before the typhoon's center arrives. In contrast, the strait, bypassing area, and sheltered area, being closer to land, displayed wind stress values that were very similar, with fluctuations ranging only between -2.26 to $1.50 \times 10^{-4} \text{ m/s}^2$. These regions were more heavily influenced by terrain, which both blocked and guided the wind, resulting in wind stress that aligned with the typhoon's wind direction as the storm approached. It is important to note that the opposite wind stress in the open sea area does not mean the wind stress was the strongest; the magnitude still depends on the absolute value of the extremes.

The dominance of wind stress in open sea regions is consistent with the findings of Large [46] who demonstrated the critical role of wind-driven forces in open ocean dynamics.

4.1.3. Convective Term

The convective term is an important term that describes the transport of mass, momentum, and energy with the movement of fluid. It usually appears in the momentum equation and is expressed as a nonlinear combination of velocity vectors. The wind field driven by typhoon storm surges increases the flow velocity, making the nonlinear effects of the convective term more pronounced, potentially leading to distortions in the flow velocity field and changes in water levels. In the two-dimensional horizontal momentum equation, the convective terms can be expressed as: $A_x = (U \cdot \nabla)U = U \frac{\partial}{\partial x} U + V \frac{\partial}{\partial y} U$ and $A_y = (V \cdot \nabla)V = U \frac{\partial}{\partial x} V + V \frac{\partial}{\partial y} V$. The convective term is uniformly represented as $(U \cdot \nabla)$ where $U = (U, V)$. In Cartesian coordinates, this expands to $U \partial / \partial x + V \partial / \partial y$, ensuring

consistency in notation across all expressions. This approach aligns with the formulation used in numerical simulations and fluid dynamics research where U and V are the respective velocity components in the x - and y -directions, and ∇ represents the gradient operator applied to scalar or vector fields.

The convective term in the strait exhibits the most dramatic variations under different conditions (Figure 11). Specifically, during the coupling of typhoon storm surges, the strait shows the largest fluctuation range among the four regions throughout the typhoon, ranging from -2.94 to $1.40 \times 10^{-4} \text{ m/s}^2$. Given the relatively shallow average depth and the concentrated water flow in this area, the increase in flow velocity directly amplifies the convective effects. The bypassing area experiences moderate fluctuations due to the interaction of flow velocity and water depth, with turbulence and shear forces shaping convective dynamics under coupled conditions. While the width of the bypassing area is relatively fixed by the presence of islands and other geographical constraints, its effective hydraulic width can vary with changes in flow velocity and topographical interactions. This implies that critical thresholds for the convective term are more closely related to local flow conditions and turbulence rather than the geometric width itself. Furthermore, greater water depth does not necessarily result in a larger convective term, as shown in similar studies of confined flows [44,45].

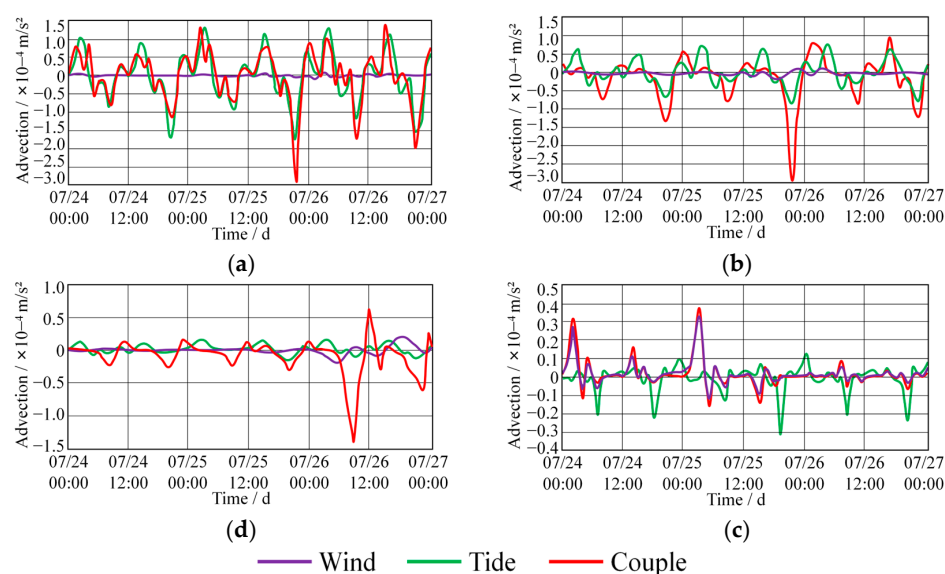


Figure 11. Temporal profiles of convective terms in four characteristic regions during Typhoon 2106. (a) Strait; (b) Bypassing area; (c) Open sea area; (d) Sheltered area.

In the open sea area, the impact under coupled conditions is relatively small. Only after the typhoon made landfall on July 26 did a notable westward convective term of $-1.37 \times 10^{-4} \text{ m/s}^2$ appear; at other times, the convective term remained almost stable around 0 m/s^2 , indicating a relatively uniform flow, with limited diffusion due to the small convective term, which may indirectly contribute to steady flow conditions in specific scenarios. Compared with the bypassing area, which is also relatively open, it can be inferred that there is a critical threshold for the sea surface width. The convective term shows a clear critical threshold for sea surface width and water depth. Specifically, the critical threshold for the sea surface width is essential in determining the convective term's contribution to flow velocity nonlinear coupling. When the sea surface width exceeds this threshold, wind stress effectively drives larger water masses, amplifying convective effects. Conversely, narrower widths may limit fluid movement and reduce the

impact of convection, as suggested by Price and Sundermeyer [45] in their study of Ekman spiral dynamics.

The convective term's significant variations in straits and bypassing zones are primarily attributed to enhanced velocity gradients caused by topographic constraints and channeling effects. While the Ekman spiral theory describes the vertical shearing of currents under steady wind forcing [49,50], it assumes no horizontal velocity gradients and does not directly account for the role of the sea surface width or local topographic effects. Increased turbulence can extend the Ekman depth by enhancing vertical mixing but does not affect Ekman transport, which remains solely dependent on wind stress and the Coriolis parameter. In contrast, the observed convective term behavior in this study is more closely related to local flow conditions, including velocity gradients induced by topography and channeling, as well as the interplay of turbulence and flow velocity changes.

4.1.4. Qualitative Analysis of Nonlinear Effects

Taking into account the three main factors influencing the “flow velocity nonlinear coupling”—wind stress, bottom friction, and the convective term—the impact on each region is shaped by its geographical location, topographical features, water depth, flow velocity, and oceanic environmental conditions. Wind stress serves as the primary driving force behind the flow direction, while bottom friction acts as the main resistance to flow velocity. The convective term comprehensively reflects the dynamic movement of water flow. The interaction of these three factors jointly determines the dynamic characteristics of the “flow velocity nonlinear coupling.” The absolute values of the extreme points of each nonlinear influencing factor indicate the magnitude of their impact in the respective regions. Naturally, real-world conditions are complex and variable, with numerous influencing factors. The extreme values after coupling can offer a simplified representation of the nonlinear characteristics in each region.

The data presented in Table 2 are derived from the Delft3D simulation results of Typhoon 2106. These extreme values were extracted by analyzing the nonlinear coupling factors—bottom friction, wind stress, and the convective term—across the four characteristic regions (strait, bypassing area, open sea, and sheltered area). Each value represents the maximum absolute magnitude recorded during the typhoon's peak period.

Table 2. Extreme value statistics of nonlinear factor coupling effects (Unit: $\times 10^{-4} \text{ m/s}^2$).

Nonlinear Term	Characteristic Region			
	Strait	Bypassing Area	Open Sea Area	Sheltered Area
Bottom friction	−0.88	−1.07	−0.25	−0.36
Wind stress	−1.89	−2.26	1.50	−2.14
Convective term	−3.21	−2.94	−1.37	0.38

The convective terms in both the strait and bypassing area are substantial, both around $3 \times 10^{-4} \text{ m/s}^2$. This can be attributed to the fact that the water depths in these two regions are quite similar. However, the bypassing area has a much wider sea surface, allowing the typhoon's driving force to have a greater impact, which is why the wind stress in the bypassing area is greater than that in the strait. The strait's effect of concentrated acceleration, combined with the strong wind driving force in the bypassing area, results in both regions having the largest contribution from the convective term among the three nonlinear factors.

In the open sea and sheltered area, the average bottom friction is only $0.31 \times 10^{-4} \text{ m/s}^2$, contributing the least among the three nonlinear factors in these regions.

However, wind stress is clearly the largest factor in the open sea, while the extremely shallow depth of the sheltered area makes wind stress the dominant factor there as well. In both regions, bottom friction, similar to the strait and bypassing area, remains the smallest among the three nonlinear factors.

4.2. Nonlinear Coupling Proportion

The previous sections analyzed the three major nonlinear factors influencing storm surges in the four characteristic sea regions. Now, we return to the central focus of this study: the impact of these three nonlinear factors on the degree of nonlinear coupling in storm surge flow velocity.

The flow velocity difference ΔU is obtained by adding the pure storm surge flow velocity U_w and pure astronomical tide flow velocity U_A , then subtracting the flow velocity under coupled conditions U_c . NCP is calculated using Equation (26) where an NCP value closer to 0 indicates a higher degree of linearity. The classification standards for the different levels of coupling proportion are provided in Table 3.

$$NCP = \left(\frac{U_w + U_A - U_c}{U_c} \right) \times 100\% = \left(\frac{\Delta U}{U_c} \right) \times 100\% \quad (26)$$

Table 3. Classification of flow velocity nonlinear coupling proportion NCP .

Coupling Conditions		Proportion of Couple Velocity (NCP)
Nonlinear	Nonlinear increase	$NCP < -50\%$
	Strong nonlinear increase	$-50\% \leq NCP < -30\%$
	Weak nonlinear increase	$-30\% \leq NCP < -15\%$
	Relatively weak increase	$-15\% \leq NCP < -5\%$
Quasi-linear (approximately referred to as linear range)	Linear	$-5\% \leq NCP \leq 5\%$
Nonlinear	Relatively weak decrease	$5\% \leq NCP < 15\%$
	Weak nonlinear decrease	$15\% \leq NCP < 30\%$
	Strong nonlinear decrease	$30\% \leq NCP < 50\%$
	Nonlinear decrease	$NCP > 50\%$

The previous analysis of the three major factors in the four regions provides insight, but the physical mechanisms of topography do not change significantly over a short period of time, while typhoon storm surges are dynamic processes. Although extreme value statistical analysis can roughly determine the influence of bottom friction, wind stress, and convective terms in a given region, it cannot reflect the relationship between the “flow velocity nonlinear effects” and these three factors in real time over different periods. However, a qualitative analysis of the nonlinear coupling proportions during the typhoon for each of the four regions can still be conducted.

In this study, nine different coupling proportions were set, and a heatmap was created (Figure 12). $NCP = 0$ strictly represents linear coupling. Values within $-5\% \leq NCP \leq 5\%$ are quasi-linear, while deviations outside this range represent varying degrees of non-linearity and are shown in white on the heatmap—the whiter the color, the closer the system is to linear coupling. (The heatmap illustrates the distribution of deviations where regions within the range $(-5\%, 5\%)$ are highlighted using white color.) As the coupling proportion decreases and falls below -50% (indicating nonlinear increase), the color shifts to red—this means the more intense the red, the smaller the sum of the pure storm surge flow velocity U_w and the pure astronomical tide flow velocity U_A compared with the flow velocity under

coupled conditions U_c . Conversely, as the coupling proportion increases (indicating a nonlinear decrease), the color turns blue.

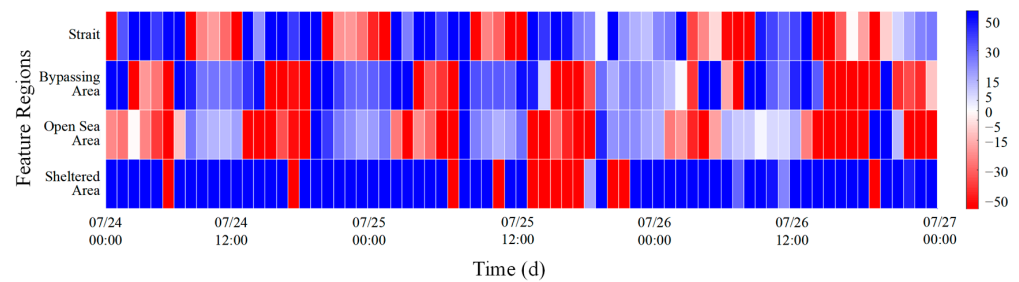


Figure 12. NCP in four characteristic regions during the typhoon.

The heatmap effectively captures the NCP during different time periods. Overall, strong nonlinear flow velocity ($NCP \geq 30\%$ and $NCP \leq -30\%$) accounts for the vast majority, making up 70.1% of the total, with a nonlinear decrease (blue) being particularly prominent, constituting 60.9% of the nonlinear coupling periods. Furthermore, the alternation of nonlinear increase and decrease (red and blue) also shows a certain periodicity, occurring approximately every six hours, while linear coupling appears infrequently.

The bypassing area and strait exhibit similar periodic patterns. The differences in geographic location result in different phases of nonlinear increase and decrease occurring at the same time in these two regions. However, the frequency of “nonlinear decrease” in the bypassing area is higher than in the strait, with a complete cycle of approximately 12 h, consisting of 8 h of a “nonlinear decrease” followed by 4 h of a “nonlinear increase.” This suggests a distinct rhythmic pattern in the coupling effects of flow velocity in these regions.

In the open sea area, the ‘linear coupling’ phenomenon occurs with the highest frequency among the four regions. ‘Nonlinear increase’ is more frequent and intense than ‘nonlinear decrease’, with the cumulative duration of ‘nonlinear increase’ accounting for 76.9% more time compared with ‘nonlinear decrease’ during the observation period. The sheltered area is distinctive, with a “nonlinear decrease” accounting for 90.3% of the time, indicating that coupling reduced flow throughout most of the period.

4.3. Quantitative Analysis of Nonlinear Effects

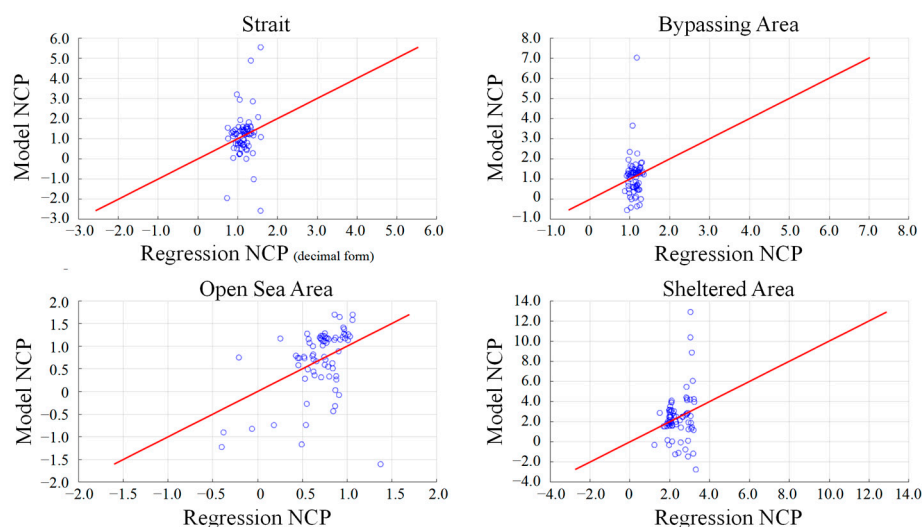
In this study, bottom friction τ_b , wind stress τ_s , and the convective term A are used as key input variables for regression analysis. Based on the response of the NCP, input patterns are established, and seven regression models are constructed to identify the most suitable regression model for quantifying the relationship between these three major nonlinear factors and NCP in the characteristic regions.

4.3.1. Input Patterns and Evaluation Metrics

To determine the optimal combination of variables for each of the seven regression models, this study inputs the three major nonlinear factors—bottom friction, wind stress, and the convective term—based on the characteristics of the flow velocity nonlinear coupling proportion (NCP) and the complex relationships between these factors. No transformations such as squaring, exponentiation, or logarithms were applied to the input variables as the three major nonlinear factors are already calculated characteristic indices and cannot be directly measured by hydrological or tidal stations. The input pattern includes all variables corresponding to the entire time period before and after the typhoon’s landfall. The evaluation metric for the output is the goodness-of-fit R^2 , which indicates the proportion of variance explained by the model relative to the total variance [51].

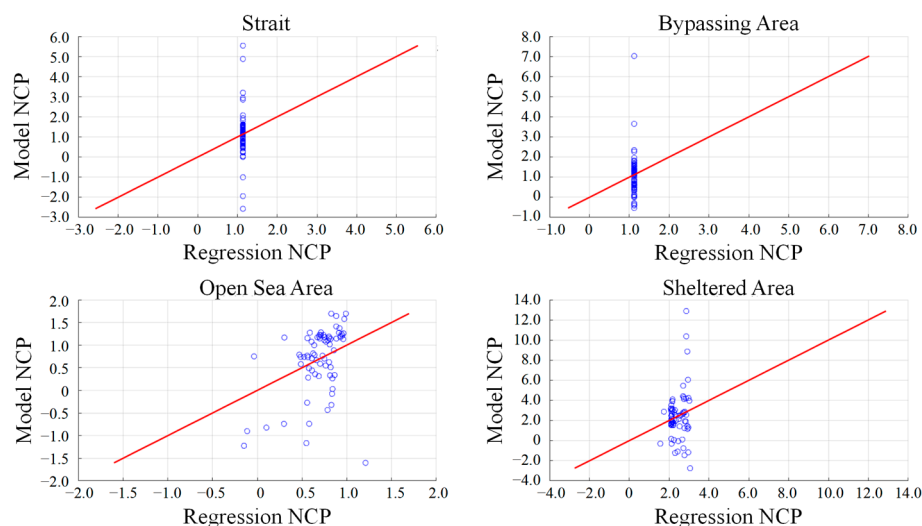
4.3.2. Model Results Comparison

The regression analyses aimed to quantify the contributions of wind stress, convection, and bottom friction to *NCP*. As expected, wind stress dominated in open sea areas due to the absence of topographical constraints, while convection emerged as the primary driver in straits and bypassing zones, reflecting the concentrated flow and increased turbulence in these regions. Although bottom friction exhibited the least influence overall, its impact was amplified in shallower regions, consistent with theoretical expectations [44,46]. These findings align well with prior studies of storm surge dynamics, reinforcing the robustness of the regression model's outcomes. The scatter plots of *NCP* (Figure 13) and the corresponding goodness-of-fit R^2 values were obtained for each model.



(a)

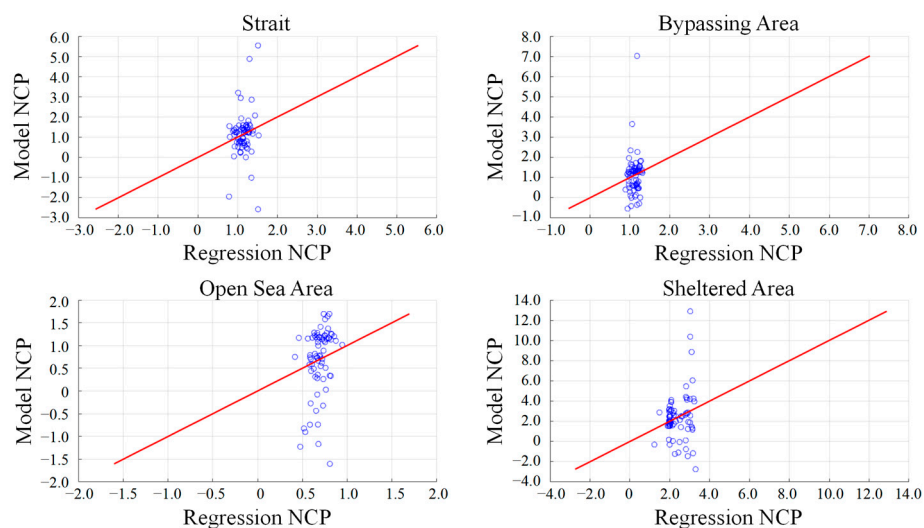
strait $R^2 = 0.0303$; bypassing area $R^2 = 0.0130$; open sea area $R^2 = 0.1766$; sheltered area $R^2 = 0.0448$



(b)

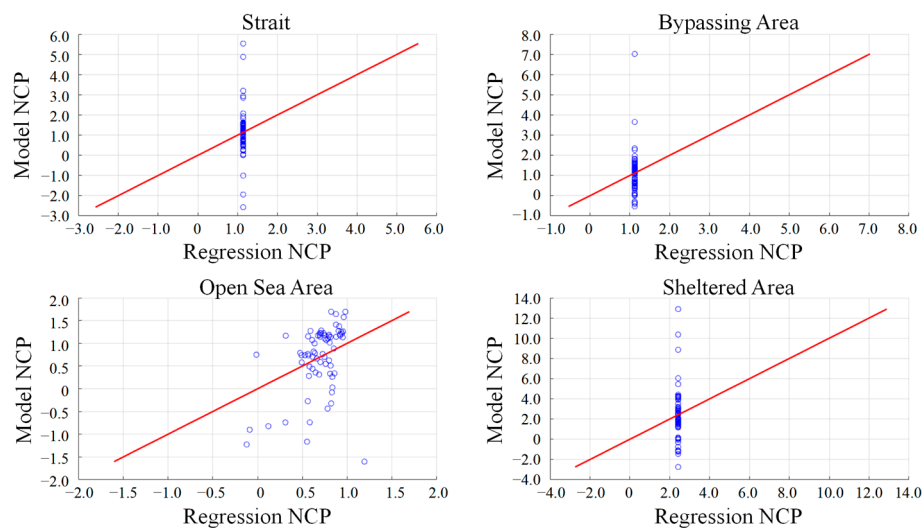
strait $R^2 = 0.0296$; bypassing area $R^2 = 0.0129$; open sea area $R^2 = 0.0692$; sheltered area $R^2 = 0.0443$

Figure 13. Cont.



(c)

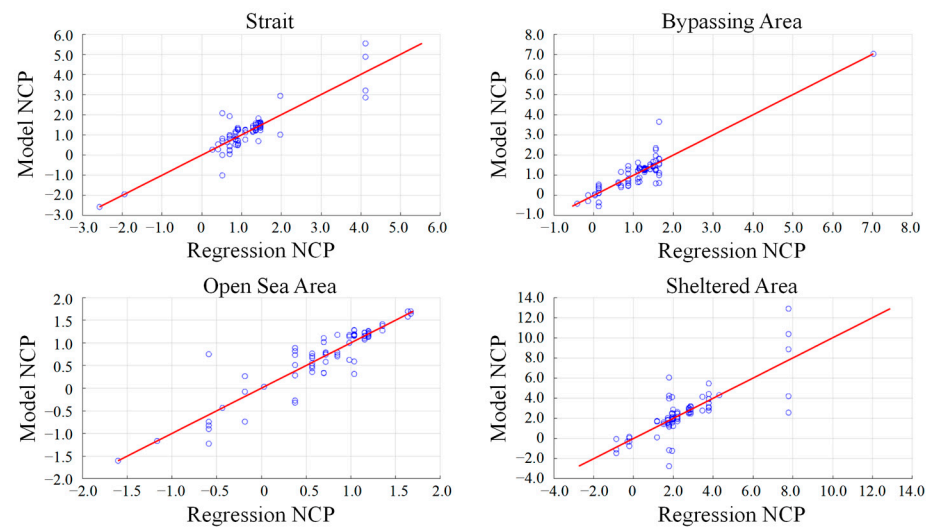
strait $R^2 = 0$; bypassing area $R^2 = 0$; open sea area $R^2 = 0.1734$; sheltered area $R^2 = 0.0416$



(d)

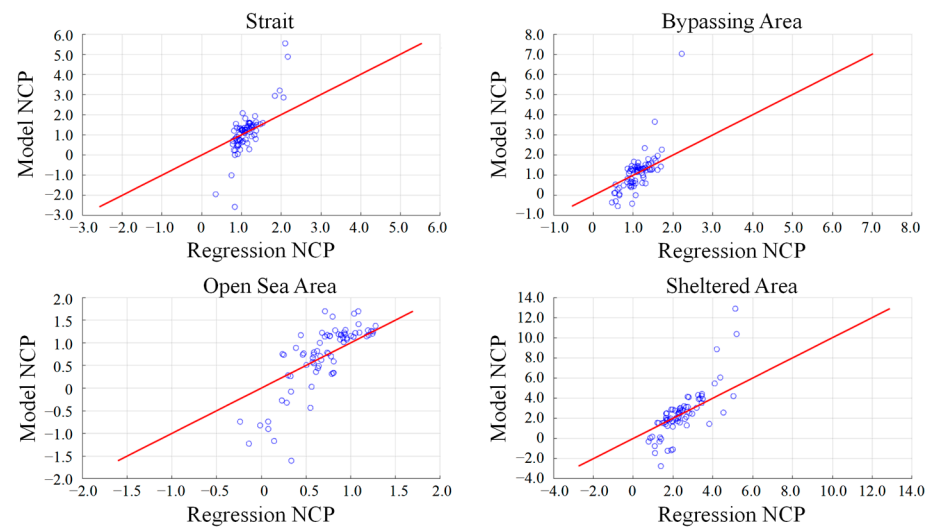
strait $R^2 = 0$; bypassing area $R^2 = 0$; open sea area $R^2 = 0.1705$; sheltered area $R^2 = 0.0390$

Figure 13. Cont.



(e)

strait $R^2 = 0.7997$; bypassing area $R^2 = 0.8279$; open sea area $R^2 = 0.8314$; sheltered area $R^2 = 0.6076$



(f)

strait $R^2 = 0.4139$; bypassing area $R^2 = 0.3489$; open sea area $R^2 = 0.5239$; sheltered area $R^2 = 0.4792$

Figure 13. Cont.

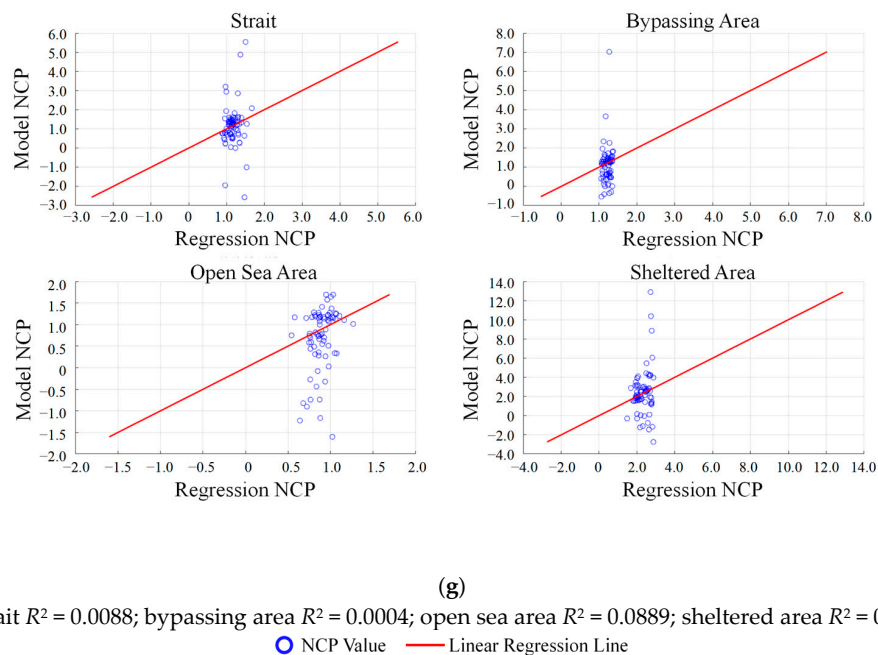


Figure 13. A comparison of actual and estimated NCP values in four regions using seven models; (a) Linear regression, (b) Ridge regression, (c) Lasso regression, (d) Elastic net regression, (e) Decision tree regression, (f) Random forest regression, and (g) Support vector regression (SVR).

The feature's importance is based on the total reduction in impurity at the split nodes where the feature participates in the tree. Here, 'impurity' refers to the measure of disorder or uncertainty in the dataset, commonly quantified using metrics such as Gini impurity or information gain. A 'feature' represents an independent variable used in the model to make predictions, with its importance determined by how effectively it reduces impurity in the splitting process. This calculation is specific to tree-based regression models, such as decision tree regression and random forest regression. Non-tree-based models, such as linear regression or support vector regression, do not involve this concept as they lack a tree structure for splitting nodes. Feature importance is typically normalized, so the sum of the importance values for all features equals 1. Let X_j represent the j -th feature, and $\Delta i(t)$ represent the reduction in Gini impurity at node t (calculated using Equation (28)). N_{trees} is the number of decision trees, and node t refers to all nodes in the decision trees associated with the feature X_j . p_i represents the probability of class i , and C is the total number of classes.

$$Importance(X_j) = \frac{1}{N_{trees}} \sum_{i=1}^{N_{trees}} \sum_{t \in \text{Nodes where } X_j \text{ is used}} \frac{\Delta i(t)}{\text{Number of sample in Node } t} \quad (27)$$

$$Gini = \sum_{i=1}^C p_i(1 - p_i) \quad (28)$$

In the study of typhoon storm surge flow velocity, a goodness-of-fit $R^2 \geq 0.7$ is typically considered a good fit, indicating that the model can explain most of the variance in the data and that the model is quite accurate. For $0.7 > R^2 \geq 0.5$, the model can explain the majority of the variance in the data, making it suitable for describing overall trends, though it may lack accuracy in predicting finer details. When $R^2 < 0.5$, it suggests that the model has a weak ability to explain the data and is not sufficiently accurate [47]. See Figure 13 for the R^2 values of each model across the different regions.

The R^2 values for linear, ridge, elastic net, and support vector regression were all below 0.2 across the four regions, indicating a limited ability to capture the complex relationships in the data. Interestingly, the random forest model, which generally outperforms decision tree models in large and diverse datasets, achieved an average R^2 of 0.4681 across the four regions, with only the open sea and sheltered area exceeding 0.5. This phenomenon may result from the relatively small sample size and the specific statistical properties of the input features, which favor simpler models like decision trees for capturing the highly nonlinear interactions among bottom friction, wind stress, and the convective term. Previous studies [52] have also reported that decision trees can outperform ensemble methods like random forests in scenarios where data complexity and sample size constrain the model's ensemble learning capacity. While the regression models provided valuable insights into the contributions of wind stress, convection, and bottom friction to NCP , we acknowledge certain limitations. For instance, the simplicity of linear models may not fully capture the complex, time-dependent interactions in storm surge dynamics. However, the decision tree model outperformed others ($R^2 \approx 0.8$ in most regions), highlighting its effectiveness in addressing nonlinear relationships. Future work could incorporate more sophisticated machine learning approaches or ensemble modeling to further enhance predictive accuracy. Among the seven regression models, the decision tree performed the best, with R^2 values of 0.7997 for the strait, 0.8279 for the bypassing area, 0.8314 for the open sea, and 0.6076 for the sheltered area. The decision tree model significantly outperformed the others in all regions. Therefore, the decision tree regression model was selected to calculate the feature importance of bottom friction, wind stress, and the convective term as explanatory variables for each region (Equation (27)), which represents the contribution of these explanatory variables to the dependent variable, NCP . The results are shown in Table 4. Unlike the qualitative ranking of the extreme impacts of the three major nonlinear factors (Table 2), this regression analysis provides a quantitative evaluation of the comprehensive influence of the three nonlinear factors over the entire time period.

Table 4. A calculation of a contribution proportion of the decision tree model to nonlinear item features.

Characteristic Regions Nonlinear Terms	Bottom Friction	Wind Stress	Convective Term
Strait	24.13%	32.37%	43.50%
Bypassing area	21.36%	38.50%	40.14%
Open sea area	12.87%	48.28%	38.85%
Sheltered area	32.68%	38.34%	28.98%

The strait, with its narrow passage and significant topographical variations, generates a strong acceleration effect on ocean currents. As the currents pass through the strait, the increased flow velocity leads to changes in momentum, which enhances the contribution of the convective term [38], reaching 43.50%. Bottom friction has a relatively smaller contribution in this region, likely due to the greater water depth, which reduces the overall impact of bottom friction on flow velocity. Scholars like Xia have also indicated that bottom friction has a weaker contribution to flow velocity in submarine canyons [53].

The bypassing area, defined by coordinates $30^{\circ}33' \text{ N}$ to $30^{\circ}38' \text{ N}$ and $122^{\circ}18' \text{ E}$ to $122^{\circ}26' \text{ E}$, has a broader sea surface width, which refers to the distance between the northern and southern extents of the sea surface in this area. This broader width allows for a larger expanse of water to be influenced by typhoon-induced wind stress and convective forces. As a result, the convective term, influenced by localized variations in wind stress, plays a crucial role [45] in redistributing momentum, particularly in regions with complex

bathymetry where the assumptions of uniformity in x and y (as per the Ekman spiral theory) are not applicable. While the Ekman spiral theory indeed assumes uniformity in the x - and y -directions, the observed flow conditions during Typhoon 2106 exhibit significant spatial heterogeneity due to local wind stress variations and topographical constraints, which invalidate this assumption and amplify the convective term's effect in redistributing flow momentum [45,46]. Similar to the strait, bottom friction in this region has a weaker contribution to the nonlinear coupling of flow velocity due to the greater water depth.

In the open sea area, wind stress contributes the most to the nonlinear increase in flow velocity, with a contribution proportion of 48.28%. The wide expanse of sea exposed to wind can effectively drive the movement of both surface and subsurface water [41]. Bottom friction has a lower contribution proportion of 12.87% to the nonlinear coupling of flow velocity as the open sea has the greatest water depth and bottom friction exerts less resistance on the overall flow velocity. The convective term contributes 38.85% to the flow velocity nonlinearity in this area, and its significance becomes particularly pronounced in open-sea areas due to steep velocity gradients caused by wind-induced surface currents interacting with underlying stratification. In such regions, the absence of significant physical barriers allows for large-scale momentum exchanges, intensifying velocity shear. Furthermore, the wide spatial scales in open-sea areas contribute to the enhanced momentum transfer through vertical and horizontal advection, which further amplifies the convective term's impact.

The ranking of influence on the nonlinear coupling of flow velocity in the first three regions, based purely on extreme values, aligns relatively well with the results. However, the sheltered area, with the shallowest depth and the most complex terrain among the four characteristic regions, presents a different scenario. Wind stress still has a significant impact, contributing 38.34%, but the differences between the three nonlinear factors in this region are the smallest among the four. Bottom friction ranks second in terms of influence, which differs from the extreme value-based qualitative ranking (Table 2). This could be due to the relatively poor fluid mobility in the sheltered area. While wind stress can affect the water flow, bottom friction plays a larger role due to the complex terrain, impeding the lower layer of the water. The convective term contributes 28.98% to the sheltered area's "nonlinear flow velocity reduction" state. Although lower, it still plays an important role in the local water movement. Stewart noted that in shallow waters with complex terrain, the combined effects of friction and the convective term are key to changes in flow velocity [44]. Comparing the contribution proportions of the convective term in the bypassing area, open sea, and sheltered area in Table 4 reveals that there are critical values for water depth and sea surface width regarding the convective term. When these depths or widths exceed or fall below certain thresholds, the contribution tends to decrease. While specific thresholds for these parameters cannot be definitively identified from the current dataset, trends suggest that changes in depth or width are associated with variations in the convective term's contribution.

By synthesizing Tables 2 and 4, it can be concluded that bottom friction is most sensitive to water depth—the greater the depth, the smaller the bottom friction. Furthermore, as bottom friction increases, the likelihood of nonlinear flow velocity reduction increases. Wind stress primarily depends on the strength of the wind, air density, and surface roughness and is not directly influenced by sea surface width. However, larger sea surface areas may enhance the effects of wind-driven processes on flow dynamics due to increased fetch and wave development. As wind stress increases, the likelihood of a nonlinear flow velocity increase becomes more pronounced. The convective term shows a critical point, which is related to both sea surface width and water depth. While a precise relationship remains

challenging to establish due to the limitations of the current dataset, larger convective term values are generally associated with increased flow velocity nonlinearity.

By combining the results of both qualitative and quantitative analyses, it is evident that flow velocity variations in different regions are influenced by the combined effects of wind stress, bottom friction, and the convective term, with varying contribution proportions. The contribution rankings of the three major nonlinear factors from the qualitative extreme value analysis are almost identical to those from the quantitative regression model analysis. Therefore, the relative importance of nonlinear factors can be reasonably explained and validated through ocean dynamics analysis and mathematical models.

The erratic performance of regression models reflects the unique complexity of the nonlinear coupling effects studied here. While decision tree regression captures localized nonlinear interactions effectively, the relatively poor performance of random forest and other models underscores the need for future work with larger datasets and more tailored feature engineering approaches. These results highlight the challenges of quantifying nonlinear storm surge dynamics using regression methods and the importance of selecting models that align with data characteristics and research objectives.

The erratic behavior observed in some regression models warrants further exploration. While decision tree regression effectively captured the nonlinear relationships in this study, random forest exhibited limitations likely tied to data sparsity and noise in localized regions. Such variability underscores the importance of tailoring model selection to the specific characteristics of the dataset. For future work, a more systematic hyperparameter tuning process and advanced methods like gradient boosting machines (GBMs) could be employed to enhance model robustness. Additionally, incorporating physical constraints into regression models may help address noise amplification in confined regions, as suggested by recent studies on storm surge dynamics [54].

5. Conclusions

This study employs the Delft3D numerical model to elucidate nonlinear flow velocity dynamics across four key marine regions during Typhoon 2106, which was selected for its extreme intensity, comprehensive observational data, and representative nonlinear characteristics. While this study focuses on a single case to establish the foundation of the *NCP* metric, future work will include multiple typhoon cases to validate its applicability. Simulations and validations of the 2106 storm surge model identified bottom friction, wind stress, and the convective term as key factors influencing nonlinear flow velocity. A novel metric, the “Flow Velocity Nonlinear Coupling proportion (*NCP*),” is introduced to quantify the nonlinear interactions between astronomical tidal flow velocities and storm surge flow velocities. This coupling reflects how the combined flow velocity diverges from the linear sum of these two components, influenced by wind stress, bottom friction, and the convective term.

Model Validation and Factor Analysis. The Delft3D model accurately captured Typhoon 2106’s effects on complex topography across the four regions, facilitated by a double-layer nested grid system. Model validation with observational data yielded a strong correlation ($R^2 \approx 0.96$) between simulated and measured values. Analyses showed wind stress exerted the most influence in open sea areas, reaching $1.50 \times 10^{-4} \text{ m/s}^2$, while the convective term was most impactful in narrow zones like straits, reaching $-3.21 \times 10^{-4} \text{ m/s}^2$. Although bottom friction’s impact was modest, with an average influence of $0.64 \times 10^{-4} \text{ m/s}^2$, it contributed notably to flow velocity changes.

Introducing the Nonlinear Coupling Proportion (*NCP*). This study introduced *NCP* as an innovative metric to quantify nonlinear coupling in typhoon-driven flow velocities. An *NCP* range of -5% to 5% indicated near-linear coupling, while values outside this

range (e.g., $NCP < -50\%$ or $NCP > 50\%$) indicated significant nonlinear increases or decreases. The analysis highlighted that a nonlinear flow reduction was particularly pronounced in nearshore areas, with reductions observed 90.3% of the time in sheltered regions. In contrast, broader and more open offshore areas exhibited a higher occurrence of flow velocity linear coupling. The analysis reveals that as distance from the shore increases, hydrodynamic complexity tends to decrease, resulting in conditions that favor near-linear coupling.

Qualitative Rankings and Regional Dynamics. Qualitative analysis of the three nonlinear factors across regions revealed that the convective term primarily drove the alternating nonlinear flow velocity increases and decreases in straits and bypassing zones, followed by wind stress, while bottom friction had the smallest influence. The open sea maintained the closest approximation to linear coupling, while the sheltered area exhibited consistent nonlinear reduction, primarily driven by wind stress, with contributions from the convective term and a minimal impact from bottom friction.

Regression Model Comparison. Seven regression models were compared to assess the relationship between NCP and the three nonlinear factors. The decision tree model, with an average goodness-of-fit ($R^2 = 0.77$), emerged as the most effective in capturing the complex nonlinear relationships among bottom friction, wind stress, and the convective term. Unlike other models, its hierarchical structure allows it to partition the input space effectively, leading to superior performance in regions with distinct nonlinearity. The robustness of the decision tree result was further confirmed through sensitivity tests, ensuring its outcome is not merely coincidental but reflects its ability to address region-specific nonlinearity. Overall, wind stress was the predominant factor driving nonlinear flow increases as the sea surface width expanded, notably in open sea regions where the convective term accounted for 48.28%. It showed critical threshold values related to water depth and sea surface width, with a nonlinear reduction in flow velocity becoming more probable as these parameters approached critical points. However, due to limitations in the data, the precise critical thresholds fall beyond the scope of this study. Bottom friction, with an average contribution of 22.76%, had a more significant influence in shallower waters, increasing the likelihood of nonlinear flow reductions.

This study provides a novel framework for evaluating nonlinear coupling effects in storm surge flow velocities, underscoring the critical roles of wind stress and the convective term during extreme events like typhoons.

Supplementary Materials: The following supporting information can be downloaded at: <https://www.mdpi.com/article/10.3390/jmse13010005/s1>, Figure S1: Distribution of flow velocity components in four characteristic regions during pure storm surge conditions. (a) X-direction flow velocity; (b) Y-direction flow velocity.

Author Contributions: Conceptualization, F.C. and Z.S.; methodology, F.C. and K.D.; software, F.C.; validation, F.C. and K.D.; formal analysis, F.C.; investigation, F.C. and K.D.; resources, F.C.; data curation, F.C. and K.D.; writing—original draft preparation, F.C. and K.D.; writing—review and editing, F.C., K.D. and Z.S.; visualization, F.C.; supervision, Z.S.; project administration, F.C. and Z.S.; funding acquisition, Z.S. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Science Technology Department of Zhejiang Province, grant number 2023C03119 and the Bureau of Science and Technology of Zhoushan, grant number 2024C03006.

Data Availability Statement: The data presented in the present study are available upon request from the corresponding author.

Conflicts of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

References

1. Flather, R.A. A storm surge prediction model for the northern Bay of Bengal with application to the cyclone disaster in April 1991. *J. Phys. Oceanogr.* **1994**, *24*, 172–190. [\[CrossRef\]](#)
2. Johns, B.; Ali, M.A. The numerical modelling of storm surges in the Bay of Bengal. *Q. J. R. Meteorol. Soc.* **1980**, *106*, 1–18. [\[CrossRef\]](#)
3. As-Salek, J.A.; Yasuda, T. Comparative study of the storm surge models proposed for Bangladesh: Last developments and research needs. *J. Wind. Eng. Ind. Aerodyn.* **1995**, *54*, 595–610. [\[CrossRef\]](#)
4. Kim, S.Y.; Yasuda, T.; Mase, H. Numerical analysis of effects of tidal variations on storm surges and waves. *Appl. Ocean Res.* **2008**, *30*, 311–322. [\[CrossRef\]](#)
5. Zhou, T.; Tan, Y.; Chu, A.; Zhang, C. Integrated model for astronomic tide and storm surge induced by typhoon for Ningbo coast. In Proceedings of the ISOPE International Ocean and Polar Engineering Conference, Sapporo, Japan, 10–15 June 2018; ISOPE: Mountain View, CA, USA, 2018; p. ISOPE-I-18-265.
6. Xu, J.; Ma, K.; Nie, Y.; Liu, C.; Bi, X.; Shi, W.; Lv, X. Numerical Study on Storm Surge Level Including Astronomical Tide Effect Using Data Assimilation Method. *Atmosphere* **2022**, *14*, 38. [\[CrossRef\]](#)
7. Prandle, D.; Wolf, J. Surge-tide interaction in the southern North Sea. In *Elsevier Oceanography Series*; Elsevier: Amsterdam, The Netherlands, 1978; Volume 23, pp. 161–185.
8. Qin, Z.; Duan, Y.; Wang, Y.; Shen, Z.; Xu, K. *Numerical Simulation and Prediction of Storm Surges and Water Levels in Shanghai Harbour and Its Vicinity*; Springer: Dordrecht, The Netherlands, 1994.
9. Wan-Kang, Y.; Qing-Ying, Y.; Feng, Z.; Ze-Kun, S. Study and numerical simulation of typhoon storm surge characteristics at the typical bay. *Mar. Sci. Bull.* **2018**, *5*, 537–547.
10. Harris, D.L.; Angelo, A. A regression model for storm surge prediction. *Mon. Weather. Rev.* **1963**, *91*, 710–726. [\[CrossRef\]](#)
11. Mulia, I.E.; Ueda, N.; Miyoshi, T.; Heidarzadeh, M. A novel deep learning approach for typhoon-induced storm surge modeling through efficient emulation of wind and pressure fields. *Sci. Rep.* **2023**, *13*, 7918. [\[CrossRef\]](#)
12. Cheng, L.; Yeow, K.; Zhang, Z.; Teng, B. Three-dimensional scour below offshore pipelines in steady currents. *Coast. Eng.* **2009**, *56*, 577–590. [\[CrossRef\]](#)
13. Draper, S.; An, H.; Cheng, L.; White, D.J.; Griffiths, T. Stability of subsea pipelines during large storms. *Philos. Trans. R. Soc. A Math. Phys. Eng. Sci.* **2015**, *373*, 20140106. [\[CrossRef\]](#)
14. Shi, Y.; Yang, H.; Wei, J.; Yu, H.; Liu, Y.; Ge, S.; Bai, Z.; Li, S. Numerical study of scour below vibrating pipelines under waves and currents. *Ocean Eng.* **2022**, *266*, 112718. [\[CrossRef\]](#)
15. Lux, J.; Olschewski, M.; Schfer, P.; Hill, W. Real-Time Determination of Depth of Burial Profiles for Submarine Power Cables. *IEEE Trans. Power Deliv.* **2019**, *34*, 1079–1086. [\[CrossRef\]](#)
16. Wang, J. Monitoring method of submarine cable fault data. *J. Coast. Res.* **2019**, *93*, 735–740. [\[CrossRef\]](#)
17. Wang, Y.; Fu, C.; Qin, X. Numerical and physical modeling of submarine telecommunication cables subjected to abrupt lateral seabed movements. *Mar. Georesources Geotechnol.* **2021**, *39*, 1307–1319. [\[CrossRef\]](#)
18. Yibing, B.; Jingyuan, S.; Chuanteng, L. Spatio-temporal distribution characteristics of surge in the reach below Nantong of Yangtze Estuary under the influence of Typhoon In-Fa. *Hydro-Sci. Eng.* **2021**, *6*, 25–33.
19. Deltares. *Delft3D-FLOW User Manual*; Deltares: Delft, The Netherlands, 2014.
20. Zhilin, S.; Shanhong, Z.; Chen, W.; Wenrong, T.; Hanqing, J. Simulation and analysis of storm surge at Zhoushan fishing port. *Haiyangxuebao* **2020**, *42*, 136–143.
21. Tritton, D.J. *Physical Fluid Dynamics*; Springer Science & Business Media: Philadelphia, PA, USA, 2012.
22. Mellor, G.L. *Users Guide for a Three Dimensional, Primitive Equation, Numerical Ocean Model*; Program in Atmospheric and Oceanic Sciences, Princeton University: Princeton, NJ, USA, 1998.
23. Warner, J.C.; Sherwood, C.R.; Signell, R.P.; Harris, C.K.; Arango, H.G. Development of a three-dimensional, regional, coupled wave, current, and sediment-transport model. *Comput. Geosci.* **2008**, *34*, 1284–1306. [\[CrossRef\]](#)
24. Blumberg, A.F.; Mellor, G.L. A description of a three-dimensional coastal ocean circulation model. In *Three-Dimensional Coastal Ocean Models*; American Geophysical Union: Washington, DC, USA, 1987; Volume 4, pp. 1–16.
25. Song, Y.; Haidvogel, D. A semi-implicit ocean circulation model using a generalized topography-following coordinate system. *J. Comput. Phys.* **1994**, *115*, 228–244. [\[CrossRef\]](#)
26. Warner, J.C.; Geyer, W.R.; Lerczak, J.A. Numerical modeling of an estuary: A comprehensive skill assessment. *J. Geophys. Res. Ocean.* **2005**, *110*, 1–13. [\[CrossRef\]](#)
27. Garratt, J.R. Review of drag coefficients over oceans and continents. *Mon. Weather. Rev.* **1977**, *105*, 915–929. [\[CrossRef\]](#)
28. Wang, F.; Qiu, L.; Liang, X.; Mo, W.; Liang, N.A.; Wang, Y. Response of human activities on tidal hydrodynamics of Hainan Qinglan tidal inlet system based on Delft 3D model. *Quat. Sci.* **2018**, *38*, 496–504.

29. Xie, L.; Bao, S.; Pietrafesa, L.J.; Foley, K.; Fuentes, M. A real-time hurricane surface wind forecasting model: Formulation and verification. *Mon. Weather. Rev.* **2006**, *134*, 1355–1370. [\[CrossRef\]](#)
30. Huang, S. Investigation on Storm Surge and Erosion-Deposition in Zhejiang Coastal Waters. Ph.D. Thesis, Zhejiang University, Hangzhou, China, 2017.
31. Zhong, S. Improved Wind-Field-Based Simulation of Storm Surge in Zhoushan Fishery Harbor. Ph.D. Thesis, Zhejiang University, Hangzhou, China, 2020.
32. Li, M.; Zhong, L.; Boicourt, W.C. Simulations of Chesapeake Bay estuary: Sensitivity to turbulence mixing parameterizations and comparison with observations. *J. Geophys. Res. Ocean.* **2005**, *110*, C12004. [\[CrossRef\]](#)
33. Gopalakrishnan, G.; Cornuelle, B.D.; Hoteit, I.; Rudnick, D.L.; Owens, W.B. State estimates and forecasts of the loop current in the Gulf of Mexico using the MITgcm and its adjoint. *J. Geophys. Res. Ocean.* **2013**, *118*, 3292–3314. [\[CrossRef\]](#)
34. Pawlowicz, R.; Beardsley, B.; Lentz, S. Classical tidal harmonic analysis including error estimates in MATLAB using T_TIDE. *Comput. Geosci.* **2002**, *28*, 929–937. [\[CrossRef\]](#)
35. Zhang, K.; Douglas, B.C.; Leatherman, S.P. Twentieth-century storm activity along the US east coast. *J. Clim.* **2000**, *13*, 1748–1761. [\[CrossRef\]](#)
36. Montgomery, D.C.; Peck, E.A.; Vining, G.G. *Introduction to Linear Regression Analysis*; John Wiley & Sons: Hoboken, NJ, USA, 2021.
37. Variyath, A.M.; Brobbey, A. Variable selection in multivariate multiple regression. *PLoS ONE* **2020**, *15*, e0236067. [\[CrossRef\]](#)
38. Hoerl, A.E.; Kennard, R.W. Ridge regression: Biased estimation for nonorthogonal problems. *Technometrics* **1970**, *12*, 55–67. [\[CrossRef\]](#)
39. Tibshirani, R. Regression shrinkage and selection via the lasso. *J. R. Stat. Soc. Ser. B Stat. Methodol.* **1996**, *58*, 267–288. [\[CrossRef\]](#)
40. Zou, H.; Hastie, T. Regularization and variable selection via the elastic net. *J. R. Stat. Soc. Ser. B Stat. Methodol.* **2005**, *67*, 301–320. [\[CrossRef\]](#)
41. Prasad, A.M.; Iverson, L.R.; Liaw, A. Newer classification and regression tree techniques: Bagging and random forests for ecological prediction. *Ecosystems* **2006**, *9*, 181–199. [\[CrossRef\]](#)
42. Breiman, L. Random forests. *Mach. Learn.* **2001**, *45*, 5–32. [\[CrossRef\]](#)
43. Drucker, H.; Burges, C.J.; Kaufman, L.; Smola, A.; Vapnik, V. Support vector regression machines. *Adv. Neural Inf. Process. Syst.* **1996**, *9*, 155–161.
44. Ruppert, D. The elements of statistical learning: Data mining, inference, and prediction. *J. Am. Stat. Assoc.* **2004**, *99*, 567. [\[CrossRef\]](#)
45. Stewart, R.H. *Introduction to Physical Oceanography*; Stewart, R.H., Ed.; Waveland Press: Long Grove, IL, USA, 2008.
46. Price, J.F.; Sundermeyer, M.A. Stratified ekman layers. *J. Geophys. Res. Ocean.* **1999**, *104*, 20467–20494. [\[CrossRef\]](#)
47. Large, W.G.; McWilliams, J.C.; Doney, S.C. Oceanic vertical mixing: A review and a model with a nonlocal boundary layer parameterization. *Rev. Geophys.* **1994**, *32*, 363–403. [\[CrossRef\]](#)
48. Prandle, D.; Wolf, J. The interaction of surge and tide in the North Sea and River Thames. *Geophys. J. Int.* **1978**, *55*, 203–216. [\[CrossRef\]](#)
49. Pugh, D. *Sea-Level Science: Understanding Tides, Surges, Tsunamis and Mean Sea-Level Changes*; Cambridge University Press: Cambridge, UK, 2014.
50. Gill, A.E. *Atmosphere—Ocean Dynamics*; Elsevier: Amsterdam, The Netherlands, 2016.
51. Cushman-Roisin, B.; Beckers, J.M. *Introduction to Geophysical Fluid Dynamics: Physical and Numerical Aspects*; Academic Press: Cambridge, MA, USA, 2011.
52. Zhang, S.; Zhang, K.; Chao, L.; Li, Y.; Zhang, Z.; Huang, Y. Development and comparison of multiple models for estimating key soil hydraulic properties considering terrain and soil physiochemical properties. *J. Hohai Univ. Nat. Sci.* **2020**, *52*, 42. [\[CrossRef\]](#)
53. Loh, W.Y. Fifty years of classification and regression trees. *Int. Stat. Rev.* **2014**, *82*, 329–348. [\[CrossRef\]](#)
54. Xia, J.; Li, C.; Ma, Y. Research Hotspots on Deep-Sea Hydrothermal Activity. *J. Geomech.* **2007**, *13*, 14. [\[CrossRef\]](#)

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.