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Distributed Neuroadaptive Formation Control for Aerial Base Station-Assisted Hovercraft Systems with Mixed Disturbances

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Abstract: Effectively addressing the formation control of ABS-assisted hovercraft systems with heterogeneities, unavailable leaders' convex combination states, nonlinearities, and mixed disturbances poses significant challenges. This paper proposes a distributed neuroadaptive formation tracking strategy of ABS-assisted hovercraft systems for the first time, where aerial base stations (ABSs) are composed of unmanned aerial vehicles (UAVs) for data distribution and computation offloading. Firstly, UAVs are designed to track the virtual-leader while shaping a fixed formation, and the observer is devised for each follower hovercraft to estimate the convex combination states of UAVs. Then, output regulation equations are employed to transform heterogeneous systems into a compact form via the Kronecker product, while neural networks (NNs) are introduced to compensate for model nonlinearities. Furthermore, based on random differential equations (RDEs) combined with Lyapunov theory, the noise-to-state practical stability in probability (NSPS-P) property of the error dynamics under mixed disturbances can be obtained. Finally, simulation examples demonstrate that the outputs of follower hovercrafts rapidly achieve a time-varying formation and rotate around convex combination states of leader UAVs simultaneously.

Keywords: ABS-assisted hovercrafts; formation control; distributed observer; output regulation equation; mixed disturbance



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1. Introduction

Unmanned hovercrafts, also known as unmanned air cushion vehicles, are highly versatile amphibious crafts that operate by generating a cushion of air beneath their hulls, enabling smooth travel over various surfaces such as water, land, and ice with minimal friction [1]. Their unique ability to glide across challenging terrains, combined with exceptional maneuverability, makes them an ideal choice for maritime transportation, scientific exploration (particularly in polar or wetland regions) [2], environmental monitoring, and disaster response operations. Compared to a single unmanned hovercraft, the coordination problems involving multiple hovercrafts have captured considerable attention. Notable studies include a multi-hovercraft wireless testbed for validating theories in coordination and distributed computation [3], a bio-inspired pursuit law for multiple hovercrafts using Lyapunov-based control to reduce energy consumption [4], and an embedded distributed model predictive control applied to a team of hovercraft for collective point-to-point transitions, trajectory tracking, and collision avoidance [5]. Moreover, research has also addressed formation-containment control for underactuated hovercrafts, accounting for ocean disturbances and model uncertainties [6].

Nevertheless, effective cooperation methods of multi-hovercraft systems engaged in offshore tasks may encounter challenges, including marine reefs hindering clear transmission channels and heavy reliance on ground stations for information, leading to issues like abnormal data and transmission delays. Aerial base stations (ABSs) are mobile or stationary platforms deployed in the air with essential capabilities to support efficient emergency communications, capable of maintaining stringent network coverage and capacity

demands [7,8]. The main types of ABSs include low-earth orbit satellites; high-altitude platforms such as airships, gliders, and balloons; and low-altitude platforms like unmanned aerial vehicles (UAVs) [9,10]. In contrast to satellites and other high-altitude platforms, which face challenges in achieving ultra-long-distance data transmission and global broadband coverage, UAVs equipped with communication infrastructure can alleviate transmission burdens within offshore unmanned marine vehicle (UMV) groups, due to their ability to dynamically adjust low-altitude locations according to communication demands. In [11], a UAV simultaneously broadcasted secondary information, safeguarded primary transmissions, and optimized the transmission rate and power allocation for maritime vessels. In [12], a UAV equipped with an intelligent reflective surface was deployed to enhance wireless communication services and provide illumination for remote ship users, thereby extending the communication coverage. In [13,14], UAVs served as edge computing units for offshore offloading. In [15], the hovering UAV fulfilled the role of a relay node to ensure timely end-to-end transmissions for ocean monitoring. In the aforementioned literature, UAVs were employed as ABSs, expanding the wireless coverage and providing communication relay solutions in a more energy-efficient and low-latency manner. To the best of our knowledge, ABS-assisted hovercraft systems have not yet been explored in existing research. Given the diverse and complex marine environments, it is both practical and reasonable to consider the application of ABS-assisted hovercraft systems for enhanced adaptability.

In the existing studies related to ABS-assisted operations in marine environments [10–15], most methods were optimum-oriented, aiming to solve the problems of complex resource allocation and task offloading. The ABS-assisted hovercrafts for collaborative operations can be viewed as a formation control challenge for heterogeneous multi-agent systems (MASs), wherein maintaining a specific distance between each agent is essential. Numerous studies have concentrated on cooperative control strategies for heterogeneous MASs. In [16–18], consensus control and formation control were achieved successfully for linear UAV-UMV systems, respectively. However, the more accurate models with the inherent nonlinearities should be considered. To solve this problem, in [19], neural networks (NNs) were employed to estimate lumped uncertainties, and a fixed-time terminal sliding formation control algorithm for the leader UAV and follower UMVs was developed. Nevertheless, the separate control of heterogeneous leaders and followers necessitates a redesign of the output reference signals for leaders. Output regulation equations that can transform heterogeneous MASs into a compact form using the Kronecker product have garnered significant attention [20,21]. Although the aforementioned research on output regulation equations has primarily focused on issues within heterogeneous MASs, it remains highly significant for addressing formation control problems in heterogeneous ABS-assisted hovercraft systems. Moreover, collaborative operations require hovercrafts not just to accomplish the given formation configuration but also to track the states of UAVs based on communication topologies. In [22], UAV-UMV-AUV systems effectively completed the path-following task by employing the asynchronous planning strategy and simulation experiments, with only a single UAV and a single UMV considered. However, ABS-assisted hovercraft systems typically require multiple UAVs to accomplish large-scale collaborative tasks, wherein unknown changes in interaction topologies, such as the breakdown of certain leaders, may lead to instability for heterogeneous systems. To address these challenges, the distributed observer was proposed in [23] to estimate convex combinations within the convex hull formed by multi-leader. From another perspective, each follower hovercraft is typically positioned on the sea surface beneath its corresponding leader ABS, which serves as the tracking target. Through Laplacian matrix analysis, the convex combinations formed by the remaining normal leader ABSs can be viewed as the new rotational center, enabling the continuation of collaborative tasks under the cases of topology changes. Therefore, introducing the distributed observer based on the neighboring relative output information, rather than relying solely on the state information of leaders, is essential for formation control in ABS-assisted hovercraft systems.

It is noteworthy that ABS-assisted hovercrafts frequently operate in random marine environments. The wave disturbances were typically regarded as bounded uncertainties in [6,19,24,25] or white noise in [26]. However, ocean waves comprise a mixture of white noise and the non-white noise, resulting in significant impacts on the control force and the torque of hovercrafts, posing challenges for suppression or compensation using traditional methods to overcome mixed disturbances. Random differential equations (RDEs) described in [27,28] offered a more accurate depiction of the mixed noise with unevenly distributed properties. Building on this, ocean waves were considered a stable random process in [29], and the colored noise spectral density was employed to model the signals received by the ship. Further, some researchers have focused on random MASs and demonstrated the results of noise-to-state stability for error systems [30–32]. Based on RDEs, further exploration on how to analyze and tolerate mixed disturbances in ABS-assisted hovercrafts is challenging yet crucial.

In light of the challenges posed by ABS-assisted hovercraft systems in offshore operations, this paper presents a distributed neuroadaptive formation control protocol that addresses heterogeneous issues, the unavailability of leaders' convex hull states, nonlinear dynamics, and mixed disturbances for the first time. This approach enables UAVs to track the virtual-leader and maintain a fixed formation, while each hovercraft is responsible for tracking the convex combination states of the UAVs and achieving a time-varying formation configuration. The major contributions of this work are outlined as follows:

- (1) Tackling heterogeneous collaboration formation in offshore operations. Output regulation equations are employed to tackle heterogeneous challenges by transforming models into a compact form using the Kronecker product for ABS-assisted hovercraft systems. Additionally, to ensure the desired distances for information reception, a distributed formation tracking protocol is proposed, with follower hovercrafts designed to shape a time-varying formation pattern and encircle leader UAVs.
- (2) Designing distributed observers and nonlinearity compensation. Distributed observers are devised for follower hovercrafts to estimate the output convex combination states of leader UAVs, where each convex combination state is considered as the formation center of the corresponding hovercrafts, effectively circumventing the unpredictable changes in interaction topologies among heterogeneous MASs. Moreover, NNs are employed to compensate for model nonlinearities, thereby broadening the applicability of more accurate actual hovercraft models.
- (3) Addressing mixed disturbances with RDEs. Mixed disturbances acting on hovercrafts are characterized a mixture of white noise and the non-white noise, providing a more accurate description of perturbations compared to solely white noise or precisely observable signals. Furthermore, based on theoretical results of Lyapunov stability for RDEs, the formation error system of hovercrafts is demonstrated to exhibit noise-to-state practical stability in probability (NSPS-P) property.

The structure of this article is organized as follows: Section 2 provides the relevant preliminaries and problem formulations. The proposed formation control protocol and stability analysis are discussed in Section 3. Section 4 gives simulation results. Section 5 offers conclusions and future works.

2. Preliminaries and Problem Formulation

Before presenting the main results of this article, Section 2.1 presents the mathematical symbols employed throughout this paper. Section 2.2 provides the mathematical models of the virtual-leader, ABS, and hovercraft that characterize the leader–follower systems, as well as assumptions regarding disturbances and heterogeneities. Additionally, the graph theory describing the communication relationships between agents, along with the relevant assumptions and lemmas is introduced in Section 2.3. Assumptions addressing nonlinearities are given in Section 2.4, and the control objectives of the entire system are provided as preliminaries in Section 2.5.

2.1. Mathematical Symbols

Throughout this paper, define $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ as the minimum and maximum eigenvalues of the relevant matrix. $\mathbf{O}_N$ and $\mathbf{I}_N$ represent $N \times N$ dimension zero and identity matrices, respectively. $\mathbf{0}$ and $\mathbf{1}$ denote column vectors of appropriate dimensions, with all elements being zero and one, respectively. $\otimes$ is used to define the Kronecker product. $\mathbb{R}$ stands for real numbers. $\mathbb{R}^+$ denotes the set of positive numbers. $\mathbb{R}^{n \times p}$ represents the $n \times p$ dimensional real matrix. The trace of a square matrix is defined as $\text{tr}(\cdot)$. $\|\cdot\|_2$ stands for the two-norm of a matrix or a vector. $\text{diag}(\cdot)$ represents a diagonal matrix. The positive definite matrix P is denoted as $P \succ 0$. For any matrix A , $\sigma(A)$ denotes as its spectrum, i.e., $\sigma(A) \triangleq \{\lambda \mid \det(A - \lambda \mathbf{I}) = 0\}$.

2.2. Mathematical Model Description

2.2.1. The Scenario of ABS-Assisted Hovercrafts

As depicted in Figure 1, a scenario for ABS-assisted multi-hovercraft within the context of collaborative offshore operations is studied in this paper. A specific number of UAVs is deployed in the ABS plane to assist hovercrafts on the sea surface. The roles of each device are delineated as follows:

- Ground station: functions as the command center, transmitting control signals to the virtual-leader, organizing and overseeing the operational scope, controlling the positions of all agents.
- Virtual-leader: determines the motion position of the entire heterogeneous system, serves as the fixed formation center and generates the desired trajectory for the leader ABSs.
- ABSs: consist of multi-UAVs that act as relay stations, receiving commands from the virtual-leader, conducting preliminary data processing, and transmitting the processed data to hovercrafts to alleviate communication burdens of the latter.
- Hovercrafts: navigate the sea to monitor the marine environment, gathering data from neighbor hovercrafts and ABSs to acquire formation instructions. Specific formations or distances must be maintained between each hovercraft and ABS to facilitate information transfer.

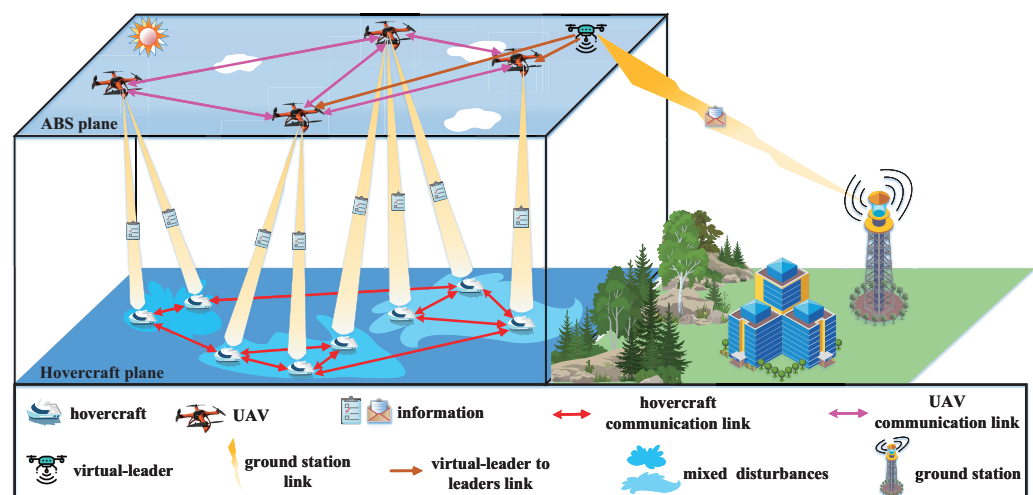


Figure 1. An illustration of the scenario for ABS-assisted hovercraft systems involved in collaborative offshore operations.

In this article, we consider the situation that the virtual-leader has already received command signals from the ground station. Our focus lies in addressing the control issue where the virtual-leader transmits information to UAVs, and UAVs relay this information to hovercrafts, which will execute the formation missions.

2.2.2. The Mathematical Model of the Leader ABS

The k th UAV can be described as follows [33]

$$\begin{cases} \dot{\chi}_k(t) = \eta_k(t) \\ \dot{\eta}_k(t) = \alpha_\chi \chi_k(t) + \alpha_\eta \eta_k(t) + u_k(t), \end{cases} \quad (1)$$

then (1) can be rewritten as

$$\begin{cases} \dot{X}_{Lk} = A_L X_{Lk} + B u_k \\ Y_{Lk} = C_L X_{Lk}. \end{cases} \quad (2)$$

The matrix pairs (A_L, B) and (C_L, A_L) are stabilizable and observable, respectively. The symbols in (1) and (2) are defined in Table 1.

Table 1. The definitions of symbols in (1) and (2).

Definitions	Symbols
the position state vector	$\chi_k(t) \triangleq \chi_k = [\chi_{kx}, \chi_{ky}, \chi_{kz}]^T \in \mathbb{R}^3$
the velocity state vector	$\eta_k(t) \triangleq \eta_k = [\eta_{kx}, \eta_{ky}, \eta_{kz}]^T \in \mathbb{R}^3$
known damping constant matrices	$\alpha_\chi \in \mathbb{R}^{3 \times 3}, \alpha_\eta \in \mathbb{R}^{3 \times 3}$
the control input	$u_k(t) \triangleq u_k \in \mathbb{R}^3$
the output state	$Y_{Lk} \in \mathbb{R}^3$
the position and velocity vector	$X_{Lk} = [\chi_k^T, \eta_k^T]^T \in \mathbb{R}^6$
coefficient matrices	$A_L = \begin{bmatrix} \mathbf{O}_3 & \mathbf{I}_3 \\ \alpha_\chi & \alpha_\eta \end{bmatrix}, B = \begin{bmatrix} \mathbf{O}_3 \\ \mathbf{I}_3 \end{bmatrix}, C_L = [\mathbf{I}_3 \quad \mathbf{O}_3]$

2.2.3. The Mathematical Model of the Virtual-Leader ABS

The virtual-leader serves as the formation center for the leader UAV, and the dynamics model is designed as

$$\begin{cases} \dot{\chi}_0(t) = \eta_0(t) \\ \dot{\eta}_0(t) = (\alpha_{v\chi} + \alpha_\chi) \chi_0(t) + (\alpha_{v\eta} + \alpha_\eta) \eta_0(t) + u_0(t), \end{cases} \quad (3)$$

then (3) can be rewritten as

$$\begin{cases} \dot{X}_0 = (A_L + B K_v) X_0 + B u_0 \\ Y_0 = C_L X_0, \end{cases} \quad (4)$$

where matrix K_v is used to regulate X_0 by adjusting the system matrix to $(A_L + B K_v, B)$, which allows the eigenvalues to be positioned at desired locations [32]. The matrix pair $(A_L + B K_v, B)$ is stabilizable. The symbols in (3) and (4) are defined in Table 2.

Table 2. The definitions of symbols in (3) and (4).

Definitions	Symbols
the position state vector	$\chi_0(t) \triangleq \chi_0 \in \mathbb{R}^3$
the velocity state vector	$\eta_0(t) \triangleq \eta_0 \in \mathbb{R}^3$
known damping constant matrices	$\alpha_{v\chi} \in \mathbb{R}^{3 \times 3}, \alpha_{v\eta} \in \mathbb{R}^{3 \times 3}$
the control input	$u_0(t) \triangleq u_0 \in \mathbb{R}^3$
the output state	$Y_0 \in \mathbb{R}^3$
the position and velocity vector	$X_0 = [\chi_0^T, \eta_0^T]^T \in \mathbb{R}^6$
the controller gain matrix	$K_v = [\alpha_{v\chi}, \alpha_{v\eta}] \in \mathbb{R}^{3 \times 6}$

Assumption 1. The control input u_0 of the virtual-leader satisfies $\|u_0\| \leq \delta$, where $\delta \in \mathbb{R}^+$.

2.2.4. The Mathematical Model of the Follower Hovercraft

The i th follower hovercraft model [34] subject to mixed disturbances is given by:

$$\begin{cases} \dot{\tilde{\chi}}_i(t) = R_i(\psi_i)\tilde{v}_i(t) \\ M_i\dot{\tilde{v}}_i(t) + C_i(\tilde{v}_i)\tilde{v}_i(t) + D_i(\tilde{v}_i)\tilde{v}_i(t) = \tilde{u}_i(t) + \tilde{g}_i\tilde{\xi}_i(t), \end{cases} \quad (5)$$

where $\tilde{\xi}_i$ affects the i th hovercraft due to mixed wave disturbances, which cannot be precisely modeled as white noise. The symbols in (5) are defined in Table 3.

Table 3. The definitions of symbols in (5).

Definitions	Symbols
the position state vector (in the earth-fixed frame)	$\tilde{\chi}_i(t) \triangleq \tilde{\chi}_i = [\chi_{xi}, \psi_i, \chi_{yi}]^T \in \mathbb{R}^3$
position states and the yaw angle	$\chi_{xi}, \chi_{yi}, \psi_i$
the velocity state vector (in the hovercraft-fixed frame)	$\tilde{v}_i(t) \triangleq \tilde{v}_i = [v_{xi}, \omega_i, v_{yi}]^T \in \mathbb{R}^3$
velocity states and the yaw angular velocity	v_{xi}, v_{yi}, ω_i
the control input	$\tilde{u}_i(t) \triangleq \tilde{u}_i = [u_{xi}, u_{\psi i}, u_{yi}]^T \in \mathbb{R}^3$
the random noise	$\tilde{\xi}_i(t) \triangleq \tilde{\xi}_i = [\xi_{xi}, \xi_{\psi i}, \xi_{yi}]^T \in \mathbb{R}^3$
the noise intensity matrix	$\tilde{g}_i = \text{diag}\{g_{xi}, g_{\psi i}, g_{yi}\} \in \mathbb{R}^{3 \times 3}$
the rotation matrix	$R_i(\psi_i) \triangleq R_i \in \mathbb{R}^{3 \times 3}$
Coriolis and centrifugal matrix	$C_i(v_i) \triangleq C_i \in \mathbb{R}^{3 \times 3}$
the inertia matrix	$M_i \in \mathbb{R}^{3 \times 3}$
the damping matrix	$D_i(v_i) \triangleq D_i \in \mathbb{R}^{3 \times 3}$

In Table 3,

$$R_i = \begin{bmatrix} \cos(\psi_i) & 0 & -\sin(\psi_i) \\ 0 & 1 & 0 \\ \sin(\psi_i) & 0 & \cos(\psi_i) \end{bmatrix}, C_i = \begin{bmatrix} 0 & -m(x_g\omega_i + v_{yi}) & 0 \\ m(x_g\omega_i + v_{yi}) & 0 & m(y_g\omega_i - v_{xi}) \\ 0 & -m(y_g\omega_i - v_{xi}) & 0 \end{bmatrix},$$

$$M_i = \begin{bmatrix} m & -my_g & 0 \\ -my_g & J_z & mx_g \\ 0 & mx_g & m \end{bmatrix}, D_i = - \begin{bmatrix} \mathfrak{D}_{v_{xi}} & \mathfrak{D}_{\omega_i} & 0 \\ \mathfrak{D}_{v_{xi}}' & \mathfrak{D}_{\omega_i}' & \mathfrak{D}_{v_{yi}}' \\ 0 & \mathfrak{D}_{\omega_i}'' & \mathfrak{D}_{v_{yi}}'' \end{bmatrix},$$

where m is the mass. J_z is the moment of inertia about the yaw rotation. x_g and y_g are the distances from the origin along the body-fixed surge and sway axes to the center of gravity, respectively. The coefficients $\{\mathfrak{D}_{(\cdot)}, \mathfrak{D}_{(\cdot)}', \mathfrak{D}_{(\cdot)}''\}$ are hydrodynamic parameters [35].

To facilitate the analysis of nonlinear dynamics and disturbance effects in a matched manner, and to more precisely characterize the influence of control inputs and external perturbations, the unmanned surface vessel model typically incorporates the transformation $M_i R_i^T$ within the control channels [36]. Given that hovercrafts are a high-performance class of unmanned surface vessels [1], (5) can be accordingly reformulated as:

$$\begin{cases} \dot{\tilde{\chi}}_i = R_i \tilde{v}_i \\ M_i \dot{\tilde{v}}_i = -C_i \tilde{v}_i - D_i \tilde{v}_i + M_i R_i^T (\tilde{u}_i + \tilde{g}_i \tilde{\xi}_i). \end{cases} \quad (6)$$

Then, it can be inferred from (6) that

$$\begin{cases} \dot{\chi}_i = \eta_i \\ \dot{\eta}_i = f_i + u_i + g_i \xi_i. \end{cases} \quad (7)$$

Subsequently, (7) can be reformulated as

$$\begin{cases} \dot{X}_{Fi} = A_F X_{Fi} + B(f_i + g_i \xi_i + u_i) \\ Y_{Fi} = C_F X_{Fi}, \end{cases} \quad (8)$$

where the matrix pair (A_F, B) is stabilizable, and the pair (C_F, A_F) is detectable. The symbols in (6)–(8) are defined in Table 4.

Table 4. The definitions of symbols in (6)–(8).

Definitions	Symbols
the transformed state vector	$\bar{\eta}_i = R_i \bar{v}_i = [\eta_{xi}, \eta_{\psi i}, \eta_{yi}]^T$
the nonlinear state	$\bar{f}_i(\bar{v}_i, \bar{\eta}_i) = [f_{xi}, f_{\psi i}, f_{yi}]^T \triangleq \bar{f}_i$ $= Q_i \bar{\eta}_i - R_i M_i^{-1} (C_i + D_i) R_i^T \bar{\eta}_i$
the coefficient matrix	$Q_i = [0, 0, -\omega_i; 0, 0, 0; \omega_i, 0, 0] \in \mathbb{R}^{3 \times 3}$
the transformed position vector	$\chi_i = [\chi_{xi}, \chi_{yi}, \psi_i]^T \in \mathbb{R}^3$
the transformed velocity vector	$\eta_i = [\eta_{xi}, \eta_{yi}, \eta_{\psi i}]^T \in \mathbb{R}^3$
the transformed nonlinear state	$f_i = [f_{xi}, f_{yi}, f_{\psi i}]^T \in \mathbb{R}^3$
the transformed control input	$u_i = [u_{xi}, u_{yi}, u_{\psi i}]^T \in \mathbb{R}^3$
the transformed general noise	$\xi_i = [\xi_{xi}, \xi_{yi}, \xi_{\psi i}]^T \in \mathbb{R}^3$
the transformed noise intensity matrix	$g_i = \text{diag}\{g_{xi}, g_{yi}, g_{\psi i}\} \in \mathbb{R}^{3 \times 3}$
the output state	$Y_{Fi} \in \mathbb{R}^3$
the position and velocity vector	$X_{Fi} = [\chi_i^T, \eta_i^T]^T \in \mathbb{R}^6$
coefficient matrices	$A_F = \begin{bmatrix} \mathbf{O}_3 & \mathbf{I}_3 \\ \mathbf{O}_3 & \mathbf{O}_3 \end{bmatrix}, C_F = [\mathbf{I}_3 \quad \mathbf{O}_3]$

For mixed disturbances, the following assumptions are required to ensure that both the noise intensity g_i and the mean square of the random process ξ_i are bounded.

Assumption 2. The intensity matrix g_i is bounded and satisfies $\|g_i\|_2 \leq g_{im}$, where $g_{im} > 0$.

Assumption 3 ([27]). The process vector ξ is $\mathcal{F}_t$ -adapted and piecewise continuous. Moreover, there exists a constant $\phi > 0$ such that

$$\sup_{t \geq t_0} E \|\xi\|^2 \leq \phi. \quad (9)$$

The following assumption is a standard condition for the regulator equations in heterogeneous MASs.

Assumption 4 ([37,38]). The following regulator equation:

$$\begin{cases} \Omega_i A_L = A_F \Omega_i + B U_i \\ C_L = C_F \Omega_i \end{cases} \quad (10)$$

has the solution pair (Ω_i, U_i) .

2.3. Graph Theory

The communication relationships between agents are described as follows. Consider a group of $N + M + 1$ agents in ABS-assisted hovercraft systems. Let 0 label the virtual-leader, $i = 1, 2, \dots, N$ represent hovercrafts as followers, and $k = N + 1, N + 2, \dots, N + M$ signify ABSs as leaders. Define $\mathbb{F} = \{1, 2, \dots, N\}$ and $\mathbb{L} = \{N + 1, N + 2, \dots, N + M\}$ as the follower set and the leader set, respectively. Let $N + M + 1$ agents communicate over a static and undirected graph $\mathcal{G}(\mathcal{V}, \Gamma, \mathcal{A})$, where $\mathcal{V} = \{v_0, v_1, \dots, v_{(N+M)}\}$ is the node set,

$\Gamma = \{(v_n, v_j) : v_n, v_j \in \mathcal{V}, n \neq j\} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set of two neighbor nodes v_n and v_j , and $\mathcal{A} = [a_{nj}]_{(N+M+1) \times (N+M+1)}$ is denoted as the adjacency matrix, respectively. The element $a_{nj} > 0$ if and only if $(v_n, v_j) \in \Gamma$. This paper assumes that $\mathcal{G}$ has no self-loops. The degree matrix of the graph $\mathcal{G}$ is labeled as $\mathcal{D} = \text{diag}\{\sum_{j=0}^{N+M} a_{0j}, \dots, \sum_{j=0}^{N+M} a_{(N+M)j}\}$. Denote $\mathcal{L} = \mathcal{D} - \mathcal{A} \in \mathbb{R}^{(N+M+1) \times (N+M+1)}$ with $l_{nj} = -a_{nj} (n \neq j)$ and $l_{nn} = \sum_{j=0, j \neq n}^{N+M} a_{nj}$ as the Laplacian matrix of $\mathcal{G}$.

The communication among the virtual-leader, ABSs, and hovercrafts can be described as follows:

$$\mathcal{L} = \begin{bmatrix} 0_{1 \times 1} & \mathbf{0}_{1 \times N} & \mathbf{0}_{1 \times M} \\ \mathbf{0}_{N \times 1} & \mathcal{L}_1 & -\mathcal{L}_2 \\ -\mathcal{L}_4 & \mathbf{0}_{M \times N} & \mathcal{L}_3 \end{bmatrix} \in \mathbb{R}^{(N+M+1) \times (N+M+1)}, \quad (11)$$

where dimensions of subsections are selected as $\mathcal{L}_2 = (\mathbf{n}_1, \dots, \mathbf{n}_M) \in \mathbb{R}^{N \times M}$, $\mathcal{L}_4 \in \mathbb{R}^{M \times 1}$, $\mathcal{L}_1 = \mathcal{L}'_{N \times N} + \sum_{l=1}^M \text{diag}(\mathbf{n}_l) \in \mathbb{R}^{N \times N}$, $\mathcal{L}_3 = \mathcal{L}'_{M \times M} + \text{diag}(\mathcal{L}_4) \in \mathbb{R}^{M \times M}$. $\mathcal{L}'_{N \times N}$ and $\mathcal{L}'_{M \times M}$ represent the Laplacian matrices for the follower hovercrafts and leader ABSs, respectively.

Assumption 5. The graph $\mathcal{G}$ is connected. The virtual-leader is the root node, and there is at least one ABS with a path to every hovercraft.

2.4. Neural Networks

Drawing upon the Stone–Weierstrass approximation theorem [39], NNs have the capability to approximate the unknown smooth function $f_i(\mathbf{x}_i)$, ($i \in \mathbb{F}$). To proceed, we introduce the following assumption.

Assumption 6 ([40]). In a sufficiently large compact set $Y_i \subset \mathbb{R}^q$, arbitrary accuracy can be achieved in approximating the unknown smooth function $f_i(\mathbf{x}_i)$

$$f_i(\mathbf{x}_i) = W_i^T \Psi_i(\mathbf{x}_i) + \epsilon_i, \quad \forall \mathbf{x}_i \in Y_i, \quad (12)$$

where $W_i \in \mathbb{R}^{s \times q}$ is the ideal weight matrix satisfying $\|W_i\|_2 \leq W_m$, $\Psi_i(\mathbf{x}_i) = [\Psi_{i1}(\mathbf{x}_i), \Psi_{i2}(\mathbf{x}_i), \dots, \Psi_{is}(\mathbf{x}_i)]^T : \mathbb{R}^q \rightarrow \mathbb{R}^s$ is the basis function vector satisfying $\|\Psi_i\|_2 \leq \Psi_m$ with $\Psi_{ip}(\mathbf{x}_i) = \exp\left(\left\|\mathbf{x}_i - \mathbf{o}_p / \mu_p\right\|\right)^2$ and $p = 1, 2, \dots, s$. The center of the receptive field is denoted as $\mathbf{o}_p$. The width of the basis function is represented as μ_p . $\epsilon_i \in \mathbb{R}^q$ is the approximation error vector confined within $\|\epsilon_i\|_2 \leq \epsilon_m$.

2.5. Control Objective

In this paper, ABS-assisted hovercrafts engaged in collaborative offshore operations are described as a formation tracking problem. Leader UAVs are designed to form a fixed formation and track the virtual-leader, while follower hovercrafts are required to accomplish a given time-varying formation pattern and simultaneously rotate around the convex combination states of leader UAVs.

The desired fixed formation for each UAV and the time-varying formation for each hovercraft are specified by the vectors $h_k \in \mathbb{R}^6 (k \in \mathbb{L})$ and $h_i(t) \triangleq h_i \in \mathbb{R}^6 (i \in \mathbb{F})$, respectively. h_i and its derivatives are both bounded, piecewise continuous and locally Lipschitz in t . $h_{ok} = C_L h_k \in \mathbb{R}^{3 \times 1}$ and $h_{oi} = C_L h_i \in \mathbb{R}^{3 \times 1}$ represent the output of the formation vectors. The two definitions of the control objectives are presented as follows.

Definition 1. For the k th leader UAV in (2) and the virtual-leader in (3) with any given bounded initial states, if

$$\lim_{t \rightarrow \infty} \|Y_{Lk} - h_{ok} - Y_0\| = 0, k \in \mathbb{L}, \quad (13)$$

then the k th leader UAV is said to achieve the fixed formation tracking target.

Definition 2. Define $\Theta = [\theta_{ik}]_{N \times M} = \mathcal{L}_1^{-1} \mathcal{L}_2$ with each entry $\theta_{ik} \geq 0, (i \in \mathbb{F}, k \in \mathbb{L})$. θ_{ik} satisfies $\sum_{k=N+1}^{N+M} \theta_{ik} = 1$. For the i th follower hovercraft with any given bounded initial states, such that

$$\lim_{t \rightarrow \infty} E \|Y_{Fi} - h_{oi} - C_L \hat{S}_i\|^2 \leq \kappa_F, \quad (14)$$

where $S_i = \sum_{k=N+1}^{N+M} \theta_{ik} X_{Lk}$. $\hat{S}_i$ is the estimation of S_i . Then, hovercrafts (8) with mixed disturbances are said to achieve the distributed time-varying formation target, where κ_F is called the mean square of the formation error bound.

3. Main Results

This section presents the control protocols for the leaders, the distributed observers, and the control protocols for the followers, respectively, ensuring the successful completion of the formation task. The fixed formation tracking protocol for leader UAVs is proposed in Section 3.1 firstly. Then in Section 3.2, the convex combination states of leaders are obtained by introducing distributed observers for follower hovercrafts. Subsequently, the formation tracking protocol and the NN adaptive law for the follower hovercraft are presented in Section 3.3. The structure of the whole designed algorithm is illustrated in Figure 2.

Remark 1. The algorithm diagram Figure 2 is described as follows. First, in the light blue section, each leader UAV acquires the fixed formation structure h_L and the virtual-leader state X_0 , forming an aerial fixed formation according to the designed control law u_L . Next, in the yellow section, follower hovercrafts obtain the UAVs' state Y_L via the distributed observers $\hat{S}_F$. Then they shape a time-varying formation h_F and the UAV states are transformed by the hovercraft controller u_F through the output regulation equation. Finally, despite the presence of mixed disturbances ζ_F , hovercrafts successfully form a formation centered on the convex combination states (five stars) of the UAVs.

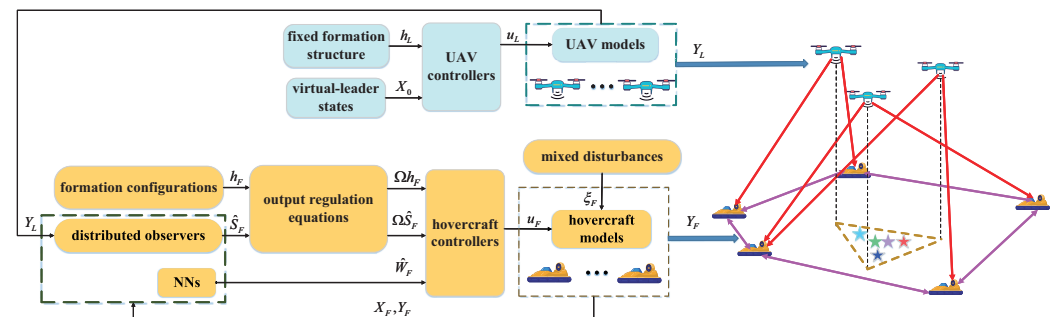


Figure 2. Diagram of the proposed distributed neuroadaptive formation control algorithm for ABS-assisted hovercrafts.

Under the framework of the formation control scheme, to accomplish control objectives (13) and (14), the following lemmas on RDEs, stability analysis, and graph theory are required.

Lemma 1 ([28]). For an RDE satisfying Assumption 3, there exist constants $q, q_1, q_2, r, \Delta \in \mathbb{R}^+$ and a function $V \in C^1$ such that

$$\begin{aligned} q_1 |\vartheta(t)|^2 &\leq V(\vartheta(t)) \leq q_2 |\vartheta(t)|^2, \\ \dot{V}(\vartheta(t)) &\leq -q V(\vartheta(t)) + r \|\tilde{\zeta}\|^2 + \Delta, \end{aligned} \quad (15)$$

where $\vartheta(t) \triangleq \vartheta$ is the state. Then, (15) exists a unique global solution that is said to be noise-to-state practical stability in probability (NSPS-P) and $\lim_{t \rightarrow \infty} EV(\vartheta) \leq \frac{r\phi + \Delta}{q}$.

Remark 2. Random differential equation (RDE) means a differential equation with random noise ξ and the noise intensity $g(\vartheta)$ like

$$\dot{\vartheta} = f(\vartheta) + g(\vartheta)\xi,$$

where functions $f(\vartheta)$ and $g(\vartheta)$ are piecewise continuous and locally Lipschitz in ϑ . If $\Delta = 0$ in (15), we can conclude that the RDE system is NSS-P. When the error systems are RDEs, Lemma 1 will be applied in the stability analysis to demonstrate that the closed-loop system is stable.

Lemma 2 ([41]). Suppose Ξ is Hurwitz and $\mathcal{B}(t) : [0, +\infty) \rightarrow \mathbb{R}^n$, then the dynamical system $\dot{V}(t) = \Xi V(t) + \mathcal{B}(t)$ satisfies $\lim_{t \rightarrow \infty} V(t) = 0$ if $\lim_{t \rightarrow \infty} \mathcal{B}(t) = 0$.

Lemma 3 ([42]). For any vectors $\mathcal{X}, \mathcal{Y} \in \mathbb{R}^\ell$, one has

$$\pm 2\mathcal{X}^T \mathcal{Y} \leq \mathcal{X}^T \mathcal{Z} \mathcal{X} + \mathcal{Y}^T \mathcal{Z}^{-1} \mathcal{Y}, \quad (16)$$

where $\mathcal{Z} \in \mathbb{R}^{\ell \times \ell}$, and ℓ is a positive integer.

Lemma 4 ([43]). The eigenvectors of the Laplacian matrix $\mathcal{L}'_{N \times N}$ and $\mathcal{L}'_{M \times M}$ with 0 as their eigenvalues are $\mathbf{1}_N$ and $\mathbf{1}_M$, respectively.

Lemma 5 ([44]). All elements of matrix $\mathcal{L}_1^{-1} \mathcal{L}_2$ are non-negative, with the sum of each row equaling one. Furthermore, matrix $\mathcal{L}_1$ and $\mathcal{L}_3$ are symmetric positive definite, and all their eigenvalues have positive real parts.

Remark 3. Brief explanations for Assumptions 1–6 and Lemmas 1–5 are given as follows: Assumptions 1 and 2 ensure the control input of the virtual-leader and the noise intensity are bounded for stability analysis. In Assumption 3 [27], the variance reflects the average power of a stochastic signal. The signal's second moment must be bounded for physical realizability. Assumption 4 [37,38] enables the models of multi-agent with different coefficient matrices to perform matrix multiplication operations. Assumption 5 ensures the connectedness of the graph $\mathcal{G}$. Assumption 6 [40] implies that NNs can approximate continuous functions on compact subsets. Lemma 1 [28] shows RDE stability under Assumption 3. Lemma 2 [41] helps with stability analysis by using constants that approach to zero. Lemma 3 [42] is a modified version of Young's inequality. Lemmas 4 and 5 [43,44] describe the properties of the connected graph for stability analysis.

3.1. Fixed Formation Tracking Protocol Design for Leader UAVs

In this part, a formation tracking protocol will be designed for the k th leader UAV to track the virtual-leader and shape a fixed formation simultaneously.

Define the formation tracking errors as follows:

$$\begin{cases} e_k = X_{Lk} - h_k - X_0, \\ \varphi_k = \sum_{j=N+1}^{N+M} a_{kj}(e_k - e_j) + a_{k0}e_k, k \in \mathbb{L}, \end{cases} \quad (17)$$

where $a_{k0} > 0$ if and only if the k th UAV can obtain the information from the virtual-leader.

Next, consider the following tracking protocol for the k th leader UAV:

$$u_k = \alpha_3 K_3 \varphi_k + K_v X_0 - \beta_3 \rho(\varphi_k), \quad (18)$$

where $\alpha_3 > 0$, $K_3 = -B^T P_3^{-1} \in \mathbb{R}^{3 \times 6}$, and $P_3 \succ 0 \in \mathbb{R}^{6 \times 6}$ remains to be devised. K_v is defined to solve the heterogeneity between the leader and the virtual-leader. $\rho(\varphi_k)$ is utilized to mitigate the impact of the bounded control input u_0 of the virtual-leader, defined as

$$\rho(\varphi_k) = \frac{B^T P_3^{-1} \varphi_k}{\|B^T P_3^{-1} \varphi_k\| + \varepsilon_3 e^{-\varepsilon_4 t}}. \quad (19)$$

Theorem 1. If Assumption 1 and 5 are valid, scalars $\beta_3, \sigma_5, \sigma_6, \varepsilon_3, \varepsilon_4 \in \mathbb{R}^+$, control parameters are appropriately adjusted to meet $\alpha_3 \geq 1/2(\sigma_5/\lambda_{N+1})$, $K_3 = -B^T P_3^{-1}$, and the subsequent linear matrix inequality (LMI) condition is satisfied as:

$$A_L P_3 + P_3 A_L^T - \sigma_5 B B^T + \sigma_6 P_3 \prec 0, \quad (20)$$

where $P_3 \succ 0$ represents the solution of (20). Then, the formation tracking issue (13) for the virtual-leader (4) and leader UAVs (2) can be accomplished under the formation tracking protocol (18).

The symbols in (17)–(20) are defined in Table 5.

Table 5. The definitions of symbols in (17)–(20).

Definitions	Symbols
the global formation error	$e_k \in \mathbb{R}^6$
the local formation error	$\varphi_k \in \mathbb{R}^6$
the element in matrix $\mathcal{L}_4$ (11)	a_{k0}
the feedback effect gain	α_3
the feedback again matrix	$K_3 \in \mathbb{R}^{3 \times 6}$
the solution of LMI (20)	$P_3 \in \mathbb{R}^{6 \times 6}$
the matrix defined in (4)	$K_v \in \mathbb{R}^{3 \times 6}$
constants in controller (18)	$\beta_3 \in \mathbb{R}^+$
the nonlinear function mitigated u_0 (19)	$\rho(\varphi_k) \in \mathbb{R}^3$
constants in nonlinear function (19)	$\varepsilon_3, \varepsilon_4 \in \mathbb{R}^+$
designed parameters in LMI (20)	$\sigma_5, \sigma_6 \in \mathbb{R}^+$
coefficient matrices in (2)	$A_L \in \mathbb{R}^{6 \times 6}, B \in \mathbb{R}^{6 \times 3}, C_L \in \mathbb{R}^{3 \times 6}$

Proof. See Appendix A. $\square$

3.2. Distributed Observers for Follower Hovercrafts

To ensure that followers remain within the leaders' influence and preserve formation integrity despite the loss of leader position information, the positions of UAVs can be represented by their convex combination states. Due to the unpredictable changes in interaction topologies among agents, not all follower hovercrafts can receive the convex combination states from leader UAVs directly. Therefore, an observer constructed by using the neighboring relative information for the i th follower is essential.

The following distributed observer $\hat{S}_i \in \mathbb{R}^6$ can be constructed

$$\dot{\hat{S}}_i = A_L \hat{S}_i + \alpha_0 K_o \left[\sum_{j=1}^N a_{ij} (\hat{Y}_{Fi} - \hat{Y}_{Fj}) + \sum_{k=N+1}^{N+M} a_{ik} (\hat{Y}_{Fi} - Y_{Lk}) \right], \quad (21)$$

where $\hat{Y}_{Fi} = C_L \hat{S}_i$, $\hat{Y}_{Fj} = C_L \hat{S}_j$, $\alpha_0 > 0$, $K_o = -P_o^{-1} C_L^T \in \mathbb{R}^{6 \times 3}$, and $P_o \succ 0 \in \mathbb{R}^{6 \times 6}$ satisfies the following LMI:

$$A_L P_o + P_o A_L^T - \sigma_1 C_L^T C_L + \sigma_2 P_o \prec 0, \quad (22)$$

where scalars σ_1 and $\sigma_2 \in \mathbb{R}^+$.

Theorem 2. Suppose Assumption 5 is valid and the LMI (22) holds. The observer parameter α_0 and matrix K_o are appropriately tuned to satisfy $\alpha_0 \geq \sigma_1/2\lambda_1$ and $K_o = -P_o^{-1} C_L^T$. Then, the distributed observer (21) will converge to the same convex combination states of leader UAVs, i.e.,

$$\lim_{t \rightarrow \infty} \left(\hat{S}_i - \sum_{k=N+1}^{N+M} \theta_{ik} X_{Lk} \right) = 0, i \in \mathbb{F}, k \in \mathbb{L}. \quad (23)$$

The symbols in (21)–(23) are defined in Table 6.

Table 6. The definitions of symbols in (21)–(23).

Definitions	Symbols
the convex combination state (14)	$S_i = \sum_{k=N+1}^{N+M} \theta_{ik} X_{Lk}$
the estimate of Y_{Fi} (8)	$\hat{Y}_{Fi} \in \mathbb{R}^3$
the estimate of Y_{Fj}	$\hat{Y}_{Fj} \in \mathbb{R}^3$
the estimate of S_i	$\hat{S}_i \in \mathbb{R}^6$
the positive gain constant	a_0
the feedback effect gain	α_3
the feedback again matrix	$K_o \in \mathbb{R}^{3 \times 6}$
the solution of LMI (22)	$P_o \in \mathbb{R}^{6 \times 6}$
designed parameters in LMI (22)	$\sigma_1, \sigma_2 \in \mathbb{R}^+$
coefficient matrices in (2)	$A_L \in \mathbb{R}^{6 \times 6}, C_L \in \mathbb{R}^{3 \times 6}$

Proof. See Appendix B. $\square$

3.3. Distributed Neuroadaptive Formation Protocol for Follower Hovercrafts

Within this section, the formation tracking control scheme and the NN adaptive law for the i th hovercraft (8) will be devised, and the NSPS-P property of the error system will be analyzed based on the RDEs.

Firstly, the formation tracking errors of the i th hovercraft are given as follows:

$$\begin{cases} e_i = X_{Fi} - \Omega_i h_i - \Omega_i \hat{S}_i \\ \varphi_i = \sum_{j=1}^{N+M} a_{ij} (e_i - e_j), i \in \mathbb{F}, \end{cases} \quad (24)$$

Then, the formation tracking protocol and the NN adaptive law for the i th hovercraft can be formulated as

$$u_i = \alpha_1 K_1 \varphi_i + K_h h_i - \hat{W}_i^T \Psi_i(X_{Fi}) - \beta_1 \rho(\varphi_i) + K_{si} \hat{S}_i, \quad (25a)$$

$$\dot{\hat{W}}_i = \pi_i \left[\Psi_i(X_{Fi}) \varphi_i^T P_1^{-1} B - \iota_i \hat{W}_i \right], \quad (25b)$$

where $\alpha_1 > 0$, $K_1 = -B^T P_1^{-1} \in \mathbb{R}^{3 \times 6}$, and $P_1 \succ 0$ remains to be devised. $K_h \in \mathbb{R}^{3 \times 6}$ for the formation configuration is determined to satisfy

$$\lim_{t \rightarrow \infty} \left[\dot{\bar{h}}_i - (A_F + B K_h) \bar{h}_i \right] = 0, \quad (26)$$

where $\bar{h}_i = \Omega_i h_i$, $K_{si} = U_i \in \mathbb{R}^{3 \times 6}$, $\rho(\varphi_i) \in \mathbb{R}^3$ is defined as

$$\rho(\varphi_i) = \frac{B^T P_1^{-1} \varphi_i}{\|B^T P_1^{-1} \varphi_i\| + \varepsilon_1 e^{-\varepsilon_2 t}}, \quad (27)$$

and $\beta_1, \pi_i, \iota_i, \varepsilon_1, \varepsilon_2 \in \mathbb{R}^+$.

Remark 4. The controller (25) consists of five components. $\alpha_1 K_1 \varphi_i$ acts as the feedback term, guiding the i th hovercraft to rotate around the convex combination state of leader UAVs and maintain the desired formation. $\hat{W}_i^T \Psi_i(X_{Fi})$ represents the NN compensation component to address the nonlinearities of the follower. $K_h h_i$ serves as the compensation term to meet the formation condition (26). $\beta_1 \rho(\varphi_i)$ is utilized to mitigate the impact of the NN approximation error.

Theorem 3. If Assumptions 2–6 are valid, the distributed neuroadaptive formation tracking problem (14) for hovercrafts (8) with mixed disturbances can be solved utilizing the distributed observer (21), the formation tracking protocol (25a), the NN adaptive law (25b), and the subsequent LMI condition:

$$A_F P_1 + P_1 A_F^T - \sigma_3 B B^T + \sigma_4 P_1 \prec 0, \quad (28)$$

where $P_1 \succ 0$ represents a positive definite matrix, scalars $\beta_1, \pi_i, \iota_i, \varepsilon_1, \varepsilon_2, \sigma_3, \sigma_4 \in \mathbb{R}^+$, and control parameters can be appropriately adjusted to meet $\alpha_1 \geq 1/2(\sigma_3/\lambda_1 + d_3/4)$ and $K_1 = -B^T P_1^{-1}$.

The symbols in (24)–(28) are defined in Table 7.

Table 7. The definitions of symbols in (24)–(28).

Definitions	Symbols
the global formation error	$e_i \in \mathbb{R}^6$
the local formation error	$\varphi_i \in \mathbb{R}^6$
the solution of the regulator equation (10)	$\Omega_i \in \mathbb{R}^{6 \times 6}, U_i \in \mathbb{R}^{3 \times 6}$
the position and velocity vector (8)	$X_{Fi} \in \mathbb{R}^6$
the time-varying formation vector	$h_i \in \mathbb{R}^6$
the element in matrices $\mathcal{L}_1$ and $\mathcal{L}_2$ (11)	a_{ij}
the control input (25a)	$u_i \in \mathbb{R}^3$
the feedback effect gain	α_1
the feedback again matrix	$K_1 \in \mathbb{R}^{3 \times 6}$
the feedback again matrix for the formation configuration	$K_h \in \mathbb{R}^{3 \times 6}$
the formation configuration vector	$h_i \in \mathbb{R}^6$
the estimation of $W_i \in \mathbb{R}^{s \times 3}$	$\hat{W}_i \in \mathbb{R}^{q \times 3}$
the NN basis function vector	$\Psi_i(X_{Fi}) \in \mathbb{R}^{s \times 1}$
the designed constant (25a)	$\beta_1 \in \mathbb{R}^+$
the nonlinear function mitigated error (27)	$\rho(\varphi_i) \in \mathbb{R}^3$
the transformed vector of the formation configuration	$\tilde{h}_i \in \mathbb{R}^6$
the transformed matrix	$K_{si} \in \mathbb{R}^{3 \times 6}$
the observer defined in (21)	$\hat{S}_i \in \mathbb{R}^6$
designed constants in nonlinear function (27)	$\varepsilon_1, \varepsilon_2 \in \mathbb{R}^+$
designed parameters in NNs (25b)	$\pi_i, \iota_i \in \mathbb{R}^+$
the solution of LMI (28)	$P_1 \in \mathbb{R}^{6 \times 6}$
designed parameters in LMI (28)	$\sigma_3, \sigma_4 \in \mathbb{R}^+$
coefficient matrices (8)	$A_F \in \mathbb{R}^{6 \times 6}, B \in \mathbb{R}^{6 \times 3}$

Proof. See Appendix C. $\square$

Remark 5. In comparison to the heterogeneous linear multi-agent systems [23], the actual heterogeneous ABS-assisted hovercraft systems in our study are addressed using output regulation equations. Unlike the linear UAV-UMV models for cooperative problems [16–18], the nonlinearities and the unavailability of leaders' output convex combination states in ABS-assisted hovercraft systems are studied by introducing NNs and distributed observers. Compared with [6,19,24,25] where wave disturbances were typically regarded as bounded uncertainties, a combination of white noise and non-white noise is studied in this paper, and RDEs are utilized to model and analyze the systems.

Remark 6. A nonlinear hovercraft model with mixed disturbances is applied to better reflect real navigation conditions in this paper. The scenario is also designed based on practical applications, where a base station transmits information to an offshore unmanned system via UAVs to enhance transmission reliability. Given the extensive equipment required for this simulation and due to current limitations in our experimental hardware, data relevant to real-world conditions could not be obtained for ABS-assisted hovercraft systems at this time. Our future work will involve real-experimental tests to further validate the algorithm's practicality.

Remark 7. The control protocols in these three sections are executed simultaneously to ensure the successful completion of the formation tracking task for the heterogeneous system. The task is divided as follows: First, leader UAVs form a formation around the virtual-leader based on the control protocol in Section 3.1. Next, follower hovercrafts estimate the convex combination of the UAVs' states using the observer from Section 3.2. Finally, hovercrafts complete the formation task based on the control protocol in Section 3.3 and rotating around the states obtained from Section 3.2.

4. Simulation Studies

For this section, numerical simulation results of the proposed distributed neuroadaptive formation tracking method applied to ABS-assisted hovercrafts will be presented, where one virtual-leader, leader ABSs comprising four UAVs ($M = 4$), and six follower hovercrafts ($N = 6$) are considered, labeled as $0, k = 7, 8, 9, 10$, and $i = 1, 2, \dots, 6$, respectively. Follower hovercrafts are deployed on the sea surface for detecting water quality and receiving data from leader UAVs, while hovering UAVs serve as relay nodes to collect data from the virtual-leader. The communication topology among the virtual-leader, UAVs, and hovercrafts is depicted in Figure 3.

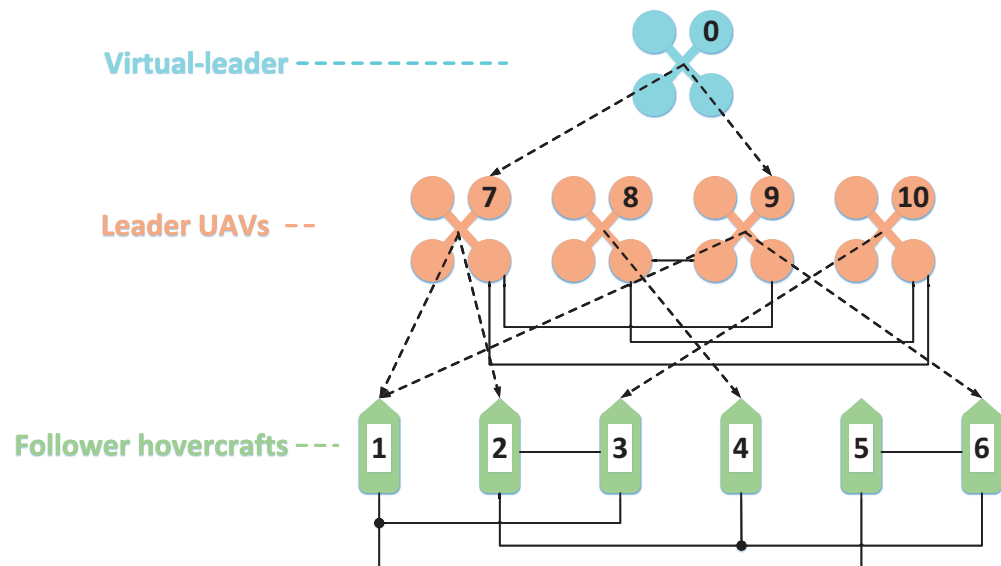


Figure 3. The communication graph among ABS-assisted hovercrafts.

For the virtual-leader system, choose $K_v = \text{diag}[-3, -3, -2; -4, -4, -3]$ and the bounded control input as $u_0 = [150\cos(0.05t), 150\sin(0.05t), 90 - 0.8t]^T$, ensuring the system remains stable while allowing for the broad movement. In leader UAV systems, the damping constant matrices are selected as $\alpha_\chi = [-0.005, 0, 0; 0, 0, 0; 0, 0, 0]$ and $\alpha_\eta = \mathbf{O}_3$. The designed parameters in leaders' controllers are set as $\sigma_5 = 1, \sigma_6 = 4, \varepsilon_3 = 0.5, \varepsilon_4 = 1, \alpha_3 = 10.2, \beta_3 = 150, K_3 = \text{diag}[-42.043\mathbf{I}_3, -16.2193\mathbf{I}_3]$. As σ_6 increases, the system error decreases but the response time slows down. The desired fixed formation structure for each UAV is given as $h_7 = [-10, 2, 0, 0, 0, 0]^T, h_8 = [4, 8, 0, 0, 0, 0]^T, h_9 = [10, -4, 0, 0, 0, 0]^T, h_{10} = [-3, -9, 0, 0, 0, 0]^T$.

The observer parameters are selected as $\sigma_1 = 2, \sigma_2 = 2.8$. The minimum eigenvalue of $\mathcal{L}_1$ can be obtained as $\lambda_1 = 0.8577$. α_0 should be large enough to stabilize the leader UAV systems and satisfy $\alpha_0 = 1.2 \geq \sigma_1/2\lambda_1$. α_0 determines the accuracy of the observer. K_0 is denoted as the feedback gain matrix and can be solved as $K_0 = [\text{diag}(-20.0405, -19.9487, -19.9487); \text{diag}(-51.7779, -51.4939, -51.4939)]$. The initial values are given by $\hat{S}_i(0) = [0.1, 0.1, 0.1, 0, 0, 0]^T, i \in \mathbb{F}$.

For follower hovercraft systems, the matrices C_i, M_i , and D_i in model (6) are selected with the same values as in [34]. The general noise ξ_i can be generated by a second-order filter as described in [45]: $\ddot{\xi}_i + \varsigma_1\dot{\xi}_i + \varsigma_2\xi_i = \varsigma_3N_i(t), \xi_i(0) = 0$, where $N_i(t) \triangleq N_i =$

$[N_{ix}, N_{iy}, N_{i\omega}]^T \in \mathbb{R}^3$ represents the white noise process. The filter parameters can be chosen as $\varsigma_1 = 0.4$, $\varsigma_2 = 0.4474$, and $\varsigma_3 = 0.1964$. The noise intensity matrix is set as $g_i = 2\mathbf{I}_3$, which is chosen to correspond with the model's mass and values of the damping matrix. A higher noise intensity may increase forces and torques on the hovercraft, which can potentially affect system stability if excessive. The desired time-varying formation pattern h_i for each hovercraft has the expression of

$$\begin{cases} h_{ix} = 15\sin(0.15t + (i-1)\pi/3), \\ h_{iy} = 30\cos(0.15t + (i-1)\pi/3), \\ h_{i\psi} = \text{atan2}(\dot{h}_{iy}, \dot{h}_{ix}), \\ h_{ivx} = \dot{h}_{ix}, \\ h_{ivy} = \dot{h}_{iy}, \\ h_{i\omega} = \dot{h}_{i\psi}. \end{cases}$$

In consideration of the practical challenges that require systems to tolerate mixed disturbances, the feedback effect gain α_1 should be large enough. We opt for $\sigma_3 = 1$, $\sigma_4 = 3$, with α_1 chosen as $\alpha_1 = \text{diag}(34, 34, 34) \geq 1/2(\sigma_3/\lambda_1 + d_3/4)$. K_1 , representing the feedback gain matrix for the controller, is solvable as $K_1 = [-17.7896\mathbf{I}_3, -9.8371\mathbf{I}_3]$. The solutions of the regulator (10) are $\Omega_i = \mathbf{I}_6$, $K_{si} = U_i = [-0.005, 0, 0; 0, 0, 0; 0, 0, 0; \mathbf{O}_3]$, $i \in \mathbb{F}$. Choose the feedback gain matrix $K_h = [-3\mathbf{I}_3, -4\mathbf{I}_3]$ to ensure that $(A_F + BK_h)$ is Hurwitz. The other designed parameters in the controller are selected as $\beta_1 = 6$, $\varepsilon_1 = 0.5$, $\varepsilon_2 = 1$. β_1 determines the ability to eliminate the neural network approximation error. The Gaussian function is chosen as the radial basis function, and $\pi_i = 2$, $\iota_i = 8$ are selected in NNs with initial values are given by 0. The selection of NN parameters follows the guidelines in reference [40]. The initial states of each agent are defined as $X_0(0) = [30(\omega - 0.5); 30(\omega - 0.5); 12; 0; 0; 0]$, $X_{L7}(0) = [20; 12; 20; 0; 0; 0]$, $X_{L8}(0) = [-4; 8; 15; 0; 0; 0]$, $X_{L9}(0) = [15; -3; 14; 0; 0; 0]$, $X_{L10}(0) = [-3; -8; 13; 0; 0; 0]$. $X_{F1}(0) = [10; 20; 0; 0; 0; 0]$, $X_{F2}(0) = [7; -1; -\pi/6; 0; 0; 0]$, $X_{F3}(0) = [7; -30; \pi/5; 0; 0; 0]$, $X_{F4}(0) = [-5; -10; -\pi/4; 0; 0; 0]$, $X_{F5}(0) = [-10; -10; \pi/3; 0; 0; 0]$, $X_{F6}(0) = [-5; 30; 0; 0; 0; 0]$, where the random value $\omega \in [0, 1]$.

The simulation results of the position snapshots at different time instants ($t = 0$ s, 5 s, 50 s, 80 s) for ABS-assisted hovercrafts in three and two dimensions are shown in Figures 4–6. Figure 4 depicts the trajectory of the virtual-leader, which determines the positions of the whole heterogeneous systems. Figure 5 illustrates that the leader UAVs form a fixed formation structure while tracking the virtual-leader, and snapshots in Figure 6 indicate the outputs of six follower hovercrafts form a time-varying ellipse shape around the different output convex combination states (depicted as hexagons with different colors) of four leader UAVs.

Figures 7 and 8 illustrate the control inputs of all UAVs and hovercrafts, indicating that forces and torques can quickly converge to the desired values and keep within bounds, demonstrating the fast response of the control system.

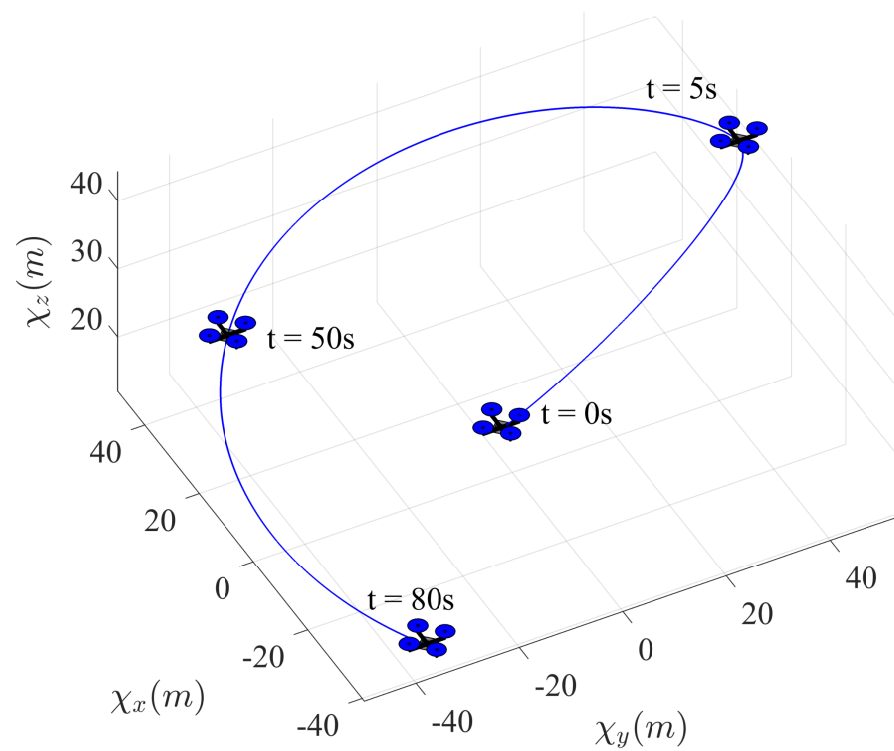


Figure 4. Position trajectories of the virtual-leader in three dimensions.

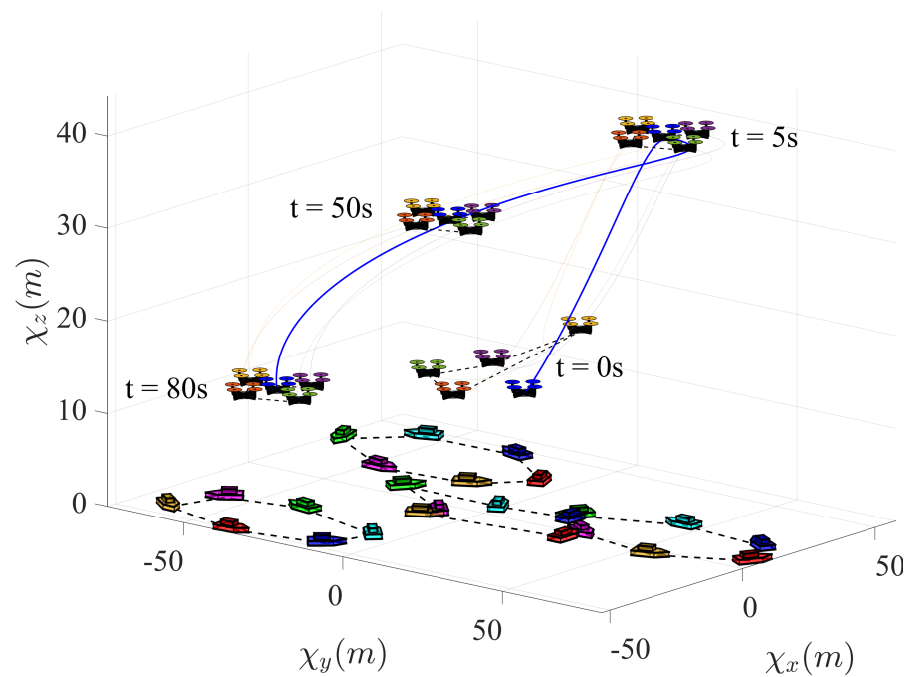


Figure 5. Position trajectories of ABS-assisted hovercraft systems in three dimensions.

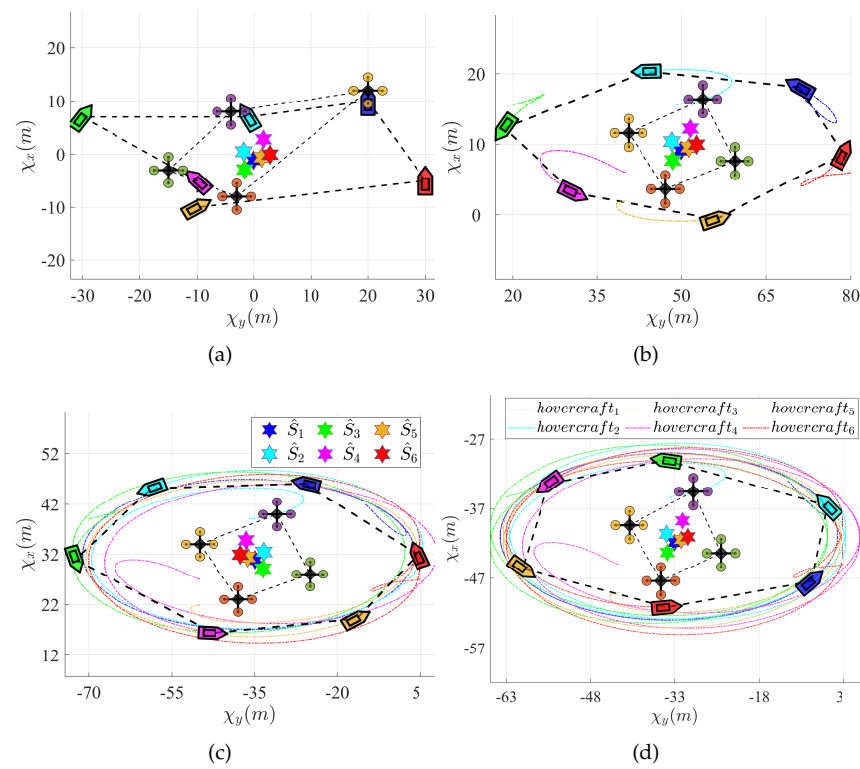


Figure 6. Position trajectories of ABS-assisted hovercrafts in two dimensions. (a) $t = 0$ s. (b) $t = 5$ s. (c) $t = 50$ s. (d) $t = 80$ s.

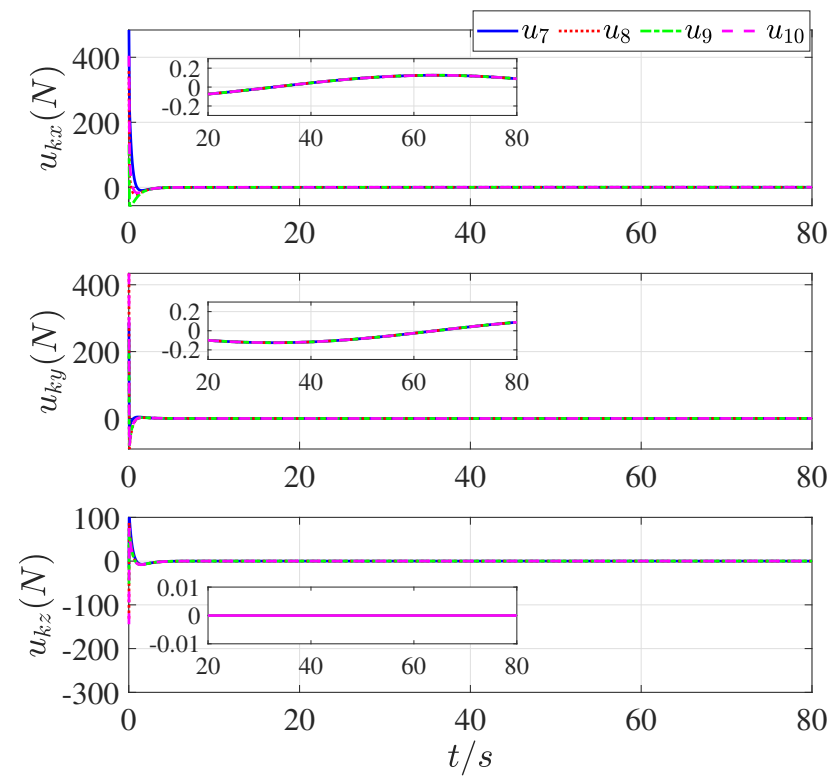


Figure 7. The forces curves of leader UAVs.

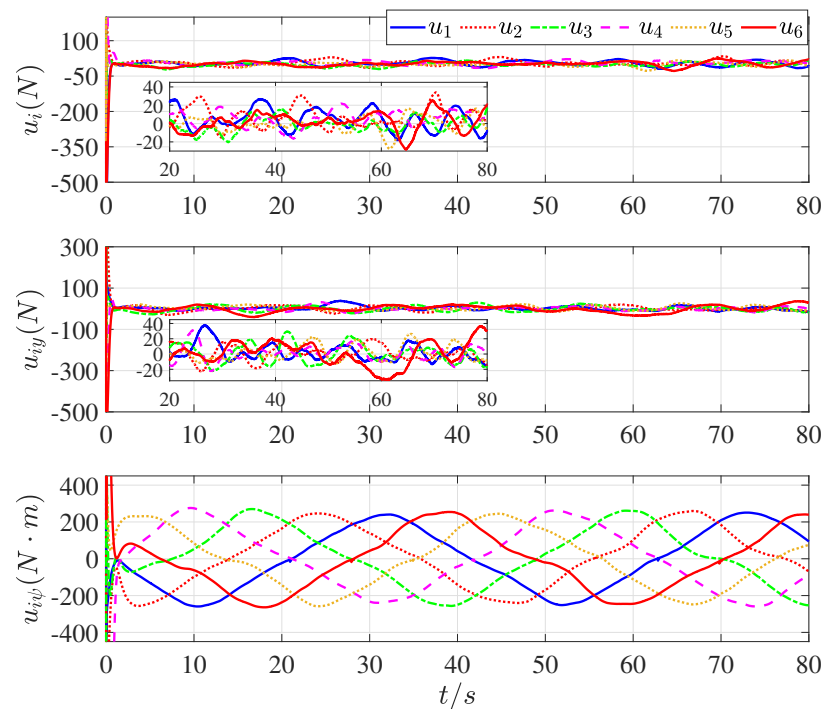


Figure 8. The forces and moment curves of follower hovercrafts.

The states of mixed disturbances affecting each hovercraft are depicted in Figure 9. The curves for errors e and observer estimation errors $\tilde{S}$ are given in Figures 10–12, where it is noticeable that errors e_k and e_i rapidly converge toward neighborhoods near the origin, while $\tilde{S}_i$ can asymptotically converge to the origin at a fast rate, highlighting the quick convergence of both tracking and estimation errors. Figure 13 illustrates the curves of the NN adaptive weights, which quickly converge and are maintained within a small bound. Thus, from Figures 5–13 and (A28), one can conclude that the distributed neuroadaptive formation tracking protocol for ABS-assisted hovercrafts with mixed disturbances is achieved by tuning $d_1, d_2, d_3, \sigma_2, \sigma_4, \pi_i$ large enough and $\alpha_o, d_4, \varepsilon_i$ small enough. This presents a valuable solution for addressing the control challenges of ABS-assisted hovercrafts engaged in monitoring sea surface operations.

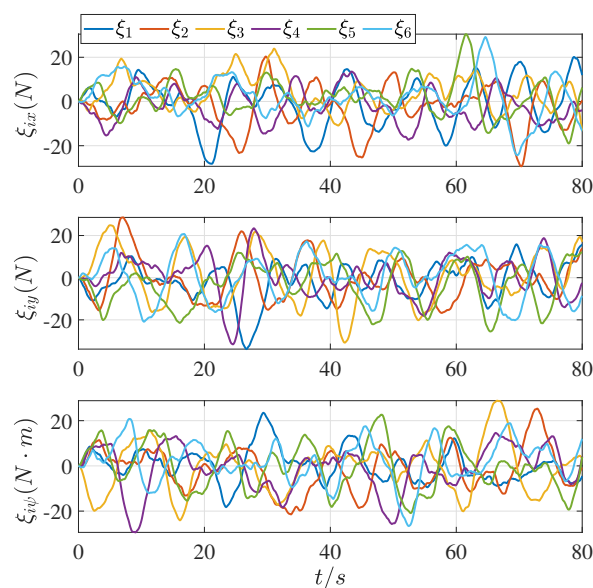


Figure 9. Curves of mixed disturbances effect on follower hovercrafts.

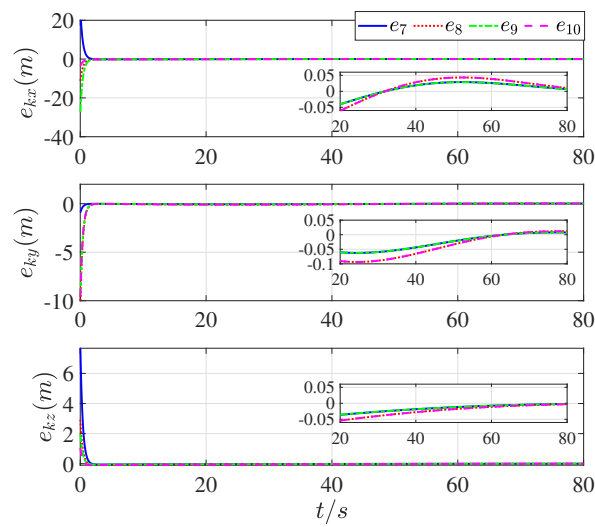


Figure 10. Curves of formation tracking errors for leader UAVs.

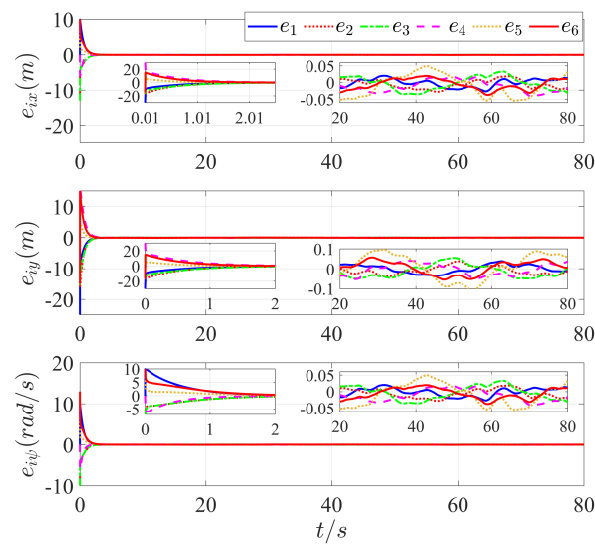


Figure 11. Curves of formation tracking errors for follower hovercrafts.

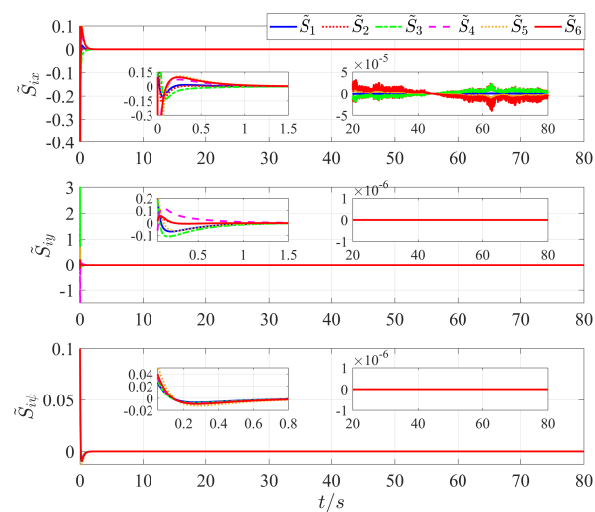


Figure 12. Curves of observer estimate errors.

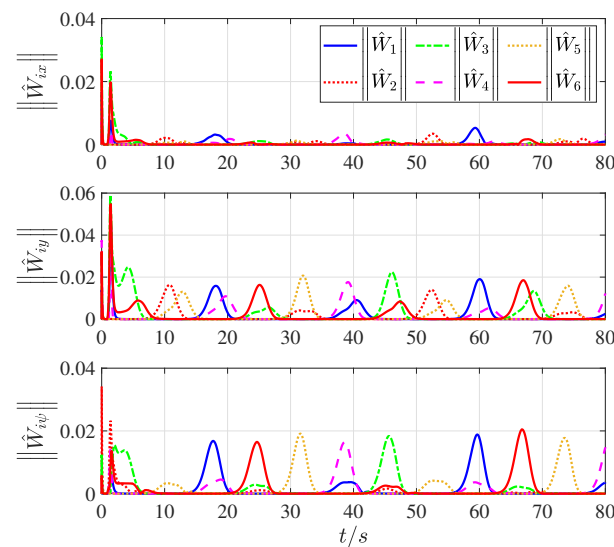


Figure 13. Curves of the NN adaptive weights.

5. Conclusions

A distributed neuroadaptive formation control scheme has been developed for ABS-assisted hovercrafts in collaborative offshore operations. The designed formation control schemes incorporate output regulation equations, distributed observers, NNs, and RDEs to address challenges such as system heterogeneities, unavailable convex combination states of the leaders, nonlinearities, and mixed disturbances in ABS-assisted hovercraft systems, enabling successful formation tracking. Based on Lyapunov stability analysis, it has been verified that the error systems exhibit the NSPS-P property. Building on the above findings, future research will focus on employing improved artificial potential field methods, distributed optimization theory, and energy-saving algorithms to address internal collision, deadlock, and energy consumption challenges with multi-constraint conditions for formation tracking in heterogeneous ABS-assisted hovercraft systems.

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Abbreviations

The following abbreviations are used in this manuscript:

ABS	Aerial base station
UAV	Unmanned aerial vehicle
NN	Neural network
RDE	Random differential equation
NSPS-P	noise-to-state-practical stability in probability
UMV	Unmanned marine vehicle
MAS	Multi-agent system

Appendix A. Proof for Theorem 1

In this part, all leader UAVs are proven to shape a fixed formation and track the virtual-leader under the protocol (18).

To analyze in a compact form, define $X_L = [X_{N+1}^T, X_{N+2}^T, \dots, X_{N+M}^T]^T \in \mathbb{R}^{6M \times 1}$, $e_L = [e_{N+1}^T, e_{N+2}^T, \dots, e_{N+M}^T]^T \in \mathbb{R}^{6M \times 1}$, $\varphi_L = [\varphi_{N+1}^T, \varphi_{N+2}^T, \dots, \varphi_{N+M}^T]^T \in \mathbb{R}^{6M \times 1}$, $L(\varphi_L) = [\rho^T(\varphi_{N+1}), \rho^T(\varphi_{N+2}), \dots, \rho^T(\varphi_{N+M})]^T \in \mathbb{R}^{3M \times 1}$, $h_L = [h_{N+1}^T, h_{N+2}^T, \dots, h_{N+M}^T]^T \in \mathbb{R}^{6M \times 1}$.

Through the expressions of (17), one sees

$$\begin{aligned} e_L &= X_L - h_L - \mathbf{1}_M \otimes X_0, \\ \varphi_L &= (\mathcal{L}_3 \otimes \mathbf{I}_6) e_L. \end{aligned} \quad (\text{A1})$$

Based on (A1), substituting (18) into (1), the compact form can be yielded as

$$\dot{X}_L = (\mathbf{I}_M \otimes A_L) X_L + (\alpha_3 \mathcal{L}_3 \otimes BK_3) e_L - (\mathbf{1}_M \otimes BK_v) X_0 - (\beta_3 \mathbf{I}_M \otimes B) L(\varphi_L). \quad (\text{A2})$$

Then, the derivative of the formation tracking error can be obtained as

$$\dot{e}_L = (\mathbf{I}_M \otimes A_L + \alpha_3 \mathcal{L}_3 \otimes BK_3) e_L - (\beta_3 \mathbf{I}_M \otimes B) L(\varphi_L) - (\mathbf{I}_M \otimes B)(\mathbf{1}_M \otimes Bu_0). \quad (\text{A3})$$

Consider $V_3 = e_L^T (\mathcal{L}_3 \otimes P_3^{-1}) e_L$; the time derivative of V_3 is given by

$$\begin{aligned} \dot{V}_3 &= 2e_L^T (\mathcal{L}_3 \otimes P_3^{-1} A_L) e_L - 2\alpha_3 e_L^T (\mathcal{L}_3 \mathcal{L}_3 \otimes P_3^{-1} B B^T P_3^{-1}) e_L \\ &\quad - 2e_L^T (\mathcal{L}_3 \otimes P_3^{-1} B) (\mathbf{1}_M \otimes u_0) - 2\beta_3 e_L^T (\mathcal{L}_3 \otimes P_3^{-1} B) L(\varphi_L). \end{aligned} \quad (\text{A4})$$

Select $\beta_3 > \delta$, one has

$$\begin{aligned} &- 2e_L^T (\mathcal{L}_3 \otimes P_3^{-1} B) (\mathbf{1}_M \otimes u_0) - 2\beta_3 e_L^T (\mathcal{L}_3 \otimes P_3^{-1} B) L(\varphi_L) \\ &\leq 2\delta \sum_{k=N+1}^{N+M} \|B^T P_3^{-1} \varphi_k\| - 2\beta_3 \sum_{k=N+1}^{N+M} \frac{\|B^T P_3^{-1} \varphi_k\|^2}{\|B^T P_3^{-1} \varphi_k\| + \varepsilon_3 e^{-\varepsilon_4 t}} \\ &\leq 2M\delta\varepsilon_3 e^{-\varepsilon_4 t}. \end{aligned} \quad (\text{A5})$$

Define $\mathcal{H}_3 \in \mathbb{R}^{6 \times 6}$ as a unitary matrix that satisfies $\mathcal{H}_3^T \mathcal{L}_3 \mathcal{H}_3 = \Lambda_3 \triangleq \text{diag}(\lambda_{N+1}, \lambda_{N+2}, \dots, \lambda_{N+M}) \in \mathbb{R}^{M \times M}$. In light of Lemma 5, the diagonal entries of Λ_3 satisfying $0 < \text{Re}(\lambda_{N+1}) \leq \text{Re}(\lambda_{N+2}) \leq \dots \leq \text{Re}(\lambda_{N+M})$. Let $\bar{e}_L = (\mathbf{I}_M \otimes P_3^{-1}) e_L$ and $\hat{e}_L = (\mathcal{H}_3^T \otimes \mathbf{I}_6) \bar{e}_L$. Choose α_3 sufficiently large such that $\alpha_3 \geq 1/2(\sigma_5/\lambda_{N+1})$. Then, it follows from (20) and (A4) that

$$\begin{aligned}
& 2e_L^T \left(\mathcal{L}_3 \otimes P_3^{-1} A_L - \alpha_1 \mathcal{L}_3 \mathcal{L}_3 \otimes P_3^{-1} B B^T P_3^{-1} \right) e_L \\
& = \bar{e}_L^T \left[\mathcal{L}_3 \otimes \left(A_L P_3 + P_3 A_L^T \right) - 2\alpha_3 \mathcal{L}_3 \mathcal{L}_3 \otimes B B^T \right] \bar{e}_L \\
& = \sum_{k=N+1}^{N+M} \lambda_k \hat{e}_k^T \left(A_L P_3 + P_3 A_L^T - 2\alpha_3 \lambda_k B B^T \right) \hat{e}_k \\
& < -\sigma_6 e_L^T \left(\mathcal{L}_3 \otimes P_3^{-1} \right) e_L.
\end{aligned} \tag{A6}$$

According to (A5) and (A6), (A4) can be rewritten as

$$\dot{V}_3 < -\sigma_6 V_3 + 2M\delta\epsilon_3 e^{-\epsilon_4 t}. \tag{A7}$$

In light of Lemma 2, one can conclude that

$$\lim_{t \rightarrow \infty} V_3 = \lim_{t \rightarrow \infty} e_L^T (\mathcal{L}_3 \otimes P_3^{-1}) e_L = 0, \tag{A8}$$

which implies

$$\lim_{t \rightarrow \infty} \|C_L\| \|e_k\| = \lim_{t \rightarrow \infty} \|Y_{Lk} - h_{ok} - Y_0\| = 0, k \in \mathbb{L}. \tag{A9}$$

Therefore, all leader UAVs can shape a fixed formation and track the virtual-leader simultaneously by using protocol (18). This completes the proof of Theorem 1. $\square$

Appendix B. Proof for Theorem 2

In this part, the designed distributed observer (21) for follower hovercrafts is proven to obtain UAVs' convex combination states.

Let $\hat{S}_F = [\hat{S}_1^T, \hat{S}_2^T, \dots, \hat{S}_N^T]^T \in \mathbb{R}^{6N \times 1}$. Then, the compact form of the distributed observer (21) can be expressed as

$$\dot{\hat{S}}_F = \left[I_N \otimes A_L - \alpha_o \left(\mathcal{L}_1 \otimes P_o^{-1} C_L^T C_L \right) \right] \hat{S}_F + \alpha_o \left(\mathcal{L}_2 \otimes P_o^{-1} C_L^T C_L \right) X_L. \tag{A10}$$

Define the compact form of the observer errors as $\tilde{S}_F = \hat{S}_F - \left(\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_6 \right) X_L$, then the time derivative of $\tilde{S}_F$ can be written as

$$\begin{aligned}
\dot{\tilde{S}}_F &= (I_N \otimes A_L) \tilde{S}_F - \alpha_o \left(\mathcal{L}_1 \otimes P_o^{-1} C_L^T C_L \right) \left[\tilde{S}_F - \left(\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes I_6 \right) X_L \right] \\
&= \left[I_N \otimes A_L - \alpha_o \left(\mathcal{L}_1 \otimes P_o^{-1} C_L^T C_L \right) \right] \tilde{S}_F.
\end{aligned} \tag{A11}$$

Take into account the following Lyapunov functional:

$$V_o = \tilde{S}_F^T (\mathcal{L}_1 \otimes P_o) \tilde{S}_F. \tag{A12}$$

Follow the same steps as in (A6), and define $\mathcal{H}_1 \in \mathbb{R}^{6 \times 6}$ as a unitary matrix that satisfies $\mathcal{H}_1^T \mathcal{L}_1 \mathcal{H}_1 = \Lambda_1 \triangleq \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_N) \in \mathbb{R}^{N \times N}$, where $0 < \text{Re}(\lambda_1) \leq \text{Re}(\lambda_2) \leq \dots \leq \text{Re}(\lambda_N)$. Let $\bar{\tilde{S}}_F = (\mathcal{H}_1^T \otimes I_6) \tilde{S}_F$, then (A12) can be expressed as

$$\begin{aligned}
\dot{V}_o &\leq \bar{\tilde{S}}_F^T \left[\Lambda_1 \otimes \left(P_o A_L + A_L^T P_o \right) - 2\alpha_o \Lambda_1^2 \otimes C_L^T C_L \right] \bar{\tilde{S}}_F \\
&= \sum_{i=1}^N \lambda_i \bar{\tilde{S}}_i^T \left(P_o A_L + A_L^T P_o - 2\alpha_o \lambda_i C_L^T C_L \right) \bar{\tilde{S}}_i.
\end{aligned} \tag{A13}$$

Let α_o be sufficiently large such that $\alpha_o \geq \sigma_1/2\lambda_1$, then substituting the LMI (22) into (A13) yields

$$\dot{V}_o < -\sigma_2 \bar{\tilde{S}}_F^T (\Lambda_1 \otimes P_o) \bar{\tilde{S}}_F = -\sigma_2 \bar{\tilde{S}}_F^T (\mathcal{L}_1 \otimes P_o) \bar{\tilde{S}}_F = -\sigma_2 V_o. \tag{A14}$$

One can obtain from (A14) that $\lim_{t \rightarrow \infty} \tilde{S}_F = \lim_{t \rightarrow \infty} [\hat{S}_F - (\mathcal{L}_1^{-1} \mathcal{L}_2 \otimes \mathbf{I}_6) X_L] = 0$. Thus, it can be concluded that $\hat{S}_F$ asymptotically approaches the convex combination states of the leaders, implying that each follower hovercraft can obtain the UAVs' convex combination states through the distributed observer (21). This completes the proof of Theorem 2. $\square$

Appendix C. Proof for Theorem 3

In this part, follower hovercrafts are proven to form the given time-varying formation while tracking convex combination states, which are estimated by distributed observers under protocols (25).

Let $X_F = [X_{F1}^T, X_{F2}^T, \dots, X_{FN}^T]^T \in \mathbb{R}^{6N \times 1}$, $\zeta_F = [\zeta_1^T, \zeta_2^T, \dots, \zeta_N^T]^T \in \mathbb{R}^{3N \times 1}$, $G = \text{diag}(g_1, g_2, \dots, g_N) \in \mathbb{R}^{3N \times 3N}$, $e_F = [e_1^T, e_2^T, \dots, e_N^T]^T \in \mathbb{R}^{6N \times 1}$, $\varphi_F = [\varphi_1^T, \varphi_2^T, \dots, \varphi_N^T]^T \in \mathbb{R}^{6N \times 1}$, $L(\varphi_F) = [\rho^T(\varphi_1), \rho^T(\varphi_2), \dots, \rho^T(\varphi_N)]^T \in \mathbb{R}^{3N \times 1}$, $h_F = [h_1^T, h_2^T, \dots, h_N^T]^T \in \mathbb{R}^{6N \times 1}$, $\Omega = \text{diag}(\Omega_1, \Omega_2, \dots, \Omega_N) \in \mathbb{R}^{6N \times 6N}$, $U = \text{diag}(U_1, U_2, \dots, U_N) \in \mathbb{R}^{3N \times 6N}$, $W_F = \text{diag}(W_1, W_2, \dots, W_N) \in \mathbb{R}^{sN \times 3N}$, $\Psi_F(X_F) = [\Psi_1^T(X_{F1}), \Psi_2^T(X_{F2}), \dots, \Psi_N^T(X_{FN})]^T \in \mathbb{R}^{sN \times 1}$, $\epsilon_F = [\epsilon_1^T, \epsilon_2^T, \dots, \epsilon_N^T]^T \in \mathbb{R}^{3N \times 1}$.

Define $\hat{W}_i = \hat{W}_i - W_i$, one has

$$\dot{\hat{W}}_i = \pi_i [\Psi_i(X_{Fi}) \varphi_i^T P_1^{-1} B - \iota_i \hat{W}_i]. \quad (\text{A15})$$

Substituting the controller (25) into (8), the compact form of hovercraft systems can be obtained as

$$\begin{aligned} \dot{X}_F &= (\mathbf{I}_N \otimes A_F) X_F + (\alpha_1 \mathcal{L}_1 \otimes B K_1) e_F - (\mathbf{I}_N \otimes B K_h) h_F \\ &\quad - (\mathbf{I}_N \otimes B) [\tilde{W}_F^T \Psi_F(X_F) - \epsilon_F] + (\mathbf{I}_N \otimes B) G \zeta_F - (\beta_1 \mathbf{I}_N \otimes B) L(\varphi_F) - (\mathbf{I}_N \otimes B) U \hat{S}_F. \end{aligned} \quad (\text{A16})$$

In light of the output regulate Equation (10) and the distributed observer error system (A11), the compact form of the dynamics of e_i can be further expressed as

$$\begin{aligned} \dot{e}_F &= \dot{X}_F - \Omega \dot{h}_F - \Omega \dot{\hat{S}}_F \\ &= (\mathbf{I}_N \otimes A_F + \alpha_1 \mathcal{L}_1 \otimes B K_1) e_F + (\mathbf{I}_N \otimes B) G \zeta_F - (\mathbf{I}_N \otimes B) [\tilde{W}_F^T \Psi_F(X_F) - \epsilon_F] \\ &\quad - (\beta_1 \mathbf{I}_N \otimes B) L(\varphi_F) + \Delta_h + \Delta_S, \end{aligned} \quad (\text{A17})$$

where $\Delta_h = [\mathbf{I}_N \otimes (A_F + B K_h)] \Omega h_F - \Omega \dot{h}_F$ and $\Delta_S = -\alpha_0 \Omega (\mathcal{L}_1 \otimes P_o^{-1} C_L^T C_L) \tilde{S}_F$.

Considering the following Lyapunov functional as

$$V = V_o + V_1, \quad (\text{A18})$$

where V_1 is defined as

$$V_1 = e_F^T (\mathcal{L}_1 \otimes P_1^{-1}) e_F + \sum_{i=1}^N \text{tr} \left(\frac{1}{\pi_i} \tilde{W}_i^T \tilde{W}_i \right), \quad (\text{A19})$$

and V_o is devised in (A12).

Since $K_1 = -B^T P_1^{-1}$, along with (25b) and (A17), the time derivation of V_1 shows

$$\begin{aligned} \dot{V}_1 &= 2e_F^T (\mathcal{L}_1 \otimes P_1^{-1} A_F - \alpha_1 \mathcal{L}_1 \mathcal{L}_1 \otimes P_1^{-1} B B^T P_1^{-1}) e_F - 2e_F^T (\beta_1 \mathcal{L}_1 \otimes P_1^{-1} B) L(\varphi_F) \\ &\quad + 2e_F^T (\mathcal{L}_1 \otimes P_1^{-1} B) G \zeta_F - 2e_F^T (\mathcal{L}_1 \otimes P_1^{-1} B) [\tilde{W}_F^T \Psi_F(X_F) - \epsilon_F] \\ &\quad + 2 \sum_{i=1}^N \text{tr} [\tilde{W}_i^T \Psi_i(X_{Fi}) \varphi_i^T P_1^{-1} B - \iota_i \tilde{W}_i^T \hat{W}_i] + 2e_F^T (\mathcal{L}_1 \otimes P_1^{-1}) \Delta_h + 2e_F^T (\mathcal{L}_1 \otimes P_1^{-1}) \Delta_S. \end{aligned} \quad (\text{A20})$$

Select $\beta_1 > \epsilon_m$, one obtains

$$-2e_F^T \left(\beta_1 \mathcal{L}_1 \otimes P_1^{-1} B \right) L(\varphi_F) + 2e_F^T \left(\mathcal{L}_1 \otimes P_1^{-1} B \right) \epsilon_F \leq 2N\epsilon_m \epsilon_1 e^{-\epsilon_2 t}. \quad (\text{A21})$$

Since $\text{tr}(Z_1 Z_2) = \text{tr}(Z_2 Z_1)$ holds for any matrices Z_1 and Z_2 , one can obtain that

$$-2e_F^T \left(\mathcal{L}_1 \otimes P_1^{-1} B \right) \tilde{W}_F^T \Psi_F(X_F) = -2 \sum_{i=1}^N \text{tr} \left[\tilde{W}_i^T \Psi_F(X_F) \varphi_i^T P_1^{-1} B \right]. \quad (\text{A22})$$

Considering (A20)–(A22), let $\bar{e}_F = \left(I_N \otimes P_1^{-1} \right) e_F$, then (A20) can be rewritten as:

$$\begin{aligned} \dot{V}_1 \leq & \bar{e}_F^T \left[\mathcal{L}_1 \otimes \left(A_F P_1 + P_1 A_F^T \right) - 2\alpha_1 \mathcal{L}_1 \mathcal{L}_1 \otimes B B^T \right] \bar{e}_F - 2 \sum_{i=1}^N \text{tr} \left(\iota_i \tilde{W}_i^T \tilde{W}_i \right) \\ & + 2e_F^T \left(\mathcal{L}_1 \otimes P_1^{-1} \right) \Delta_h + 2e_F^T \left(\mathcal{L}_1 \otimes P_1^{-1} \right) \Delta_S + 2\bar{e}_F^T \left(\mathcal{L}_1 \otimes B \right) G \xi_F + 2N\epsilon_m \epsilon_1 e^{-\epsilon_2 t}. \end{aligned} \quad (\text{A23})$$

According to the Young's inequality and Lemma 3, one has

$$\begin{aligned} 2e_F^T \left(\mathcal{L}_1 \otimes P_1^{-1} \right) \Delta_h & \leq d_1 e_F^T \left(\mathcal{L}_1 \otimes P_1^{-1} \right) e_F + \frac{\lambda_{\max} \left(\mathcal{L}_1 \otimes P_1^{-1} \right) \|\Delta_h\|^2}{d_1}, \\ 2e_F^T \left(\mathcal{L}_1 \otimes P_1^{-1} \right) \Delta_S & \leq d_2 e_F^T \left(\mathcal{L}_1 \otimes P_1^{-1} \right) e_F + \frac{\lambda_S \|\tilde{\xi}_F\|^2}{d_2}, \\ 2\bar{e}_F^T \left(\mathcal{L}_1 \otimes B \right) G \xi_F & \leq d_3 \bar{e}_F^T \left(\mathcal{L}_1 \mathcal{L}_1 \otimes B B^T \right) \bar{e}_F + \frac{g^2 \|\xi_F\|^2}{d_3}, \\ -2 \sum_{i=1}^N \text{tr} \left(\iota_i \tilde{W}_i^T \tilde{W}_i \right) & \leq - \left(2 - \frac{1}{d_4} \right) \sum_{i=1}^N \text{tr} \left(\iota_i \tilde{W}_i^T \tilde{W}_i \right) + d_4 \iota_m W_{Fm}, \end{aligned} \quad (\text{A24})$$

where $g = \max\{g_{1m}, g_{2m}, \dots, g_{Nm}\} \geq \|G\|_1$, $\iota_m = \max\{\iota_1, \iota_2, \dots, \iota_N\}$, $W_{Fm} = \text{tr}(W_F^T W_F)$, $\lambda_S = \lambda_{\max} \left\{ \alpha_o^2 \left(\mathcal{L}_1 \otimes P_1^{-1} \right) \left[\Omega^2 \left(\mathcal{L}_1 \mathcal{L}_1 \otimes P_o^{-1} C_L^T C_L C_L^T P_o^{-1} \right) \right] \right\}$. Thus, it can be obtained from (A24) that

$$\begin{aligned} \dot{V}_1 \leq & \bar{e}_F^T \left[\mathcal{L}_1 \otimes \left(A_F P_1 + P_1 A_F^T \right) - 2 \left(\alpha_1 - \frac{1}{2} d_3 \right) \mathcal{L}_1 \mathcal{L}_1 \otimes B B^T \right] \bar{e}_F \\ & + (d_1 + d_2) e_F^T \left(\mathcal{L}_1 \otimes P_1^{-1} \right) e_F + \frac{\lambda_{\max} \left(\mathcal{L}_1 \otimes P_1^{-1} \right) \|\Delta_h\|^2}{d_1} + \frac{\lambda_S \|\tilde{\xi}_F\|^2}{d_2} + \frac{g^2 \|\xi_F\|^2}{d_3} \\ & - \left(2 - \frac{1}{d_4} \right) \sum_{i=1}^N \text{tr} \left(\iota_i \tilde{W}_i^T \tilde{W}_i \right) + 2N\epsilon_m \epsilon_1 e^{-\epsilon_2 t} + d_4 \iota_m W_{Fm}. \end{aligned} \quad (\text{A25})$$

Let $\hat{e}_F = (\mathcal{H}_1^T \otimes I_6) \bar{e}_F$. Choose α_1 sufficiently large such that $\alpha_1 \geq 1/2(\sigma_3/\lambda_1 + d_3)$. Then, it follows from (28) and (A25) that

$$\bar{e}_F^T \left[\mathcal{L}_1 \otimes \left(A_F P_1 + P_1 A_F^T \right) - 2 \left(\alpha_1 - \frac{1}{2} d_3 \right) \mathcal{L}_1 \mathcal{L}_1 \otimes B B^T \right] \bar{e}_F < -\sigma_4 e_F^T \left(\mathcal{L}_1 \otimes P_1^{-1} \right) e_F. \quad (\text{A26})$$

Select $\sigma_4 > d_1 + d_2$, then the time derivation of V_1 can be given as follows:

$$\begin{aligned}
\dot{V} &= \dot{V}_0 + \dot{V}_1 \\
&< -[\sigma_4 - (d_1 + d_2)]e_F^T(\mathcal{L}_1 \otimes P_1^{-1})e_F - \sigma_2 \tilde{S}_F^T(\mathcal{L}_1 \otimes P_0)\tilde{S}_F + \frac{g^2 \|\tilde{\zeta}_F\|^2}{d_3} + d_4 \iota_m W_{Fm} \\
&+ \frac{\lambda_S \|\tilde{S}_F\|^2}{d_2} + \frac{\lambda_{max}(\mathcal{L}_1 \otimes P_1^{-1}) \|\Delta_h\|^2}{d_1} + 2N\epsilon_m \epsilon_1 e^{-\epsilon_2 t} - \left(2 - \frac{1}{d_4}\right) \sum_{i=1}^N \text{tr}(\iota_i \tilde{W}_i^T \tilde{W}_i) \\
&\leq -\varrho V + r \|\tilde{\zeta}_F\|^2 + \Delta,
\end{aligned} \tag{A27}$$

where $\varrho = \min\left\{\sigma_4 - (d_1 + d_2), \frac{\sigma_2 \lambda_{max}(\mathcal{L}_1 \otimes P_0) - \lambda_S/d_2}{\lambda_{min}(\mathcal{L}_1 \otimes P_0)}, (\pi_m \iota_m) \left(2 - \frac{1}{d_4}\right)\right\}$, $\pi_m = \max\{\pi_1, \pi_2, \dots, \pi_N\}$, $r = g^2/d_3$, $\Delta = \lambda_S \|\tilde{S}_F\|^2/d_2 + \lambda_{max}(\mathcal{L}_1 \otimes P_1^{-1}) \|\Delta_h\|^2/d_1 + d_4 \iota_m W_{Fm} + 2N\epsilon_m \epsilon_1 e^{-\epsilon_2 t}$. From Lemma 1, it can be deduced that the mean square of the solution for error systems (A15) and (A17) are NSPS-P.

From Theorem 2, one can obtain $\lim_{t \rightarrow \infty} \tilde{S}_F = 0$. Since $\lim_{t \rightarrow \infty} 2N\epsilon_m \epsilon_1 e^{-\epsilon_2 t} = 0$, and condition (26) holds, we can conclude that $\lim_{t \rightarrow \infty} \Delta_h = 0$. Therefore, the formation error e_F satisfies $\lim_{t \rightarrow \infty} E\|e_F\|^2 \leq (rN\phi + \Delta_W) / \left[\varrho \lambda_{min}(\mathcal{L}_1 \otimes P_1^{-1})\right] = \bar{\kappa}_F$, where $\phi = \sum_{i=1}^N \phi_i^2$, $\Delta_W = d_4 \iota_m W_{Fm}$. Thus,

$$\lim_{t \rightarrow \infty} E \left\| Y_{Fi} - h_{oi} - \sum_{k=N+1}^{N+M} \theta_{ik} \hat{Y}_{Lk} \right\|^2 = \|C_F\|^2 \lim_{t \rightarrow \infty} E \|e_i\|^2 \leq \kappa_F, \tag{A28}$$

where $\kappa_F = \|C_F\|^2 \bar{\kappa}_F$, which can be reduced to small values of the origin by tuning $d_1, d_2, d_3, \sigma_2, \sigma_4, \pi_i$ large enough and $\alpha_o, d_4, \epsilon_i$ small enough. Hence, this leads us to conclude that follower hovercrafts can form the given time-varying formation while tracking convex combination states that are estimated by distributed observers. This completes the proof of Theorem 3. $\square$

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