Journal of
Marine Science and Engineering

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A High-Efficiency Theorical Model of Von Karman–Generalized Wagner Model–Modified Logvinovich Model for Solving Water-Impacting Problem of Wedge

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Abstract: The water-impacting behavior of a wedge is often studied in the slamming phenomenon of ships and aircraft. Many scholars have proposed theoretical models for studying the water-impacting problem of a wedge, but these models still have some shortcomings. This study combines Von Karman's method, the Generalized Wagner Model (GWM), and Modified Logvinovich Model (MLM) to establish a converged theoretical Von Karman-GWM-MLM (VGM) model. The VGM model utilizes added mass to replace the fluid influence, which is derived from the velocity potential and boundary conditions. Considering the influence of impulse, the velocity is determined by the momentum theorem. Subsequently, the pressure, resultant force, and acceleration of the wedge can be calculated. By comparing with the published test data of other scholars, it is found that the velocity, acceleration, pressure, and force of the wedge obtained by the VGM model reached a consensus with experiments. The validity and accuracy of the VGM model are also verified. The efficiency and accuracy of problem-solving are both balanced when using the VGM model. The establishment of the VGM model is significant for solving water-impacting problems related to wedges.

Keywords: water impacting; wedge experiment; VGM model; comparison; pressure; force; acceleration; velocity

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Citation: Liu, W.; Liu, T.; Hu, Q.; Wang, M.; Song, X.; Chen, H. A High-Efficiency Theorical Model of Von Karman–Generalized Wagner Model–Modified Logvinovich Model for Solving Water-Impacting Problem of Wedge. *J. Mar. Sci. Eng.* **2024**, *12*, 1125. https://doi.org/10.3390/jmse12071125

Academic Editor: Kamal Djidjeli

Received: 10 May 2024
Revised: 21 June 2024
Accepted: 27 June 2024
Published: 4 July 2024

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1. Introduction

Water impacting on wedge-shaped bodies is crucial for naval architecture engineering, surface aircraft, and ground-effect vessels. Due to the periodic change in buoyancy caused by waves passing through the hull, the ship will experience a heaving motion. The vessel's hull rises above the water surface and then plunges back into the water due to gravity. During impact with the water surface, the vessel's bottom experiences a significant and sudden load. Water-impacting loads are a significant concern for waterborne vessels. Numerous accidents happen due to vessels experiencing significant impact loads. The container ship "MSC Carla" was struck by high waves, which resulted in a structural failure in the cargo hold area of the ship, causing the hull to split into two parts [1]. Defects in the design of the "MOL Comfort" led to insufficient hull strength, resulting in the hull breaking after encountering a significant slamming load [2]. The "PK-GWA" aircraft made an emergency landing on the "Bengawan Solo River" due to engine failure, resulting in structural damage from the impact force on the underside of the aircraft [3]. Many accidents were also caused by impact loads, so it is crucial to study the impact of water on structures.

1.1. Literature Review

The study of water impacting has attracted the attention of many researchers. The bow of water-going vessels is an important aspect that has a significant impact. Typically, the bow of a vessel is designed as a wedge to optimize its speed. Therefore, it is common to conduct studies on the water impacting on wedges. The wedge has a V-shaped section consisting of two bottom plates that form the impact angle. Bottomley [4] conducted water-impacting tests using a V-shaped wedge, which is similar to a seaplane. Then, he measured the acceleration curve and analyzed the maximum impact loads on the wedge. Sedov [5] calculated the water-impacting load of a wedge with a large inclination in the vertical direction and obtained good agreement with the test results. Faltinsen et al. [6,7] utilized the linear Bernoulli equation to calculate the impact loads of a wedge when it entered calm water. They also employed nonlinear potential flow theory to establish the relationship between velocity and pressure during the process of a wedge entering into water. Since the 19th century, numerous scholars have conducted theoretical research, model experiments, and numerical simulations of the water impacting on wedges.

Theoretical methods have been proposed to study the impact loads of wedges, and many scholars have proposed famous mathematical theories. Von Karman [8] first proposed the most famous impacting theory to study the impacting problem of wedges. He used an added mass instead of fluid to analyze the water impacting of the wedge and derived the formula of the water-impacting loads of the wedge through the law of conservation of momentum. According to Von Karman's theoretical research on water entry, Mayo [9] analyzed the problem of seaplane water landing and revised the theoretical results according to the actual situation, once again verifying the feasibility of Von Karman's theory and method. On the basis of the Von Karman model, Wagner [10] modified Von Karman's method and proposed the Generalized Wagner Model (GWM), which was widely used by subsequent scholars. Wagner's research provided a theoretical basis for the academic research on the water impacting of a wedge.

The Generalized Wagner Model (GWM) was proposed by Wagner to consider the phenomenon of water rise based on potential flow theory [10]. The GWM considers the nonlinear term related to the slamming force and calculates the pressure distribution on the wet surface of the wedge through potential flow theory, which provides a new approach for subsequent studies on water impacting. Monaghan [11] applied the GWM to study the problem of seaplane landings; he found that the GWM ignored the loss of momentum when an aircraft landed on the water. Garabedian [12] introduced a method of asymptotic analysis and further studied the GWM when calculating similar solutions. He found a new way for the academic community to study the water-impacting problem. Borg [13] transformed the potential flow problem into a self-similar flow problem by introducing variables. This was another attempt to use a similar flow method. Logvinovich [14] modified the calculation of the velocity potential based on the GWM and obtained the pressure distribution on the wet surface of the wedge body, which also provided a theoretical basis for the MLM.

Korobkin proposed the Modified Logvinovich Model (MLM) by considering a nonlinear higher-order term in the Bernoulli equation. Korobkin et al. [15,16] calculated the higher-order terms of the Bernoulli equation using Logvinovich's method to solve the pressure. And he derived the higher-order terms of Taylor expansion when calculating velocity potential on the surface of the wedge, resulting in highly accurate calculation results. Qin et al. [17] improved the MLM and applied it to the study of water impacting of asymmetric wedges with a rolling motion. Hulin et al. [18] extended the MLM and applied the expanded mathematical theory to study the water impacting of wedges and cones. Qin and Hulin broadened the application of MLM theory. Tassin et al. [19] further expanded the application of the MLM. They enhanced Von Karman's method to accurately depict the water-impacting phase and developed a comprehensive analytical model for offshore engineering. By integrating the MLM with Von Karman's method, Tassin achieved a significant advancement in the theoretical understanding of water impact. The development of the MLM provided a relatively reasonable theoretical method for subsequent scholars to study

the water impacting of the wedge, and accurate data on the surface pressure on the wedge were acquired.

A number of scholars have used Computational Fluid Dynamics (CFDs) simulations to study the water impacting of wedges. Kleefsman et al. [20] used the Finite Volume Method (FVM) and the Volume of Fluid (VOF) to capture the nonlinearities of the free surface when the wedge enters the water and compared the vertical velocity and impact force with experimental results. It was found that the method had good accuracy. Chen et al. [21] carried out a simulation of the water-impacting problem of the wedge, and the relationships between the shape, speed, and maximum impact of the wedge were analyzed. Oger et al. [22] used the Smooth Particle Hydrodynamics (SPHs) method to simulate the water-entering process of a wedge, and it was found that the numerical results agreed well with Faltinsen's experiment, indicating that the SPH method can be applied to the water-impacting problem of a wedge. Zhang et al. [23] combined the implicit interface tracking (level set) method and the Immersed Boundary Method (IBM) to study the water-entering problem, but the pressure distribution on the wedge was not ideal. To study the resonant phenomena in water bodies/waves, Gao et al. [24] used open-source CFD software OpenFOAM to simulate transient fluid resonances in a narrow gap between two adjacent boxes excited by incident focused waves with various spectral peak periods and focused wave amplitudes. To investigate the impact loads induced by incident waves on the dynamic response of symmetrical and asymmetric ships, Southall et al. [25] proposed an approach for predicting impact loads using the open-source CFD code OpenFOAM. They employed an incompressible VOF to capture the liquid level effect. The results were compared with the experimental findings of WILS JIP-III.

The structure may cause vibrations in the water when the ship enters the water, so the water slamming problem has a great impact on marine buildings and coastal structures. To analyze the impact loads of ships navigating in ice channels, Yang et al. [26] used CFDs and the Wagner-type theoretical model to conduct a series of calculations with varying ice sizes and locations. The study examined the hydrodynamic force due to impact, pressure distribution on the wedge surface, and the pile-up phenomenon. To investigate the impact behavior of ship hull structures and ocean waves, Chen et al. [27] employed the CFD-FEM two-way coupling method to explore the fluid-structure interaction problem and slamming loads on a wedge grillage structure when it is falling into waves freely. Additionally, they conducted a model experiment to validate the method's feasibility, and the experimental results aligned with the numerical simulation results. Gao et al. [28] used open-source CFD software, OpenFOAM, to conduct a numerical study on the fluctuation phenomenon of the water body in an enclosed harbor caused by the fall of a wedge. To investigate the effect of dam-break wave on coastal structure, Lo et al. [29] integrated the level set (LS) and immersed boundary (IB) method into the Navier-Stokes equations to setup the two-phase flow solver. The wave-structure interactions and induced motions of floating bodies in two dimensions were studied. The impact load of the free-falling wedge was calculated. To solve the transient gap resonance problem triggered by nonlinear focused wave groups, Gao et al. [30] established a two-dimensional viscous wave flume based on OpenFOAM. The influence of nonlinearity free-surface and heave-freedom on the coupled effects between the fluid and the box heave motion in the clearance was also investigated. To reduce the damage to structures in waves, Li et al. [31] used the CFDs method to simulate the slamming process of floating bodies with different shapes. The impacting loads of floating body under extreme wind and waves were studied. In general, the continuous development and improvement of CFDs technology have greatly improved the technology and level of numerical simulation methods. Scholars can more easily study the wedge water impact problem in more complex situations. However, the computational cost of numerical simulation methods is expensive. Its calculation speed is related to the performance of the computer and takes a long time. Therefore, a high-efficiency method for calculating the water impact problem of wedge is essential.

1.2. Aims and Objectives

Von Karman's method, the GWM, and the MLM are dominating theoretical models to study water-impacting problems of wedges, but they all have shortcomings. Von Karman's theory [8] solved the problem of water impacting on the wedge by using added mass instead of fluid action. However, it neglects the influence of the liquid surface rise on the pressure distribution of the wedge, which leads to much larger calculated values than the actual situation. The GWM [10] considers the effect of liquid surface rise on the calculation and uses potential flow theory to solve the water-entering problem of a wedge, but the boundary conditions are different from the actual conditions, resulting in the failure to obtain the correct load and pressure. The MLM [15] adds the calculation of nonlinear correlation terms on the basis of potential flow theory, and the pressure distribution results are close to the experimental results. However, the calculation of the higher-order terms of the velocity potential is very complicated, the value obtained on the solid-liquid boundary is negative, and it takes a large amount of time and cost to complete the calculation of the wedge entering water.

To improve the efficiency and accuracy of water-impacting problems of wedges for theoretical analysis, this study combines three important theoretical models, Von Karman's method, the Generalized Wagner Model (GWM), and the Modified Logvinovich Model (MLM), to establish a joint theoretical model called the Von Karman-GWM-MLM (VGM) model. The VGM model overcomes the shortcomings of Von Karman's method, the GWM, and the MLM, while making good use of their advantages. The VGM model uses the way of the GWM to calculate the added mass of the wedge as it enters water. On the premise of considering the impulse from water, the instantaneous velocity of the wedge is solved by the momentum conservation theorem. Then, the distribution of pressure and impact load time domain of the wedge are calculated using the relevant formulas in the MLM. The establishment of the VGM model is shown in Figure 1.

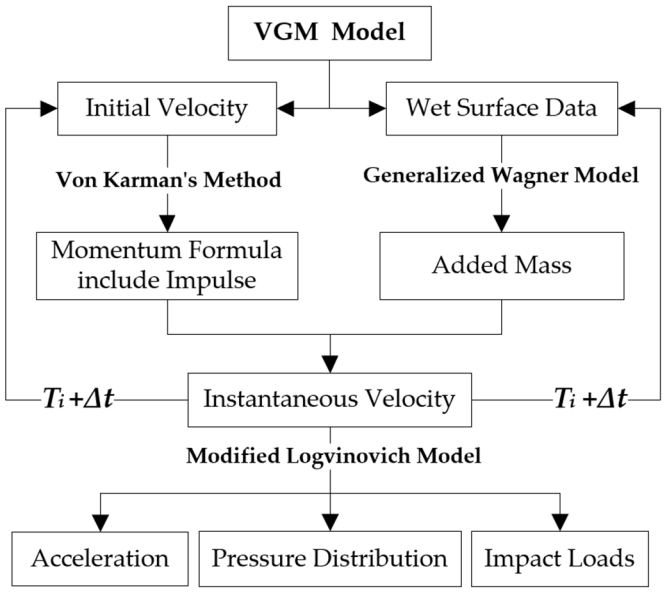


Figure 1. The calculation of the VGM model.

2. Theoretical Model

In this section, the process from establishment to solution of Von Karman's momentum analysis, the Generalized Wagner Model (GWM) and the Modified Logvinovich Model (MLM) will be briefly introduced. Then, the VGM model proposed in this study will be established according to the process and principles in Figure 1.

2.1. Von Karman's Method

The water entry process of the wedge established by Von Karman [8] is shown in Figure 2. Von Karman suggests that the wedge would produce an additional mass of $0.5\rho\pi x^2$ when moving in an infinite basin. So, the total momentum M of the moving system is

$$M = mV + \frac{1}{2}\rho\pi x^2 \cdot V \tag{1}$$

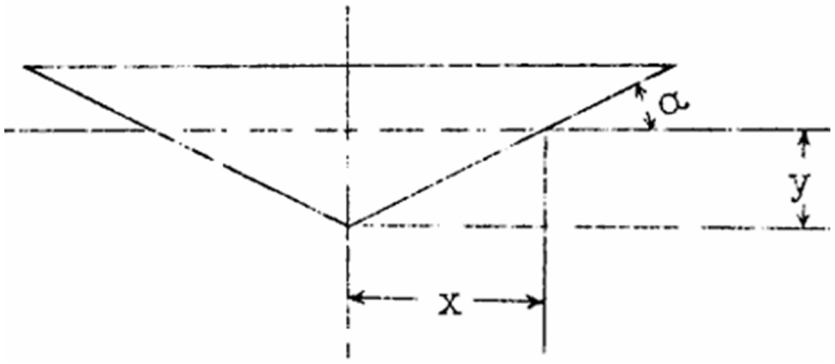


Figure 2. Diagram of the entry process by Von Karman.

Since the velocity V is satisfied:

$$V = \frac{dy}{dt} = tan\alpha \cdot \frac{dx}{dt} \tag{2}$$

Substituting it into Equation (1), the following can be obtained:

$$m \cdot V_0 = m \cdot tan\alpha \cdot \frac{dx}{dt} + \frac{1}{2}\rho\pi x^2 \cdot tan\alpha \cdot \frac{dx}{dt} \tag{3}$$

The relation between sinking velocity and depth is as follows:

$$\frac{dx}{dt}\left(1 + 0.5\rho\pi x^2/m\right) = V_0 \cdot tan\alpha \tag{4}$$

And

$$\frac{d^2x}{dt^2} = \frac{V_0^2 \cdot cot^2\alpha}{(1 + 0.5\rho\pi x^2/m)^3} \cdot \left(-\frac{\rho\pi x}{m}\right) \tag{5}$$

Then, the expression for the force of impact F as

$$F = m\frac{d^2y}{dt^2} = \frac{V_0^2 \cdot cot^2\alpha}{(1 + 0.5\rho\pi x^2/m)^3}\rho\pi x \tag{6}$$

And the average pressure as

$$p = \frac{P}{2x} = \frac{\rho V_0^2}{2} \cdot \frac{\pi \cdot cot\alpha}{(1 + 0.5\rho\pi x^2/m)^3} \tag{7}$$

The above are all the establishment and solution process of Von Karman's theory. However, because Von Karman did not take into account the influence of water surface changes, his method results in large errors. Nevertheless, applying the momentum conservation theorem allows for calculation results to be obtained quickly.

2.2. Generalized Wagner Model

Based on Von Karman's research, Wagner [10] considered the phenomenon of water surface lifting during the process of the wedge entering water. He introduced the potential flow theory to build the Generalized Wagner Model (GWM). The velocity field constructed by Wagner is shown in Figure 3.

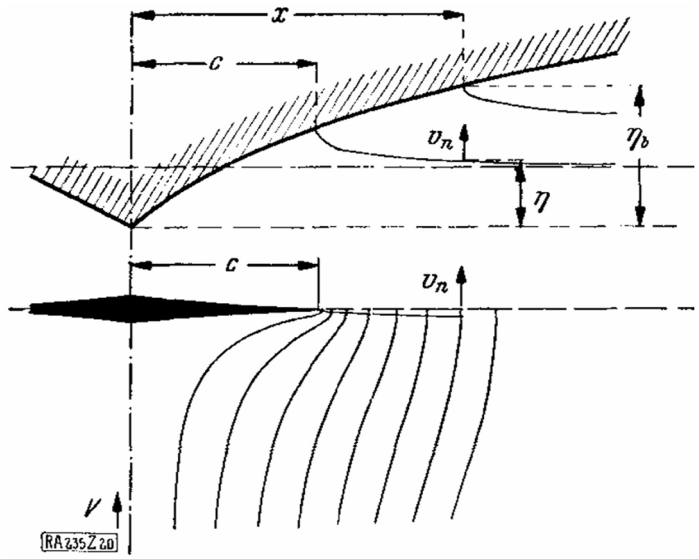


Figure 3. Field of velocity on impact.

Wagner assumed that the fluid was an ideal fluid, irrotational, inviscous, and incompressible. The structure has a symmetrical geometry. The governing equations and boundary conditions in GWM are

$$\begin{array}{ll} \nabla^2\varphi = 0 & y < 0 \\ \varphi = 0 & y = 0 \;\&\; |x| > c \\ \partial\varphi/\partial y = -V & y = 0 \;\&\; |x| < c \\ \partial\eta/\partial t = \partial\varphi/\partial y & |x| > c \;\&\; \eta = 0 \\ p = -\rho \cdot \partial\varphi/\partial t & y \le 0 \end{array} \tag{8}$$

Wagner introduced the complex velocity potential:

$$\Phi = \phi + i\varphi = iVZ - iV\left(Z^2 - c^2\right)^{\frac{1}{2}} \tag{9}$$

He obtained the velocity potential on the surface of the wedge as

$$\phi = -V\left(c^2 - x^2\right)^{\frac{1}{2}} \tag{10}$$

Then, the fluid pressure P is expressed as follows:

$$P = \rho\left(c^2 - x^2\right)^{\frac{1}{2}}\frac{dv}{dt} + \rho v\frac{c}{\sqrt{c^2 - x^2}}\frac{dc}{dt} \tag{11}$$

By integrating the pressure against the wedge area, the impact force F can be obtained as

$$F = \int_{-c}^{c} p dx = \frac{1}{2}\rho\pi \cdot \frac{d}{dt}\left(-vc^2\right) \tag{12}$$

Through the boundary conditions, the GWM can calculate the influence of water surface change on the wedge, but the GWM is only applicable to the wedge with a small dead-rise angle. The GWM is capable of calculating the impact of water entering at a constant speed. However, for the water impact problem of free-falling motion, it is necessary for the GWM to take into account the influence of gravity and the variation in velocity during water entry.

2.3. Modified Logvinovich Model

The Modified Logvinovich Model (MLM) uses the same assumptions as Wagner and solves the wedge water impact problem. The MLM ignores the influence of surface tension and gravity of water. Korobkin [15] added a $\varepsilon = f(L)/L$ when deriving the boundary conditions. The governing equation and boundary conditions of the MLM are

$$\begin{array}{ll} \Delta\overline{\varphi} = 0 & \left(\text{in } \overline{\Omega}(\overline{t})\right) \\ \frac{\partial\overline{\varphi}}{\partial\overline{t}} + \varepsilon\frac{1}{2}|\nabla\overline{\varphi}|^2 = 0 & \\ \frac{\partial\overline{\eta}}{\partial\overline{t}} + \varepsilon\frac{\partial\overline{\eta}}{\partial\overline{x}}\frac{\partial\overline{\varphi}}{\partial\overline{x}} = \frac{\partial\overline{\varphi}}{\partial\overline{y}} & \left(\overline{y} = \varepsilon\overline{\eta}(\overline{x},\overline{t})\right) \\ \frac{\partial\overline{\varphi}}{\partial\overline{y}} = \varepsilon\overline{f}\prime(\overline{x})\frac{\partial\overline{\varphi}}{\partial\overline{x}} - \frac{\partial\overline{h}}{\partial\overline{t}} & \left(\overline{y} = \varepsilon\left[\overline{f}(\overline{x}) - \overline{h}(\overline{t})\right]\right) \\ \overline{\varphi} \to 0 & \left(\overline{x}^2 + \overline{y}^2 \to \infty\right) \end{array} \tag{13}$$

Through the higher-order term of velocity potential of Taylor's expansion wedge into water, Korobkin obtained the velocity potential expression as follows:

$$\Phi(x,t) \approx -\dot{h}(t)\sqrt{c^2(t) - x^2} - \dot{h}(t)[f(x) - h(t)] \tag{14}$$

Therefore, the expression of pressure P and impact force F in MLM are

$$P(x,t) = \frac{1}{2}\rho V^2\left[\frac{\pi}{tan\beta}\frac{c}{\sqrt{c^2 - x^2}} - cos^2\beta\frac{c^2}{c^2 - x^2} - sin^2\beta\right] \tag{15}$$

$$F(t) = \rho V^2 c(t)\left[\frac{\pi^2}{2tan\beta} - K(\beta)\right] \tag{16}$$

$$K(\beta) = \frac{\pi}{tan\beta}\left(\frac{\pi}{2} - arcsin\xi\right) + \frac{1}{2}cos^2\beta\ln\left[\frac{1+\xi}{1-\xi}\right] + \xi sin^2\beta \tag{17}$$

Based on potential flow theory, the MLM utilizes Taylor expansion to incorporate higher-order terms in the calculation of velocity potential and Bernoulli equations. In comparison with the GWM, the MLM demonstrates a broader application range at the dead-rise angle. Nevertheless, the computational process of the MLM is notably intricate, which makes it impossible to obtain the result of wedge water impact quickly.

2.4. VGM Model

The Von Karman-GWM-MLM combined model of the wedge's impact of entry water in this study is constructed according to the approach shown in Figure 1. First, the water-impacting process lasts for a short time, so the momentum conservation theorem, including the impulse from the water, can be used for the velocity after the wedge enters water. The theoretical model in this study learns from Von Karman's method [8], using added mass instead of fluid action. Assuming that the upward force is positive, the force analysis of the wedge body shows that it is affected by gravity and the impact force at the moment of entering the water. The cross section of the wedge when it enters the water is shown in Figure 4.

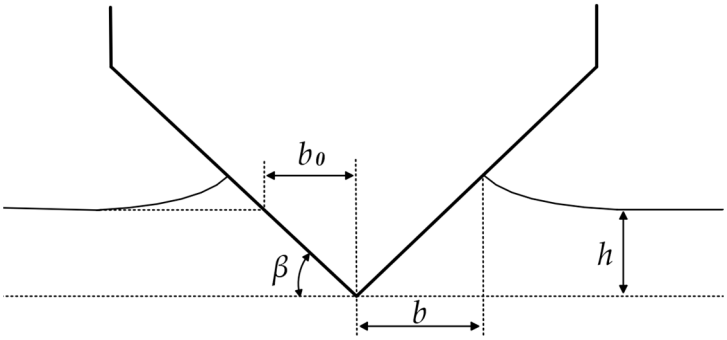


Figure 4. Diagram of the wedge entering water.

In Figure 4, h is the displacement of the dropped wedge, b is the half-width that is wetted, b_0 represents the half-width of the wedge without water rise, and β is the dead-rise angle of the wedge. According to the law of momentum conservation, the momentum of the moving system is

$$M\frac{dV}{dt} = Mg - \frac{d}{dt}(mV) \tag{18}$$

Among them, M is the mass of the wedge, m is the added mass generated during the process of entering water, $m_0 = 0$, and g is the gravitational acceleration. The integration times on both sides of the equal sign in Equation (18) are

$$V = \frac{MV_0 + Mgt}{M + m} \tag{19}$$

where V_0 is the instantaneous velocity in front of the wedge entering water and V is the velocity of the system of motion (wedge and added mass) after the wedge enters the water. The cross section of the wedge when it enters water is shown in Figure 4. The geometric shape of the wedge is as follows:

$$f(x) = x \cdot tan\beta \tag{20}$$

As shown in Figure 4, the half-width b_0 has the following relationship without the water rise phenomenon:

$$b_0 = h/tan\beta \tag{21}$$

To calculate the added mass m, it is necessary to determine the geometric relationship between b_0 and b. Payne [32] proposed a coefficient C_{Payne} to correct the wetted half-width of the wedge, which can be obtained as follows:

$$b = C_{Payne} \cdot b_0 \tag{22}$$

C_{Payne} was calculated as follows:

$$C_{Payne} = \frac{\pi}{2} - \beta(1 - \frac{2}{\pi}) \tag{23}$$

Therefore, the formula for calculating the wet half-width b with the water-rising phenomenon is as follows:

$$b = [\frac{\pi}{2} - \beta(1 - \frac{2}{\pi})]\frac{h}{tan\beta} \tag{24}$$

By analyzing the velocity potential and boundary conditions of the motion system, the expression of the added mass in the GWM [10] was derived as follows:

$$m = \rho b^2(\frac{\pi}{2} - \frac{4tan\beta}{\pi}) \tag{25}$$

The velocity V and the added mass of the wedge at some point in time were determined. Then, this study used the MLM established by Korobkin [15] to calculate the surface pressure distribution and the force of the wedge as follows:

$$P(x,t) = \frac{1}{2}\rho V^2[\frac{\pi}{tan\beta}\frac{b}{\sqrt{b^2-x^2}} - cos^2\beta\frac{b^2}{b^2-x^2} - sin^2\beta] \tag{26}$$

$$F(t) = \rho V^2 b[\frac{\pi^2}{2tan\beta} - K(\beta)] \tag{27}$$

$K(\beta)$ in Equation (27) is calculated as follows:

$$K(\beta) = \frac{\pi}{tan\beta}(\frac{\pi}{2} - arcsin\xi) + \frac{1}{2}cos^2\beta\ln\left[\frac{1+\xi}{1-\xi}\right] + \xi sin^2\beta \tag{28}$$

$$\xi = \sqrt{1-X^2}\,,\ X = \frac{sin(2\beta)}{\pi(1+\sqrt{1-4\frac{sin^2\beta}{\pi^2}})} \tag{29}$$

Since V is a term related to time, both the pressure distribution $P(x, t)$ and the force $F(t)$ change with time. The velocity, pressure distribution, and force of the wedge are obtained. By substituting the time step into V, the time-domain variations in the velocity, pressure, and force parameters can be obtained. Now, the high-efficiency theoretical model (VGM model) for solving the water-impacting problem of wedges is established in this section. The model is verified and discussed in Sections 3 and 4, respectively. The calculation of higher-order terms in the MLM is too complicated and cumbersome, which makes it difficult to reproduce the process. For reasons of time and computational costs, this study uses Von Karman's method, the GWM, and experiments published by other scholars to compare and verify the VGM model.

3. Validation of Analytical Model by Wu's Experiment

3.1. Experiment and Equipment Setup

This study proposes the Von Karman-GWM-MLM (VGM) combined model for solving the impacting loads of a 2D wedge, and it is necessary to validate the VGM model. The study selected a publicized water-impacting experiment as the simulation objective, as in Wu's experiment. Wu et al. [33] carried out a study on a 2D wedge entering water through a free-fall motion based on the theory, analyses, and model experiments. The velocity potential theory was considered in Wu's theory, which ignored the gravity effect of the fluid, the initial condition for the subsequent solution, which was obtained by the same Boundary Element Method (BEM) in a stretched coordinate system, and the time marching technique to follow the body motion and free surface deformation. Many sets of experimental models with varying dead-rise angles were designed and manufactured, and water-impacting experiments were performed. These water-impacting experiments were conducted at Shanghai Jiao Tong University [34]. The acceleration and strain data were recorded using relevant experimental equipment. These data were used to verify the accuracy of the VGM model, and the acceleration of the VGM model was calculated as follows:

$$a = \frac{F(t) - M\cdot g}{M} \tag{30}$$

An outline of the simulation model is shown in Figure 5, and the experimental model and facilities are presented in Figure 6. The experiment is carried out using two wedge models with different dead-rise angles β. The two models are both 60 cm long and 20 cm wide, with a plate thickness of 3 mm. Owing to the presence of friction, the acceleration of the model is not equal to the acceleration of gravity. Wu et al. [33] assumed that the model was subjected to the same friction in water as in air, and that the mass, acceleration, and speed of the wedge drop experiments can be adjusted by changing the initial height and

adding weight to the model. Eight experimental cases were performed in Wu's experiments. In this section, all the cases are verified by comparing the theoretical VGM model proposed in this study with the experimental results from Wu's research. The eight cases and the experimental parameters are given in Table 1.

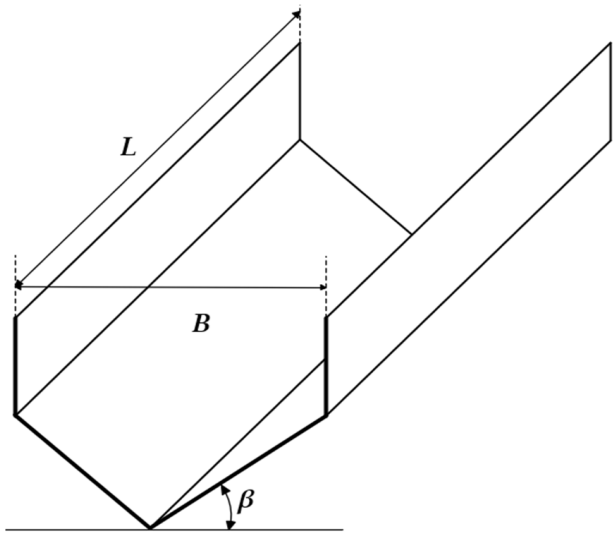


Figure 5. Outline of simulating model.



Figure 6. Experimental model and facilities.

Table 1. The main parameters of experimental model by Wu.

Case	Dead-Rise Angle/β	Acceleration/g	Mass/M	Entry Speed/V
1	$\pi/4$	8.2015 m/s^2	13.522 kg	1.57974 m/s
2	$\pi/4$	8.0062 m/s^2	13.522 kg	0.95623 m/s
3	$\pi/4$	8.9716 m/s^2	30.188 kg	1.69673 m/s
4	$\pi/4$	9.3523 m/s^2	30.188 kg	1.03634 m/s
5	$\pi/9$	7.9228 m/s^2	13.492 kg	1.54405 m/s
6	$\pi/9$	7.8144 m/s^2	12.952 kg	0.86165 m/s
7	$\pi/9$	8.6103 m/s^2	29.618 kg	1.54405 m/s
8	$\pi/9$	9.1091 m/s^2	29.618 kg	0.85462 m/s

3.2. Comparison and Verification

Figure 7 illustrates the comparisons and verifications between the accelerations calculated from three theoretical models and the experimental results of Cases 1 to 4 by Wu. The dead-rise angle β of the wedge in Cases 1 to 4 is $\pi/4$. Observing the results in Figure 7, it is evident that the acceleration calculated using Von Karman's method is consistently smaller than the acceleration recorded in the four cases of Wu's experiments. This outcome is not suitable for engineering applications, which necessitate accurate prediction of wedge water impact. The results of the VGM model in this study closely align with Wu's experiment in Cases 1 and 3. However, in the results of Cases 2 and 4, although the acceleration obtained from the VGM model and the experiment is close in value, it is not exactly the same in the time term of acceleration. Comparing the results obtained by Wu's results and the GWM in Figure 7, it can be found that the acceleration derived from the GWM is larger than that measured by the experiments, especially when the acceleration of the wedge is at its maximum.

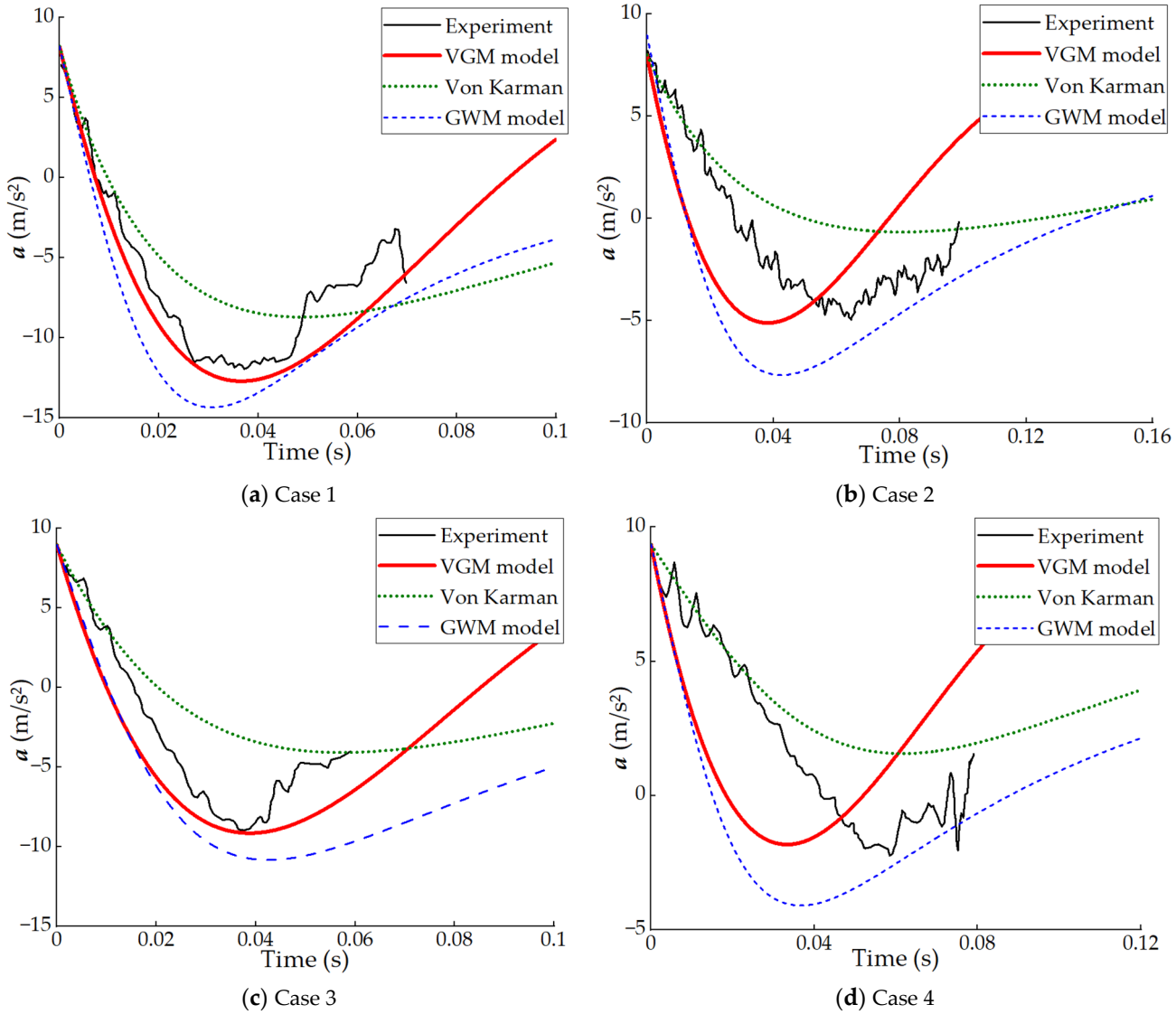


Figure 7. Comparisons between three theoretical models and Wu's experiment in Case 1~4: (**a**) Case 1; (**b**) Case 2; (**c**) Case 3; (**d**) Case 4.

Figure 8 illustrates the comparisons and verifications of the three theoretical models with Cases 5 to 8 in Wu's experiments. The dead-rise angle β of the wedge from Cases 5 to

8 is $\pi/9$. By analyzing the comparison results between theoretical models and experiments in the four cases depicted in Figure 8, it can be observed that the three theoretical models align better with the acceleration measured in Wu's experiment for the calculation of water impact of wedges with a small dead-rise angle. Although the numerical value of Von Karman's method is smaller than the experimental results, the variation trend is more consistent with the experimental results than that of Cases 1 to 4. Regarding the calculated results obtained by the VGM model, it is evident that the value and variation trend of the acceleration obtained by the theoretical model are more consistent with the results of Wu's experiment than Von Karman and the GWM. The GWM calculated in Cases 5 to 8 is greater than that measured by Wu's experiments, especially the acceleration of the wedge after it has reached its peak.

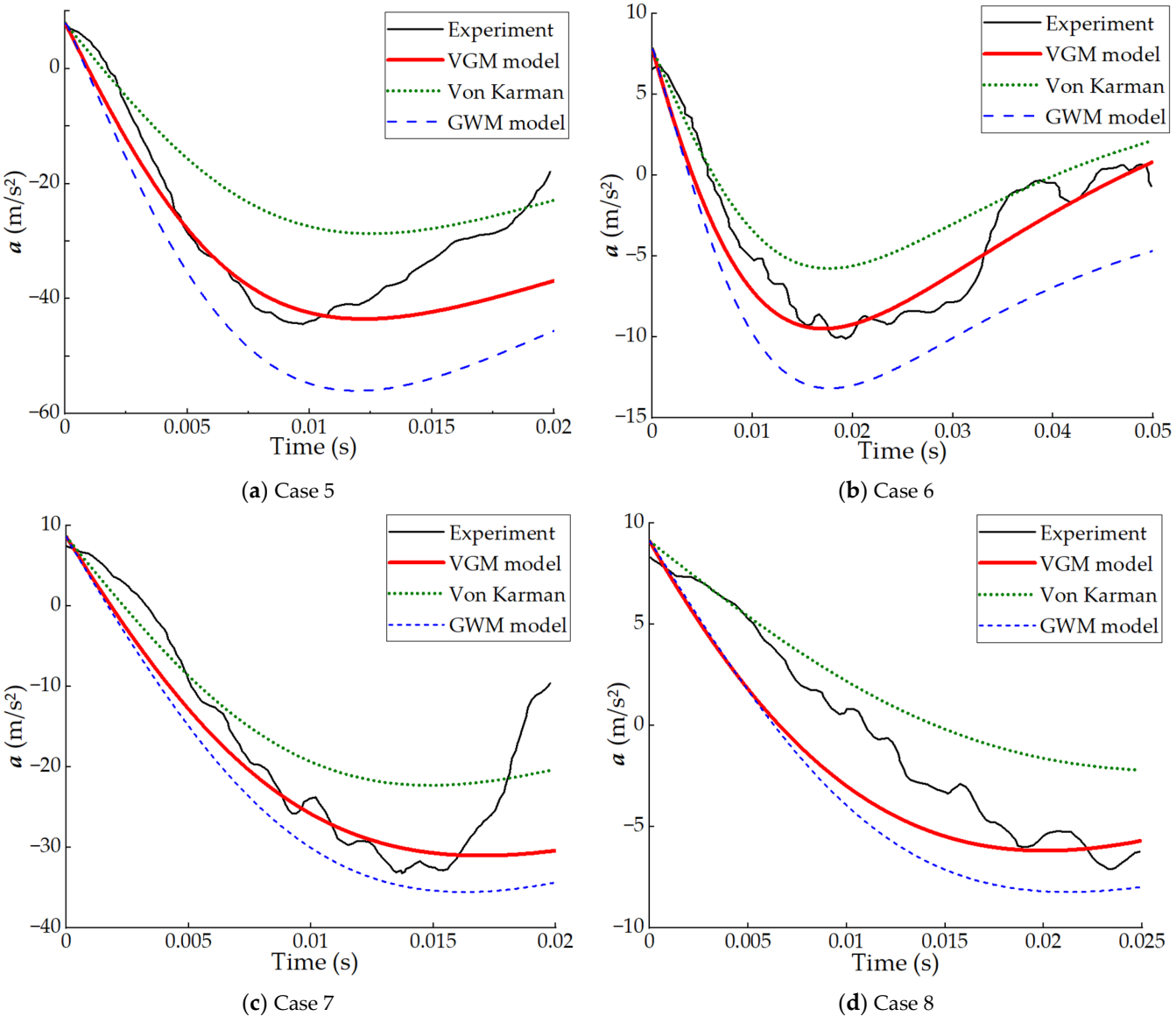


Figure 8. Comparisons between three theoretical models and Wu's experiment in Case 5~8: (**a**) Case 5; (**b**) Case 6; (**c**) Case 7; (**d**) Case 8.

3.3. Analysis and Summary

Figures 7 and 8 show comparisons between the accelerations calculated by three theoretical models and the experimental results obtained in Wu's research. Since the acceleration data provided in Wu's research are not comprehensive, the comparison for each case can

only utilize the data within the corresponding time frame. Additionally, due to the influence of friction in Wu's experimental setup, the magnitude of the gravitational acceleration provided by Wu in each case is not exactly 9.8 m/s^2. This discrepancy contributes to the differences between the theoretical and the experimental results. By observing Cases 2 and 4, it can be found that the acceleration calculated by the VGM model does not entirely align with the experimental results in terms of timing. The VGM model reaches the minimum value earlier than Wu's experiment. The discrepancy may be attributed to the acceleration measured in the experiment being altered by friction on the experimental support.

In Section 3, the results of three theoretical models are compared with those of Wu's experiments. Overall, the accelerations in most cases showed good agreement between the VGM model and Wu's experiments. Because only the acceleration is verified by the VGM model in Section 3, to fully verify the accuracy of the VGM model, more motion and load results, including pressure distribution, force, and velocity, will be analyzed and verified in the next section.

4. Validation of the Analytical Model by WILS JIP-III

4.1. Experimental Setup

Because the value of acceleration is related to the force, the impact force obtained by the VGM model in this study is verified above. The free-falling water impact of a wedge is considered an important part of the third iteration of the Wave Induced Loads on Ships Joint Industry Project (WILS JIP-III). The experimental results are compared to the VGM model to verify the pressure, impacting force, and velocity. WILS JIP-III carried out a series of tests, and wedge experiments were conducted using the equipment of the Korean Maritime and Ocean Engineering Research Institute by Korea Research Institute of Ships and Ocean Engineering (KRISO). The WILS JIP-III provided various data for the free-falling water impact experiment of the wedge model. KRISO varied the drop height and tilting angle and designed wedge models with different dead-rise angles. By arranging force sensors (50×50 mm^2) and pressure sensors, the impacting force and pressure on the bottom were measured dynamically.

The main dimensions and sensor arrangement of the WILS JIP-III wedge model are shown in Figure 9, and the experimental setup is depicted in Figure 10. Two pressure sensors and two force sensors are positioned on the bottom surface of the wedge. The pressure sensor is located on the bottom surface 0.05 m and 0.1 m away from the longitudinal section in the center plane of the wedge, and the force sensor is situated on the bottom surface at a distance of 0.05 m between the middle transverse section and the longitudinal section in the center plane. The wedge measures 0.8 m in length and 0.6 m in width, weighs 85.375 kg, and the dead-rise angles are designed to be 20° and 30°. The drop height of the wedge was determined to be 0.5 m and 0.25 m. KRISO assumed that the experimental tank would be infinitely long and 1.2 m wide. The water depth is 1.0 m. A water blocking plate ensures the two-dimensional flow of the wedge when it enters the water. The impacting phenomenon of the wedge was recorded by a high-speed camera and accelerometer measurements. More details on the experimental setup are available from Kim et al. [35].

The details of WILS JIP-III's experimental parameters are presented in Table 2. KRISO designed ten typical cases to investigate the water impact problem of wedges. Among these cases, a tilting angle θ was incorporated in Cases 13 to 18. These cases cannot be applied to the VGM, Von Karman's method, and the GWM for the calculation of wedge water impact problems. Moreover, it is difficult to find clear data on Cases 11 and 12, so this study only utilizes Cases 9 and 10 for comparing and verifying the three theoretical models with the experimental results.

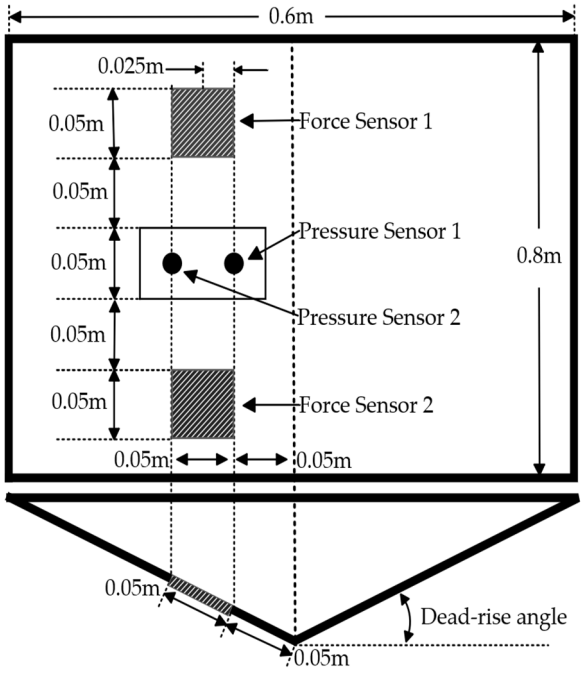


Figure 9. Main scales and sensor arrangement.

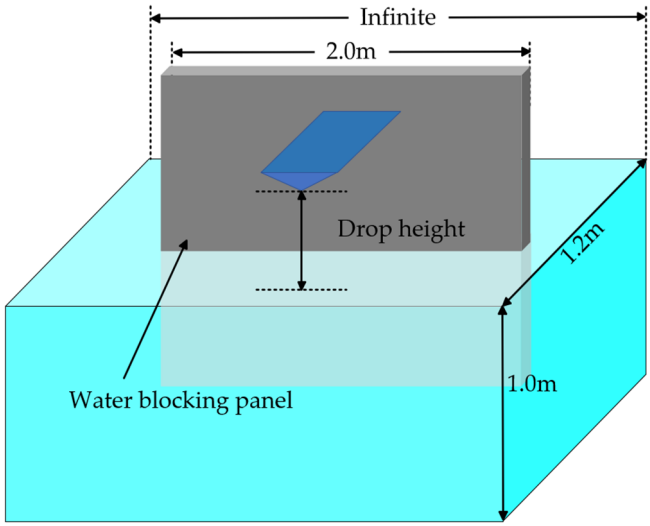


Figure 10. Experimental setup.

Table 2. The relevant parameters by WILS JIP-III.

Case	Dead-Rise Angle/β	Tilting Angle/θ	Drop Height/h
9	30°	0°	0.5 m
10	20°	0°	0.25 m
11	30°	0°	0.25 m
12	20	0°	0.5 m
13	30°	10°	0.5 m
14	30°	−10°	0.5 m
15	30°	20°	0.5 m
16	30°	−20°	0.5 m
17	20°	10°	0.5 m
18	20°	−10°	0.5 m

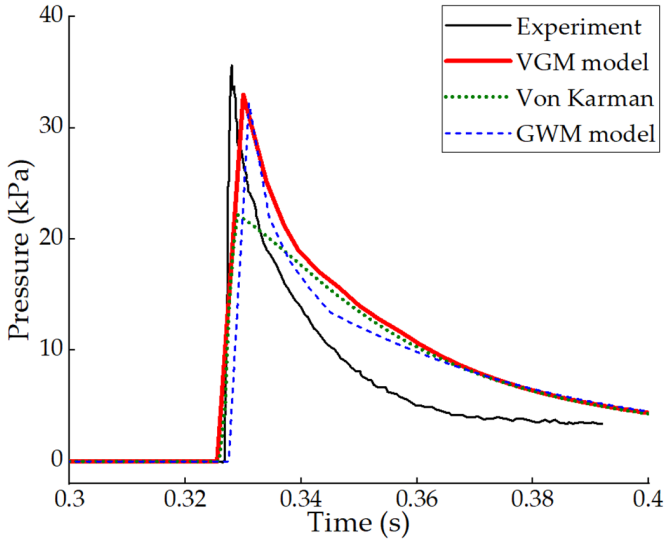
In the WILS JIP-III experiment, the center position of the force sensor is 0.075 m away from the longitudinal plane and 0.075 m away from the transverse plane in the wedge. The size of the force sensor is 50 × 50 mm^2. In this study, the formula for calculating the force is given in Equation (31) as follows:

$$F = P \cdot S_{sensor} \tag{31}$$

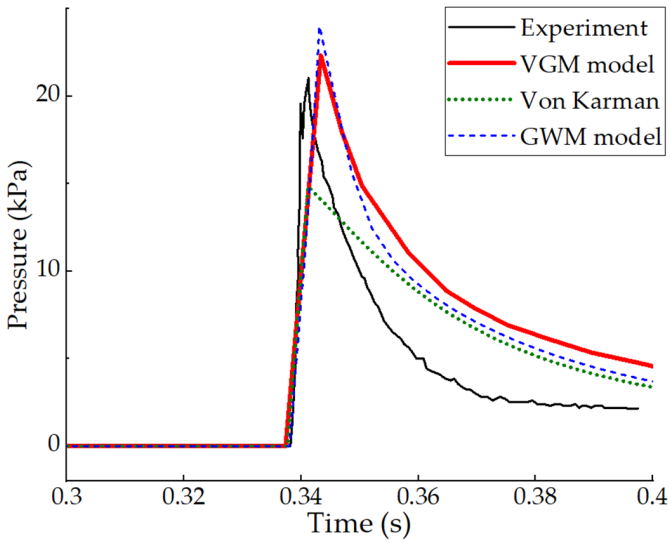
The VGM model in this study first calculates the pressure generated at the center position of the force sensor (0.075 m away from the longitudinal plane) and then multiplies it by the area of the sensor to obtain the force result in this region. Since the VGM model can only calculate the pressure of the transverse section, and the pressure is equal at each longitudinal position, the two force sensors in WILS JIP-III only need to use the data from force sensor 1.

4.2. Comparison and Verification

Figure 11 illustrates the pressure histories recorded by pressure sensors No. 1 and No. 2 for Cases 9 and 10, comparing the experimental data with the numerical results obtained from three theoretical models. The positions of sensors No. 1 and No. 2 are 0.05 m and 0.1 m away from the longitudinal plane, respectively. For the pressure histories of the two locations in Cases 9 and 10, it can be observed from the results in Figure 11 that the calculated values of the VGM model are closest to the experimental measurements of WILS JIP-III. The second is the GWM. There is only a small difference between the calculated results of the GWM and the VGM model in the four comparison graphs. Notably, as the influence of water surface change not taken into account, the pressure values calculated using Von Karman's method are significantly lower than those measured by WILS JIP-III in all four comparison results.

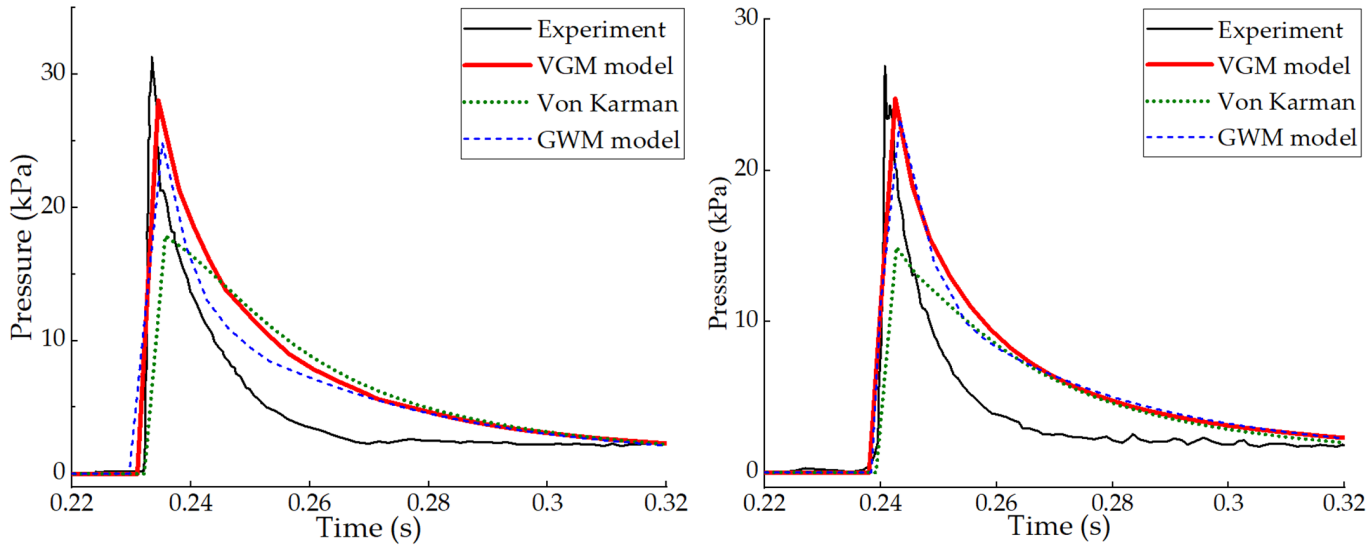


(**a**) Pressure sensor 1 of Case 9



(**b**) Pressure sensor 2 of Case 9

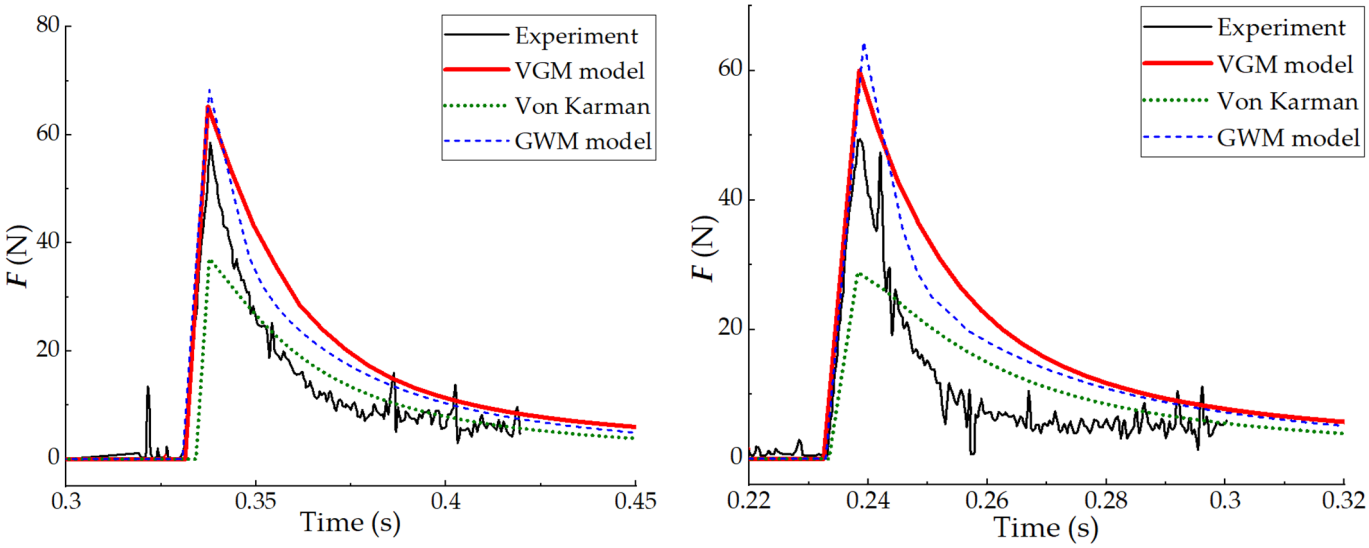
Figure 11. *Cont.*



(**c**) Pressure sensor 1 of Case 10 (**d**) Pressure sensor 2 of Case 10

Figure 11. Comparisons of pressure between WILS JIP-III and three theoretical models: (**a**) Pressure sensor 1 of Case 9; (**b**) pressure sensor 2 of Case 9; (**c**) pressure sensor 1 of Case 10; (**d**) pressure sensor 2 of Case 10.

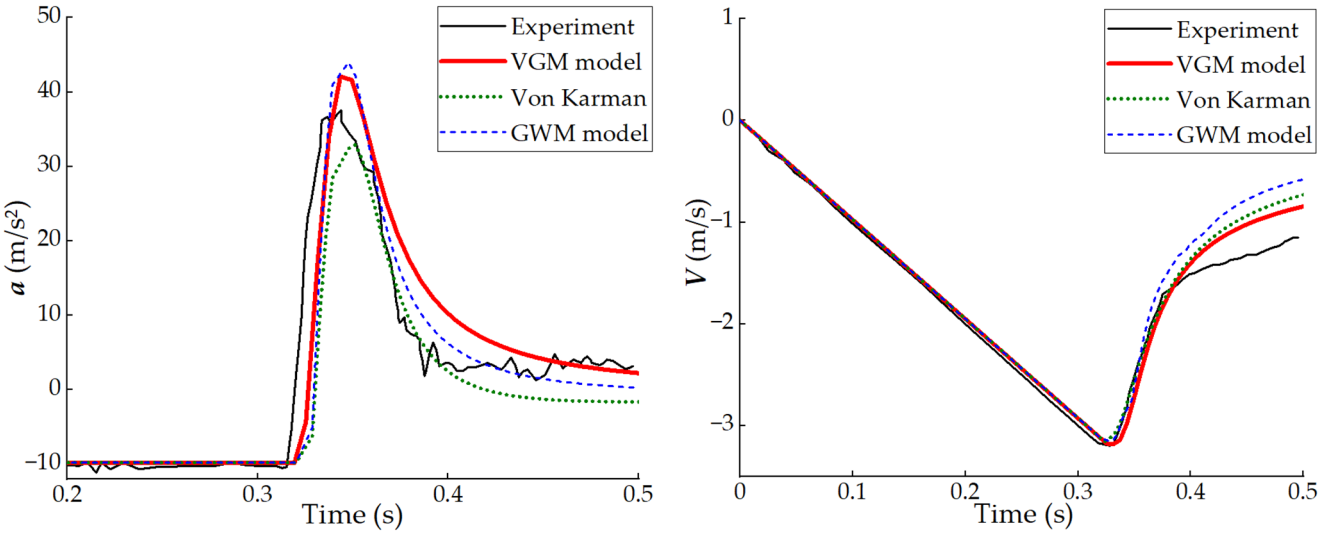
Figure 12 shows the comparisons and verifications between the three theoretical models and the measured values of the force sensor in the WILS JIP-III experiment. It can be observed from Figure 12 that the force values calculated by Von Karman's method are smaller than those measured experimentally. However, the calculated results of the VGM model and the GWM are slightly larger than the experimental measurements of WILS JIP-III. Moreover, the value of the VGM model is closer to the experimental result than the GWM. In both cases of comparing force histories, the VGM model demonstrates good agreement with the WILS JIP-III experiment, and the numerical values obtained by the VGM model are slightly higher than those from the WILS JIP-III experiment results. It indicates that the VGM model can be effectively applied in practical engineering for measuring force histories.



(**a**) Force sensor of Case 9 (**b**) Force sensor of Case 10.

Figure 12. Comparisons of force between WILS JIP-III and three theoretical models: (**a**) Force sensor of Case 9; (**b**) force sensor of Case 10.

Figure 13 shows the comparisons of acceleration and velocity between the data from the WILS JIP-III experiment and the numerical values calculated by three theoretical models in Case 9. Due to the unavailability of acceleration and velocity data for Case 10, only Case 9 is analyzed. It can be observed from Figure 13a that the contrast of acceleration is the same as that of the force sensor. The results obtained by the VGM model and the GWM are greater than the experimental values. Von Karman's results are smaller than the experimental results. It can be seen from Figure 13b that the velocity of the wedge increases at the same rate before contact with water and decreases rapidly after contact with water. The three theoretical models align well with the WILS JIP-III experiment.



(**a**) Acceleration of Case 9 (**b**) Velocity of Case 9

Figure 13. Comparisons of motions between experiment and three theoretical models: (**a**) Acceleration of Case 9; (**b**) Velocity of Case 9.

4.3. Analysis and Summary

The three theoretical models and the experimental measurements of WILS JIP-III are compared in Figures 11–13. The experimental results of pressure sensors compared with the three theoretical models are shown in Figure 11. The experimental results of the force sensor and the three theoretical models are examined in Figure 12. The three theoretical models' numerical results are compared to the experimentally measured acceleration and velocity, as depicted in Figure 13. The findings indicate that the VGM model aligns well with the experimental data.

In Section 4, three theoretical models are evaluated against the WILS JIP-III experiments. Since not all experiments in WILS JIP-III involve vertical free-fall motion, only cases applicable to Von Karman's method, the GWM, and the VGM model are selected for comparison calculations. Simultaneously, the motion and load data obtained by the VGM model are analyzed and verified. The results of Section 4 indicate that the VGM model aligns well with the experiments.

5. Conclusions

This study establishes a high-efficiency theoretical model (VGM model) for solving the water-impacting problem of wedge quickly. The research methods and the strengths and weaknesses of many scholars on the water impacting of a wedge are reviewed in Section 1. The combined model of Von Karman theory-GWM-MLM (VGM) is developed in Section 2. Sections 3 and 4 discussed two experiments conducted by Wu [31] and WILS JIP-III [33]. The experimental results are contrasted and analyzed against the numerical results of the three theoretical models. The pressure, impact force, acceleration, and velocity obtained from the VGM model have been verified. Several conclusions are obtained as follows:

(1) This paper presents an effective VGM theoretical model for solving the wedge water impact problem. Compared with the experiments of Wu and WILS JIP-III and two other theoretical methods, the feasibility and accuracy of the VGM model are verified.
(2) By comparing the results of Cases 1 to 4 and Cases 5 to 8 in Section 3 and Cases 9 and 10 in Section 4, it can be found that the accuracy of the three theoretical models with a small dead-rise angle is better than that with a large dead-rise angle. Therefore, the accuracy of solving with a large dead-rise angle should be improved in future research.
(3) The theoretical models, including the VGM model, Von Karman's method, and the GWM, are only suitable for calculating the vertical water entry problem of a wedge. The influence of asymmetric structure, non-free fall motion, and velocity in other directions on a wedge can be expanded in future research.

This study proposes a high-efficiency theoretical model (the VGM model) for solving the water-impacting problem of wedges. The VGM model is compared with two published impacting experiments, and good agreements are found and obtained. It is anticipated that the VGM model will be utilized to assess practical engineering issues and will be improved as much as possible in the future.

Author Contributions: Conceptualization, T.L. and W.L.; methodology, T.L. and X.S.; software, T.L.; validation and analysis, T.L.; investigation, T.L.; resources, T.L. and H.C.; data curation, T.L.; writing—original draft preparation, W.L.; writing—review and editing, X.S.; supervision, Q.H. and M.W.; project administration, Q.H.; All authors have read and agreed to the published version of the manuscript.

Funding: Wuhan Science and Technology Plan Key Project (2023010402010602), National Science Foundation of China (No. 52101371 and 52071243).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The raw data supporting the conclusions of this article will be made available by the authors on request.

Acknowledgments: All authors gratefully acknowledge the fund support of Wuhan Science and Technology Plan Key Project (2023010402010602) and National Science Foundation of China (No. 52101371 and 52071243).

Conflicts of Interest: The authors declare no conflict of interest.

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