

## Article

# Multidisciplinary Design Optimization of Alignment and Whirling Vibration Characteristics of a Submarine Propulsion Shafting Using Kriging Surrogate Model

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**Abstract:** To improve the performance indexes, such as safety, reliability and acoustic stealth, of a submarine, it is significant to optimize the dynamic characteristics of its propulsion shafting. The alignment state of a shafting has a coupling effect on its whirling vibration characteristics, and the multidisciplinary design optimization (MDO) theory can fully consider the coupling relationships between different disciplines like this, which is a scientific and effective method to achieve the design optimization of shafting dynamics. However, the iterative calculation of high-precision numerical models greatly restricts the optimization efficiency of this method. Aiming at this problem, in this paper, an MDO model was established based on the coupling dynamic analysis of submarine propulsion shafting, and the Kriging surrogate model was used to predict the state variables within each subdiscipline. Along with the reduction of computational expense, the MDO of the alignment and whirling vibration characteristics of the shafting was achieved. The studied results can be applied to the design process of submarine propulsion shafting, which can provide technical and theoretical support for improving the optimization efficiency of its coupling dynamics.

**Keywords:** submarine propulsion shafting; shafting alignment; whirling vibration; kriging surrogate model; multidisciplinary design optimization

**Citation:** Gu, Z.; Liu, J.Multidisciplinary Design  
Optimization of Alignment and  
Whirling Vibration Characteristics of a  
Submarine Propulsion Shafting Using  
Kriging Surrogate Model. *J. Mar. Sci.  
Eng.* **2024**, *12*, 1812. [https://doi.org/  
10.3390/jmse12101812](https://doi.org/10.3390/jmse12101812)

Academic Editor: Joško Parunov

Received: 11 September 2024

Revised: 4 October 2024

Accepted: 9 October 2024

Published: 11 October 2024



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## 1. Introduction

Propulsion shafting is a key part of a submarine's power plant, and its dynamic characteristics are directly related to the safety, reliability, and acoustic stealth of the entire submarine. The technical design of a shafting includes the calculation and check of its alignment and vibration characteristics, which is an important basis for the evaluation of its dynamic performance. At present, studies on the design optimization of shafting dynamics are mainly carried out around these two aspects.

A poor shafting alignment state will cause uneven load distribution among the bearings, resulting in abnormal wear of the bearings, especially for the rear stern bearing. In severe cases, accidents, such as heat damage and fracture of shafts, may occur during the operation of a shafting [1]. For the calculation and optimization of shafting alignment, current studies mainly focus on the dynamic alignment, which means the alignment state is optimized with dynamic factors considered, such as support stiffness, many kinds of excitation force, and hull deformation. Zhou et al. [2] calculated the global and local hull deformation and proposed a matrix displacement method to evaluate the influence of the combined results on shafting alignment. Considering the oil film stiffness, elastic deformation, and dynamic stiffness, Yin et al. [3] analyzed the alignment characteristics of an elastically supported marine propulsion shafting and optimized the alignment state using the response surface method. However, the dynamic alignment includes many inputs, and the model used is relatively complex, resulting in a long optimization process and solution difficulty. Therefore, cutting-edge optimization algorithms or models are gradually being applied to the alignment calculation. Magalhães et al. [4] created a dataset by the

finite-element method (FEM) and used machine-learning algorithms to efficiently define the optimum set of bearings' vertical displacements. Using bearing elevations and material parameters as random input variables, Lu et al. [5] established a dynamic alignment model under uncertainty using the FEM combined with Latin hypercube sampling (LHS). The reliability of this model was verified by the measurement of bearing loads and axis orbits in four different alignment states. These proposed methods have provided references for dynamic alignment.

Whirling vibration is a key part of shafting vibration that requires special attention. Currently, due to the lower support stiffness of the stern shaft and the larger span between two stern bearings, the natural frequencies of the whirling vibration of a shafting have been decreased. Meanwhile, the use of multi-blade propellers may make the critical speeds of blade frequency fall within the shafting's working speed range, resulting in resonance. Severely, it can cause safety accidents, such as shaft crack and engine damage. In addition, if a submarine propulsion shafting experiences severe whirling vibration at critical speeds of blade frequency, strong underwater line spectrum noise will be induced in the low-frequency band, threatening the submarine's acoustic stealth performance [6]. So, the analysis and reduction control of a shafting's whirling vibration is currently a hot issue. Kandouci et al. [7] used the relationship between the vectors of solution coefficients to calculate the critical speeds and global response of a rotor-bearing system. The presented approach can be used to analyze the whirling vibration with complex boundaries, such as mass imbalance, elastic deformation, and gyroscopic torque. Vuong et al. [8] studied a hybrid propulsion shafting and analyzed the influence of propeller excitation force on the shafting whirling vibration under various maneuvering conditions. The vibration-reduction measures during rapid turning maneuvers were proposed. The studies above mainly focus on the single aspect of whirling vibration. However, studies have shown that there is a coupling relationship between the alignment and whirling vibration characteristics. Alignment parameters, such as deflection [9] and bearing displacements [10], have varying degrees of impact on whirling vibration characteristics. When the alignment state is optimized, the whirling vibration response will correspondingly increase. Therefore, it is necessary to consider the coupling effect between them and achieve the coupling design optimization of shafting dynamics.

The design optimization of shafting dynamics includes the calculation of parameters like bearing load, stress, rotation angle, vibration response, etc. The whole process involves multiple disciplines, such as alignment, strength, and vibration. It is difficult to consider the coupling effect between some disciplines using the traditional serial design method. Moreover, the obtained result may not meet the constraints within multiple disciplines. The MDO theory is a scientific methodology for the design of a complicated system, which can comprehensively consider the coupling or collaborative relationships between various subdisciplines in the system and obtain the optimal design scheme based on a suitable strategy [11]. The field of ship design often involves coupling the design optimization problems of complex objects. MDO is useful and efficient in tackling these design challenges, which has huge development potentialities in this field and has gradually attracted more attention in academia [12]. Spain et al. [13] performed a concurrent multidisciplinary analysis to identify the optimal configurations for the structural and acoustic performances of a submarine pressure hull. A set-based design method was used to find the optimal region of the design space. Using a multidisciplinary and configurable modeling approach, Bennabi et al. [14] studied the design optimization of a hybrid propulsion system, with the aim of reducing energy consumption and pollution emissions. In view of the high-dimensional design space in hull form optimization, Chang et al. [15] proposed a multidisciplinary platform combined with the Karhunen–Loève transform method, which reduced the number of design variables and iterations during optimization simultaneously. To reduce the weight under structural safety constraints, Diez et al. [16] studied the fluid–structure interaction (FSI) MDO for a deep-V planing-hull grillage panel subject to slamming loads. The effec-

tiveness of the optimization methods was verified by comparison of computational and experimental data, reflecting the advantages of MDO in hydro-structural optimization.

The application of MDO in the design of shafting dynamics is currently in the exploration stage. With the improvement of numerical calculation, dynamic models are becoming more and more accurate, and more inputs are considered in alignment and vibration calculations, which brings a higher computational expense than ever. If high-precision numerical models are used for iteration, much time will be consumed, and algorithms and models are hard to adjust during optimization, which is a key problem that needs to be solved urgently in the process of MDO. At present, the commonly adopted solution to this problem is using surrogate models, like the response surface, the Kriging model, and the neural network. These models predict the response values of unknown points by sample fitting and analysis, thus reducing computational expense. Studies and applications of surrogate models in the field of ship design have become relatively mature. Hefazi et al. [17] proposed a synthesis-level MDO method suitable for the design of multihull ships. A neural network-based response surface method was used to surrogate the time-consuming numerical calculation. Hu et al. [18] established an MDO framework for a vessel hull form, and a support vector regression-based surrogate model was used to replace the expensive hydrodynamic simulation. The accuracy of the surrogate model was checked by calculating the coefficients of determination. Serani et al. [19] built a multi-fidelity metamodel based on stochastic radial basis functions and assessed its global optimization performance using four adaptive sampling methods. Their research results provide an effective method for reducing the high computational cost in CFD-based optimization design.

In this paper, taking a submarine propulsion shafting as the object of study, an MDO model of its alignment and whirling vibration characteristics was established based on the coupling dynamic analysis results. Subsequently, as a precondition of meeting the accuracy requirements, the Kriging model was used to surrogate the high-precision numerical model, in order to solve the problems of high computational expense and low optimization efficiency. Finally, the optimal solution was obtained after optimization, and the effectiveness of the presented methods was verified by results comparison.

## 2. Finite-Element Analysis Model

The general layout of a submarine propulsion shafting is shown in Figure 1, which mainly includes a seven-blade propeller, a stern shaft, an intermediate shaft, a thrust shaft, a rear stern bearing (#1), a front stern bearing (#2), a thrust bearing (#3), flanges and couplings, etc. Due to the short length of the shafting, there are only three bearings. Two stern bearings are used to support the stern shaft. The intermediate shaft is connected to the stern shaft and the thrust shaft by two flanges. The thrust bearing is used to support the thrust shaft and bear the axial load transmitted in the system. The shafting is connected to the motor by a flexible coupling.

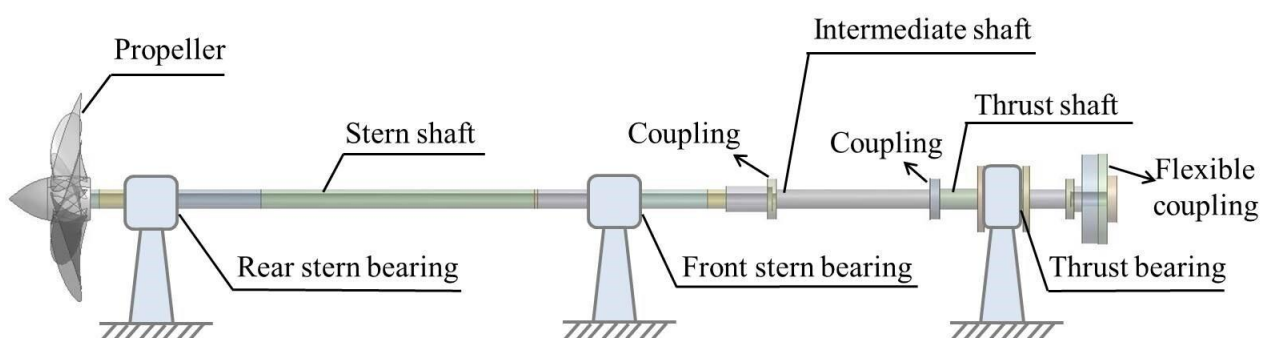


**Figure 1.** Layout diagram of a submarine propulsion shafting.

The main material properties and structural parameters of each shafting part are listed in Table 1. The numerical analysis model of the shafting was built using the FEM. On the premise of guaranteeing the accuracy, the shafting was simplified as follows:

1. Each shaft segment was modeled using the beam188 elements, with the internal density, modulus, and stiffness uniformly distributed;
2. Considering the mass of the attached water, the propeller was equivalent to a homogeneous disc. The attached mass coefficient was taken as 1.15;
3. The propeller and stern shaft are submerged in water, and the thrust shaft is oil-lubricated. So, the density for calculation was taken as the actual density with buoyancy considered, as listed in Table 1. The hollowness represents the ratio of the inner diameter to the outer diameter of a shaft segment;
4. The combi214 elements were set at each bearing to simulate the bearing support, with the stiffness and damping values rationally set [1]. The element of the rear stern bearing was set at the rear one-third of the axial bearing length, while that of the front stern bearing and thrust bearing were set at the midpoint.

**Table 1.** Load and material properties of each shafting part.

| Part               | Young's Modulus (MPa) | Poisson's Ratio | Actual Density ( $\text{kg}\cdot\text{m}^{-3}$ ) | Density for Calculation ( $\text{kg}\cdot\text{m}^{-3}$ ) | Hollowness |
|--------------------|-----------------------|-----------------|--------------------------------------------------|-----------------------------------------------------------|------------|
| Propeller          | $1.27 \times 10^5$    | 0.300           | 7206                                             | 6181                                                      | —          |
| Stern shaft        | $2.10 \times 10^5$    | 0.266           | 7850                                             | 6825                                                      | 0.581      |
| Intermediate shaft | $2.10 \times 10^5$    | 0.266           | 7850                                             | 7850                                                      | 0.589      |
| Intermediate shaft | $2.10 \times 10^5$    | 0.266           | 7850                                             | 7000                                                      | 1.000      |

After simplification, a finite-element analysis model of the shafting was built, as shown in Figure 2. Herein,  $k_i$ ,  $c_i$  ( $i = 1, 2, 3$ ) represent the stiffness and damping at each bearing, respectively.  $k_{yy}$ ,  $k_{zz}$  represent the vertical and lateral stiffness, respectively.  $c_{yy}$ ,  $c_{zz}$  represent the vertical and lateral damping, respectively. The model was built based on the Cartesian coordinate system. The  $x$  axis was set along the axial direction, and pointing to the motor is positive. The negative direction of the  $y$  axis was set in the gravity direction. The  $z$  axis was set along the direction perpendicular to the plane of the paper, and pointing outward is positive.

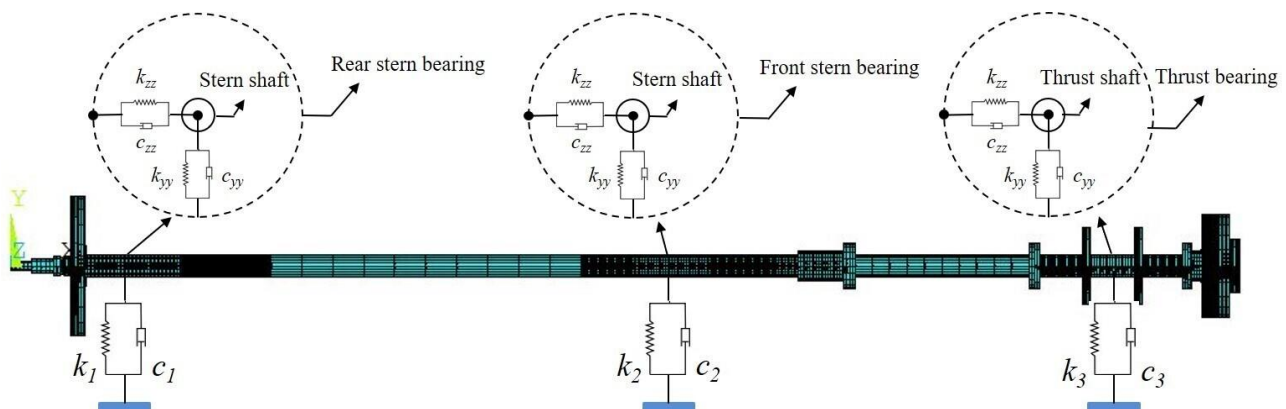


**Figure 2.** Finite-element analysis model of shafting.

### 3. Kriging Surrogate Model

The disciplinary models will be iterated many times during the process of MDO. The mesh of the finite-element analysis model is very dense, which ensures the accuracy of calculation, but it brings high computational expense.

Based on known sample points, the surrogate model can use approximate methods to predict the response values of unknown sample points on the premise of accuracy requirements satisfied and thereby replace the numerical model. The Kriging model is a kind of interpolation surrogate model using random statistical methods, which can fully

consider the correlation between the design variables in the sample set and predict the unknown values by building approximate functional relationships.

Supposing a point in a sample set is  $x_i$  ( $i = 1, 2, \dots, n$ ) and its response value is  $y_r$  ( $j = 1, 2, \dots, l$ ), the mathematical expression of the Kriging model is:

$$\hat{y}_r(x_i) = P(a_{ir}, x_i) + z_r(x_i) \quad (1)$$

$$P(a_{ir}, x_i) = a_{1r}p_1(x_i) + a_{2r}p_2(x_i) + \dots + a_{pr}p_p(x_i) \quad (2)$$

where  $p_p(x_i)$  is the polynomial basis function;  $a_{pr}$  is the undetermined coefficient; and  $z_r(x_i)$  is the realization of a stochastic process with mean zero. Supposing  $x_i$  has one response value ( $l = 1$ , this can be inferred in the same way when there are multiple response values), Equation (1) can be simplified as:

$$\hat{y}(x_i) = p^T(x_i)a + z(x_i) \quad (3)$$

$$p^T(x_i)a = a_1p_1(x_i) + a_2p_2(x_i) + \dots + a_pp_p(x_i) \quad (4)$$

The covariance of  $z(x_i)$  can be expressed as:

$$\text{cov}[z(x_i), z(x_j)] = \sigma^2\lambda(x_i, x_j) \quad (5)$$

where  $\sigma^2$  is the variance and  $\lambda(x_i, x_j)$  is the Gaussian correlation function that represents the correlation between sample points  $x_i$  and  $x_j$ , which can be expressed as:

$$\lambda(x_i, x_j) = \prod_{k=1}^w \exp\left(-\theta_k |x_{ik} - x_{jk}|^2\right) \quad (6)$$

where  $w$  is the dimension of the sample points and  $\theta_k$  is the correlation parameter. The response value of a certain point to be predicted by the Kriging model can be expressed as:

$$\hat{y}(x) = \hat{a} + r^T(x)\lambda^{-1}(Y - P\hat{a}) \quad (7)$$

$$r^T(x) = [l(x, x_1), l(x, x_2), \dots, l(x, x_n)]^T \quad (8)$$

where  $r^T(x)$  represents the correlation between the points to be predicted and the sample points;  $Y = [y_1(x_1), y_2(x_2), \dots, y_n(x_n)]^T$  is the column vector that contains the response values of the sample set;  $P = [p_1(x_1), p_2(x_2), \dots, p_n(x_n)]^T$  is the column vector which contains the basis functions of the sample set; and  $\hat{a}$  can be calculated by the generalized least squares estimation:

$$\hat{a} = (P^T\lambda^{-1}P)^{-1}P^T\lambda^{-1}Y \quad (9)$$

The estimate of the variance of  $Y$  can be expressed as:

$$\hat{\sigma}^2 = \frac{1}{n}(Y - P\hat{a})^T\lambda^{-1}(Y - P\hat{a}) \quad (10)$$

As mentioned above, it is known that both  $\hat{y}(x)$  and  $\hat{\sigma}^2$  are related to  $\theta_k$ .  $\theta_k$  can be calculated by the maximum likelihood estimation [20]. That is, to find the maximum value of the likelihood function specified by:

$$L(\theta_k) = -\frac{1}{2}\left[n\ln(\hat{\sigma}^2) + \ln(|\lambda|)\right], \theta_k > 0 \quad (11)$$

By solving the  $k$ -dimensional nonlinear unconstrained optimization problem above, the Kriging model with the optimum fitting effect is found. Faced with a nonlinear fitting problem, the Kriging model has excellent adaptability. In addition, it can provide the coeffi-

cients of determination of the test set, which is crucial for judging the accuracy. Therefore, the Kriging model was used as the surrogate model and applied to the MDO subsequently.

#### 4. Coupling Analysis of Shafting Alignment and Whirling Vibration Characteristics

##### 4.1. Alignment Characteristics

The straight-line alignment state was analyzed based on the finite-element analysis model of the shafting. The stern bearings contain sealing materials and other accessories, and their gravity was considered as an even load applied to the corresponding positions. The state variables of each bearing in the straight-line alignment state are listed in Table 2, and the deflection and bending stress distribution of the shafting are shown in Figure 3.

**Table 2.** State variables of each bearing in straight-line alignment state.

| Name                | Rotation Angle (Rad)    | Bearing Load (kN) | Specific Pressure (MPa) | Allowable Specific Pressure (MPa) |
|---------------------|-------------------------|-------------------|-------------------------|-----------------------------------|
| Rear stern bearing  | $1.0041 \times 10^{-4}$ | 47.724            | 0.132                   | 0.300                             |
| Front stern bearing | $1.2206 \times 10^{-5}$ | 22.963            | 0.086                   | 0.300                             |
| Thrust bearing      | $5.1709 \times 10^{-5}$ | 44.185            | 0.320                   | 0.600                             |

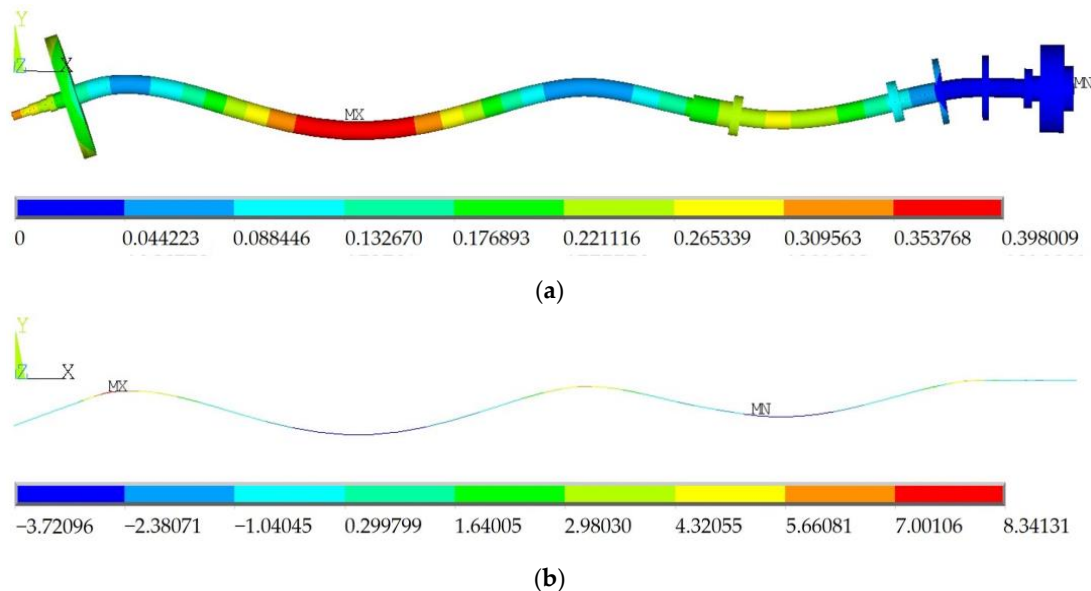


**Figure 3.** Deflection and bending stress distribution of shafting in straight-line alignment state. (a) Deflection distribution (mm); (b) bending stress distribution (MPa).

According to the straight-line alignment results, the rotation angle of the rear stern bearing is less than the required upper limit. The specific pressure of each bearing and the bending stress of each shaft segment are also within the allowable range [21]. However, influenced by the cantilever beam, the load difference between the front and rear stern bearings was as high as 24.761 kN. A long time in this state will exacerbate the wear of the rear stern bearing and the stern shaft. Therefore, it is necessary to further improve the working state of the shafting by alignment optimization.

The rational alignment is a method that adjusts the vertical position of bearings, other than the rear stern bearing, under the constraints of the relevant shafting design standards to improve the alignment state. The vertical displacement of the front stern bearing  $\Delta h_2$  and the vertical displacement of the thrust bearing  $\Delta h_3$  were taken as variables, and the values range for both was  $[-5.00, 5.00]$  mm. Taking the minimization of the rear stern bearing load and the load difference between two stern bearings as the objectives, the

genetic algorithm [22] was used to interact with the parameterized finite-element model for rational alignment. The following constraints have been added to the fitness functions:

$$\begin{aligned} \text{s.t.} \quad & 0.2G_i \leq F_i \leq F_{i\max} \\ & \sigma_{1\max} \leq [\sigma_1] \\ & \sigma_{2\max} \leq [\sigma_2] \\ & \varphi_1 \leq [\varphi] \end{aligned} \quad (12)$$

where  $F_i$  and  $F_{i\max}$  are the load and maximum allowable load of the  $i$ -th ( $i = 1, 2, 3$ ) bearing, respectively;  $G_i$  is the total weight of two adjacent shaft segments of the  $i$ -th bearing spans;  $\sigma_{1\max}$  and  $[\sigma_1]$  are the maximum bending stress and allowable bending stress of the stern shaft, respectively;  $\sigma_{2\max}$  and  $[\sigma_2]$  are the maximum bending stress and allowable stress of the thrust shaft, respectively; and  $\varphi_1$  and  $[\varphi]$  are the rotation angle and allowable rotation angle at the rear stern bearing, respectively.

The genetic algorithm terminated the iterations at the 52nd generation. The output variables were  $\Delta h_2 = 1.06$  and  $\Delta h_3 = 0.15$ , which means that the vertical position of the front stern bearing was elevated by 1.06 mm, while that of the thrust bearing was elevated by 0.15 mm. The state variables of each bearing in the rational alignment state are listed in Table 3, and the deflection and bending stress distribution of the shafting are shown in Figure 4.

**Table 3.** State variables of each bearing in rational alignment state.

| Name                | Rotation Angle (Rad)     | Bearing Load (kN) | Specific Pressure (MPa) | Allowable Specific Pressure (MPa) |
|---------------------|--------------------------|-------------------|-------------------------|-----------------------------------|
| Rear stern bearing  | $3.4793 \times 10^{-4}$  | 47.037            | 0.130                   | 0.300                             |
| Front stern bearing | $-5.3229 \times 10^{-5}$ | 25.244            | 0.094                   | 0.300                             |
| Thrust bearing      | $-6.4735 \times 10^{-5}$ | 40.098            | 0.291                   | 0.600                             |

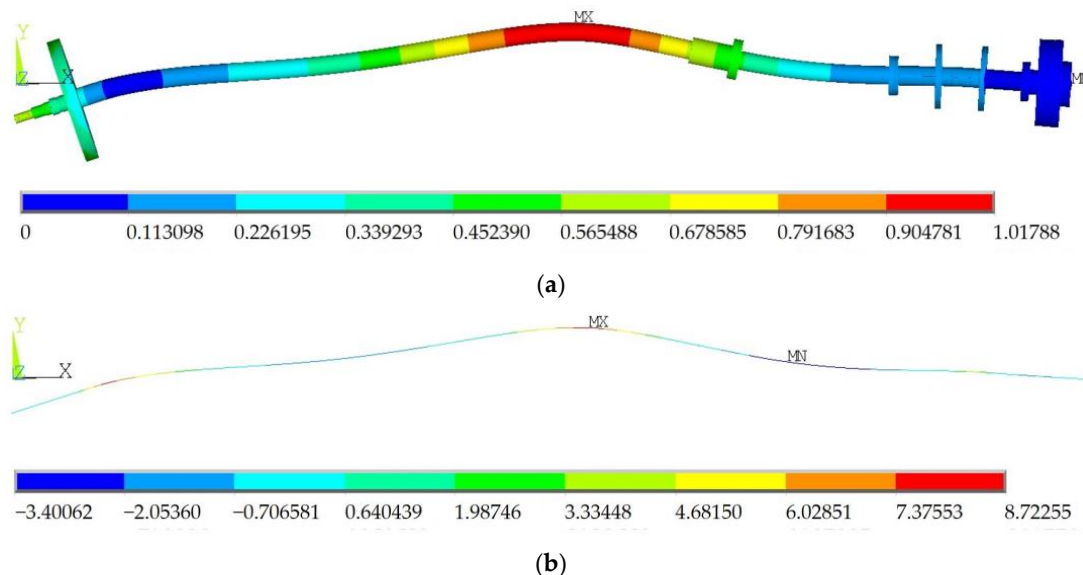


**Figure 4.** Deflection and bending stress distribution of shafting in rational alignment state. (a) Deflection distribution (mm); (b) bending stress distribution (MPa).

Due to the elevation of the front stern bearing's vertical position after rational alignment, comparing Figure 4a with Figure 3a, the deflection near the front stern bearing increased. From Figure 4b, the maximum bending stress of the stern shaft was also found near the front stern bearing but still meets the shafting design standards. According to Table 3, with the rotation angles and specific pressures of each bearing allowable, the rear

stern bearing load was reduced by 1.44%, and the load difference between the front and rear stern bearings was reduced by 11.99% after rational alignment. The load distribution among bearings became more reasonable. Thus, the shafting working state was improved.

#### 4.2. Whirling Vibration Characteristics

To study the coupling effect of the alignment states on the whirling vibration characteristics, first of all, the natural frequencies and the critical speeds of the shafting's whirling vibration were analyzed. A mass element was added to the finite-element model to consider the moment of inertia of the propeller and its attached water. The attached water coefficient of the polar moment of inertia was taken as 1.3 and that of the radial moment of inertia was taken as 1.6. Different from lateral vibration, influenced by the gyroscopic effect, the natural frequencies of whirling vibration change with the rotating speed. The rotor dynamics equation, considering the gyroscopic effect, can be expressed as:

$$M\ddot{x} + (C + \Omega G_c)\dot{x} + Kx = F \quad (13)$$

where  $M$ ,  $C$ , and  $K$  are the generalized mass matrix, damping matrix, and stiffness matrix, respectively;  $G_c$  is the Coriolis damping matrix;  $\Omega$  is the rotating speed;  $\ddot{x}$ ,  $\dot{x}$  and  $x$  are the nodal acceleration vector, nodal velocity vector, and nodal displacement vector, respectively; and  $F$  is the external excitation force vector. In practical engineering, only the natural frequencies and the critical speeds of shafting frequency and blade frequency that may cause resonance need to be calculated [1]. The natural frequencies and the critical speeds of the first three-order forward whirling vibration (FWV) and backward whirling vibration (BWV) of the shafting are listed in Table 4, and the corresponding Campbell diagram is shown in Figure 5.

**Table 4.** Natural frequencies and critical speeds of first three-order whirling vibration.

| Order               | Natural Frequency (Hz) |                 | Critical Speed (rpm) |                 |
|---------------------|------------------------|-----------------|----------------------|-----------------|
|                     | Shafting Frequency     | Blade Frequency | Shafting Frequency   | Blade Frequency |
| First order of FWV  | 12.5                   | 11.49           | 749.94               | 98.48           |
| First order of BWV  | 10.44                  | 11.2            | 626.62               | 96              |
| Second order of FWV | —                      | 30.51           | —                    | 261.48          |
| Second order of BWV | —                      | 30.43           | —                    | 260.79          |
| Third order of BWV  | —                      | 42.53           | —                    | 364.56          |
| Third order of FWV  | —                      | 42.59           | —                    | 365.09          |

The shafting rotating speed under the design operating condition is 210 rpm (termed as the design rotating speed) and that of the cruising condition is 70 rpm. According to Table 4 and Figure 5, the critical speed of the shafting frequency is 1.2 times higher than the design rotating speed, and there is no critical speed of blade frequency within the range of 0.85~1.00 times the design rotating speed, which meets the check criteria for the whirling vibration of a marine propulsion shafting [21].

However, from the cruising condition to the design condition, the rotating speed will inevitably pass through the critical speed of the first-order whirling vibration corresponding to blade frequency, which causes short-term resonance. Figure 6 shows the first three modal shapes of the whirling vibration of the shafting (vertical direction). It can be seen that the whirling vibration of the stern shaft is much more severe than other shaft segments when passing through the critical speed because of the large mass of the propeller and the low support stiffness of the water-lubricated rear stern bearing. Heavy stern vibration not only causes abnormal wear of the stern bearings and the stern shaft but also induces strong underwater-line spectrum noise when transmitted to the hull through the bearings. Therefore, reducing the vibration response of the stern part of the shafting by optimization is of great significance for improving the acoustic stealth performance of the submarine.

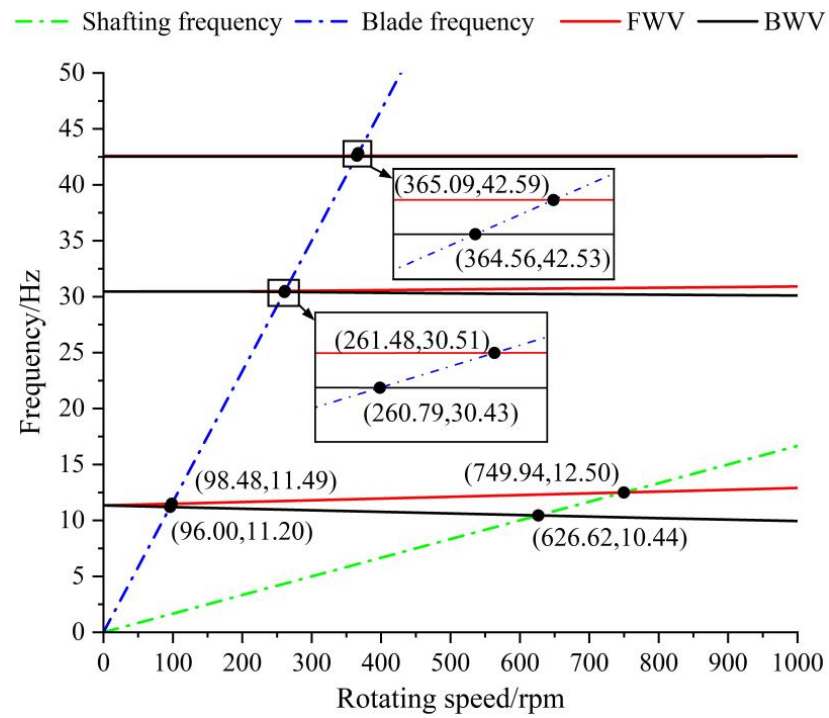


Figure 5. Campbell diagram.

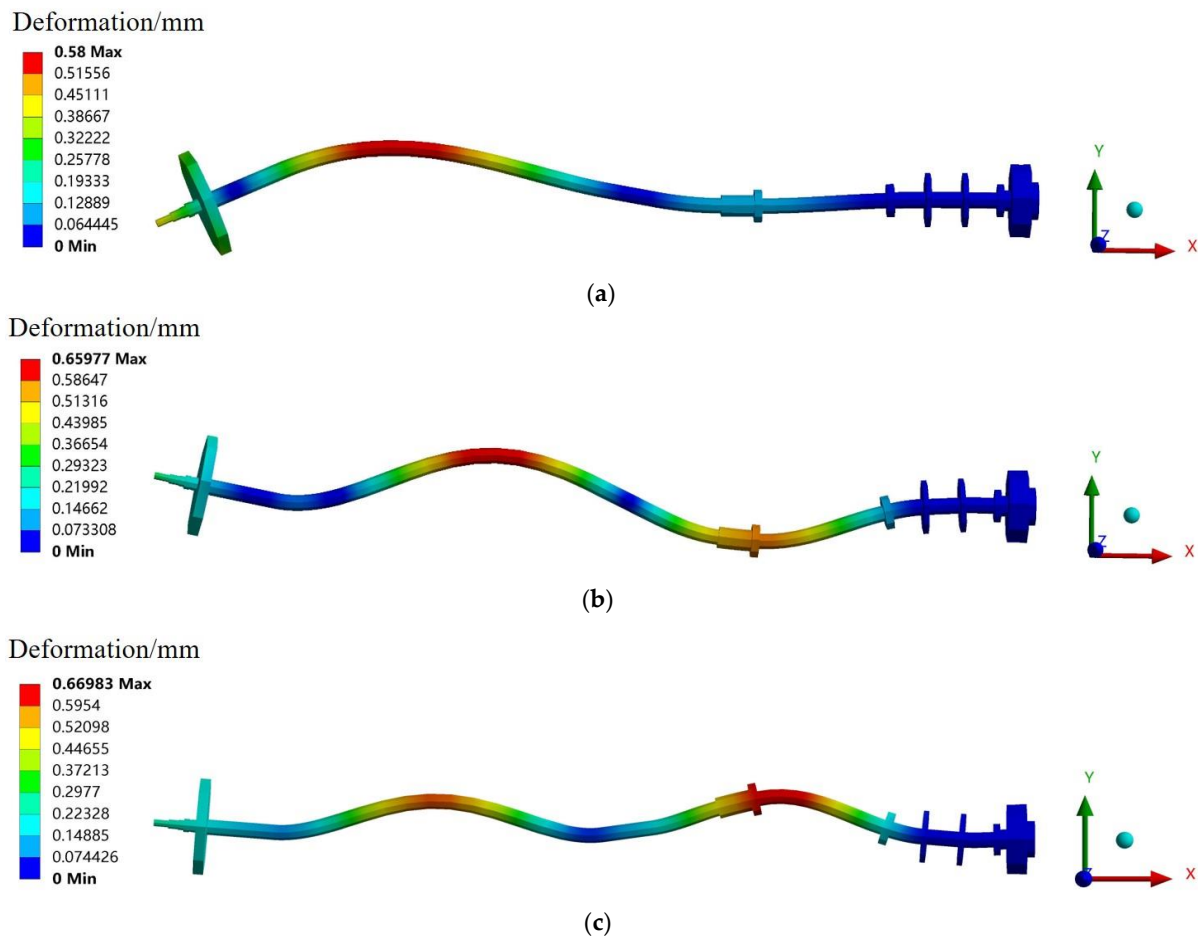


Figure 6. First three modal shapes of whirling vibration. (a) First order; (b) second order; and (c) third order.

#### 4.3. Whirling Vibration Response in Different Alignment States

According to Figure 4, after rational alignment,  $\Delta h_1$  and  $\Delta h_2$  changed the distribution of deflection and bending stress all over the shafting. Elastic deflection will make the rotating centerline of the shafting deviate from that in the static state. With elastic deflection, Equation (13) can be changed to:

$$M\ddot{x} + (C + \Omega G_c)\dot{x} + Kx = Kx_e(t) + F \quad (14)$$

where  $x_e(t)$  represents the nodal bending displacements, which are a function of time, and  $Kx_e(t)$  represents the bending disturbance caused by the elastic deflection during rotation. When the shafting rotates at the speed of  $\Omega$ , this disturbance also rotates along with it [23]. Therefore, when the vertical position of the bearings changes, the whirling vibration response of each node in the finite-element model will change as well. It is necessary to take the coupling effect of this change into account in the analysis of whirling vibration characteristics.

The excitation force generated by the rotation of a propeller in the non-uniform wake field is one of the main external excitation sources for the forced whirling vibration of a propulsion shafting. Therefore, a transverse ( $z$ -direction) resonant load of 1 kN and a vertical ( $y$ -direction) resonant load of 1 kN were applied to the propeller as the external input excitation. Referring to the modal shapes in Figure 6, the midpoint of the stern shaft and the connection between the stern shaft and the intermediate shaft were selected as the vibration monitoring points a and b, respectively. The whirling vibration response of the shafting at the monitoring points under different alignment states was obtained by a harmonic response analysis, as shown in Figure 7.

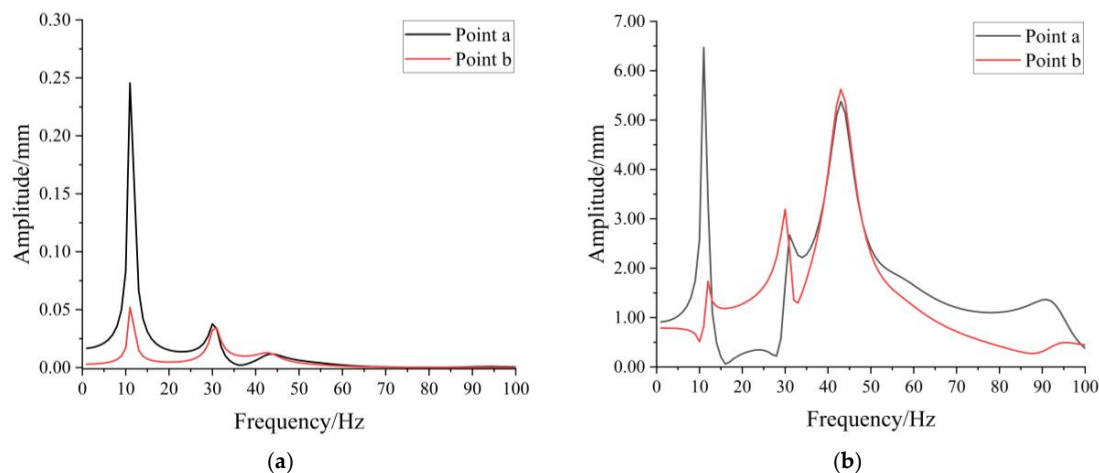


**Figure 7.** Amplitude–frequency response curves of whirling vibration under different alignment states. (a) Straight-line alignment state; (b) rational alignment state.

In Figure 7, the resonance peaks appear at the natural frequencies of the first three-order whirling vibration. From Figure 7b, it can be observed that the amplitude is large at the natural frequency of the third-order whirling vibration (near 42.5 Hz). This is because the third-order whirling vibration of the two monitoring points is relatively intense, and the vertical position of the two points was elevated after rational alignment. However, given the analysis in Section 4.2, the working speed range of the shafting only includes the critical speed of the first-order whirling vibration corresponding to the blade frequency. So, only the resonance response at the frequency of this speed needs to be focused on in the optimization of the dynamic characteristics of the shafting.

Consequently, although the rational alignment made the load distribution among the bearings more uniform, it increased the whirling vibration response at the critical speeds of blade frequency, which is not good for the stable operation of the shafting and will threaten the acoustic stealth performance of the whole submarine. So, the alignment state of the

shafting has an obvious impact on its whirling vibration characteristics. It is essential to obtain a design scheme of the shafting dynamics that has a good alignment state and low vibration response by coupling optimization.

## 5. MDO Using Kriging Surrogate Model

### 5.1. MDO Model

Based on the analysis above, the optimization objectives mainly include the minimization of the bearing load, bearing load difference, and whirling vibration response. Additionally, it is worth noting that the constraints specified in Equation (12) involve the strength check results (bending stress). So, the optimization problem was divided into three subdisciplines, namely alignment, vibration, and strength, and the vertical displacements of the front stern bearing and thrust bearing were selected as the design variables ( $\Delta h = (\Delta h_2, \Delta h_3)^T$ ). The coupling relationships between the subdisciplines are shown in Figure 8.

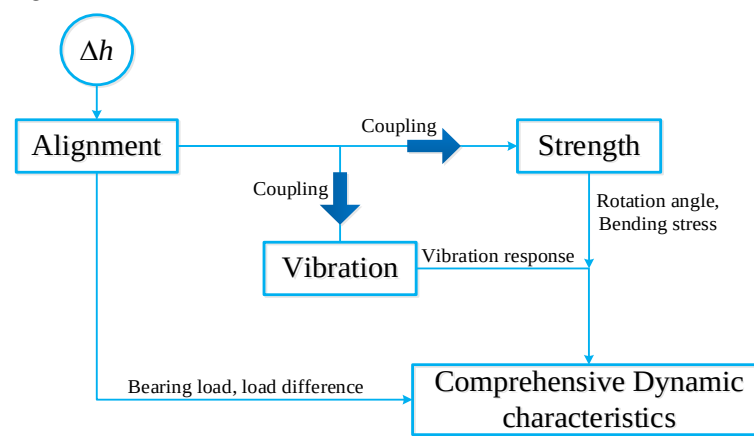


Figure 8. Coupling relationships between subdisciplines.

Referring to Figure 8, there are coupling relationships but no hierarchical relationships between the three subdisciplines. Consequently, the multidisciplinary feasible (MDF) strategy [24] was adopted to establish the MDO model, which can effectively control all of the design variables, objective functions, and constraints, and has the advantages of strong targeting, global optimization, and easy implementation. To solve the problem of low efficiency in iterations of the finite-element model, the Kriging model was adopted to surrogate the numerical calculation. The MDO framework using the Kriging model is shown in Figure 9.

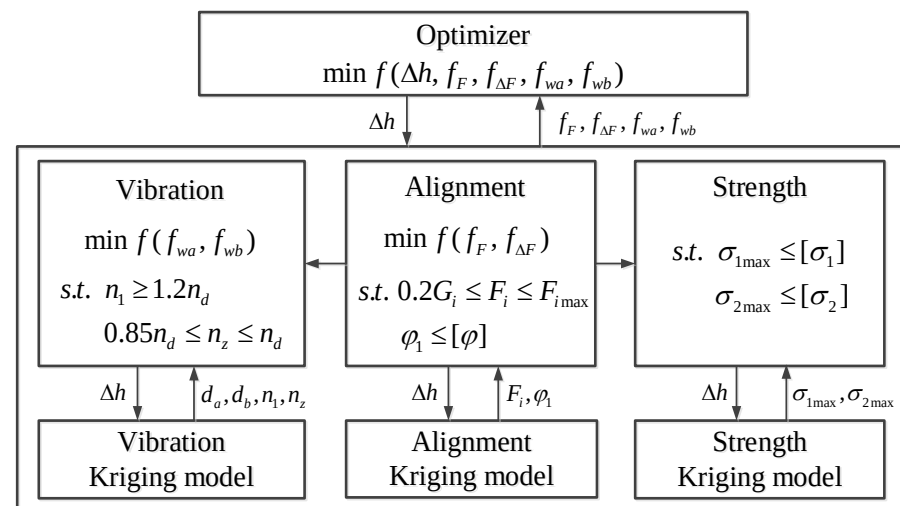


Figure 9. MDO framework using Kriging model.

This framework includes the Kriging models for data interaction with three subdisciplines. The objective functions and constraints contained in each subdiscipline are listed as follows:

- Alignment:

$$f_F = \frac{F_1 - F_1^*}{F_1^*}, f_{\Delta F} = \frac{|F_1 - F_2| - |F_1^* - F_2^*|}{|F_1^* - F_2^*|} \quad (15)$$

$$\begin{aligned} \text{s.t. } 0.2G_i &\leq F_i \leq F_{i\max} \\ \varphi_1 &\leq [\varphi] \end{aligned} \quad (16)$$

- Vibration:

$$f_{wa} = \frac{d_a - d_a^*}{d_a^*}, f_{wb} = \frac{d_b - d_b^*}{d_b^*} \quad (17)$$

$$\begin{aligned} \text{s.t. } n_1 &\geq 1.2n_d \\ 0.85n_d &\leq n_z \leq n_d \end{aligned} \quad (18)$$

- Strength:

$$\begin{aligned} \text{s.t. } \sigma_{1\max} &\leq [\sigma_1] \\ \sigma_{2\max} &\leq [\sigma_2] \end{aligned} \quad (19)$$

where  $f_F$  is the index function representing the reduction of the rear stern bearing load;  $f_{\Delta F}$  is the index function representing the reduction of load difference between the front and rear stern bearings;  $F_1^*$ ,  $F_2^*$  and  $F_1$ ,  $F_2$  are the load of two stern bearings before and after MDO, respectively;  $f_{wa}$  and  $f_{wb}$  are the index functions representing the reduction of whirling vibration response, respectively;  $d_a^*$ ,  $d_b^*$  and  $d_a$ ,  $d_b$  are the resonance peak values of the first-order whirling vibration corresponding to the blade frequency at the monitoring points a and b before and after MDO, respectively;  $n_1$  and  $n_z$  are the critical speeds of the first-order whirling vibration of the shafting frequency and blade frequency, respectively; and  $n_d$  is the design rotating speed.

## 5.2. Surrogate Models of Each Subdiscipline

The Kriging surrogate models of each subdiscipline were established using the method mentioned above. The sample set is crucial, as its size and the spatial correlation between the sample points influence the prediction accuracy of the surrogate model. Here, the LHS in the design of the experiment (DOE) was used for sampling in the design space. The distribution of the sample points selected by this method is relatively uniform, which can improve the accuracy. The establishment process of the surrogate model is shown in Figure 10, which mainly includes the following steps:

1. Determine the sample space, including the values range of the design variables. Then, the DOE is used to obtain the sample set. The response values of each sample point are calculated by the finite-element model;
2. Divide the sample set into training and testing sets and normalize the data, respectively;
3. Use the initial trained Kriging model to predict the response values of the test set, and implement the anti-normalization on them;
4. Calculate the coefficients of determination between the predicted values and true values to test the accuracy of the surrogate model. If the accuracy is satisfied, the Kriging model will be output. Otherwise, the sample set will be expanded and updated, and the model will be trained again until the accuracy meets the requirements.

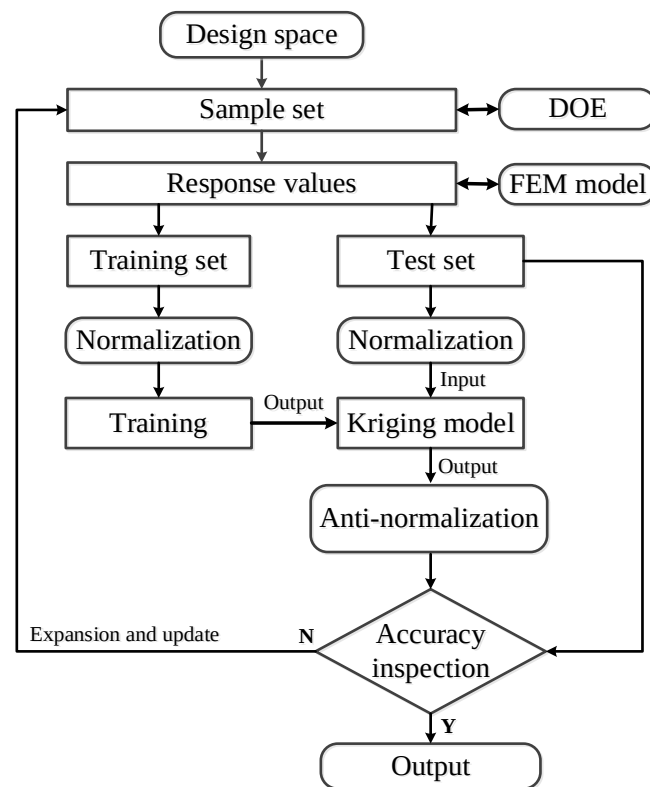


**Figure 10.** Establishment process of surrogate model.

First, a surrogate model of the alignment subdiscipline was established, and the values range of the design variables  $\Delta h_2$  and  $\Delta h_3$  were  $[-5.00, 5.00]$  mm. Setting the number of sample points to 500, the points sampled by the LHS are shown in Figure 11.

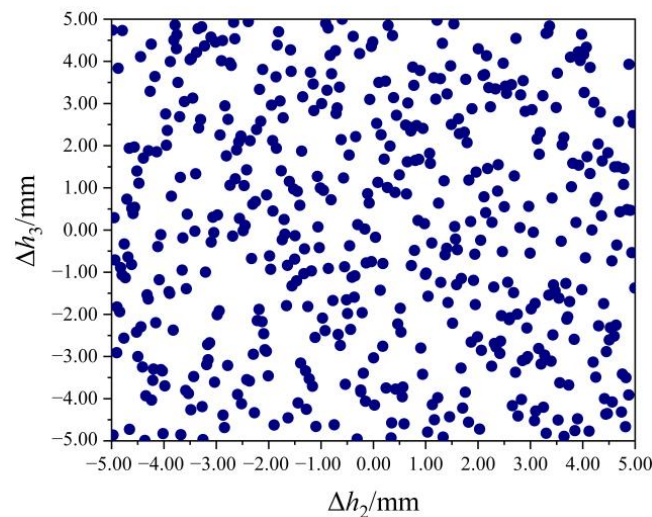


**Figure 11.** Sample set obtained by LHS.

Ninety-six percent of the sample points were randomly selected as the training set, and the rest of them were selected as the test set. After training, the final Kriging model was output. Compare the predicted state variables within the alignment subdiscipline with the actual values, including the bearings' load and the rotation angle of the rear stern bearing, as shown in Figure 12.

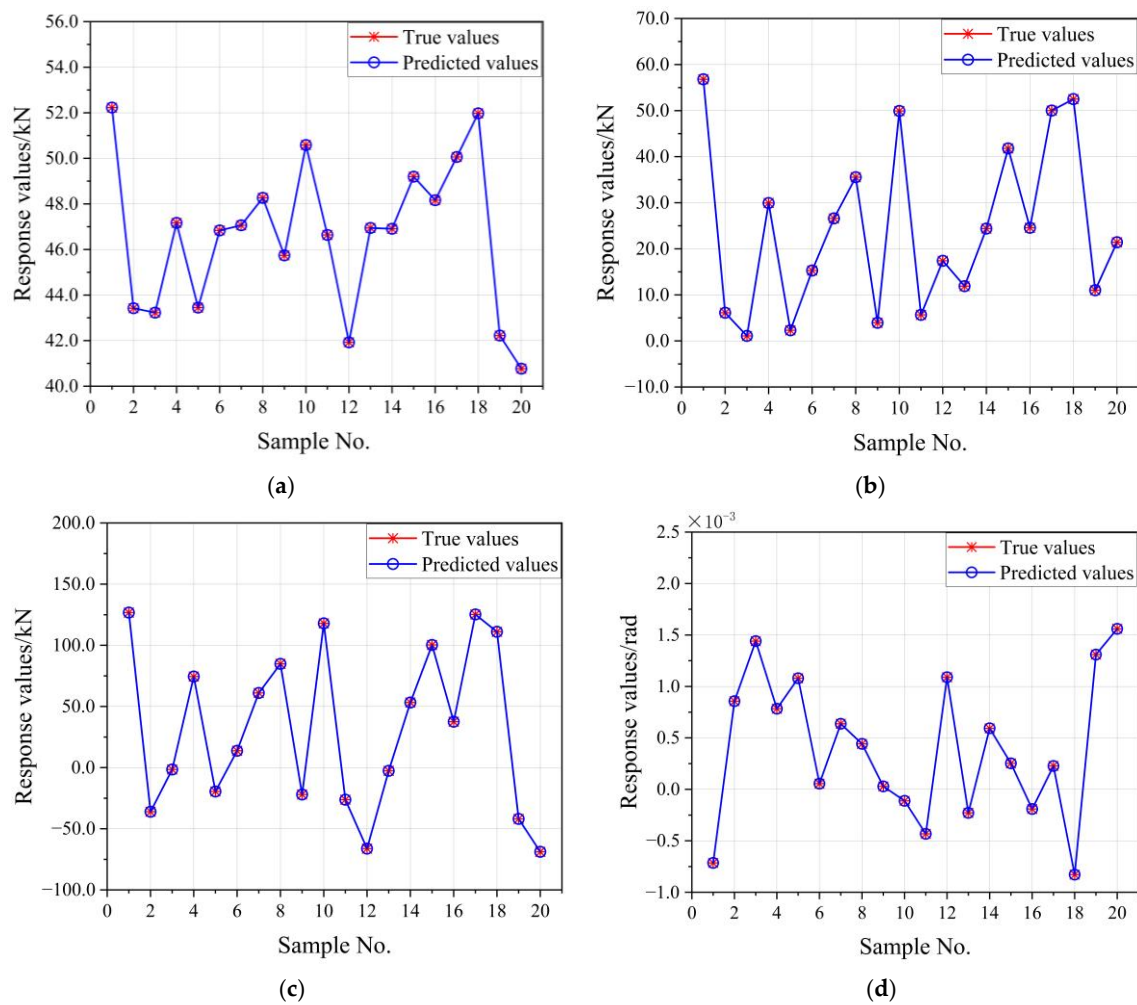


**Figure 12.** Predicted results of state variables within alignment subdiscipline. (a)  $F_1$ ; (b)  $|F_1 - F_2|$ ; (c)  $F_3$ ; and (d)  $\phi_1$ .

In the same way, the Kriging models of the other two subdisciplines were established. The state variables of the vibration subdiscipline are the resonance peak values of the first-order whirling vibration corresponding to the blade frequency, while that of the strength subdiscipline are the maximum bending stress of the shaft segments. The response values of these state variables were predicted, as shown in Figure 13.

Based on the true and predicted response values of the sample points in the test set, the coefficients of determination  $R_j^2$  were calculated to test the accuracy of the Kriging models for each subdiscipline, which can be expressed as:

$$R_j^2 = 1 - \frac{\sum_{i=1}^{20} (y_{ij} - \hat{y}_{ij})^2}{\sum_{i=1}^{20} (y_{ij} - \bar{y}_{ij})^2} \quad (20)$$

where  $y_{ij}$ ,  $\hat{y}_{ij}$ , and  $\bar{y}_{ij}$  represent the true, predicted, and average response values of the  $i$ -th sample point corresponding to the  $j$ -th ( $j = 1, 2, \dots, 8$ ) subdisciplinary state variable, respectively. The calculated results for  $R_j^2$  are listed in Table 5.

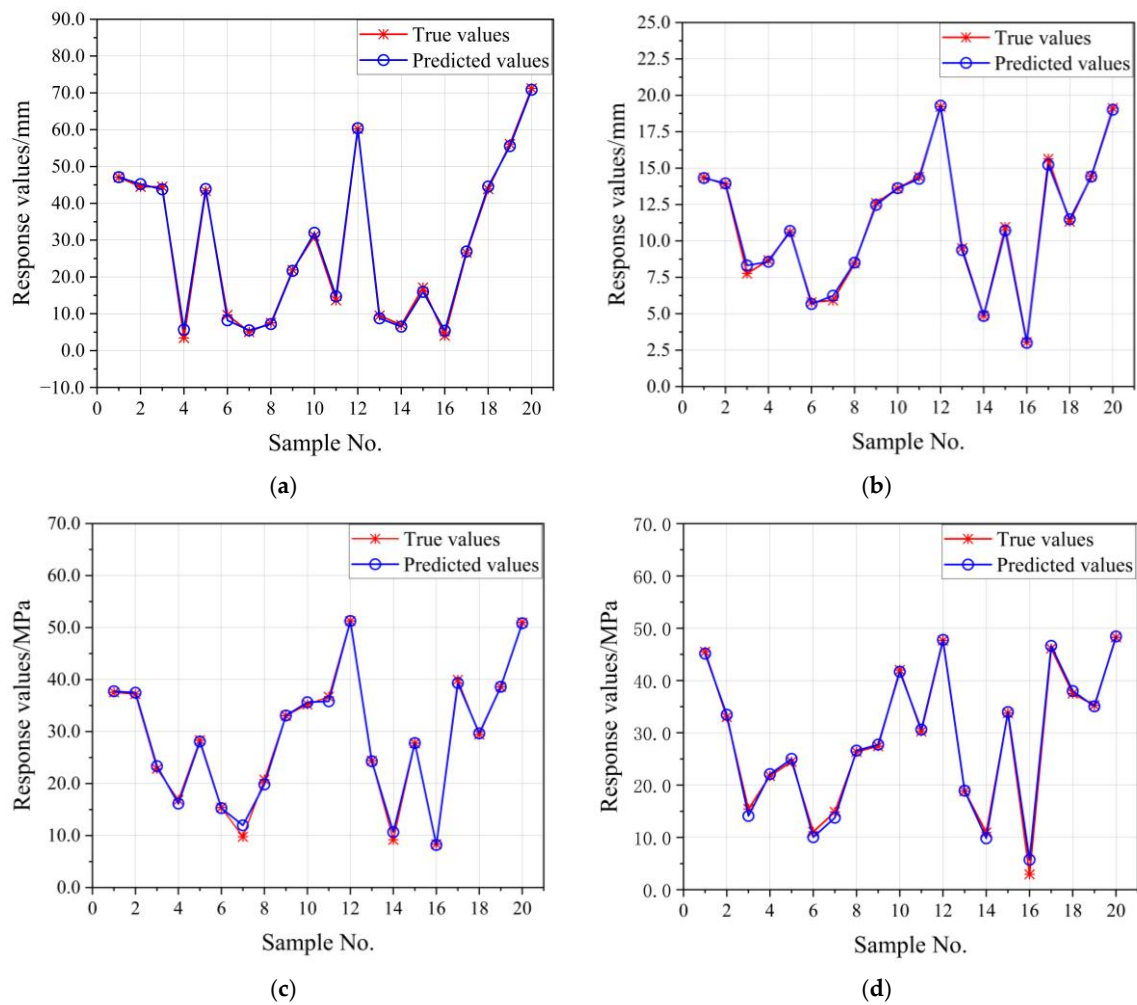


**Figure 13.** Predicted results of state variables within vibration and strength subdisciplines. (a)  $d_a$ ; (b)  $d_b$ ; (c)  $\sigma_{1max}$ ; and (d)  $\sigma_{2max}$ .

**Table 5.** Coefficients of determination.

| Subdiscipline | State Variable  | $R_j^2$ |
|---------------|-----------------|---------|
| Alignment     | $F_1$           | 1.0000  |
|               | $ F_1 - F_2 $   | 1.0000  |
|               | $F_3$           | 1.0000  |
| Vibration     | $d_a$           | 1.0000  |
|               | $d_b$           | 0.9982  |
| Strength      | $\sigma_{1max}$ | 0.9979  |
|               | $\sigma_{2max}$ | 0.9967  |

The values range of  $R_j^2$  is [0, 1], and the larger its value, the more accurately the Kriging model predicts the state variable. It is generally agreed that the accuracy meets the requirement when  $R_j^2 \geq 0.99$  [25]. According to Table 5, the established Kriging models provided good prediction accuracy and could be used to surrogate high-precision numerical models in the MDO model.

### 5.3. MDO Results

As expressed by Equations (15)–(19), solving the MDO model belongs to a multi-objective optimization problem with multiple constraints, and there are coupling relationships between the objective functions to some extent. In light of this, the non-dominated

sorting genetic algorithm-II (NSGA-II) was used as the optimization algorithm to interact with the Kriging models. This algorithm incorporates an elite individual reservation mechanism during the iterative process, which is beneficial for finding the global optimal solution [26,27]. With regard to the constraints in the MDO model, they were managed in the fitness function of the algorithm. The fitness values of the individuals that do not satisfy all the constraints were set to the minimum value in order to eliminate them during the optimization process.

The parameters of the algorithm were set as follows. The initial population size was 200. The maximum number of iterations was 100. The Pareto fraction was 0.7, and the function tolerance was 0.001. After iterative calculation, the dimensionless objective function values were transformed into actual values. Then, the Pareto solution set was obtained. The Pareto fronts for  $d_a$ ,  $d_b$  with respect to  $F_1$ ,  $|F_1 - F_2|$  were plotted, as shown in Figure 14.

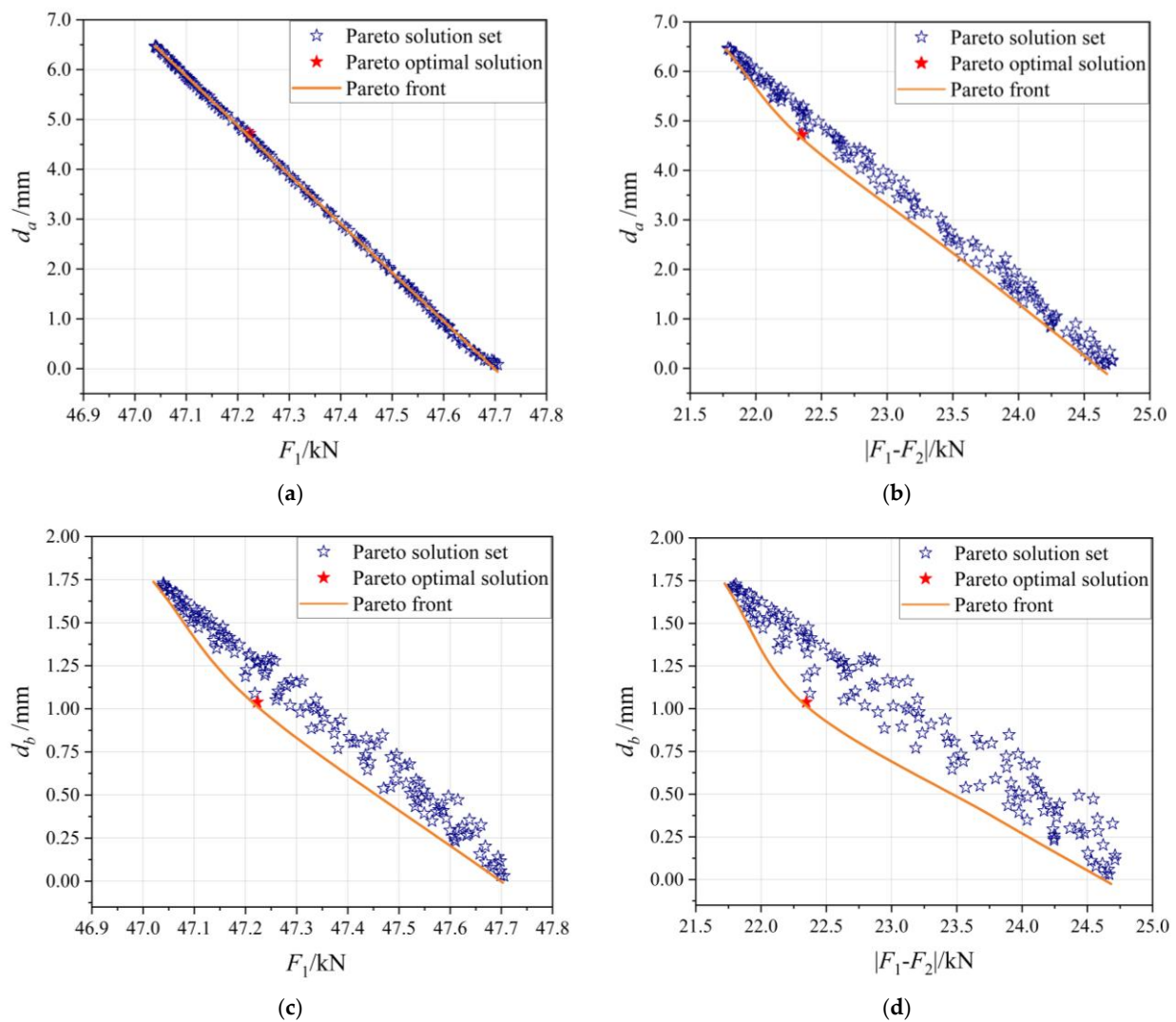


**Figure 14.** Pareto fronts. (a)  $d_a - F_1$ ; (b)  $d_a - |F_1 - F_2|$ ; (c)  $d_b - F_1$ ; and (d)  $d_b - |F_1 - F_2|$ .

There are 140 non-dominated solutions contained in the solution set. In Figure 14a, the Pareto front is similar to a straight line, indicating a negative linear correlation between two state variables, approximately. In Figure 14b–d, the Pareto fronts are convex, and each of them also shows a negative correlation. Based on the minimum distance selection method presented by Sun et al. [28], the 123rd Pareto solution was defined to be the knee point,

which is the optimal solution marked in Figure 14, and its corresponding vector of design variables is:

$$\Delta h = (\Delta h_2, \Delta h_3)^T = (0.66, 0.02)^T \quad (21)$$

Update the vertical position of each bearing according to the optimization results and calculate the state variables within each subdiscipline once again. The state variables of each bearing after the MDO are listed in Table 6. The comparison of the whirling vibration responses at monitoring points a and b before and after the MDO is shown in Figure 15, and the resonance peaks of the first-order whirling vibration corresponding to blade frequency have been marked out.

**Table 6.** State variables of each bearing after MDO.

| Name                | Rotation Angle (Rad)     | Bearing Load (kN) | Specific Pressure (MPa) | Allowable Specific Pressure (MPa) |
|---------------------|--------------------------|-------------------|-------------------------|-----------------------------------|
| Rear stern bearing  | $2.6489 \times 10^{-4}$  | 47.223            | 0.131                   | 0.300                             |
| Front stern bearing | $-5.1514 \times 10^{-5}$ | 24.876            | 0.093                   | 0.300                             |
| Thrust bearing      | $1.0684 \times 10^{-6}$  | 40.095            | 0.291                   | 0.600                             |

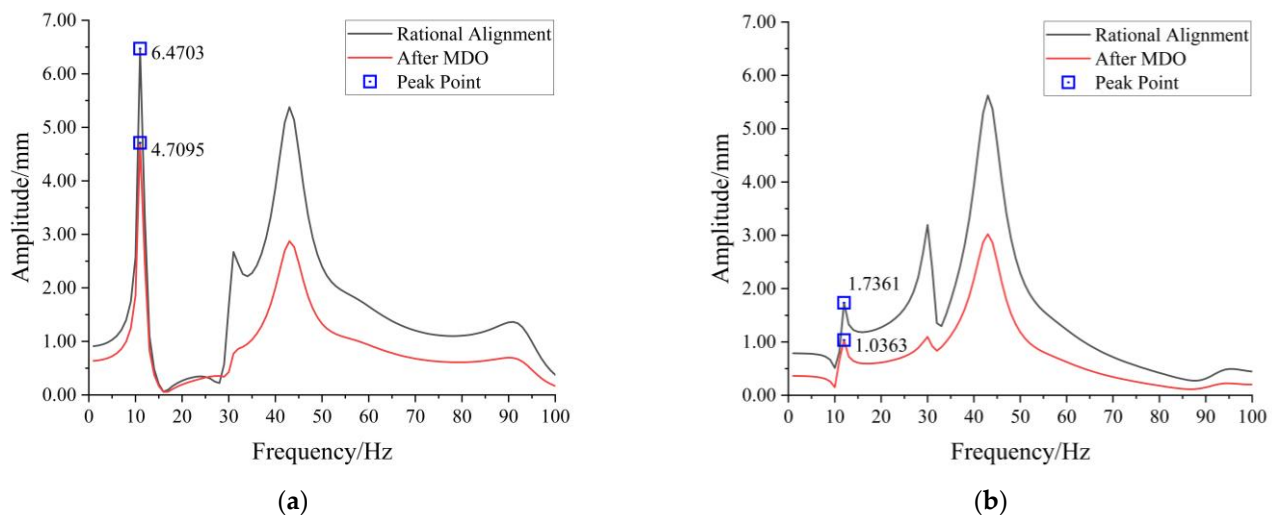


**Figure 15.** Comparison of amplitude-frequency response of whirling vibration at monitoring points before and after MDO. (a) Point a; (b) Point b.

After the MDO, compared with the straight-line alignment state, the rear stern bearing load was reduced by 1.05%, and the load difference between the front and rear stern bearings was reduced by 9.75%. The resonance peak values of the first-order whirling vibration, corresponding to blade frequency at monitoring points a and b, were clearly reduced. Compared with the existing optimization methods that optimize the alignment or vibration characteristics of a shafting from a single perspective, on the condition of meeting the shafting design standards and reducing the high computational expense, the studied method not only improved its alignment state but also significantly reduced the whirling vibration response of the shafting when passing through the critical speeds. The MDO of the alignment and whirling vibration characteristics of the submarine propulsion shafting was finally achieved, and the performance indexes, such as safety, reliability, and acoustic stealth, of the submarine were improved as well.

## 6. Conclusions

In this study, a submarine propulsion shafting was taken as the object of study, and the coupling effect of its alignment states on its whirling vibration response was analyzed. On this basis, using the Kriging model to surrogate the numerical model, the MDO of the

alignment and whirling vibration characteristics of the shafting was carried out. The main conclusions are summarized below.

1. The whirling vibration responses in different alignment states of the shafting were analyzed. The response of the stern shaft was increased after rational alignment, which reflected the coupling effect of the alignment states on the whirling vibration characteristics. It is necessary to consider this coupling effect in the design of a submarine propulsion shafting;
2. An MDO model of the alignment and whirling vibration characteristics of the shafting was established based on the results of coupling dynamic analysis. The Kriging surrogate model was used to predict the state variables within each subdiscipline, in order to decrease the high computational expense brought by the finite-element model. After calculation, the coefficients of determination of each state variable were all above 0.99. Thus, the surrogate model had sufficient accuracy;
3. The MDO model was solved by combining it with the NSGA-II algorithm. The optimization results show that the rear stern bearing load and the load difference between the front and rear stern bearings were reduced by 1.05% and 9.75%, respectively, and the resonance peak values of the first-order whirling vibration corresponding to blade frequency at monitoring points a and b were reduced. On the whole, the alignment and whirling vibration characteristics of the shafting were comprehensively optimized.

A complete and effective MDO method of the alignment and whirling vibration characteristics of a submarine propulsion shafting using the Kriging surrogate model is proposed in this work, which can provide technical and theoretical support for improving the efficiency of the coupling design optimization of shafting dynamics. In future work, the adaptive sampling strategies will be studied and used to establish a more accurate dynamic Kriging surrogate model, which can further reduce the computational expense caused by a large-scale sample set. Additionally, more shafting vibration forms, such as axial vibration, torsional vibration, and even multidirectional vibration, will be explored. On this basis, more performance indexes will be analyzed and then added to the MDO model.

**Author Contributions:** Conceptualization, Z.G. and J.L.; Methodology, Z.G.; Validation, Z.G. and J.L.; Formal analysis, Z.G.; Investigation, Z.G. and J.L.; Resources, Z.G. and J.L.; Data curation, Z.G.; Writing—original draft preparation, Z.G.; Writing—review and editing, Z.G. and J.L.; Funding acquisition, J.L. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was funded by the Natural Science Foundation of Hubei Province, grant number 2017CFB584, and the National Defense Science and Technology Foundation, grant number 20191C080744.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** Data sharing is not applicable due to privacy restrictions.

**Conflicts of Interest:** The authors declare no conflicts of interest.

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