

Article

On the Added Modal Coefficients of a Rotating Submerged Cylinder Induced by a Whirling Motion—Part 1: Experimental Investigation

Rafel Roig , Xavier Sánchez-Botello , Esteve Jou and Xavier Escaler * 

Barcelona Fluids & Energy Lab, Universitat Politècnica de Catalunya, 08034 Barcelona, Spain; rafel.roig@upc.edu (R.R.); xavier.sanchez.botello@upc.edu (X.S.-B.); esteve.jou@upc.edu (E.J.)

* Correspondence: xavier.escaler@upc.edu

Abstract: The operation of submerged rotating machines, such as marine current or tidal turbines, can present deleterious fluid phenomena that may provoke extreme structural vibrations. To predict their dynamic responses, it is necessary to know the added modal coefficients of their runners under a whirling motion. For that purpose, a bespoke test rig was designed to investigate the added modal coefficients of a submerged cylinder, which could rotate at different speeds both in air and completely submerged in water inside a cylindrical tank. First, the modes of vibration were experimentally measured by exciting the cylinder with a push-release method during steady tests or with ramps in rotating speed during transient tests. The calculated natural frequencies and damping ratios were then used in a mathematical model of the dynamic system to calculate the added modal coefficients. During steady tests, the natural frequencies and damping ratios of the whirling modes changed significantly as a function of the rotating speed. Additionally, a whirling mode was observed to change its direction at a given rotating speed. During transient tests, rotating speed ramps with high accelerations were found to present lower lock-in amplitude and frequencies. Moreover, fast downward ramps presented lock-in amplitudes four times higher than fast upward ramps. Consequently, the added modal coefficients changed accordingly as a function of the rotating speed, ramp acceleration, and ramp direction. For these reasons, it was confirmed that the modal responses of submerged rotating bodies must be calculated for each operational rotating speed, even at low velocities, and for each transient event in order to precisely predict their vibration behaviors.

Keywords: experimental modal analysis; hydraulic machines; fluid–structure interaction; added modal coefficients; whirling motion; transient operation; lock-in



Citation: Roig, R.; Sánchez-Botello, X.; Jou, E.; Escaler, X. On the Added Modal Coefficients of a Rotating Submerged Cylinder Induced by a Whirling Motion—Part 1: Experimental Investigation. *J. Mar. Sci. Eng.* **2023**, *11*, 1758. <https://doi.org/10.3390/jmse11091758>

Academic Editor: María Isabel Lamas Galdo

Received: 27 July 2023

Revised: 31 August 2023

Accepted: 5 September 2023

Published: 8 September 2023



Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Even though rotating machines submerged in water have been widely used since a long time ago, their operation still represents a challenge today. For instance, rotating vortex ropes and vanless space vortex structures [1–5] threaten the safe operation of hydraulic turbines. And large relative variations in head [6], free surface vortices or upstream and downstream water level fluctuations [7,8] compromise the operation of marine and run-of-river turbines. In order to minimize the risks of these phenomena, it is of paramount importance to determine the modal responses, i.e., the natural frequencies, mode shapes, and damping ratios, of the machines. However, calculating their modal responses is always complex as the rotor is often composed of a runner, a shaft, an alternator, bearings, and seals, and this type of machine always has certain parts submerged in water, significantly affecting their modal responses [9].

Historically, turbine manufacturers have studied their modal responses by means of rotor dynamic analysis considering that the turbines are in air or at most considering increases in the runner masses due to the added mass effect. Jasper [10], Benkö et al. [11], and Hofstad [12] proposed a value for this added mass calculated through a rule of thumb.

Later, Gustavsson et al. [13] and Keto-Tokio et al. [14] calculated the added mass by means of acoustic structural simulations and a method derived from Theodorsen's unsteady thin airfoil theory. The added mass was found to significantly influence the natural frequencies of a rotor [13–15].

In addition to the added mass, which is associated with the body's acceleration, the fluid–structure interaction (FSI) also depends on the body's displacement and velocity through the added stiffness and the added damping, respectively [16,17].

There have been a large number of investigations that have focused on the influence of the added coefficients on the modal responses of non-rotating structures submerged in water. For instance, Leblond et al. [18] studied the added mass and damping of a non-rotating circular cylinder oscillating with a low amplitude and a high frequency in cylindrical confinement. The authors presented analytical equations for the added mass and damping, considering the fluid as incompressible and inviscid as well as incompressible and viscous. It was concluded that the analytical solutions underestimated both the added mass and damping.

The standing mode shapes, which are characteristics of non-rotating structures, become whirling mode shapes when a structure rotates. Both the rotation and the whirling motion may then influence the added coefficients. In this sense, Gandjalikhan et al. [19] numerically investigated the added forces associated with body displacement, which were induced on a rotating cylinder with static eccentricity in cylindrical confinement. The pressure in the laminar flow was observed to increase as the fluid passed through the diverging part, such that the maximum and minimum pressure occurred upstream and downstream of the minimum film thickness. Moreover, the unfavorable pressure gradient in the circumferential direction was found to cause flow separation in the vicinity of the maximum fluid thickness. Similarly, Adibi et al. [20] numerically investigated the incompressible viscous flow between eccentric rotating cylinders. The maximum and minimum pressures were also found downstream and upstream of the minimum film thickness. They concluded that the amount of shear stress and its gradient increased on the cylinder with an increase in eccentricity.

Magnus [21] experimentally investigated the added forces associated with body velocity, which were induced by a flow passing through a rotating cylinder. The combined effects of cylinder rotation and translation inside a fluid led to an asymmetrical pressure distribution at the top and bottom of the cylinder, which flipped as the rotating direction changed. It was concluded that this asymmetrical pressure distribution resulted in an additional lift force called the Magnus effect. Prandtl [22] also conducted experiments on a flow passing through a rotating cylinder. He stated that the maximum intensity of the lift coefficient due to the Magnus effect could not exceed 4π . Later, Glauert [23] concluded that the intensity of the lift coefficient due to the Magnus effect might exceed 4π because a fluid body with a high velocity and high vorticity could break due to the centrifugal force and subsequently be carried away by the stream, increasing the circulation of the cylinder. More recently, Yao et al. [24] numerically studied the flow passing through a rotating cylinder at supercritical Reynolds numbers. It was concluded that there was a linear relationship between the Magnus effect and the rotating speed ratio, i.e., the ratio of rotating speeds to flow velocities. On the other hand, it was observed that the drag coefficient decreased gradually to a certain value and subsequently remained constant when the rotating speed of the cylinder was increased. Ma et al. [25] experimentally investigated the flow around a cylinder at Reynolds numbers from 4.6×10^4 to 5.1×10^5 for rotating speed ratios from 0 to 2. They concluded that the evolution of the drag coefficient and the Magnus effect depended on the flow state. For instance, in the subcritical flow state, the drag coefficient decreased at rotating speed ratios below one whilst it increased at rotating speed ratios above one whenever the speed ratio was increased. On the other hand, the Magnus effect continuously increased.

Most of the previous studies have focused on the added coefficients of oscillating non-rotating structures [10–14,18] or those of non-oscillating rotating structures [19–25]

when submerged in water. However, they have not studied rotating submerged structures presenting a whirling motion, which is the actual case of turbine runners. Despite the potential new added coefficients induced due to the rotation and whirling motion, no results have been published on their identification, origin, and/or evolution as a function of the rotating speed.

This work presents a study on the added modal coefficients of a rotating submerged cylinder when it presents a whirling motion. For this reason, a bespoke test rig was built, which consisted of a cylinder submerged in a cylindrical water tank and clamped to a shaft that could be rotated. The identification of added modal coefficients, their origin, their values, and their evolutions is presented and discussed as a function of the rotating speed. The direct measurement of the added modal forces and their corresponding added modal coefficients is complex and it implies an interaction between the dynamic response of the structure and the excitation system attached to it, which might influence their values. In order to minimize this interaction, the added modal coefficients were calculated indirectly from measured natural frequencies and damping ratios as well as from a proposed mathematical model of the system.

2. Experimental Setup

2.1. Test Rig

The test rig consisted of a long vertical stainless-steel shaft coupled to a stainless-steel cylinder through a screwed connection of six bolts, which could be partially submerged in a cylindrical water tank made of transparent methacrylate (see Figure 1a,b). The shaft was composed of two parts: (i) a slim part, with external and internal diameters of 12 and 10 mm, respectively, and (ii) a thick part, with external and internal diameters of 30 and 24 mm, respectively. Both parts were coupled by means of six bolts (see the red square in Figure 1a). The thickest part, supported by two ball bearings placed on an aluminum frame, was equipped with a driven pulley connected to a variable-speed motor (Crouzet SQ75) through a cog belt. The motor permitted the rotation of the system at different speeds (Ω) and carrying out ramps in Ω at different accelerations and in different frequency ranges.

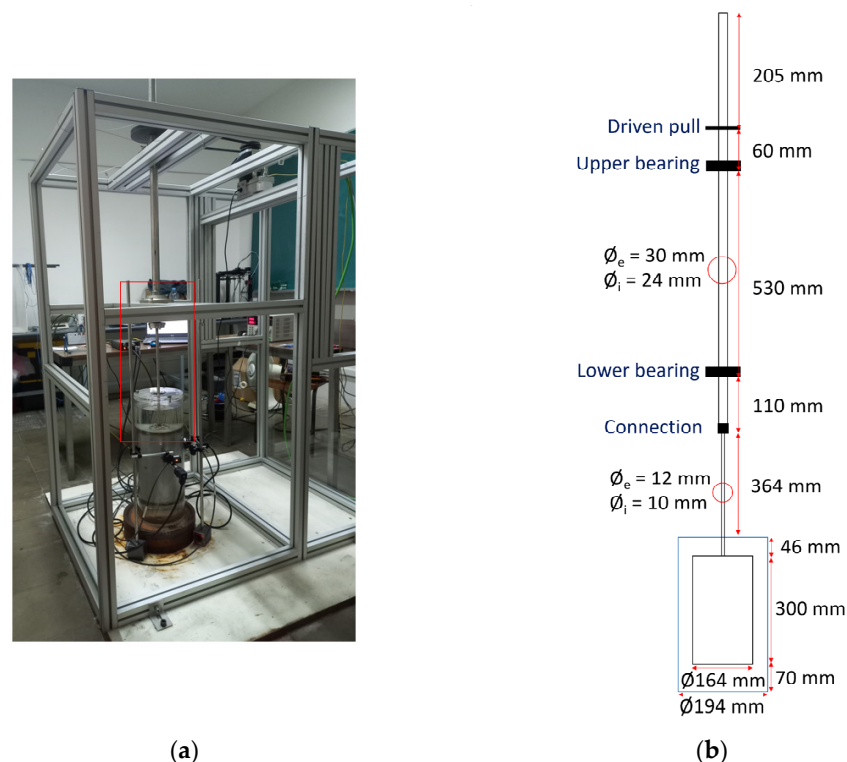


Figure 1. Test rig mounted in the laboratory (a). Geometry and dimensions of the test rig (b).

Due to the measuring range of the laser displacement sensors, the gap between the external wall of the cylinder and the external wall of the water tank had to be around 25 mm. Since the water tank wall thickness was 10 mm, a maximum gap distance of 15 mm had to be chosen between the cylinder and the inner wall of the tank.

2.2. Instrumentation

In order to measure the radial movement at the midplane of the cylinder's lateral wall, two laser displacement sensors, D0 and D90 (ILD1320-10), were fixed outside the water tank in off-board positions 90° apart from each other with their laser beams crossing the transparent methacrylate wall. Similarly, three pressure transducers, P0, P90, and P180 (UNIK5000: PMP5076-TA-A2-CA-H0-PB), were mounted flush on the inner surface of the water tank wall, also measuring the pressure fluctuations on the midplane 90° apart from each other, as shown in Figure 2. These signals were digitalized and recorded using the National Instruments compact DAQ NI-cDAQ-9185 chassis equipped with the input current module NI-9253 for the laser displacement sensors and the input voltage module NI-9223 for the pressure sensors. The signals were measured and synchronized with a LabVIEW program and sampled at a frequency of 800 Hz.

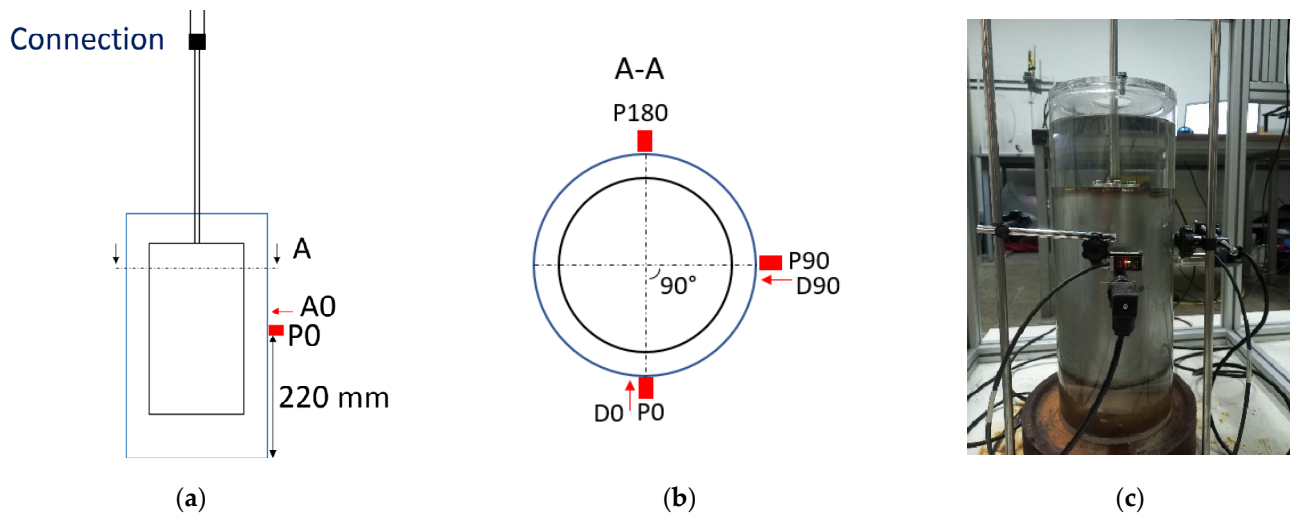


Figure 2. Lateral view of the positions of the sensors (a). Section A-A view of the positions of the sensors (b). D0 and P0 sensors (c).

On one hand, the 24-bit NI-9253 module had a measuring range of ± 20 mA, a gain uncertainty of 0.41%, and an offset uncertainty of 0.08%, resulting in a total uncertainty of ± 0.114 mA when measuring the maximum current of 20 mA. Similarly, the 16-bit NI-9223 module had a measuring range of ± 10 V, a gain uncertainty of 0.2%, and an offset uncertainty of 0.1%, resulting in a total uncertainty of ± 0.04 V when measuring the maximum voltage of 10 V.

On the other hand, the laser displacement sensors had a linearity uncertainty of 0.12%, a repeatability uncertainty of 0.1%, and an accuracy of 0.3%. Assuming a confidence interval of 95%, the random uncertainty of the laser displacement sensors was $\pm 9.05 \times 10^{-4}$ mm and the total uncertainty was ± 0.09 mm when measuring the maximum displacement of 30 mm. Similarly, the pressure sensors had a linearity uncertainty of 0.1%, a repeatability uncertainty of 0.11%, and an accuracy of 0.1%. Assuming again a confidence interval of 95%, the random uncertainty of the pressure sensors was ± 0.03 mbar and the total uncertainty was ± 0.77 mbar when measuring the maximum pressure of 200 mbar.

3. Mathematical Model

Assuming that the two first identical but perpendicular eigenmodes of the dynamic system, Φ_{s1} and Φ_{s2} , are independent and orthogonal, the equations of motion of the

cylinder are described using mode superposition. Consequently, the mathematical model only has two degrees of freedom with their corresponding modal displacements, q_1 and q_2 ; modal velocities, $\dot{q}_1$ and $\dot{q}_2$; and modal accelerations, $\ddot{q}_1$ and $\ddot{q}_2$. When the cylinder does not rotate, the two degrees of freedom are not coupled and the system has two standing mode shapes, Φ_{s1} and Φ_{s2} ; two repeated natural frequencies, f_s in air and f_f in water; and two repeated damping ratios, ζ_s in air and ζ_f in water. When the cylinder rotates, the two degrees of freedom couple, resulting in two whirling modes. One of them whirls in the direction of the rotation and is referred to as the forward whirling mode (FW), and the other one whirls in the opposite direction to the rotation and is referred to as the backward whirling mode (BW). Each FW and BW mode has corresponding natural frequencies (f_{FWs} and f_{BWf} in air and f_{FWf} and f_{BWf} in water) and damping ratios (ζ_{FWf} and ζ_{BWf} in water).

3.1. Non-Rotating Cylinder in Air

Assuming that there are no external excitation forces, when the cylinder does not rotate in air, the dynamic equations are

$$\begin{bmatrix} M_s & 0 \\ 0 & M_s \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \begin{bmatrix} C_s & 0 \\ 0 & C_s \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & K_s \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = 0, \quad (1)$$

where M_s is the structural modal mass, K_s is the structural modal stiffness, and C_s is the structural modal damping.

Assuming that the influence of C_s on the natural frequencies is negligible:

$$f_s = f_{s1} = f_{s2} = \frac{\pm \sqrt{\frac{K_s}{M_s}}}{2\pi}, \quad (2)$$

$$\zeta_s = \zeta_{s1} = \zeta_{s2} = \frac{C_s}{2M_s 2\pi f_s}, \quad (3)$$

where f_{s1} and f_{s2} are the natural frequencies and ζ_{s1} and ζ_{s2} are the damping ratios in air. M_s can also be expressed as

$$M_s = \iiint \rho_s \Phi_{s1}^2 dV_s, \quad (4)$$

where ρ_s and V_s are the density and volume of the structure, respectively.

3.2. Cylinder Rotating in Air

Assuming that, in the range of Ω tested, the gyroscopic effects and other structural forces associated with the rotation and coupling of the two degrees of freedom have a negligible influence on the natural frequencies of the FW and BW, f_{FWs} and f_{BWf} have to be equal to f_s and can be calculated using Equation (2).

3.3. Non-Rotating Cylinder in Water

When the cylinder submerged in water does not rotate and assuming that the structural damping is negligible compared to the fluid damping [26], the dynamic equations are

$$\begin{bmatrix} M_s & 0 \\ 0 & M_s \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & K_s \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = F_f(t), \quad (5)$$

where F_f is the added modal force exerted on the structure, which can also be expressed as

$$F_f(t) = \iint \tau_f(t) \cdot \Phi_{s1} dS, \quad (6)$$

where τ_f is the total added force exerted on the structure and S is the fluid–structure interface. Assuming that there are no external excitation forces and that there is a lineal

dependency between the added modal damping, C_f , and the $\dot{q}_1$ and $\dot{q}_2$ as well as between the added modal mass, M_f , and the $\ddot{q}_1$ and $\ddot{q}_2$, then F_f can be described as

$$F_f(t) = - \begin{bmatrix} M_f & 0 \\ 0 & M_f \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} - \begin{bmatrix} C_f & 0 \\ 0 & C_f \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix}, \quad (7)$$

Finally, Equation (5) can be rewritten as

$$\begin{bmatrix} M_s + M_f & 0 \\ 0 & M_s + M_f \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \begin{bmatrix} C_f & 0 \\ 0 & C_f \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & K_s \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = 0, \quad (8)$$

Assuming that the influence of C_f on the natural frequencies is negligible:

$$f_f = f_{f1} = f_{f2} = \frac{\pm \sqrt{\frac{K_s}{M_s + M_f}}}{2\pi}, \quad (9)$$

$$\xi_f = \xi_{f1} = \xi_{f2} = \frac{C_f}{2(M_s + M_f)2\pi f_f}, \quad (10)$$

where f_{f1} and f_{f2} are the natural frequencies and ξ_{f1} and ξ_{f2} are the damping ratios in water.

With the non-rotating cylinder oscillating in a cylindrical water tank while modeling the water as incompressible and viscous, a new added mass, $\overline{M}_f$, and added damping, $\overline{C}_f$, can be expressed based on the work from Chen et al. [27,28] as

$$\overline{M}_f = L\rho_f\pi R_i^2 \left(\frac{\left(\frac{R_e}{R_i}\right)^2 + 1}{\left(\frac{R_e}{R_i}\right)^2 - 1} + \frac{4}{\sqrt{\pi \frac{(2R_i)^2 f}{\nu}}} \right), \quad (11)$$

$$\overline{C}_f = L2\pi\mu \sqrt{\pi \frac{(2R_i)^2 f}{\nu}} \frac{\frac{R_e}{R_i} \left(\left(\frac{R_e}{R_i}\right)^3 + 1 \right)}{\left(\left(\frac{R_e}{R_i}\right)^2 - 1 \right)^2}, \quad (12)$$

where L is the length of the cylinder, ρ_f is the density of the fluid, R_i is the radius of the cylinder, R_e is the radius of the cylindrical water tank, ν is the kinematic viscosity of the fluid, μ is the dynamic viscosity of the fluid, and f is the oscillating frequency of the cylinder. It must be noted that $\overline{M}_f$ and $\overline{C}_f$ differ from M_f and C_f because they are defined assuming a pure radial displacement of the cylinder rather than based on the displacement induced by Φ_{s1} .

3.4. Cylinder Rotating in Water

To the authors' knowledge, no dynamic equation for a rotating cylinder submerged in a cylindrical water tank has been published in the literature up to now. In order to better reproduce experimental observations of the dynamic system, the incorporation of two new matrices in the dynamic equation, which are composed of two new added modal coefficients, A_f and B_f , was considered. It must be noted that both matrices couple the two degrees of freedom of the system. A_f induces an added modal force proportional and perpendicular to the modal velocity. This added modal force may have its origin in the Magnus effect [21–25]. B_f induces another added modal force proportional and perpendicular to the modal displacement. Assuming that the structural damping is negligible compared to the fluid damping [26], the following dynamic equations are proposed:

$$\begin{bmatrix} M_s & 0 \\ 0 & M_s \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & K_s \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = F_f(t), \quad (13)$$

$$F_f(t) = - \begin{bmatrix} M_f & 0 \\ 0 & M_f \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} - \left(\begin{bmatrix} 0 & +A_f \\ -A_f & 0 \end{bmatrix} + \begin{bmatrix} C_f & 0 \\ 0 & C_f \end{bmatrix} \right) \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} - \left(\begin{bmatrix} 0 & +B_f \\ -B_f & 0 \end{bmatrix} + \begin{bmatrix} K_f & 0 \\ 0 & K_f \end{bmatrix} \right) \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}, \quad (14)$$

$$\begin{bmatrix} M_s + M_f & 0 \\ 0 & M_s + M_f \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \left(\begin{bmatrix} 0 & +A_f \\ -A_f & 0 \end{bmatrix} + \begin{bmatrix} C_f & 0 \\ 0 & C_f \end{bmatrix} \right) \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} + \left(\begin{bmatrix} 0 & +B_f \\ -B_f & 0 \end{bmatrix} + \begin{bmatrix} K_s + K_f & 0 \\ 0 & K_s + K_f \end{bmatrix} \right) \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = 0, \quad (15)$$

where K_f is the added modal stiffness.

Equation (15) can be transformed to complex coordinates defining r as

$$r = q_1 + jq_2, \quad (16)$$

and adding j times the second equation to the first equation:

$$(M_s + M_f)\ddot{r} - jA_f\dot{r} + C_f\dot{r} - jB_fr + (K_s + K_f)r = 0, \quad (17)$$

Assuming that the influence of C_f and B_f on the natural frequencies is negligible, then:

$$f_{FWf} = \frac{\frac{A_f}{2(M_s + M_f)} + \sqrt{\frac{A_f^2}{4(M_s + M_f)^2} + \frac{K_s + K_f}{(M_s + M_f)}}}{2\pi}, \quad (18)$$

$$\begin{aligned} \zeta_{FWf} &= \frac{C_f}{2(M_s + M_f)2\pi f_{FWf}} \\ &- \frac{1}{\sqrt{22}\pi f_{FWf}} \sqrt{\sqrt{\left(\frac{C_f^2}{4(M_s + M_f)^2} - \left(\frac{A_f^2}{4(M_s + M_f)^2} + \frac{K_s + K_f}{(M_s + M_f)}\right)\right)^2 + \left(-\frac{A_f C_f}{2(M_s + M_f)^2} + \frac{B_f}{(M_s + M_f)}\right)^2}} \\ &\quad + \left(\frac{C_f^2}{4(M_s + M_f)^2} - \left(\frac{A_f^2}{4(M_s + M_f)^2} + \frac{K_s + K_f}{(M_s + M_f)}\right)\right), \end{aligned} \quad (19)$$

$$f_{BWf} = \frac{\frac{A_f}{2(M_s + M_f)} - \sqrt{\frac{A_f^2}{4(M_s + M_f)^2} + \frac{K_s + K_f}{(M_s + M_f)}}}{2\pi}, \quad (20)$$

$$\begin{aligned} \zeta_{BWf} &= \frac{C_f}{2(M_s + M_f)2\pi f_{BWf}} \\ &+ \frac{1}{\sqrt{22}\pi f_{BWf}} \sqrt{\sqrt{\left(\frac{C_f^2}{4(M_s + M_f)^2} - \left(\frac{A_f^2}{4(M_s + M_f)^2} + \frac{K_s + K_f}{(M_s + M_f)}\right)\right)^2 + \left(-\frac{A_f C_f}{2(M_s + M_f)^2} + \frac{B_f}{(M_s + M_f)}\right)^2}} \\ &\quad + \left(\frac{C_f^2}{4(M_s + M_f)^2} - \left(\frac{A_f^2}{4(M_s + M_f)^2} + \frac{K_s + K_f}{(M_s + M_f)}\right)\right), \end{aligned} \quad (21)$$

From Equations (18) and (20), it must be noted that f_{FWf} is always higher than f_{BWf} .

Using complex coordinates, the natural frequencies can be positive or negative. A positive natural frequency means that the corresponding mode whirls in the direction of the rotation, and a negative one means that it whirls in the opposite direction to the rotation. Like in real coordinates, positive and negative damping ratios mean stable and unstable motion, respectively.

Assuming that A_f is associated with the Magnus effect, a new $\overline{A}_f$ can be expressed based on the work from the authors of [21–23], who defined the Magnus force as

$$F_M = 2\pi R_i^2 \rho_f L (2\pi\Omega) V, \quad (22)$$

where F_M is the Magnus force exerted on a non-oscillating rotating cylinder located in a flow with velocity V . From Equation (15), it can be seen that A_f induces a force, F_{A_f} , defined as

$$F_{A_f} = A_f \dot{q}_1, \quad (23)$$

Assuming that V is equivalent to $\dot{q}_1$ and that F_M is equivalent to F_{A_f} , the new $\overline{A}_f$ can be expressed as

$$\overline{A}_f = 2\pi R_i^2 \rho_f L (2\pi\Omega), \quad (24)$$

It must be noted that $\overline{A}_f$ differs from A_f because it is defined based on the equation of the Magnus effect, which assumes that (i) the cylinder does not oscillate and is submerged in a water flow, i.e., a pure radial displacement of the cylinder; (ii) the cylinder is not confined in a cylindrical water tank; and (iii) the flow velocity is constant rather than oscillating due to the whirling motion.

Due to the complexity of the flow, no analytical equations can be developed for C_f , K_f , and B_f for the case of the cylinder rotating in water.

4. Experimental Methodology

During steady conditions, the values of M_f , C_f , A_f , K_f , and B_f were calculated indirectly from measured natural frequencies and damping ratios together with the proposed mathematical models in order to minimize the interaction with the dynamic system and to simplify the experimental setup. The natural frequencies and damping ratios of the cylinder rotating and not rotating in air and water were first excited through a push release and determined via a spectral analysis. Subsequently, the added modal coefficients were calculated by means of the mathematical models using the measured values of the natural frequencies and damping ratios. The steady tests were carried out at Ω values of 0.00, 0.83, and 1.25 Hz in air and between 0.00 and 6.25 Hz in steps of approximately 0.20 Hz in water. In order to calculate averaged results, each test was repeated three times.

During transient conditions, ramps in Ω with different accelerations were performed. In this case, the exact values of the added coefficients were not calculated but were assessed qualitatively based on the values of the natural frequencies and damping ratios excited by the system's mechanical unbalance and determined via a spectral analysis. Each test was also repeated three times in order to calculate averaged results.

4.1. Measurement of Natural Frequencies, Damping Ratios, and Whirling Directions during Steady Conditions

Natural frequencies were excited through a push release with a manual push applied to a slim shaft and its subsequent release, leaving the entire structure to oscillate freely. During all tests, with the cylinder not rotating and rotating in air and water, the cylinder was displaced from its equilibrium position by around 1 mm at the midplane by the push action (see Figure 3).

The values of the natural frequencies were identified from the averaged spectra of the cylinder displacement responses to the push release, calculated by means of a Fast Fourier Transform (FFT) with a Hanning window. All the signals had a length of 100 s and were acquired with a sampling rate of 800 Hz, resulting in a frequency resolution of 0.01 Hz in the spectra (see Figure 4). Based on Equations (18) and (20), the peak at a lower frequency has to correspond to the f_{BWf} and that at a higher frequency has to correspond to the f_{FWf} .

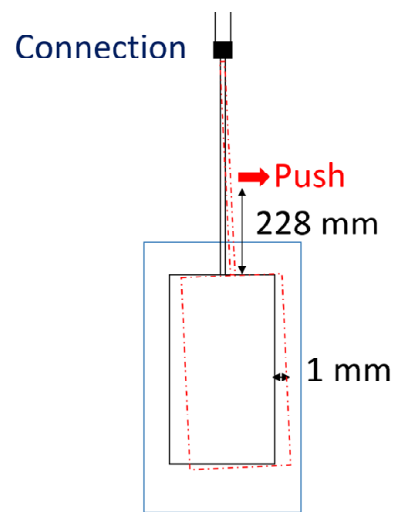


Figure 3. Scheme of the push release.

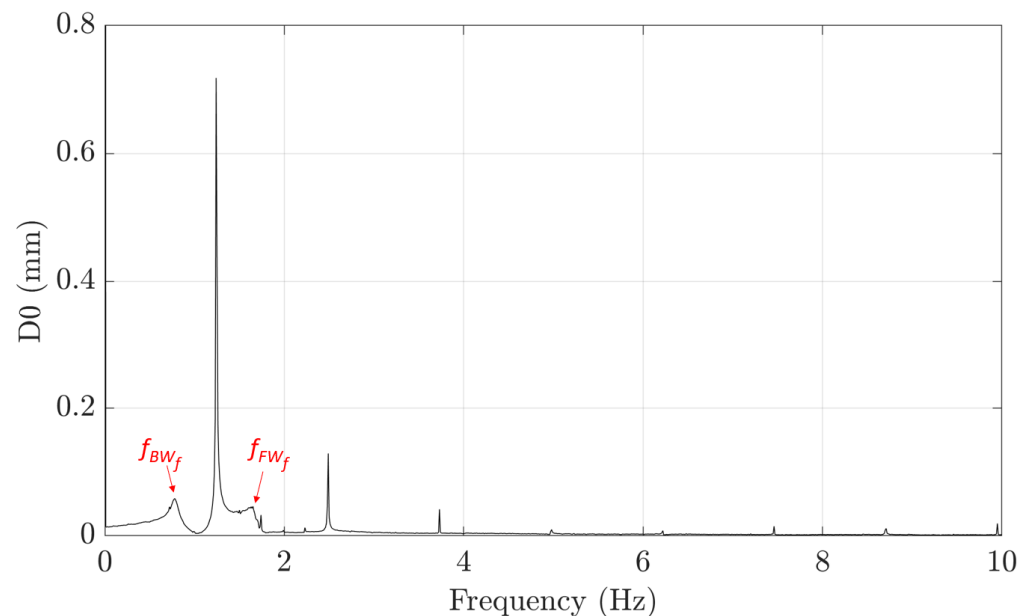


Figure 4. Spectrum of the D0 displacement showing the f_{FWf} and f_{BWf} when the cylinder rotates at 1.25 Hz in water.

The damping ratios were calculated using a curve-fitting algorithm applied to the displacement time signals induced by the push release and bandpass-filtered (see Figure 5). The selected filter was an infinite impulse response filter of the Bessel type with one order. The center of the filter was adjusted to the value of the natural frequency and the band limits were set 0.25 Hz above and 0.25 Hz below the natural frequency. The curve-fitting algorithm is programmed to find the best fit for the filtered experimental signal based on Equation (25), which represents an ideal free decay motion. From the fitted curve, the value of the damping ratio, ζ_{Xf} , can be extracted given the known value of $d0_0$, which corresponds to the initial displacement; the value of $\dot{d}0_0$, which corresponds to the initial velocity; and the value of f_{Xf} , which corresponds to the natural frequency.

$$d0(t) = \left(d0_0 \cos(2\pi f_{Xf} t) + \left[\frac{\dot{d}0_0}{2\pi f_{Xf}} + \frac{d0_0 \zeta_{Xf}}{\sqrt{1 - \zeta_{Xf}^2}} \right] \sin(2\pi f_{Xf} t) \right) e^{-\zeta_{Xf} 2\pi f_{Xf} t}, \quad (25)$$

where $d0(t)$ is the temporal evolution of the amplitude.

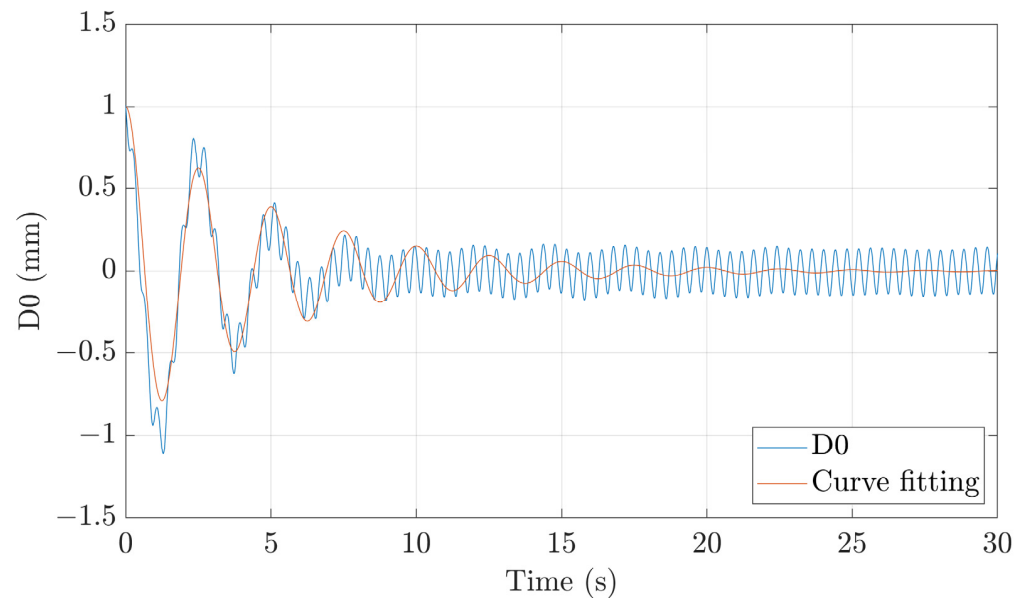


Figure 5. D0 filtered signal (blue) and curve-fitting signal (orange) of the BW when the cylinder rotates at 2.5 Hz in water.

The direction of the whirling motion, corresponding to f_{FWf} and f_{BWf} , was identified from an orbit graph, which was plotted using the bandpass-filtered D0 and D90 displacement signals. More specifically, the center of the filter was adjusted to f_{FWf} or f_{BWf} and the band limits were set 0.25 Hz above and 0.25 Hz below the natural frequency. The filtered D0 and D90 signals were plotted on the horizontal and vertical axes, respectively, of a polar graph, as shown in Figure 6.

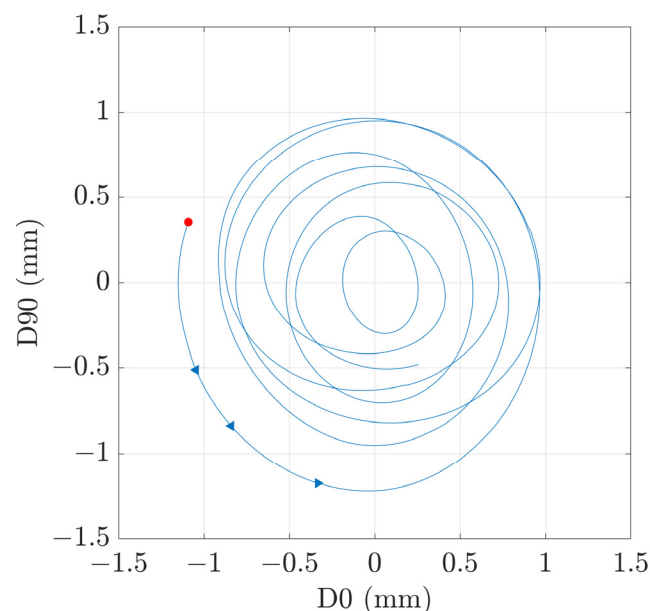


Figure 6. Orbit plot corresponding to the FW when the cylinder rotates at 1.25 Hz in water. The red dot indicates the beginning of the orbit.

4.2. Fluid-Added Modal Coefficient Calculation during Steady Conditions

M_S was calculated using Equation (4) and using the Φ_{s1} of the non-rotating cylinder in air, which was computed by means of a numerical structural modal analysis. Only the

cylinder and the slim shaft were included in the numerical model since the eigenmodes of interest only affected these two parts of the test rig. This could be confirmed by the fact that the top of the slim shaft presented zero displacement, as shown in Figure 7, which in turn confirmed that the system's dynamic response was isolated from the non-linearities of the bearings. It must be noted that a slim shaft was designed specifically for this purpose and that, in addition, it resulted in lower natural frequencies, which were also more convenient for carrying out the tests. Both the cylinder and the slim shaft were made of a standard stainless steel and modeled using tetrahedral elements (SOLID185). A mesh refinement process was carried out to determine the optimal mesh size that gives an accurate solution with a low computational cost based on the evolution of f_s with the number of mesh nodes (see Table 1). The selected mesh had a total of 359,954 nodes. Figure 7 presents the numerical mode shapes, Φ_{s1} and Φ_{s2} , of the cylinder before unity normalization.

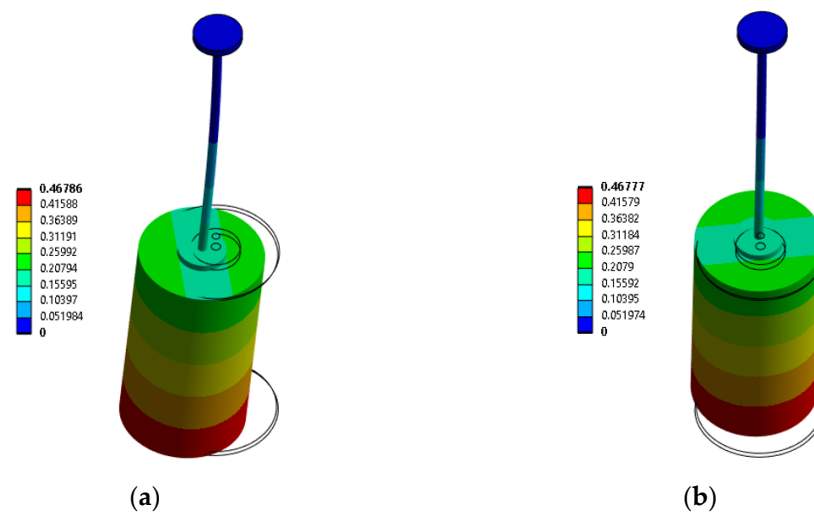


Figure 7. Numerical mode shapes of the cylinder in air: Φ_{s1} (a) and Φ_{s2} (b).

Table 1. Mesh sensitivity study of the numerical structural modal analysis with Δf_s (%) indicating a reduction in the natural frequency relative to that calculated using the previous mesh.

Node Number (-)	88,073	135,514	359,954	567,761	1,254,242	2,771,720	3,427,578
f_s (Hz)	3.461	3.385	3.303	3.300	3.299	3.298	3.298
Δf_s (%)		−2.20	−2.42	−0.09	−0.03	−0.03	0

Once M_s was calculated, first, K_s and C_s were computed using Equations (2) and (3), respectively, based on M_s and the measured f_s and ξ_s . Second, M_f was calculated with Equation (9) and C_f was calculated with Equation (10) based on the measured f_f and ξ_f . Finally, A_f and K_f were calculated with Equations (18) and (20), and C_f and B_f were calculated with Equations (19) and (21) based on the measured f_{FWf} , f_{BWf} , ξ_{FWf} , and ξ_{BWf} values, assuming that M_f does not depend on the flow surrounding the structure [26,29–31], i.e., on the flow induced by the rotation, and that M_f , C_f , A_f , K_f , and B_f are independent of the natural frequency value.

4.3. Fluid-Added Modal Coefficient Assessment during Transient Conditions

Upward and downward ramps of Ω at different accelerations were performed in order to investigate the added modal coefficients under transient conditions. For that, the system was not excited using a push release but was excited with its own mechanical unbalance. Tests were carried out for Ω values from 0.85 to 2.50 Hz with durations of 1.0, 2.5, 5.0, 10.0, 50.0, and 100.0 s and for Ω values from 0.00 to 6.25 Hz with a duration of 375 s. The frequency content and amplitudes of the ramp signals were calculated with a Short-Time

Fourier Transform (STFT). All the values composing the waterfall spectra in each frequency bin, as shown in Figure 8, were summed to construct a single overall maximum spectrum, P_{max} , with a resolution of 0.016 Hz, as shown in Figure 8b, using a Hanning window with an FFT block size of 50,000 points and a sliding time segment with a 5% time increment between two consecutive FFT blocks.

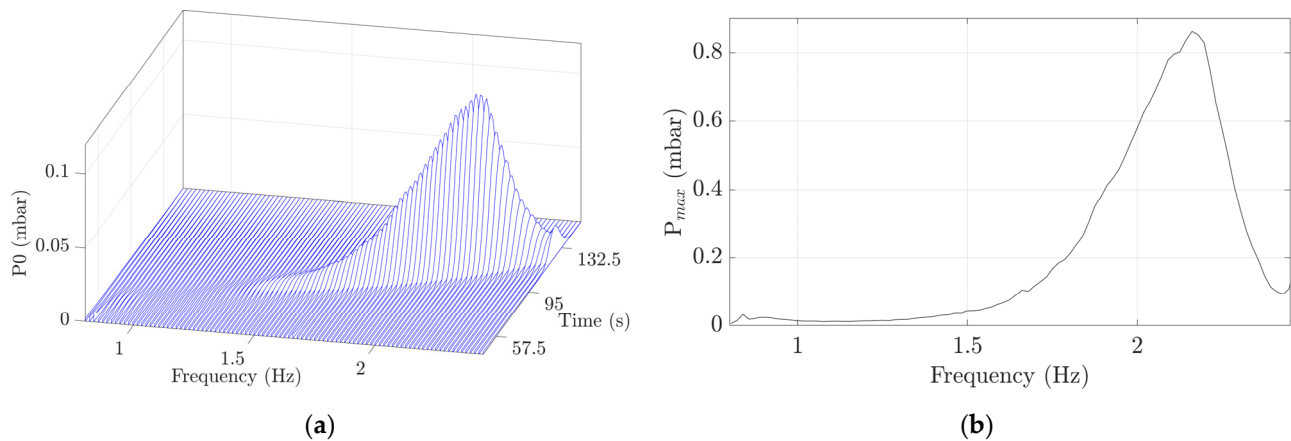


Figure 8. Waterfall obtained from the STFT analysis of a ramp-up test with a duration of 100 s (a). Maximum spectrum of the STFT (b).

5. Results

5.1. Natural Frequencies and Damping Ratios of the Cylinder in Air and Water during Steady Conditions

Figure 9 shows f_s and f_f as well as the evolution of f_{FWs} , f_{BW_s} , f_{FWf} , and f_{BWf} as a function of Ω . When the cylinder does not rotate, f_s is 3.1 Hz and f_f is 1.2 Hz, presenting a frequency reduction ratio (FRR) of 61.3% due to M_f .

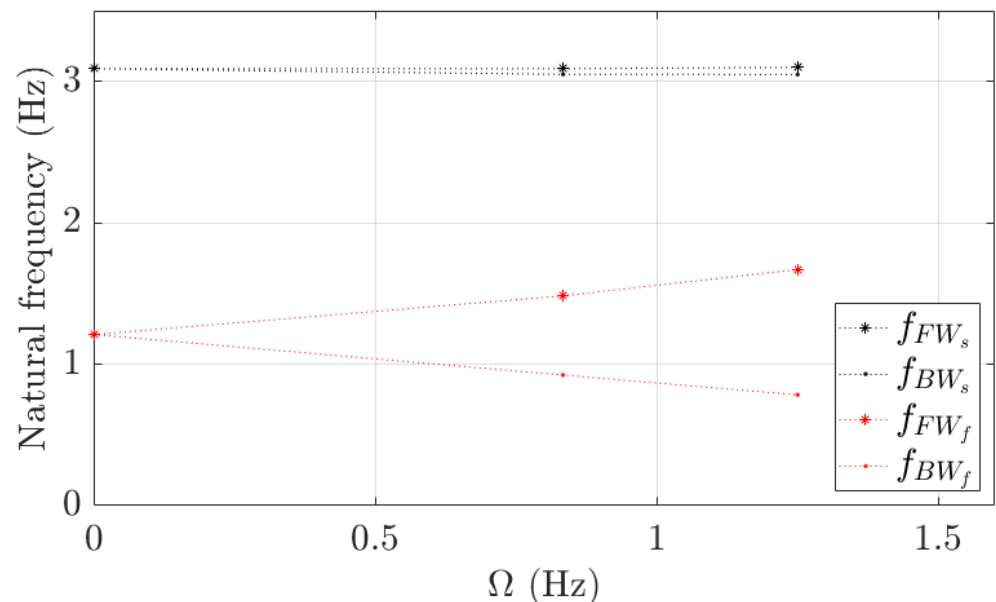


Figure 9. Natural frequencies in air (black lines) and in water (red lines) as a function of Ω .

When the cylinder rotates in air, f_{FWs} and f_{BW_s} are approximately equal to each other and f_s , which agrees with the assumption noted in Section 3.2. Nonetheless, f_{FWf} and f_{BWf} change in water depending on Ω . f_{FWf} increases and f_{BWf} decreases as Ω is increased, with f_{FWf} equal to 1.67 Hz and f_{BWf} equal to 0.78 Hz at an Ω value equal to 1.25 Hz. These results suggest that there are more forces than those due to the gyroscopic effect

inducing the frequency shift when the cylinder is in water compared to air. Based on Equations (18) and (20), these forces have to be induced mainly by A_f , which must have an effect significantly stronger than the gyroscopic effect.

Figure 10 shows the temporal response of the displacement time signals to a push release and the curve fitting in air and water when the cylinder does not rotate. ζ_s is 0.0014 and ζ_f is 0.02, thus presenting an increase of about 1330% from air to water. It must be noted that the R-square values of the curve fittings are 0.85 in air and 0.83 in water. This result agrees with previous investigations concluding that the structural damping is negligible compared to the viscous damping associated with the fluid [26].

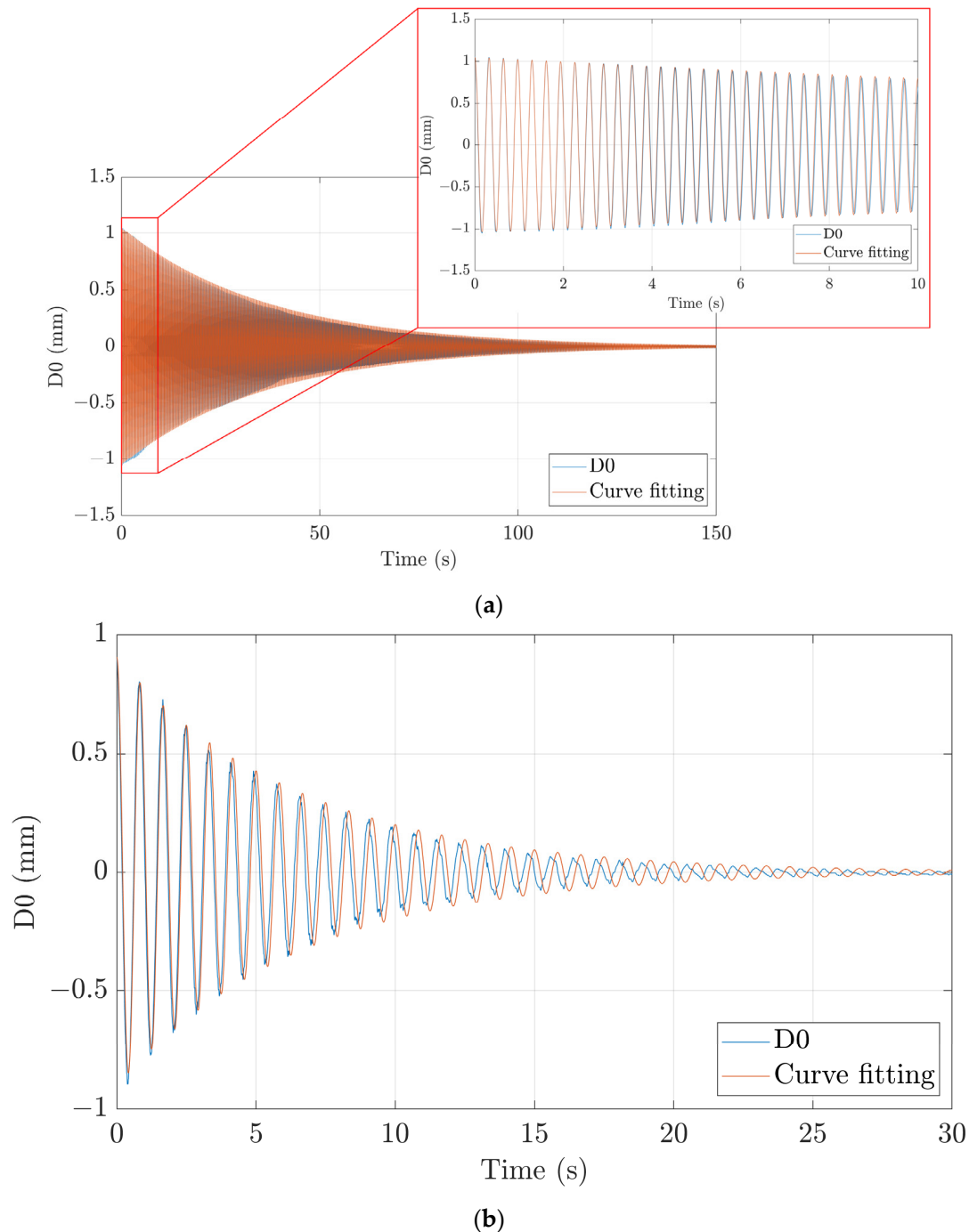


Figure 10. D_0 filtered signal (blue) and curve-fitting signal (orange) of the response of q_1 to a push release in air (a) and water (b).

5.2. Natural Frequencies, Damping Ratios, and Whirling Directions of the Cylinder in Water during Steady Conditions

Figure 11 shows f_f and Ω as well as the evolutions of f_{FWf} and f_{BWf} as a function of Ω . f_{FWf} always increases, whereas f_{BWf} decreases with Ω from 0 to 4.5 Hz, reaches a value of 0 Hz for an Ω equal to 4.5 Hz, and begins to increase as Ω is further increased. Based on Equations (18) and (20), when f_{BWf} decreases it implies that the structural modal stiffness is higher than the absolute added modal stiffness ($K_s > |K_f|$), when f_{BWf} is 0 Hz it implies that both the structural and absolute added modal stiffness are equal ($K_s = |K_f|$), and when f_{BWf} increases it implies that the structural modal stiffness is lower than the absolute added modal stiffness ($K_s < |K_f|$).

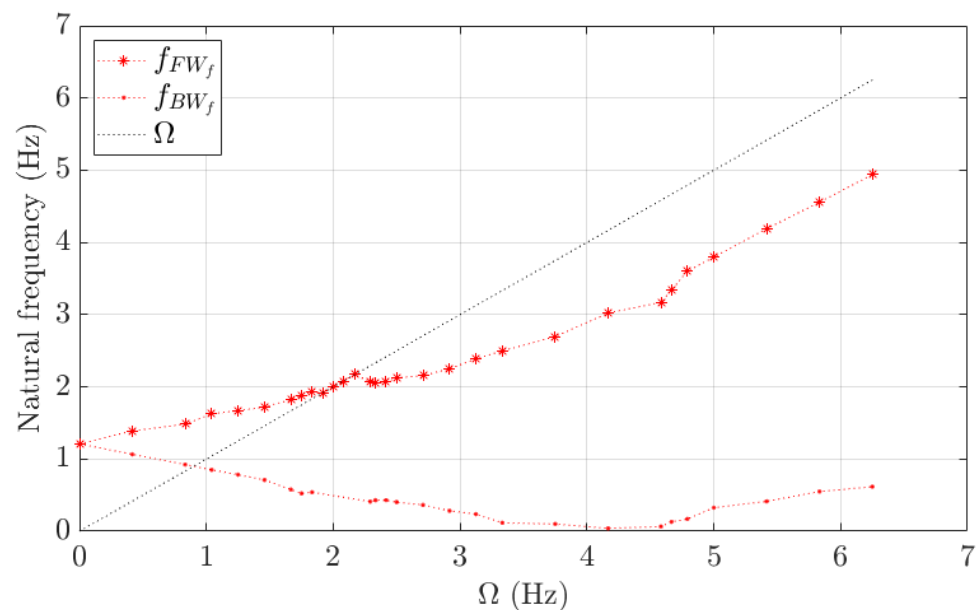


Figure 11. Natural frequencies of FW and BW in water as well as Ω as a function of Ω .

f_{BWf} evolves continuously at $\Omega < 4.5$ Hz, whereas f_{FWf} presents a discontinuity for Ω from 1.8 to 2.3 Hz due to a lock-in between f_{FWf} and Ω . f_{FWf} presents a constant value for Ω from 1.8 to 1.9 Hz; increases with Ω from 1.9 to 2.16 Hz, during the lock-in; drops and presents a constant value for Ω from 2.16 to 2.3 Hz, when the lock-in is overcome; and increases continuously again at higher Ω values. It must be noted that at 0.83 Hz, the f_{BWf} does not lock in with Ω because, in contrast to FW, BW whirles in the opposite direction to the rotation. In this case, the lock-in is governed by the mechanical unbalance and f_{FWf} takes the value of Ω . During the lock-in, f_{BWf} is not excited when performing the push release, leading to a structure resonating only with the FW.

Figure 12 presents the temporal responses of the cylinder displacement due to the push release and the curve fitting. Figure 13 shows ζ_f as well as the evolutions of ζ_{BWf} and ζ_{FWf} as a function of Ω . ζ_{BWf} increases from 0.02 up to 0.23, representing an increase of around 1000% when Ω is also increased from 0 to 5 Hz. ζ_{FWf} was only calculated up to an Ω of 1.25 Hz, and based on the current measurements, it has not been possible to determine its trend. However, ζ_{BWf} can be assumed to always be higher than ζ_{FWf} , which agrees with Equations (19) and (21). It must be noted that the R-square of the curve fitting at an Ω equal to 0.42 Hz is 0.76 for the FW and 0.79 for the BW, at an Ω equal to 1.25 Hz, it is 0.21 for the FW and 0.35 for the BW; and at Ω values equal to 2.5 and 5 Hz, it is 0.64 and 0.98 for the BW. The R-square value decreases when the natural frequencies are close to the rotating frequency of the cylinder since its contribution cannot be removed from the signal

through the filter. Although the exact values of the added damping present uncertainty, at least we can believe its calculated trend as a function of the rotation.

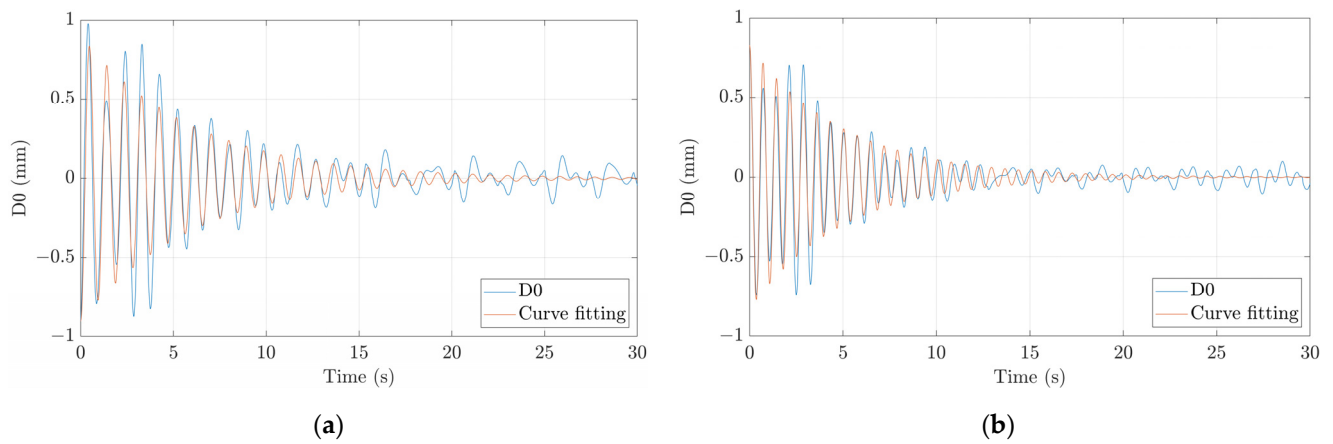


Figure 12. D0 filtered signal (blue) and curve-fitting signal (orange) of the response of the BW (a) and FW (b) to push release when the cylinder rotates at 0.42 Hz.

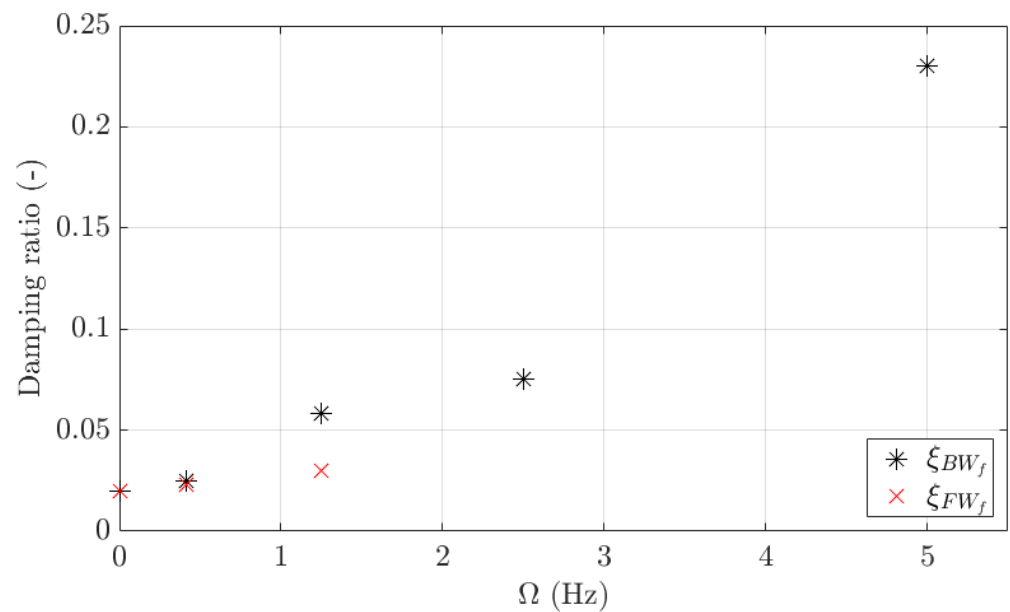


Figure 13. ξ_{BWf} (in black) and ξ_{FWf} (in red) as a function of Ω .

Figure 14 presents some orbits corresponding to the FW and BW modes. The FW mode whirls in the same direction as the rotation and the BW mode whirls in the opposite direction to the rotation for Ω values from 0 to 4.5 Hz. At higher Ω values, both the FW and BW modes whirl in the same direction as the rotation. Based on Equation (20), this change in the BW whirling direction must imply that $K_s < |K_f|$.

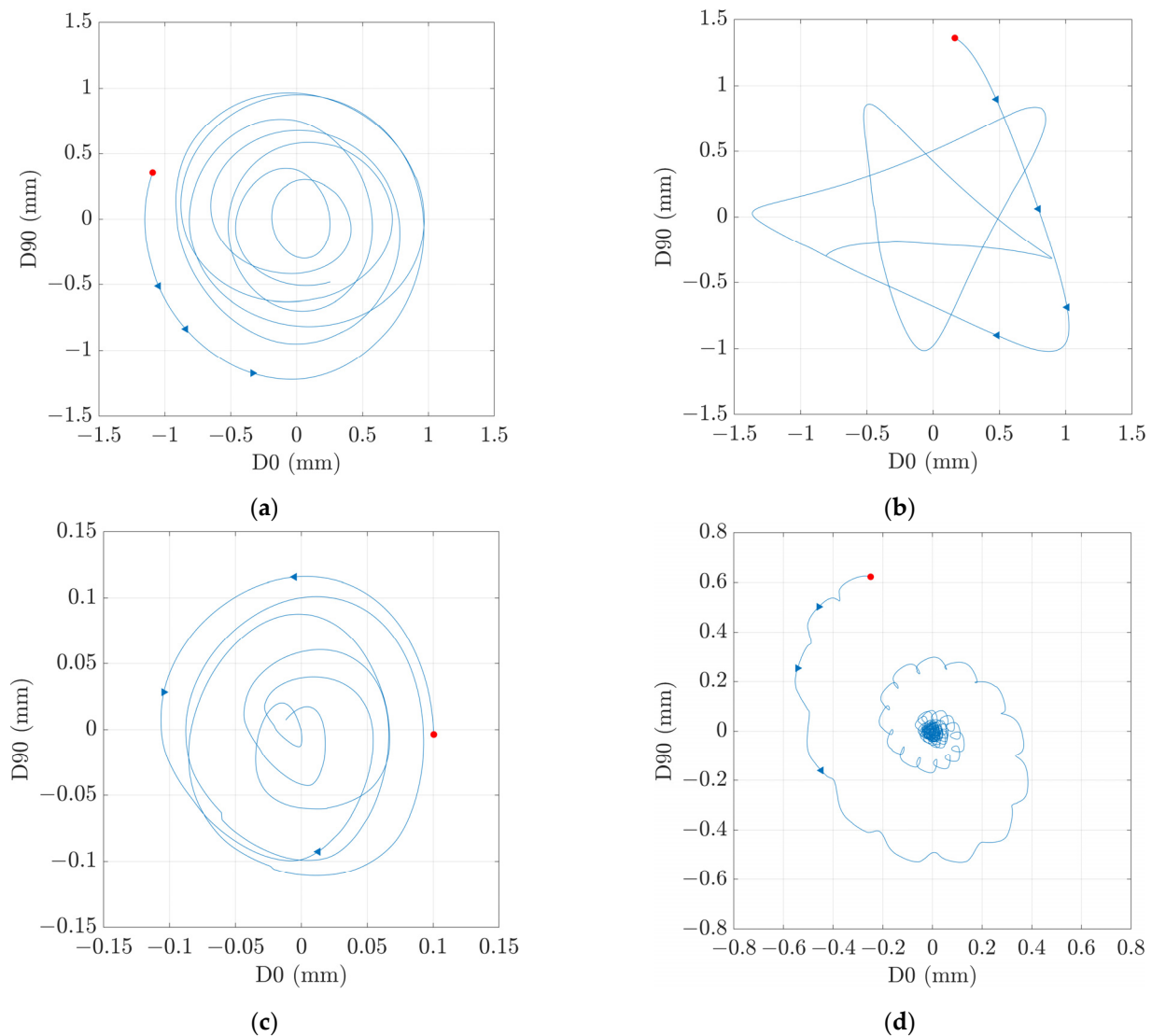


Figure 14. Orbits corresponding to the FW (a) and BW (b) at Ω of 1.25 and to the FW (c) and BW (d) at Ω of 5.4 Hz. The red dot indicates the beginning of the orbit.

5.3. Fluid-Added Coefficients during Steady Conditions

Table 2 presents the values of M_f , C_f , A_f , K_f , and B_f , when the non-rotating cylinder is oscillating at 1.2 Hz according to Φ_{s1} in water as well as those of $\overline{M}_f$ [27,28], $\overline{C}_f$ [27,28], and $\overline{A}_f$ [21–23], calculated using equations extracted from the literature. A_f , $\overline{A}_f$, K_f , and B_f are zero, whereas M_f and $\overline{M}_f$ are 13.1 and 38 kg, respectively, and C_f and $\overline{C}_f$ are 4.23 and 11.9 Ns/m, respectively. The deviations between M_f and $\overline{M}_f$ and between C_f and $\overline{C}_f$ are 190% and 181%, respectively. $\overline{M}_f$ and $\overline{C}_f$ are higher than M_f and C_f because Equations (11) and (12) consider that the cylinder moves radially. However, as presented in Figure 7, Φ_{s1} reassembles a pendulum movement. At the top of the cylinder, the fluid presents lower acceleration and velocity than at the bottom, resulting in lower M_f and C_f values.

Table 2. Values of M_f , $\overline{M}_f$, C_f , $\overline{C}_f$, A_f , $\overline{A}_f$, K_f , and B_f when the cylinder is not rotating and it is oscillating at 1.2 Hz in water.

M_f (kg)	$\overline{M}_f$ (kg)	C_f (Ns/m)	$\overline{C}_f$ (Ns/m)	A_f (Ns/m)	$\overline{A}_f$ (Ns/m)	K_f (N/m)	B_f (N/m)
13.1	38	4.23	11.9	0	0	0	0

Figure 15a presents constant linear increases in A_f and $\overline{A_f}$ with Ω . Equation (24) considers that the cylinder moves radially rather than as a pendulum, which should result in higher values of $\overline{A_f}$ than A_f . Nonetheless, Equation (24) does not consider the boundaries of the cylindrical water tank, which should result in lower values of $\overline{A_f}$ than A_f , as the boundaries are expected to cause higher flow velocities. In this specific case, both simplifications compensate for each other, leaving Equation (24) predicting A_f accurately. The minimum deviation between both values occurs for an Ω of 5 Hz and is 0.5%, and the maximum deviation occurs for an Ω of 2.7 Hz and is 24% with an average deviation of 11%. However, it must be noted that the evolution of A_f with Ω agrees with the results of the Magnus effect obtained in previous studies [21–23].

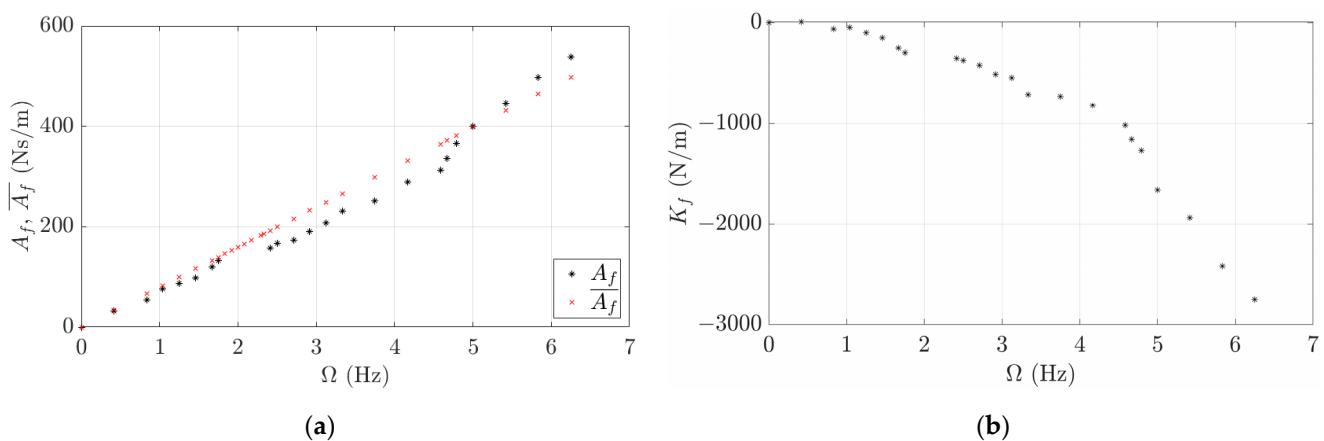


Figure 15. Evolution of A_f (in black) and $\overline{A_f}$ (in red) (a) and K_f (b) as a function of Ω .

Figure 15b plots K_f and shows that it is negative and decreases quadratically with Ω . A negative value of K_f is expected since an eccentricity of the rotating cylinder implies a smaller fluid thickness on one side of the cylinder. In this smaller gap, there are higher velocities and consequently lower pressures, resulting in a force towards the smaller fluid thickness, which pushes the cylinder away from its equilibrium position. The evolution of K_f agrees with previous studies concluding that an increase in Ω results in an increase in the forces exerted on a cylinder [19,20].

Table 3 presents C_f and B_f at different Ω values. C_f increases with Ω , which is in line with previous studies concluding that C_f increases with the flow velocity [32–34], i.e., the flow induced by the rotation. Based on the present measurements, it can only be concluded that B_f is lower for lower Ω values. Assuming that C_f and B_f increase with Ω , the FW could become unstable if $\frac{A_f^2}{4(M_s+M_f)^2} + \frac{K_s+K_f}{(M_s+M_f)} < \frac{B_f^2}{C_f^2} + \frac{A_f^2}{4(M_s+M_f)^2} - \frac{B_f A_f}{C_f(M_s+M_f)}$ (see Equation (19)). Consequently, the stability of the FW depends on the values of the added modal coefficients, which in turn depend on Ω . The FW may be unstable at some specific Ω value and stable at others. In contrast, the BW is always stable.

Table 3. C_f and B_f when the cylinder is rotating in water.

Ω (Hz)	C_f (Ns/m)	B_f (N/m)
0	4.23	0
0.4	5.45	5.76
1.25	9.26	24

Based on the presented experimental methodology, it can be summarized that (i) C_f and A_f are positive and increase, (ii) K_f is negative and decreases, and (iii) K_f decreases quadratically relative to the increase in A_f as Ω is increased. Additionally, it must be noted

that the effect of radial confinement must influence all the added modal coefficients since it causes higher flow accelerations and velocities and consequently higher values of M_f as well as C_f , A_f , K_f , and B_f , respectively.

5.4. Lock-in between the FW Natural Frequency and the Rotating Frequency during Transient Conditions

Figure 16 shows a colormap of the P0 pressure signal during an upward ramp from 0 to 6.25 Hz and a downward ramp from 6.25 to 0 Hz with a duration of 375 s for each of them. During both ramps, P0 presents an increase in amplitude between 1.8 and 2.3 Hz due to the lock-in. It must be noted that the lock-in resonance presents a second harmonic of lower amplitude. Additionally, the P0 presents another increase in the amplitude of f_{FWf} from 3 to 3.5 Hz that could be related to a decrease in ζ_{FWf} .

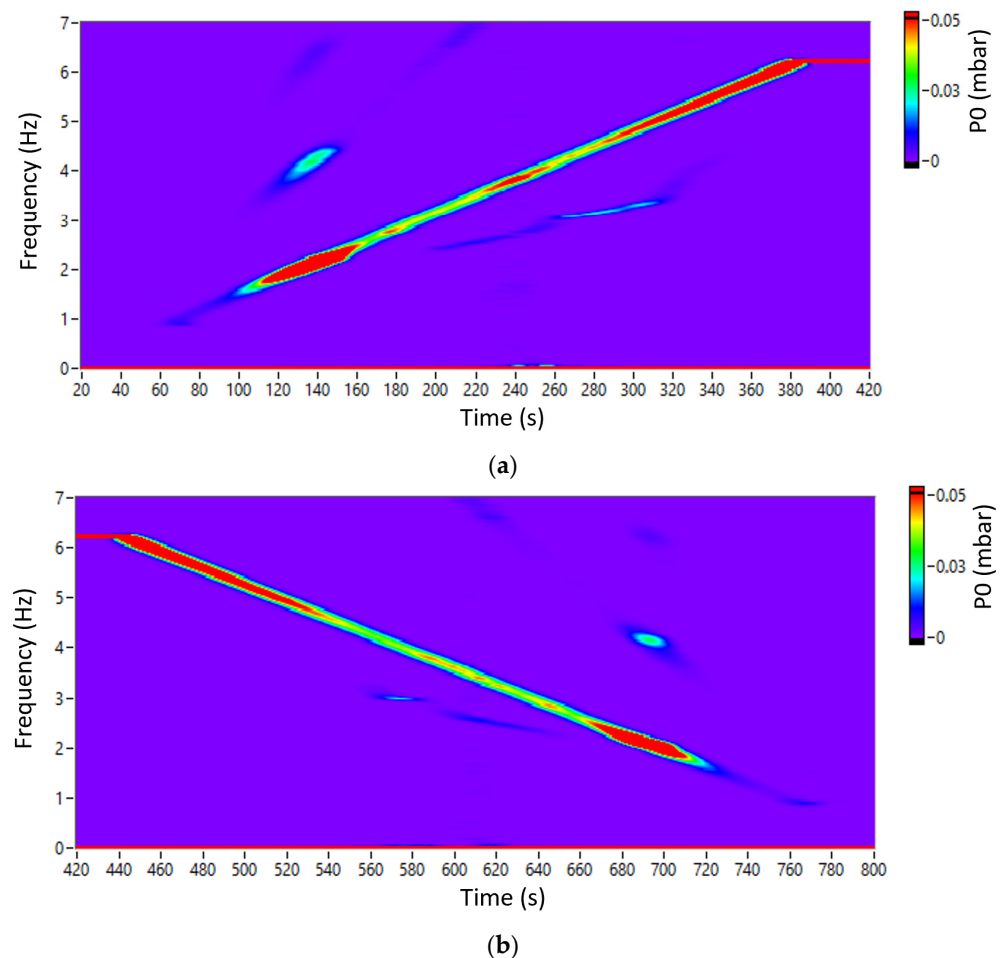


Figure 16. Colormap of the P0 signal during an upward ramp (a) and a downward ramp (b) of the rotating frequency of the cylinder, accelerating from 0 to 6.25 Hz with a duration of 375 s.

Figure 17 presents the P_{max} values of P0 during ramps performed at different accelerations, with an enlargement of the frequency range from 1 to 2.4 Hz in order to visualize the lock-in amplitude in detail. The lock-in amplitude of the 1 s ramp is 180 times lower than that of the 100 s ramp, suggesting that the lock-in requires a certain amount of time to develop. The lock-in peaks during ramps with higher accelerations are wider than those observed during ramps with lower accelerations, indicating that an increase in the ramp acceleration may also imply an increase in the damping ratio. The reduction in lock-in amplitudes observed with higher accelerations could therefore occur due to both of the following conditions: (i) the lock-in is not fully developed and (ii) the damping ratio of the system is higher.

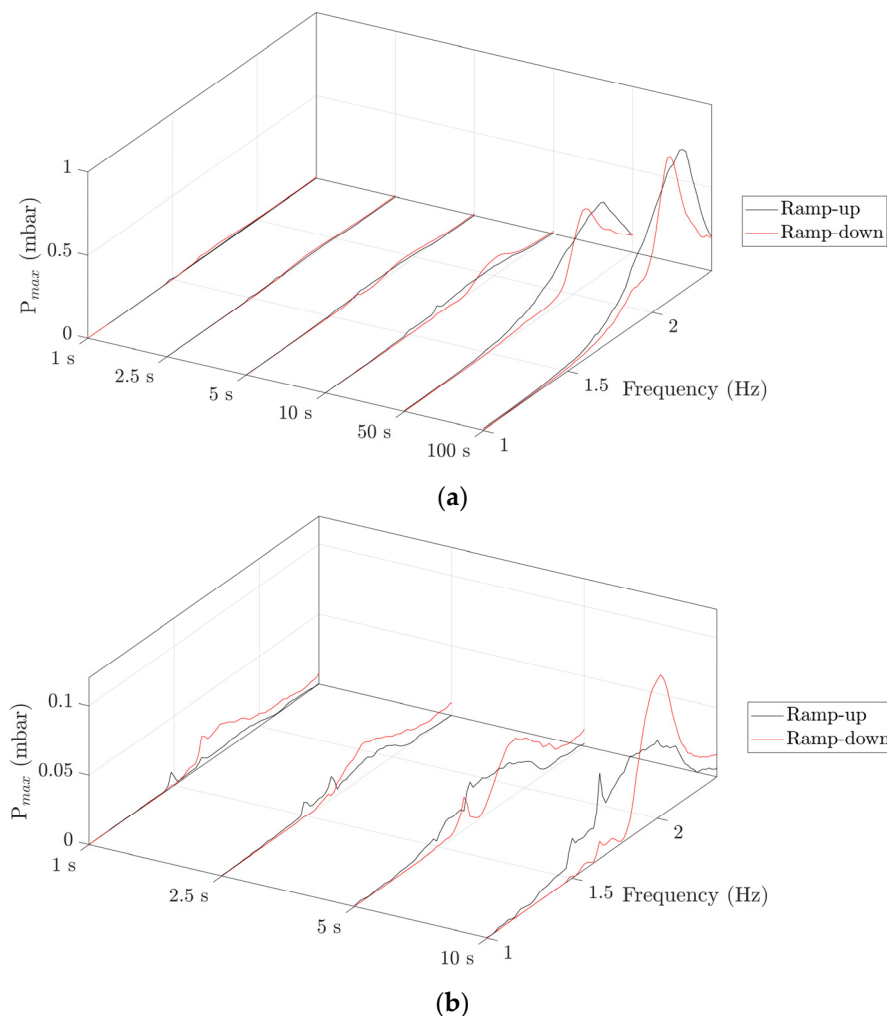


Figure 17. Spectra of upward ramps (in black) and downward ramps (in red) performed with durations of 1, 2.5, 10, 50 and 100 s (a) and with durations of 1, 2.5, 5 and 10 s (b) with an enlargement of the frequency range from 1 to 2.4 Hz.

For high accelerations, the lock-in amplitudes during downward ramps are higher than those during upward ramps. For instance, during a 1 s ramp, the lock-in amplitude during a downward ramp is around four times higher than that during an upward ramp. The lock-in amplitude is damped more slowly during a downward ramp because after the resonance the mechanical unbalance decreases in intensity as the rotating frequency also decreases and it cannot govern the flow and the cylinder, which continue whirling together at the lock-in frequency. On the other hand, during an upward ramp, after the resonance, the mechanical unbalance increases in intensity as the rotating frequency also increases and it governs the flow and the cylinder, which is forced to whirl at higher frequencies away from the lock-in frequency, resulting in lower amplitudes. At low accelerations, the transient case resembles a stationary case and both upward and downward ramps result in similar lock-in amplitudes.

Figure 18 shows the P_{max} values of P0, normalized by the lock-in amplitude, during ramps performed at different accelerations, with an enlargement of the frequency range of 1 to 2.4 Hz in order to visualize the exact lock-in frequency in detail. During ramps of 1 s, plotted in blue, the lock-in frequencies are 1.77 Hz for downward ramps and 1.87 Hz for upward ramps. However, during ramps of 100 s, plotted in black, the lock-in frequencies are 2.12 Hz for downward ramps and 2.18 Hz for upward ramps. This represents lock-in frequency increases of 19.8% for downward ramps and 16.6% for upward ramps. The decrease in the ramp acceleration results in a continuous increase in the lock-in frequency.

In both upward and downward ramps, the acceleration may alter the modal coefficients of the system and consequently f_{FWf} . Higher ramp accelerations imply a lower f_{FWf} and consequently a lower lock-in frequency due to an increase in the M_f and/or a decrease in A_f and/or K_f .

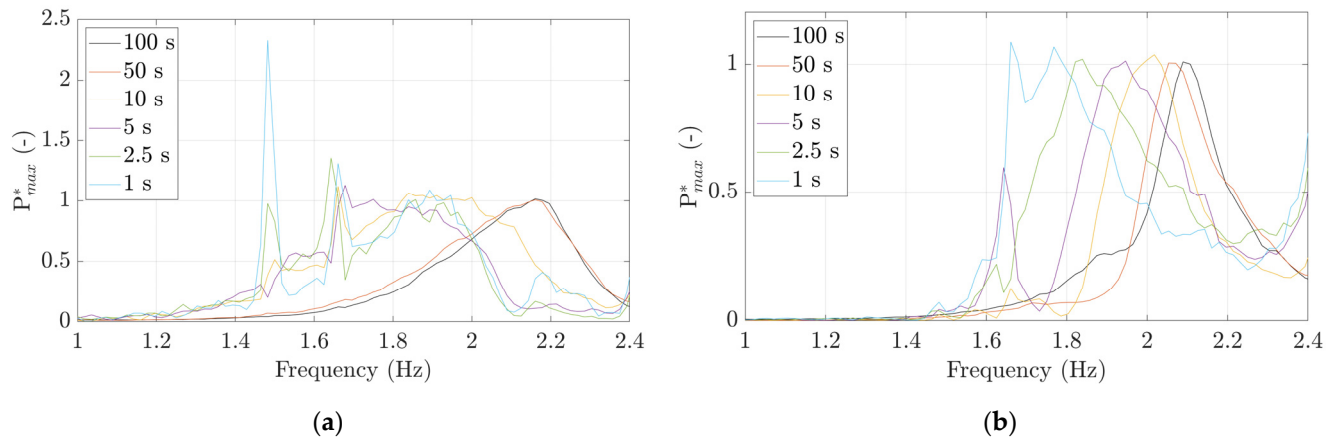


Figure 18. Spectra of upward ramps (a) and downward ramps (b), normalized by the maximum lock-in amplitude, performed with durations of 1, 2.5, 10, 50, and 100 s with an enlargement of the frequency range from 1 to 2.4 Hz.

6. Conclusions

A test rig was designed to investigate the added modal coefficients of a rotating submerged cylinder with a whirling motion, based on the values of measured natural frequencies and damping ratios as well as a proposed two-degree-of-freedom mathematical model of the dynamic system.

Two new added modal coefficients, A_f and B_f , were incorporated into the mathematical model of the rotating cylinder in water to better reproduce experimental observations of the dynamic system. A_f induces a force proportional and perpendicular to the modal velocity and influences both the natural frequencies and damping ratios. B_f induces a force proportional and perpendicular to the modal displacement and mainly influences the damping ratios. Both A_f and B_f couple the two degrees of freedom of the system.

Under rotating conditions, the system presents the FW and BW modes as a result of the coupling of the two degrees of freedom. In air, their corresponding natural frequencies are independent of Ω and equal to each other with a value of 3.1 Hz. In water, the FW natural frequency increases from 1.2 to 4.9 Hz as Ω is increased from 0 to 6.25 Hz. Nonetheless, the BW natural frequency decreases from 1.2 to 0 Hz as Ω is increased from 0 to 4.5 Hz and then it increases from 0 to 0.6 Hz as Ω is further increased from 4.5 to 6.25 Hz. In water, the FW mode always whirls in the direction of Ω . However, the BW mode whirls opposite to Ω for values of Ω lower than 4.5 Hz and in the direction of Ω for values of Ω higher than 4.5 Hz. Both the changes in the BW natural frequency trend and the whirling direction are predicted by the proposed mathematical model. In water, the BW damping ratio increases from 0.02 to 0.23 as the value of Ω is also increased from 0 up to 5 Hz, whilst the trend of the FW damping ratio has not been determined. Based on the proposed mathematical model, all these trends of the natural frequencies and damping ratios imply that C_f and A_f must increase and K_f must decrease when Ω is increased.

During ramps in the rotating speed, the BW natural frequency does not lock in the rotating frequency, whereas the FW natural frequency does lock in. This is probably because the FW mode whirls in the cylinder's rotation direction and the BW mode whirls in the opposite direction. During these transient tests, higher accelerations of the rotating speed ramps resulted in 16 to 20% lower lock-in frequencies and 180 times lower lock-in amplitude. Additionally, for higher accelerations, the downward ramps resulted in lock-in amplitudes four times higher than the upward ramps. All these observations suggest

that the acceleration of Ω and the directions of the ramps significantly alter the added modal coefficients.

Author Contributions: Conceptualization, R.R., E.J. and X.E.; methodology, R.R. and X.E.; software, R.R. and X.S.-B.; validation, R.R.; formal analysis, R.R. and X.S.-B.; investigation, R.R.; resources, X.E.; writing—original draft preparation, R.R.; writing—review and editing, R.R., E.J. and X.E.; supervision, X.E.; project administration, X.E.; funding acquisition, X.E. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the European Union’s Horizon H2020 research and innovation program under grant agreement No. 814958.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Data sharing not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Trivedi, C. Investigations of Transient Pressure Loading on a High Head Francis Turbine. Ph.D. Thesis, Luleå Tekniska Universitet (LTU), Luleå, Sweden, 2015.
2. Trivedi, C.; Cervantes, M.J.; Gandhi, B.K.; Dahlhaug, O.G. Pressure measurements on a high-head Francis turbine during load acceptance and rejection. *J. Hydraul. Res.* **2014**, *52*, 283–297.
3. Trivedi, C.; Gandhi, B.K.; Cervantes, M.J. Effect of transients on Francis turbine runner life: A review. *J. Hydraul. Res.* **2013**, *51*, 121–132. [[CrossRef](#)]
4. Goyal, R.; Gandhi, B.K.; Cervantes, M.J. PIV measurements in Francis turbine—A review and application to transient operations. *Renew. Sustain. Energy Rev.* **2018**, *81*, 2976–2991. [[CrossRef](#)]
5. Roig, R.; Sánchez-Botello, X.; Escaler, X.; Mulu, B.; Höglström, C.-M. On the Rotating Vortex Rope and Its Induced Structural Response in a Kaplan Turbine Model. *Energies* **2022**, *15*, 6311. [[CrossRef](#)]
6. Luo, Y.; Presas, A.; Wang, Z. Operating conditions leading to crack propagation in turbine blades of tidal barrages. Influence of head and operating mode. *Eng. Fail. Anal.* **2020**, *108*, 104254. [[CrossRef](#)]
7. Ahn, S.-H.; Xiao, Y.; Wang, Z. Numerical prediction on the effect of free surface vortex on intake flow characteristics for tidal power station. *Renew. Energy* **2017**, *101*, 617–628. [[CrossRef](#)]
8. Ahn, S.-H.; Xiao, Y.; Wang, Z. Performance prediction of a prototype tidal power turbine by using a suitable numerical model. *Renew. Energy* **2017**, *113*, 293–302. [[CrossRef](#)]
9. Egusquiza, E.; Valero, C.; Huang, X. Failure investigation of a large pump-turbine runner. *Eng. Fail. Anal.* **2012**, *23*, 27–34. [[CrossRef](#)]
10. Jasper, N.H. A design approach to the problem of critical whirling speeds of shaft-disk systems. *Int. Shipbuild. Progr.* **1956**, *3*, 37–60. [[CrossRef](#)]
11. Benkö, G.B.; Holmén, E.K. Paper 3: Parametric resonances in umbrella-type generating units. *Proc. Inst. Mech. Eng.* **1966**, *181*, 39–49. [[CrossRef](#)]
12. Hofstad, Å. Rotordynamic behaviour of hydropower units. In Proceedings of the 22nd IAHR Symposium on Hydraulic Machinery and Systems, Stockholm, Sweden, 29 June–2 July 2004.
13. Gustavsson, R.; Ahlsén, D.; Fagerberg, L.; Höglström, C.-M. Effects of added mass and moments of inertia on hydroelectric turbines for dynamic applications using structural acoustic simulation. *J. Fluids Struct.* **2022**, *115*, 103757. [[CrossRef](#)]
14. Keto-Tokoi, J.M.; Matusiak, J.E.; Keskinen, E.K. Hydrodynamic added mass and damping of the Kaplan turbine. In Proceedings of the ASME 2011 International Mechanical Engineering Congress & Exposition, Denver, CO, USA, 11–17 November 2011.
15. Gustavsson, R.; Nässelqvist, M.; Österud, J. Radial dampers impact on shaft vibration at resonance. *Proc. 29th IAHR Symp. Hydraul. Mach.* **2019**, *240*, 022011. [[CrossRef](#)]
16. Paidoussis, M. *Fluid–Structure Interactions: Slender Structures and Axial Flow*; Elsevier Science: San Diego, CA, USA, 1998.
17. Blevins, R. *Flow-Induced Vibration*; Krieger Publishing Company: Malabar, FL, USA, 2001.
18. Leblond, C.; Francois, J.-F.; Laine, C.; Auvity, B.; Peerhossaini, H. A Review of Fluid Forces Induced by a Circular Cylinder Oscillating at Low Amplitude and High Frequency in Cylindrical Confinement. In Proceedings of the ASME 2006 Pressure Vessels and Piping/ICPVT-11 Conference, Vancouver, BC, Canada, 23–27 July 2006.
19. Gandjalikhan, S.A.; Mehrabian, M.A. Three-dimensional hydrodynamic analysis of fluid flow between two eccentric rotating cylinders. *Adv. Fluid Mech. IV* **2002**, *36*, 10.
20. Adibi, T.; Razavi, S.E.; Ahmed, S.F.; Amrikachi, A.; Saha, S.C. Characteristic-based fluid flow modelling between two eccentric cylinders in laminar and turbulent regimes. *Hindawi Geofluids* **2022**, *2022*, 8265671.

21. Magnus, H.G. On the deviation of projectiles; and on a remarkable phenomenon of rotating bodies. *Taylor's Foreign Sci. Mem.* **1853**, *804*, 10. [[CrossRef](#)]
22. Prandtl, L. Application of the "Magnus Effect" to the wind propulsion of ships. *NACA Tech. Memo.* **1926**, *367*, 1–12.
23. Glauert, M.B. The flow past a rapidly rotating circular cylinder. *Proc. R. Soc. Lond. Ser. A Math. Phys. Sci.* **1957**, *242*, 108–115.
24. Yao, Q.; Zhou, C.Y.; Wang, C. Numerical Study of the Flow past a Rotating Cylinder at Supercritical Reynolds Number. In Proceedings of the 4th International Conference on Mechanical Materials and Manufacturing Engineering, Wuhan, China, 15–16 October 2016.
25. Ma, W.; Liu, J.; Jia, Y.; Jin, L.; Ma, X. The aerodynamic forces and wake flow of a rotating circular cylinder under various flow regimes. *J. Wind Eng. Ind. Aerodyn.* **2022**, *224*, 104977. [[CrossRef](#)]
26. Seeley, C.; Coutu, A.; Monette, C.; Nennemann, B.; Marmont, H. Characterization of hydrofoil damping due to fluid structure interaction using piezocomposite actuators. *Smart Mater. Struct.* **2012**, *21*, 035027. [[CrossRef](#)]
27. Chen, S.; Wambsganss, M.; Jendrzejczyk, J. Added mass and damping of a vibrating rod in confined viscous fluids. *J. Appl. Mech.* **1976**, *43*, 325–329. [[CrossRef](#)]
28. Chen, S. *Flow Induced Vibrations*; Hemisphere Publishing Corporation: London, UK, 1987.
29. Coutu, A.; Seeley, C.; Monette, C.; Nennemann, B.; Marmont, H. Damping measurements in flowing water. In Proceedings of the 26th IAHR Symposium on Hydraulic Machinery and Systems, Beijing, China, 9–23 August 2012.
30. Seeley, C.; Coutu, A.; Monette, C.; Nennemann, B.; Marmont, H. Determination of Hydrofoil Damping Due to Fluid Structure Interaction Using MFC Actuators. In Proceedings of the 54th Structures, Structural Dynamics and Materials Conference, Boston, MA, USA, 8–11 April 2013.
31. Gauthier, J.P.; Giroux, A.M.; Etienne, S.; Gosselin, F.P. A numerical method for the determination of flow-induced damping in hydroelectric turbines. *J. Fluids Struct.* **2017**, *69*, 341–354. [[CrossRef](#)]
32. Monette, C.; Nennemann, B.; Seeley, C.; Coutu, A.; Marmont, H. Hydro-dynamic damping theory in flowing water. In Proceedings of the 27th IAHR Symposium on Hydraulic Machinery and Systems, Montreal, QC, Canada, 22–26 September 2014.
33. Zeng, Y.S.; Yao, Z.F.; Yang, Z.J.; Wang, F.J.; Hong, Y.P. The prediction of hydrodynamic damping characteristics of a hydrofoil with blunt trailing edge. In Proceedings of the Asian Working Group—IAHR's Symposium on Hydraulic Machinery and Systems, Beijing, China, 16–19 November 2017.
34. Roth, S.; Calmon, M.; Farhat, M.; Muench, C.; Huebner, B.; Avellan, F. Hydrodynamic damping identification from an impulse response of a vibrating blade. In Proceedings of the 3rd IAHR International Meeting of the Workgroup on Cavitation and Dynamic Problems in Hydraulic Machinery and Systems, Brno, Czech Republic, 14–16 October 2009.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.