

Article

Erosion–Seepage System (ESS) for Flow-Induced Soil Erosion Rate with Seepage

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Abstract: Critical shear stress and erosion rate are two key factors for the prediction of the incipient motion of sediment and the transport of sediment. Seabed seepage can significantly alter the pore pressure gradient within the soil and the hydrodynamics around the surface of the seabed, further affecting erosion processes. Previous studies attempted to theoretically clarify the effect of the seepage force on sediment incipient motion. In this study, a newly designed erosion–seepage system (ESS) that considers the effect of seepage under steady or oscillatory flow is used to simulate the erosion process. Through the designed ESS, the erosion height per unit time was measured directly on the Yellow River sand, and the upward seepage force was applied at the bottom of the soil sample in the process. Then, the relationship between the erosion rate and seepage was established. The experimental results show that upward seepage reduces the critical shear stress of the sand bed and increases the erosion rate of the soils under both steady flow and oscillatory flow conditions. The erosion coefficients in the erosion models decrease with increasing seepage gradient. The effect of seepage on erosion is more obvious when the flow velocity of the steady stream is large, while the effect of seepage on erosion is relatively small under the oscillatory state with a shorter period. However, when violent erosion of soil samples occurs, seepage under both flow conditions greatly increases the erosion rate.

Keywords: sediment incipient motion; critical shear stress; erosion rate; erosion–seepage system; seepage



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1. Introduction

Sediment incipient motion is a key process in the study of sediment transportation, in which the static equilibrium state of the sediment on the surface of a sand bed is disrupted under specific water flow conditions, causing the sediment to transition from a static to a dynamic state. The activation of sediment particles can lead to instability in coastal engineering structures, even resulting in the failure of such structures, such as sediment washout on beaches near ports and local scour around the foundations of offshore wind turbines or artificial fishing reefs.

In general, seepage in coarse bed material (granular or loose soil) or fine bed material (fine and dense soil) has different hydrodynamic characteristics. The two kinds of bed materials have completely different behaviours, such as shear thickening and shear thinning [1], under oscillatory flow actions. In soil erosion, shear thickening occurs in sediment

mixtures where the particles are densely packed or where the interaction between the particles becomes more pronounced at higher shear rates. This leads to the development of more cohesive and less fluid-like behaviour as the flow rate or stress increases. However, shear thinning occurs in sediment mixtures that are more fluid-like or where the particles can slip past each other more easily under higher shear rates. In a sand or granular sediment mixture, the flow can become more fluid as shear stress increases, allowing more rapid erosion or sediment transport. In this study, we focus only on the shear thinning characteristics, because the bed soil is loose sand.

The concept of sediment initiation was introduced as early as the nineteenth century. Systematic studies were carried out in the early twentieth century and continue to the present day, and several experimental instruments have been developed to determine soil erosion rate. Hanson [2] invented the vertical jet apparatus (JET) to study the scour characteristics of cohesive and non-cohesive soils. The JET must be submerged in water during the test, which may not exclude larger soil particles from scouring the scour hole, making it unsuitable for soils with large particles. Normal open-channel flume devices are widely used in the study of soil erosion characteristics due to their alignment with actual water flow conditions [3–5]. However, the velocity of water flow achievable in an open-channel flume is typically low, which is inadequate to study the initiation and erosion rates of cohesive soils. Consequently, they are suitable only for investigating non-cohesive or low-cohesive soils [6,7]. Briaud et al. [8] developed a device called the erosion function apparatus (EFA) for site-specific scour studies. In these tests, the sample was placed in a cylinder under a transparent closed rectangular pipeline, with the sample surface layer positioned 1 mm above the surface of the pipeline. The erosion rate was obtained by measuring the time required to remove the 1 mm soil layer. Gao et al. [9] and Gao et al. [10] conducted erosion tests using the erosion function apparatus (EFA) to investigate the effect of soil properties on the critical shear stress of remoulded cohesive soils and mixed cohesive and non-cohesive soils. Kimiaghalam et al. [11] used the erosion measuring device (EMD) to quantitatively analyse the critical shear stress and erosion rate. Several scholars have relied on experiments to establish various empirical formulas. Among them, the Shields parameter has been well recognised as a criterion for the initiation of cohesionless sediments and has been continuously modified to be applicable to more general river beds [12–14].

The critical shear stress is the maximum flow shear stress that the soil can withstand without damage caused by erosion, that is, the flow shear stress to which the soil particles are subjected at initiation. There are several methods to determine the critical shear stress. It can be estimated from soil parameters such as particle size. For cohesionless soils, the Shields curve can be used to estimate the critical shear stress. However, for fine-grained cohesive soils, the critical shear stress would be underestimated by using the Shields curve because it ignores soil cohesion. Kramer [15] classifies sediment movement into four phases: quiescent, weak, intermediate, and generalized. McNeil et al. [16] conducted a study on scour in soils in situ using a sediment flume to quantify the critical shear stress. They determined that the shear stress of the water flow corresponding to an erosion rate of 10^{-4} cm/s represents the critical shear stress. Later, Perera et al. [17] conducted erosion tests on sediment mixtures using a closed pressure pipe and also defined the flow shear stress corresponding to an erosion rate of 10^{-4} cm/s as the critical shear stress.

Most previous studies have simplified the process by ignoring the effect of seepage on the mobility of sediment particles [18–21]. Several studies have shown that the presence of seepage forces affects the incipient motion of the sediment and the erosion process. Zhang et al. [22] conducted experiments in an annular flume with reference to advanced laboratory [23] or in situ flumes for erosion research [24] to explore the influence of the direction and magnitude of the seepage flow on the erodibility of fluidized sediment.

Several studies have been performed using a device consisting of a closed recirculation tube with a structure similar to the EFA device, a sediment box with a cohesionless sample subjected to seepage flow, and a deposition box [25–28]. Among these, Jewel et al. [28] studied the influence of upward seepage on the critical shear stress of a cohesionless soil bed subjected to surface flow through a permeability test.

Recently, Guo et al. [29] investigated the effect of the oscillatory seepage force on sediment initiation around a free-pass pipeline by coupling a shear stress transport turbulence model with a porous seabed model. Li et al. [30] considered the influence of upward seepage on the local scour of steady current under the pipeline through the CFD model. More recently, Zhai et al. [31,32] further proposed modified Shields numbers in two and three dimensions to explore the effect of seabed seepage on sediment initiation for the scouring problem around pipelines and pile foundations, and examined the effect of seepage on the Shields number using analytical and numerical methods. Based on these, Zhai and Jeng [33] incorporated the wave-induced seabed response and its associated seepage into the conventional scour model and demonstrated the significant impact on the prediction of the scour profile around an offshore pipeline. Upward seepage creates seepage channels within the seabed, causing upward transport of fine particulate matter away from the soil skeleton to the bed and facilitating bed sediment initiation and sediment transport. However, to date, there have been relatively fewer experimental studies on the relationship between erosion rate and seabed seepage.

The scientific research gaps of the research and the innovation of this paper are summarised here.

- Design of a new ESS system: Existing experimental studies for one-dimensional erosion seepage have been limited to steady flow [8–10]. The new ESS system designed further added two features: (1) the oscillatory flow, and (2) additional seepage flux caused by pressure drop from the external pressure pump. The new design allowed us to further clarify the relationship between soil erosion rate and seepage over a wider range.
- Relationship between soil erosion, seepage, and critical shear stresses: Another new scientific contribution of this article is the new established relationship between the erosion rate, seepage, and critical stresses in both steady and oscillatory flows. With the experimental results, we further examine the existing erosion formulae for the relationship between erosion rate and critical shear stresses. This study provides a more accurate theoretical basis for soil erosion under different seepage gradients through a new experimental design and elucidates the systematic influence of seepage on the erosion rate. Although this study only limits itself to shear thinning characteristics, the bed soil is loose sand. This provides a framework for future studies for different kinds of hydrodynamic characteristics.

This paper is structured as follows. In Section 2, the design concept of the in-house-developed erosion–seepage system (ESS) is provided, the experimental procedure is outlined and the properties of the soil samples are provided. In Sections 3 and 4, different upward seepage flows are first quantitatively simulated by applying pressure from the lower part of the soil sample through a pressure device. Then, the erosion characteristics of sandy soils under different flows (both steady and oscillatory) are investigated. The critical shear stress of this soil sample is obtained under different seepage gradients. The relationship between the seepage gradient and the erosion rate is then established under the various flow conditions. In Section 5, two types of erosion formulae are examined based on the experimental data. Finally, key finding of the experimental results are summarised in Section 6, together with the limitations of this study.

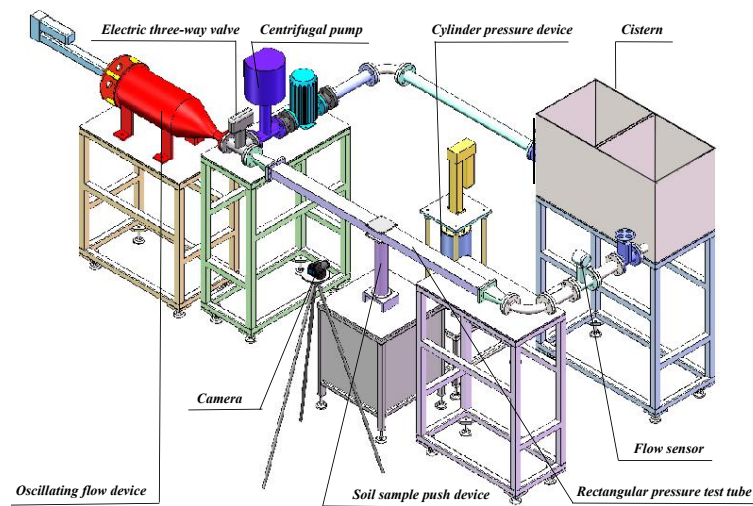
2. Experimental Setup

2.1. Overview of Experiment Facility

The experiments were carried out at the Modern Marine Geotechnical Engineering Laboratory of Qingdao University of Technology. The in-house erosion–seepage system (ESS) is shown in Figure 1, with overall dimensions of 4.6 m in length and 2.9 m in width. The conceptual design of the ESS originated from Briaud et al. [8], in which only steady flow without seepage was considered. Furthermore, only the erosion rate (directly measured) and the shear stress (determined by a theoretical formula) were determined in Briaud et al. [8]. The erosion–seepage system proposed in this study (ESS) includes a water flow circulation device, a cistern filtration device, a soil sample pushing device, and a cylinder pressure device, as shown in Figure 1. An ESS is generally used to study the erosion characteristics of soil particles with a grain size less than 8 mm. It can perform seepage–erosion experiments on cohesive and cohesionless soils under steady and oscillatory flow conditions.



(a) Photo of ESS



(b) 3-D diagram of ESS

Figure 1. The in-house-designed erosion–seepage system: (a) a photo of the system and (b) a 3-D view of the ESS.

2.1.1. Water Flow Circulation Device

The water flow circulation device is composed of a rectangular tube, a centrifugal pump, an oscillating flow device, a flow sensor, an electric three-way valve, a cistern, and a camera (Figure 2). The dimensions of the rectangular tube are $1.6 \text{ m} \times 1.0 \text{ m} \times 0.5 \text{ m}$ (length $\times$ width $\times$ height). It is constructed from fully transparent acrylic, making it airtight and waterproof, and is symmetrical along the centre of the observation area. The centrifugal pump provides a constant flow rate for steady-flow experiments. The flow sensor, located between the cistern and the rectangular tube, records the steady flow velocity through the cross-sectional tube. For oscillatory flow experiments, the electric three-way valve is utilized. The oscillating flow device generates regular and irregular waves via a piston with an oscillation period adjustable from 2 to 20 s. In the designed ESS, to meet the requirements for a higher flow rate, a $1.2 \text{ m} \times 0.6 \text{ m} \times 0.7 \text{ m}$ cistern is designed and the cistern is divided into two parts by a sediment filter. Sand-infused water is pumped to the right side of the cistern, where the sediment filter removes the sediment. This is to ensure that the water flowing to the left side of the tank is clear and without sediment. The camera monitors the experiment and records the experimental data.

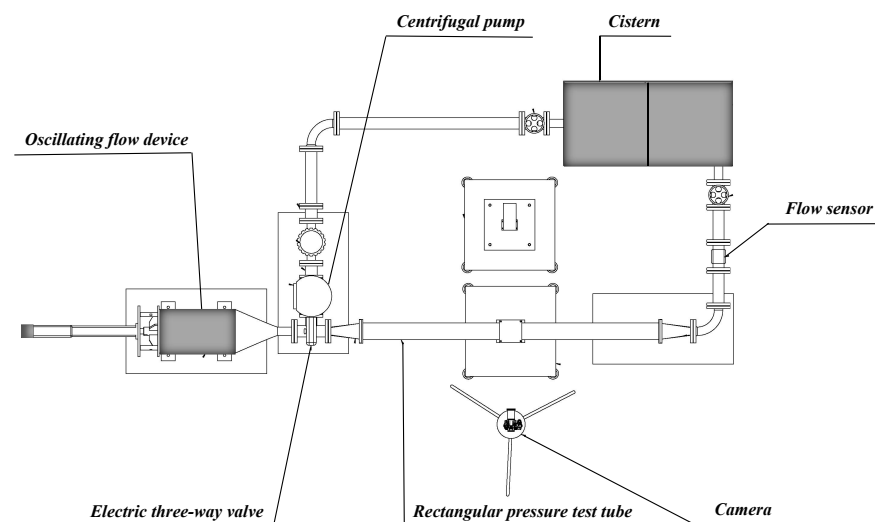


Figure 2. Sketch of the water flow circulation device.

2.1.2. Soil Sample Pushing Device

As shown in Figure 3, the soil sample pushing device allows the upward movement of the soil within the sample tube using a push piston. This device automatically and precisely adjusts the distance to which the soil sample is pushed, with millimetre-level accuracy. Unlike the classic erosion function apparatus (EFA) described by Briaud et al. [8], this design features two pore water pressure transducers mounted on top of the sample tube and above the piston, respectively, to monitor the pore water pressure within the soil sample. The soil sample pushing device is connected to the bottom of the rectangular tube. By controlling the upward movement of the top soil piston during the experiment, a specific height of the soil sample is flushed during the test. Pore water pressure transducers mounted at the top and bottom positions of the sample tubes are used to measure changes in pore pressure in the soil during the experiment, within a range of 0–30 kPa and an accuracy of 0.1%.

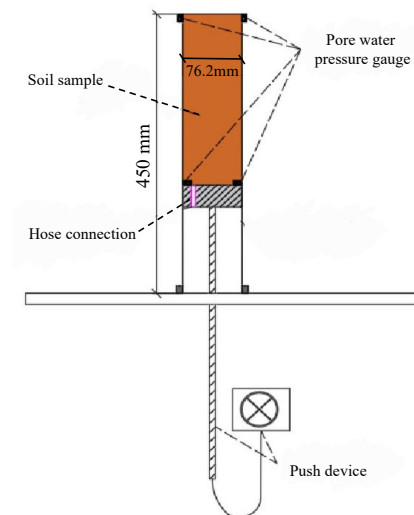


Figure 3. Sketch of the soil sample pushing device.

2.1.3. Cylindrical Pressure Device

Recognising the deficiencies of the erosion function apparatus (EFA) in accounting for seepage within the soil sample, an ESS cylinder pressurisation device (Figure 4) was designed. The bottom of the piston on the soil sample pushing device is connected to the measuring cylinder by a hose. The pressure of the gas inside the cylinder is adjusted by moving the position of the pressurised plate up and down. The upward movement of the water flow within the sample soil body is performed using a connected hose. In addition, an air pressure sensor is positioned 400 mm from the bottom on the right side to accurately measure the air pressure inside the cylinder.

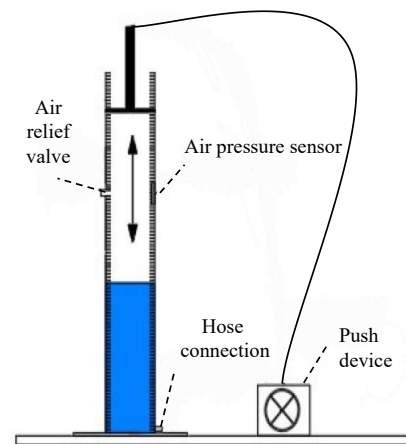


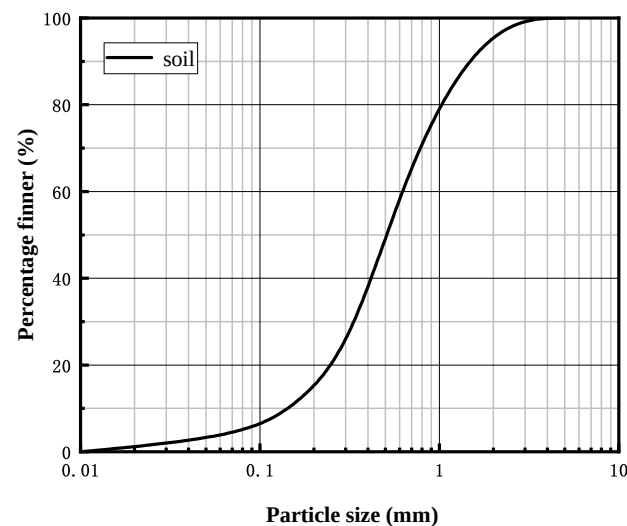
Figure 4. Sketch of the cylindrical pressure device.

2.2. Soil Samples

In this study, soil samples comprising non-uniform sand with a mean particle size (d_{50}) of 0.5 mm were collected from the Yellow River Delta. The particle size distribution of sandy soil was determined using an electric vibrating sieve machine, as shown in Figure 5. The coefficient of permeability of the sandy soil (K) is 3.34×10^{-2} cm/s, the maximum dry density (ρ_{max}) is 1.737 g/cm³ (as measured via the vibration method), and the minimum dry density (ρ_{min}) is 1.273 g/cm³ (as measured via the hopper method), while the specific gravity (G_s) is 2.65. The basic parameters of sandy soils are summarized in Table 1.

Table 1. Basic parameters of sandy soil.

d_{50}	0.5 mm
K	3.34×10^{-2} cm/s
ρ_{max}	1.737 g/cm ³
ρ_{min}	1.273 g/cm ³
G_s	2.65

**Figure 5.** Grain size distribution of sand sample.

2.3. Experimental Process

Before testing, the soil sample was first left outside the test cylinder to saturate. The experiments were carried out using the self-designed erosion–seepage system with the following steps:

- (1) Place the soil sample in the soil sample tube and push it to the bottom of the rectangular test tube.
- (2) Open the cistern valve and fill the tube with water. Let the soil sample sit for an hour so that the water in the water circulation device passes through the soil sample to the measuring cylinder. The water inside the measuring cylinder can be adjusted via the vent valve so that it is in the middle of the measuring cylinder and remains stable.
- (3) To conduct a steady flow test, turn on the water pump to provide a steady flow to the water flow circulation device and observe the flow meter indication until the flow rate remains constant.
- (4) Start the test: push the soil sample 1 mm upward, observe the erosion of the soil sample during the test, and record the time needed for erosion in all the soil samples at this height. Then, continue to push the soil sample upward and cycle the process until the erosion time exceeds half an hour or the total height of the raised soil sample exceeds 10 mm.
- (5) At the end of the set of tests, remove the soil sample and place a new sample in the soil sample tube, repeating steps #1–4. Continue step #5, gradually increasing the pressure applied while maintaining a constant water flow velocity; then, change the water flow velocity.
- (6) To conduct an oscillating flow test, turn the electric three-way valve towards the oscillatory flow device to perform the oscillatory flow experiment.

- (7) Repeat step #5 and keep the pressure value and oscillatory period in the pressure device of the measuring cylinder unchanged for each group of tests, and vary the oscillatory flow velocities.
- (8) After the tests have been performed for each set of flow velocities, remove the soil sample and replace it with a new one. Repeat steps #1–4 and #8, varying the oscillatory flow period and applied pressure values.

2.4. Determination of Erosion Rate (E) and Seepage (j_z)

For each flow velocity, the erosion rate (mm/h) can be determined as

$$E = \frac{h}{t} \quad (1)$$

where h is the height of the soil sample eroded during the time t .

In the ESS system, two pore pressure transducers (nos. 1 and 2) are installed at the top of the sample tube and two (nos. 3 and 4) at the bottom. Since the soil sample pushing device controls the upward movement of the piston, determining the seepage with the pressure gradient is straightforward. The upward seepage gradient (j_z) is calculated by dividing the pore pressure difference (Δp) between the top and bottom of the soil sample by its height (Δh):

$$j_z = \frac{\Delta p}{\Delta h} = \frac{\Delta p}{H - h} \quad (2)$$

where Δp is the pressure difference between the top and bottom of the soil sample (values read from sensors 1 and 3, or 2 and 4), Δh is the vertical distance between transducers, H is the distance between the pore pressure transducers before the erosion test, and h is the distance that the soil sample is pushed upwards. As the pressure applied at the bottom of the soil sample by the pressurization equipment increases, the pore pressure gradient increases. Consequently, the seepage force and its effect become more pronounced. To minimize errors, each flow condition was tested 3–5 times. The final values were then determined by averaging the data, ensuring minimal variation.

3. Erosion Rate and Critical Shear Stress Under Steady Flow (U_s)

3.1. Erosion Rate Under Steady Flow

The relationships between the erosion rate (E) and the seepage gradient (j_z) under steady flow at various flow velocities are presented in Figure 6. In the tests, additional pressures of 0–2.5 kPa (0 kPa, 1 kPa, 1.5 kPa, 2 kPa, and 2.5 kPa) were applied to measure their erosion rates at seven flow velocities (U_s), where the subscript “s” denotes steady flow. The flow velocities were 0.264 m/s, 0.278 m/s, 0.298 m/s, 0.323 m/s, 0.350 m/s, 0.373 m/s, and 0.404 m/s. As shown in Figure 6, the erosion rate increases with the seepage gradient at a constant flow rate. In addition, as the seepage gradient increases, the erosion rates become more pronounced at all flow velocities.

Figure 6 further illustrates the effect of the seepage gradient on the erosion rate at different flow velocities in steady flow. Since the flow velocity is the main reason for the different degrees of erosion of the soil samples, the erosion rate of the soil sample increases with increasing flow velocity. Meanwhile, an increase in the seepage gradient under the same flow velocity is also observed to lead to drastic erosion. This is due to the fact that seepage increases the local pressure change in flow, especially in looser sediments such as Yellow River sand. Flow infiltration can weaken the consolidation of the sand bed, making the surface sediment more susceptible to erosion, which promotes the erosion rate.

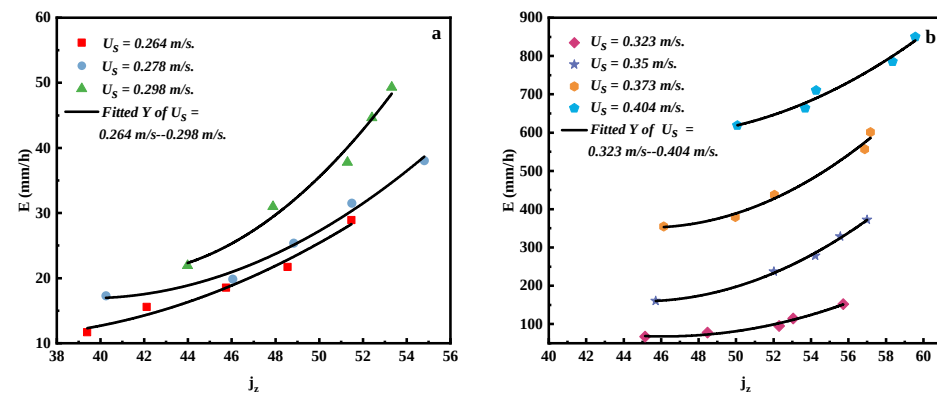


Figure 6. Relationship between erosion rate (E) and seepage gradient (j_z) under steady flow: (a) $U_s = 0.264$ – 0.298 m/s, and (b) $U_s = 0.323$ – 0.404 m/s.

To further investigate the relationship between the erosion rate (E) and the seepage gradient (j_z), the fitting curves of the quadratic function for seven different flow velocities are adopted, as indicated by the solid black line in Figure 6. The unified expression can be expressed as follows:

$$E = a j_z^2 + b j_z + c. \quad (3)$$

In (3), the values of a , b , and c are presented in detail in Table 2. It is clear that the transverse coordinates of the first point in each set of data are greater than the axis of symmetry of the fitted function, which implies that not only does the erosion rate increase as the seepage gradient increases, but also that the rate of increase is increasing.

Since the relationship between soil erosion rate and seepage gradient is not been available in the literature, based on the results presented in Figure 6, we decided to apply a quadratic function in the curve fitting process after several trials with different functions. As shown in Section 4, this quadratic function can also be used for oscillatory flow. However, since this curve fitting was only based on the experimental data for the Yellow Rive Delta, the question of how soil types affect the curve fitting function should be further examined with different soil samples in the future.

In this study, the correlation coefficient R^2 is used to examine the results of the above curve fitting process. In general, the correlation merely indicates the strength of the relationship between two variables, which are the erosion rate and the seepage gradient in this study. Although a high correlation coefficient does not imply the existence of a causal relationship, R^2 values have been commonly used to justify the behaviour of the curve fitting process in engineering applications. Therefore, the correlation coefficient (R^2) is used to show the goodness of the curve fitting process.

Table 2. Coefficients (a , b , and c), correlation coefficients of the fitted function (R^2) and the friction factor (f) under steady flow in (3).

U_s (m/s)	a	b	c	R^2	f
0.264	0.06027	−4.15444	82.43159	0.97986	0.030
0.278	0.09099	−7.15922	157.72309	0.98885	0.028
0.298	0.2381	−19.64198	425.06737	0.99867	0.027
0.323	0.38342	−27.50682	519.91839	0.99858	0.026
0.350	0.61388	−43.3775	861.06014	0.99944	0.025
0.373	0.60747	−39.25821	862.82616	0.99767	0.024
0.404	0.78232	−63.30472	1777.8601	0.99151	0.023

Simultaneous observation of the figures reveals that the seepage gradient at the first point of each data set increases as the water flow velocity increases. According to Bernoulli's equation, when other conditions remain constant, the pressure in the upper part of the soil body reduces when the water flow velocity increases. This reduction increases the pressure difference between the upper and lower parts of the soil body, consequently increasing the seepage gradient.

3.2. Critical Shear Stress Under Steady Flow

In steady flow, the shear stress can be calculated as follows:

$$\tau = \frac{1}{8} f \rho \omega u^2, \quad (4)$$

where f is the friction factor, which is a function of the Reynold's number R_e and the roughness of the pipe ϵ / D , ρ ($=1000 \text{ kg/m}^3$) is the density of water, and u is the mean flow velocity in the tube. Briaud et al. [8] obtained shear stresses by comparing standpipe manometers and transducer calculations and found that the best way to calculate τ for the EFA was to use the Moody diagram [34]. Therefore, in this paper, this approach is also chosen to calculate the shear stress. The hydraulic diameter $D = 67 \text{ mm}$, the mean roughness ϵ is $0.5 d_{50}$, and d_{50} is the mean particle size of the soil (see Figure 5).

As mentioned in Section 1, there are several methods to determine the critical shear stress. In this study, the methodology outlined in McNeil et al. [16] was used to determine the critical shear stress. According to the erosion rate–shear stress curve, the shear stress used is the critical shear stress under the erosion rate of 10^{-4} cm/s (that is, 3.6 mm/h). To reduce the error, the two smallest data points where the erosion rate is greater than 3.6 mm/h are connected, and the straight line formed by the two points is extended. The shear stress corresponding to the intersection of the straight line and the horizontal line $E = 3.6 \text{ mm/h}$ is the critical shear stress (Figure 7). For example, according to Figure 6a, the erosion rate under different seepage gradients is obtained when the water flow velocity is 0.264 m/s and 0.278 m/s , and the water flow velocities are converted into water flow shear stresses according to (4). Finally, the critical shear stress under different seepage gradients can be obtained by substituting this method.

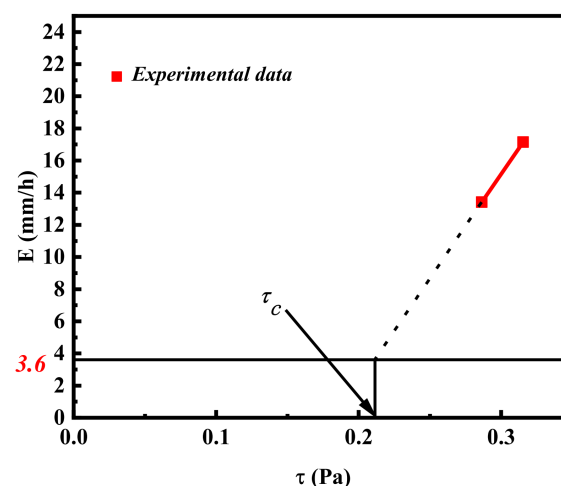


Figure 7. Calculation diagram of critical shear stress.

To reduce the error caused by the fitting function, the seepage gradient was taken from the data points within the tests on the fitted curves with water velocities of 0.264 m/s and 0.278 m/s . Finally, the critical shear stress values calculated according to the method

mentioned above are shown in Figure 8, and a quadratic fit is applied to the data to obtain the following equation:

$$\tau_c = -0.00023j_z^2 + 0.01057j_z + 0.18586 \quad (R^2 = 0.99578) \quad (5)$$

Based on the experimental data, it can be concluded that the critical shear stress is inversely related to the seepage gradient. On the one hand, the upward seepage force increases the mobility of the water within the soil sample, thus reducing the bonding force between the particles; on the other hand, the upward seepage force reduces the combined force of the soil particles in the vertical direction. Both of these make soil particles more prone to initiation, which is also consistent with most of the experimental findings in the literature [28,35,36].

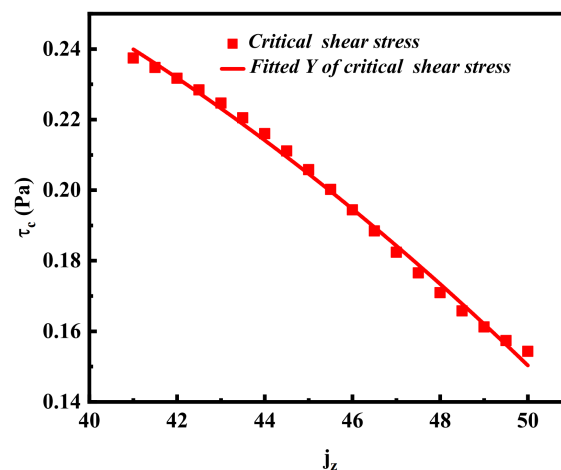


Figure 8. Relationship between seepage gradient and critical shear stress.

4. Erosion Rate and Critical Shear Stress Under Oscillatory Flows

4.1. Erosion Rate Under Oscillatory Flows

In oscillatory flow, the flow direction of the fluid is periodically reversed, creating a series of vortices or eddies in the trapped region [37]. In this study, the experiments were also carried out with different velocities of oscillating flow. In this section, experiments in which the upward seepage force was gradually increased to investigate the effect of different seepage gradients on the erosion process of non-uniform sand are presented. As mentioned in Section 2, oscillatory flow can be implemented in functions with user-defined types such as sine, cosine, and triangle. Sine wave flows were selected in the presented study:

$$U = U_o \sin\left(\frac{2\pi}{T}t + \phi\right) \quad (6)$$

where U_o is the velocity amplitude of the oscillatory flow, T is the oscillation period, t is the time, and ϕ is the phase angle.

The process of change in the surface sediment of the soil samples was first analysed using a video recording of the test (Figure 9). At lower velocities, the finer particles of the surface sediment exhibit slight wobbling and creeping. Over time, larger particles begin to show slight movement. As velocity increases, more sediments become involved in a rocking motion, including particles of the median grain size. At weak points, small groups of sediment can roll up and detach from the soil. When the velocity reaches a certain threshold, large particles also begin to sway and a large amount of sediment on the surface participate in the movement. Under the continuous action of oscillatory flows, some of the

sediment is suspended in the upper water body, causing turbidity and leading to complete erosion of a sediment layer on the surface of the soil body.

Three sets of oscillatory flow experiments were conducted with various wave periods: 2 s, 2.5 s, and 3 s, respectively. Similarly to for the steady flow experiments, pressures of 0–2.5 kPa (0 kPa, 1 kPa, 1.5 kPa, 2 kPa, and 2.5 kPa) were applied to the interior of the soil samples using the cylinder pressurisation device to achieve different seepage gradients. An increase in velocity significantly increased the erosion rate. As can be seen in Figure 10a, the erosion rate at the maximum velocity is 50 times higher than at the minimum velocity. For clearer presentation, the data for each period are plotted as two graphs, as seen in Figure 10.

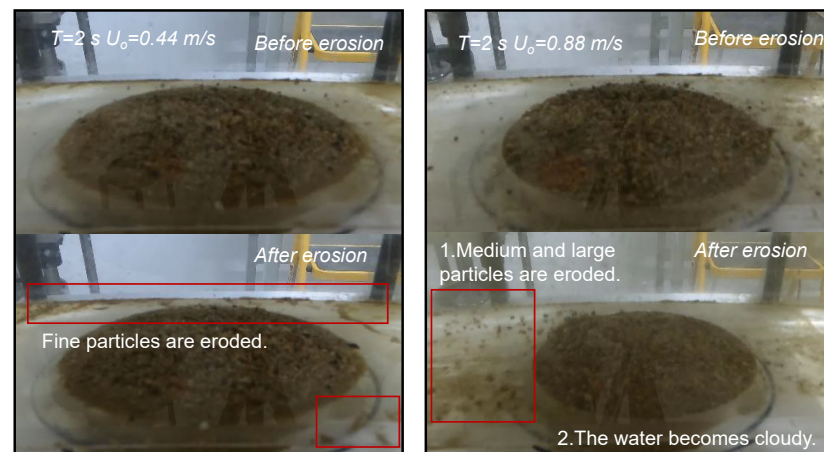


Figure 9. The process of change in the surface sediment of the soil samples.

Figure 10 shows the effect of the seepage gradient on the erosion rate during wave periods of 2 s, 2.5 s, and 3 s, corresponding to Figure 10a–c for velocities of 0.044–0.115 m/s, 0.046–0.092 m/s, and 0.050–0.077 m/s, respectively. The gap in the number of experimental groups is due to the fact that the velocity required for erosion to occur in the surface soil samples is not the same at different periods, and the larger the period, the larger the required velocity. The relationship between the seepage gradient and the erosion rate under oscillatory flow can be fitted by (3), and the coefficients (a , b , and c) are tabulated in Tables 3–5. Based on the results in these tables, it is concluded that the correlation of each fitted curve is greater than 0.89 and can reach a maximum of 0.999, thus demonstrating a strong correlation and indicating that the chosen correlation function is appropriate.

Table 3. Equation coefficients and correlation coefficients of the fitted function under the oscillatory flow period of 2 s.

U_o (m/s)	a	b	c	R^2
0.044	−0.00243	0.69722	−13.12664	0.99275
0.048	−0.01849	1.9852	−23.66683	0.99517
0.053	0.01818	0.32011	8.40298	0.99639
0.062	0.02032	0.43551	17.09479	0.98142
0.071	0.07644	−4.04357	108.71639	0.9788
0.080	0.36104	−22.28846	417.71375	0.99737
0.088	0.75746	−44.33791	742.39741	0.97429
0.097	1.34949	−76.01884	1204.40599	0.93127
0.106	1.04999	−54.54461	922.7167	0.90487
0.115	0.08637	9.4452	124.90174	0.99664

Table 4. Equation coefficients and correlation coefficients of the fitted function under the oscillatory flow period of 2.5 s.

U_o (m/s)	a	b	c	R^2
0.046	−0.01508	1.3226	−23.91969	0.99028
0.049	−0.00264	0.23828	3.2064	0.89144
0.056	0.00652	0.27616	−0.65038	0.99142
0.064	0.05295	−2.42761	42.19304	0.97242
0.071	0.01925	3.20439	−78.57401	0.99724
0.078	0.74206	−48.66713	855.9505	0.99676
0.085	0.53365	−28.20605	399.19072	0.92652
0.092	0.42921	−18.07706	321.58962	0.99057

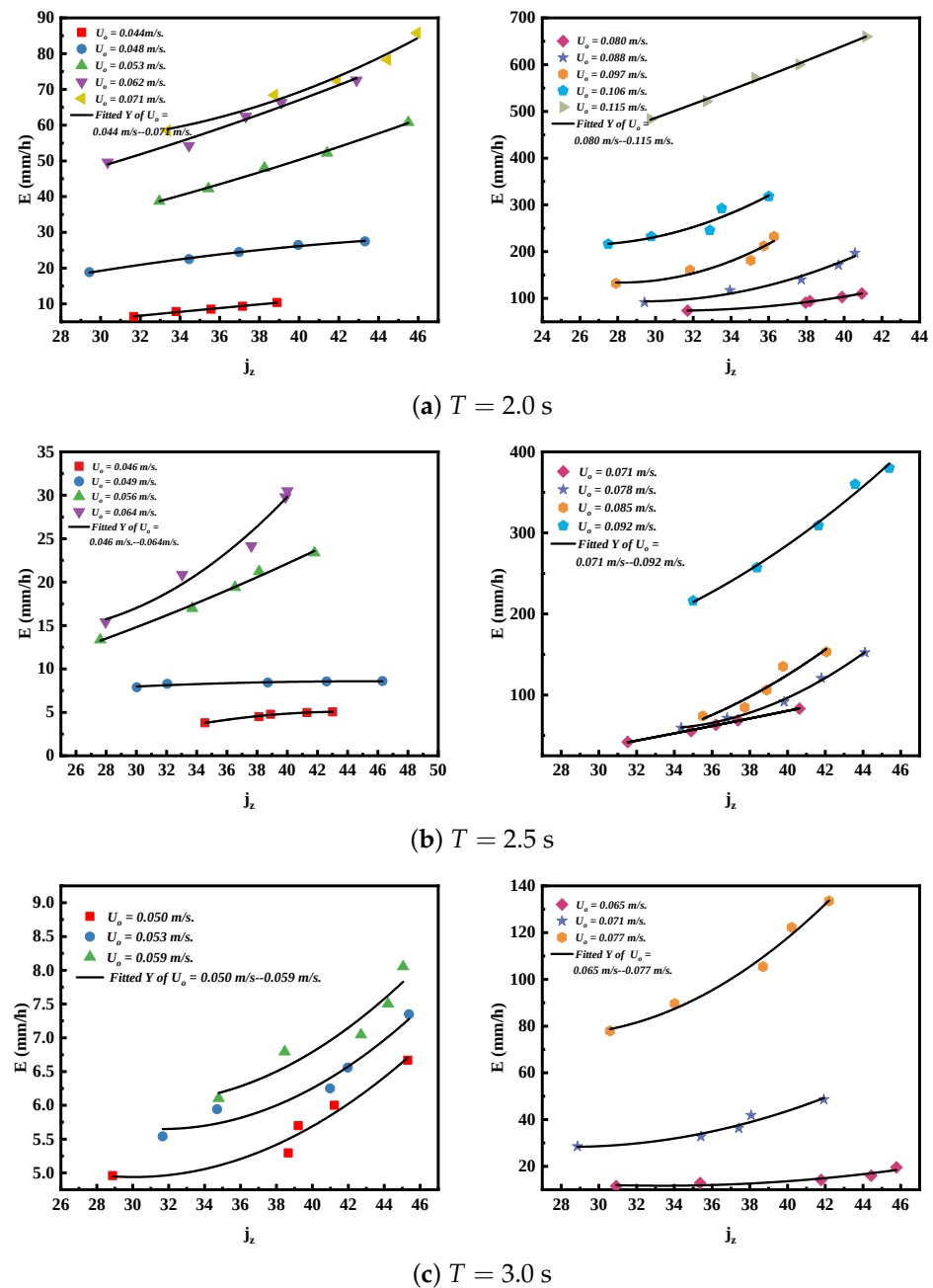
**Figure 10.** Relationship between erosion rate and seepage under oscillatory seepage conditions. (a) $T = 2$ s; (b) $T = 2.5$ s; (c) $T = 3$ s.

Table 5. Equation coefficients and correlation coefficients of the fitted function under the oscillatory flow period of 3 s.

U_o (m/s)	a	b	c	R^2
0.050	0.00761	−0.4573	11.81117	0.95877
0.053	0.00861	−0.54467	14.26148	0.95447
0.059	0.00872	−0.53568	14.26422	0.91357
0.065	0.00838	0.10514	−1.57912	0.97983
0.071	0.09361	−4.88125	91.28363	0.9624
0.077	0.26844	−14.80208	280.32355	0.98526

The shape of the erosion rate curve is usually related to the change in erosion. It can be used to predict the erosion rate at high seepage gradients. Based on the results in Figure 10 and the fitted equations, the fitted curves with velocities of 0.044 m/s–0.048 m/s in a period of 2 s are convex functions, while those with velocities of 0.046 m/s–0.049 m/s in a period of 2.5 s are convex functions. This means that although the erosion rate increases with the seepage gradient, this effect has a decreasing trend. Furthermore, the oscillatory flow rates of these functions are all within 0.050 m/s. At this time, the velocity is small, the surface of the soil sample is in the slow-start state, even though the pressure is applied to the bottom of the soil sample; in addition, the velocity at this time can only remove the smaller particles and the larger particles cannot still be shaken. This may be due to the fact that for the early erosion of the soil surface, the magnitude of the velocity has the main influence, while the change in the seepage gradient plays only an auxiliary role.

In Figure 10, all the curves except the above curve show a concave functional relationship. Furthermore, the water flow velocities of these curves are greater than 0.050 m/s. At this time, the flow rate can make the surface of the soil sample more uniform. The pressure applied to the bottom of the soil sample via a cylinder pressure device enhances the fluidity of the water in the soil. The upward seepage force reduces the combined force on soil particles in the vertical direction, resulting in a significant increase in the rate of erosion. Due to the limited number of tests, it is impossible to know how the erosion rate will change if it continues to increase. However, from the available data it can be seen that the effect of the seepage gradient on erosion is obvious.

When comparing oscillatory flow experiments in three periods, the oscillatory period directly affects the erosion rate of the Yellow River sand. It is found that the erosion growth rate decreases with an increase in the seepage gradient when the flow velocity is less than 0.05 m/s, and increases with increasing seepage gradient when the flow velocity is greater than 0.05 m/s. Thus, this flow velocity appears to be critical as it causes the correlation function between the seepage gradient and the erosion rate to change from a convex to a concave function. That is, the contribution of seepage to the erosion rate goes from increasingly pronounced to weaker. It may thus be difficult to obtain an exact value in an experiment, but the experiments herein indicate the possibility that this relationship exists.

With the same oscillatory flow velocity, the erosion rate is larger with a short period. According to the fitted relationship curves, taking the oscillatory flow velocities of 0.062 m/s, 0.064 m/s, and 0.065 m/s as an example, the change in erosion rate caused by the increase in unit seepage gradient in three cycles can be calculated. The period of 2 s is the largest, 2.5 s is the second, and 3 s is the smallest. The small period means that fluctuation is frequent and that the wave will impact the soil surface more frequently per unit time, causing rapid movement and loss of surface soil particles. When the oscillatory flow velocity is 0.071 m/s, the period of 2.5 s is greater than the amount of change of 2 s. Therefore, the difference in the effect of seepage on the erosion rate between the period of 2 s and 2.5 s is not very significant. However, it is obvious that the effect of seepage on

erosion is the weakest when the period is 3 s; when the frequency of fluctuation is low, the periodic shock produced by the oscillatory flow on the surface is small, and the extrinsic penetration effect cannot significantly strengthen the erosion process.

4.2. Critical Shear Stress Under Oscillatory Flows

The sinusoidal regular wave shown in (6) was chosen for the oscillatory flow experiment. U_o represents the maximum velocity amplitude of the oscillatory flow and the relationship between U_o and the mean flow velocity in the tube (u) can be expressed as follows:

$$u = 2U_o. \quad (7)$$

According to Jonsson [38], the Reynolds number for a wave boundary layer can be expressed as

$$Re = \frac{U_m A_m}{\nu}, \quad (8)$$

where U_m is the maximum velocity of the free-flow wave water quality point, and in this study U_m is U_o ; A_m is the amplitude of the bottom track of the wave water quality point; ν is the viscosity coefficient of water. The relative roughness of the surface of the bed is defined as A_m/k_s , where k_s is the equivalent roughness height of the bed surface.

The boundary layer of laminar flow at the bottom of the waves has a theoretical solution, and its friction coefficient f_w was determined by Jonsson [38], and can be expressed as

$$f_w = \frac{2}{\sqrt{Re}}. \quad (9)$$

By substituting (7) and (9) into (4), we can obtain the shear stress of the bed surface under oscillatory flow:

$$\tau = \frac{1}{2} f_w \rho U_o^2 \quad (10)$$

Following the same procedure used for steady flow, the critical shear stress in oscillatory flows can be obtained. Figure 11 represents the relationship between the seepage gradient and the critical shear stress for periods of 2 s, 2.5 s, and 3 s, respectively. A quadratic fit was made to the data and the following equations were obtained:

$$(a) \tau_c = -0.0025j_z + 0.14189 \quad \text{for } T = 2.0 \text{ s } (R^2 = 0.999)$$

$$(b) \tau_c = -0.00001j_z^2 - 0.00045j_z + 0.08695 \quad \text{for } T = 2.5 \text{ s } (R^2 = 0.999)$$

$$(c) \tau_c = -0.00005j_z^2 + 0.0027j_z + 0.02968 \quad \text{for } T = 3.0 \text{ s } (R^2 = 0.999)$$

Regardless of whether the oscillatory period is 2 s, 2.5 s, or 3 s, the increase in the seepage gradient decreases the critical shear stress to varying degrees. The seepage generated within the seabed exerts a vertical upward seepage force on the sediments located on the seabed surface, which can reduce the critical shear stress on the bed particles by counteracting a portion of the particles' self-weight, further affecting bed sediment initiation and transport. Similarly to steady flow, the presence of seepage forces in oscillatory flow lowers the threshold for sediment initiation, making it easier for sediment initiation to occur and accelerating the sediment erosion rate.

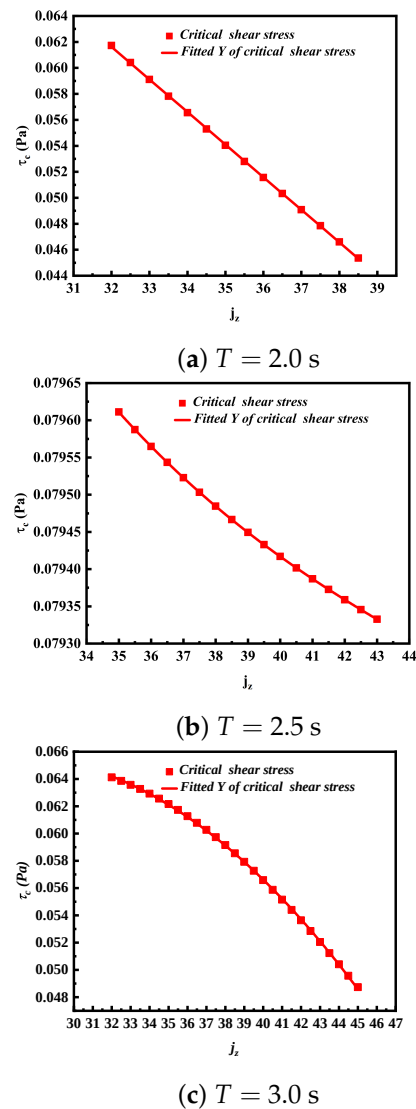


Figure 11. Relationship between critical shear stress and seepage under oscillatory seepage conditions. (a) $T = 2$ s; (b) $T = 2.5$ s; (c) $T = 3$ s.

4.3. Comparison of Seepage Gradient in Steady and Oscillatory Flows

In this section, we discuss the seepage gradient in steady and oscillatory flows. In steady flow, pressures inside soil samples are usually stable and the flow velocity is constant under certain conditions. Therefore, the erosion rate of the soil surface is also stable. The main cause of erosion damage on the surface of soil samples is the change in flow velocity, and external seepage will also have an effect. Through experiments related to the Yellow River sand, it can be seen that the influence of seepage is more obvious in the case of high flow velocity.

In oscillatory flow, the surface erosion of the soil body shows a more complicated pattern of changes due to the periodic change in the flow velocity, and the pressures of the upper and lower parts inside the soil sample also change periodically, although the difference is relatively stable. With increases in flow velocity fluctuations, the erosion rate also changes with the fluctuations of the fluid. The magnitude of the oscillatory flow velocity affects the effect of seepage on erosion. At low flow rates, the experimental data for the three periods show that the effect of seepage on the erosion rate is weak. As the flow rate gradually increases, this effect also expands. High-frequency oscillatory flows can lead to frequent erosion, with the addition of upward seepage resulting in localised

increases in erosion of the soil surface. In contrast, low-frequency oscillatory flow allows for a slower erosion process at the surface of the soil.

5. Relationship of Erosion Rate, Critical Shear Stress, and Seepage Gradient

Critical shear stress (τ_c) is one of the important parameters for sediment erosion and depends on the erosion rate. In this section, the relationship between erosion rate, critical shear stress, and seepage gradient is further examined. In general, two models for sediment erosion have been commonly employed in the literature. They are as follows:

$$\text{Type I: } E = \alpha \exp[\epsilon(\tau - \tau_c)^\varphi] \quad (11)$$

$$\text{Type II: } E = M(\tau - \tau_c) \quad (12)$$

Among these, the type I erosion formulation, in exponential form, is suitable for newly deposited sediments that contain soil through upward seepage [39]. Type II, in a linear form, is usually observed in consolidated beds that have insignificant variations in the consolidation depth with depth [40]. Based on this concept, the erosion cases in the present experiment belong to type I erosion, because the sediments are consolidating beds. This will be further confirmed by analysis of the experimental data.

To further clarify which model should be used for the present study, the relationships between $\tau - \tau_c$ and E for steady flow with seepage gradients of 47 and 49.5 were fitted using type I and II, and the expressions are plotted in Figure 12. Note that the coefficient φ in (11) is set as one, as reported in the literature [22]. Similarly, the relationship between $\tau - \tau_c$ and E at seepage gradients of 35 and 37.5 at different periods of oscillatory flow was fitted using these two models and is illustrated in Figure 13. The numerical values are tabulated in Tables 6 and 7. By comparing the correlation coefficients R^2 , the type I erosion formula is more suitable for sands for steady and oscillatory flow.

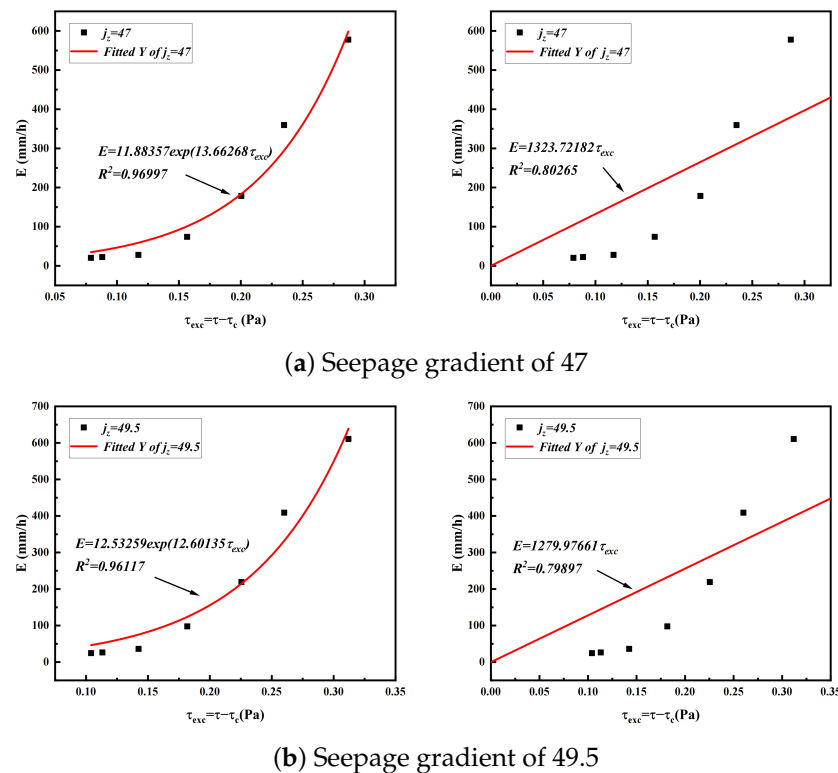


Figure 12. Relationship between erosion rate and $\tau_{exc} = \tau - \tau_c$ for steady flow. (a) $j_z = 47$; (b) $j_z = 49.5$.

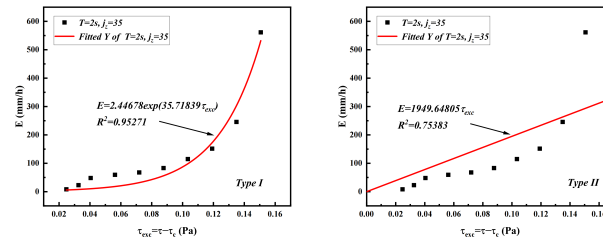
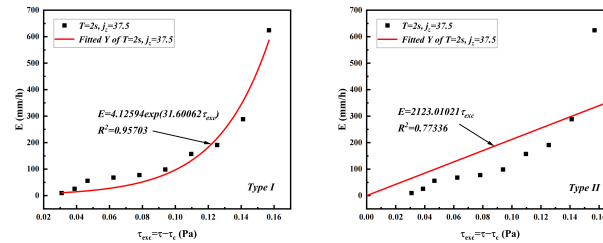
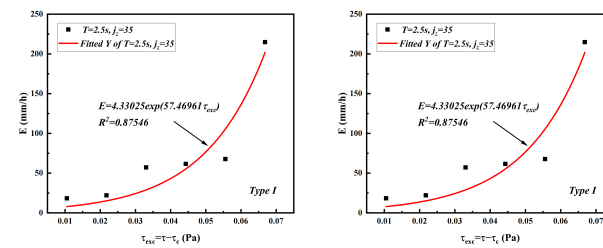
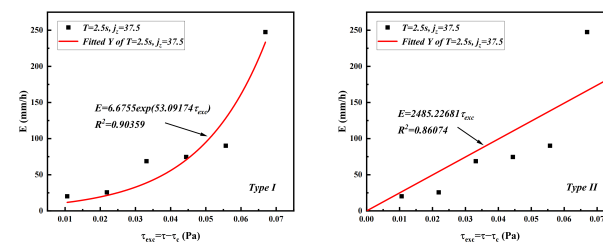
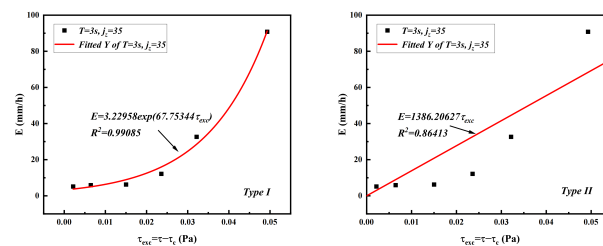
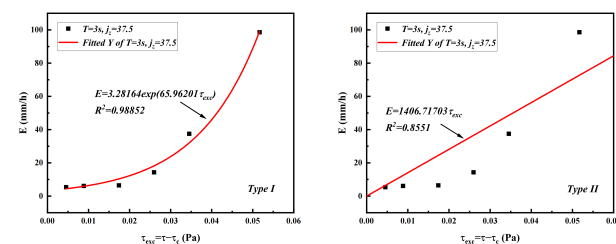
(a) Seepage gradient of 35 under $T = 2$ s(b) Seepage gradient of 37.5 under $T = 2$ s(c) Seepage gradient of 35 under $T = 2.5$ s(d) Seepage gradient of 37.5 under $T = 2.5$ s(e) Seepage gradient of 35 under $T = 3$ s(f) Seepage gradient of 37.5 under $T = 3$ s

Figure 13. Relationship between erosion rate and $\tau_{exc} = \tau - \tau_c$ for oscillatory flow. (a,b) $T = 2$ s; (c,d) $T = 2.5$ s; (e,f) $T = 3$ s.

Table 6. Parameters obtained by fitting using type I and II erosion equations and related parameters under steady flow.

j_z	Type I			Type II	
	α	ϵ	R^2	M	R^2
47	11.88357	13.66268	0.96997	1323.72182	0.80265
47.5	11.83014	13.44287	0.96844	1304.69406	0.80027
48	11.84157	13.22874	0.96686	1290.04824	0.79857
48.5	11.9291	13.02049	0.96522	1280.10442	0.79761
49	12.13599	12.81828	0.96351	1276.56157	0.79768
49.5	12.53259	12.60135	0.96117	1279.97661	0.79897

Table 7. Parameters obtained by fitting using type I and II erosion equations and related parameters under oscillatory flow.

j_z	Type I			Type II	
	α	ϵ	R^2	M	R^2
$T = 2 \text{ s}$					
35	2.44678	35.71839	0.95271	1949.64805	0.75383
35.5	2.63195	35.06471	0.95298	1976.76258	0.7559
36	2.90024	34.2866	0.95356	2010.06657	0.75927
36.5	3.2359	33.43643	0.95443	2045.57998	0.76339
37	3.63585	32.53255	0.9556	2081.78473	0.76799
37.5	4.12594	31.60062	0.95703	2123.01021	0.77336
$T = 2.5 \text{ s}$					
35	4.33025	57.46961	0.87546	2091.65418	0.83553
35.5	4.6711	56.74287	0.87986	2160.86565	0.83948
36	5.07605	55.91274	0.88496	2234.86983	0.84408
36.5	5.54494	55.00996	0.89067	2313.56443	0.84923
37	6.07836	54.06202	0.89692	2397.0535	0.85482
37.5	6.6755	53.09174	0.90359	2485.22681	0.86074
$T = 3 \text{ s}$					
35	3.22958	67.75344	0.99085	1386.20627	0.86413
35.5	3.20657	67.66245	0.99122	1389.47152	0.86193
36	3.28432	66.89066	0.98961	1399.5989	0.86228
36.5	3.24633	66.85636	0.98981	1399.5342	0.85885
37	3.32611	66.06139	0.98819	1406.91371	0.85869
37.5	3.28164	65.96201	0.98852	1406.71703	0.8551

Based on the discussion above, the experimental data were further analysed using the type I erosion equation. Based on a series of relationships between the erosion rate (E) and the corresponding critical shear stress ($\tau_{exc} = \tau - \tau_c$) for various seepage gradient (j_z), the erosion coefficients (α and ϵ) for each round were obtained by exponential regression. Then, a parametric equation between the seepage gradient and the erosion coefficient could be established.

Figure 14 illustrates the erosion coefficients (α) and empirical parameters (ϵ) versus seepage gradient for steady and oscillatory flows. Figure 14a shows the relationship between the seepage gradient and the erosion coefficient in the steady flow in parabolic form. Although there is a slight decrease between $j_z = 47$ and 47.5, the difference in α is 0.05, as seen in Table 6. This relatively small difference is considered negligible because the general trend is that the erosion coefficient increases with the seepage gradient. Therefore, it can be concluded that as the seepage gradient increases, the erodibility of the sandy soil

increases and it is more susceptible to erosion. Similarly to steady flow, as in Figure 14a, oscillatory flow also exhibits a positive relationship between the erosion coefficient (α) and seepage gradient (j_z) for the three periods. However, in comparison, this relationship is not shown as strongly for longer periods. This is probably due to the fact that with a long period, the flow velocity and shear stress are small, which limits the erosion of the sand within the same seepage gradient interval.

The empirical parameters (ϵ) in the type I erosion equation are displayed versus seepage gradient (j_z) in Figure 14b. It is clear that both exhibit an inverse relationship and a strong correlation for both steady and oscillatory flows.

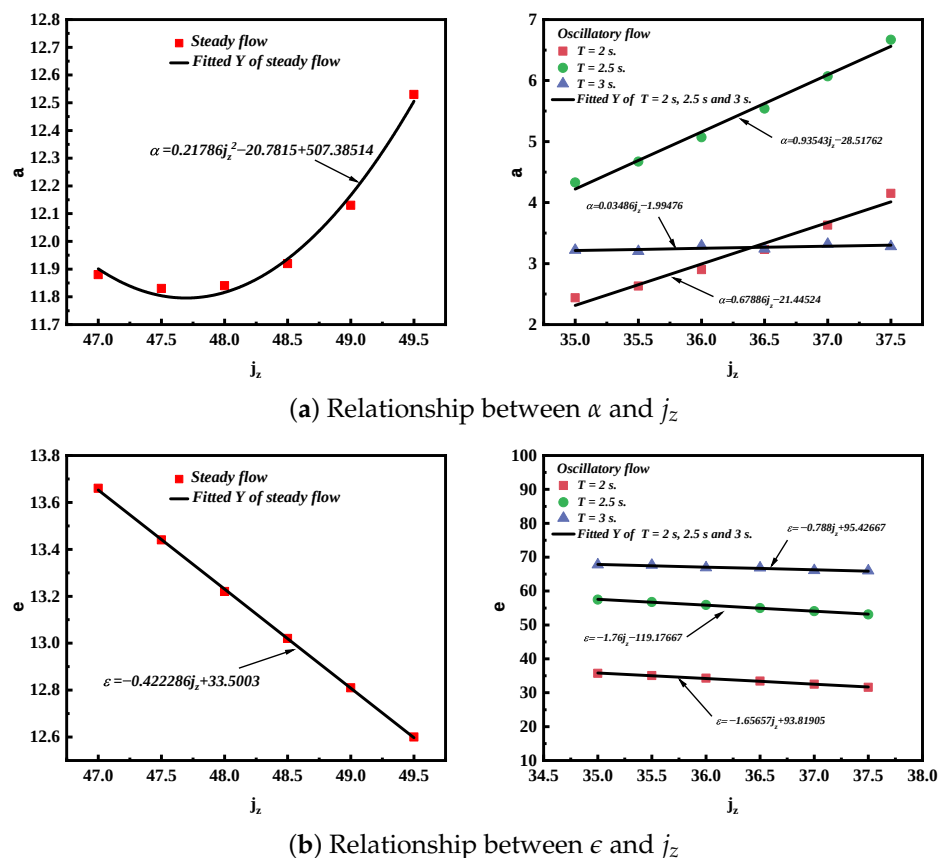


Figure 14. Relationship between erosion coefficients and seepage gradients j_z fitted by the type I erosion equation. (a) α versus j_z ; (b) ϵ versus j_z .

6. Conclusions

In this study, a new erosion–seepage system (ESS) facility is proposed to establish the relationship between erosion rate and seepage in marine sediment under oscillatory and steady flow conditions. The erosion rate and the critical shear stress are determined from the measurement data. Soil samples of medium-grain sand were obtained from the Yellow River Delta. This series of studies is important for elucidating the control mechanisms of soils under the coupled action of seepage and erosion. This will provide an important experimental basis for erosion protection and invaluable information for the study of coastal erosion and localised fluvial and coastal structural scour.

On the basis of the experimental data, the following conclusions can be drawn.

1. Under steady flow, the erosion rate increases with the seepage gradient. The effect is different at different water flow velocities: the effect of seepage on erosion is more pronounced at higher flow velocities of steady flow water. In addition, the critical shear stress decreases with an increasing seepage gradient.

2. Under oscillatory flow, at different velocities, the erosion rate is affected by the seepage gradient with different functions. When the velocity is small, the erosion rate increases with the seepage gradient but shows a slow increase. When the velocity is greater than a certain value, the rate of increase in the erosion rate accelerates. The effect of seepage on erosion is relatively small when the flow is in oscillatory state and over a short period. However, regardless of the velocity, the increase in the seepage gradient increases the erosion rate.
3. Under oscillatory flow, upward seepage can have an effect on the critical shear stress, with slightly different effects at different oscillatory flow periods. The critical shear stress decreases as the seepage gradient increases.
4. The type I erosion equation was used to fit the seepage gradient to the erosion rate, and equations for the erosion coefficients (α) and empirical coefficients (ϵ) versus the seepage gradient for steady and oscillatory flow were obtained. An increase in the seepage gradient was seen to make the sand more erodable.

In this study, only medium-grain sand is considered, but different marine sediments such as coral sand, silt, clay, and mixtures will be studied in the future.

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Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

a, b, c	Coefficients of the fitted equations
d_{50}	The mean grain size
f	The friction factor
f_w	Friction coefficient of the boundary layer of laminar flow at the bottom of the waves
h	The height of the eroded sample
j_z	Pore pressure gradient
k_s	The equivalent roughness height of the bed surface
u	The mean flow velocity in the pipe
A_m	The amplitude of the bottom track of the wave water quality point
E	Erosion rate
H	Original distance between pore pressure transducers before erosion
R^2	Correlation coefficient of the fitted equation
Re	Reynolds number
T	The oscillation period
U_m	The maximum velocity of free-flow wave water quality point
U_o	The maximum velocity of the oscillating flow
Δp	The pressure difference between the top and bottom of the soil sample
Δh	The vertical distance between transducers

ν	The viscosity coefficient of water
ϕ	The phase angle
ρ	The density of water
τ	The flow shear stress
τ_c	Critical shear stress

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