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A Block Sparse-Based Dynamic Compressed Sensing Channel Estimator for Underwater Acoustic Communication

Lingji Xu [1], Lixing Chen [1], Yaan Li [2,3] and Weihua Jiang [4,*]

[1] School of Marine Engineering and Technology, Sun Yat-sen University, Zhuhai 519082, China; xulj26@mail.sysu.edu.cn (L.X.); chenlx76@mail2.sysu.edu.cn (L.C.)
[2] Shenzhen Research Institute, Northwestern Polytechnical University, Shenzhen 518057, China; liyaan@nwpu.edu.cn
[3] School of Marine Science and Technology, Northwestern Polytechnical University, Xi'an 710072, China
[4] Hatter Department of Marine Technologies, University of Haifa, Haifa 3498838, Israel
* Correspondence: wjiang@campus.haifa.ac.il

Abstract: Due to the complex ocean propagation environments, the underwater acoustic (UWA) multipath channel often exhibits block sparse time-varying features, and while dynamic compressed sensing (DCS) can mitigate the time-varying effects of the UWA channel, DCS-based algorithms have limited performance for the UWA channel with block sparsity. In this study, by formulating the UWA channel with blocks concatenation, a block sparse-based DCS approach (BS-CS) is proposed to explore the block and time-varying sparsity of UWA channel simultaneously. In detail, we firstly adopt a block sparse recovery algorithm, block orthogonal matching pursuit (BOMP), to compute the temporary estimate. Then, the CS approach is applied to compute the support additions, which are caused by the time-varying components of the UWA channel. Next, we use the selected support to perform the BOMP estimate, and obtain the estimated channel response. Finally, the numerical simulation and the sea experiment were carried out to verify the superior performance of the proposed BS-CS algorithm in the block sparse time-varying UWA channels.

Keywords: block sparse; time-varying; underwater acoustic communication; dynamic compressed sensing

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1. Introduction

With the increasing demand of ocean exploration, pollution monitoring, marine data collection, etc, the role of underwater acoustic (UWA) communication becomes more crucial [1–3]. However, compared to the terrestrial wireless channel, the UWA channel exhibits serious multipath, Doppler, and time-varying characteristics, which is a time-space-frequency varying channel [4,5]. The extreme adverse and complex marine environment raises many challenging for the UWA communication technology.

The traditional non-sparse channel estimation algorithms, such as least squares (LS) [6,7], can achieve better channel estimation performance in dense multipath scenarios. The LS algorithm estimates all the tap coefficients directly, which results in the relatively high channel estimation errors, as it is well recognized in the research community that UWA channels are commonly sparse [8]. Under the compressed sensing (CS) framework, it is capable of improving the channel estimation performance by exploiting the inherent channel sparsity. The most popular CS-based algorithms are the l_0-norm constraint algorithms [9–11] and greedy algorithms [12–14], such as matching pursuit (MP) [7] and orthogonal matching pursuit (OMP) [13]. Compared with the MP, the main difference of the OMP is that the coefficients are the orthogonal projection of the signal on the selected subspace, and the OMP presents more accuracy on the sparse recovery. However, the time-varying characteristic of the UWA channel would cause the performance degradation for the above algorithms.

Dynamic compressed sensing theory (DCS) has attracted considerable attention in recent years, as it provides a new approach to the reconstruction of time-varying sparse signals [15–20]. Kalman-filtered compressed sensing (KF-CS) [15] was proposed to tackling the time-varying sparsity recovery problem. The key idea of KF-CS is to estimate the initial support set by KF; then, the possible support additions are computed by CS to track the time-varying components. Built on the KF-CS framework, a serial DCS-based works have been proposed to improve the performance of time-varying sparsity reconstruction, such as least squares CS (LS-CS) [17], modified-CS [18], regularized modifled-CS [19], and minimum mean square estimation CS (MMSE-CS) [20]. Furthermore, by formulating the hybrid multipath UWA channels as sparse set consisting of static and time-varying supports, a static-dynamic discriminative compressed sensing (SDD-CS) [21] approach was proposed to explore the hybrid sparsity of UWA channels.

From the viewpoint of UWA multipath channels, except the channel time variation introduced by the dynamic ocean environment or the platform motion, the paths of the UWA channel often present in a cluster, where it may have several small paths centering around the eigen-paths due to the surface/bottom reflections and the medium refractions [22]. Thus, while the classic DCS-based algorithms only consider the time-varying sparsity recovery problem, the ignorance of block sparsity will limit the estimation performance of the KF-CS algorithm for the block sparse time-varying UWA channel. Moreover, to exploit the block sparsity of channel, [23] proposed the block OMP (BOMP) algorithm, which shows better recovery performance than the OMP for a block sparse channel. A block sparse recursive least squares (BS-RLS) algorithm was proposed to improve the performance of channel estimation under block sparse condition [24], which introduces an approximate mixed $l_{2,0}$-norm to take advantage of the underlying block sparse property of the system. However, the time-varying feature of the UWA channel is neglected by the BOMP and the BS-RLS, which may result in a degradation of performance. Moreover, to enhance performance, the adaptive filter based channel estimation algorithm requires a relatively long training sequence and brings extra computational burdens.

Inspired by the special characteristics of the UWA channel and the existing DCS algorithms for time-varying sparse recovery, in this paper, we introduce the block sparse reconstruction into DCS algorithm and propose a block sparse DCS (BS-CS) algorithm. The proposed BS-CS algorithm can make use of the inherent block sparse and time-varying property of the UWA channel and improve the estimation performance for the block sparse time-varying UWA channel. Specifically, we firstly adopt a block orthogonal matching pursuit (BOMP) to compute the initial channel estimate. Then, the CS approach is applied on the BOMP error to compute the possible support additions, which are introduced by the time-varying components of UWA channel. Finally, we use the selected support to perform the BOMP estimate, and receive the estimated channel response.

A channel impulse response (CIR) generated by the parabolic Equation (PE) propagation model [25] and the recorded signals obtained from a shallow sea experiment are applied to demonstrate the behavior of the proposed BS-CS algorithm, respectively. The numerical simulation and sea experiment results show that for the block sparse time-varying UWA channel, the proposed BS-CS performs better than the benchmark sparse and non-sparse channel estimation algorithms, namely, the least squares with the QR-factorization (LSQR) [26], the OMP [13], the BOMP [23], the BS-RLS [24], the KF-CS [15], and the SDD-CS [21].

The rest of the paper is structured as follows. In Section 2, we describe the problem formulation of UWA channel estimation and block sparse representation of UWA channel. Section 3 describes our proposed BS-CS algorithm. Sections 4 and 5 present the numerical simulation and sea experiment results, respectively. Finally, Section 6 draws the conclusions.

Notation 1. *Capital bold letters denote matrices, and small bold letters denote vectors.* $\cup$ *denotes the set union operation.* T *and* H *denote the transpose, and conjugate transpose operator, respectively.* $||\mathbf{x}||$ *and* $||\mathbf{x}||_1$ *denote* l_2 *norm and* l_1 *norm of the vector* $\mathbf{x}$*, respectively.* $\mathbf{X}^{\dagger}$ *denotes the pseudo inverse of the matrix* $\mathbf{X}$*.* $|supp(\mathbf{x})|$ *denotes the number of non-zero elements of the vector* $\mathbf{x}$*.*

2. Problem Formulation

2.1. Problem Statement

In this study, we consider a block sparse time-varying UWA channel estimation problem. For an UWA multipath channel, expect the channel time variation introduced by the dynamic ocean environment or the platform motion, the paths of UWA channel often present in a cluster. The sample of experimental UWA channel obtained from [27] is shown in Figure 1, from which we can observe a block sparse time-varying structure.

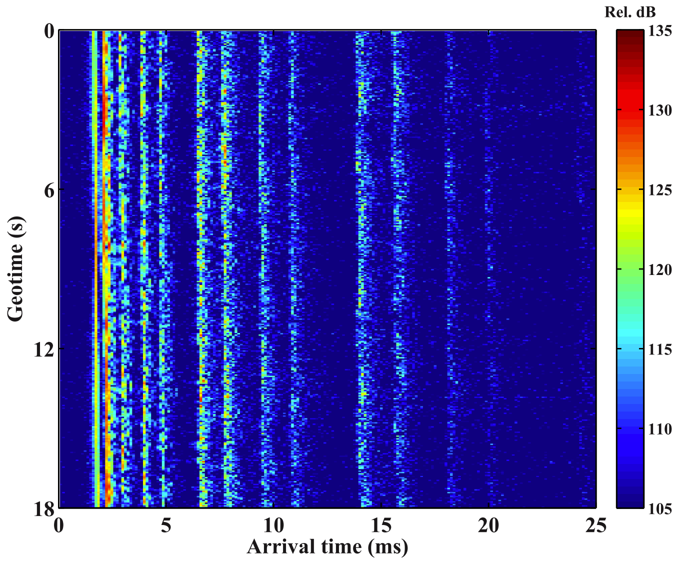

Figure 1. The sample of experimental UWA channel with a block sparse time-varying structure.

The classic CS-based channel estimation algorithm is subject to performance degradation due to the ignorance of the time-varying structure and block sparsity of the UWA channel. Moreover, to exploit the block sparsity of the channel, the block sparse-based algorithms were proposed to improve the performance of channel estimation under the block sparse condition. On the other hand, the DCS-based algorithms were proposed to tackling the time-varying sparsity recovery problem. However, to our knowledge, very few studies have reported channel estimation on the block sparse time-varying channel. Aiming at this gap, we propose the BS-CS algorithm to enhance the estimation performance, which can exploit the block sparsity for DCS estimation in the block sparse time-varying UWA channel.

2.2. UWA Communication Model

The UWA communication model in noisy environment can be written as [28,29]:

$$y(i) = \sum_{j=0}^{N-1} x(i-j)h(j) + w(i), \quad i = 0, \ldots, M-1, \tag{1}$$

where x, y, w, and h are the transmitted signal, the received signal, the sea noise, and the CIR, respectively. Thus, (1) can be rewritten by a matrix representation as:

$$\mathbf{y} = \mathbf{X}\mathbf{h} + \mathbf{w}, \tag{2}$$

where the received signal $\mathbf{y}$ and the noise $\mathbf{w}$ are the size of $M \times 1$ in the vector form and the UWA CIR $\mathbf{h}$ is a size of $N \times 1$. The matrix $\mathbf{X} \in \mathbb{C}^{M \times N}$ is the measurement matrix with a size of $M \times N$ which is made by the transmitted signal, as follows:

$$\mathbf{X} = \begin{pmatrix} x(0) & x(-1) & \cdots & x(-N+1) \\ x(1) & x(0) & \cdots & x(-N+2) \\ \vdots & \vdots & \ddots & \vdots \\ x(M-1) & x(M-2) & \cdots & x(M-N) \end{pmatrix}. \tag{3}$$

Hence, to find the sparse solution of the UWA channel $\mathbf{h}$ in (2), the CS algorithm can be applied to reconstruct the UWA channel $\mathbf{h}$ from the received signal $\mathbf{y}$ for a UWA communication system.

2.3. Block Sparse Representation of UWA Channel

For a UWA channel with block sparsity, block sparse recovery algorithm is capable of further improving the estimation performance by exploiting the intra-block correlation [23,30]. The block sparse UWA channel can be treated as a concatenation of ζ blocks with a block size of d; hence, the channel $\mathbf{h}$ can be written as:

$$\mathbf{h} = [\mathbf{h}^T[1], \mathbf{h}^T[2], \cdots, \mathbf{h}^T[\zeta]]^T, \tag{4}$$

where:

$$\mathbf{h}[i] = [h((i-1)d+1), h((i-1)d+2), \cdots, h(id)]^T,\ 1 \leq i \leq \zeta \tag{5}$$

denotes the i-th sub-block channel of the channel $\mathbf{h}$, and $N = \zeta d$. Similarly, the sub-matrix $\mathbf{X}[i] \in \mathbb{C}^{M \times d}$ consists of the columns of $\mathbf{X}$ with indices $(i-1)d+1$ to id for $1 \leq i \leq \eta$. Next, we define the block sparsity κ of the channel $\mathbf{h}$ as:

$$\kappa = ||\mathbf{h}||_{2,0} = |\mathrm{supp}(\mathbf{h}[i])|,\ 1 \leq i \leq \zeta. \tag{6}$$

Thus, the optimization problem of the block sparse channel estimation can be expressed by [9]:

$$\arg\min_{\mathbf{h}} (||\mathbf{h}||_{2,0}) \quad s.t. \quad ||\mathbf{y} - \mathbf{X}\mathbf{h}||_2^2 \leq \varepsilon, \tag{7}$$

where ε is the noise factor. In order to effectively recover the block sparse UWA channel $\mathbf{h}$ from (7), the BOMP algorithm is commonly applied to exploit block sparsity of UWA channel. In particular, when $d = 1$, BOMP gives the same results as OMP.

3. The Proposed BS-CS Method

While the DCS algorithm was proposed to explore the time-varying characteristic of UWA channel, the sparse multipath with block sparsity tends to be underrated. Hence, in our work, to exploit the block sparsity of the time-varying UWA channel, the BS-CS algorithm is proposed.

The measurement model at time t can be expressed by [15,16]:

$$\mathbf{y}_t = \mathbf{X}\mathbf{h}_t + \mathbf{w}_t, \tag{8}$$

where $\mathbf{y}_t$, $\mathbf{h}_t$, and $\mathbf{w}_t$ are the received signal, the CIR, and the sea noise at time t, respectively. Let T_t denote the estimate support set of $\hat{\mathbf{h}}_t$, and T_t^c denote the complement of T_t.

Figure 2 shows the flowchart of the proposed BS-CS algorithm. We describe the proposed BS-CS algorithm below.

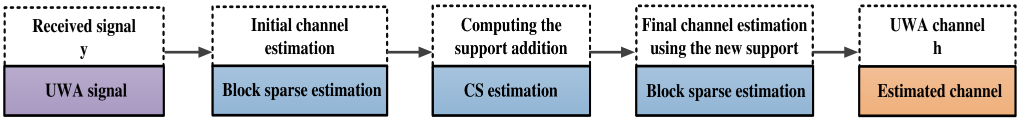

Figure 2. The flowchart of the proposed BS-CS algorithm.

(1) Running the BOMP. At time t, we adopt the BOMP [23] algorithm to perform an initial channel estimation. Firstly, we compute the inner product between residual error $\mathbf{r}$ and sub-matrix $\mathbf{X}[j]$, where j takes from T_{t-1}, namely, the support set of the estimated channel $\hat{\mathbf{h}}_{t-1}$ at time $t-1$. Furthermore, the location γ_k is given by maximum absolute value of the inner product of each sub-block. With the initialization of $k = 1, \mathbf{r}^0 = \mathbf{y}_t$, we have:

$$\gamma^k = \arg\max_{j=T_{t-1}} ||\mathbf{X}[j]^T \mathbf{r}^{k-1}||_2, \tag{9}$$

where k denotes the iteration number of BOMP. Then, we add γ_k into the block index $\hat{T}_t$ as:

$$\hat{T}_t^k = \hat{T}_t^{k-1} \cup \gamma^k. \tag{10}$$

Thus, using the updated block index $\hat{T}_t$, we can have the multipath magnitude of the UWA channel with LS algorithm:

$$\hat{\mathbf{h}}_{t,tmp} = (\mathbf{X}[\hat{T}_t^k])^{\dagger} \mathbf{y}_t. \tag{11}$$

Next, the residual error $\mathbf{r}^k$ is updated by:

$$\mathbf{r}^k = \mathbf{y}_t - \mathbf{X}[\hat{T}_t^k]\hat{\mathbf{h}}_{t,tmp}. \tag{12}$$

Iteration stops when the maximum iteration number κ is reached, and an initial channel estimation result $\hat{\mathbf{h}}_{t,tmp}$ is obtained. Otherwise, the above iterations continue.

(2) Computing the additions. The residual error of the BOMP is:

$$\hat{\mathbf{y}}_{t,res} = \mathbf{y}_t - \mathbf{X}\hat{\mathbf{h}}_{t,tmp}. \tag{13}$$

We can see that the residual error $\hat{\mathbf{y}}_{t,res}$ in (13) can be rewritten as:

$$\hat{\mathbf{y}}_{t,res} = \mathbf{X}\mathbf{b}_t + w_t, \tag{14a}$$

$$\mathbf{b}_t = \mathbf{h}_t - \hat{\mathbf{h}}_{t,tmp}, \tag{14b}$$

where $\mathbf{b}_t$ is a $|\hat{T}_t \cup \Delta_t|$-sparse vector with $(\mathbf{b}_t)_{(\hat{T}_t \cup \Delta_t)^c} = \mathbf{0}$. Thus, the CS algorithm [16] can be used on $\hat{\mathbf{y}}_{t,res}$ and followed by thresholding α to calculate its new support at time t as:

$$\Delta_t = \{i \in \hat{T}_t^c : |(\mathbf{b}_t)_i| > \alpha\}, \tag{15a}$$

$$\tilde{T}_t = \hat{T}_t \cup \Delta_t. \tag{15b}$$

(3) BOMP update. We use the new support $\tilde{T}_t$ to perform the BOMP [23] estimate. With the initialization of $k = 1, \mathbf{r}^0 = \mathbf{y}_t$, we have:

$$\gamma^k = \arg\max_{j=\tilde{T}_t} ||\mathbf{X}[j]^T \mathbf{r}^{k-1}||_2, \tag{16}$$

where k denotes the iteration number of BOMP. Then, we add γ_k into the block index T_t as:

$$T_t^k = T_t^{k-1} \cup \gamma^k. \tag{17}$$

Thus, using the updated block index T_t, we can have the multipath magnitude of the UWA channel with LS algorithm:

$$\hat{\mathbf{h}}_t = (\mathbf{X}[T_t^k])^{\dagger}\mathbf{y}_t. \tag{18}$$

Next, the residual error $\mathbf{r}^k$ is updated by:

$$\mathbf{r}^k = \mathbf{y}_t - \mathbf{X}[T_t^k]\hat{\mathbf{h}}_t. \tag{19}$$

Iteration stops when the maximum iteration number κ is reached, and the channel estimation result $\hat{\mathbf{h}}_t$ is obtained. Otherwise, the above iterations continue.

Finally, if there are some false detections in computing the additions step due to the small addition threshold α, their elements need to be removed from the support estimate.

The proposed BS-CS algorithm is summarized in Algorithm 1. The computational complexity of BS-CS is equal to that of simple CS, which is $\mathcal{O}(N^3)$, where N is the length of CIR [17].

Algorithm 1. $\mathbf{x}, \mathbf{y}_t, d, \kappa, \alpha$

(1) Running the BOMP: initialization of $\mathbf{r}^0 = \mathbf{y}_t$;
for $k = 1 : \kappa$
$\gamma^k = \arg\max_{j=T_{t-1}} ||\mathbf{X}[j]^T\mathbf{r}^{k-1}||_2$;
$\hat{T}_t^k = \hat{T}_t^{k-1} \cup \gamma^k$;
$\hat{\mathbf{h}}_{t,tmp} = (\mathbf{X}[\hat{T}_t^k])^{\dagger}\mathbf{y}_t$;
$\mathbf{r}^k = \mathbf{y}_t - \mathbf{X}[\hat{T}_t^k]\hat{\mathbf{h}}_{t,tmp}$;
end for
(2) Computing the additions:
$\hat{\mathbf{y}}_{t,res} = \mathbf{X}\mathbf{b}_t + w_t$;
$\mathbf{b}_t = \mathbf{h}_t - \hat{\mathbf{h}}_{t,tmp}$;
$\Delta_t = \{i \in \hat{T}_t^c : |(\mathbf{b}_t)_i| > \alpha\}$;
$\tilde{T}_t = \hat{T}_t \cup \Delta_t$;
(3) BOMP update: initialization of $\mathbf{r}^0 = \mathbf{y}_t$;
for $k = 1 : \kappa$
$\gamma^k = \arg\max_{j=\tilde{T}_t} ||\mathbf{X}[j]^T\mathbf{r}^{k-1}||_2$;
$T_t^k = T_t^{k-1} \cup \gamma^k$;
$\hat{\mathbf{h}}_t = (\mathbf{X}[T_t^k])^{\dagger}\mathbf{y}_t$;
$\mathbf{r}^k = \mathbf{y}_t - \mathbf{X}[T_t^k]\hat{\mathbf{h}}_t$;
end for
(4) Deleting some small false detections.
Output $\hat{\mathbf{h}}_t$.

4. Numerical Simulation

In this section, numerical simulations are presented to evaluate the estimation performance. In this paper, we assume that the non-zero coefficients' time-varying set of the UWA channel changes slowly over time, namely, the UWA channel is a slow time-varying channel. Moreover, the addictive noise $\mathbf{w}$ is assumed to be Gaussian. We compare the estimation performance of the proposed algorithm to the following benchmark algorithms.

(1). LSQR: The classic non-sparse channel estimation algorithm.
(2). OMP: The classic CS channel estimation algorithm.
(3). BOMP: The classic block sparse-based CS channel estimation algorithm.
(4). BS-RLS: The novel block sparse-based channel estimation algorithm, which was proposed to exploit the block sparsity of UWA channels.
(5). KF-CS: The classic DCS channel estimation algorithm.
(6). SDD-CS: The novel DCS-based channel estimation algorithm, which was proposed to explore the hybrid sparsity of UWA channels.

(7). BS-CS: The proposed channel estimation algorithm, which is designed for the block sparse time-varying UWA channels.

To verify and compare the estimation performance of the BS-CS algorithm with the benchmarks, a known signal is transmitted through a simulated block sparse time-varying channel, and the channel estimation is performed on the received signal. Our numerical simulation performance matrix is the channel-to-reconstruction error ratio (CRER). The CRER is defined by:

$$\mathrm{CRER} = 10\log_{10}\frac{||\mathbf{h}||_2^2}{||\mathbf{h}-\tilde{\mathbf{h}}||_2^2}, \tag{20}$$

where $\mathbf{h}$ is the true CIR and $\tilde{\mathbf{h}}$ is the estimated CIR. The simulations were performed using MATLAB R2019b running on a computer with an Intel i7-1165G7 processor and 8 GB of memory.

We firstly use the PE propagation model to generate a block sparse time-varying channel with a water depth of 30 m and a distance of 2000 m. Our propagation model adopts a zero or negative gradient sound speed profile with an uneven seabed, while the sound speed on the surface is given by a uniform distribution between 1500 m/s and 1540 m/s. The transmitter and the receiver are deployed at the depth of 5 m and 10 m under the sea surface, respectively. Figure 3 shows the sound speed profile and the corresponding simulated UWA channel, which has time-varying and block sparse characteristics. Note that the delay and the magnitude of two surface paths are imposed with zero-mean random variations and sinusoidal variations, respectively, to artificially simulate the time-varying multipath induced by the sea surface, while the remaining paths remain static.

In our numerical simulation, the lengths of the CIR N and the observation window M are set to 200 and 150, respectively, with a sampling rate of 4000 Hz. The received signal-to-noise ratio (SNR) is set to 15 dB. The length of the training sequence for BS-RLS is set to 1000. The continuous measurement number for SDD-CS is set to 2, and the addition threshold for KF-CS, SDD-CS, and BS-CS is set to 0.03. Specifically, larger block size d would exploit better the intra-block correlation of the channel, which is more appropriate for the UWA channel with less sparsity. When the block size $d = 1$, the BS-CS becomes a classic DCS-based algorithm. Thus, in this paper due to the sparse simulated channel, a small block size d is adopted for our simulation. The sparsity factor and block sparsity factor κ are set to 16 and 8, respectively, while the block size is set to $d = 2$ for BOMP, BS-RLS, and BS-CS.

Figure 4 shows the CRER results obtained by different estimators, while the iteration number is set to 1000 for each 1 ms time slot. We observe that the LSQR estimator has significantly lower CRER output than all the six sparse estimators, namely OMP, SOMP, BS-RLS, KF-CS, SDD-CS, and BS-CS, due to the considerable estimation noise. However, while OMP only considers the sparsity of UWA channel, the CRER output of OMP is still inferior to that of BOMP, which is capable of taking advantage of the block sparsity of the UWA channel. Furthermore, the time-varying characteristic of the UWA channel would deteriorate the intra-block correlation of BOMP and cause the performance degradation. KC-CS and BS-RLS achieve roughly equivalent CRER results by exploiting the time-varying sparsity and block sparsity. However, the BS-RLS requires a relatively long training sequence and brings much computational burdens. The SDD-CS achieves the second-highest CRER results by exploiting the joint sparsity and time-varying characteristics simultaneously, while this type algorithm is more suitable for the hybrid channel. The proposed BS-CS outperforms the other six algorithms with the highest CRER output in the block sparse time-varying UWA channel. Specifically, the BS-CS not only utilizes the correlation among intra-block of the UWA channel, but also tracks possible addition new support caused by time-varying multipath while performing CS on the block sparse estimation error.

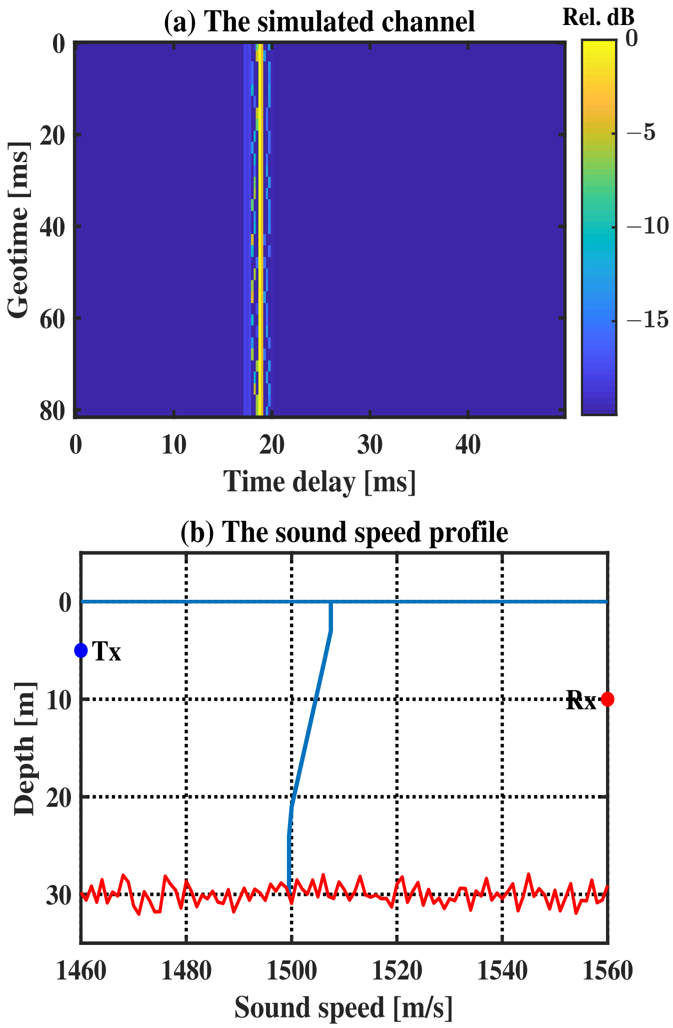

Figure 3. The simulated block sparse time-varying UWA channel and the corresponding sound speed profile. The red line and the light blue line represent the seabed topography and sea surface, respectively.

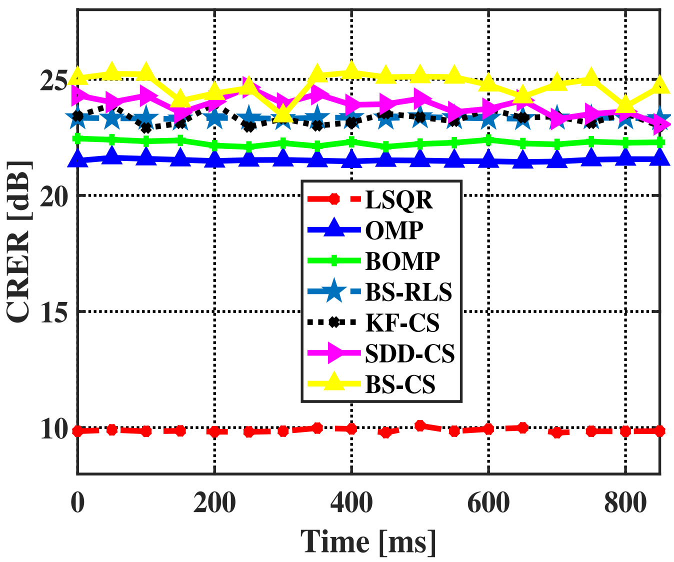

Figure 4. The CRER output of different estimators.

5. Sea Experiment

5.1. Experiment Setup

To further demonstrate the effectiveness of the proposed BS-CS algorithm, the experimental data was collected from a shallow water field at Wuyuan bay, Xiamen, China, to perform the channel estimation and equalization. Figure 5 shows the deployment of four receivers and one transmitter at a communication distance 1000 m in the experiment. The water depth of the transmitter is 2 m, as the water depth of experiment area is 10 m. Moreover, a four-element vertical receiver array was deployed at a water depth from 2 m to 8 m with an element spacing of 2 m.

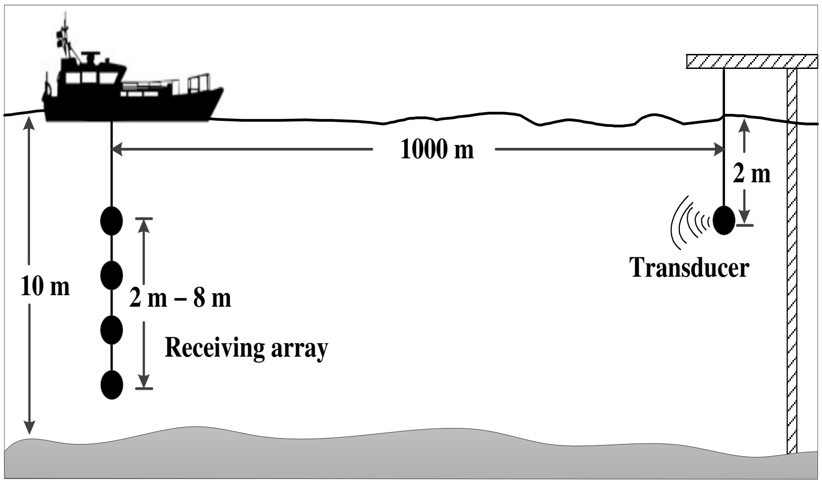

Figure 5. The deployment of receivers and transmitter.

In our experiment, a QPSK modulation signal is transmitted at a bit rate of 1 kbps and a carrier frequency of 16 kHz. Figure 6 shows the response of four channels obtained in our experiment. We observe that these four channels exhibit block sparse structure and different degrees of time-varying characteristics at a time delay of 16 ms. Moreover, to perform the communication performance evaluation of different algorithms, the channel estimation-based decision feedback equalizer (CE-DFE) [31] is adopted in this study and a periodic training scheme is taken to avoid error propagation. In particular, we divide the received data into several data blocks, while the received data include 6000 information bits and 1500 training bits. The parameters of sea experiment are given in Table 1. In addition, our experiment's communication performance metrics are the symbol-to-reconstruction error ratio (SRER), the bit error rate (BER), and the constellation. The SRER is defined by:

$$\mathrm{SRER} = 10\log_{10}\frac{||\mathbf{s}||_2^2}{||\mathbf{s}-\tilde{\mathbf{s}}||_2^2}, \tag{21}$$

where $\mathbf{s}$ is the transmitted symbol and $\tilde{\mathbf{s}}$ is the soft output from the CE-DFE receiver.

Table 1. The parameters of the sea experiment.

Description	Value
Carrier frequency	16,000 Hz
Bandwidth	5000 Hz
Channel impulse response duration	60 ms
Length of discrete observation window	30
Length of feedforward filter	120
Length of feedback filter	59
Error tolerance of LS	10^{-4}

Table 1. *Cont.*

Description	Value
Length of training sequence for BS-RLS	400
Channel sparsity for OMP and KF-CS	6
Block size for BOMP, BS-RLS, and BS-CS	2
Block sparsity for BOMP, BS-RLS, and BS-CS	3
Continuous measurement number for SDD-CS	2
The addition threshold for KF-CS, SDD-CS, and BS-CS	0.07

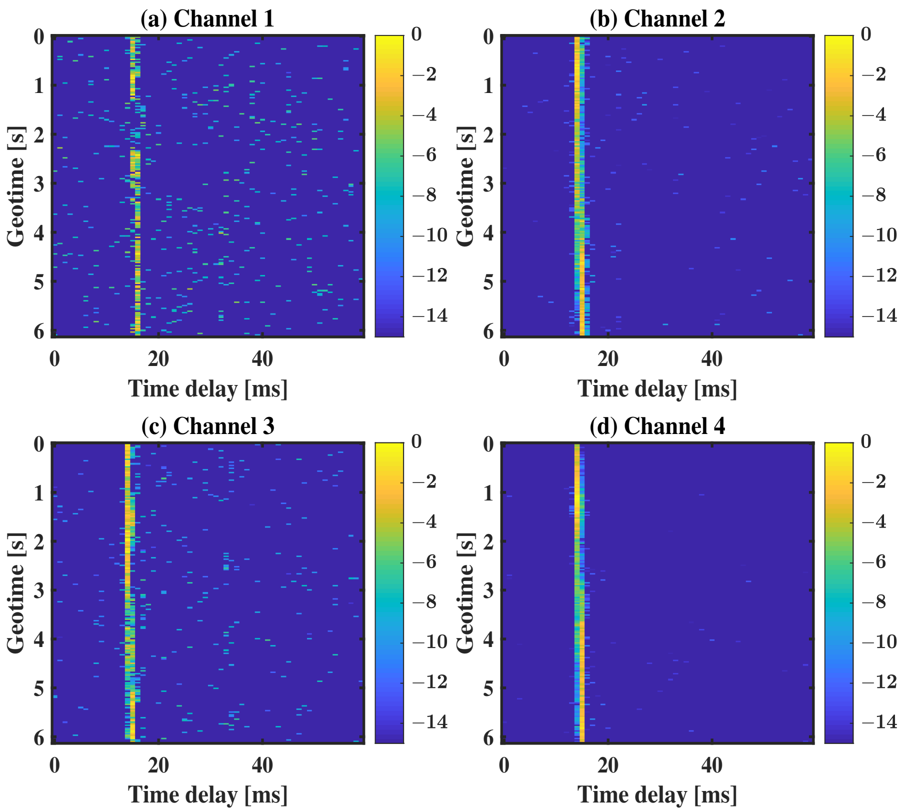

Figure 6. Response of UWA channels associated with four receiving elements. (**a**) Channel 1. (**b**) Channel 2. (**c**) Channel 3. (**d**) Channel 4.

5.2. Result Analysis

In this subsection, we report the results of the communication performance evaluation driven by different channel estimators. Figure 7 and Table 2 show the SRER results and the average SRER of the CE-DFE for different channel estimation algorithms, respectively. We observe that compared with the benchmark algorithms, the BS-CS algorithm achieves the highest SRER output with an average gain of 4.33 dB by LSQR, 0.92 dB by OMP, 0.64 dB by BOMP, 1.22 dB by BS-RLS, 0.52 dB by KF-CS, and 0.38 dB by SDD-CS. Affected by the considerable estimation noise, the LSQR estimator performs the worst for the SRER output. Moreover, due to the underrated of block and time-varying sparsity of the UWA channel, other CS-based benchmark algorithms display an obvious performance degradation when compared with the BS-CS algorithm.

Next, Figure 8 and Table 2 show a histogram for the BER and the average BER of the CE-DFE for different channel estimation algorithms, respectively. Observing the results in Figure 8, we can see that the proposed BS-CS estimator outperforms the benchmark estimators in BER with an average gain of 6.59%, 0.57%, 0.50%, 0.63%, 0.25%, and 0.17% over LSQR, OMP, BOMP, RLS-CS, KF-CS, and SDD-CS, respectively. Due to the limitation of the length of the training sequence, BS-RLS has the second-worst BER performance. By exploiting the block sparsity, BOMP performs better than OMP; however, the time-varying feature of the UWA channel would cause performance degradation severely for the classic CS-based algorithm. Moreover, compared with OMP and BOMP estimators, KF-CS has relatively stable performance due to the time-varying sparsity exploitation of the UWA channel, but there is still a room for improvement for KF-CS as the limited role for block sparsity exploitation. Although the performance of SDD-CS is better than that of KF-CS by modeling the UWA channel as hybrid sparsity, the block sparse time-varying feature of the channel would also cause the model mismatch and performance degradation for SDD-CS. It is evident that the proposed BS-CS estimator can simultaneously exploit the time-varying characteristics and block sparsity for the UWA channel, and achieves the best estimation performance in the block sparse time-varying channel.

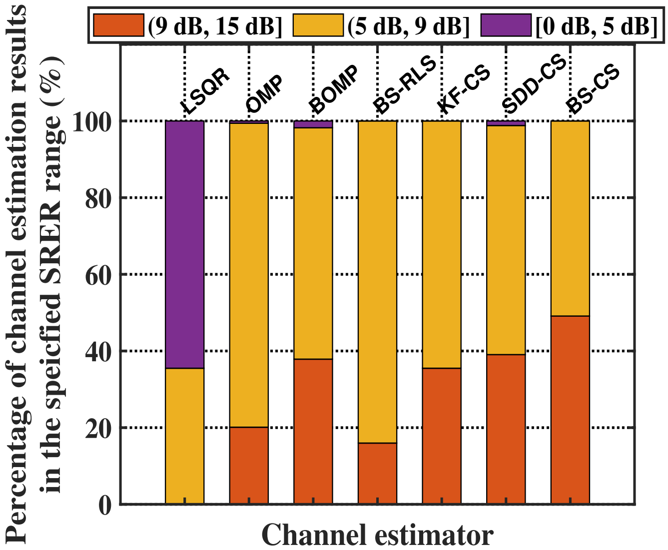

Figure 7. The SRER output of CE-DFE.

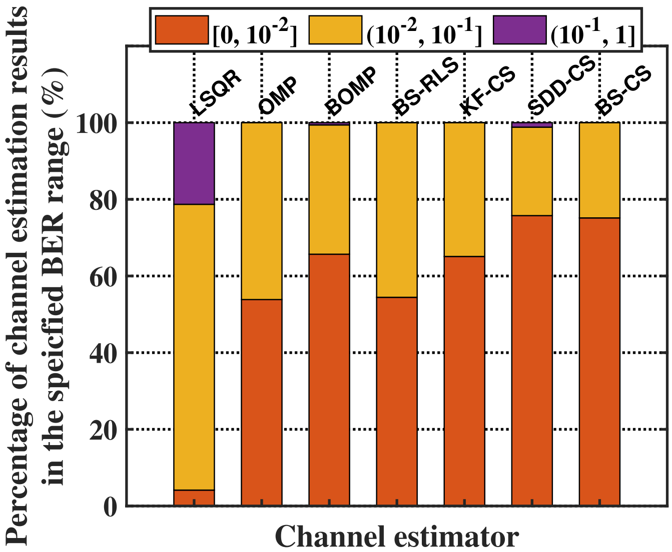

Figure 8. The BER output of CE-DFE.

Finally, Figure 9 shows the constellation outputs driven by the LSQR, OMP, BOMP, BS-RLS, KF-CS, SDD-CS, and BS-CS estimators. We observe that the six CS-based estimators generally outperforms the LSQR estimator that does not exploit the channel sparsity. Compared with the LSQR estimator, the constellations obtained by OMP, BOMP, BS-RLS, and KF-CS have different degrees of performance improvement due to the exploitation of sparsity, block sparsity, block sparsity, and time-varying sparsity, respectively. Moreover, the SDD-CS achieves the second-highest constellation results by exploiting the hybrid sparsity of the UWA channel. Furthermore, it is obvious that BS-CS outperforms the other five sparse estimators with respect to the constellation results. Thus, we conclude that due to the exploitation of time-varying characteristics and the block sparsity in the UWA channel, the proposed BS-CS algorithm achieves the communication performance improvement for the block sparse time-varying channel.

Table 2. The average SRER and BER of CE-DFE driven by different estimators.

Algorithms	SRER (dB)	BER (%)
LSQR	4.71	7.13
OMP	8.12	1.11
BOMP	8.40	1.04
BS-RLS	7.82	1.17
KF-CS	8.52	0.79
SDD-CS	8.66	0.72
BS-CS	9.04	0.54

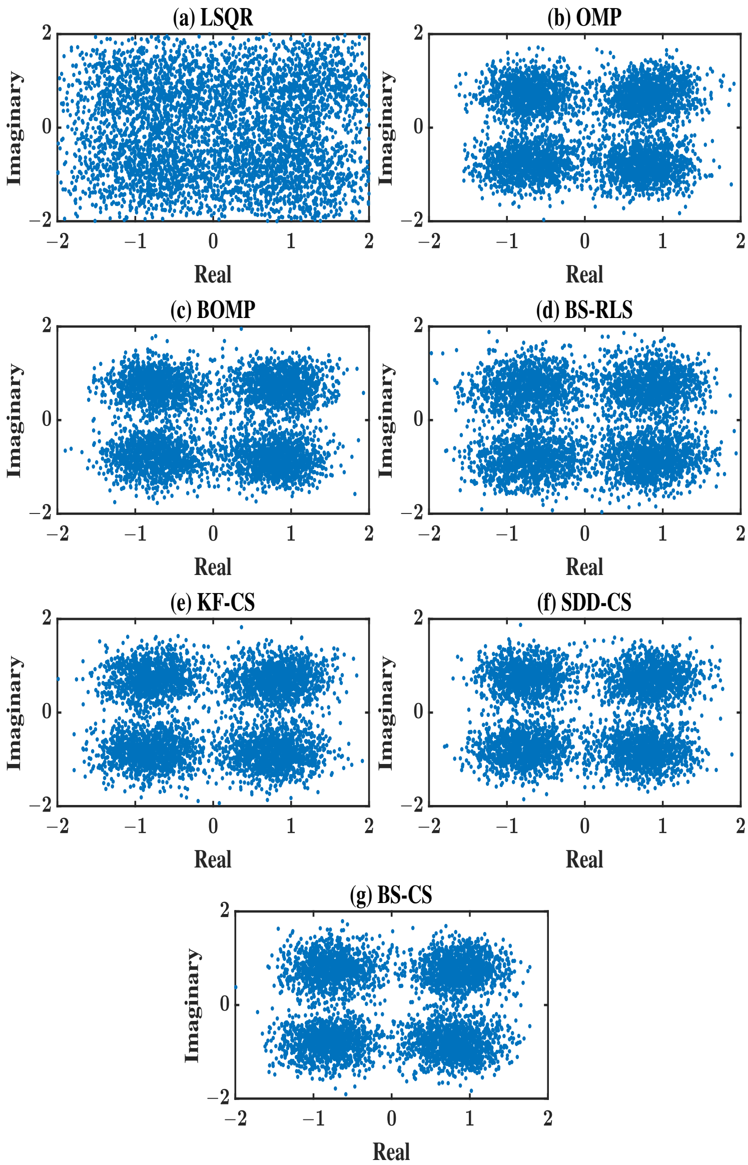

Figure 9. The constellation outputs driven by different channel estimators. (**a**) LSQR. (**b**) OMP. (**c**) BOMP. (**d**) BS-RLS. (**e**) KF-CS. (**f**) SDD-CS. (**g**) BS-CS.

6. Conclusions

While the classic CS-based algorithm is subject to performance degradation due to the ignorance of time-varying characteristics and block sparsity of the UWA channel, in this study, we investigate the exploitation of block sparsity for DCS estimation in the block sparse time-varying UWA channel. The key idea is that by modeling the time-varying UWA channel with blocks concatenation, the BOMP algorithm is applied to exploit the block sparsity of UWA channel. Furthermore, the support additions are computed to track the time-varying multipath. Note that the performance of block sparse channel estimation is affected by the block size, and we adopt a larger block size when the sparsity of channel is smaller. Otherwise, a smaller block size is more appropriate.

Numerical simulations and a sea experiment are provided to demonstrate the effectiveness of the proposed algorithm, and the results show that for the block sparse time-varying UWA channels, the proposed BS-CS method outperforms the benchmark algorithms due to the exploitation of the block and time-varying sparsity. In future research, we will further test our proposed BS-CS algorithm using the watermark benchmark [32], so that a standard sound channel can be used as a reference for our estimations. Moreover, we will design the channel equalizer to handle the block sparse time-varying features in the UWA communication.

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