

Article

Error Analysis and Correction of ADCP Attitude Dynamics under Platform Swing Conditions

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Abstract: The Acoustic Doppler Current Profiler (ADCP) on a platform generates rotational linear velocity due to dynamic factors in attitude changes, leading to measurement errors in vessel and water flow velocities. This study derives and analyzes these errors, focusing on factors such as emission angle, transducer position, water depth, and measured depth, while also accounting for the variation in linear velocity and radial direction during each transmit–receive pulse cycle in the simulations. A method is proposed that introduces the concept of an equivalent radial radius to correct vessel and flow velocities, specifically designed for the common scenario where the ADCP is installed on the central longitudinal section of a vessel undergoing free roll motion. This method is suited for shallow water conditions without waves, with measurements taken vertically downward. It uses least squares fitting with an exponentially decaying sinusoidal model to process low-sampling-rate inclinometer data from the ADCP. This approach requires only the processing of measured data based on existing ADCP hardware, without the need for additional equipment. Field tests in a pool demonstrate that the proposed method significantly reduces vessel velocity errors, outperforming the traditional attitude static correction method.

Keywords: ADCP; rolling platform; attitude dynamic error analysis; attitude dynamic error correction; radial radius



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1. Introduction

An Acoustic Doppler Current Profiler (ADCP) is an instrument widely used to measure the water flow velocity profiles in natural water bodies and artificial waterways [1,2]. Due to their high measurement accuracy, fast speed, and low measurement cost [3–5], ADCPs have been extensively applied in various fields, including oceanographic research [6–9], marine development [10,11], hydrological surveys [12–14], underwater protection [15,16], and navigation [17].

Depending on the installation platform, installation method, application scenario, and data acquisition approach, ADCPs can be categorized into various types. Among them, the more commonly used vessel-mounted ADCP can be installed on platforms such as ships to measure both vessel velocity and flow velocity profiles [18]. When the attitude of the platform changes, the attitude of the ADCP mounted on it also changes, affecting the accuracy of flow velocity measurements. Therefore, vessel-mounted ADCPs are typically equipped with inclinometers and electronic compasses to correct for tilt angles and determine the direction of navigation. This allows for the transformation of transducer coordinates for vessel velocity and flow velocity into the Earth coordinate system [19,20].

In fact, the impact of attitude changes includes not only changes in angles, but also additional linear velocities generated by rotation. However, the influence of ADCP attitude changes is typically addressed by correcting only the tilt angle, often neglecting the effect of linear velocity caused by the transducer's sway. Some researchers believe that the impact of linear velocity due to vessel sway with an amplitude within 5° can be entirely

ignored [21], while others suggest that the Janus [22] symmetrical transducer structure can cancel out the effects of sway. Indeed, in such cases, the impact of sway can sometimes be negligible, and the traditional attitude correction method of the ADCP can still be effective. However, in certain scenarios, the influence of transducer sway on flow measurement can still be significant. For example, in the commonly used moored observation method for deep-sea ocean monitoring, the current measuring instruments within the buoy system may tilt and sway with the water flow, and the accuracy of flow measurement can be affected by displacement, attitude changes, and the linear velocity generated during these changes [9–13]. Additionally, due to the time interval between the emission and reception of ADCP signals, changes in linear velocity and the radial direction of the transducer during this interval can also affect velocity measurement.

Traditional ADCPs typically correct only for attitude angles and often overlook changes in attitude angles during the echo transmission and reception period. To clarify and differentiate between methods, this paper refers to the traditional correction approach as the “attitude static correction method”. In contrast, the method that accounts for additional linear velocities caused by sway and considers changes in the transducer’s motion during the echo transmission and reception period is termed the “attitude dynamic correction method”.

Some researchers have already conducted studies to reduce the impact of platform sway on flow measurements. For example, Raye [23] aimed to mitigate the impact of vessel sway and reduce the average number of pings required to obtain high-spatiotemporal-resolution water flow velocities by using data from an inertial navigation system (INS) and differential GPS (DGPS) to correct ADCP-measured water flow velocities. Terray [24] focused on correcting the impact of AUV (Autonomous Underwater Vehicle) motion under wave action on ADCP performance, achieving certain results by replacing the costly tactical-grade INS with an industrial-grade AHRS (attitude and heading reference system). Zhang Hongmei [25] applied linear velocity correction for ADCP velocity measurements using differential GPS. In the field of Doppler Velocity Log (DVL), which operates on principles nearly identical to those of ADCP and is used for underwater navigation, existing Kalman filter-based online calibration methods have often overlooked the lever arm between the Inertial Measurement Unit (IMU) and the DVL [26–28]. It is only in recent years that attention has been given to the impact of the lever arm between the IMU and the DVL, leading to the development of some solutions [29,30].

While some of these approaches have achieved good results in linear velocity correction, they require at least one of the following systems during measurement: INS, DGPS, or AHRS. This increases the cost and complexity of velocity measurements, reducing flexibility and convenience. Moreover, these methods do not include a detailed quantitative analysis of the impact of sway.

This study thoroughly investigates the impact of dynamic factors caused by platform sway on ADCP flow measurements, analyzing the effects of factors such as emission angle, the relative position of the sway center to the transducers, water depth, and measured depth on the errors through simulations. Subsequently, a flow velocity measurement method based on radial radius estimation for linear velocity correction is proposed to address the dynamic attitude effects. The proposed method is designed for the common application scenario where the ADCP is installed on the central longitudinal section of a vessel undergoing free roll motion, with measurements taken vertically downward in shallow water without waves, such as when towing the ADCP-equipped vessel for measurements on relatively calm lakes or reservoirs. The method involves fitting low-sampling-rate inclinometer data to an exponential decay sinusoidal oscillation model to derive the roll angle expression, and it introduces the concept of an equivalent radial radius for linear velocity correction, all without requiring any additional equipment beyond the ADCP itself.

Compared with other related research, this study provides a more in-depth and detailed simulation analysis of attitude dynamic errors, taking into account the effects of

changes in linear velocity and radial direction during transducer pulse transmission and reception. Unlike methods that require additional hardware such as an inertial navigation system (INS), differential GPS, or attitude and heading reference system (AHRS) to correct attitude dynamic errors, the attitude dynamic correction method proposed in this paper effectively corrects these errors without the need for additional hardware on the ADCP. Although more complex attitude dynamic error correction techniques are widely used in ship and vehicle navigation systems, they are less commonly applied in the field of ADCP flow velocity measurement, and most methods require additional attitude measurement or navigation equipment. Additionally, although many large vessels at sea are equipped with accurate, high-velocity motion sensors, ADCPs are also often used to measure flow velocity in rivers, lakes, and other water bodies, where the vessels are usually smaller and typically lack such precise and fast motion sensors. The method proposed in this study can also be applied to post-process ADCP data that have already been collected without attitude dynamic error correction or accurate, high-velocity attitude information, offering a way to retroactively correct the attitude dynamic errors and salvage existing data.

The remainder of this paper is organized as follows: Section 2 derives the errors in the attitude static correction method under the influence of dynamic attitude factors. Section 3 proposes a flow velocity measurement method based on radial radius estimation for linear velocity correction to address the velocity measurement errors caused by platform sway dynamics. Section 4 uses the attitude dynamic error formulas derived in Section 2 to analyze the impact of various factors on these errors through simulations, followed by an experimental comparison of the performance of the method proposed in Section 3 with that of the attitude static correction method. Finally, Section 5 provides a summary of the entire paper.

2. Theoretical Derivation of Attitude Dynamic Error

The traditional ADCP correction for transducer sway is limited to addressing static factors of attitude changes and does not consider the impact of dynamic factors. Therefore, the following section derives the errors introduced by the attitude static correction method under transducer sway conditions, with a simulation analysis provided in Section 4.1.

In this section, we first substitute the roll motion equation, assumed to be simple harmonic, into the acoustic ray model of a vessel-mounted ADCP that accounts for sway, generating Doppler frequency shifts. Then, using the attitude static correction method, we derive the radial velocities for both vessel velocity and flow velocity. The vessel velocity and flow velocity are then determined from the radial velocities of the three beams. By subtracting the actual vessel velocity and flow velocity from the derived values, we obtain the errors in vessel velocity and flow velocity. Finally, based on the derived error formulas, a detailed quantitative analysis is conducted through a simulation to examine how various factors influence the errors introduced by the attitude static correction method.

When using a vessel-mounted ADCP for measurements, the transducers sway along with the platform on which they are installed. This swaying causes an additional relative velocity between the transducers and the water, introduced by the platform's motion. This additional relative velocity affects the measured Doppler frequency shift, leading to errors in the radial velocities of the transducers, which, in turn, impacts the accuracy of the measured vessel velocity and flow velocity.

The primary reasons for the measurement errors in the traditional ADCP under swaying conditions are as follows:

1. The attitude static correction method does not account for the linear velocity generated by the sway.
2. It does not consider the changes in sway-induced linear velocity or radial direction during the transmission and reception of acoustic pulses, assuming that the velocity and direction of the transducers remain constant during a single transmission–reception cycle.

Next, we take these two factors into account to derive the error formulas. This paper only considers the case of four transducers in a Janus configuration or a phased array transducer that, after beamforming, can be equivalent to four discrete transducers. For the phased array transducer, the term “transducer” used later in this paper refers to the discrete transducers that are equivalent to the phased array transducer after beamforming. It is important to note that the central axis of the four transducers refers to the symmetrical axis of the transducers’ acoustic axes, with the acoustic axis being the centerline of each transducer’s acoustic beam.

For the swaying motion of the vessel, only the roll motion is considered here. As shown in Figure 1, transducers 3 and 4 lie within the yOz plane. The central axis of the four transducers passes through the rotation center o in the yOz plane, and the transducers undergo roll motion around a rotational axis parallel to the x axis that passes through point o . The water depth is H . The transducer emits sound waves at position 2, receives the echo from a depth of H_n at position 3, and receives the bottom echo at position 4. It is known that the flow velocity at depth H_n , V_n , and the vessel velocity V_a have directions along the positive y axis. The angles ϕ_{V_a} and ϕ_{V_n} , which represent the angles of V_a and V_n relative to the vertically downward direction, are both $\pi/2$, indicating that they are along the positive y axis.

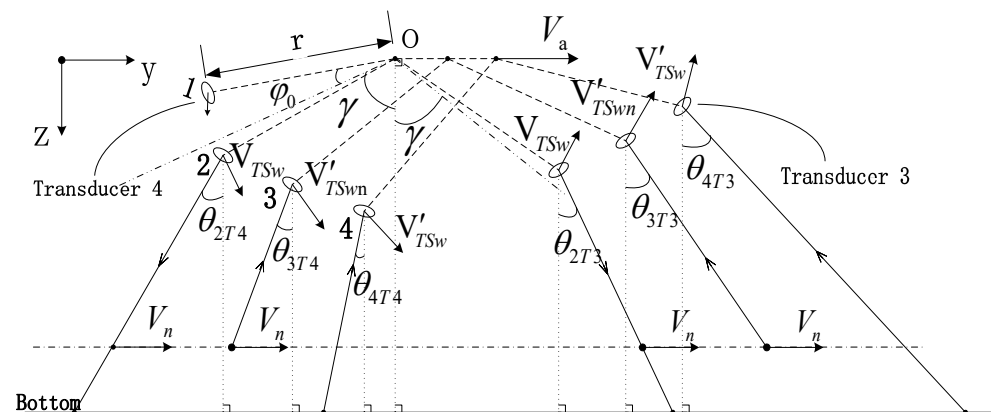


Figure 1. A schematic diagram of pulse transmission and reception during transducer sway.

Let the linear velocities of the transducers at positions 2, 3, and 4 be V_{TSw} , V'_{TSwn} , and V'_{TSw} , respectively, corresponding to the times t_2 , t_3 , and t_4 . The roll angles at these positions are ϕ_3 and ϕ_4 . The angles between the radial direction of transducer 3 and the vertical direction are θ_{2T3} , θ_{3T3} , and θ_{4T3} , and, for transducer 4, the corresponding angles are θ_{2T4} , θ_{3T4} , and θ_{4T4} . When the roll angle is 0, the angle between the roll radius r and the vertically downward direction is γ .

Although ships at sea are generally larger, with roll periods typically ranging from 6 to 14 s, ADCPs are also frequently used to measure the flow velocity of rivers, lakes, and other water bodies, where the vessels are usually smaller. For example, in this study, the RiverRay ADCP was mounted on a small boat with dimensions of only 0.96 m in length, 0.2 m in width, and 0.18 m in height. Based on the natural roll periods of various types of vessels, the natural roll period of the towed monohull equipped with the ADCP was set to 1 s, with a roll amplitude ϕ_0 of 10° [31]. At the initial roll moment $t = 0$, transducer 4 is at position 1, with an initial roll angle of $-\phi_0$ and an initial roll angular velocity of $\dot{\phi}_0 = 0$. According to the ship’s roll motion law [31], the function of the transducer’s roll angle with respect to time t is given by

$$\phi(t) = -\phi_0 \cos n_\phi t = -\frac{\pi}{18} \cos 2\pi t \quad (1)$$

where $\varphi(t)$ is the angle measured counterclockwise from the vertically downward direction. By differentiating it, the roll angular velocity of the transducer $\dot{\varphi}(t)$ is given by

$$\dot{\varphi}(t) = \frac{\pi^2}{9} \sin 2\pi t \quad (2)$$

Next, based on the positions, attitudes, and motion states described above, the actual echo frequency under the influence of dynamic attitude changes is derived. Then, using the echo frequency, the vessel velocity and flow velocity measured using the traditional ADCP are obtained using the attitude static correction method described in Section 2, allowing for a comparison with the actual vessel velocity and flow velocity.

First, consider the one-dimensional case of a single transducer under the two-way Doppler effect, where the transducer velocity and water flow velocity are aligned on a single straight line. According to the instantaneous nature of the Doppler effect, the two-way Doppler effect formula changes from its original form

$$f_1 = f_0 \cdot \frac{(c + v_s)(c - v_r)}{(c + v_r)(c - v_s)} \quad (3)$$

to the new form

$$f = f_0 \cdot \frac{(c + v_s)(c - v'_r)}{(c + v_r)(c - v_s)} \quad (4)$$

where c is the velocity of sound, v_s is the water flow velocity, and v_r and v'_r are the transducer velocities during the transmission and reception of sound waves, respectively. f_0 is the frequency of the emitted sound wave, and f_1 and f are the received frequencies without and with the consideration of the difference in transducer velocities during sound wave transmission and reception, respectively.

Next, based on the one-dimensional two-way Doppler formula for a single transducer, we derive a three-dimensional Doppler formula for three transducers and subsequently analyze the errors introduced by the sway.

By selecting any three transducers, it is possible to derive the vessel velocity and flow velocity. In this study, transducers 2, 3, and 4 are used as examples for the derivation. Therefore, it is only necessary to determine the components of each velocity along the radial directions of transducers 2, 3, and 4. Substituting these into Equation (4) yields the echo frequencies received by the three transducers.

The direction along the radial axis pointing upward is defined as the positive direction for the velocity in the beam direction. Since the sway-induced linear velocity and radial direction change during the transmission and reception of sound waves by the transducer, the transducer's velocity at times t_2 , t_3 , and t_4 will differ, and their components in the beam direction may also vary. Let us first examine transducer 4. The components of its velocity in the beam direction at times t_2 , t_3 , and t_4 are denoted as V_{TRadT4} , $V'_{TnRadT4}$, and V'_{TRadT4} , respectively. Since the component of the transducer's velocity in the beam direction can be considered as the sum of the vessel velocity and the linear velocity components in the beam direction, V_{TRadT4} , $V'_{TnRadT4}$, and V'_{TRadT4} are given by

$$V_{TRadT4} = -V_a \cos(\theta_{2T4} - \phi_{Va}) - V_{TSw} \sin(\gamma - \alpha) \quad (5)$$

$$V'_{TnRadT4} = -V_a \cos(\theta_{3T4} - \phi_{Va}) - V'_{TSwn} \sin(\gamma - \alpha) \quad (6)$$

$$V'_{TRadT4} = -V_a \cos(\theta_{4T4} - \phi_{Va}) - V'_{TSw} \sin(\gamma - \alpha) \quad (7)$$

where V_{TSw} , V'_{TSwn} , and V'_{TSw} are the transducer's linear velocities at times t_2 , t_3 , and t_4 , respectively. The formulas for these velocities are given as

$$V_{TSw} = r\dot{\varphi}(t_2) \quad (8)$$

$$V'_{TSwn} = r\dot{\varphi}(t_3) \quad (9)$$

$$V'_{TSw} = r\dot{\phi}(t_4) \quad (10)$$

Due to the changes in the radial direction during the transmission and reception of sound waves, the paths of the sound waves received and scattered by the scatterer at depth H_n are different, with the directions of these paths aligning with the radial directions at times t_2 and t_3 , respectively. Therefore, the components of the flow velocity in the beam direction of transducer 4 during the incidence and scattering of the sound waves, V_{nRadT4} and V'_{nRadT4} , are given by

$$V_{nRadT4} = -V_n \cos(\phi_{Vn} - \theta_{2T4}) \quad (11)$$

$$V'_{nRadT4} = -V_n \cos(\phi_{Vn} - \theta_{3T4}) \quad (12)$$

Therefore, by substituting V_{TRadT4} , $V'_{TnRadT4}$, V_{nRadT4} , and V'_{nRadT4} into Equation (4), the frequency of the echo received by transducer 4 from depth H_n , denoted as f_{nT4} , is given by

$$f_{nT4} = f_0 \frac{(c + V_{nRadT4})(c - V'_{TnRadT4})}{(c + V_{TRadT4})(c - V'_{nRadT4})} \quad (13)$$

and by substituting V_{TRadT4} and V'_{TRadT4} into Equation (4), the frequency of the bottom echo received by transducer 4, denoted as f_{BTT4} , is given by

$$f_{BTT4} = f_0 \frac{c - V'_{TRadT4}}{c + V_{TRadT4}} \quad (14)$$

Similarly, the frequencies of the echo from depth H_n and the bottom echo for transducers 2 and 3 can be obtained as follows:

$$f_{nT2} = f_0 \frac{(c - V_n \cos(\phi_{Vn} + \phi_2) \cos \alpha)(c + V_a \cos(\phi_{Va} + \phi_3) \cos \alpha)}{(c - V_a \cos(\phi_{Va} + \phi_2) \cos \alpha)(c + V_n \cos(\phi_{Vn} + \phi_3) \cos \alpha)} \quad (15)$$

$$f_{BTT2} = f_0 \frac{c + V_a \cos(\phi_{Va} + \phi_4) \cos \alpha}{c - V_a \cos(\phi_{Va} + \phi_2) \cos \alpha} \quad (16)$$

and

$$f_{nT3} = f_0 \frac{(c - V_n \cos(\phi_{Vn} + \theta_{2T3}))(c - V'_{TnRadT3})}{(c + V_{TRadT3})(c + V_n \cos(\phi_{Vn} + \theta_{3T3}))} \quad (17)$$

$$f_{BTT3} = f_0 \frac{c - V'_{TRadT3}}{c + V_{TRadT3}} \quad (18)$$

where

$$V_{TRadT3} = -V_a \cos(\theta_{2T3} + \phi_{Va}) + V_{TSw} \sin(\gamma - \alpha) \quad (19)$$

$$V'_{TnRadT3} = -V_a \cos(\theta_{3T3} + \phi_{Va}) + V'_{TSwn} \sin(\gamma - \alpha) \quad (20)$$

$$V'_{TRadT3} = -V_a \cos(\theta_{4T3} + \phi_{Va}) + V'_{TSw} \sin(\gamma - \alpha) \quad (21)$$

After obtaining the actual echo frequencies of each transducer under sway conditions, the flow velocity and vessel velocity are then calculated using the attitude static method, which does not account for the dynamic effects of sway. First, using the Doppler formula, the radial vessel velocities $V_{aRadT2Tra}$, $V_{aRadT3Tra}$, and $V_{aRadT4Tra}$, and the radial flow velocities $V_{nRadT2Tra}$, $V_{nRadT3Tra}$, and $V_{nRadT4Tra}$ for transducers 2, 3, and 4, respectively, are obtained through the attitude static method:

$$V_{aRadTiTra} = c \frac{f_0 - f_{BTTi}}{f_0 + f_{BTTi}} \quad (22)$$

$$V_{nRadTiTra} = c \frac{(f_{nTi} - f_0)c + (f_{nTi} + f_0)V_{aRadTiTra}}{(f_{nTi} + f_0)c + (f_{nTi} - f_0)V_{aRadTiTra}} \quad (23)$$

Here, i takes the values 2, 3, and 4. Based on the principle of converting from beam coordinates to transducer coordinates [32], the vessel velocity vector $\vec{V}_{aTra}$ and the flow velocity vector $\vec{V}_{nTra}$ in the transducer coordinate system are given by

$$\vec{V}_{aTra} = \begin{bmatrix} \frac{V_{aRadT4Tra} + V_{aRadT3Tra} - 2V_{aRadT2Tra}}{2 \sin \alpha} \\ \frac{-V_{aRadT3Tra} + V_{aRadT4Tra}}{2 \sin \alpha} \\ \frac{V_{aRadT4Tra} + V_{aRadT3Tra}}{2 \cos(\alpha)} \end{bmatrix} \quad (24)$$

$$\vec{V}_{nTra} = \begin{bmatrix} \frac{V_{nRadT4Tra} + V_{nRadT3Tra} - 2V_{nRadT2Tra}}{2 \sin \alpha} \\ \frac{-V_{nRadT3Tra} + V_{nRadT4Tra}}{2 \sin \alpha} \\ \frac{V_{nRadT4Tra} + V_{nRadT3Tra}}{2 \cos \alpha} \end{bmatrix} \quad (25)$$

Based on the coordinate transformation principle [33], let the transformation matrix from the transducer coordinates to the vessel coordinates be Λ_{tb} , and let the transformation matrix from the vessel coordinates to the Earth coordinates be $\Lambda(t)$. Then, the vessel velocity vector $\vec{V}_{aTra}$ and the flow velocity vector $\vec{V}_{nTra}$ in the Earth coordinate system, obtained using the attitude static correction method, are given by

$$\vec{V}_{aTra} = \Lambda(t) \Lambda_{tb} \vec{V}_{aTra} \quad (26)$$

$$\vec{V}_{nTra} = \Lambda(t) \Lambda_{tb} \vec{V}_{nTra} \quad (27)$$

In this section, Λ_{tb} is the identity matrix. Let $\vec{V}_{aTra} = [V_{aTrax}, V_{aTray}, V_{aTraz}]^T$. Since, in actual measurements, only the horizontal component of $\vec{V}_{aTra}$ is generally taken as the vessel velocity—meaning that only the horizontal component of the bottom-tracking velocity is considered—the final bottom-tracking velocity scalar V_{BTra} and flow velocity scalar V_{nTra} obtained using the attitude static correction method are given by

$$V_{BTra} = \text{sgn}(V_{aTray}) \sqrt{V_{aTrax}^2 + V_{aTray}^2} \quad (28)$$

$$V_{nTra} = \text{sgn}(V_{nTray}) \left| \vec{V}_{nTra} \right| \quad (29)$$

The sign function sgn is used to distinguish the general direction of the vessel velocity V_{BTra} and the flow velocity V_{nTra} for analytical purposes. A positive value of the sign function indicates that the velocity is directed toward the positive side of the y axis, while a negative value indicates that the velocity is directed toward the negative side of the y axis.

Thus, the vessel velocity and flow velocity errors ΔV_a and ΔV_n caused by the attitude static correction method, which does not consider the dynamic effects of sway, are given by

$$\Delta V_a = V_{BTra} - V_a \quad (30)$$

$$\Delta V_n = V_{nTra} - V_n \quad (31)$$

Based on the known positions, attitudes, and velocities of the transducers during platform sway, as well as the water flow velocity and depth, the vessel velocity V_{BTra} and flow velocity V_{nTra} , along with the errors ΔV_a and ΔV_n under the attitude static method, are derived. Next, the derivation of a flow velocity measurement method based on radial radius estimation for linear velocity correction is presented, specifically for a towed ADCP installed on the central longitudinal section of a vessel in shallow water.

3. Flow Velocity Measurement Method Based on Radial Radius Estimation for Linear Velocity Correction

A common application scenario involves installing the ADCP on the central longitudinal section of the vessel, measuring vertically downward, such as in towed measurements with a non-powered trimaran, powered measurements with an unmanned trimaran, or powered measurements with an unmanned monohull. For this typical situation under free roll conditions, this paper proposes a flow velocity measurement method based on radial radius estimation for linear velocity correction, specifically designed for a vessel-mounted ADCP installed on the central longitudinal section of the vessel in shallow water.

In shallow water, the linear velocity can be considered constant during a single transmit–receive pulse cycle (for a detailed analysis, refer to Section 4.1.4). Therefore, this method first lets the vessel, without sailing velocity, freely roll in shallow water, using the traditional ADCP with the attitude static correction method to measure the bottom-tracking radial velocity and attitude angles. These measurements are then used to estimate the effective radial radius of the roll. This effective radial radius can subsequently be used to correct the attitude dynamic errors due to roll in future measurements. The definition of the effective radial radius is provided in Equation (42) in the following derivation. The specific steps are as follows.

As shown in the block diagram of the flow velocity measurement method based on radial radius estimation for linear velocity correction in Figure 2, as the vessel's free roll motion in water closely approximates simple harmonic oscillation, the roll angle curve can be considered an exponentially decaying oscillation curve [31]. Therefore, this method first conducts a single measurement (the measurement used for radial radius estimation indicated by the red dashed box). In this measurement, the measured roll angle is fitted using the exponentially decaying sine oscillation model formula and the least squares method to obtain the roll angle expression. Then, the average bottom-tracking radial velocity measured by the four transducers is fitted using the derivative of the exponentially decaying sine oscillation model formula and the least squares method to obtain the expression for the average bottom-tracking radial velocity of the transducers. Finally, the estimated radial radius is obtained by dividing the amplitude of the transducer's average bottom-tracking radial velocity expression by the amplitude of the roll angle expression.

After obtaining the radial radius, it can be used to correct other measurements (vessel velocity and flow velocity corrections in the green dashed box). The same method is applied to determine the roll angle expression for the current measurement, and then its derivative is taken to obtain the roll angular velocity expression. By multiplying the roll angular velocity expression by the radial radius, the radial component of the roll-induced linear velocity is obtained. Subtracting this radial component of the roll-induced linear velocity from the measured bottom-tracking and intermediate-layer radial velocities gives the radial velocity corrected for linear velocity effects. Finally, by applying the coordinate transformations, the vessel velocity and flow velocity corrected for linear velocity are obtained.

The detailed derivation process is as follows:

Since the ADCP is installed on the central longitudinal section of the vessel and the four transducers are typically arranged in an Xform configuration with $\psi_{tb} = 45^\circ$ [32], the roll radii of the four transducers are equal due to symmetry. Taking transducer 1 as an example, let the roll radius be r . The angle between the roll-induced linear velocity and the direction of the radial velocity for transducer 1 is denoted as η_1 , with the positive direction of the radial velocity defined as pointing radially inward toward the transducer.

In shallow water, the linear velocity can be considered constant during a single transmit–receive pulse cycle. Additionally, as the vessel velocity is zero, the traditionally measured radial bottom-tracking velocity by the ADCP, $V_{BTRad1Tra}$, can be expressed as

$$V_{BTRad1Tra} = \omega r \cos \eta_1 \quad (32)$$

Here, ω represents the roll angular velocity. ω can be obtained by differentiating the roll angle measured by the inclination sensor. However, because the sensor data from

the ADCP are generally transmitted and recorded in unified data packets, the maximum update rate of these data packets is typically only around 10 Hz. As a result, the roll angle data have a maximum sampling rate of 10 Hz, often around 2 Hz, which is insufficient for the required accuracy.

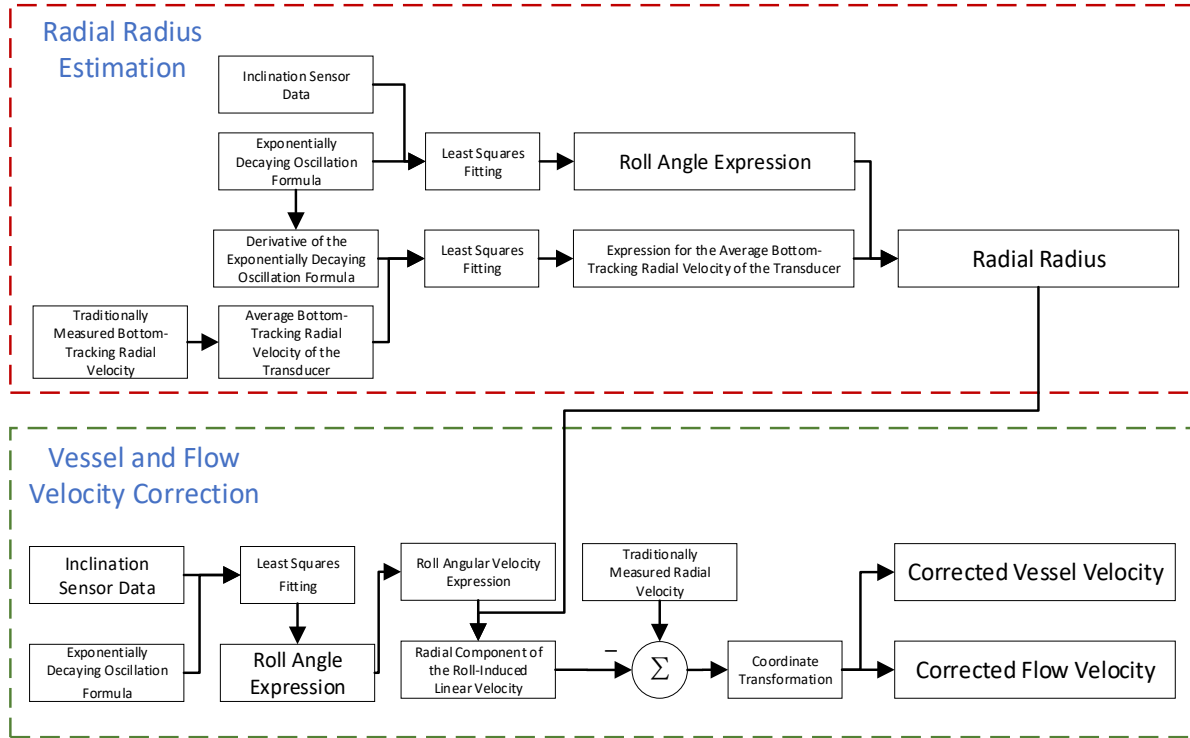


Figure 2. Block diagram of the flow velocity measurement method based on radial radius estimation for linear velocity correction.

Therefore, it is necessary to first process the roll angle data obtained from the inclination sensor. Since the roll angle data, packaged together with the ADCP data, typically have a sampling rate of only about 2 Hz, and the roll frequency of the unmanned monohull vessel carrying the ADCP is often around 1 Hz, they easily fail to meet the Nyquist sampling theorem and are insufficient for the differentiation needed. Given that the vessel's free roll motion in water closely approximates simple harmonic oscillation, the curve can be considered an exponentially decaying oscillation curve [31]. Thus, the issue of an insufficient sampling rate can be addressed by estimating the parameters in the exponentially decaying sine oscillation model to derive the roll angle expression. The expression for the roll angle during free roll motion without waves in the water is defined as follows:

$$\phi = \phi_0 e^{-v_\phi(t-t_0)} \left[\cos n_\phi(t-t_0) + \frac{v_\phi}{n_\phi} \sin n_\phi(t-t_0) \right] \quad (33)$$

Here, ϕ_0 , t_0 , n_ϕ , and v_ϕ are the parameters to be estimated. Therefore, by using Equation (33) as the fitting formula and applying the least squares method to the measured roll angle data, the roll angle expression $\hat{\phi}(t)$ can be obtained. The roll angular velocity ω is then given by

$$\omega = \dot{\hat{\phi}}(t) \quad (34)$$

Next, let us determine η_1 . Let the angle between the roll radius and the z_b axis be γ . It is known that the angle between the transducer's acoustic axis and the z_t axis is α .

As shown in Figure 3a, the vessel undergoes roll motion around the x_b axis, with transducers 1, 2, 3, and 4 positioned at the four vertices of a square. The x_b axis is parallel to the line connecting transducers 1 and 4, and its projection on the square passes through the

center of the square. The angle between the radial direction and the positive direction of the z_b axis is α . The blue arrows represent the linear velocities corresponding to each transducer, and the red line segments represent the roll radii. Figure 3b provides an enlarged view of the area around transducer 1. In this figure, KM is parallel to the linear velocity of transducer 1 (indicated by the blue arrow), E_1E and KL are parallel to the z_b axis, and AE_1 and ML are parallel to the y_b axis. Therefore, the angle $\angle MKE$ in triangle $\triangle KEM$ is the desired angle η_1 . The solution is given by

$$\cos \eta_1 = \cos \alpha \sin \gamma - \frac{\sqrt{2}}{2} \sin \alpha \cos \gamma \quad (35)$$

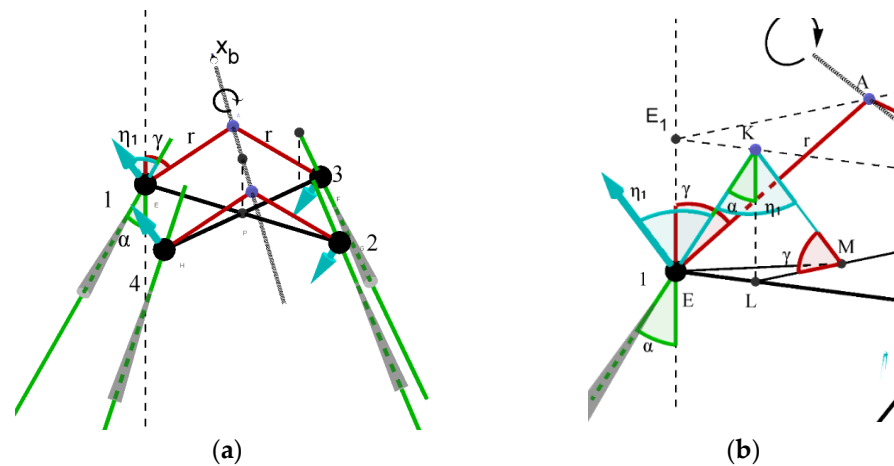


Figure 3. Diagram of transducer positions on the vessel. (a) Diagram of four transducers swaying around the x_b axis. (b) Geometric diagram of Transducer 1 swaying around the x_b axis.

Due to the symmetry in the arrangement of the transducers, it can be determined that

$$\eta_2 = \eta_3 = \pi - \eta_1 \quad (36)$$

$$\eta_4 = \eta_1 \quad (37)$$

Substituting Equations (34) and (35) into Equation (32) yields

$$V_{BTRad1Tra}(t) = \dot{\phi}(t)r \left(\cos \alpha \sin \gamma - \frac{\sqrt{2}}{2} \sin \alpha \cos \gamma \right) \quad (38)$$

By simplifying, we obtain

$$r \sin(\gamma - \beta_1) \sqrt{\frac{\cos^2 \alpha + 1}{2}} = \frac{V_{BTRad1Tra}(t)}{\dot{\phi}(t)} \quad (39)$$

where

$$\beta_1 = \arctan \left(\frac{\sqrt{2} \tan \alpha}{2} \right) \quad (40)$$

β_1 is an acute angle.

Let

$$r_{rad1} = r \sin(\gamma - \beta_1) \sqrt{\frac{\cos^2 \alpha + 1}{2}} \quad (41)$$

then Equation (39) becomes

$$r_{rad1} = \frac{V_{BTRad1Tra}(t)}{\dot{\phi}(t)} \quad (42)$$

Let r_{rad1} be called the effective radial radius of transducer 1. The radial radius in this study is a quantity that directly converts angular velocity into the radial component of the transducer's linear velocity. Its function is similar to that of a radius, hence the term "radial radius". Unlike common concepts such as the radius vector or slant range, the radial radius introduced here is not a measure of distance in three-dimensional space and cannot simply be understood as the projection of the radius vector onto the radial direction. To simplify expression and facilitate calculation, this new concept is introduced.

Similarly, for transducers 2, 3, and 4, we have

$$r_{radi} = -\frac{V_{BTRadiTra}(t)}{\dot{\hat{\phi}}(t)} \quad (43)$$

Here, i takes the values 2, 3, and 4. Since the transducers are installed in an "X" shape on the central longitudinal section of the vessel, due to symmetry, r_{rad1} , r_{rad2} , r_{rad3} , and r_{rad4} are equal. Let them all be denoted as r_{rad} . Therefore, to improve accuracy, the mean value of the radial radius across all transducers and all time instances can be taken. Since the radial radii are equal, according to Equations (42) and (43), we have

$$V_{BTRad1} = V_{BTRad4} = -V_{BTRad2} = -V_{BTRad3} \quad (44)$$

Sometimes, due to the time alignment between the roll angle and radial velocity or issues with the measurement accuracy of the radial velocity, the effectiveness of calculating the radial radius using Equations (42) and (43) may be poor. In such cases, the average radial velocity of the four transducers can first be calculated according to Equation (44), yielding

$$V_{BTRadTra} = \frac{V_{BTRad1Tra} + V_{BTRad4Tra} - V_{BTRad2Tra} - V_{BTRad3Tra}}{4} \quad (45)$$

then, according to Equation (42), we have

$$r_{rad} = \frac{V_{BTRadTra}(t)}{\dot{\hat{\phi}}(t)} \quad (46)$$

According to Equation (46), it can be seen that the ratio of $V_{BTRadTra}(t)$ to $\dot{\hat{\phi}}(t)$ is a constant r_{rad} . Therefore, the expression for $V_{BTRadTra}(t)$ can be written in the same form as that for $\dot{\hat{\phi}}(t)$, differing only by a constant multiplier. The expression for $\dot{\hat{\phi}}(t)$ can be obtained by differentiating Equation (33), as shown below:

$$\dot{\hat{\phi}}(t) = -\phi_0 e^{-v_\varphi(t-t_0)} \left(n_\varphi + \frac{v_\varphi^2}{n_\varphi} \right) \sin[n_\varphi(t-t_0)] \quad (47)$$

Therefore, the expression for $V_{BTRadTra}(t)$ can be written as

$$V_{BTRadTra}(t) = -V_{BTRadTra0} e^{-v_\varphi(t-t_0)} \left(n_\varphi + \frac{v_\varphi^2}{n_\varphi} \right) \sin[n_\varphi(t-t_0)] \quad (48)$$

By using Equation (48) to construct the objective function, the parameters are estimated using the least squares method to obtain $V_{BTRadTra0}$. Then, based on Equation (46), the radial radius r_{rad} can be determined as

$$r_{rad} = \frac{V_{BTRadTra0}}{\phi_0} \quad (49)$$

Using this method to determine r_{rad} can effectively address the issue of large fluctuations in r_{rad} values at different times when calculated using Equation (42) or Equation (43), which can lead to an inaccurate determination of r_{rad} .

After obtaining the radial radius r_{rad} , the traditionally measured bottom-tracking radial velocities $V'_{BTRad1Tra}$, $V'_{BTRad2Tra}$, $V'_{BTRad3Tra}$, and $V'_{BTRad4Tra}$ from other or the current measurements can be corrected to obtain

$$V'_{BTRadi} = V'_{BTRadiTra} - \dot{\hat{\phi}}'(t)r_{rad} \quad (50)$$

Next, the intermediate-layer radial velocities $V'_{nRad1Tra}$, $V'_{nRad2Tra}$, $V'_{nRad3Tra}$, and $V'_{nRad4Tra}$ obtained from other or the current measurements using the traditional ADCP can be corrected to obtain

$$V'_{nRadi} = V'_{nRadiTra} - \dot{\hat{\phi}}'(t)r_{rad} \quad (51)$$

Here, i takes the values 1, 2, 3, and 4. Then, by performing the coordinate transformation, the vessel and flow velocities corrected using the attitude dynamic method can be obtained.

This method, under the existing hardware conditions of the ADCP, uses the low-sampling-rate data from the inclination sensor to estimate the roll angle expression of the transducer. It then estimates the roll radial radius through fixed-point bottom-tracking velocity measurements in shallow water, and it finally uses this radial radius to correct the vessel velocity and flow velocity. Considering that, in practical situations, it may not be convenient to add inertial navigation systems or other attitude or angular velocity measurement instruments to the existing ADCP equipment, this method provides a convenient approach for dynamic attitude correction.

4. Pool Test

This section begins by utilizing the attitude dynamic error derived in Section 2 and performs a simulation analysis, using the method of controlled variables, to assess the impact of different factors on the error. Then, to verify the effectiveness of the flow velocity measurement method based on radial radius estimation for linear velocity correction, vessel velocity measurement tests are conducted in the anechoic pool at Southeast University. These tests involve stationary free roll and moving free roll of the ADCP-equipped small vessel, validating the effectiveness of this method in correcting the dynamic attitude errors of vessel velocity measurements during free roll in shallow water.

4.1. Error Simulation Analysis

As shown in the theoretical error derivation in Section 2, considering the variation in linear velocity during a single transmit–receive cycle results in different linear velocities at various water depths and flow measurement depths. The linear velocity is also influenced by the sway radius, which is primarily determined by the transducer's radial direction and the position of the rotation center.

Therefore, this study focuses on conducting a simulation analysis of the impact of the following factors on the magnitude of errors introduced by attitude dynamics: the angle between the transducer's radial direction and the central axis, the roll angle at the time of transmission, the relative position of the transducer to the roll rotation center, and the water depth, as well as the depth at which the flow velocity is measured.

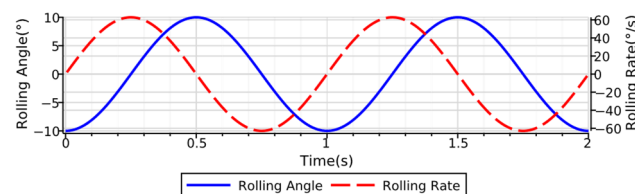
The following section uses the formulas derived in Section 2 to perform a simulation analysis, using the method of controlled variables, to study the impact of various parameters on attitude dynamic errors.

Table 1 below provides the initial settings for the simulation parameters related to the error analysis. In the subsequent simulation analysis, only the parameters that need to be varied are specified; all other parameters are set according to the values in Table 1. The parameter settings in Table 1 correspond to the small boat paired with the RiverRay ADCP, which has a length of 0.96 m, a width of 0.2 m, a height of 0.18 m, and a total mass of 18 kg including the ADCP.

Table 1. Initial values of parameters for error simulation analysis.

Parameter	Description	Value	Unit	Parameter	Description	Value	Unit
c	Velocity of Sound in Water	1500	m/s	ϕ_{Va}	Angle between Ship Velocity (Counterclockwise) and Vertically Downward Direction	$\pi/2$	rad
f_0	Transmission Frequency	600	kHz	ϕ_{Vn}	Angle between Flow Velocity (Counterclockwise) and Vertically Downward Direction	$\pi/2$	rad
H	Water Depth	65	m	γ	Angle between Sway Radius and Central Axis of Four Transducers	$\pi/3$	rad
H_n	Depth of the Water Layer for Flow Velocity Measurement	35	m	r	Sway Radius	0.2	m
α	Angle between Transducer Radial Direction and Central Axis of Four Transducers	$\pi/6$	rad	ϕ_0	Initial Rolling Angle	$\pi/18$	rad
V_a	Vessel Velocity	1	m/s	T	Rolling Period of the Vessel	1	s
V_n	Flow Velocity	1	m/s	t_2	Time of Transducer Emission	0.4	s

Figure 4 shows curves of the roll angle and angular velocity as they vary with time. In Figure 4, the blue and red curves represent the changes in the roll angle and roll angular velocity over time when the roll period T is 1 s. It can be observed that the maximum roll angular velocity can reach 60 degree/s.

**Figure 4.** Curves of roll angle and angular velocity over time.

4.1.1. The Impact of Different Angles between the Transducer Radial Direction and the Central Axis

In Figure 5a, the red curve represents the vessel velocity V_{BTra} obtained using the attitude static correction method as a function of the angle α between the radial direction and the central axis, when the actual vessel velocity $V_a = 0$ and the water depth $H = 2$ m. The blue and green curves represent the transducer's linear velocity at the time of transmission V_{TSw} and at the time of receiving the bottom echo V'_{TSw} , respectively. Since the actual vessel velocity is zero, the non-zero portion of the vessel velocity obtained using the attitude static correction method is entirely due to the contribution of the sway-induced linear velocity.

As shown in Figure 5a, when $\alpha < \pi/3$, the smaller the α , the larger the magnitude of V_{BTra} , leading to a greater vessel velocity error, and V_{BTra} is negative, indicating a direction toward the negative side of the y axis. When the transmission time is $t_2 = 0.4$ s, the general direction of the transducer's roll should be toward the positive side of the y axis. However, V_{BTra} is directed toward the negative side of the y axis, which might seem counterintuitive. This is because the linear velocity affects V_{BTra} through its component in the radial direction, and V_{BTra} is determined by the combined contributions of the linear velocity components in the radial directions of each transducer, which are influenced by α .

As shown in Figure 1, when $\alpha < \pi/3$, the components of the linear velocity in the radial directions of transducers 3 and 4 are both directed toward the negative side of the y axis, while the component in the radial direction of transducer 2 is zero. Therefore, the resulting V_{BTra} is also directed toward the negative side of the y axis.

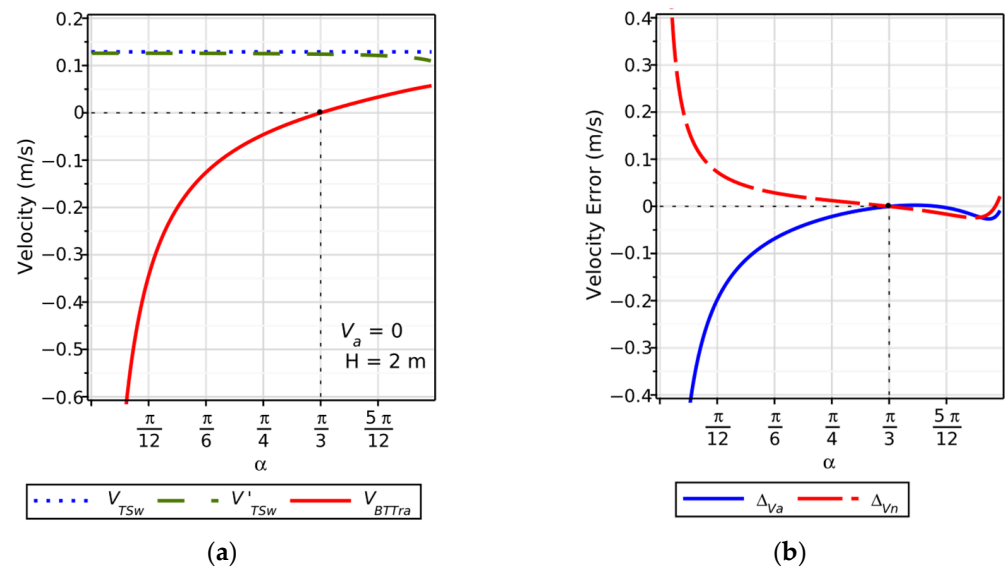


Figure 5. Curves of velocity and velocity error as a function of angle α between the radial direction and the central axis. (a) Attitude static corrected vessel velocity at 0 true velocity and 2 m depth. (b) Error in velocity from attitude static correction method at initial setup.

As indicated by the red curve in Figure 5a, when $\alpha = \pi/3$, that is, $\alpha = \gamma$, $V_{BTTr a} = V_a = 0$, because, at this point, the linear velocities of all transducers are perpendicular to the radial direction, resulting in zero components of the linear velocity in the radial direction. Therefore, $V_{BTTr a}$ is not affected by attitude dynamics (sway dynamics) at this point. Similarly, the flow velocity is also unaffected by attitude dynamics at this point. As shown in Figure 5b, when $\alpha = \gamma = \pi/3$, both the vessel velocity error Δ_{Va} and the flow velocity error Δ_{Vn} are zero.

4.1.2. The Impact of Transmission at Different Roll Angles

In Figure 6, the curves show the vessel velocity and flow velocity obtained using the attitude static correction method as a function of the roll angle, with sound waves transmitted at different roll angles and directions. The long dashed line with a right-pointing arrow indicates the velocity measured when the transducer sways in the counterclockwise direction (increasing φ_2 , as shown in Figure 1), while the dotted line with a left-pointing arrow represents the velocity measured when the transducer sways in the clockwise direction (decreasing φ_2).

It can be observed that, for most roll angles, the vessel velocity $V_{BTTr a}$ obtained when the transducer sways to the right is less than the actual vessel velocity, while the vessel velocity $V_{BTTr a}$ obtained when the transducer sways to the left is greater than the actual vessel velocity. The reason for this phenomenon has already been explained in the analysis of Figure 5, so it is not repeated here.

The curves for $V_{BTTr a}$ and $V_{nTr a}$ are not symmetric about $\varphi_2 = 0$ and the actual vessel velocity or flow velocity curve (blue straight line) because the difference in the radial direction during the transmission and reception of sound waves affects the symmetry. As a result, the point of maximum error does not occur at $\varphi_2 = 0$. In Figure 6a,b, it can be seen that the maximum relative error in the vessel velocity is approximately 22%, and the maximum relative error in the flow velocity is about 3.3%. The reason for the flow velocity error being much smaller than the vessel velocity error is mainly that the linear velocity component in the vessel velocity partially cancels out the linear velocity component in the flow velocity.

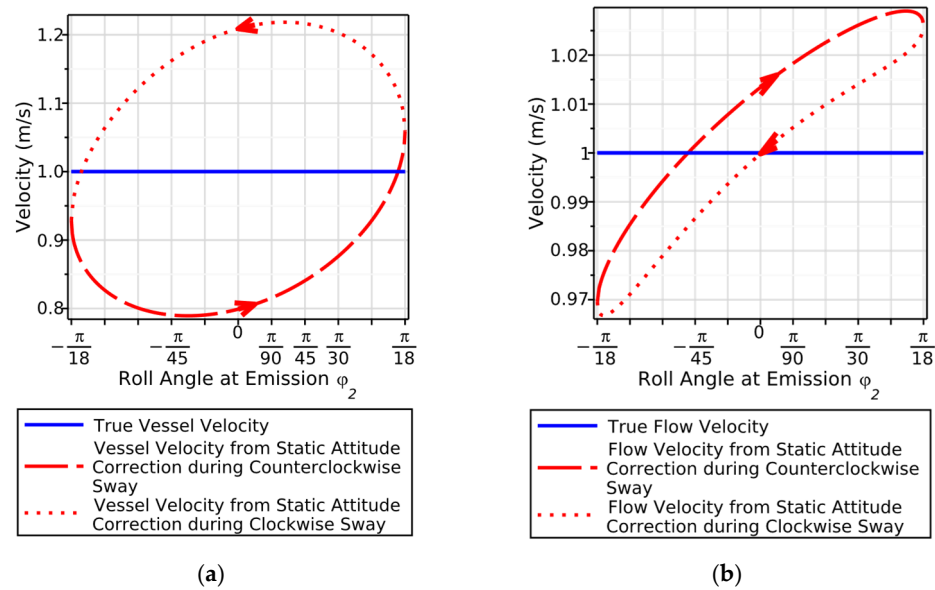


Figure 6. Curves of vessel velocity and flow velocity as a function of roll angle obtained using the attitude static correction method under different roll directions. (a) Vessel velocity from attitude static correction during different sway directions. (b) Flow velocity from attitude static correction during different sway directions.

4.1.3. The Impact of Different Relative Positions between the Sway Rotation Center and the Transducer

Figure 7 illustrates the impact of the relative position between the sway center O and the transducer on the velocity measurement errors. This relative position is primarily determined by the angle and magnitude of the sway radius. As shown in Figure 7a, the curves represent the vessel velocity and flow velocity errors varying with γ , where γ is the angle between the transducer's rotation radius and the central axis of the Janus structure. It can be observed that, when $\gamma = \alpha = \pi/6$, the errors are nearly zero because, at this point, the rotation radius and radial direction lie on the same straight line, making the linear velocity perpendicular to the radial direction, thus eliminating its influence. However, during the transmission and reception of sound waves, the radial direction changes, and the attitude static correction method does not account for this variation. As a result, there is still some error when $\gamma = \alpha$, as shown in the inset of Figure 7a.

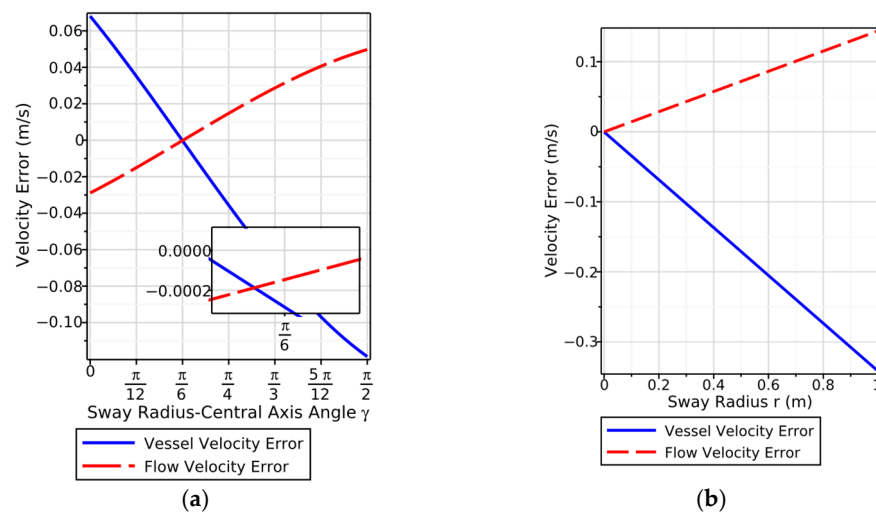


Figure 7. The impact of the relative position between the sway center and the transducer on velocity measurement errors. (a) Curve of vessel and flow velocity errors vs. radius-central axis angle. (b) Curve of vessel and flow velocity errors vs. sway radius.

As shown in Figure 7b, the vessel velocity and flow velocity errors are approximately proportional to the sway radius r , indicating that reducing the sway radius r can effectively decrease the errors caused by sway. Therefore, by combining the observations from Figure 7a,b, it can be concluded that the velocity measurement errors caused by sway are influenced by the transducer's installation position on the vessel. On the one hand, γ should be as close to α as possible, and, on the other hand, the sway radius r should be minimized. However, these two measures are often contradictory, so α or r must be determined based on the specific situation.

Additionally, it is generally assumed that the vessel sways around an inertia axis passing through its center of gravity. Therefore, the problem of determining the installation position can be translated into determining the relative position of the transducer with respect to the inertia axis.

4.1.4. The Impact of Different Water Depths and Measurement Depths on Flow Velocity

The simulation curves in Figure 8 explore the relationship between the dynamic effects caused by sway and water depth. In Figure 8a, the curves show how the vessel velocity and flow velocity errors vary with the water depth when the measurement depth is $H_n = 2\text{ m}$. It can be observed that both the vessel velocity and flow velocity errors change with depth. The flow velocity error (red curve) initially increases as the water depth H increases because a greater $H - H_n$ leads to a longer time interval between receiving the scattered echo and the bottom echo ($t_4 - t_3$). This increases the difference between the linear velocities V'_{TSwn} at time t_3 and V'_{TSw} at time t_4 , resulting in an increase in the flow velocity error as the water depth increases.

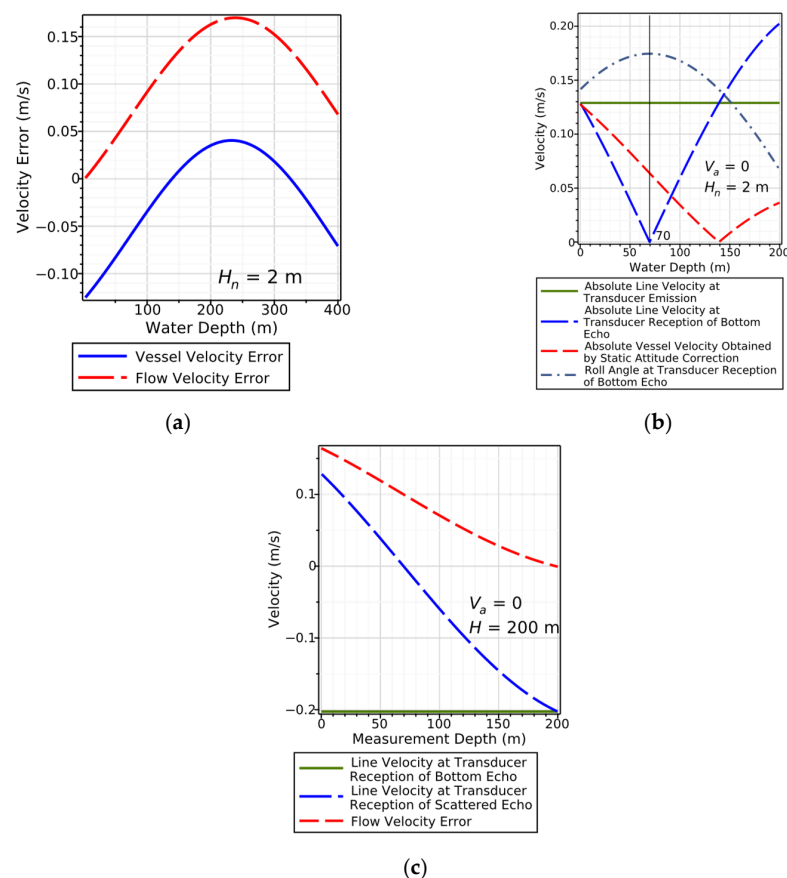


Figure 8. The relationship between the dynamic effects of sway and water depth. (a) Velocity error vs. water depth. (b) Absolute traditional vessel velocity and line velocity at emission and bottom echo reception vs. water depth. (c) Flow velocity error and line velocity at scattered and bottom echo reception vs. measurement depth.

However, when the depth increases beyond a certain point, due to the periodic nature of roll motion, the difference between V'_{TSwn} at time t_3 and V'_{TSw} at time t_4 begins to decrease, leading to a reduction in the flow velocity error. Therefore, the curve showing the variation in the error with the water depth is actually periodic, which is a consequence of the roll motion's periodicity.

Similarly, in Figure 8c, when $V_a = 0$ and $H = 200\text{m}$, as H_n increases, V'_{TSwn} becomes closer to V'_{TSw} , resulting in a decrease in the flow velocity error. When $H_n = H$, $V'_{TSwn} = V'_{TSw}$, and the flow velocity error becomes zero.

In Figure 8b, the variation in the transducer's linear velocities during pulse transmission and bottom echo reception, V_{TSw} and V'_{TSw} , as well as the vessel velocity V_{BTTr} obtained using the attitude static correction method, with respect to the water depth, is explored. It can be observed that, when $V_a = 0$, although the relative magnitudes of V_{TSw} , V'_{TSw} , and V_{BTTr} exhibit periodicity influenced by the sway cycle, the values of V_{TSw} , V'_{TSw} , and V_{BTTr} are very close at shallow depths. For instance, at $H = 3\text{ m}$, $V_{TSw} \approx V'_{TSw} \approx V_{BTTr}$. This suggests that, when the vessel is stationary and the water depth is very shallow, the transducer's linear velocity and the vessel velocity obtained using the attitude static correction method are approximately equal.

4.1.5. Simulation Summary

In summary, under the conditions described, vessel velocity errors are generally greater than flow velocity errors. When $\alpha < \gamma$, the smaller the α , the greater the velocity measurement error; when $\alpha = \gamma$, the error is zero. The magnitude of the error is approximately proportional to the sway radius, indicating that the velocity measurement error caused by sway is influenced by the transducer's installation position. On the one hand, γ should be made as close as possible to α ; on the other hand, the sway radius r should be minimized. However, these two measures often conflict, requiring specific adjustments of α or r based on the situation. The error varies cyclically with the water depth, decreasing as the measured flow velocity depth approaches the total water depth. When the vessel is stationary and the water depth is very shallow, the transducer's linear velocity is approximately equal to the vessel velocity obtained using the attitude static correction method.

4.2. Error Correction Pool Test

4.2.1. Test Scenario

The test utilized a RiverRay ADCP from Teledyne RD Instruments (TRDI). This ADCP has a bottom-tracking flow velocity measurement range of $\pm 9.5\text{ m/s}$ and a bottom-tracking depth range of 0.4 to 100 m. It uses a planar phased-array transducer with four beams, each with a beam angle of 30° . During the test, the radial velocity, inclination, and compass data measured using the RiverRay ADCP were exported using the accompanying software for further processing. The RiverRay ADCP packages various data and sends them to a PC at regular intervals, with the data packet update interval randomly fluctuating between 0.64 and 0.95 s. The positive directions of the roll, pitch, and yaw angles are shown in Figure 3, with the roll and pitch angles being 0 when the vessel is level and the yaw angle being 0 when the bow points due north. The ADCP was installed on the small vessel. The vessel has dimensions of 0.96 m in length, 0.2 m in width, and 0.18 m in height, with a total mass of 18 kg including the ADCP. The vessel's actual velocity was determined using motion tracking on video footage.

A schematic of the test scenario is shown in Figure 9. The pool is rectangular, measuring 8 m in length, 5 m in width, and 5 m in depth. The positions of the four beams (1, 2, 3, and 4) of the ADCP transducer are arranged in an "X" configuration, as shown in the diagram. The physical appearance of the transducer is depicted in Figure 10. The actual test scenario is shown in Figure 11.

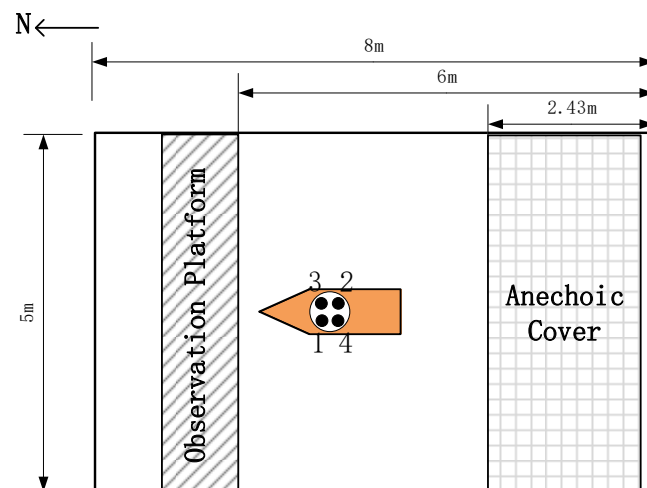


Figure 9. Pool test schematic diagram.

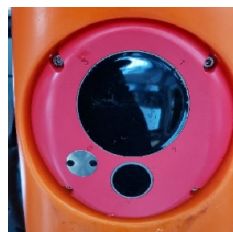


Figure 10. RiverRay ADCP transducer.



Figure 11. Pool test scenario diagram. (a) Method for moving the vessel. (b) Method for inducing vessel sway.

The small boat was moved by pulling it with a rope, as shown in Figure 11a. The method depicted in Figure 11b was used to induce roll motion in the boat.

To estimate the radial radius and use it for attitude dynamic error correction, two measurements were conducted using the ADCP in this test, resulting in two data sets. The measurement where the boat performed a free roll while stationary, used to estimate the radial radius, is referred to as Measurement 1. The measurement where the boat moved while performing a free roll is referred to as Measurement 2. The test first used the data from Measurement 1 to estimate the radial radius, and then this radial radius was used to apply attitude dynamic correction to the vessel velocity in both Measurements 1 and 2. The specific results of the test are detailed below.

4.2.2. Radial Radius Estimation

First, the radial radius of the transducers is estimated using the method described in the previous section with the data from Measurement 1. The boat is made to perform

only a stationary free roll in the water, and the bottom-tracking radial velocity and attitude angle data for all four beams are collected for radial radius estimation. The roll angle data from this measurement are first fitted to obtain the roll angle expression. Then, the average bottom-tracking radial velocity of the transducers, calculated according to Equation (45), is fitted to derive the expression for the average bottom-tracking radial velocity of the transducers.

Tables 2 and 3 below show the parameter estimation results for the roll angle and the average bottom-tracking radial velocity of the transducers in Measurement 1, respectively. According to Equation (49), the radial radius is determined to be 0.0425 m. From the estimated value of $\hat{n}_\varphi$ in the table, it can be observed that the approximate frequency of the roll motion and the transducers' average bottom-tracking radial velocity is 1.02 Hz.

Table 2. Roll angle parameter estimation results.

$\hat{\phi}_0$ (rad)	$\hat{t}_0$ (s)	$\hat{n}_\varphi$ (rad/s)	$\hat{v}_\varphi$ (s ^{−1})	RSS (rad ²)
−0.1947	−0.4580	6.1592	0.0894	0.0127

Table 3. Parameter estimation results for the average bottom-tracking radial velocity of the transducers.

$\hat{V}_{\text{BTRadTra0}}$ (m/s)	$\hat{t}_0$ (s)	$\hat{n}_\varphi$ (rad/s)	$\hat{v}_\varphi$ (s ^{−1})	RSS (m ² /s ²)
−0.0083	−0.0761	6.1592	0.0894	4.3921×10^{-4}

Figures 12 and 13 show the fitting results for the roll angle and the average bottom-tracking radial velocity of the transducers, respectively. The dashed line represents the roll angular velocity. In the figures, it can be observed that the phase difference between the roll angle and the roll angular velocity is approximately $\pi/2$, and the phase of the average bottom-tracking radial velocity aligns with that of the roll angular velocity. The fitting curves almost pass through all the original data points, with very few data points deviating from the curve, and the residual sum of squares (RSS) is also small. This indicates that the fitting is effective for radial radius estimation. Therefore, it can be concluded that the obtained radial radius of 0.0425 m is reliable.

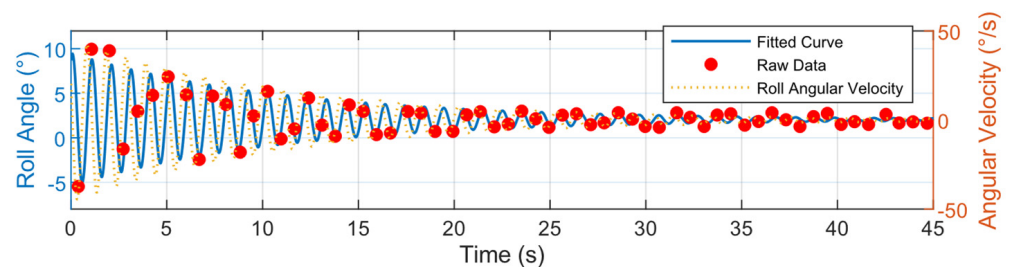


Figure 12. Fitting results for the roll angle and roll angular velocity.

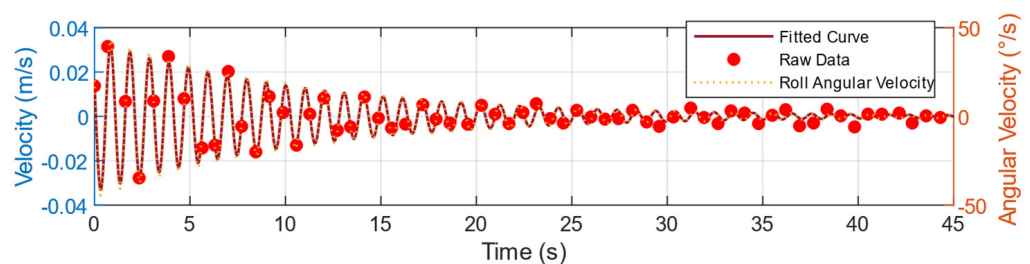


Figure 13. Fitting results for the average bottom-tracking radial velocity of the transducers and roll angular velocity.

4.2.3. Vessel Velocity Correction during Stationary Free Roll

After estimating the radial radius, the vessel velocity from this measurement is first corrected for attitude dynamics. According to Equation (50), the bottom-tracking radial velocities of the four transducers, after subtracting the radial component of the roll-induced linear velocity, are compared with the bottom-tracking radial velocities measured using the attitude static correction method, as shown in Figure 14. The figure shows that the bottom-tracking radial velocity fluctuations are significantly reduced with the proposed method, but, over time, they gradually converge with the bottom-tracking radial velocity obtained from the attitude static method. As shown in Figure 12, this occurs because the roll angle and angular velocity gradually decrease over time, reducing the impact of the roll-induced linear velocity on the radial velocity. This indicates that the proposed method is effective in correcting the radial velocity.

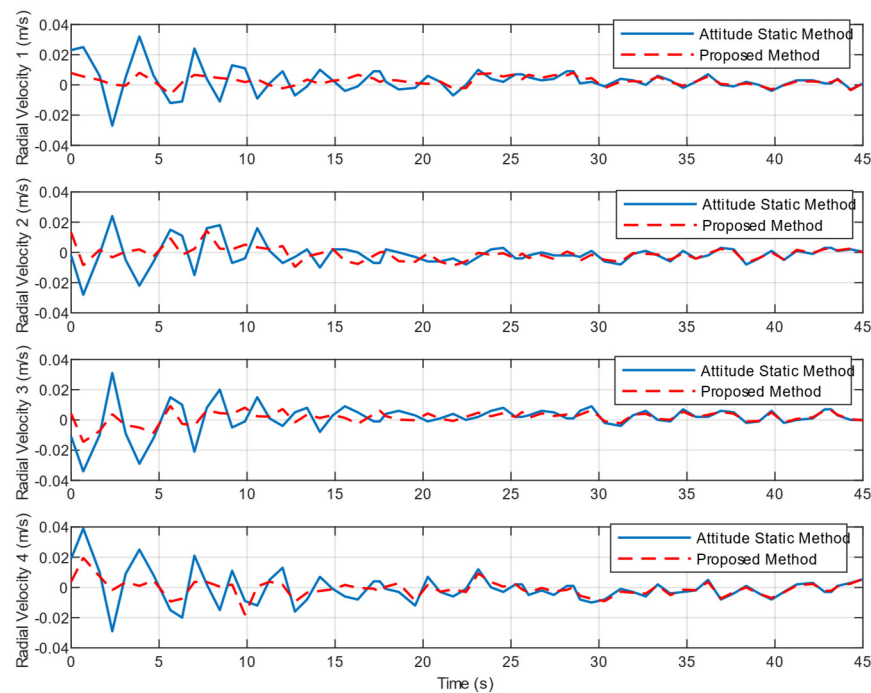


Figure 14. Bottom-tracking radial velocity of the four transducers: comparison between the attitude static method and the proposed method.

After obtaining the bottom-tracking radial velocity corrected for the influence of linear velocity, a coordinate transformation is performed to derive the final vessel velocity corrected for attitude dynamics. Figure 15 shows the magnitude of the roll angular velocity and the vessel velocity obtained using the attitude static method. By comparing the magnitude of the roll angular velocity with the vessel velocity from the attitude static method in the figure, it is evident that the fluctuations in the vessel velocity measured using the attitude static method are consistent with the magnitude of the roll angular velocity. This consistency indicates that the vessel's roll has a significant impact on the vessel velocity measured using the attitude static method.

A comparison of the vessel velocity magnitudes obtained using the attitude static method and the proposed method is shown in Figure 16. The figure clearly demonstrates that the fluctuations in the vessel velocity magnitude are significantly reduced with the proposed method. Although efforts were made to ensure that the vessel remained stationary during free roll in this measurement, the vessel still had a very small velocity, as indicated by the black dashed line in the figure. By comparing the curves in the figure, it is evident that the vessel velocity magnitude curve obtained with the proposed method is much closer to the actual vessel velocity magnitude, particularly when the roll angular velocity magnitude is high. In such cases, the vessel velocity magnitude obtained using the attitude

static method is significantly higher than that obtained using the proposed method and the actual vessel velocity magnitude.

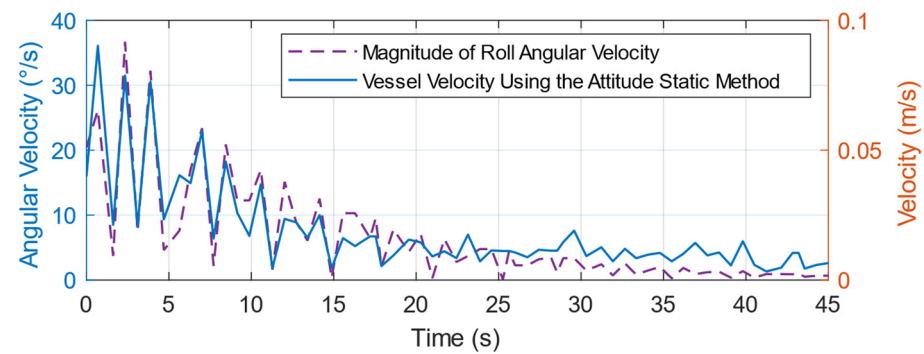


Figure 15. Magnitude of the roll angular velocity and vessel velocity using the attitude static method in Measurement 1.

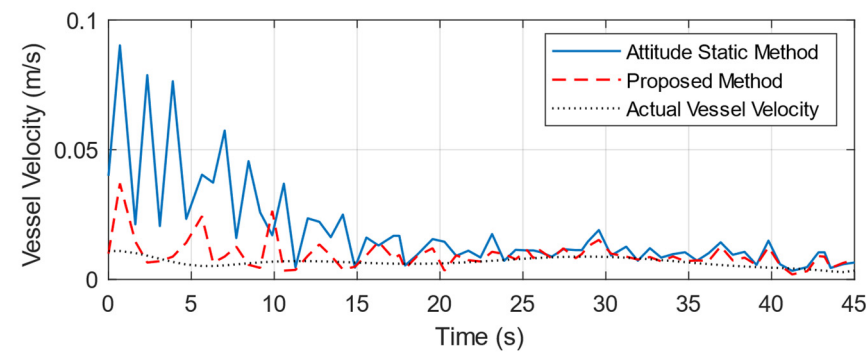


Figure 16. Vessel velocity using the attitude static method and the proposed method in Measurement 1.

A comparison of the vessel velocity directions obtained using the attitude static method and the proposed method is shown in Figure 17. The directions of the vessel velocity, as measured using both the attitude static method and the proposed method, reflect the scenario of the vessel freely rolling with its bow facing north. According to the black dashed arrows in the figure, representing the actual vessel velocity direction, and considering the vessel velocity magnitude in Figure 16, it can be seen that the vessel initially moves slightly northwest and then northeast at a very slow velocity.

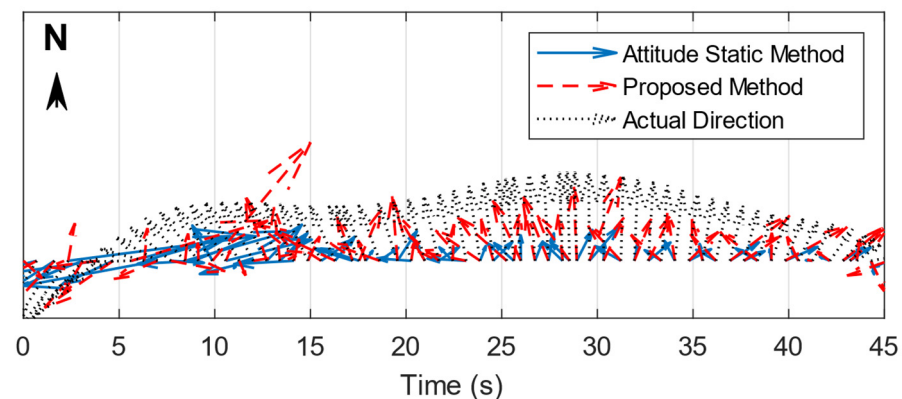


Figure 17. Vessel velocity direction using the attitude static method and the proposed method in Measurement 1.

The blue arrows in the figure indicate that the vessel velocity direction measured using the attitude static method fluctuates significantly between the east and west directions,

with a much larger angle variation than the proposed method. As time progresses and the roll amplitude decreases, the fluctuation in the vessel velocity direction measured using the attitude static method also gradually reduces, eventually aligning with the direction measured using the proposed method. The red dashed arrows represent the vessel velocity direction as measured using the proposed method. Compared with the attitude static method, the proposed method's vessel velocity direction is much less affected by the vessel's roll and is closer to the actual vessel velocity direction.

Next, the mean, standard deviation, relative standard deviation (RSD), mean absolute error (MAE), mean relative error (MRE), and standard deviation of the errors for the vessel velocity magnitudes obtained using the attitude static method, the proposed method, and the actual vessel velocity are calculated. These vessel velocities are then integrated to obtain the vessel's displacement, and the magnitude of the displacement, along with other relevant metrics, is computed. The calculated results are presented in Table 4, where RSD refers to the relative standard deviation, which is the ratio of the standard deviation (SD) to the mean (Mean). Error SD represents the standard deviation of the error, which is the standard deviation of the difference between the measured value and the actual value. The vessel displacement is calculated as the displacement at 45 s relative to the position at 0 s, and the distance error represents the difference between the measured vessel displacement and the actual displacement. Distance RE is the ratio of the distance error to the magnitude of the actual displacement.

Table 4. Statistics for vessel velocity magnitude and vessel displacement in Measurement 1.

	Vessel Velocity Magnitude					Vessel Displacement			
	Mean (m/s)	SD (m/s)	RSD (%)	MAE (m/s)	MRE (%)	Error SD (m/s)	Magnitude (m)	Distance Error (m)	Distance RE (%)
Attitude Static Method	0.0184	0.0181	98.1	0.012	178.68	0.0176	0.1986	0.0241	12.18
Proposed Method	0.0094	0.0057	60.4	0.0037	59.67	0.0053	0.1932	0.0067	3.41
Actual Value	0.0066	0.0017	25.87	0	0	0	0.1982	0	0
Reduction Amount	0.009	0.0124	37.7	0.0083	119.02	0.0123	-	0.0174	8.77

In Table 4, it can be seen that, in this measurement, because the mean value of the actual vessel velocity magnitude is very small, only 0.0066 m/s, the proportion of the linear velocity caused by roll is relatively large, significantly affecting the accuracy of the attitude static method. Consequently, the mean value and standard deviation of the vessel velocity magnitude obtained using the attitude static method are both significantly higher than the corresponding values of the actual vessel velocity magnitude. In contrast, the proposed method's vessel velocity magnitude has a mean value and standard deviation that are much closer to the true values. Since the vessel's movement in this measurement is almost entirely due to its free roll, with very little actual translational velocity, the mean relative error of the vessel velocity magnitude obtained using the attitude static method and the proposed method are as high as 178.68% and 59.67%, respectively, and the relative standard deviations are as high as 98.1% and 60.4%, respectively. Therefore, in the stationary free roll measurement, the mean relative error and relative standard deviation of the proposed method are reduced by 119.02% and 37.7%, respectively, compared with those of the attitude static method.

Although the proposed method significantly improves the accuracy of the vessel velocity in this measurement, Table 4 also shows that the magnitude of the displacement obtained by integrating the velocity vectors from both the attitude static method and the proposed method differs by less than 5 mm from the actual displacement magnitude. The distance between the displacements calculated using both methods and the actual displacement is less than 3 cm. Given that the actual displacement in this measurement is less than 20 cm, the distance relative error (Distance RE) of the proposed method is only reduced by 8.77% compared with that of the attitude static method. This indicates that, while the proposed method provides a significant improvement in vessel velocity measurement performance, its impact on vessel displacement measurement performance

is limited. This is because the vessel's free roll causes the velocity to approximately follow a decaying sinusoidal oscillation pattern. When integrating the velocity vector over time to obtain the vessel's displacement, the linear velocities caused by free roll at different times mostly cancel each other out, resulting in a limited improvement in displacement accuracy when using the attitude dynamic correction method.

Next, the effectiveness of the proposed method for correcting vessel velocity during free roll motion while the vessel is moving in shallow water is validated, and a comparison is made with the attitude static method.

4.2.4. Vessel Velocity Correction during Free Roll While Moving

In this measurement, the bow of the vessel faces north while the vessel moves southward and performs a free roll. Following the method described in the previous section, the roll angle expression is first determined for this measurement, and then its derivative is taken to obtain the roll angular velocity.

The parameter values and residuals obtained by fitting the roll angle are shown in Table 5. The estimated value of $\hat{n}_\varphi$ indicates that the approximate roll frequency is 1.05 Hz. Figure 18 below shows the corresponding roll angle fitting results and the roll angular velocity. In the figure, it can be observed that the phase difference between the roll angle and the roll angular velocity is approximately $\pi/2$. Although the maximum roll amplitude is only about 9.4° , the roll angular velocity reaches a peak of approximately $56^\circ/\text{s}$. The fitting curve passes through almost all the original data points, with very few data points deviating from the curve, and the residual sum of squares is small, indicating that the fitting is sufficient for determining the angular velocity.

Table 5. Roll parameter estimation results for Measurement 2.

$\hat{\phi}_0$ (rad)	$\hat{t}_0$ (s)	$\hat{n}_\varphi$ (rad/s)	$\hat{v}_\varphi$ (s^{-1})	RSS (rad^2)
0.1929	−0.1907	5.9708	0.0671	0.0365

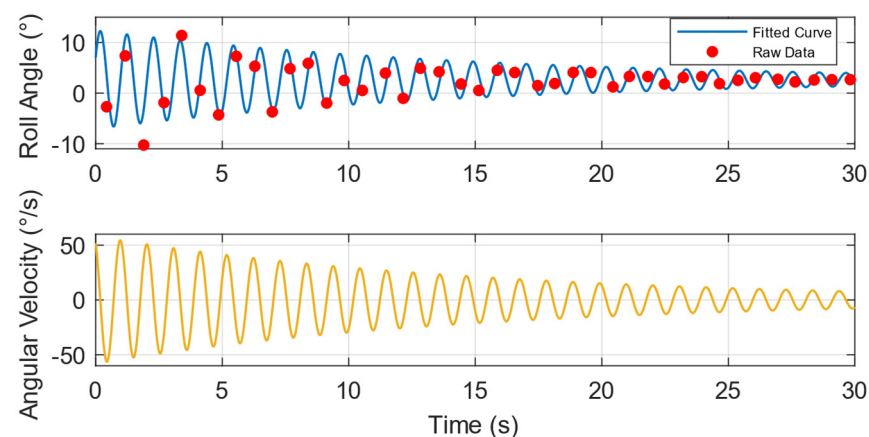


Figure 18. Fitting results for the roll angle and roll angular velocity in Measurement 2.

With the radial radius now available, the vessel velocity can be corrected for attitude dynamics. First, according to Equation (50), the bottom-tracking radial velocities of the four transducers, after subtracting the radial component of the roll-induced linear velocity, are compared with the bottom-tracking radial velocities measured using the attitude static method, as shown in Figure 19. The figure demonstrates that the fluctuations in the bottom-tracking radial velocity are significantly reduced when using the proposed method, but, over time, they gradually converge with the bottom-tracking radial velocity obtained from the attitude static method. As shown in Figure 18, this occurs because the roll angular velocity gradually decreases over time, which, in turn, reduces the impact of the roll-

induced linear velocity on the radial velocity. This indicates that the proposed method is effective in correcting radial velocity during free roll while the vessel is in motion.

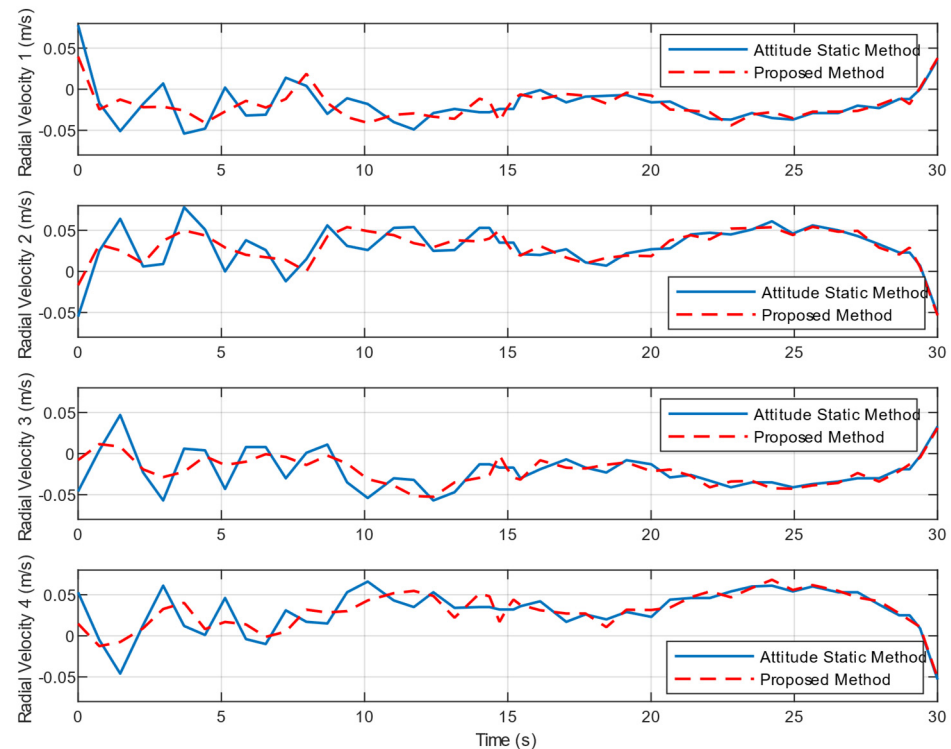


Figure 19. Bottom-tracking radial velocity of the four transducers: comparison between the attitude static method and the proposed method in Measurement 2.

After obtaining the bottom-tracking radial velocity corrected for the influence of linear velocity, a coordinate transformation is performed to derive the final vessel velocity corrected for attitude dynamics. The magnitude of the roll angular velocity and the vessel velocity obtained using the attitude static method are shown in Figure 20. A comparison between the magnitude of the roll angular velocity and the vessel velocity from the attitude static method in the figure shows that the fluctuations in the vessel velocity measured using the attitude static method are consistent with the magnitude of the roll angular velocity. This consistency indicates that the vessel's roll has a significant impact on the vessel velocity measured using the attitude static method.

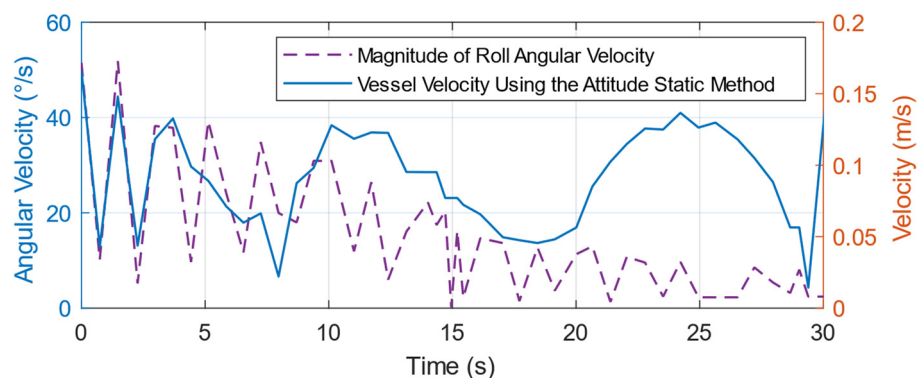


Figure 20. Magnitude of the roll angular velocity and vessel velocity using the attitude static method in Measurement 2.

A comparison of the vessel velocity magnitudes obtained using the attitude static method and the proposed method is shown in Figure 21. The figure clearly demonstrates that the fluctuations in the vessel velocity magnitude are significantly reduced with the

proposed method. In the beginning, due to the large roll amplitude, the vessel velocity magnitude obtained using the attitude static method shows much larger fluctuations than the actual vessel velocity magnitude, with the maximum fluctuation reaching nearly 0.1 m/s, almost twice the magnitude of the actual vessel velocity at the corresponding time. In contrast, the vessel velocity magnitude obtained using the proposed method only shows minor fluctuations around the actual value. As time progresses and the roll angular velocity gradually decreases, the vessel velocity magnitude curves from both methods converge with the actual values. This indicates that the proposed method effectively corrects for the dynamic effects of the attitude.

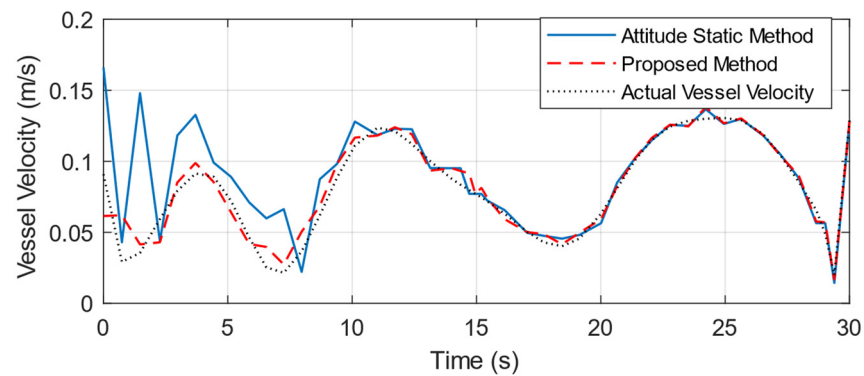


Figure 21. Vessel velocity using the attitude static method and the proposed method in Measurement 2.

A comparison of the vessel velocity directions obtained using the attitude static method and the proposed method is shown in Figure 22. The black dashed arrows in the figure represent the actual vessel velocity direction, reflecting the scenario where the vessel heads north, rolling freely while moving southward. The figure shows that, initially, the vessel velocity direction measured using the attitude static method deviates from the actual velocity direction by a larger angle than the proposed method. As time progresses and the roll amplitude decreases, the deviation in the vessel velocity direction measured using the attitude static method also gradually diminishes. The red dashed arrows indicate the vessel velocity direction as measured using the proposed method. Compared with the attitude static method, the proposed method shows significantly less deviation due to the vessel's roll and is closer to the actual vessel velocity direction.

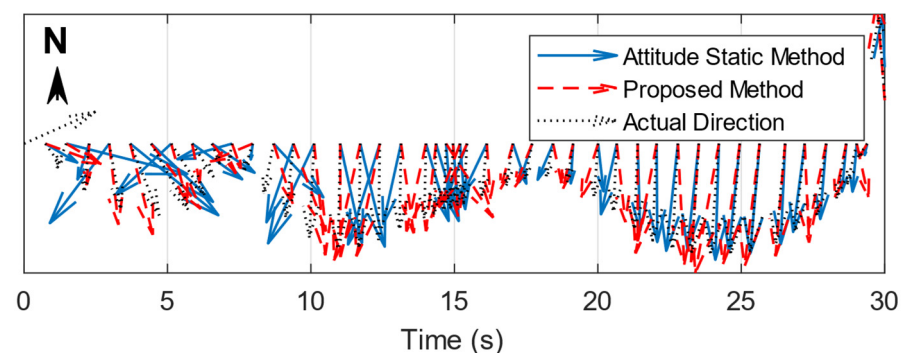


Figure 22. Vessel velocity direction using the attitude static method and the proposed method in Measurement 2.

Next, the mean, mean absolute error (MAE), mean relative error (MRE), standard deviation of the errors (Error SD), and relative standard deviation of the errors (Error RSD) for the vessel velocity magnitudes obtained using the attitude static method, the proposed method, and the actual vessel velocity are calculated. These vessel velocities are then integrated to obtain the vessel's displacement, and the magnitude of the displacement, along with other relevant metrics, is computed. The calculated results are presented in

Table 6, where Error SD and Error RSD represent the standard deviation and relative standard deviation of the error, respectively, with Error RSD being the ratio of Error SD to the Mean. The vessel displacement is calculated as the displacement at 45 s relative to the position at 0 s. The distance error represents the difference between the measured vessel displacement and the actual displacement, while Distance RE is the ratio of the distance error to the magnitude of the actual displacement.

Table 6. Statistics for vessel velocity magnitude and vessel displacement in Measurement 2.

	Vessel Velocity Magnitude					Vessel Displacement		
	Mean (m/s)	MAE (m/s)	MRE (%)	Error SD (m/s)	Error RSD (%)	Magnitude (m)	Distance Error (m)	Distance RE (%)
Attitude Static Method	0.091	0.0136	29.37	0.0228	25.03	2.1874	0.0219	1
Proposed Method	0.0828	0.0062	12.09	0.0086	10.36	2.1899	0.0166	0.76
Actual Value	0.0808	0	0	0	0	2.1933	0	0
Reduction Amount	0.0081	0.0073	17.28	0.0142	14.67	-	0.0053	0.24

In Table 6, it can be observed that the mean value of the actual vessel velocity magnitude in this measurement is 0.0808 m/s, significantly higher than the 0.0066 m/s in the previous measurement. As a result, the proportion of roll-induced linear velocity in the total velocity is noticeably reduced. Therefore, the difference between the mean vessel velocity magnitudes obtained using the attitude static method and the proposed method compared with the actual vessel velocity magnitude is not significant, with the proposed method's vessel velocity magnitude being slightly smaller and closer to the actual value.

In this measurement of free roll while moving, the mean relative error (MRE) of the vessel velocity magnitude obtained using the attitude static method and the proposed method is 29.37% and 12.09%, respectively, showing a reduction of 17.28%. Since the actual vessel velocity itself has some fluctuation, using the standard deviation of the error (Error SD) is a more reasonable measure to assess the impact of roll on the fluctuation of the vessel velocity magnitude. The table shows that the Error SD for the vessel velocity magnitude obtained using the attitude static method and the proposed method is 0.0228 m/s and 0.0086 m/s, respectively, representing a reduction of 0.0142 m/s. The Error RSD is 25.03% and 10.36%, respectively, showing a reduction of 14.67%. This significant reduction indicates that the proposed method effectively reduces the degree of dispersion in vessel velocity.

Therefore, it is evident that the proposed method significantly improves the performance of vessel velocity measurement.

Similarly, because the vessel velocity follows an approximate decaying sinusoidal oscillation pattern, causing most of the linear velocity components to cancel each other out during the integration of the velocity vector, the improvement in vessel displacement accuracy when using the attitude dynamic correction method in this measurement is also limited. As shown in Table 6, the magnitude of the displacement obtained by integrating the velocity vector using both the attitude static method and the proposed method differs by less than 6 mm from the actual displacement magnitude. The distance between the displacement calculated using both methods and the actual displacement is less than 3 cm. Given that the actual displacement in this measurement is 2.1933 m, the distance relative error (Distance RE) of the proposed method is only reduced by 0.24% compared with that of the attitude static method, indicating a minimal improvement in displacement measurement performance.

4.2.5. Test Summary

This test validated the effectiveness of vessel velocity correction in two scenarios: a stationary free roll and a moving free roll in shallow water. First, in Measurement 1, where the vessel performed a stationary free roll, the radial radius was estimated. The estimated radial radius was then used for vessel velocity correction in both Measurements 1 and 2.

The experimental results indicate that the roll angle, obtained by fitting the exponential decay sinusoidal oscillation model to the inclinometer data using the least squares method, performed well in both radial radius estimation and linear velocity correction. The successful correction of attitude dynamic errors in vessel velocity using the estimated roll angle expression and radial radius in both measurements further validates the effectiveness of the roll angle estimation and radial radius estimation methods.

In the stationary free roll measurement, the mean relative error of the vessel velocity obtained using the attitude static method and the proposed method was 178.68% and 59.67%, respectively, resulting in a reduction of 119.02%. The relative standard deviation was 98.1% and 60.4%, respectively, showing a reduction of 37.7%.

In the moving free roll measurement, the mean relative error of the vessel velocity obtained using the attitude static method and the proposed method was 29.37% and 12.09%, respectively, with a reduction of 17.28%. The relative standard deviation of the error was 25.03% and 10.36%, respectively, showing a reduction of 14.67%.

These results indicate that the proposed method significantly reduces the vessel velocity error and the degree of dispersion, leading to a substantial improvement in velocity accuracy and an effective correction of attitude dynamic errors.

5. Conclusions

To analyze and correct the errors caused by dynamic factors in the attitude changes of an ADCP mounted on a rolling platform, this study derived the vessel velocity and flow velocity errors introduced by the attitude static correction method under transducer roll conditions. Through simulations, a detailed verification analysis was conducted on the influence of various factors on the errors, including the angle between the transducer's radial direction and the central axis, the roll angle at the time of transmission, the relative position between the swaying center and the transducer, the water depth, and the depth at which the flow velocity is measured.

For the common application scenario where an ADCP is installed vertically downward on the central longitudinal section of a vessel undergoing free roll motion, this paper proposes a flow velocity measurement method based on radial radius estimation for linear velocity correction. Compared with the traditional attitude static correction method, the proposed method demonstrates higher velocity measurement accuracy and reduced fluctuation under free roll conditions. Additionally, this method does not require any additional hardware, making it more cost-effective and less complex to use than other methods that require inertial navigation systems (INSs), differential GPS, or AHRS to correct for dynamic attitude errors. Experimental results show that, during a stationary free roll, the proposed correction method reduces the average relative error of the vessel velocity by 119.02% and the relative standard deviation by 37.7% compared with the attitude static method. During a moving free roll, the proposed method reduces the average relative error by 17.28% and the relative standard deviation of the error by 14.67% compared with the attitude static method.

Although this study focuses on the analysis and correction of vessel velocity errors using roll motion as an example, the conclusions and the proposed correction method can also be applied to pitch motion.

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