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Fixed-Time Average Consensus of a Dynamic Event-Triggered Mechanism in a System of Multiple Underactuated Autonomous Underwater Vehicles Based on a Body Frame Spherical Coordinate System

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Abstract: This paper is concerned with the consensus of a system involving multiple underactuated autonomous underwater vehicles (AUVs). Combined with a dynamic event-triggered mechanism and a fixed-time stability theorem, the backstepping average consensus controllers are designed. Firstly, the new consensus control objective on the system for multiple underactuated AUVs in a body frame (BF) spherical coordinate system is proposed, and the tracking error kinematic equations are established based on the kinematic characteristics of the underactuated AUV. The fixed-time consensus controller is designed by the backstepping method, and the average consensus theorem is proposed to improve the Lyapunov function. Furthermore, the dynamic event-triggered mechanism is adopted to reduce the communication requirements and energy consumption. This is the first solution to the problem of a consensus controller design for a system of multiple underactuated AUVs. Finally, numerical simulation results demonstrate that the proposed method has superior effectiveness over alternatives.

Keywords: system for multiple underactuated AUVs; dynamic event-triggered mechanism; fixed-time stability; average consensus; body frame spherical coordinate system



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1. Introduction

In recent years, increasing attention has been focused on systems for multi-AUVs due to their widespread application. The more complex the task of a system for multi-AUVs, the higher the cooperation requirement. Formation control is a fundamental research topic in systems for multi-AUVs. In addition to the challenges of highly coupled nonlinear hydrodynamics, complex ocean disturbances and limited hardware performance, the formation controller of a system for multi-AUVs must also take the communication topology, communication constraints and information utilization into consideration.

Many scholars have studied the formation control of multi-AUVs. In order to improve the convergence speed of AUV formation control, finite-time and fixed-time stability theories are applied to design the formation controller. Ge Guo [1] presented a tracking control strategy of AUV formation with model parameter uncertainties and external disturbances. With a disturbance observer and fixed-time sliding control scheme, the follower vehicle can track the leader vehicle with all states globally stabilized within a given settling time. A novel control scheme was presented to accomplish the leader–follower formation control of unmanned surface vehicles (USVs) under model uncertainties, roll motion and environmental disturbances [2]. A three-dimensional coordination control scheme was proposed by combining the sliding mode control, backstepping technique and leader–follower strategy to deal with coordination control of multiple AUVs [3].

Multi-agent system consensus refers to the task of a group of agents trying to achieve a common state via local distributed control executed over a communication network, which

is an effective method to design a multi-agent system formation controller. Compared with the leader–follower method, consensus control can effectively utilize the information transmitted between neighboring agents, instead of just the leader. Researchers have published many studies on consensus control [4–10], such as the fixed-time leader–follower consensus for a high-order time-varying multi-agent system [4], and on the consensus of multi-agent systems with and without input saturation constraints [5]. For a multi-AUV system, a leaderless fault-tolerant consensus strategy based on a heterogeneous condensation/communication topology was proposed [11,12]. Consensus was applied to a leader–follower recovery system, using the mothership as a leader [13]. A finite-time velocity-free position consensus tracking control method was investigated for a multi-AUV system taking into consideration the uncertainties of heterogeneous dynamics [14].

In practical applications, the underwater acoustic communication bandwidth between AUVs is limited due to the performance of communication hardware equipment, which leads to problems with communication constraints. Event-triggered control strategies can use the effective information between neighboring nodes, reducing the requirements for communication conditions and unnecessary energy consumption. An event-triggered fixed-time leader–follower formation controller was designed for a USV [15]. A sliding mode controller was designed in the presence of model uncertainties, roll motion and environmental disturbances, ensuring a small formation error in finite-time [2]. Ge Guo [1] investigated a fixed-time leader–follower formation control method for a group of AUVs with event-triggered acoustic communications and designed a disturbance observer to estimate the compound disturbance, which can be achieved within a finite amount of time independent of the initial estimation error. A finite-time velocity-free rendezvous control method is considered for a multi-AUV system with intermittent and undirected communications [16].

Underactuated AUVs are widely used in practical applications. The main problem in the design of an underactuated AUV controller is that the number of actuators is fewer than the degrees of freedom, so that the control of the tracking error without an actuator depends on the coupling. Numerous relevant results are obtained for these challenges. The line of sight (LOS) guidance algorithm plays a pivotal role due to its simplicity and intuitiveness, and has been widely applied to path-following for a single AUV or USV. A series of LOS algorithms (proportional LOS, integral LOS and adaptive LOS) were proposed to reduce the external environment disturbances [17]. Moreover, in the literature on LOS algorithms, an output feedback controller based on linear stability theory and the backstepping technique was proposed to track the desired 3-dimensional trajectory of an underactuated AUV in the Earth reference frame [18]. For multiple underactuated AUVs, an adaptive formation control of underactuated AUVs was presented [19], which detailed the additional control inputs and proved the stability of the system by using the Lyapunov stability theory. A 3-dimensional coordinated formation controller was proposed for a system of multiple underactuated AUVs [20], which combined the singular perturbation theory.

The kinematic characteristics of the underactuated AUV demonstrate that there are few researches currently working on the consensus control of systems for multiple underactuated AUVs. Thrusters and rudders are usually used as the actuators of underactuated AUVs. Most stabilization controller designs for underactuated AUVs are based on the angle errors in a BF spherical coordinate system, such as the heading angle error and pitch angle error. However, the control objective for consensus control is based on the position errors in the Earth reference frame coordinate system. When designing the stabilization controller, it is necessary to transform the position errors in the Earth reference frame coordinate system to the angle errors in the BF spherical coordinate system. The transformation between the different coordinate systems will cause coupling, therefore it is difficult to design the consensus controller for a system of multiple underactuated AUVs.

Motivated by the previous results, this study proposed an average consensus controller for multiple underactuated AUVs based on the BF spherical coordinate system. Firstly, the control objective is proposed on account of the kinetic characteristics of a system of

multiple underactuated AUVs. By designing the average consensus control objective, the tracking error kinematic equations of an underactuated AUV are constructed. We decouple the position errors in the Earth reference frame coordinate system into three controllable degrees of freedom in the BF spherical coordinate system, namely the distance, pitch angle and yaw angle errors are stabilized by the thrust, pitching torque and yaw torque controllers, respectively. Then, the backstepping method and fixed-time stability theory are used to design the average consensus controller. The dynamic event-triggered mechanism is adopted in the controller to reduce resource consumption. To the best of our knowledge, this is the first time that the problem of consensus control for a system of multiple underactuated AUVs has been tackled. The principle contributions of this study are as follows:

1. The new consensus control objective for a system of multiple underactuated AUVs based on a BF spherical coordinate system is proposed, which is more adaptive to the kinematic and kinetic characteristics of underactuated AUVs. The tracking error kinematics equation is constructed in a BF spherical coordinate system to reduce the accumulated error that results from the transformation between different coordinate systems.
2. The average consensus strategy is proposed to design a consensus controller for a system of multiple underactuated AUVs. Under the new control objective, the expression between the actuator control variable and the control objective can be constructed, which simplifies the controller design. Moreover, the angle consensus for a system of multiple underactuated AUVs has practical physical significance. Under this control strategy, the states of AUVs can achieve a consensus faster than in the leader–follower strategy.
3. The dynamic event-triggered mechanism is adopted to reduce the unnecessary triggers resulting from the decrease of the system operation-triggered threshold, which reduces the communication requirements and energy consumption in the consensus controller.

2. Mathematical Model and Preliminaries

In this section, graph theory, fixed-time stability theory and useful lemmas are introduced, kinematic and kinetic equations of the AUV model are constructed, the concept of average consensus is proposed and a description of the problem is presented.

2.1. Graph Theory

An undirected graph $G = (v, \varepsilon, A)$ with N nodes is considered, and $\varepsilon \subseteq v \times v$ is the set of edges. If $(v_i, v_j) \in \varepsilon$, they are neighbors and get information from each other. The adjacency matrix $A = [a_{ij}]$ satisfies that $a_{ii} = 0$, $a_{ij} = a_{ji}$, $i \neq j$, and if $(v_j, v_i) \in \varepsilon$, then $a_{ij} = 1$, otherwise $a_{ij} = 0$. The degree matrix $D = \text{diag}\{d_1, \dots, d_N\}$, $d_i = \sum_{j=1}^N a_{ij}$. The Laplacian matrix $L = D - A = [l_{ij}] \in R_{N \times N}$.

Lemma 1 ([21]). *For an undirected graph G , the Laplacian matrix is positive definite. The eigenvalues of the Laplacian matrix L are $0, \lambda_2, \dots, \lambda_N$, and $0 < \lambda_2 < \dots < \lambda_N$. If $X = [x_1, x_2, \dots, x_N]^T$ satisfies $\mathbf{1}_N^T X = 0$, one can obtain that*

$$\lambda_2 X^T X \leq X^T L X \leq \lambda_N X^T X \tag{1}$$

where $X^T L X = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N a_{ij} (x_i - x_j)^2$.

2.2. Fixed-Time Control Stability Theory

Suppose in the system exists a Lyapunov function $V(x) > 0$, and the derivative of $V(x)$ satisfies the following inequality:

$$\dot{V}(x) \leq -aV(x)^\alpha - bV(x)^\beta + \delta, 0 < \alpha < 1, 1 < \beta, a, b > 0 \tag{2}$$

The system is globally stable in fixed-time. There exists a finite constant T_{\max} , and the settling time function T can be estimated by (3):

$$T \leq T_{\max} = \frac{1}{a(1-\alpha)} + \frac{1}{b(\beta-1)} \tag{3}$$

2.3. Useful Lemmas

Lemma 2 ([22]). Let $c_1, c_2 \dots c_n \geq 0$, the following inequalities hold:

$$\sum_{i=1}^N c_i^p \geq \left(\sum_{i=1}^N c_i \right)^p, 0 < p \leq 1 \tag{4}$$

$$N^{1-q} \left(\sum_{i=1}^N c_i \right)^q \leq \sum_{i=1}^N c_i^q \leq \left(\sum_{i=1}^N c_i \right)^q, 1 < q < \infty$$

Lemma 3 ([15]). For $\phi, \vartheta \in \mathcal{R}$ and any real numbers $p > 0, q > 0, \zeta > 0$, the following inequality holds

$$|\phi|^p |\vartheta|^q \leq \frac{p}{p+q} \zeta |\phi|^{p+q} + \frac{q}{p+q} \zeta^{-\frac{p}{q}} |\vartheta|^{p+q} \tag{5}$$

Lemma 4. (Young’s inequality) For $a, b > 0, p > 1, q > 1$ and $\frac{1}{p} + \frac{1}{q} = 1$, the following inequality holds:

$$ab \leq \frac{a^p}{p} + \frac{b^q}{q} \tag{6}$$

2.4. The AUV Model

As shown in Figure 1, it is convenient to use the Earth reference frame and body frame coordinate system to describe the motion of an AUV in 3-D space. Particularly, there are two generalized coordinate vectors $\eta, \vartheta \in R_6$ that need to be explained. $\eta = [x, y, z, \varphi, \theta, \psi]^T$ denotes the kinematic state of the AUV in the Earth reference frame coordinate system, where x, y, z represent the positions, and φ, θ, ψ represent the attitudes. $\vartheta = [u, v, w, p, q, r]^T$ denotes the kinetic state of the AUV in the body frame coordinate system, where u, v, w are the linear velocities and p, q, r are the angular velocities.

The kinematic and kinetic equations of AUVs can be described as follows:

$$\dot{\eta} = J(\eta)\vartheta$$

$$(M_{RB} + M_A)\dot{\vartheta} + (C_{RB}(\vartheta) + C_A(\vartheta))\vartheta + D(\vartheta)\vartheta + g(\eta) = \tau_\vartheta + \tau_d \tag{7}$$

$$J(\eta) = \begin{bmatrix} J_1(\eta) & 0^{3 \times 3} \\ 0^{3 \times 3} & J_2(\eta) \end{bmatrix}$$

$$J_1(\eta) = \begin{bmatrix} c\psi c\theta & -s\psi c\varphi + c\psi s\theta s\varphi & s\psi s\varphi + c\psi c\varphi s\theta \\ s\psi c\theta & c\psi c\varphi + s\psi s\theta s\varphi & -c\psi s\varphi + s\psi c\varphi s\theta \\ -s\theta & c\theta s\varphi & c\theta c\varphi \end{bmatrix} \tag{8}$$

$$J_2(\eta) = \begin{bmatrix} 1 & s\varphi t\theta & c\varphi t\theta \\ 0 & c\varphi & -s\varphi \\ 0 & s\varphi/c\theta & c\varphi/c\theta \end{bmatrix}$$

where $J(\eta)$ denotes the rotation matrix. The inertia matrix and additional inertia matrix are expressed as M_{RB} and M_A , respectively. C_{RB} and C_A represent the Coriolis centripetal force matrix, and $D(\vartheta)$ denotes the damping matrix, where $g(\eta)$ is the restoring force vector, i.e., the generalized buoyancy and gravitational torque.

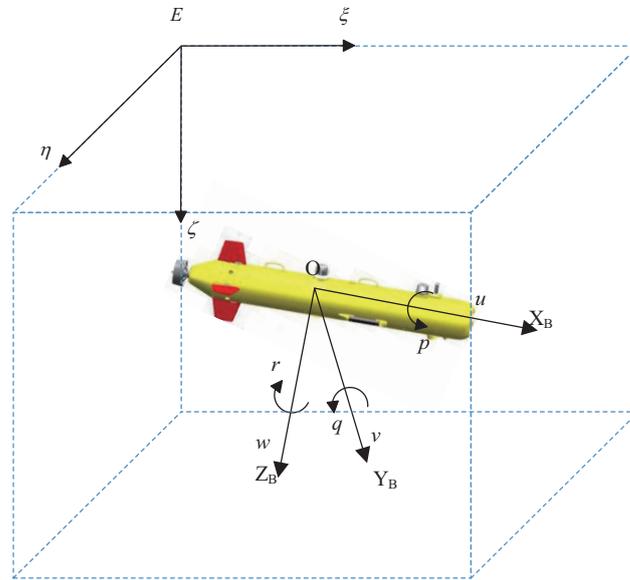


Figure 1. AUV coordinate system.

Remark 1. In practical applications of underactuated AUVs, the actuator inputs are $\tau_\vartheta = [\tau_u, 0, 0, 0, \tau_q, \tau_r]$, where τ_u , τ_q and τ_r are the thrust, pitching torque and yaw torque, respectively. The controllable state variables of the AUV are $\vartheta_c = [u, q, r]^T$. The design of the position error stabilization controller is complicated because of the coupling between the controllable variables and the control objective.

2.5. Average Consensus

Consensus controller design is a challenging problem in a system of multiple underactuated AUVs. Based on the kinematic characteristics of the underactuated AUV, this paper proposes the concept of average consensus protocol to simplify the design of the consensus controller for a system of multiple underactuated AUVs.

Definition 1. The system of multiple underactuated AUVs is said to achieve average consensus if for any given bounded initial states (where p_i are the states of the AUV, i) there exists an average consensus protocol, such that

$$\lim_{t \rightarrow \infty} \sum_{i=1}^N \sum_{j=1}^N a_{ij} (p_i - \bar{p}_i)^2 = 0 \tag{9}$$

where \bar{p}_i satisfies the following equation:

$$\bar{p}_i = \frac{\sum_{j=1}^n a_{ij} p_j}{\sum_{j=1}^n a_{ij}} \tag{10}$$

Average consensus protocol is an adaptive expression of consensus protocol for a system of multiple underactuated AUVs. For AUV i , the average consensus protocol generates a virtual reference trajectory by preprocessing the neighboring information.

Therefore, the tracking error kinematic equations can be constructed, which simplify the design of the controller. The average consensus protocol and consensus protocol satisfy the following inequality:

Theorem 1. Suppose $A = \{a_{ij}\}$ is a symmetric matrix, $D = \text{diag}\{d_1, d_2, \dots, d_N\}$, $d_1 = d_2 = \dots = d_N = k \leq N$. If the eigenvalues of the Laplacian matrix L are $\lambda_1, \lambda_2, \dots, \lambda_N$, and $\lambda_1 < \lambda_2 < \dots < \lambda_N$, the following inequality between the average consensus and consensus protocol holds:

$$\sum_{i=1}^N \sum_{j=1}^N a_{ij}(p_i - p_j)^2 \leq \frac{\lambda_N}{\lambda_2 - k + 1} \sum_{i=1}^N \sum_{j=1}^N a_{ij}(p_i - \bar{p}_i)^2 \tag{11}$$

Proof of Theorem 1. Define $V_1 = \sum_{i=1}^N \sum_{j=1}^N a_{ij}(p_i - p_j)^2$ and $V_2 = \sum_{i=1}^N \sum_{j=1}^N a_{ij}(p_i - \bar{p}_i)^2$. V_1 satisfies the following Equation (12):

$$\begin{aligned} V_1 &= \sum_{i=1}^N \sum_{j=1}^N a_{ij}(p_i - p_j)^2 \\ &= 2P^T L P \end{aligned} \tag{12}$$

V_2 satisfies the following Equation (13):

$$\begin{aligned} V_2 &= \sum_{i=1}^N \sum_{j=1}^N a_{ij}(p_i - \bar{p}_i)^2 \\ &= \sum_{i=1}^N \sum_{j=1}^N a_{ij} \left(p_i - \frac{\sum_{j=1}^n b_{ij} p_j}{\sum_{j=1}^n b_{ij}} \right)^2 \\ &= P^T (D - A) P + P^T (E - A) P \\ &= 2P^T L P - P^T (D - E) P \end{aligned} \tag{13}$$

where E is expressed as follows:

$$E = \text{diag}\{c_1, c_2, \dots, c_N\}, c_i = \sum_{p=1}^N \frac{a_{pi}}{d_p} \tag{14}$$

$A = \{a_{ij}\}$ is a symmetric matrix, and $D = \text{diag}\{d_1, d_2, \dots, d_N\}$, $d_1 = d_2 = \dots = d_N = k \leq N$, so the parameter c_i can be transformed to the following equation:

$$c_i = \sum_{p=1}^n \frac{a_{pi}}{d_p} = \sum_{p=1}^n \frac{a_{pi}}{k} = \frac{1}{k} \sum_{p=1}^n a_{ip} = \frac{k}{k} = 1 \tag{15}$$

the eigenvalues of the Laplacian matrix L are $\lambda_1, \lambda_2, \dots, \lambda_N$, and $\lambda_1 < \lambda_2 < \dots < \lambda_N$. From (12) and (13), one can obtain (16):

$$V_1 - V_2 = P^T (D - \Lambda) P \tag{16}$$

where $(D - \Lambda)$ is diagonal matrix. Suppose the eigenvalues of the Laplacian matrix of V_2 are $\lambda_1 - (k - 1), \lambda_2 - (k - 1), \dots, \lambda_N - (k - 1)$, then the following inequality can be obtained:

$$[\lambda_2 - (k - 1)] P^T P \leq V_2 \leq \delta \tag{17}$$

From Lemma 1 and (17), the following inequality holds:

$$V_1 \leq \lambda_N P^T P \leq \lambda_N \frac{\delta}{\lambda_2 - (k - 1)} = \frac{\lambda_N}{\lambda_2 - k + 1} V_2 \tag{18}$$

This concludes the proof. \square

2.6. Problem Description

The consensus control of a system of multiple underactuated AUVs is a challenging research topic. Compared with the full-actuated AUV, the number of actuators in the underactuated AUV is less than the degrees of freedom, so the controllability of an underactuated AUV is affected. The consensus control of a system of multiple underactuated AUVs cannot be directly and completely controlled based on the position error, as that of a full-actuated system of AUVs, so the control objective based on position error convergence is not completely suitable for an underactuated AUV. Based on the dynamic characteristics of the system of underactuated AUVs, the control objective based on the distance and angle errors is designed.

In this paper, a new consensus control objective for a system of multiple underactuated AUVs based on a BF spherical coordinate system is proposed. Different from the traditional consensus control objective, which uses the position error in a Earth reference frame coordinate system, we use the distance $d_{i\bar{i}}$, pitch angle $\gamma_{i\bar{i}}$ and yaw angle $\delta_{i\bar{i}}$ in a BF spherical coordinate system to describe the consensus control objective. As shown in Figure 2, the consensus of a system of multiple underactuated AUVs can be transformed into the average consensus based on Theorem 1. In order to achieve the average consensus of a system of multiple underactuated AUVs, the control objective is designed as follows.

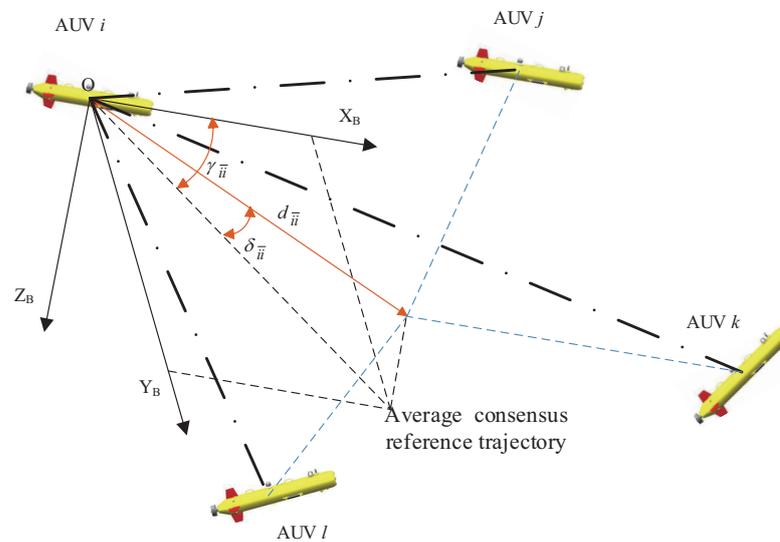


Figure 2. Trajectory tracking average consensus of AUVs.

Definition 2. (Control Objective) For each AUV i , the measured states in a BF spherical coordinate system are $p = [d_{i\bar{i}}, \gamma_{i\bar{i}}, \delta_{i\bar{i}}]^T$. The system of multiple underactuated AUVs is said to achieve an

average consensus if the tracking error satisfies $\lim_{t \rightarrow \infty} p < \sigma$, where σ is a positive constant. The detailed expression of these errors is shown in (19).

$$\begin{aligned}
 d_{i\bar{i}} &= (x_i - \bar{x}_i)^2 + (y_i - \bar{y}_i)^2 + (z_i - \bar{z}_i)^2 \\
 \gamma_{i\bar{i}} &= 0.5[1 - \text{sgn}(\bar{x}_i - x_i)]\text{sgn}(\bar{y}_i - y_i)\pi + \arctan \frac{\bar{y}_i - y_i}{\bar{x}_i - x_i} - \psi_i \\
 \delta_{i\bar{i}} &= \arctan \frac{\bar{z}_i - z_i}{\sqrt{(\bar{x}_i - x_i)^2 + (\bar{y}_i - y_i)^2}} - \theta_i
 \end{aligned}
 \tag{19}$$

Remark 2. Tracking error refers to the difference between the measured signal and the predicted signal. The measured signal used in this paper is p , and the expected signal is indicated as $p = 0$ in the control objective, so the difference between the measured signal and the predicted signal is the tracking error $e = p - 0 = p = [d_{i\bar{i}}, \gamma_{i\bar{i}}, \delta_{i\bar{i}}]^T$.

In this article, we will discuss a kind of system of multiple underactuated AUVs, including N underactuated AUVs. For each AUV, the kinematic and kinetic characteristics satisfied the equations composed in (7). Combined with the backstepping method and dynamic event-triggered mechanism, we design a dynamic event-triggered average consensus control law with fixed-time convergence for the system of multiple underactuated AUVs based on a BF spherical coordinate system.

3. Multiple Underactuated AUVs System Fixed-Time Average Consensus Controller Design Based on Dynamic Event Triggering Mechanism

This section mainly introduces the design method of a fixed-time average consensus controller with a dynamic event-triggered mechanism. The kinematic equations of the tracking error in a BF spherical coordinate system are constructed, and the Lyapunov function is designed based on the new control objective. The controller is designed according to backstepping and fixed-time stability theory. Aiming at the integral explosion caused by the derivation of auxiliary variables in the backstepping method, the fixed-time filter is used for approximation. Additionally, the dynamic event-triggered mechanism is used to solve the communication constraints and energy consumption problems.

3.1. Tracking Error Kinematic Equations in BF Spherical Coordinate System

It is necessary to obtain the equations pertaining to the tracking error kinematics for designing the control objective stabilization controller. Firstly, the coordinates of average consensus reference trajectory are obtained in the BF spherical coordinate system. Then, according to the characteristics of the kinematics of an underactuated AUV, the tracking error kinematics equations are established. The relative translation and rotation between the AUV and the average consensus reference trajectory is considered in the deduction of the kinematics equations, based on a BF spherical coordinate system. The tracking error kinematics equations are described as (20) to (22).

$$\begin{aligned}
 \dot{d}_{i\bar{i}} &= -u_i \cos \gamma_{i\bar{i}} \cos \delta_{i\bar{i}} + \frac{\sum_{j=1}^N a_{ij} u_j \cos \gamma_{j\bar{i}} \cos \delta_{j\bar{i}}}{\sum_{j=1}^N a_{ij}} - v_i \sin \gamma_{i\bar{i}} \cos \delta_{i\bar{i}} \\
 &+ \frac{\sum_{j=1}^N a_{ij} v_j \sin \gamma_{j\bar{i}} \cos \delta_{j\bar{i}}}{\sum_{j=1}^N a_{ij}} - w_i \sin \delta_{i\bar{i}} + \frac{\sum_{j=1}^N a_{ij} w_j \sin \delta_{j\bar{i}}}{\sum_{j=1}^N a_{ij}}
 \end{aligned}
 \tag{20}$$

$$\delta_{i\bar{i}} = \frac{1}{d_{i\bar{i}}} \left(\begin{array}{c} u_i \cos \gamma_{i\bar{i}} \sin \delta_{i\bar{i}} - \frac{\sum_{j=1}^N a_{ij} u_j \cos \gamma_{j\bar{i}} \sin \delta_{j\bar{i}}}{\sum_{j=1}^N a_{ij}} + v_i \sin \gamma_{i\bar{i}} \sin \delta_{i\bar{i}} \\ - \frac{\sum_{j=1}^N a_{ij} v_j \sin \gamma_{j\bar{i}} \sin \delta_{j\bar{i}}}{\sum_{j=1}^N a_{ij}} - w_i \cos \delta_{i\bar{i}} + \frac{\sum_{j=1}^N a_{ij} w_j \cos \delta_{j\bar{i}}}{\sum_{j=1}^N a_{ij}} \end{array} \right) - q_i \quad (21)$$

$$\gamma_{i\bar{i}} = \frac{1}{d_{i\bar{i}}} \left(\begin{array}{c} u_i \sin \gamma_{i\bar{i}} \cos \delta_{i\bar{i}} - \frac{\sum_{j=1}^N a_{ij} u_j \sin \gamma_{j\bar{i}} \cos \delta_{j\bar{i}}}{\sum_{j=1}^N a_{ij}} \\ - v_i \cos \gamma_{i\bar{i}} \cos \delta_{i\bar{i}} + \frac{\sum_{j=1}^N a_{ij} v_j \cos \gamma_{j\bar{i}} \cos \delta_{j\bar{i}}}{\sum_{j=1}^N a_{ij}} \end{array} \right) - r_i \quad (22)$$

where $\gamma_{j\bar{i}}$ and $\delta_{j\bar{i}}$ satisfy (23):

$$\begin{aligned} \gamma_{j\bar{i}} &= 0.5[1 - \text{sgn}(\bar{x}_i - x_j)]\text{sgn}(\bar{y}_i - y_j)\pi + \arctan \frac{\bar{y}_i - y_j}{\bar{x}_i - x_j} - \psi_i \\ \delta_{j\bar{i}} &= \arctan \frac{\bar{z}_i - z_j}{\sqrt{(\bar{x}_i - x_j)^2 + (\bar{y}_i - y_j)^2}} - \theta_i \end{aligned} \quad (23)$$

Remark 3. The Earth reference frame coordinate system kinematics state variable is transformed into the BF spherical coordinate system, and according to the physical model in the BF spherical coordinate system, the tracking error kinematics expression in a BF spherical coordinate system is established. The control objective $p = [d_{i\bar{i}}, \gamma_{i\bar{i}}, \delta_{i\bar{i}}]^T$ is partially decoupled from the controllable states $\vartheta_c = [u, q, r]^T$, which simplifies the design of the controller.

3.2. Fixed-Time Average Consensus Controller Design

The Lyapunov function is derived by the average consensus method to design the controller. According to the average consensus theorem, the \bar{p}_i followed by that of each AUV is obtained to achieve consensus. The stability of the fixed-time average consensus control system is stated as below.

Theorem 2. The actuator control inputs $\tau_{ui}(t)$, $\tau_{qi}(t)$ and $\tau_{ri}(t)$ can be designed as follows:

$$\begin{aligned} \tau_{ui} &= M \frac{\left[-ae_{ui}^\alpha - be_{ui}^\beta + u_i \sin \gamma_{i\bar{i}} \cos \delta_{i\bar{i}} \dot{\gamma}_{i\bar{i}} + u_i \cos \gamma_{i\bar{i}} \sin \delta_{i\bar{i}} \dot{\delta}_{i\bar{i}} \right] + \frac{-a(\Lambda_i - \bar{\Lambda}_i)^\alpha - b(\Lambda_i - \bar{\Lambda}_i)^\beta}{\tau_i} + L_{ii} d_{i\bar{i}}}{\cos \gamma_{i\bar{i}} \cos \delta_{i\bar{i}}} + Cv_i + Dv_i + g_{ui} \\ \tau_{qi} &= M \left[-ae_{qi}^\alpha - be_{qi}^\beta + \frac{-a(\Phi_i - \bar{\Phi}_i)^\alpha - b(\Phi_i - \bar{\Phi}_i)^\beta}{\tau_i} + L_{ii} \delta_{i\bar{i}} \right] + Cq_i + Dq_i + g_{qi} \\ \tau_{ri} &= M \left[-ae_{ri}^\alpha - be_{ri}^\beta + \frac{-a(\Gamma_i - \bar{\Gamma}_i)^\alpha - b(\Gamma_i - \bar{\Gamma}_i)^\beta}{\tau_i} + L_{ii} \gamma_{i\bar{i}} \right] + Cr_i + Dr_i + g_{ri} \end{aligned} \quad (24)$$

where $0 < \alpha < 1, \beta > 1$ and $\alpha\beta = 1$. The system of multiple underactuated AUVs under the above control scheme reaches a fixed-time average consensus.

Remark 4. In Equation (24), $\bar{\Lambda}_i, \bar{\Phi}_i$ and $\bar{\Gamma}_i$ are the auxiliary variables; Λ_i, Φ_i and Γ_i are the outputs of fixed-time filter; and e_{ui}, e_{qi} and e_{ri} are the second-order tracking errors. All parameters will be detailed in the following study.

Proof of Theorem 2. Based on the control objective, the Lyapunov functions V_1, V_2, V_3 are designed as follows:

$$\begin{aligned} V_1 &= \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N a_{ij} d_{ii}^2 \\ V_2 &= \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N a_{ij} \gamma_{ii}^2 \\ V_3 &= \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N a_{ij} \delta_{ii}^2 \end{aligned} \tag{25}$$

Remark 5. The design of the pitch and yaw controller is similar to that of the thrust controller. In this study, the thrust controller is designed as a demonstrative example.

Derivation of the Lyapunov function:

$$\dot{V}_1 = \sum_{i=1}^N \sum_{j=1}^N a_{ij} d_{ii} \left(\begin{aligned} & - u_i \cos \gamma_{ii} \cos \delta_{ii} + \frac{\sum_{j=1}^N a_{ij} u_j \cos \gamma_{ji} \cos \delta_{ji}}{\sum_{j=1}^N a_{ij}} - v_i \sin \gamma_{ii} \cos \delta_{ii} \\ & + \frac{\sum_{j=1}^N a_{ij} v_j \sin \gamma_{ji} \cos \delta_{ji}}{\sum_{j=1}^N a_{ij}} - w_i \sin \delta_{ii} + \frac{\sum_{j=1}^N a_{ij} w_j \sin \delta_{ji}}{\sum_{j=1}^N a_{ij}} \end{aligned} \right) \tag{26}$$

$\bar{\Lambda}_i$ is designed as an auxiliary control variable based on the backstepping method. In the derivation of the second-order backstepping controller, the derivation of the auxiliary variables will lead to differential explosion. Therefore, this study uses the fixed-time filter to approximate the auxiliary control variables to eliminate this problem.

$$\begin{aligned} \bar{\Lambda}_i &= \frac{\sum_{j=1}^N a_{ij} u_j \cos \gamma_{ji} \cos \delta_{ji}}{\sum_{j=1}^N a_{ij}} - v_i \sin \gamma_{ii} \cos \delta_{ii} + \frac{\sum_{j=1}^N a_{ij} v_j \sin \gamma_{ji} \cos \delta_{ji}}{\sum_{j=1}^N a_{ij}} \\ & - w_i \sin \delta_{ii} + \frac{\sum_{j=1}^N a_{ij} w_j \sin \delta_{ji}}{\sum_{j=1}^N a_{ij}} + ad_{ii}^\alpha + bd_{ii}^\beta \end{aligned} \tag{27}$$

Remark 6. To address the so called "explosion of complexity", the filter technique is employed to avoid the analytical derivative of the auxiliary control variable.

Define the fixed-time filter as follows:

$$\begin{aligned} f_i &= \Lambda_i - \bar{\Lambda}_i \\ \tau_i \dot{\Lambda}_i &= -a(\Lambda_i - \bar{\Lambda}_i)^\alpha - b(\Lambda_i - \bar{\Lambda}_i)^\beta \end{aligned} \tag{28}$$

The second order backstepping controller is designed as (29):

$$\begin{aligned} e_{ui} &= u_i \cos \gamma_{i\bar{i}} \cos \delta_{i\bar{i}} - \Lambda_i \\ \dot{e}_{ui} &= \dot{u}_i \cos \gamma_{i\bar{i}} \cos \delta_{i\bar{i}} - u_i \sin \gamma_{i\bar{i}} \cos \delta_{i\bar{i}} \dot{\gamma}_{i\bar{i}} - u_i \cos \gamma_{i\bar{i}} \sin \delta_{i\bar{i}} \dot{\delta}_{i\bar{i}} - \dot{\Lambda}_i \end{aligned} \tag{29}$$

Recombine and construct the Lyapunov functions as follows:

$$V_1 = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N a_{ij} d_{i\bar{i}}^2 + \frac{1}{2} \sum_{i=1}^N f_i^2 + \frac{1}{2} \sum_{i=1}^N e_{ui}^2 \tag{30}$$

Derivation of the Lyapunov function:

$$\begin{aligned} \dot{V}_1 &= \sum_{i=1}^N \sum_{j=1}^N a_{ij} d_{i\bar{i}} \dot{d}_{i\bar{i}} + \sum_{i=1}^N f_i (\dot{\Lambda}_i - \dot{\Lambda}_i) + \sum_{i=1}^N e_{ui} \dot{e}_{ui} \\ &= \sum_{i=1}^N \sum_{j=1}^N a_{ij} \left[-a(d_{i\bar{i}}^2)^{\frac{\alpha+1}{2}} - b(d_{i\bar{i}}^2)^{\frac{\beta+1}{2}} \right] + \sum_{i=1}^N f_i \left(\frac{-a(\Lambda_i - \bar{\Lambda}_i)^\alpha - b(\Lambda_i - \bar{\Lambda}_i)^\beta}{\tau_i} - \dot{\Lambda}_i - L_{ii} d_{i\bar{i}} \right) \\ &+ \sum_{i=1}^N e_{ui} (\dot{u}_i \cos \gamma_{i\bar{i}} \cos \delta_{i\bar{i}} - u_i \sin \gamma_{i\bar{i}} \cos \delta_{i\bar{i}} \dot{\gamma}_{i\bar{i}} - u_i \cos \gamma_{i\bar{i}} \sin \delta_{i\bar{i}} \dot{\delta}_{i\bar{i}} - \dot{\Lambda}_i - L_{ii} d_{i\bar{i}}) \end{aligned} \tag{31}$$

According to (21), put the actuator inputs $\tau_{u_i}(t)$ into (7) to obtain (32):

$$\begin{aligned} \dot{V}_1 &= \sum_{i=1}^N L_{ii} \left[-a(d_{i\bar{i}}^2)^{\frac{\alpha+1}{2}} - b(d_{i\bar{i}}^2)^{\frac{\beta+1}{2}} \right] + \sum_{i=1}^N \left[\frac{-a(f_i^2)^{\frac{\alpha+1}{2}} - b(f_i^2)^{\frac{\beta+1}{2}}}{\tau_i} \right] \\ &+ \sum_{i=1}^N \left[-a(e_{ui}^2)^{\frac{\alpha+1}{2}} - b(e_{ui}^2)^{\frac{\beta+1}{2}} \right] - \sum_{i=1}^N (f_i \dot{\Lambda}_i + L_{ii} d_{i\bar{i}} f_i) \end{aligned} \tag{32}$$

According to Lemma 3, the term $f_i \dot{\Lambda}_i$ can be transformed as (33):

$$f_i \dot{\Lambda}_i \leq \frac{|f_i \dot{\Lambda}_i|^{\alpha+1}}{\alpha+1} \zeta + \frac{\alpha}{\alpha+1} \zeta^{-\frac{1}{\alpha}} \tag{33}$$

By utilizing Lemma 4, we have the inequality (34):

$$d_{i\bar{i}} f \leq |d_{i\bar{i}}| |f| \leq \frac{|d_{i\bar{i}}|^{\alpha+1}}{\alpha+1} + \frac{|f|^{\beta+1}}{\beta+1} = \frac{(d_{i\bar{i}}^2)^{\frac{\alpha+1}{2}}}{\alpha+1} + \frac{(f_i^2)^{\frac{\beta+1}{2}}}{\beta+1} \tag{34}$$

Substituting (33), (34) into (32), one can obtain (35):

$$\begin{aligned} \dot{V}_1 &\leq \sum_{i=1}^N L_{ii} \left[-\left(a_{1i} - \frac{1}{\alpha+1}\right) (d_{i\bar{i}}^2)^{\frac{\alpha+1}{2}} - b_{1i} (d_{i\bar{i}}^2)^{\frac{\beta+1}{2}} \right] \\ &+ \sum_{i=1}^N \left[-\left(\frac{a_{2i}}{\tau_i} - \frac{(\dot{\Lambda}_i^2)^{\frac{\alpha+1}{2}}}{\alpha+1} \zeta\right) (f_i^2)^{\frac{\alpha+1}{2}} - \left(\frac{b_{2i}}{\tau_i} - \frac{L_{ii}}{\beta+1}\right) (f_i^2)^{\frac{\beta+1}{2}} \right] \\ &+ \sum_{i=1}^N \left[-a_{3i} (e_{ui}^2)^{\frac{\alpha+1}{2}} - b_{3i} (e_{ui}^2)^{\frac{\beta+1}{2}} \right] + \frac{\alpha}{\alpha+1} \zeta^{-\frac{1}{\alpha}} \end{aligned} \tag{35}$$

According to Lemma 2, the derivation of the Lyapunov function satisfies the inequality (36):

$$\dot{V}_1 \leq \kappa_1 V^{\frac{\alpha+1}{2}} + \kappa_2 V^{\frac{\beta+1}{2}} + \delta \tag{36}$$

The parameters κ_1 and κ_2 are as follows:

$$\begin{aligned} \kappa_1 &= \min\left\{L_{ii}\left(a_{1i} - \frac{1}{\alpha + 1}\right), \frac{a_{2i}}{\tau_i} - \frac{\left(\bar{\Lambda}_i^2\right)^{\frac{\alpha+1}{2}}}{\alpha + 1} \zeta, a_{3i}\right\} \\ \kappa_2 &= N^{1-\beta} \min\left\{L_{ii}b_{1i}, \frac{b_{2i}}{\tau_i} - \frac{L_{ii}}{\beta + 1}, b_{3i}\right\} \end{aligned} \tag{37}$$

The proof of the fixed-time average consensus is completed. \square

3.3. Dynamic Event-Triggering Mechanism

To reduce the execution frequency of the actuators, a dynamic event-triggered mechanism is introduced. The actuator control inputs are written as $\tau_{pi}(t)$, the controllable state of the AUV is denoted as p , where $p = u, q, r$. The state $s_{pi}(t_k)$ can be designed as follows:

$$s_{pi}(t_k) = \kappa_{p1}e_{pi}^\alpha(t_k) + \kappa_{p2}e_{pi}^\beta(t_k) \tag{38}$$

Design measurement error of AUV i is as follows:

$$X_{pi}(t) = s_{pi}(t_k) - \left(\kappa_{p1}e_{pi}^\alpha + \kappa_{p2}e_{pi}^\beta\right) \tag{39}$$

Then, one can obtain (40)

$$s_{pi}(t_k) = X_{pi}(t) + \kappa_{p1}e_{pi}^\alpha + \kappa_{p2}e_{pi}^\beta \tag{40}$$

We then construct the following triggering function:

$$\begin{cases} t_{k+1}^i = \inf\{t > t_k^i | \Delta(t) \geq 0\} \\ \Delta(t) = X_{pi}^2(t) - \mu_i^2 \left(\kappa_{p1}e_{pi}^\alpha + \kappa_{p2}e_{pi}^\beta\right)^2 - \sigma_i \varepsilon_i \eta_i(t) \\ \dot{\eta}_i(t) = -\varepsilon_i \eta_i(t) - X_{pi}^2(t) + \mu_i^2 \left(\kappa_{p1}e_{pi}^\alpha + \kappa_{p2}e_{pi}^\beta\right)^2 \\ \eta_i(0) > 0 \end{cases} \tag{41}$$

where $0 < \mu_i < 1, 0 < \sigma_i < 1$, and $\varepsilon_i > 0$.

Theorem 3. The actuator control inputs $\tau_{ui}(t)$, $\tau_{qi}(t)$ and $\tau_{ri}(t)$ can be designed as follows:

$$\begin{aligned} \tau_{ui} &= M \frac{\begin{bmatrix} -s_{ui}(t_k) + u_i \sin \gamma_{i\bar{i}} \cos \delta_{i\bar{i}} \dot{\gamma}_{i\bar{i}} + u_i \cos \gamma_{i\bar{i}} \sin \delta_{i\bar{i}} \dot{\delta}_{i\bar{i}} \\ -a(\Lambda_i - \bar{\Lambda}_i)^\alpha - b(\Lambda_i - \bar{\Lambda}_i)^\beta \\ + \frac{L_{ii}d_{i\bar{i}}}{\tau_i} \end{bmatrix}}{\cos \gamma_{i\bar{i}} \cos \delta_{i\bar{i}}} + Cv_i + Dv_i + g_{ui} \\ \tau_{ri} &= M(-s_{ri}(t_k) + \frac{-a(\Gamma_i - \bar{\Gamma}_i)^\alpha - b(\Gamma_i - \bar{\Gamma}_i)^\beta}{\tau_i} + L_{ii}\dot{\gamma}_{i\bar{i}}) + Cr_i + Dr_i + g_{ri} \\ \tau_{qi} &= M(-s_{qi}(t_k) + \frac{-a(\Phi_i - \bar{\Phi}_i)^\alpha - b(\Phi_i - \bar{\Phi}_i)^\beta}{\tau_i} + L_{ii}\dot{\delta}_{i\bar{i}}) + Cq_i + Dq_i + g_{qi} \end{aligned} \tag{42}$$

The system of multiple underactuated AUVs under the above control scheme reaches a fixed-time consensus with t_{k+1}^i satisfying the triggering function (41). In addition, Zeno behavior can be ruled out for each AUV under the proposed control protocol.

Proof of Theorem 3. Similar to Theorem 2, the proof of dynamic event-triggered mechanism stability and exclusion of Zeno behavior on the thrust controller is introduced as an example.

Equation (43) can be derived by (41):

$$s_{ui}(t_k) = \rho_i \left(\kappa_{u1} e_{ui}^\alpha + \kappa_{u2} e_{ui}^\beta \right) \tag{43}$$

$$\rho_i \in (1 - \mu_i, 1 + \mu_i)$$

The Lyapunov function can be constructed as $V'_1 = V_1 + \sum_{i=1}^N \eta_i$. Derivation of the Lyapunov function:

$$\begin{aligned} \dot{V}'_1 &= \sum_{i=1}^N L_{ii} \left[-\left(a_{1i} - \frac{1}{\alpha + 1}\right) (d_{ii}^2)^{\frac{\alpha+1}{2}} - b_{1i} (d_{ii}^2)^{\frac{\beta+1}{2}} \right] \\ &+ \sum_{i=1}^N \left[-\left(\frac{a_{2i}}{\tau_i} - \frac{(\dot{\Lambda}^2_i)^{\frac{\alpha+1}{2}}}{\alpha + 1} \zeta\right) (f_i^2)^{\frac{\alpha+1}{2}} - \left(\frac{b_{2i}}{\tau_i} - \frac{L_{ii}}{\beta + 1}\right) (f_i^2)^{\frac{\beta+1}{2}} \right] \\ &+ \sum_{i=1}^N e_{ui} [-s_{ui}(t_k)] + \frac{\alpha}{\alpha + 1} \zeta^{-\frac{1}{\alpha}} + \sum_{i=1}^N \dot{\eta}_i \end{aligned} \tag{44}$$

According to (43), we can obtain

$$\begin{aligned} \dot{V}'_1 &= \sum_{i=1}^N L_{ii} \left[-\left(a_{1i} - \frac{1}{\alpha + 1}\right) (d_{ii}^2)^{\frac{\alpha+1}{2}} - b_{1i} (d_{ii}^2)^{\frac{\beta+1}{2}} \right] \\ &+ \sum_{i=1}^N \left[-\left(\frac{a_{2i}}{\tau_i} - \frac{(\dot{\Lambda}^2_i)^{\frac{\alpha+1}{2}}}{\alpha + 1} \zeta\right) (f_i^2)^{\frac{\alpha+1}{2}} - \left(\frac{b_{2i}}{\tau_i} - \frac{L_{ii}}{\beta + 1}\right) (f_i^2)^{\frac{\beta+1}{2}} \right] \\ &+ \sum_{i=1}^N \rho_i \left[\kappa_{u1} (e_{ui}^2)^{\frac{\alpha+1}{2}} + \kappa_{u2} (e_{ui}^2)^{\frac{\beta+1}{2}} \right] + \frac{\alpha}{\alpha + 1} \zeta^{-\frac{1}{\alpha}} + \sum_{i=1}^N -(1 - \sigma_i) \varepsilon_i \eta_i \end{aligned} \tag{45}$$

The proof of fixed-time consensus is completed.

Next, the exclusion proof of Zeno behavior for a controller update is given. Firstly, this study proposes the following sufficient conditions. From the event-triggered function, $\|X_{ui}(t)\| \leq \mu \|\kappa_{u1} e_{ui}^\alpha + \kappa_{u2} e_{ui}^\beta\|$ can be guaranteed. It is obtained that

$$\begin{aligned} \|X_{ui}(t)\|^2 &\leq \mu^2 \|\kappa_{u1} e_{ui}^\alpha + \kappa_{u2} e_{ui}^\beta\|^2 = \mu^2 \|s_{ui}(t_k) + X_{ui}(t)\|^2 \\ \|X_{ui}(t)\| &\leq \frac{\mu}{\sqrt{1 + \mu^2}} \|s_{ui}(t_k)\| \end{aligned} \tag{46}$$

Then, the time derivative of $\|X_{ui}(t)\|$ over the interval $[t_k, t_{k+1})$ satisfies

$$\frac{d}{dt} \|X_{ui}(t)\| \leq \left\| \left(\kappa_{u1} \alpha e_{ui}^{\alpha-1} + \kappa_{u2} \beta e_{ui}^{\beta-1} \right) \right\| \|\dot{e}_{ui}\| \tag{47}$$

It is concluded that e_{ui} is non-increasing for the interval $[0, +\infty)$, implying that e_{ui} is bounded. So that $\left\| \left(\kappa_{u1} \alpha e_{ui}^{\alpha-1} + \kappa_{u2} \beta e_{ui}^{\beta-1} \right) \right\| \leq \sigma$. From (7) and (29),

$$\begin{aligned} \frac{d}{dt} \|X_{ui}(t)\| &\leq \sigma \|X_{ui}(t)\| + \sigma \left\| \kappa_{i31} e_{ui}^\alpha + \kappa_{i32} e_{ui}^\beta \right\| \\ &\leq \sigma \|X_{ui}(t)\| + \frac{\sigma}{\rho} \|s_{ui}(t_k)\| \leq \sigma \|X_{ui}(t)\| + \frac{\sigma}{1 - \mu} \|s_{ui}(t_k)\| \end{aligned} \tag{48}$$

Applying the comparison principle and considering $X_{ui}(t_k) = 0$, one obtains

$$\|X_{ui}(t)\| \leq \frac{1}{1-\mu} \|s_{ui}(t_k)\| \left\{ e^{\sigma(t-t_k)} - 1 \right\} \tag{49}$$

According to the event-triggered mechanism (41) and the sufficient condition above, the event cannot trigger before $\|X_{ui}(t)\| \leq \frac{\mu}{\sqrt{1+\mu^2}} \|s_{ui}(t_k)\|$. Thus

$$\frac{1}{1-\mu} \|s_{ui}(t_k)\| \left\{ e^{\sigma(t-t_k)} - 1 \right\} \geq \frac{\mu}{\sqrt{1+\mu^2}} \|s_{ui}(t_k)\| \tag{50}$$

Through calculation, one obtains

$$t_{k+1} - t_k \geq \frac{1}{\sigma} \ln \left(\frac{\mu(1-\mu)}{\sqrt{1+\mu^2}} + 1 \right) \tag{51}$$

Thus, the inter-event time intervals of the controller update are also strictly greater than zero. The proof is completed. \square

4. Simulation Results

This section presents the simulation results of the fixed-time average consensus for a system of multiple underactuated AUVs using a dynamic event-triggered mechanism. Numerical simulation experiments verified the effectiveness of the new control objective in the BF spherical coordinate system, the control effect of the fixed-time average consensus controller and the effect of the dynamic event-triggered mechanism proposed in this paper. Furthermore, the average consensus strategy and leader–follower strategy are compared on the matter of the formation effect.

The parameter settings of the AUV model are provided in Table 1. The communication topology shown in Figure 3 demonstrates that the topology is connected and comprises five AUVs. The communication topology has a spanning tree, and AUV 2, AUV 3 and AUV 4 can obtain the reference trajectory directly.

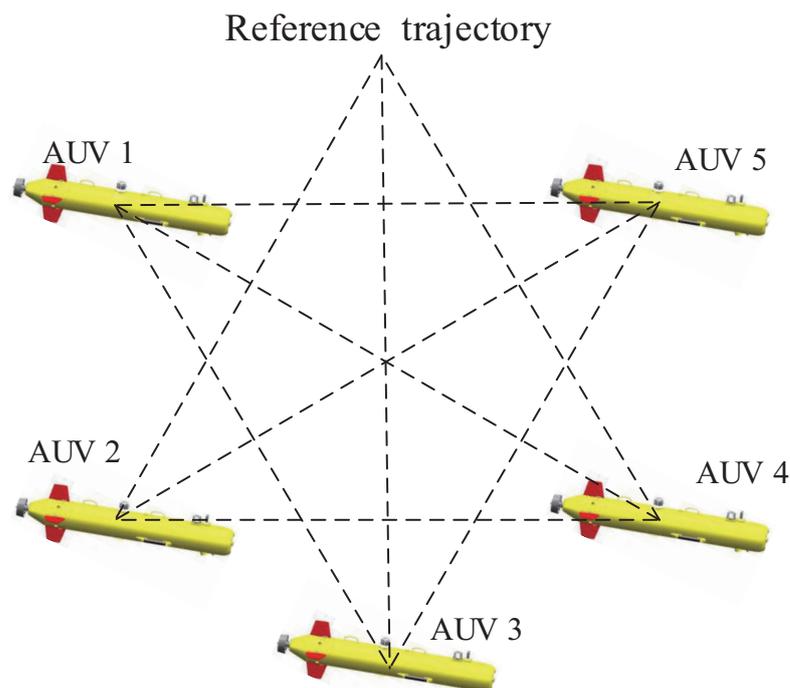


Figure 3. Communication graph.

Table 1. AUV parameter settings.

Variables	Values
m	2146.3 kg
$X_{\dot{u}}$	-0.00158
$X_{u u }$	-0.00686
Y_v	-0.06973
$Y_{\dot{v}}$	-0.08154
$Y_{v \dot{v} }$	-0.1844
N_r	-0.0261
$N_{\dot{r}}$	-0.00168
$N_{r r }$	-0.000092
I_x	542
I_y	7580
I_z	7620

Simulation results are depicted as follows. The parameter settings for the initial position are presented in Table 2. Five AUVs started at the initial position and are able to track the reference trajectory and build the desired consensus based on the proposed algorithm. The reference trajectory is expressed as follows:

$$\begin{aligned}
 x(t) &= 50 - 50 \cos(0.004\pi t) \\
 y(t) &= 50 + 50 \sin(0.004\pi t) \\
 z(t) &= 10 + 0.05t
 \end{aligned}
 \tag{52}$$

3-dimensional helix tracking consensus results are shown in Figure 4. Five AUVs beginning at random positions are able to track the reference trajectories and build the desired consensus based on the proposed algorithm. The above figure demonstrates that the yaw angle controller and pitch angle controller can operate efficiently.

Table 2. Parameter settings for the initial position.

Initial Position	x(m)	y(m)	z(m)
AUV1	-30	30	5
AUV2	30	30	6
AUV3	0	15	7
AUV4	30	0	8
AUV5	-30	0	9

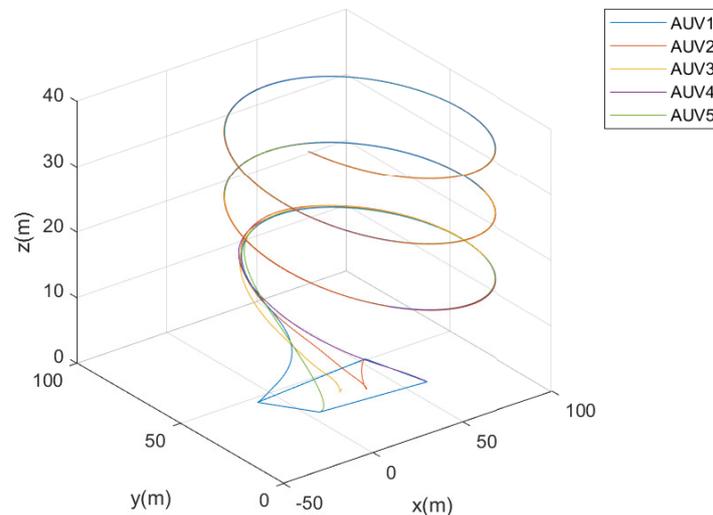


Figure 4. 3-dimensional helix trajectory tracking results for the average consensus of 5 AUVs.

The kinematics state of five AUVs is shown in Figure 5. The kinematics state and kinetics state converged within 200 s, and five AUVs arrived at the reference trajectory within 200 s. The experimental results demonstrate that the proposed algorithm can achieve good consensus performance, and the fixed-time average consensus strategy proposed in this study can effectively achieve the consensus control of multi-AUV systems.

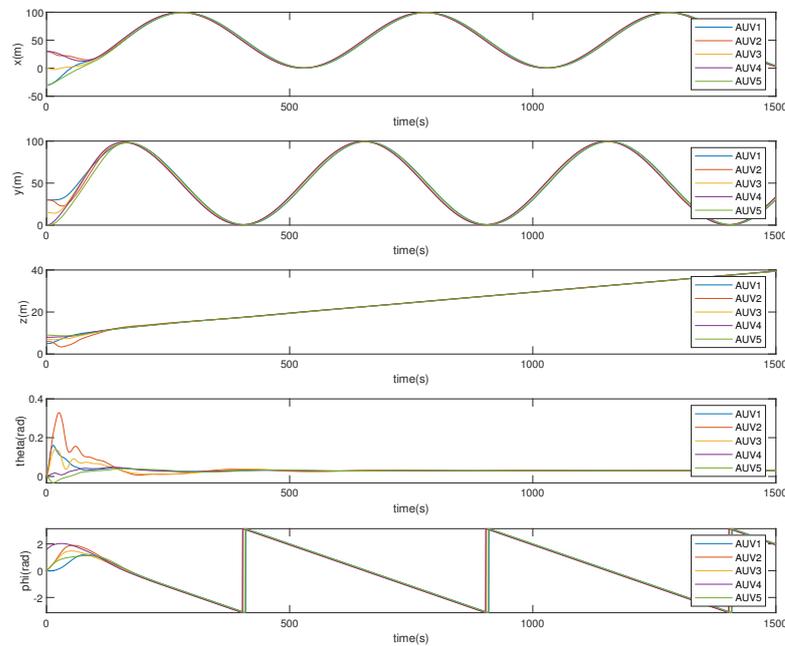


Figure 5. Kinematics states of five AUVs.

The slightly different states of the AUVs is caused by the communication connection state between them. All the AUVs could achieve consensus by following the state of the neighboring AUVs by communicating with the reference trajectory. Moreover, because the reference trajectories are helix, the existence of steady-state errors can generate actuator inputs to keep the average consensus.

Distance error, pitch angle error and yaw angle error of each AUV under different initial conditions are illustrated in Figure 6. Each controller has its own convergence time. The pitch angle and yaw angle errors converge to zero within 20 s. The position error converges in 200 s.

The calculation of the tracking error is the sum of the distance between AUV i and the neighboring AUV j . Because the communication topology of an AUV system is not completely connected with the reference trajectory, if AUV i does not obtain the reference trajectory, its response speed will be slightly slower than the AUV that obtains the reference trajectory when the reference trajectory is transformed. Therefore, the delay will cause some tracking error.

The experiment demonstrates that the control objective proposed in this study can satisfy the control requirement for multi-AUV average consensus. The control objective based on distance and angle errors can track the reference trajectory. The yaw angle, pitch angle and thrust actuators can respectively stabilize the control objective, thus the control objective proposed in this study can effectively simplify the controller design process, and the actuators under different degrees of freedom can be controlled separately, which provides a new scheme for the consensus controller design for a system of multiple underactuated AUVs.

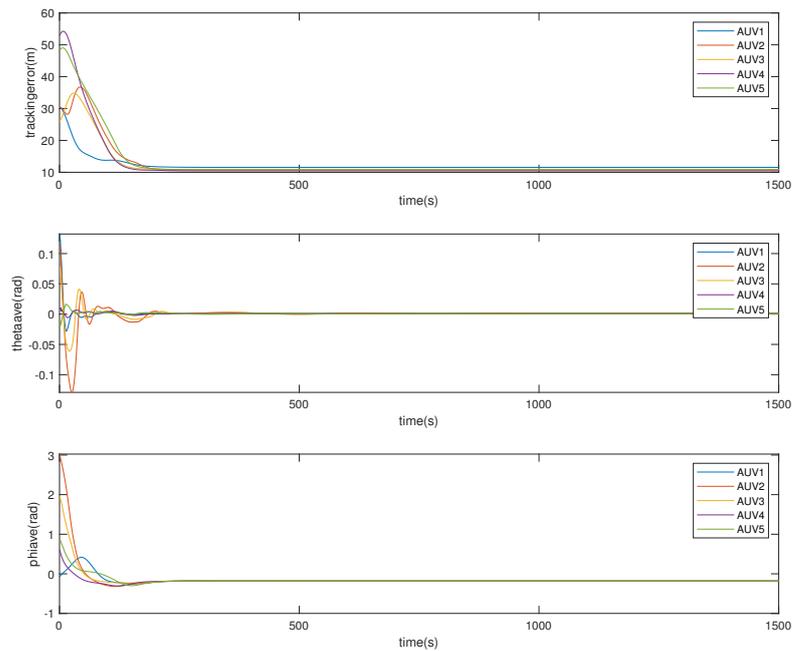


Figure 6. Distance error, pitch angle error and yaw angle error of the five AUVs.

In Table 3, we show the number of triggers under different conditions, and the triggered instants of control signals are shown in Figure 7. The continuous execution of actuators is avoided, thus the communication and energy consumption requirements are reduced.

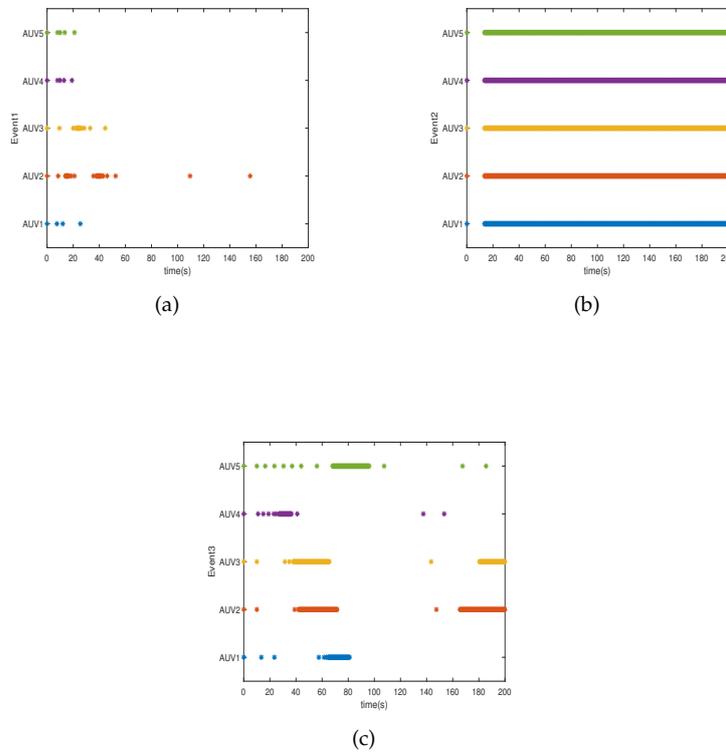


Figure 7. Triggering instants of control signals. (a) Event-triggered time in τ_u controllers. (b) Event-triggered time in τ_q controllers. (c) Event-triggered time in τ_r controllers.

Table 3. Number of triggers under different conditions.

Initial Position	Event1	Event2	Event3	Time Sample
AUV1	4	374	40	400
AUV2	24	374	133	400
AUV3	14	374	100	400
AUV4	5	374	29	400
AUV5	5	374	68	400

The root-mean-square error (RMSE) of the spatial position of the five AUVs using the average consensus formation strategy and leader–follower strategy are shown in Figure 8. The average consensus formation strategy is clearly superior to the leader–follower formation strategy in the convergence process. The reason is that compared with the leader–follower method, the average consensus control method proposed in this study has advantages in the speed of state consensus convergence. Because the actuator input obtained by the consensus control law not only depends on the desired reference trajectory, but is also related to the current state of neighboring AUVs, the AUV state between neighboring AUVs can reach consensus faster. The steady-state error of the average consensus method is slightly larger than that of the leader–follower method, due to receiving the information between neighboring AUVs.

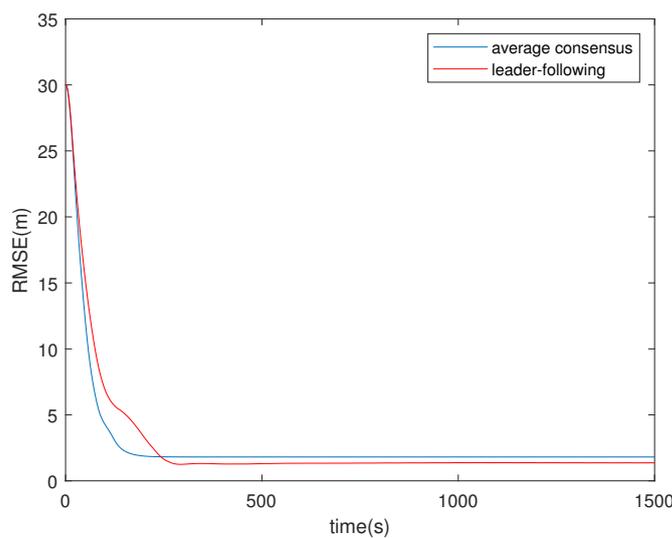


Figure 8. RMSE of the spatial position of the five AUVs.

5. Conclusions

This study is dedicated to solving the consensus for a system of multiple underactuated AUVs. The new control objective is suitable for the dynamic model of an underactuated AUV as it decouples the states of the underactuated AUV. The fixed-time average consensus controller can ensure the stable convergence of the system. The dynamic event-triggering mechanism can effectively reduce the communication pressures and energy consumption of the system. Detailed theoretical derivation and numerical simulation experiments demonstrate that the proposed method has superior control performance over alternatives. These reflect the great advantages of the proposed method in this research on the consensus control theory of a system of multiple underactuated AUVs. Our future research will devote to solving topology switching, affine formation and time-varying formation control, and explore how to strengthen the adaptive ability of a system of multiple underactuated AUVs. In addition, a new event-triggered mechanism will be designed to improve the convergence speed of the system. This algorithm will be applied into an actual engineering project of systems of multiple underactuated AUVs as soon as possible.

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