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A Novel Variable Weight VIKOR Grade Assessment Method for Waterway Navigation Safe Routes Selection

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Abstract: According to the characteristics of waterway navigation safe routes selection, and considering the individual feelings and group benefits of information, as well as no-compensation information between indexes, this paper describes the safe rating of waterway navigation routes, and then puts forward an evaluation model of and method for waterway navigation safe route selection based on variable weight VIKOR. First of all, from the concept and connotation of grade assessment, this paper describes the safe rating of waterway navigation routes, so as to avoid confusing the two essential different problems of safe rating and ranking. Then, the evaluation indexes and membership function of the appropriate grade of the safe rating of waterway navigation route are constructed, and the weights of an evaluation index based on entropy are proposed. Secondly, a variable weight VIKOR evaluation model and a binary semantic evaluation method for the safe grading of waterway navigation safe routes are proposed. Finally, through case study and comparative analysis, the rationality and feasibility of the model and method proposed in this paper are illustrated. This model can better reflect the connotation and characteristics of the appropriate grade assessment of waterway navigation safe routes, and provides new approaches and methods to support the development and management of waterway navigation safe route selection.

Keywords: VIKOR; variable weight decision making; waterway navigation

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1. Introduction

As the world economy has been developing at a fast speed, the economic exchanges between countries and regions have occurred even more often. Statistics show that approximately 80 percent of global trade contacts are made through marine transport. Thus, the importance of marine transport becomes more and more prominent. While the global trade volume and the total number of ships are gradually increasing, the pace of making ships larger, more intelligent and more complex is accelerating. Energy infrastructure and port construction supporting marine transport are upsizing with each passing year. In addition, the higher ship navigation density, the continuous improvement of navigation aids in waters near to ports and the continuous increase of ship scale contribute to the gradual increase of potential risks for ships entering and leaving ports as well as navigating in waterways. According to statistics, such accidents as colliding, running into rocks, stranding and being on fire have tended to take place in various ports and nearby waters. This leads to life and property loss and water pollution to various degrees. For the purpose of navigation safety in waterways and waters near to ports, it is significant to assess the safety of marine navigation, which is a crucial part of waterway safety management.

In the research field of route selection for safe waterway navigation, the problem has been formulated as a multi-attributes decision making problem [1]. For example, Zhu and Huang [2] used the matter-element comprehensive evaluation method to make a scientific assessment of the night navigation environment risks for the waters of the fairway. Rong et al. [3] proposed a ship navigational risk assessment method in the waters of offshore wind farms based on a multi-factor fuzzy analytic hierarchy process. Gao et al. [4] proposed a multi-criteria group decision-making method based on the intuitionistic linguistic aggregation operators and applied it to the site selection decision-making process for waters of offshore wind farms. Deveci et al. [5] integrated interval rough numbers and best worst method to choose the best waters for siting offshore wind farms.

The VIKOR method is a MCDM technique designed to rank a set of alternatives in the presence of conflicting criteria by proposing a compromise solution [6,7]. Ren et al. [8] proposed VIKOR-based decision support systems in fuzzy environments. Büyüközkan et al. [9] proposed some VIKOR-based GDM methods under intuitionistic fuzzy environment. Wu et al. [10] proposed a VIKOR-based GDM approach under an interval type-2 fuzzy environment.

Chen [11] proposed VIKOR-based methods for multiple criteria decision analysis under Pythagorean fuzzy information. Liang et al. [12] proposed a new perspective of a compromise solution based on the traditional VIKOR for handling the decision maker's psychological behavior by inducing TODIM. Wu et al. [13] proposed hesitant Pythagorean fuzzy VIKOR methods for enhancing fuzzy related problems flexibility. Çalı et al. [14], Gupta et al. [15] and Zeng et al. [16] proposed MADM methods based on VIKOR with application to plant location selection. Wu et al. [17] and You et al. [18] proposed extended VIKOR methods with possibility distributions of linguistic information and interval 2-tuple linguistic information. Yue [19] and Wang Çalı et al. [20] proposed an extended VIKOR approach with Picture fuzzy normalized information. Leila [21] and Çalı et al. [22] proposed extended VIKOR models with TOPSIS and ELECTRE for classification problems. Tavana et al. [23] proposed an extended stochastic VIKOR model considering the decision maker's attitude towards risk. Luo et al. [24] proposed a variable weigh VIKOR evaluation modeland method for libraries emergency ability rating.

Social network group decision methods are proposed by Wu et al. [25] and Liu et al. [26,27]. Gong et al. [28] and Xu et al. [29] proposed social network group decision methods based on uncertainty theory. Gao et al. [30] proposed group consensus decision methods with non-cooperative behavior management for social network group decision problems. Wu et al. [31,32], Cao et al. [33], Wang et al. [34] and Sun et al. [35] proposed group consensus models with feedback mechanisms for social network group decision problems.

The above MCDM methods with VIKOR help to enrich the research on multi-attribute decision-making methods and their applications. However, a limitation is that the above VIKOR methods only considered the individual feelings and group benefits of information and gave the grades of alternates; they cannot distinguish the ranking of the safe grades of waterway navigation routes, which does not facilitate quick decisions. However, many practical situations such as waterway navigation safe routes selection require reasonable determination of the grade assessment.

Another limitation of the above MCDM methods with VIKOR is that the weights of indexes are constant and pay little attention to no-compensation information between indexes. The constant weighted comprehensive VIKOR methods for waterway navigation safe routes selection problems can thus lead to irrational results. Therefore, the no-compensation information between indexes must be considered.

Motivated by the above limitations, this paper puts forward the evaluation model and method of the waterway navigation safe routes selection based on variable weight VIKOR. The membership function of the safe grade of waterway navigation routes based on fuzzy sets are constructed, and the weights of an evaluation index based on entropy are put forward. A variable weight VIKOR evaluation model based a two tuple linguistic method for the safe grade of waterway navigation safe route is proposed. The proposed method not only solves the above limitations and improves VIKOR methods and the constant weighted approach, but also can better reflect the connotation and characteristics of the appropriate grade assessment of waterway navigation safe routes, and provide new approaches and methods to support the development and management of waterway navigation safe routes selection.

The rest of the paper is structured as follows. The waterway navigation safe route selection problem is described in Section 2. A method and procedure for the waterway navigation safe route selection problem is solved by the variable weight VIKOR grade assessment method in Section 3. Section 4 applies the proposed method, illustrated with a waterway navigation safe route selection example and comparison analysis. Section 5 shows conclusions and some remarks.

2. Description of the Waterway Navigation Safe Route Selection Problems

In order to assess the light environment of ship navigation at night, let us start with the definition, characteristics and origin of light pollution at sea. Then photometrics, colorimetry and principles of visual performance in combination with basic photometric and colorimetric information are employed to analyse the effects of light pollution at sea on the visibility of ship lights and the visual performance of navigators. Based on this, indices which affect ship navigation at night are sifted out in accordance with basic principles of screening out evaluation indices, so as to construct the index system for assessing the light environment of ship navigation at night, as shown in Table 1.

Table 1. The index system and categorisation criterion for assessing the safety grade of a waterway environment.

Indexes		Safety Grade				
		e_1	e_2	e_3	e_4	e_5
Natural factors c_1	Visibility c_{11}	90	50	40	25	15
	Wind c_{12}	200	150	100	60	30
	Current velocity c_{13}	7	4	2.5	1.5	0.5
Waterway conditions c_2	Width of waterways c_{21}	0.91	0.93	0.67	0.5	0.33
	Length of waterways c_{22}	0.77	0.50	0.17	0.1	0.07
	Curvature of waterways c_{23}	90	60	45	30	15
	Intersection of waterways c_{24}	90	70	60	45	20
	Obstacles in waterways c_{25}	0.02	0.13	0.72	1.3	2.02
Traffic situations c_3	Traffic volume c_{31}	650	500	300	150	70
	Traffic control c_{32}	O_{321}	O_{322}	O_{323}	O_{324}	O_{325}
	Navigation aids c_{33}	O_{331}	O_{332}	O_{333}	O_{334}	O_{335}

See Table 1 for more details. The index system for assessing the safety grade for a waterway environment is a two-level hierarchical structure of indices. The fist level represented as Level I Index Set $C = \{c_1, c_2, c_3\}$ includes 3 assessment indices. Level I Index $c_i (i = 1, 2, 3)$ comprises m_i Level II indices. These indices are represented as Level II Index Set $C_i = \{c_{i1}, c_{i2}, \ldots, c_{im_i}\}$, where $m_i = 4$ $(i = 1, 2, 3)$. b_{isk} stands for the benchmark criterion for Level II index c_{is} with respect to e_k, as shown in Table 1.

For the convenience of description, the safety grade set for a waterway navigation safe route is denoted by $E = \{e_1, e_2, \ldots, e_5\}$, where e_k means the k th safety grade for waterway routes and $e_k < e_{k+1}$ is prescribed, indicating that the $k + 1$ th safety grade e_{k+1} is better than the k th one e_k. The safety grade eigenvalue $v_j (j = 1, 2, \cdots, m)$ for waterway routes based on variable weight VIKOR is computed, If $v_j \in [4.5, 5.5)$, the safety grade for waterway routes a_j is e_5. If $v_j \in [3.5, 4.5)$, the safety grade for waterway routes a_j is e_4. If $v_j \in [2.5, 3.5)$, the safety grade for waterway routes a_j is e_3. If $v_j \in [1.5, 2.5)$, the safety grade for waterway routes a_j is e_2. If $v_j \in [0, 1.5)$, the safety grade for waterway routes a_j is e_1.

The eigenvalue of waterway a_j with regard to Level II index c_{is} is $y_{jis}(j = 1, 2, \cdots, n; i = 1, 2, 3; s = 1, 2, \cdots, m_i)$. That is to say, the information for assessment of waterway a_j with respect to Level II Index Set C_i can be expressed by the matrix below.

$$\boldsymbol{Y}_i = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{array} \begin{array}{c} \begin{array}{cccc} c_{i1} & c_{i2} & \cdots & c_{im_i} \end{array} \\ \begin{pmatrix} y_{1i1} & y_{1i1} & \cdots & y_{1im_i} \\ y_{2i1} & y_{2i2} & \cdots & y_{2im_i} \\ \vdots & \vdots & \cdots & \vdots \\ y_{ni1} & y_{ni2} & \cdots & y_{nim_i} \end{pmatrix} \end{array} \quad (1)$$

In Matrix $\boldsymbol{Y}_i$, the s th column refers to the eigenvalue of all waterways with respect to Level II index c_{is}, whereas the j th row shows the eigenvalue of waterway a_j with regard to Level II Index Set C_i, denoted by $\boldsymbol{Y}_{ji} = (y_{ji1}, y_{ji2}, \cdots, y_{jim_i})^T$ $(j = 1, 2, \cdots, n)$.

Grade division is a concept of fuzziness. Different grade assessment problems may have different division attributes. As the key information of multi-attribute grade division, attribute grade threshold values involve both quantitative and qualitative information. They are classified into upper-bound quantitative B_1, lower-bound quantitative B_2 and and language B_3 grade threshold values. Then, they satisfy the constraints: $B_{t_1} \cap B_{t_2} = \varnothing$ and $\bigcup_{t=1}^{4} B_t = B$, where $\varnothing$ is an empty set.

Lower bound grade threshold B_1: The eigenvalue y_{ji} of alternative a_j on attribute c_{is} is no less than the criterion b_{isk} of grade e_k $(k = 1, 2, \cdots, 5)$ on attribute c_{is}, i.e., $y_{jis} \geq b_{isk}$, which satisfies the condition of $b_{is1} \leq b_{is2} \leq \cdots \leq b_{is5}$.

Upper bound grade threshold B_2: The eigenvalue y_{ji} of alternative a_j concerning attribute c_{is} is no more than the criterion b_{isk} of grade e_k $(k = 1, 2, \cdots, 5)$ of attribute c_{is}, i.e., $y_{jis} \leq b_{isk}$, which satisfies the condition of $b_{is1} \geq b_{is2} \geq \cdots \geq b_{is5}$.

Linguistic grade threshold B_3: the eigenvalue y_{ji} of alternative a_j on qualitative attribute c_{is} is linguistic information, i.e.,$y_{ji} \in O_{is}$, where $O_i = \{o_{i1}, o_{i2}, \cdots, o_{i5}\}$ is the grade threshold of the quantitative attribute c_{is}.

3. The Variable Weight VIKOR Assessment Method for the Safety Grade of a Waterway Environment

3.1. The Construction of Membership Function for the Safety Grade of a Waterway Environment

For index $c_{is} \in B_1 (i = 1, 2, 3; s = 1, 2, \cdots, m_s)$, the membership function $\widetilde{\mu}_{jisk}$ for safety grade $e_k (k = 1, 2, \cdots, 5)$ of route $a_j (j = 1, 2, \cdots, n)$ with respect to Level II index c_{is} is defined as follows:

$$\mu_{jis1} = \begin{cases} 1 & (y_{jis} \geq b_{is1}) \\ y_{jis}/b_{is1} & (0 \leq y_{jis} < b_{is1}) \end{cases} \quad (c_{is} \in B_1) \quad (2)$$

$$\widetilde{\mu}_{jis2} = \begin{cases} b_{is1}/y_{jis} & (y_{jis} > b_{is1}) \\ 1 & (b_{is2} \leq y_{jis} \leq b_{is1}) \\ y_{jis}/b_{is2} & (0 \leq y_{jis} < b_{is2}) \end{cases} \quad (c_{is} \in B_1) \quad (3)$$

$$\widetilde{\mu}_{jis3} = \begin{cases} b_{is2}/y_{jis} & (y_{jis} > b_{is3}) \\ 1 & (b_{is4} \leq y_{jis} \leq b_{is3}) \\ y_{jis}/b_{is3} & (0 \leq y_{jis} < b_{is3}) \end{cases} \quad (c_{is} \in C_1) \quad (4)$$

$$\widetilde{\mu}_{jis4} = \begin{cases} b_{is3}/y_{jis} & (y_{jis} > b_{is3}) \\ 1 & (b_{is4} \leq y_{jis} \leq b_{is3}) \\ y_{jis}/b_{is4} & (0 \leq y_{jis} < b_{is4}) \end{cases} \quad (c_{is} \in B_1) \quad (5)$$

$$\widetilde{\mu}_{jis5} = \begin{cases} b_{is4}/y_{jis} & (y_{jis} > b_{is4}) \\ 1 & (b_{is5} \leq y_{jis} \leq b_{is4}) \\ y_{jis}/b_{is5} & (0 \leq y_{jis} < b_{is5}) \end{cases} \quad (c_{is} \in B_1) \quad (6)$$

Similarly, for index $c_{is} \in B_2(i = 1, 2, 3; s = 1, 2, \cdots, m_s)$, the membership function $\widetilde{\mu}_{jisk}$ for safety grade $e_k(k = 1, 2, \cdots, 5)$ of route $a_j(j = 1, 2, \cdots, n)$ with regard to Level II index c_{is} is defined as follows:

$$\widetilde{\mu}_{jis1} = \begin{cases} 1 & (0 \leq y_{jis} \leq b_{is1}) \\ b_{is1}/y_{jis} & (y_{jis} > b_{is1}) \end{cases} \quad (c_{is} \in B_2) \tag{7}$$

$$\widetilde{\mu}_{jis2} = \begin{cases} y_{jis}/b_{is1} & (0 \leq y_{jis} < b_{is1}) \\ 1 & (b_{is1} \leq y_{jis} \leq b_{is2}) \\ b_{is2}/y_{jis} & (y_{jis} > b_{is2}) \end{cases} \quad (c_{is} \in B_2) \tag{8}$$

$$\widetilde{\mu}_{jis3} = \begin{cases} y_{jis}/b_{is2} & (0 \leq y_{jis} < b_{is2}) \\ 1 & (b_{is2} \leq y_{jis} \leq b_{is3}) \\ b_{is3}/y_{jis} & (y_{jis} > b_{is3}) \end{cases} \quad (c_{is} \in B_2) \tag{9}$$

$$\widetilde{\mu}_{jis4} = \begin{cases} y_{jis}/b_{is3} & (0 \leq y_{jis} < b_{is3}) \\ 1 & (b_{is3} \leq y_{jis} \leq b_{is4}) \\ b_{is4}/y_{jis} & (y_{jis} > b_{is4}) \end{cases} \quad (c_{is} \in B_2) \tag{10}$$

$$\widetilde{\mu}_{jis5} = \begin{cases} y_{jis}/b_{is4} & (0 \leq y_{jis} < b_{is4}) \\ 1 & (b_{is4} \leq y_{jis} \leq b_{is5}) \\ b_{is5}/y_{jis} & (y_{jis} > b_{is5}) \end{cases} \quad (c_{is} \in B_2) \tag{11}$$

For index $c_{is} \in B_3(i = 1, 2, 3; s = 1, 2, \cdots, m_s)$, the membership function $\widetilde{\mu}_{jisk}$ for safety grade $e_k(k = 1, 2, \cdots, 5)$ of route $a_j(j = 1, 2, \cdots, n)$ with regard to Level II index c_{is} is defined as follows:

$$\widetilde{\mu}_{jisk} = \begin{cases} 1 & (y_{jis} = o_{isk}) \\ 0 & (y_{jis} \neq o_{isk}) \end{cases} \quad (c_{is} \in B_3) \tag{12}$$

With Equations (2)–(12), it can be seen that the membership degree of route $a_j(j = 1, 2, \cdots, n)$ with respect to index $c_{is} \in C_3(i = 1, 2, 3; s = 1, 2, \cdots, m_s)$ belonging to the k th grade $e_k(k = 1, 2, \cdots, 5)$ is $\widetilde{\mu}_{jisk}$, which is expressed by the matrix below.

$$\widetilde{\boldsymbol{\mu}}_{ji} = \begin{array}{c} \\ c_{i1} \\ c_{i2} \\ \vdots \\ c_{im_i} \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & \cdots & e_5 \end{array} \\ \begin{pmatrix} \widetilde{\mu}_{ji11} & \widetilde{\mu}_{ji12} & \cdots & \widetilde{\mu}_{ji15} \\ \widetilde{\mu}_{ji21} & \widetilde{\mu}_{ji22} & \cdots & \widetilde{\mu}_{ji25} \\ \vdots & \vdots & \cdots & \vdots \\ \widetilde{\mu}_{jim_i1} & \widetilde{\mu}_{jim_i2} & \cdots & \widetilde{\mu}_{jim_i5} \end{pmatrix} \end{array}$$

This function is sometimes abbreviated as $\widetilde{\boldsymbol{\mu}}_{ji} = (\widetilde{\mu}_{jisk})_{m_i \times 5}$ and addressed as the grade eigenvalue matrix for route $a_j(j = 1, 2, \cdots, n)$ concerning Level II Index Set C_i. In the matrix, the s th column refers to the eigenvalue vector of route a_j with respect to Level II Index Set c_{is} belonging to $e_k(k = 1, 2, \cdots, 5)$ and is denoted as

$$\widetilde{\boldsymbol{\mu}}_{jis} = (\widetilde{\mu}_{jis1}, \widetilde{\mu}_{jis2}, \cdots, \widetilde{\mu}_{jis5})(j = 1, 2, \cdots, n)$$

Generally, $\sum\limits_{k=1}^{5} \widetilde{\mu}_{jisk} \neq 1$. For the convenience of computing, $\widetilde{\boldsymbol{\mu}}_{jis}$ is normalized. The membership degree of route a_j with respect to Level II index c_{is} belonging to grade $e_k(k = 1, 2, \cdots, 5)$ is:

$$\mu_{jisk} = \widetilde{\mu}_{jisk} / (\sum_{k=1}^{5} \widetilde{\mu}_{jisk}) \tag{13}$$

Using Equation (13), $\widetilde{\boldsymbol{\mu}}_{ji} = (\widetilde{\mu}_{jisk})_{m_i \times 5}$ can be transformed into the following matrix:

$$\boldsymbol{\mu}_{ji} = \begin{array}{c} \\ c_{i1} \\ c_{i2} \\ \vdots \\ c_{im_i} \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & \cdots & e_5 \end{array} \\ \begin{pmatrix} \mu_{ji11} & \mu_{ji12} & \cdots & \mu_{ji15} \\ \mu_{ji21} & \mu_{ji22} & \cdots & \mu_{ji25} \\ \vdots & \vdots & \cdots & \vdots \\ \mu_{jim_i1} & \mu_{jim_i2} & \cdots & \mu_{jim_i5} \end{pmatrix} \end{array}$$

3.2. The Principles and Procedure of the Variable Weight VIKOR Assessment Method

Considering the influence of grades (i.e., information of location distribution), the subscript k (the grade variable) of grade e_k is deemed as "variable weight" and the grade eigenvalue of route $a_j (j = 1, 2, \cdots, n)$ regarding to Level II Index c_{is} is:

$$v_{jis} = (1, 2, \cdots, 5)(\mu_{jis1}, \mu_{jis2}, \cdots, \mu_{jis5})^T = \sum_{k=1}^{5} k\mu_{jisk} (s = 1, 2, \cdots, m_i) \tag{14}$$

The entropy of index c_{is} is defined as

$$\phi_{is} = -[\sum_{j=1}^{n} (v_{jis}/5) \ln(v_{jis}/5)] / \ln n \quad (s = 1, 2, \cdots, m_i)$$

With the entropy of index c_{is}, the weight of Level II index c_{is} can be computed as

$$w_{is} = (1 - \phi_{is}) / \sum_{s=1}^{m_i} (1 - \phi_{is}) (s = 1, 2, \cdots, m_i) \tag{15}$$

According to the variable weighted decision method [36,37], suppose that $\boldsymbol{u}_i(\boldsymbol{v}_{ji})$ is a variable weight state vector, which is written as $\boldsymbol{u}_i(\boldsymbol{v}_{ji}) = (u_{i1}(\boldsymbol{v}_{ji}), u_{i2}(\boldsymbol{v}_{ji}), \cdots, u_{im_i}(\boldsymbol{v}_{ji}))^T$. Then, the constant weight vector $\boldsymbol{w}_i = (w_{i1}, w_{i2}, \cdots, w_{im_i})^T$ is multiplied by vector $\boldsymbol{u}_i(\boldsymbol{v}_{ji})$ (the normalisation) and their Hardarmard is defined as vector $\boldsymbol{w}_i(\boldsymbol{v}_{ji}) = (w_{i1}(\boldsymbol{v}_{ji}), w_{i2}(\boldsymbol{v}_{ji}), \cdots, w_{im_i}(\boldsymbol{v}_{ji}))^{\mathrm{T}}$. In other words,

$$\boldsymbol{w}_i(\boldsymbol{v}_{ji}) = \frac{\boldsymbol{w}_i \otimes \boldsymbol{u}_i(\boldsymbol{v}_{ji})}{\sum\limits_{t=1}^{m_i} w_{it} u_{it}(\boldsymbol{v}_{ji})} = (\frac{w_{i1} u_{i1}(\boldsymbol{v}_{ji})}{\sum\limits_{t=1}^{m_i} w_{it} u_{it}(\boldsymbol{v}_{ji})}, \frac{w_{i2} u_{i2}(\boldsymbol{v}_{ji})}{\sum\limits_{t=1}^{m_i} w_t u_{it}(\boldsymbol{v}_{ji})}, \cdots, \frac{w_{im_i} u_{im_i}(\boldsymbol{v}_{ji})}{\sum\limits_{t=1}^{m_i} w_{it} u_{it}(\boldsymbol{v}_{ji})})^{\mathrm{T}} \tag{16}$$

In the above equation, sign $\otimes$ refers to the Hardarmard multiplication of the two vectors.

As $0 \leq \mu_{jisk} \leq 1$ $(j = 1, 2, \cdots, n;\ i = 1, 2, 3;\ s = 1, 2, \cdots, m_i;\ k = 1, 2, \cdots, 5)$, it is easy to know that $\boldsymbol{\mu}^{+}_{isk} = (1, 1, \cdots, 1)^T$ and $\boldsymbol{\mu}^{-}_{isk} = (0, 0, \cdots, 0)^{\mathrm{T}}$ are the positive and negative ideal vectors for index c_{is} with respect to safety grade e_k. The similarity values S_{jik} and R_{jik} of route a_j with respect to index c_i belonging to safety grade e_k are

$$S_{jik} = \sum_{t=1}^{m_i} (\boldsymbol{w}_{it}(\boldsymbol{v}_{ji}) \frac{\mu_{jitk} - \mu^{-}_{itk}}{\mu^{+}_{itk} - \mu^{-}_{itk}})^p = \sum_{t=1}^{m_i} (\frac{w_{it} u_{it}(\boldsymbol{v}_{ji}) \mu_{jitk}}{\sum\limits_{t=1}^{m_i} w_{it} u_{it}(\boldsymbol{v}_{ji})})^p = \frac{\sum\limits_{t=1}^{m_i} (w_{it} u_{it}(\boldsymbol{v}_{ji}) \mu_{jitk})^p}{(\sum\limits_{t=1}^{m_i} w_{it} u_{it}(\boldsymbol{v}_{ji}))^p}$$

$$R_{jik} = \max_{1 \leq t \leq m_i} \{((w_{it}(\boldsymbol{v}_{ji}) \frac{\mu_{jitk} - \mu^{-}_{itk}}{\mu^{+}_{itk} - \mu^{-}_{itk}}))^p\} = \max_{1 \leq t \leq m_i} \{(\frac{w_{it} u_{it}(\boldsymbol{v}_{ji}) \mu_{jitk}}{\sum\limits_{t=1}^{m_i} w_{it} u_{it}(\boldsymbol{v}_{ji})})^p\} = \max_{1 \leq t \leq m_i} \{\frac{(w_{it} u_{it}(\boldsymbol{v}_{ji}) \mu_{jitk})^p}{(\sum\limits_{t=1}^{m_i} w_{it} u_{it}(\boldsymbol{v}_{ji}))^p}\}$$

where p is the parameter of distance. This parameter is selected in terms of real situations, so $p = 1$ is chosen in this paper. The similarity measure Q_{jik} of safety grade $e_k(k = 1, 2, \cdots, 5)$ of route a_j with respect to Level I index c_i is

$$\begin{aligned} Q_{jik} &= \lambda \frac{S_{jik} - S_{ji}^-}{S_{ji}^+ - S_{ji}^-} + (1-\lambda) \frac{R_{jik} - R_{ji}^-}{R_{ji}^+ - R_{ji}^-} \\ &= \lambda \frac{\sum\limits_{t=1}^{m_i} (w_{it} u_{it}(\boldsymbol{v}_{ji}) \mu_{jitk})^p - \max\limits_{1 \le k \le 5} \{ \sum\limits_{t=1}^{m_i} (w_{it} u_{it}(\boldsymbol{v}_{ji}) \mu_{jitk})^p \}}{\max\limits_{1 \le k \le 5} \{ \sum\limits_{t=1}^{m_i} (w_{it} u_{it}(\boldsymbol{v}_{ji}) \mu_{jitk})^p \} - \min\limits_{1 \le k \le 5} \{ \sum\limits_{t=1}^{m_i} (w_{it} u_{it}(\boldsymbol{v}_{ji}) \mu_{jitk})^p \}} + (1-\lambda) \\ &\quad \frac{\max\limits_{1 \le t \le m_i} \{ (w_{it} u_{it}(\boldsymbol{v}_{ji}) \mu_{jitk})^p \} - \max\limits_{1 \le k \le 5} \{ \max\limits_{1 \le t \le m_i} \{ (w_{it} u_{it}(\boldsymbol{v}_{ji}) \mu_{jitk})^p \} \}}{\max\limits_{1 \le k \le 5} \{ \max\limits_{1 \le t \le m_i} \{ (w_{it} u_{it}(\boldsymbol{v}_{ji}) \mu_{jitk})^p \} \} - \min\limits_{1 \le k \le 5} \{ \max\limits_{1 \le t \le m_i} \{ (w_{it} u_{it}(\boldsymbol{v}_{ji}) \mu_{jitk})^p \} \}} \end{aligned} \tag{17}$$

where $S_{ji}^+ = \max\limits_{1 \le k \le 5} \{S_{jik}\}$, $S_{ji}^- = \min\limits_{1 \le k \le 5} \{S_{jik}\}$, $R_{ji}^+ = \max\limits_{1 \le k \le 5} \{R_{jik}\}$ and $R_{ji}^- = \min\limits_{1 \le k \le 5} \{R_{jik}\}$. $\lambda \in [0, 1]$ is a mixed coefficient, indicating a decision-maker's preference; $\lambda > 0.5$ shows that the decision-maker prefers to make decisions from the perspective of maximum population effect, while $\lambda < 0.5$ means that the decision-maker prefers to make decisions from the perspective of minimum individual regret; and $\lambda = 0.5$ suggests that the decision-maker make decisions from the perspective of equilibrium, representing both maximum population effect and minimum individual regret of equal importance.

The membership degree of route a_j with respect to index $c_i(i = 1, 2, 3)$ belonging to all safety grades $e_k(k = 1, 2, \cdots, 5)$ is

$$\boldsymbol{\mu}_{\mathbf{ji}} = (\mu_{ji1}, \mu_{ji2}, \cdots, \mu_{ji5}) = \left(\frac{Q_{ji1}}{\sum\limits_{k=1}^{5} Q_{jik}}, \frac{Q_{ji2}}{\sum\limits_{k=1}^{5} Q_{jik}}, \cdots, \frac{Q_{ji5}}{\sum\limits_{k=1}^{5} Q_{jik}} \right)$$

Therefore, the membership degree of route a_j with respect to Level I Index Set $C = \{c_1, c_2, c_3\}$ belonging to safety grade $e_k(k = 1, 2, \cdots, 5)$ can be expressed as the following matrix:

$$\boldsymbol{\mu}_j = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & \cdots & e_5 \end{array} \\ \begin{pmatrix} \mu_{j11} & \mu_{j12} & \cdots & \mu_{j15} \\ \mu_{j21} & \mu_{j22} & \cdots & \mu_{j25} \\ \mu_{j31} & \mu_{j32} & \cdots & \mu_{j35} \end{pmatrix} \end{array}$$

Similar to Equations (7) and (8), the grade eigenvalue of route $a_j(j = 1, 2, \cdots, n)$ with respect to Level I index c_i is defined as follows:

$$v_{ji} = (1, 2, \cdots, 5)(\mu_{ji1}, \mu_{ji2}, \cdots, \mu_{ji5})^{\mathrm{T}} = \sum_{k=1}^{5} k \mu_{jik} \ \ (i = 1, 2, 3) \tag{18}$$

The collectivity entropy of Level I index c_i is defined as:

$$\phi_i = -[\sum_{j=1}^{n} (v_{ji}/5) \ln(v_{ji}/5)] / \ln n \ (i = 1, 2, 3)$$

In combination with the collectivity entropy above, its weight can be computed:

$$w_i = (1 - \phi_i) / \sum_{i=1}^{3} (1 - \phi_i) \ (i = 1, 2, 3)$$

Then, the constant weight vector $\boldsymbol{w} = (w_1, w_2, w_3)^{\mathrm{T}}$ is multiplied by variable weight state vector $\boldsymbol{u}(\boldsymbol{v}_j) = (u_1(\boldsymbol{v}_j), u_2(\boldsymbol{v}_j), u_3(\boldsymbol{v}_j))^{T}$(the normalisation) and their Hardarmard is defined as vector $\boldsymbol{w}(\boldsymbol{v}_j) = (w_1(\boldsymbol{v}_j), w_2(\boldsymbol{v}_j), w_3(\boldsymbol{v}_j))^{\mathrm{T}}$. In other words,

$$\boldsymbol{w}(\boldsymbol{v}_j) = \frac{\boldsymbol{w} \otimes \boldsymbol{u}(\boldsymbol{v}_j)}{\sum\limits_{i=1}^{3} w_i u_i(\boldsymbol{v}_j)} = (\frac{w_1 u_1(\boldsymbol{v}_{ji})}{\sum\limits_{i=1}^{3} w_i u_i(\boldsymbol{v}_j)}, \frac{w_2 u_2(\boldsymbol{v}_{ji})}{\sum\limits_{i=1}^{3} w_i u_i(\boldsymbol{v}_j)}, \frac{w_3 u_3(\boldsymbol{v}_{j3})}{\sum\limits_{i=1}^{3} w_i u_i(\boldsymbol{v}_j)})^{\mathrm{T}} \tag{19}$$

In the above equation, sign $\otimes$ refers to the Hardarmard multiplication of the two vectors.

As $0 \leq \mu_{jik} \leq 1$ $(j = 1, 2, \cdots, n;\ i = 1, 2, 3;\ k = 1, 2, \cdots, 5)$, it is easy to know that $\mu_{ik}^{+} = (1, 1, \cdots, 1)^{\mathrm{T}}$ and $\boldsymbol{\mu}_{ik}^{-} = (0, 0, \cdots, 0)^{\mathrm{T}}$ are the positive and negative ideal vectors of Level I index c_i with respect to e_k. The similarity degrees S_{jk} and R_{jk} of route a_j in regard to safety grade e_k are

$$S_{jk} = \sum_{i=1}^{3} (\frac{w_i u_i(\boldsymbol{v}_j)}{\sum\limits_{i=1}^{3} w_i u_i(\boldsymbol{v}_j)} \frac{\mu_{jik} - \mu_{ik}^{-}}{\mu_{ik}^{+} - \mu_{ik}^{-}})^{p} = \sum_{i=1}^{3} (\frac{w_i u_i(\boldsymbol{v}_j)}{\sum\limits_{i=1}^{3} w_i u_i(\boldsymbol{v}_j)} \mu_{jik})^{p} = \frac{\sum\limits_{i=1}^{3} (w_i u_i(\boldsymbol{v}_j) \mu_{jik})^{p}}{(\sum\limits_{i=1}^{3} w_i u_i(\boldsymbol{v}_j))^{p}}$$

$$R_{jk} = \min_{1 \leq i \leq 3} \{ (\frac{w_i u_i(\boldsymbol{v}_j)}{\sum\limits_{i=1}^{3} w_i u_i(\boldsymbol{v}_j)} \frac{\mu_{jik} - \mu_{ik}^{-}}{\mu_{ik}^{+} - \mu_{ik}^{-}})^{p} \} = \min_{1 \leq i \leq 3} \{ (\frac{w_i u_i(\boldsymbol{v}_j) \mu_{jik}}{\sum\limits_{i=1}^{3} w_i u_i(\boldsymbol{v}_j)})^{p} \}$$

The similarity measure Q_{jk} of route a_j concerning safety grade $e_k (k = 1, 2, \cdots, 5)$ is

$$\begin{aligned} Q_{jk} &= \lambda \frac{S_{jk} - S_j^{-}}{S_j^{+} - S_j^{-}} + (1 - \lambda) \frac{R_{jk} - R_j^{-}}{R_j^{+} - R_j^{-}} \\ &= \lambda \frac{\sum\limits_{i=1}^{3} (w_i u_i(\boldsymbol{v}_{ji}) \mu_{jik})^{p} - \max\limits_{1 \leq k \leq 5} \{ \sum\limits_{i=1}^{3} (w_i u_i(\boldsymbol{v}_{ji}) \mu_{jik})^{p} \}}{\max\limits_{1 \leq k \leq 5} \{ \sum\limits_{i=1}^{3} (w_i u_i(\boldsymbol{v}_{ji}) \mu_{jik})^{p} \} - \min\limits_{1 \leq k \leq 5} \{ \sum\limits_{i=1}^{3} (w_i u_i(\boldsymbol{v}_{ji}) \mu_{jik})^{p} \}} + (1 - \lambda) \\ &\quad \frac{\max\limits_{1 \leq i \leq 3} \{ (w_i u_i(\boldsymbol{v}_{ji}) \mu_{jik})^{p} \} - \max\limits_{1 \leq k \leq 5} \{ \max\limits_{1 \leq i \leq 3} \{ (w_i u_i(\boldsymbol{v}_{ji}) \mu_{jik})^{p} \} \}}{\max\limits_{1 \leq k \leq 5} \{ \max\limits_{1 \leq i \leq 3} \{ (w_i u_i(\boldsymbol{v}_{ji}) \mu_{jik})^{p} \} \} - \min\limits_{1 \leq k \leq 5} \{ \max\limits_{1 \leq i \leq 3} \{ (w_i u_i(\boldsymbol{v}_{ji}) \mu_{jik})^{p} \} \}} \end{aligned} \tag{20}$$

where $S_j^{+} = \max\limits_{1 \leq k \leq 5} \{S_{jk}\}$, $S_j^{-} = \min\limits_{1 \leq k \leq 5} \{S_{jk}\}$, $R_j^{+} = \max\limits_{1 \leq k \leq 5} \{R_{jk}\}$ and $R_j^{-} = \min\limits_{1 \leq k \leq 5} \{R_{jk}\}$.

In accordance with Equation (16), the comprehensive membership vector of route a_j concerning all safety grades $e_k (k = 1, 2, \cdots, 5)$ is

$$\boldsymbol{\mu}_j = (\mu_{j1}, \mu_{j2}, \cdots, \mu_{j5}) = (\frac{Q_{j1}}{\sum\limits_{k=1}^{5} Q_{jk}}, \frac{Q_{j2}}{\sum\limits_{k=1}^{5} Q_{jk}}, \cdots, \frac{Q_{j5}}{\sum\limits_{k=1}^{5} Q_{jk}}) \tag{21}$$

At the same time, the similarity degree vector of route a_j with respect to all safety grades $e_k (k = 1, 2, \cdots, 5)$ can be normalised as, respectively:

$$\begin{aligned} \boldsymbol{S}_j &= (s_{j1}, s_{j2}, \cdots, s_{j5}) = (\frac{S_{j1}}{\sum\limits_{k=1}^{5} S_{jk}}, \frac{S_{j2}}{\sum\limits_{k=1}^{5} S_{jk}}, \cdots, \frac{S_{j5}}{\sum\limits_{k=1}^{5} S_{jk}}) \\ &= (\frac{\sum\limits_{i=1}^{3} (w_i u_i(\boldsymbol{v}_j) \mu_{ji1})^{p}}{\sum\limits_{k=1}^{5} \sum\limits_{i=1}^{3} (w_i u_i(\boldsymbol{v}_j) \mu_{jik})^{p}}, \frac{\sum\limits_{i=1}^{3} (w_i u_i(\boldsymbol{v}_j) \mu_{ji2})^{p}}{\sum\limits_{k=1}^{5} \sum\limits_{i=1}^{3} (w_i u_i(\boldsymbol{v}_j) \mu_{jik})^{p}}, \cdots, \frac{\sum\limits_{i=1}^{3} (w_i u_i(\boldsymbol{v}_j) \mu_{ji5})^{p}}{\sum\limits_{k=1}^{5} \sum\limits_{i=1}^{3} (w_i u_i(\boldsymbol{v}_j) \mu_{jik})^{p}}) \end{aligned}$$

$$\boldsymbol{r}_j = (r_{j1}, r_{j2}, \cdots, r_{j5}) = (\frac{R_{j1}}{\sum\limits_{k=1}^{5} R_{jk}}, \frac{R_{j2}}{\sum\limits_{k=1}^{5} R_{jk}}, \cdots, \frac{R_{j5}}{\sum\limits_{k=1}^{5} R_{jk}})$$
$$= (\frac{\min\limits_{1 \le i \le 3}\{(w_i u_i(\boldsymbol{v}_j)\mu_{ji1})^p\}}{\sum\limits_{k=1}^{5}\min\limits_{1 \le i \le 3}\{(w_i u_i(\boldsymbol{v}_j)\mu_{jik})^p\}}, \frac{\min\limits_{1 \le i \le 3}\{(w_i u_i(\boldsymbol{v}_j)\mu_{ji2})^p\}}{\sum\limits_{k=1}^{5}\min\limits_{1 \le i \le 3}\{(w_i u_i(\boldsymbol{v}_j)\mu_{jik})^p\}}, \cdots, \frac{\min\limits_{1 \le i \le 3}\{(w_i u_i(\boldsymbol{v}_j)\mu_{ji5})^p\}}{\sum\limits_{k=1}^{5}\min\limits_{1 \le i \le 3}\{(w_i u_i(\boldsymbol{v}_j)\mu_{jik})^p\}})$$

Thus, the eigenvalue of safety grade for the environment of route a_j is

$$v_j = (1, 2, \cdots, 5)\boldsymbol{\mu}_j{}^{\mathrm{T}} = \sum_{k=1}^{5} k\mu_{jk} = \sum_{k=1}^{5} k\frac{Q_{jk}}{\sum\limits_{k=1}^{5} Q_{jk}} \tag{22}$$

To any $v_j \in [1, 5]$, its two-tuple linguistic model is defined as

$$\tau(v_j) = (e_{k_j}, \Delta_j) \in E' \times [-0.5, 0.5) \tag{23}$$

where $E' = \{1, 2, \cdots, 5\}$ is the subscript set for safety grade set $E = \{e_1, e_2, \cdots, e_5\}$ defined before. In $k_j = \mathrm{Round}(v_j)$, Round is a bracket function. $\Delta_j = v_j - k_j$ stands for the deviation between safety grade eigenvalues v_j and k_j. Afterwards, the safety grade for the navigation environment of route a_j and its deviation can be determined, respectively, according to the positive integers k_j and Δ_j of Equation (23).

4. The Calculation and Analysis of Waterway Navigation Safe Route Selection

4.1. Description of Waterway Navigation Safe Route Selection

To demonstrate the superiority of the proposed method, literature data [2] are used for reference, including three selected routes, namely Channels a_1 (the main course), a_2 (Dagusha course) and a_3 (compound course) of Tianjin Port. Channel A is the only route that vessels must take and where merely one-way navigation is available for over 10,000 DWT ships. With a total length of 44 km, it winds in broken lines. Channel C is composed of the small-ship waterways and alert areas on the north and south sides of Channel A and the two-side waterways follow the mode of entry through the northern sides and exit through the southern ones. Channel B has a total length of 27.5 km, in which most ships are from the fishing sector. These constitute the only way for ships to reach and leave the Dagusha, Nanjing-Nangang harbour areas. The data of the three routes are shown in Table 2.

Table 2. Environment data of route.

Indexes	Routes		
	a_1	a_2	a_3
c_{11}	28	25	26
c_{12}	48	42	49
c_{13}	0.8	0.6	0.7
c_{21}	0.28	0.3	0.29
c_{22}	0.09	0.09	0.09
c_{23}	28	25	35
c_{24}	47	47	48
c_{25}	0.82	1.02	1.1
c_{31}	97.4	61.6	75.6
c_{32}	O_{325}	O_{325}	O_{325}
c_{33}	O_{335}	O_{335}	O_{335}

4.2. Determination of Waterway Navigation Safe Route Selection

According to Table 1, such quantitative indexes as c_{11} are lower bound grade threshold, so the membership functions of grade $e_k (k = 1, 2, \cdots, 5)$ for alternative a_j concerning index c_{11} are:

$$\mu_{j111} = \begin{cases} 1 & (y_{j11} \ge 90) \\ y_{j11}/90 & (0 \le y_{j11} < 90) \end{cases}$$

$$\widetilde{\mu}_{j112} = \begin{cases} 90/y_{j11} & (y_{j11} > 90) \\ 1 & (50 \leq y_{j11} \leq 90) \\ y_{j11}/50 & (0 \leq y_{j11} < 50) \end{cases}$$

$$\widetilde{\mu}_{j113} = \begin{cases} 50/y_{j11} & (y_{j11} > 50) \\ 1 & (40 \leq y_{j11} \leq 50) \\ y_{j11}/40 & (0 \leq y_{j11} < 40) \end{cases}$$

$$\widetilde{\mu}_{j114} = \begin{cases} 40/y_{j11} & (y_{j11} > 40) \\ 1 & (25 \leq y_{j11} \leq 40) \\ y_{j11}/25 & (0 \leq y_{j11} < 25) \end{cases}$$

$$\widetilde{\mu}_{j115} = \begin{cases} 25/y_{j11} & (y_{j11} > 25) \\ 1 & (15 \leq y_{j11} \leq 25) \\ y_{j11}/15 & (0 \leq y_{j11} < 15) \end{cases}$$

Analogously, the membership functions of grade $e_k(k = 1, 2, \cdots, 5)$ for routes $a_j\ (j = 1, 2, 3)$ on indexes $c_{is} \in C_2(i = 1, 2, 3; s = 1, 2, \cdots, m_i)$ can be constructed. Thus, we can get through calculation the membership of indexes $c_{is} \in C_2(i = 1, 2, 3; s = 1, 2, \cdots, m_i)$ for routes $a_j\ (j = 1, 2, 3)$, and $\tilde{\boldsymbol{\mu}}_j(j = 1, 2, 3)$ is normalized, which can be expressed as follows

$$\boldsymbol{\mu}_{11} = \begin{pmatrix} 0.100 & 0.180 & 0.225 & 0.322 & 0.172 \\ 0.085 & 0.113 & 0.169 & 0.282 & 0.352 \\ 0.053 & 0.092 & 0.148 & 0.264 & 0.461 \end{pmatrix} \quad \boldsymbol{\mu}_{12} = \begin{pmatrix} 0.124 & 0.136 & 0.169 & 0.227 & 0.343 \\ 0.043 & 0.066 & 0.194 & 0.330 & 0.367 \\ 0.093 & 0.140 & 0.187 & 0.280 & 0.300 \\ 0.153 & 0.197 & 0.230 & 0.294 & 0.125 \\ 0.100 & 0.065 & 0.410 & 0.259 & 0.166 \end{pmatrix}$$

$$\boldsymbol{\mu}_{13} = \begin{pmatrix} 0.065 & 0.084 & 0.140 & 0.280 & 0.431 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \boldsymbol{\mu}_{21} = \begin{pmatrix} 0.093 & 0.167 & 0.208 & 0.333 & 0.200 \\ 0.080 & 0.107 & 0.161 & 0.268 & 0.383 \\ 0.046 & 0.080 & 0.128 & 0.213 & 0.533 \end{pmatrix}$$

$$\boldsymbol{\mu}_{22} = \begin{pmatrix} 0.124 & 0.136 & 0.169 & 0.227 & 0.343 \\ 0.043 & 0.066 & 0.194 & 0.330 & 0.367 \\ 0.090 & 0.135 & 0.180 & 0.270 & 0.324 \\ 0.153 & 0.197 & 0.230 & 0.294 & 0.125 \\ 0.075 & 0.049 & 0.383 & 0.300 & 0.193 \end{pmatrix} \quad \boldsymbol{\mu}_{23} = \begin{pmatrix} 0.055 & 0.072 & 0.120 & 0.240 & 0.513 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boldsymbol{\mu}_{31} = \begin{pmatrix} 0.095 & 0.171 & 0.214 & 0.329 & 0.190 \\ 0.085 & 0.113 & 0.170 & 0.284 & 0.347 \\ 0.049 & 0.087 & 0.138 & 0.231 & 0.495 \end{pmatrix} \quad \boldsymbol{\mu}_{32} = \begin{pmatrix} 0.124 & 0.136 & 0.169 & 0.227 & 0.343 \\ 0.043 & 0.066 & 0.194 & 0.330 & 0.367 \\ 0.099 & 0.149 & 0.199 & 0.298 & 0.255 \\ 0.155 & 0.200 & 0.233 & 0.291 & 0.121 \\ 0.068 & 0.044 & 0.372 & 0.314 & 0.202 \end{pmatrix}$$

$$\boldsymbol{\mu}_{33} = \begin{pmatrix} 0.057 & 0.075 & 0.125 & 0.249 & 0.494 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Combining with Equation (6), the weights of the indexes $c_{is}(i = 1, 2, 3; s = 1, 2, \cdots, m_i)$ with Equations (7) and (8) are calculated as follows:

$$\boldsymbol{w}_1 = (0.363, 0.333, 0.304)$$

$$\boldsymbol{w}_2 = (0.304, 0.199, 0.181, 0.199, 0.219)$$

$$\boldsymbol{w}_3 = (0.433, 0.284, 0.284)$$

According to the characteristics of waterway navigation safe routes selection, the variable weight state function of index $c_{is}(i = 1,2,3; s = 1,2,\cdots,m_i)$ is constructed as follows: $u_{is}(v_{jis}) = e^{-0.5v_{jis}}$. Combining with $\boldsymbol{w}_i(i = 1,2,3)$, the variable weight function of index $c_{is}(i = 1,2,3; s = 1,2,\cdots,m_i)$ can be expressed as follows:

$$w_{is}(\boldsymbol{v}_{ji}) = \frac{w_{is}u_{is}(v_{jis})}{\sum\limits_{t=1}^{m_i} w_{it}u_{it}(v_{jit})} = \frac{w_{is}e^{-0.5v_{jis}}}{\sum\limits_{t=1}^{m_i} w_{it}e^{-0.5v_{jit}}}$$

Combining with Equation (17) and $\lambda = 0.5$, the relative closeness degrees of routes $a_j\ (j = 1,2,3)$ for indexes $c_i(i = 1,2,3)$ on grade $e_k(k = 1,2,\cdots,5)$ can be computed and normalized; the membership degrees of routes $a_j\ (j = 1,2,3)$ for indexes $c_i(i = 1,2,3)$ on grade $e_k(k = 1,2,\cdots,5)$ can be expressed as follows:

$$\boldsymbol{\mu}_1 = \begin{pmatrix} 0.000 & 0.068 & 0.157 & 0.270 & 0.506 \\ 0.000 & 0.024 & 0.383 & 0.307 & 0.285 \\ 0.000 & 0.010 & 0.037 & 0.107 & 0.846 \end{pmatrix}$$

$$\boldsymbol{\mu}_2 = \begin{pmatrix} 0.000 & 0.051 & 0.130 & 0.240 & 0.579 \\ 0.000 & 0.021 & 0.193 & 0.212 & 0.574 \\ 0.000 & 0.011 & 0.042 & 0.119 & 0.828 \end{pmatrix}$$

$$\boldsymbol{\mu}_3 = \begin{pmatrix} 0.000 & 0.062 & 0.144 & 0.259 & 0.536 \\ 0.000 & 0.021 & 0.206 & 0.234 & 0.539 \\ 0.000 & 0.010 & 0.038 & 0.109 & 0.843 \end{pmatrix}$$

Combining with Equation (13), the weights of the indexes $c_i(i = 1,2,3)$ are calculated as follows:

$$\boldsymbol{w}= (0.252, 0.365, 0.283)$$

Similarity, the variable weight state function of index $c_i(i = 1,2,3)$ is constructed as follows: $u_i(v_{ji}) = e^{-0.5v_{ji}}$. Combining with $\boldsymbol{w}$, the variable weight function of index $c_i(i = 1,2,3)$ can be expressed as follows:

$$w_i(\boldsymbol{v}_j) = \frac{w_i u_i(v_{ji})}{\sum\limits_{t=1}^{3} w_t u_t(v_{ji})} = \frac{w_i e^{-0.5v_{ji}}}{\sum\limits_{t=1}^{3} w_t e^{-0.5v_{jt}}}$$

Combining with Equation (20) and $\lambda = 0.5$, the membership and grade assessment eigenvalues of grade assessment $e_k(i = 1,2,\cdots,5)$ for routes $a_j\ (j = 1,2,3)$ are computed as shown in Table 3.

Table 3. Results for grade assessment of waterway navigation safe routes ($\lambda = 0.5$).

Routes	The Membership Degree of Safe Rating					Grade Assessment Eigenvalues	Grade Assessment	The Maximum Membership Degree Method
	e_1	e_2	e_3	e_4	e_5			
a_1	0.000	0.042	0.288	0.260	0.409	4.036	e_3	e_5
a_2	0.000	0.039	0.161	0.211	0.589	4.351	e_3	e_5
a_3	0.000	0.046	0.171	0.227	0.555	4.292	e_3	e_5

Similarily, combining with Equations (14)–(16), when $\lambda = 1$ and $\lambda= 0$, the membership and grade assessment eigenvalues of grade assessment $e_k(i = 1,2,\cdots,5)$ for routes $a_j\ (j = 1,2,3)$ are computed as shown in Tables 4 and 5.

According to Tables 4–6, it is unfeasible to distinguish the only, accurate, and proper grade through adopting the maximum membership principle. On the contrary, the two-tuple linguistic method not only evaluates information reasonably and assumes grade information and deviation, but also demonstrates security degrees for night ships in different waterways. This paper uses a variable VIKOR method to assess security grades of night ships in various channels, taking into account both group benefits and individual regrets, while overcoming the flaws of previous VIKOR methods which merely address the problem of ranking and information compensation among diverse indices.

Table 4. Results for grade assessment of waterway navigation safe routes ($\lambda = 1$).

Routes	The Membership Degree of Safe Rating					Grade Assessment Eigenvalues	Grade Assessment	The Maximum Membership Degree Method
	e_1	e_2	e_3	e_4	e_5			
a_1	0.000	0.034	0.214	0.294	0.458	4.175	e_3	e_5
a_2	0.000	0.021	0.292	0.187	0.500	4.166	e_3	e_5
a_3	0.000	0.031	0.158	0.244	0.567	4.346	e_3	e_5

Table 5. Results for grade assessment of waterway navigation safe routes ($\lambda = 0$).

Routes	The Membership Degree of Safe Rating of					Grade Assessment Eigenvalues	Grade Assessment	The Maximum Membership Degree Method
	e_1	e_2	e_3	e_4	e_5			
a_1	0.000	0.058	0.383	0.182	0.377	3.878	e_3	e_3
a_2	0.000	0.059	0.149	0.192	0.600	4.332	e_3	e_5
a_3	0.000	0.067	0.163	0.206	0.563	4.266	e_3	e_5

The waterway navigation safe route selection problem of Section 4.1 is solved by Wang et al. [38] and the membership and grade assessment eigenvalues of grade assessment $e_k(i = 1, 2, \cdots, 5)$ for routes a_j $(j = 1, 2, 3)$ are shown as Table 6.

Table 6. Results for grade assessment of waterway navigation safe routes.

Routes	The Membership Degree of Safe Rating of					Grade Assessment Eigenvalues	Grade Assessment	The Maximum Membership Degree
	e_1	e_2	e_3	e_4	e_5			
a_1	0.177	0.106	0.172	0.115	0.430	3.514	e_3	e_5
a_2	0.171	0.098	0.159	0.114	0.458	3.588	e_3	e_5
a_3	0.174	0.102	0.165	0.117	0.441	3.549	e_3	e_5

5. Conclusions

In order to select safe courses for waterway navigation, the evaluation model and method of the waterway navigation safe route selection based on variable weight VIKOR considering the individual feelings and group benefits of information, taking account of no-compensation information between indexes. The advantages of the proposed method can be summed up as:

(1) Security grade division, evaluation index systems and grade thresholds for the navigation waterways of night ships are constructed.

(2) The method to determine the index weight based on entropy and then the variable weight VIKOR method are proposed, the latter of which gives consideration to both group benefits and individual regrets. It not only overcomes the problem that the previous ranking evaluation methods only consider group benefits, but also overcomes the shortcomings that VIKOR method itself only solves the problem of ranking and information compensation among indices. This is an expansion and development of VIKOR methods.

(3) The results of using two-tuple linguistic information to measure the security grade of ships' night navigation channels reflect the grade information and deviation, judge the security level of each ship's night navigation waterways, and overcome the

shortcomings of the maximum-membership-principle method, and further improve the evaluation method.

The variable weight VIKOR method to assess security grades for the navigation routes of night ships proposed in this paper can also be used to solve other management decision-making problems such as those about ecological, supply chain and network security, or to study issues of group consensus [39], large group risk decision making [40], the green economy [41–44] and portfolio selection [45].

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