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Ship's Wake on a Finite Water Depth in the Presence of a Shear Flow

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Abstract: The ship's wake in the presence of a shear flow of constant vorticity at a finite water depth is investigated by expanding the Whitham-Lighthill kinematic theory. It has been established that the structure of a wave ship wake radically depends on Froude number Fr (in terms of water depth) and especially on the critical Froude number Fr_{cr} , which depends on the magnitude and direction of the shear flow. At its subcritical values $Fr < Fr_{cr}$ two types of waves are presented: long transverse and short divergent waves inside the wedge area with an angle depending on Fr . For the supercritical range $Fr > Fr_{cr}$ only divergent waves are presented inside the wake region. Critical Froude number Fr_{cr} is variable and mostly depends on collinear with the ship path component of the shear flow: it decreases with the unidirectional shear flow and increases on the counter shear current. The wedge angle of the ship wake expands with an increase in the unidirectional shear flow and narrows in the oncoming flow in the subcritical mode of the ship's motion $Fr < Fr_{cr}$. Wake angle decreased with Froude number for $Fr > Fr_{cr}$ and only divergent waves with the crests almost collinear with ship path are finally presented in the narrow ship wake. For a critical value of the Froude number $Fr = Fr_{cr}$, the ship's wake has a total wedge angle of 180° with waves directed parallel to the ship's motion. The presence of a shear flow crossing the path of the ship gives a strong asymmetry to the wake. An increase in the perpendicular shear flow leads to an increase in the difference between the angles of the wake arms.

Keywords: ship wake; wake angle; transverse and divergent waves; shear current; finite water depth

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1. Introduction

Managing the maneuvering of ships in the coastal waters of the ocean is an important practical task. Estimation of the speed and direction of the ship's movement can be made using remote sensing methods of the sea surface. SAR methods are able to reliably record ship wakes extending over kilometers, according to the characteristics of which it is possible to monitor ships, which is an important part of shipping management. SAR techniques used to locate ships have evolved significantly over the past few decades. Vessels may be almost invisible on radar images due to their small size, but a large-scale ship trail can provide important information about the ship, its length, speed, direction of movement, etc. [1,2]. In recent years, a number of effective methods for recognizing radar images have been proposed to analyze the trajectories of ships. The direction of the vessel's movement and its speed can be estimated both by the angle of the ship's wake, and by its asymmetry, the wavelengths along the edges of the wake, etc. [2–5]. Thus, the quantitative assessment of the geometrical properties of the Kelvin ship wake is an urgent task in demand. A significant part of the ship's energy is spent in the wave wake [6–8], which is why the wave resistance properties are so important in shipbuilding.

A modern idea of a ship wake includes several regions. Around and directly behind the ship, the wake is quite complex, with bow and stern waves, vortexes, currents, and

foam. Starting at about three ship lengths behind the ship, the main features of a ship's wake are somewhat universal and do not depend critically on the shape of the ship or the propeller. Here, the wake of a ship combines two phenomena: a turbulent wake, i.e., foam, a strip of turbulent water, and a Kelvin wake, i.e., a characteristic wave pattern far behind the ship.

The wake structure on the water surface behind the ship was first studied by Lord Kelvin (Thomson) in 1887 [9]. According to his theory, the far wake is quite universal and does not depend on the speed and shape of the ship in deep water conditions. The wake consists of two types of waves: transverse waves with crests across the path of the vessel and divergent waves moving outward. The wake is wedge-shaped, and the wake angle is approximately 39° . Waves of greatest amplitude are located at the edges of the wake, called Kelvin arms, and waves in this region are called cusp waves.

Numerous observations, however, have shown that, along with a qualitative agreement with the theoretical predictions of Kelvin, the wake of a ship can have significant variability [10–12]. The wake angle can be either narrower or much wider than the Kelvin angle [12,13]. Wake angle statistics [13] based on the results of remote sensing shows that in most observations the wake angles are less than those predicted by Kelvin. Another difference may be the asymmetry of the wake of the ship—quite often, on SAR images, the angles of the wake arms differ or only one of the Kelvin arms is visible [13–16].

There are several reasons for the existing discrepancies between the theoretical Kelvin model and the results of observations of the ship wake structure. First of all, the classical model is based on the approximation of deep water and the movement of the ship in the absence of wind waves and currents in calm water. Accounting for the finite depth of the sea and the presence of wind surface waves can significantly change the structure of the ship wake [17,18]. The angle of the wake and the characteristics of the ship's waves in this case fundamentally depend on the speed of the ship, the depth of the water and the field of wind surface waves.

The use of the theory of linear waves on water also greatly simplified the problem [19,20] and, in addition to nonlinearity, did not take into account all the effects of the hull shape, the interaction of bow and stern waves, which does not allow adequate modeling of various types of waves near the ship.

Another reason for the variability of the ship wake can be the presence of shear currents that are variable in depth. Such currents are often observed in coastal areas, caused by wind drift, tides, variable bottom topography, etc. Such shear currents can radically change the characteristics of surface waves, including ship waves [21,22]. A large bank of works is devoted to the influence of arbitrary shear flows on the propagation of surface waves in two and three dimensions [21–27]. The dispersion properties of waves reveal their significant dependence on the magnitude and direction of the shear currents.

It should be emphasized the huge contribution to the theory and experimental observations of ship waves made by Ellingsen and co-authors over the past decade [7,8,23,26–30]. The three-dimensional problem of the ship wake on a shear flow of constant vorticity in the deep-water approximation was studied in [28]. The linear model based on considering the ship as a moving pressure source was analyzed and the asymptotic method of the stationary phase was used to calculate the far wake. The paper shows a strong dependence of the wake structure on the characteristics of the shear flow: the ship wake angle can be both large and small, and the wake can be asymmetric due to three-dimensional interaction with the shear flow. The effects of finite water depth in the same type of simulation were studied for this problem in [7]. Surface waves on currents with arbitrary vertical shear were analyzed by Smeltzer and Ellingsen in [29]. Experimental observations of ship waves distorted by subsurface shear currents were published in [30], which confirmed the effects predicted by Ellingsen's theory. Ship waves moving across the subsurface current on a stationary surface show a striking asymmetry with respect to the line of motion of the ship, and large differences in transverse wavelengths for upstream and downstream movement are demonstrated, which is fully consistent with

theoretical predictions. The ship wave resistance in the presence of shear current was calculated by Ellingsen and coauthors containing a non-zero lateral component [7,8]. Solutions were derived for ship waves generated by a wave source acting on the free surface, with the source's shape and time-dependence is being arbitrary. The effect of shear flow on wave resistance has been analyzed in detail, including the effect of finite water depth. Another theoretical ship wake model was originally proposed by Lighthill and Whitham [31,32]. The presented kinematic model reproduced the main features of the Kelvin solution and attracted attention with its simplicity and clarity. An extension of the ship's wake kinematic model for a finite water depth and in the presence of wind waves was carried out in [17,18]. More recently, we presented a model of a ship wake on a shear flow at an infinite water depth [33].

This study continues the series of studies on the ship wake based on a kinematic model and is devoted to the wave wake structure on a shear flow of constant vorticity in water of finite depth. The presented model has a completely different approach from the classical one, based on the consideration of the ship as a moving pressure distribution on the sea surface. Nontrivial numerical integration for each specific oceanographic situation in the classical solution scheme requires a lot of time and has its own limitations. Despite its relative simplicity, our kinematic model reproduces the main results similar to those obtained using the classical Ellingsen scheme [7,28], which, in turn, can serve as a test of the applicability of the model. Here we can give typical patterns of the phase lines of ship waves, characteristics of cusp waves, critical values of the shear flow gradient during transverse interaction, etc. As an advantage of the presented model, it can be noted that its analytical solution makes it possible to explicitly describe the structure of ship waves in the entire range changes in the defining parameters of the problem, and not just its typical behavior and limiting cases.

The paper is organized as follows. Section 2 presents a theoretical model of the ship wake in the presence of a horizontal shear flow at a finite water depth and describes a method for solving the problem. Section 3 describes the results of wave modelling for the following variants of the mutual ship path and shear current directions:

1. Collinear (Section 3.1),
2. Cross (Section 3.2),
3. Inclined (Section 3.3).

Conclusions are presented in Section 4.

2. Ship Wake Model

We'll analyze stationary waves behind a ship at a finite water depth $H < \infty$ in the presence of a horizontal shear flow. The geometric scheme of the problem is shown in Figure 1. Surface waves with a wavenumber vector $\vec{k} = (k_X, k_Y) = (-k \cos \psi, k \sin \psi)$ (see Figure 1), form the ship's wake, in the presence of a shear flow $\vec{U}_c = \vec{U}_0 + \vec{U}'_0 z$, which varies linearly with water depth z . The moving frame of reference is connected with the ship fixed at its beginning (P) flown around by a uniform flow $\vec{U} = -(\vec{U}_{ship} + \vec{U}_0)$ moving in the positive direction along the X axis. $\vec{U}_{ship}$ is the ship's speed relative to the moving water surface, $\vec{U}'_0 = (\vec{U}'_{0X}, \vec{U}'_{0Y})$ is the horizontal vector of a shear flow gradient. We also assume the approximation of an incompressible and inviscid fluid.

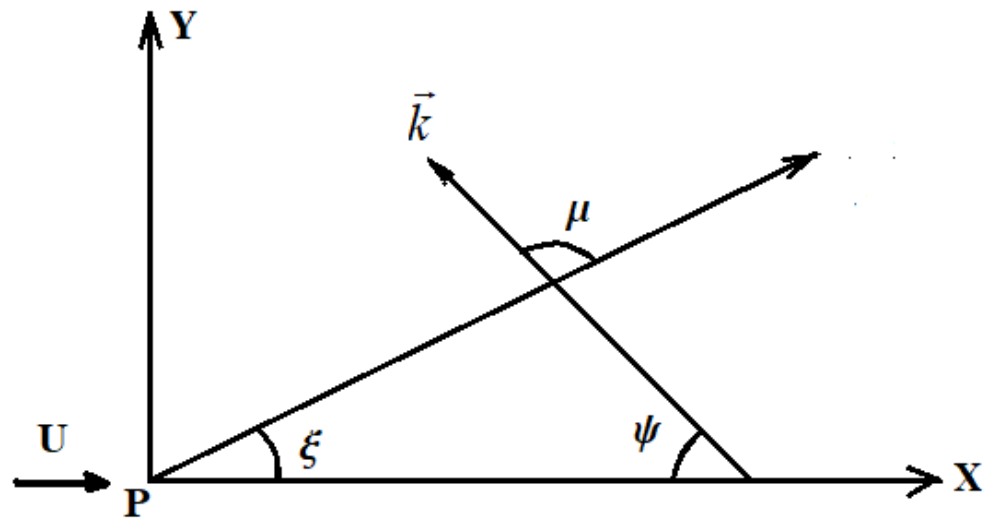


Figure 1. The geometry of waves behind the ship. $\vec{U}$ is the flow velocity, $\vec{k}$ —wave number vector of surface waves.

The problem will be analyzed by the methods of kinematics wave theory [31,32]. The wave field behind the ship is governed by the conservation law for the number of waves [32,34,35]:

$$\frac{\partial \vec{k}}{\partial t} + \nabla \omega = 0 \quad (1)$$

where ω —is the frequency of waves, and consistency equation for the phase of waves function [32,34,35]:

$$\frac{\partial k_X}{\partial Y} = \frac{\partial k_Y}{\partial X} \quad (2)$$

The linear dispersion equation for surface gravity waves at a finite water depth, including the Doppler frequency shift and the influence of the shear current, has the well-known form [7,28]:

$$G(k_X, k_Y) = (\omega - \vec{k} \cdot \vec{U})^2 + (\omega - \vec{k} \cdot \vec{U}) \frac{(\vec{k} \cdot \vec{U}_0)'}{|\vec{k}|} \tanh(|\vec{k}|H) - |\vec{k}| g \tanh(|\vec{k}|H) = 0 \quad (3)$$

where g —gravity acceleration.

We consider a stationary field of surface waves with zero frequency in the selected coordinate system; therefore, Equation (1) is identically valid.

So, we have two Equations (2) and (3) for a pair of unknown functions (k_X, k_Y) . Equation (3), in principle, allows us to determine the function $k_X = f(k_Y)$:

$$f'(k_Y) = -\frac{G_{k_Y}(k_X, k_Y)}{G_{k_X}(k_X, k_Y)} \quad (4)$$

Equation (2) can therefore be interpreted as an equation for the k_Y function and may be represented in the following form:

$$\frac{\partial k_Y}{\partial X} = f'(k_Y) \frac{\partial k_Y}{\partial Y} \quad (5)$$

The system of characteristics for Equation (4) is:

$$\frac{dY}{dX} = -f'(k_Y), \quad \frac{dk_Y}{dX} = 0 \quad (6)$$

Characteristics are the straight lines with a constant wavenumber along it:

$$\tan \xi = Y/X = -f'(k_Y) = \frac{G_{k_Y}(k_X, k_Y)}{G_{k_X}(k_X, k_Y)}, \quad k_Y, k_X = \text{const} \quad (7)$$

here ξ is the angle of characteristic (see Figure 1).

3. Results

3.1. Ship Wake at Finite Water Depth

First, we analyze the stationary structure of the wake of a ship for a finite water depth without considering the shear current ($\vec{U}'_0 = 0$). The dispersion relation for surface gravity waves (3) in this case is simplified as follows:

$$k_X^2 U^2 - \left| \vec{k} \right| g \tanh\left(\left| \vec{k} \right| H\right) = 0, \quad (8)$$

where U is the speed of the surface flow.

The equation of characteristics (5) takes the form:

$$\tan \xi = Y/X = \frac{\sqrt{K Fr^2 / \tanh[K] - 1} \left(\text{sech}[K]^2 + \tanh[K]/K \right)}{-\text{sech}[K]^2 - \tanh[K]/K + 2 Fr^2}, \quad (9)$$

where $K = H\sqrt{k_X^2 + k_Y^2}$ and $Fr = U/\sqrt{gH}$ —is a Froude number for the considering problem.

The ship wake waves geometry critically depends from the value of Froude number.

The direction of ship waves: $-k_Y/k_X = \tan(\psi)$ depending on the ray angle $\tan \xi$ is shown in Figure 2 for low ($Fr < 1$) and high ($Fr > 1$) Froude numbers.

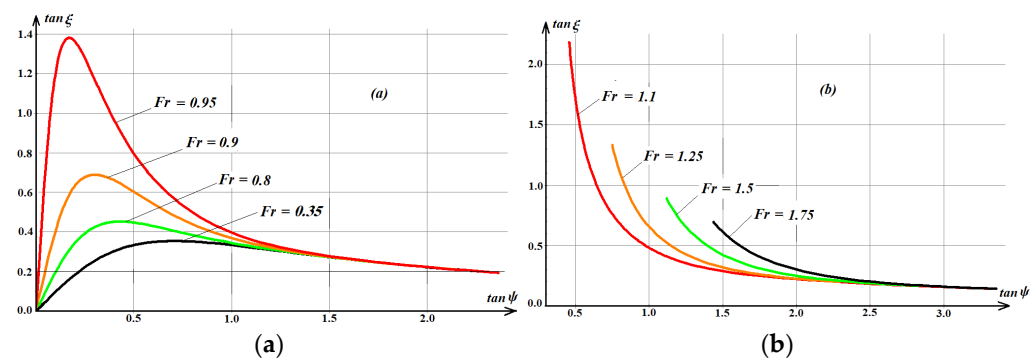


Figure 2. Directions of ship waves ψ : $-k_Y/k_X = \tan(\psi)$ depending on the ray angle $\tan \xi$ for different values of Froude number: $Fr = U/\sqrt{gH}$: (a) $Fr < 1$; (b) $Fr > 1$.

For every value of $Fr < 1$ two types of waves are distinguished: long transverse and short divergent waves covering the wedge area with an angle at the origin depending on Fr (Figure 2a). The ship's total wake angle increases with Fr number up to 180° as it approaches one. The ship wake is principally modified for $Fr > 1$ (Figure 2b). Only divergent waves are presented for every ray inside the region. Wake angle decreased with Froude number from 180° for $Fr \sim 1$ to really small values for $Fr \gg 1$ and only short waves with the crests almost collinear with ship track direction are finally presented in the narrow ship wake. For a critical value of the Froude number $Fr_{cr} = 1$, the ship's wake has a total wedge angle of 180° with waves directed parallel to the ship's motion. The direction of

cuspl waves at the edges of wake in dependence on the Fr number is shown in Figure 3. The angle of their propagation decreases with increasing Froude number from 35.3° up to zero in the interval $(0 < Fr < 1)$ and, conversely, increases up to 90° for $(Fr > 1)$.

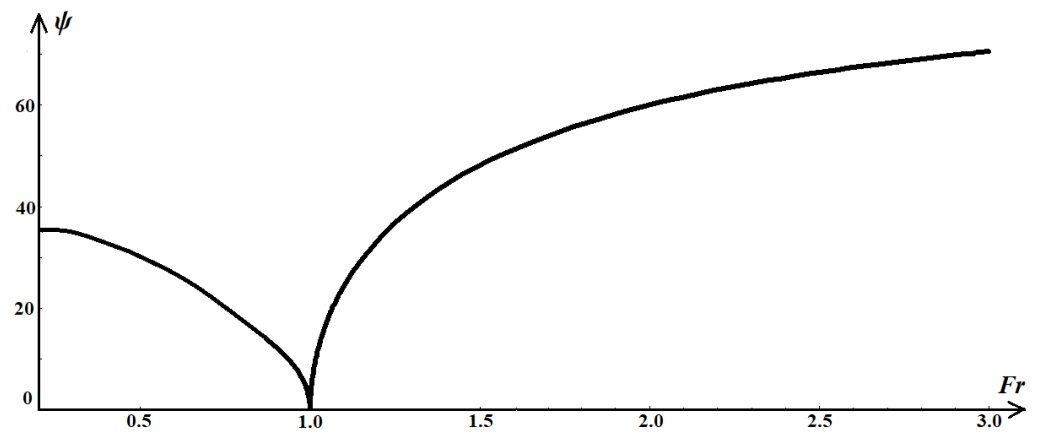


Figure 3. The direction of cuspl waves at the edges of wake in dependence of Fr number.

3.2. Collinear Propagation of the Ship and Shear Flow

The dispersion relation (3) for the collinear ship propagation and shear current along the X-axis takes the form:

$$G(k_x, k_y) = k_x^2 (Fr^2 + W_X \frac{\tanh(\frac{|\vec{k}|}{H})}{|\vec{k}|}) - |\vec{k}|^2 \frac{\tanh(\frac{|\vec{k}|}{H})}{|\vec{k}|} = 0, \quad W_X = -\frac{U U_{0X}'}{g}, \quad (10)$$

where W_X is a dimensionless parameter that characterizes the shear flow.

The ray Equation (5) takes the form:

$$\tan \zeta = \frac{\sqrt{KR/\tanh[K]-1} \left(\text{sech}[K]^2 + \tanh[K]/K - W_X \tanh[K]/K/R / (\text{sech}[K]^2 - \tanh[K]/K) \right)}{-\text{sech}[K]^2 - \tanh[K]/K + W_X \tanh[K]/K/R (\text{sech}[K]^2 - \tanh[K]/K) + 2R}; \quad (11)$$

$$R = Fr^2 + W_X \tanh[K]/K.$$

The structure of a ship's wake in a shear flow, collinear to the ship's motion, has both common features and fundamental differences compared to a ship's wake in calm water. Depending on the Froude number, two completely different wave patterns of the ship's wake are still distinguished: in the subcritical region $(Fr < Fr_{cr})$, two systems of waves are represented—transverse and divergent, and in the supercritical region $(Fr > Fr_{cr})$ —only divergent waves. Dependence of wake angle ζ_m from the Froude number for different values of the shear flow W_X is presented in Figure 4. The ship's total wedge angle increases with Fr number for $Fr < Fr_{cr}$ decreased with Froude number for $Fr > Fr_{cr}$.

The value of the critical Froude number Fr_{cr} itself is variable and depends on the magnitude of the shear flow $Fr_{cr} = Fr_{cr}(W_X)$; this function is presented in Figure 5.

Critical Froude number decreases with the unidirectional shear flow ($W_X > 0$) up to zero for $W_X = 1$ and increases on the counter shear current.

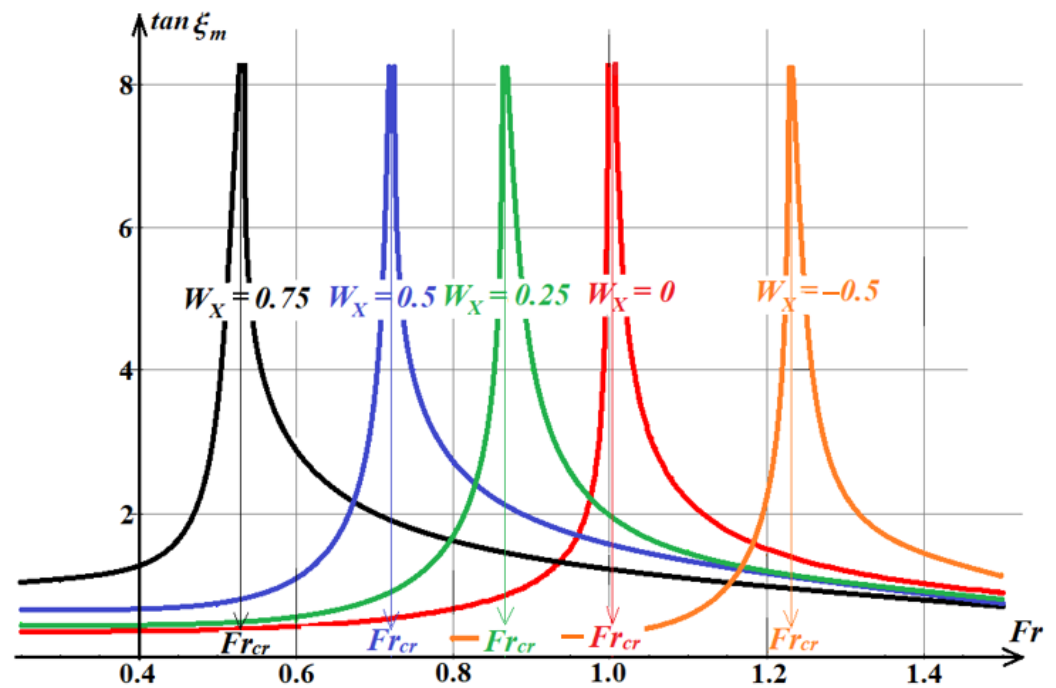


Figure 4. Dependence of wake angle ξ_m from the Fr number for different values of the shear flow W_X .

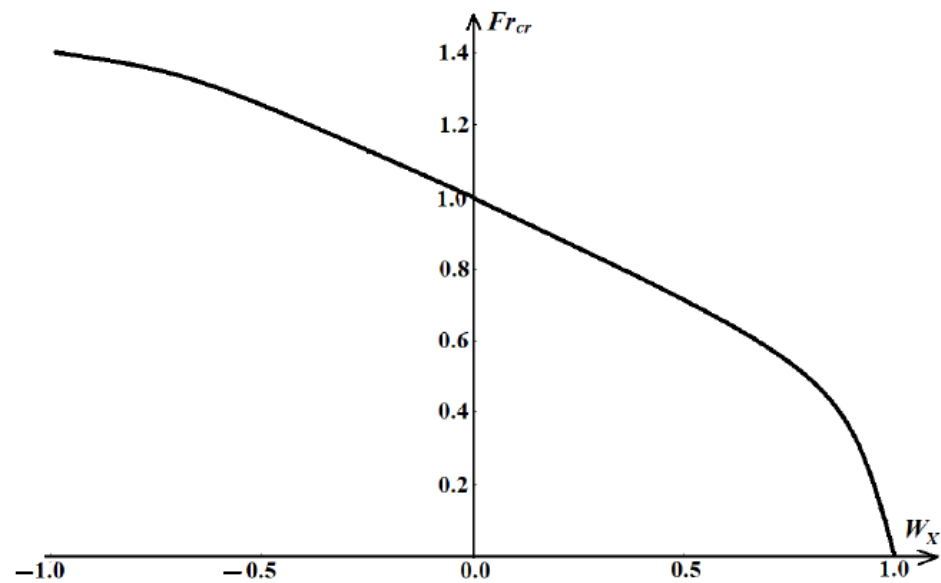


Figure 5. Dependence of the critical Froude number Fr_{cr} from the shear flow gradient W_X .

3.3. Cross Shear Flow

The dispersion relation (3) for the ship waves in the presence of the shear current perpendicular to the ship's speed ($U_{0Y}' \neq 0, U_{0X}' = 0$) has the form:

$$G(k_x, k_y) = k_X^2 Fr^2 + W_Y \frac{\tanh[K]}{K} - \left| \vec{k} \right|^2 \frac{k_X k_Y \tanh[K]}{K} = 0, W_Y = -\frac{U U_{0Y}'}{g}, \quad (12)$$

where $K = H \sqrt{k_X^2 + k_Y^2}$.

The ray Equation (5) will be the following:

$$\tan \xi = Y/X = k_Y/k_X \frac{-\operatorname{sech}[K]^2 + \frac{W_Y(k_Y/k_X)(\operatorname{sech}[K]^2 - \tanh[K]/K)}{1+(k_Y/k_X)^2} + \frac{W_Y \tanh[K]/K}{K(k_Y/k_X)} - \frac{\tanh[K]}{K}}{2Fr^2 - \operatorname{sech}[K]^2 + \frac{W_Y(k_Y/k_X)(\operatorname{sech}[K]^2 - \tanh[K]/K)}{1+(k_Y/k_X)^2} + \frac{W_Y(k_Y/k_X) \tanh[K]/K}{K} - \frac{\tanh[K]}{K}} \quad (13)$$

We will consider only positive values of $W_Y > 0$ since this equation is invariant under the transformation: $W_Y \rightarrow -W_Y$, $\psi \rightarrow -\psi$, $\xi \rightarrow -\xi$. Typical patterns of ship waves directions $-k_Y/k_X = \tan(\psi)$ depending on the ray angle ξ for different Froude numbers and shear flow gradient $W_Y = 0.5$ are shown in Figure 6a,b. The dependence is clearly asymmetric about the X-axis; therefore, we presented the entire range of variation $-\pi/2 < \psi < \pi/2$. In the subcritical region ($Fr < 1$) there are two systems of waves—divergent and transverse, and in the supercritical region ($Fr > 1$)—only divergent waves.

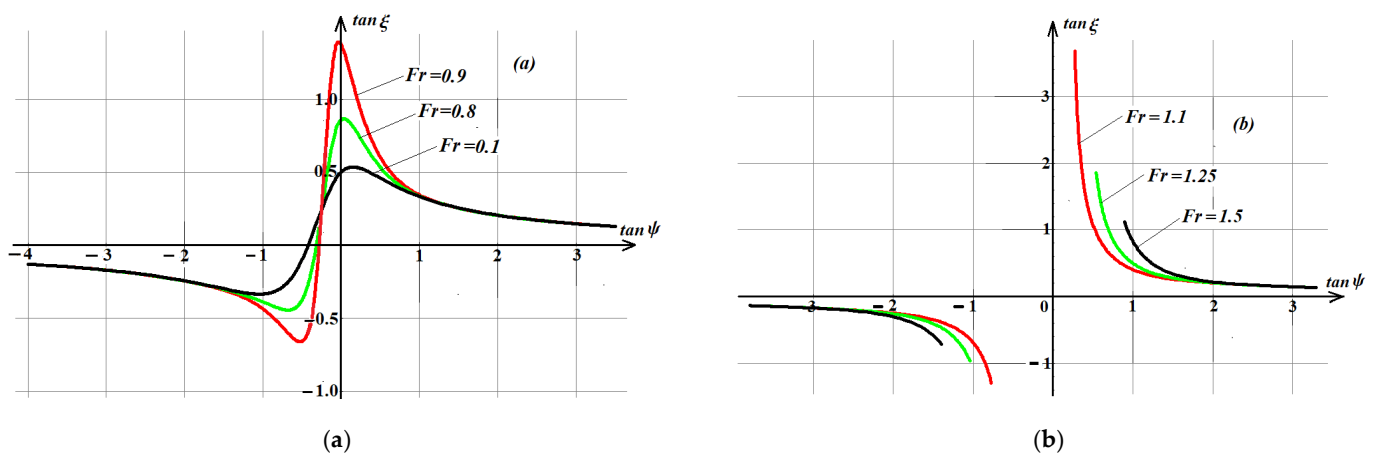


Figure 6. Dependence of ship waves: $-k_Y/k_X = \tan(\psi)$ from the $\tan \xi$ for shear gradient $W_Y = 0.5$ and different values of Froude number: (a) $Fr < 1$; (b) $Fr > 1$.

Transverse waves have a negative direction ($\psi < 0$, $\xi = 0$) along the trajectory of the ship. Two wake arms have different angles (see Figure 6a,b). The positive wake arm angle ξ_m increases with the subcritical Froude number $Fr < 1$ while the negative arm angle is limited by the value. The image of wave crest lines behind the ship can be represented by lines with the same phase of the waves [32,35] $\theta(X, Y) = \text{const}$:

$$\theta(X, Y) = \int_0^{\vec{X}} \vec{k} \cdot d\vec{X} = r \left| \vec{k} \right| \cos(\mu), \quad \mu = \pi - \psi - \xi, \quad (14)$$

where r —is the distance from the origin, μ is the angle between the vectors of the ray and the wavenumber (see Figure 1). A characteristic pattern of phase lines for the shear current ($W_Y = 0.5$) and Froude number ($Fr = 0.9$) is shown in Figure 7.

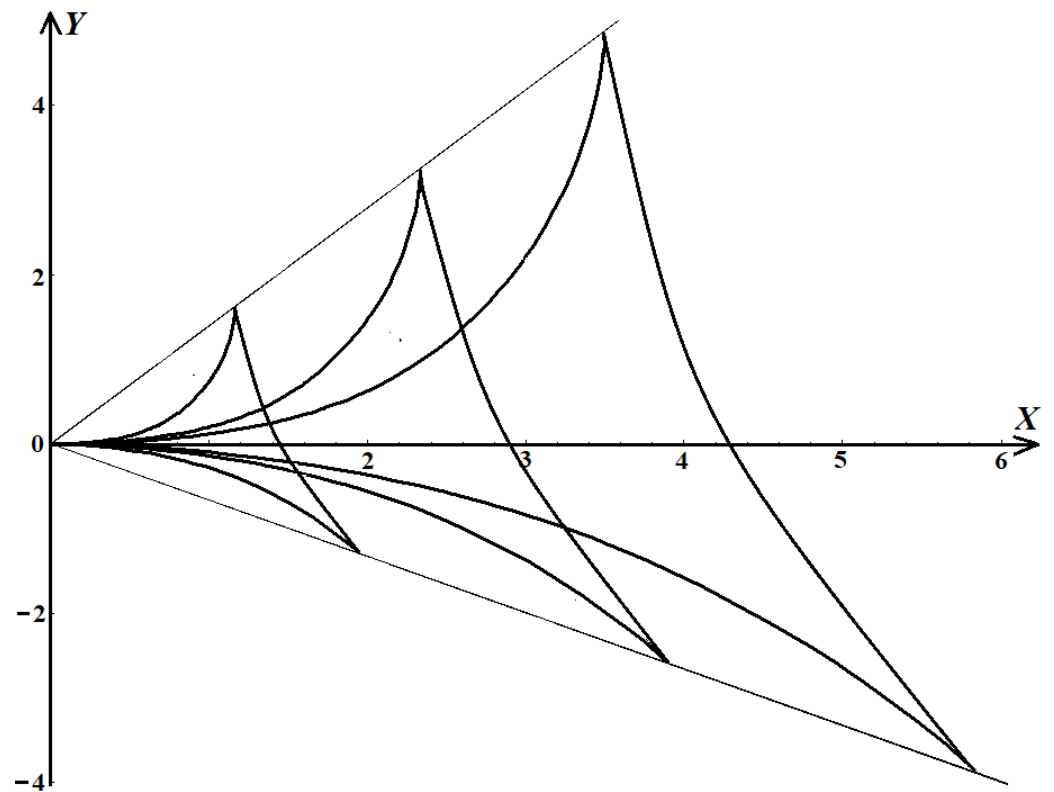


Figure 7. Phase lines $\theta(X, Y) = \text{const}$ for the shear current value ($W_Y = 0.5$) and Froude number ($Fr = 0.9$).

3.4. Inclined Direction of the Shear Current

In the common case, ($W_Y \neq 0, W_X \neq 0$) dispersion relation (3) is

$$G(k_x, k_y) = k_X^2 Fr^2 + k_X^2 W_X \frac{\tanh[K]}{K} + k_X k_Y W_Y \frac{\tanh[K]}{K} - \left| \vec{k} \right|^2 \frac{\tanh[K]}{K} = 0. \quad (15)$$

The ray direction Equation (5) will take the form:

$$\tan \zeta = k_Y / k_X \frac{-\text{sech}[K]^2 + \frac{(W_X + W_Y k_Y / k_X)(\text{sech}[K]^2 - \tanh[K]/K)}{1 + (k_Y / k_X)^2} + \frac{W_Y \tanh[K]/K}{(k_Y / k_X)} - \frac{\tanh[K]}{K}}{2Fr^2 - \text{sech}[K]^2 + \frac{(W_X + W_Y k_Y / k_X)(\text{sech}[K]^2 - \tanh[K]/K)}{1 + (k_Y / k_X)^2} + \frac{W_Y (k_Y / k_X) \tanh[K]}{K} - \frac{\tanh[K](1 - 2W_X)}{K}} \quad (16)$$

The Equation (6) obviously admits an invariant transformation:

$$W_Y \rightarrow -W_Y, k_Y / k_X \rightarrow -k_Y / k_X, \zeta \rightarrow -\zeta, \quad (17)$$

Therefore, we will consider only positive values of $W_Y > 0$. The main features of the structure of ship waves on an inclined shear flow remain qualitatively similar to those considered above. The component of the shear flow collinear with the movement W_X of the ship leads to a change in the critical Froude number Fr_{cr} , which almost does not differ in the presence of cross component of the shear flow. The angle of the ship wake expands with an increase in the unidirectional shear flow and narrows in the oncoming flow in the subcritical mode of the ship's motion $Fr < Fr_{cr}$. The presence of the cross component of the shear flow W_Y leads to a significant asymmetry of the ship wake. The typical directions of ship waves $-k_Y / k_X = \tan(\psi)$ depending on the ray angle ζ for Froude number $Fr = 0.5$ and different shear flow gradients $W_X = \pm W_Y > 0$ are shown in Figure 8a,b.

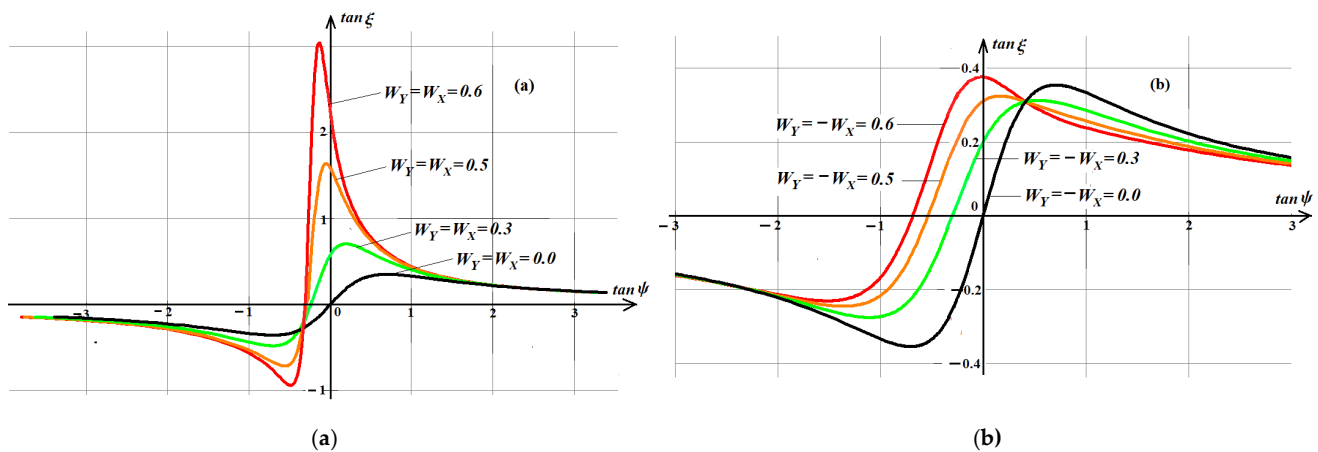


Figure 8. Dependence of ship waves: $-k_Y/k_X = \tan(\psi)$ from the $\tan \xi$ for Froude number $Fr = 0.5$ and different values of shear gradient: (a) $W_Y = W_X > 0$; (b) $W_Y = -W_X > 0$.

A sketch of the phase lines for the $Fr = 0.5$ and shear current gradient $W_Y = \pm W_X = 0.5$ is shown in Figure 9a,b. As can be seen, all phase lines have a cusp that separates transverse and divergent waves.

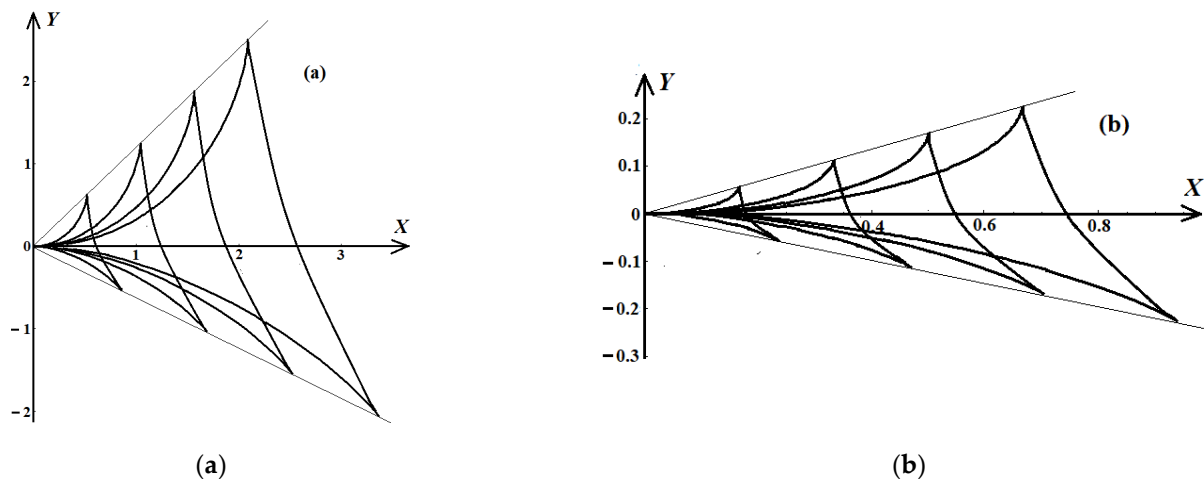


Figure 9. The lines of the constant phase $\theta(X, Y) = \text{const}$ for $Fr = 0.5$ and different values of shear current: (a) positive gradient components ($W_X = W_Y = 1/2$); (b) opposite X- shear gradient components ($W_X = -W_Y = -1/2$).

4. Conclusions

The ship's wake at finite water depths in the coastal regions of the sea and in the presence of a horizontal shear current can differ significantly from the classical model of Kelvin ship waves and exhibits a number of unusual properties. The developed simplified kinematic model of the ship's wave wake allows one to reasonably describe its main features.

The ship wake in calm water critically depends on the value of the Froude number. For every value of $Fr < 1$ two types of waves are presented: long transverse and short divergent waves inside the wedge area with an angle depending on Fr . The ship's total wake angle increases with Fr number up to 180° as it approaches one. For $Fr > 1$ only divergent waves are presented for every ray inside the region. Wake angle decreased with Froude number from 180° for $Fr \sim 1$ to very small values for $Fr \gg 1$ and only short waves with the crests almost collinear with ship track direction are finally presented in the narrow ship wake. For a critical value of the Froude number $Fr_{cr} = 1$, the ship's wake has a total wedge angle of 180° with waves directed parallel to the ship's motion. The angle of

cusp waves decreases with increasing Froude number from 35.3° up to zero in the interval $(0 < Fr < 1)$ and, conversely, increases up to 90° for $(Fr > 1)$.

A ship's wake in a shear flow, collinear to the ship's motion, has both common features and essential differences compared to a ship's wake in calm water. In the subcritical region $(Fr < Fr_{cr})$, two systems of waves are represented—divergent and transverse, and in the supercritical region $(Fr > Fr_{cr})$ —only divergent waves. The value of the critical Froude number Fr_{cr} itself is variable and depends on the magnitude of the collinear shear flow: it decreases with the unidirectional shear flow ($W_X > 0$) up to zero for $W_X = 1$ and increases on the counter shear current.

The presence of a shear current perpendicular to the path of the ship leads to the asymmetry of the wake of the ship, in particular, the two arms of the wake have different angles. For a positive shear flow ($W_Y > 0$) divergent waves have a negative direction along the trajectory of the ship. The positive wake arm angle increases with the subcritical Froude number $Fr < 1$ while the negative arm angle is limited by the value.

The main properties of a ship wake in the presence of an arbitrary inclined shear flow are qualitatively similar to those described above. The component of the shear flow, collinear to the motion of the ship, leads to a change in the critical Froude number, which changes insignificantly in the presence of a transverse component of the shear flow. The wake angle expands with an increase in the unidirectional shear current and narrows when the ship moves in the subcritical mode. The presence of the transverse component of the shear current leads to a significant asymmetry of the ship wake.

In conclusion, it should be noted that the kinematic model has a fundamentally different approach from the classical one, based on the representation of a ship as a moving source of pressure on the sea surface. Rather voluminous and non-trivial numerical integration for each particular oceanographic situation in traditional modeling requires a lot of effort and has its own limitations.

Despite its relative simplicity, our kinematic model reproduces some of the results corresponding to those obtained according to the traditional scheme [7,28], which, in turn, can serve as a test of the applicability of the model. The analytical form of the solution makes it possible to explicitly describe the structure of ship waves in the entire range of change in the defining parameters of the problem.

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