

Article

Parameter Sensitivity Analysis and Irrigation Regime Optimization for Jujube Trees in Arid Regions Using the WOFOST Model

Shihao Sun, Yingjie Ma, Pengrui Ai , Ming Hong  and Zhenghu Ma

College of Hydraulic and Civil Engineering, Xinjiang Agricultural University, Urumqi 830052, China; s1319889410@163.com (S.S.); xj-myj@163.com (Y.M.); hongming1109@163.com (M.H.); zhenghu_ma@126.com (Z.M.)

* Correspondence: xjaipengrui@xjau.edu.cn

Abstract

In arid regions, water scarcity and soil potassium destruction are major constraints on the sustainable development of the jujube industry. In this regard, the use of crop models can compensate for time-consuming and costly field trials to screen for better irrigation regimes, but their predictive accuracy is often compromised by parameter uncertainty. To address this issue, we utilized data from a three-year (2022–2024) field trial (with irrigation at 50%, 75%, and 100% of evapotranspiration and potassium applications of 120, 180, and 240 kg/ha) to simulate the growth process of jujube trees in arid regions using the WOFOST model. In this study, parameter sensitivity analyses were conducted to determine that photosynthetic capacity maximization (A_{max}), the potassium nutrition index (K_{status}), the water stress factor (SWF), the water–potassium photosynthetic coefficient of synergy (α), and potassium partitioning weight coefficients (β_i) were the important parameters affecting the simulated growth process of the crop. Path analysis using segmented structural equations also showed that water stress factor (SWF) and potassium nutrition index (K_{status}) indirectly controlled yield by significantly affecting photosynthesis (path coefficients: 0.72 and 0.75, respectively). The ability of the crop model to simulate the growth process and yield of jujube trees was improved by the introduction of water and potassium parameters ($R^2 = 0.94$ – 0.96 , NRMSE = 4.1–12.2%). The subsequent multi-objective optimization of yield and crop water productivity of dates under different combinations of water and potassium treatments under a bi-objective optimization model based on the NSGA-II algorithm showed that the optimal strategy was irrigation at 80% ET_c combined with 300 kg/ha of potassium application. This management model ensures yield and maximizes crop water use efficiency (CWP), thus providing a scientific and efficient irrigation and fertilization regime for jujube trees in arid zones.

Keywords: jujube trees; WOFOST model; parameter sensitivity analysis; path contribution; water-potassium synergy



Academic Editor: Dengpan Xiao

Received: 26 June 2025

Revised: 4 August 2025

Accepted: 6 August 2025

Published: 7 August 2025

Citation: Sun, S.; Ma, Y.; Ai, P.; Hong, M.; Ma, Z. Parameter Sensitivity Analysis and Irrigation Regime Optimization for Jujube Trees in Arid Regions Using the WOFOST Model. *Agriculture* **2025**, *15*, 1705. <https://doi.org/10.3390/agriculture15151705>

Copyright: © 2025 by the authors.

Licensee MDPI, Basel, Switzerland.

This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

As the core bottleneck constraining its sustainable socio-economic development, the competitive pressure on water resources is mainly reflected in the small total amount, uneven distribution, rapid growth in demand, and the serious crowding of ecological water [1]. The competitive pressure on water resources in the Xinjiang region of southern China presents a typical “production–life–ecology” triad of structural contradictions. The

absolute dominance of agricultural water use is the core source of pressure [2]. The rapid development of industry and urbanization has continuously raised new water demands and intensified the competition. It has become a consensus that competition for water resources poses a serious threat to the forest and fruit industries as well as to the steady development of the economy. As an important part of the economy of Xinjiang, the development of specialty forest and fruit industries is crucial to ensure the sustainable development of the region's economy [3]. In response to these problems, Xinjiang has widely promoted water deficit irrigation in recent years, and research has shown that moderate water deficit irrigation can promote crop growth and development by improving crop water use efficiency [4].

Jujube (*Ziziphus jujuba* Mill.) serves as an economically important fruit crop in the arid and semi-arid regions of China as well as globally [5]. Its cultivation not only plays an important role in ecological restoration and soil conservation but is also an economic lifeline as far as the development of local farmers is concerned [6]. Soil nitrogen, phosphorus, and potassium, as mineral elements necessary for the growth and development of fruit trees, have an irreplaceable role in the process of organ development, fruit yield, and quality formation of the crop [7,8]. The blind pursuit of maximizing the yield of date palm plantations exacerbates the competition for water resources and the destruction of soil nutrients in the region [9]. Studies have found that potash fertilizer is significantly less important for crop research compared with nitrogen and phosphorus [10,11]. Therefore, research on crop impacts based on water–potassium synergism is of great significance for the sustainable development of specialty forest and fruit industries located in arid zones [12].

However, as research continues, it has been found that traditional crop yield prediction methods, such as statistical models based on historical data or simple empirical models, often lack a reliable mechanistic basis [13,14]. As a result, the complex effects of complex environmental factors on the physiological processes, growth, and eventual yield formation of jujube cannot be effectively captured and elucidated by traditional empirical formulas alone. This shortcoming leads to significant prediction inaccuracies, especially under different environmental factor conditions, thus limiting the strategies for increasing yield and efficiency in the specialty forestry and fruit industry [15]. Crop growth models, as an alternative to field trials, have become an important tool for optimizing resource allocation by effectively connecting crop and environmental factors [16]. Models such as APSIM, DSSAT, AquaCrop, and WOFOST are used to overcome the spatial–temporal constraints, long experimental cycles, and uncertainties inherent in large-scale field trials [17,18]. As a result, they have become indispensable tools for a range of applications, from yield prediction and farm management optimization to climate change adaptation assessment. In this study, we consider that the WOFOST model can simulate in detail the key physiological and ecological processes of crops, such as photosynthesis, respiration, transpiration, and dry matter partitioning [19]. This makes it possible not only to predict yield but also to explain in depth the processes and causes of yield formation [20]. Current research has shown that key parameters affecting wheat growth and yield, such as optimum temperature, thermal time, and photoperiod sensitivity, have been identified through model simulations of wheat growth [21]. Through cassava crop studies, radiation use efficiency and maximum leaf area per leaf were identified through sensitivity analysis as key parameters affecting model outputs, which helped to determine the optimum amount of irrigation [22].

In summary, the sensitivity analysis of crop growth model parameters is an important tool for improving crop simulation accuracy and optimizing agricultural management [23,24]. Current sensitivity analysis methods can be broadly categorized into two types: local sensitivity and global sensitivity [25,26]. Unlike the Sobol method, which is

used to find the total and partial variances, the EFAST method simplifies the calculation process of the sensitivity index by designing the transfer function to transform the multidimensional integral into a one-dimensional one, thus making the parameter tuning process more structured and evidence-based [27]. However, despite extensive research on model sensitivity [22,23], there is still a gap in the current literature in the quantification of different parameter contributions. To bridge this gap, this study proposes a novel approach [28]. We use coefficient partitioning equations to combine water stress factors calculated within the WOFOST model with externally obtained data on potassium nutritional status. Segmented structural equation modeling (P-SEM) is used to quantitatively partition and reveal the extent to which the parameters contribute to the pathways affecting crop growth and yield, thus providing a theoretical basis for model calibration and subsequent research [29].

This research was carried out in Aksu Prefecture, Xinjiang. Its main objectives were as follows: (1) to decompose and screen the parameters of the WOFOST model based on field data, and to quantify the contribution of these parameters to the growth of jujube trees; (2) to calibrate and verify the WOFOST model, and then to simulate the local jujube orchard agricultural production process; (3) to simulate the crop growth process based on the model and combine it with the Non-dominated Sorting Genetic Algorithm II (NSGA-II). This study took yield and crop water productivity as the target constraints. Through the combination of irrigation system simulation and multi-objective optimization, the optimal irrigation system for increasing production and improving efficiency in this region was screened out, providing a reasonable scientific basis for the sustainable development of the characteristic forest and fruit industry in southern Xinjiang.

2. Materials and Methods

2.1. Study Location

The field experiment was conducted from 2022 to 2024 at the Fruit Tree Experiment Station of Xinjiang Agricultural University, which is located at Hongqipo Farm, Aksu Region, Xinjiang, China. The geographic coordinates of the site are 41°16' N, 80°14' E, and it is 1133 m above sea level. It is located in the central part of the southern foothills of the Tianshan Mountains, and the experimental area is characterized by a typical temperate continental climate.

The region receives an annual mean total solar radiation ranging from 544.12 to 590.16 kJ/cm², with an average of 2855 to 2967 h of sunshine annually. The frost-free period typically lasts for 205 to 219 days. The site is arid, with an annual mean precipitation of only 42.4 to 94.4 mm. The annual mean temperature is 11.2 °C, and the annual mean effective accumulated temperature is 3950 °C. The initial soil organic matter content and other key soil properties of the experimental plots are detailed in Table 1.

Table 1. Physicochemical parameters of the soil.

Soil Layer Depth (m)	Organic Carbon (g·kg ⁻¹)	Ammonium N (mg·kg ⁻¹)	Available P (mg·kg ⁻¹)	Exchangeable K (mg·kg ⁻¹)	Salt Content (mg·g ⁻¹)	pH	Soil Properties
0.00~0.10	7.76	43.28	130.55	295.04	223.02	7.40	Silt loam
0.10~0.20	6.42	24.57	20.52	215.21	175.41	7.91	Silt loam
0.20~0.30	5.58	24.81	26.22	197.25	124.83	8.01	Silt loam
0.30~0.40	5.21	18.39	12.31	193.45	117.65	7.91	Silt loam
0.40~0.50	4.53	17.33	11.4	190.29	92.82	7.96	Sandy loam
0.50~0.60	4.15	16.11	17.56	154.81	90.21	7.89	Sandy loam
0.60~0.70	3.45	15.49	16.67	146.20	84.34	8.01	Sandy soil
0.70~0.80	3.11	14.52	15.97	134.57	80.11	8.12	Sandy soil
0.80~0.90	2.34	13.78	15.42	120.35	78.56	8.15	Sandy soil
0.90~1.00	2.11	13.26	14.62	107.59	77.63	8.31	Sandy soil

Notes: The soil data were measured using a soil nutrient analyzer and soil reagents. The pH measurement was conducted using a soil: water ratio of 1:1 (*w/v*).

2.2. Experimental Design

The experiment was conducted using perennial ‘Huizao’ jujube trees, planted at a spacing of 4 m between rows and 1 m between individual plants. The field plots were managed under a drip irrigation system, with irrigation scheduling guided by intelligent soil moisture monitoring. Drip lines were positioned parallel to the tree rows at a distance of 40 cm from the trunks, featuring emitters spaced 30 cm apart, each with a nominal flow rate of 2.3 L/h. Irrigation events were scheduled at 10-day intervals, encompassing a 70 mm application for spring irrigation and a 30 mm application during the fruit maturation stage. For all other irrigation cycles, the specific irrigation quota was determined based on the cumulative crop evapotranspiration (ET_c) calculated for the preceding 10-day period using the Penman–Monteith equation. The detailed experimental treatments are provided in Table 2. Regarding fertilization, all treatments received a basal application of a compound fertilizer (N-P₂O₅-K₂O ratio of 17:17:17; total nutrients $\geq 51\%$) at a rate of 200 kg/ha in the spring. Subsequently, potassium fertilizer (as K₂O, $\geq 52\%$) was top-dressed at the flowering, young fruit, and fruit expansion stages. The amount of irrigation and fertilizer application was determined by consulting local research and other agronomic measures were the same as those practiced by other local farmers. All fertilizers were applied manually at designated points beneath the drip irrigation lines for each tree.

$$ET_{ci} = K_{ci} \cdot ET_{0i} \quad (1)$$

In the formula: ET_{ci} —The water requirement of crops in a certain period, mm.

ET_{0i} —The water requirement of reference crops in a certain period, mm.

K_{ci} —Represent the crop coefficient of a certain period.

Table 2. Experimental design treatment.

Handle	Spring Irrigation/(mm)	I / (mm)	II / (mm)	III / (mm)	IV / (mm)	V / (mm)
W1	70	ET_c	ET_c	ET_c	ET_c	30
W2	70	ET_c	$75\% ET_c$	$75\% ET_c$	$75\% ET_c$	30
W3	70	ET_c	$50\% ET_c$	$50\% ET_c$	$50\% ET_c$	30
Handle	Spring Irrigation/(kg·ha ^{−1})	I / (kg·ha ^{−1})	II / (kg·ha ^{−1})	III / (kg·ha ^{−1})	IV / (kg·ha ^{−1})	V / (kg·ha ^{−1})
A	200	-	240	240	240	-
B	200	-	180	180	180	-
C	200	-	120	120	120	-
Kc	-	0.42	0.78	0.88	1.00	0.81

Notes: I—germination stage; II—flowering period; III—young fruit stage; IV—fruit expansion period; V—maturity period. These are consistent with the following text.

2.3. Field Data Measurement

The comprehensive dataset acquired in this study was utilized for both the development and evaluation of the model. Specifically, data for the calibration and validation of the WOFOST model included meteorological records, soil properties, soil moisture content, time series measurements of the leaf area index (LAI), and key photosynthetic parameters (Table 3). Following calibration, the model’s predictive performance was rigorously evaluated by comparing its simulated outputs against a suite of independent field measurements. This evaluation dataset comprised the final yield, leaf area index, water use efficiency (WUE), potassium content in both leaves and soil, and critical fruit quality indicators.

Table 3. Main parameters of WOFOST growth simulation for jujube trees.

Parameter Name	Definition	Threshold	Critical	Unit
TBASE	Substrate Temperature	−1.00	1.00	°C
TFI	Initial accumulated temperature	200	400	°C
FLWR	Flowering day	10	15	Day
AWC	Soil available moisture content	10	40	mm/m
K0	Saturated soil conductance	10	30	cm/day
CDC	Canopy attenuation coefficient	5	10	%/d
DLC	Optical cycle	23	24	hr
DLO	Optimal day length	12	14	hr
AMAX	Maximization of photosynthetic capacity	30	40	$\mu\text{mol}/\text{m}^2\cdot\text{s}^1$
Kc	Crop transpiration coefficient	0.42	1.00	-
α	Photosynthetic synergy coefficient of water and potassium	0	1	
βi	Potassium allocation weight coefficient	0	1	
PRDEL	Output time steps	1.00	1.00	Day
WD	Water deficiency	50%	100%	mm
CWP	Crop moisture productivity	0	1	%
TBASEM	Minimum temperature required for crop growth	5.00	10.00	°C
SWF	Water stress factor	0	0.00	-
Kstatus	Potassium nutritional index	Kleaf	Kcrit	g/kg
DVS_K_STOP	Developmental stage when potassium absorption stops	0.50	1.50	-
KPART	Influence coefficient of potassium stress on leaf allocation	0.50	1.00	-

Soil moisture content: Soil moisture content was determined using an intelligent monitoring sensor (Model RS-WS, Jianda Renke, Jinan, China) and calibrated using the soil drying method. The range of measurement depth was 0 to 100 cm, and measurements were taken at 20 cm intervals. The observation levels of the measurement points were as follows: 0–20 cm, 20–40 m, 40–60 cm, 60–80 cm, and 80–100 cm.

Leaf area index and photosynthesis index: Measurements were made using a leaf area index meter (LAI-2200C, LI-COR, Cambridge, UK). Three jujubes with almost the same growth conditions were selected for each plot. Photographs were taken three times from the east, west, south, and north directions under the canopy of the jujubes, and the average value was taken. Using a photosynthesis meter (CIRAS-3, PP SYSTEMS, Amesbury, MA, USA), the net photosynthetic rate, stomatal conductance, intercellular CO₂ concentration, and transpiration rate of jujube leaves were measured. Measurements were taken every 7 to 10 days until the end of the growing season, and data were obtained from three jujubes in each treatment group.

Among them, the water use efficiency of the leaves WUE ($\mu\text{mol}/\text{mmol}$) was calculated by adopting the following calculation formula: $WUE = P_n / T_r$. In the formula: P_n represents the net photosynthetic rate; T_r is the transpiration rate.

The determination of the potassium content in the leaves: Leaves from the fourth row of the next year's jujube branches, in the east–west direction, were selected for processing. A total of 12 leaves of the same growth conditions were taken. They were immediately brought indoors for weighing, washed with distilled water, blanched in a 105 °C oven for half an hour, dried at 80 °C, and weighed again. Then they were ground, passed through a 1 mm sieve, and analyzed using an intelligent nutritional analyzer (TRX-03, Jianda Renke, Jinan, China). Sampling was conducted at the midpoint of each growth period until the end of the growth period.

The determination of yield and its constituent factors: The individual fruit mass of jujubes collected from each treatment was weighed using an electronic balance, and the longitudinal and transverse diameters of the corresponding jujubes were measured with a vernier caliper. Three plants were randomly selected from each repeat to measure the yield per unit area and converted into an output to represent the total output of the repeat. The volume of the fruit was approximately calculated by multiplying the harmonic parameters using the sphere formula. The crop water productivity CWP (kg/m^3) is calculated using the formula $CWP = Y / ET_C$. The partial productivity of fertilizer (PFP) (kg/kg) is calculated using the formula $PFP = Y / F$. In the formula: Y represents the output; ET_C is the filling water volume; and F represents the amount of potassium applied.

Meteorological module: Meteorological data, including temperature, radiation, and rainfall, were measured every 30 min using a Watchdog small automatic weather station (Model 2700, Spectrum Technologies, Inc., Aurora, IL, USA). The meteorological data for 2022–2024 is shown in Figure 1.

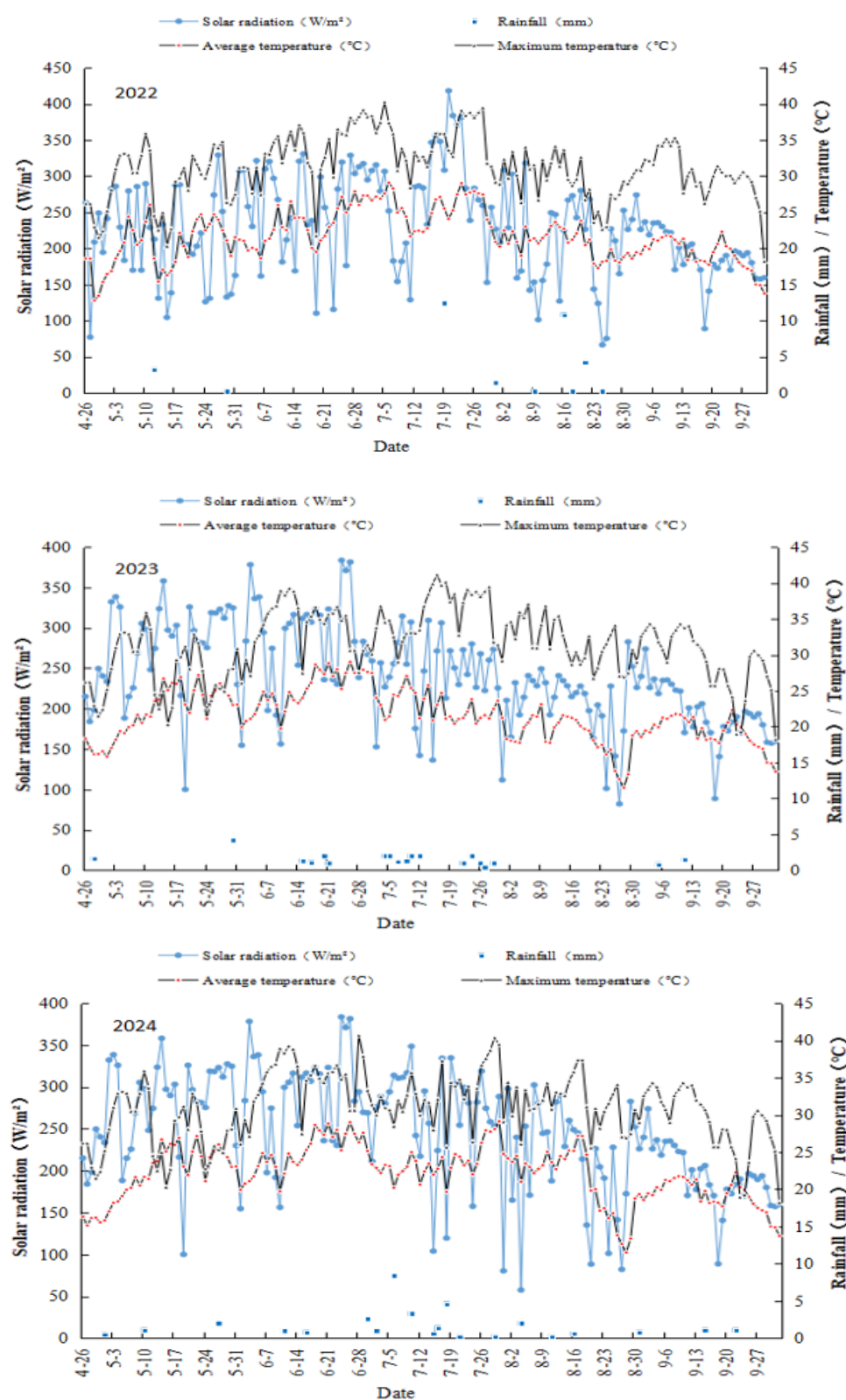


Figure 1. Meteorological data of the test station from 2022 to 2024.

2.4. Steps for Analyzing the WOFOST Model

Based on the meteorological data, local soil data, and yield data of Aksu Prefecture, Xinjiang, from 2022 to 2024, the model workflow is illustrated by Figure 2. In this study, the WOFOST model was employed to simulate jujube growth across a range of treatments involving different water and potassium levels. The Extended Fourier Amplitude Sensitivity Test (EFAST) method was subsequently utilized to conduct a sensitivity analysis, identifying the key parameters that most significantly influence the simulated growth processes and final yield of jujube. Within the water balance subroutine of the WOFOST model, water stress is quantified by the ratio of actual transpiration to potential transpiration ($SWF = T_{act}/T_{pot}$). This ratio represents the degree of physiological stress experienced by the crop under a soil moisture deficit. The assessment process of potassium nutrient status converts discrete and unitized field-measured leaf potassium content data into a standardized, dimensionless potassium nutrient index ranging from severe deficiency to adequate nutrition through piecewise functions, facilitating mathematical modeling.

$$K_{status} = \begin{cases} 0 & \text{if } K_{leaf} \leq K_{def} \\ \frac{K_{leaf} - K_{def}}{K_{crit} - K_{def}} & \text{if } K_{def} < K_{leaf} < K_{crit} \\ 1 & \text{if } K_{leaf} \geq K_{crit} \end{cases} \quad (2)$$

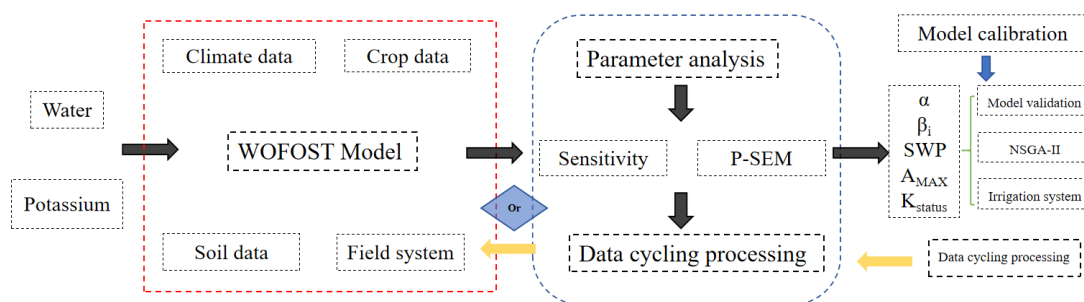


Figure 2. Experiment and crop model flowchart.

Among them, K_{status} represents the potassium nutritional index, K_{leaf} represents the potassium content in the leaves measured in field experiments (k/kg), and K_{crit} represents the threshold for potassium content deficiency in the leaves (k/kg).

The effective maximum assimilation rate ($A_{max} = A \times SWF^{(1-\alpha \cdot K_{status})}$) simulates the compensatory or mitigating effect of adequate potassium nutrition on alleviating the decline in photosynthetic rate caused by water stress. Among them, A represents the photosynthetic capacity of leaves, and α is the photosynthetic synergy coefficient of water and potassium, which determines the intensity of the alleviating effect of potassium on water stress. The dry matter distribution coefficient ($F = F(DVS) \times [(1 - \beta_i) \cdot SWF + \beta_i \cdot K_{status}]^\rho$) represents the dynamic adjustment of photosynthetic product distribution strategies in crops due to the synergistic effect of water and potassium, β represents the potassium distribution weight coefficient, and $i = 1, 2$, and 3 represent leaves, jujube stems, and branches. ρ represents the curvature parameter of the stress response, which is used to adjust the nonlinearity degree of the response and is usually set to 1 to simplify it to linear weighting. Finally, these parameters were imported into the WOFOST model to indicate the changes in the growth process and yield of jujube trees.

2.5. Parameter Sensitivity Correlation Analysis

To systematically identify the key parameters that influence model predictions and to elucidate their underlying mechanisms, this study employed a dual-analysis strategy. First,

a Global Sensitivity Analysis (GSA) was conducted using the Extended Fourier Amplitude Sensitivity Test (EFAST). The EFAST method quantifies the contribution of each parameter to the total variance of the key model output—namely, yield—by calculating the total-order sensitivity indices. This process allows for the ranking of all parameters and the subsequent screening of a core set of sensitive parameters that predominantly govern the model's behavior. A brief introduction to the algorithm is as follows:

The equation is defined as $y = f(x_1, x_2, \dots, x_k)$ which is transformed into $y = f(s)$ through the transformation function, and the Fourier transform is performed on $f(s)$ as follows:

$$y = f(s) = \sum_{i=-\infty}^{\infty} \{A_i \cos(is) + B_i \sin(is)\} \quad (3)$$

$$A_i = \frac{1}{N_s} \sum_{k=1}^{N_s} f(s_k) \cos(p_i s_k) \quad (4)$$

$$B_i = \frac{1}{N_s} \sum_{k=1}^{N_s} f(s_k) \sin(p_i s_k) \quad (5)$$

In the formula, N_s represents the number of samples, $i \in \bar{Z} = \left\{-\frac{N_s-1}{2}, \dots, -1, 0, 1, \dots, \frac{N_s-1}{2}\right\}$.

The spectral curve of the Fourier series is defined as $\Lambda_p = A_p^2 + B_p^2$. Then the variance of the model results caused by the input change in parameter x_i is $V_i = \sum_{p \in \bar{Z}} \Lambda_p \times \omega_i = 2 \sum_{p=1}^{\infty} \Lambda_p \times \omega_i$. The total variance of the model is decomposed as follows:

$$V = \sum_i V_i + \sum_{i \neq j} V_{ij} + \sum_{i \neq j \neq m} V_{ijm} + \dots + V_{i,j,\dots,k} \quad (6)$$

In the formula, V_i is the model variance caused solely by the input change in parameter x_i , V_{ij} is the coupled variance contributed by parameter x_i through the action of parameter x_j , V_{ijm} is the variance contributed by parameter x_i through the action of parameters x_j and x_m , and so on, and $V_{1,2,\dots,k}$ is the parameter x_i through $x_{1,2,\dots,k}$, the variance contributed by k . The contribution of the total variance of the model output can be represented by normalizing the first-order sensitivity index S_i of the processing parameter x_i as follows:

$$S_i = \frac{V_i}{V} \quad S_{T,i} = \frac{V - V_{-i}}{V} \quad (7)$$

In the formula, $S_{T,i}$ represents the global sensitivity. V_{-i} represents the sum of the variances of all other parameters excluding parameter x_i .

2.6. Parameter Contribution Degree Analysis

Building upon the parameters identified by the sensitivity analysis, this study employed Piecewise Structural Equation Modeling (P-SEM) to perform a parameter contribution analysis. By fitting the model, standardized path coefficients for each causal pathway are obtained, with the magnitude of each coefficient representing the relative importance of that path. A key strength of P-SEM is its ability to partition the total effect of a parameter on yield into a direct effect (the path directly linking the parameter to yield) and indirect effects (the sum of paths mediated through one or more intermediate variables). This decomposition reveals the regulatory mechanisms of the model's sensitive parameters on intermediate physiological processes, thereby clarifying how they subsequently influence the growth process and yield variations in jujube trees.

2.7. Model Evaluation and Testing Methods

In this study, the determination coefficient (R^2), root mean square error (RMSE), standardized root mean square error (NRMSE), fit index (d), and Nash efficiency coefficient (EF) were adopted to test the validity of the model. The calculation formulas are as follows:

$$R^2 = \frac{[\sum_{i=1}^n (S_i - \bar{S})(O_i - \bar{O})]^2}{\sum_{i=1}^n (S_i - \bar{S})^2 \sum_{i=1}^n (O_i - \bar{O})^2} \quad (8)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (O_i - S_i)^2}{n}} \quad (9)$$

$$NRMSE = \frac{RMSE}{\bar{O}} \times 100\% \quad (10)$$

$$d = 1 - \frac{\sum_{i=1}^n (S_i - O_i)^2}{\sum_{i=1}^n (|S_i - \bar{O}| + |O_i - \bar{O}|)^2} \quad (11)$$

$$EF = 1 - \frac{\sum_{i=1}^n (S_i - O_i)^2}{\sum_{i=1}^n (O_i - \bar{O})^2} \quad (12)$$

In the formula, n —sample size; O_i —measured value, $i = 1, 2, \dots, n$; $\bar{O}$ —mean of O_i ; S_i —model simulation value; and $\bar{S}$ —mean of S_i .

The closer the determination coefficient R^2 is to 1, the better the fitting effect of the equation is. If $NRMSE$ is $\leq 10\%$, it indicates that the model performs extremely well. Additionally, $10\% < NRMSE \leq 20\%$ indicates that the model performs well. Meanwhile, $20\% < NRMSE \leq 30\%$ indicates that the model performance is average. If $NRMSE$ is greater than 30% , it indicates that the model performs very poorly. In practical applications, the closer the d value is to 1, the better the fitting effect of the model is. When the d value is close to 0 or negative, it indicates that the predictive ability of the model is poor. An EF value close to 1 indicates that the model's prediction is accurate. A value close to 0 indicates that although there is consistency, the error is large. A value between 0 and 1 is usually acceptable, but less than 0 is not. The actual measurement is better than the simulation.

2.8. Multi-Objective Optimization of Irrigation System

In practical irrigation management, producers aim to not only achieve high yields but also to reduce irrigation water consumption, thereby enhancing crop water productivity. Therefore, this study conducts a multi-objective optimization of the irrigation strategy, using both yield (Y) and crop water productivity (CWP) as the optimization objectives. The decision variables for this optimization are the total irrigation amount and the potassium application rate. The optimization objectives are defined by the following equations:

$$\text{Optimization objective: } [Y, CWP] = \text{WOFOST}(W_1, \dots, W_j, F_1, \dots, F_j) \quad (13)$$

$$\text{Constraint conditions: } \begin{cases} W_{\min} \leq W \leq W_{\max} & (W \in N^*) \\ F_{\min} \leq F \leq F_{\max} & (F \in N^*) \end{cases} \quad (14)$$

In the formula, W —irrigation volume, mm; F —Potassium application rate, $\text{kg} \cdot \text{ha}^{-1}$.

The Non-dominated Sorting Genetic Algorithm II (NSGA-II) was employed to solve this multi-objective optimization model. In the NSGA-II framework, while the precision of the optimization results generally improves with larger population sizes and a greater number of generations, a balance must be struck to maintain computational feasibility. For this study, the number of generations was set to 100, and the population size was set to 50. This

configuration was used to generate a set of non-dominated solutions, from which suitable irrigation strategies for the region were identified.

3. Results

3.1. Effects of Different Water and Potassium Treatments on the Growth Indicators of Jujube Trees

As shown in Table 4, different irrigation levels had a significant effect on the physiological indicators of the crop ($p < 0.05$). Compared with the water deficit treatments, full irrigation (W1) significantly increased the leaf area index (LAI) by 34.8% and enhanced the net photosynthetic rate by 32.6%. Under a mild water deficit (W2), the LAI exhibited a slight decrease of 8.67%, while the leaf potassium content was reduced by 7.89%. Conversely, a severe water deficit (W3) resulted in a substantial 32.7% reduction in LAI, a sharp decline in the net photosynthetic rate to $16.27 \mu\text{mol}\cdot\text{m}^{-2}\cdot\text{s}^{-1}$, and a 38.88% drop in leaf potassium content. Similarly, the potassium fertilization treatments significantly influenced the growth and development of the jujube trees ($p < 0.05$). The high-potassium (A) treatment markedly boosted yield by 56.77% and increased the LAI by 22.70%. The W2-A treatment (mild water deficit combined with high potassium) emerged as the optimal strategy, demonstrating the highest water use efficiency and crop water productivity. A moderate synergistic application of water and potassium promoted the overall growth and development of jujube trees by coordinating nutrient transport and allocation among various organs, thereby enhancing resource use efficiency.

Table 4. Contents of various indicators of Jujube trees from 2022 to 2024.

Year	W	F	Yield /(kg·ha ⁻¹)	WUE /($\mu\text{mol}\cdot\text{m}^{-2}\cdot\text{s}^{-1}$)	CWP /(kg·ha ⁻¹ ·mm ⁻¹)	Fruit Index /(mm·mm ⁻¹)	LAI /(m ² ·m ⁻²)	Potassium Content /(g·kg ⁻¹)	A _{MAX} /($\mu\text{mol}\cdot\text{m}^{-2}\cdot\text{s}^{-1}$)
2022	W1	A	7356 a	2.51 a	8.17 b	1.41 a	2.27 a	26.55 a	28.24 a
		B	7079 a	2.43 b	7.87 c	1.40 a	2.11 b	24.37 a	26.34 b
		C	6520 b	2.05 d	7.24 d	1.44 a	1.88 c	21.19 b	21.48 c
	W2	A	6828 ab	2.73 a	8.88 a	1.33 b	2.45 a	25.23 a	26.84 b
		B	6620 b	2.53 b	8.56 a	1.32 b	2.03 b	22.57 b	25.41 b
		C	6156 c	2.35 c	8.21 b	1.31 b	1.73 c	20.36 c	19.34 c
	W3	A	4692 d	2.47 c	7.82 c	1.46 a	1.85 c	21.22 b	19.84 c
		B	4459 d	2.47 c	7.43 d	1.38 b	1.62 d	19.68 d	19.84 c
		C	4330 d	2.32 c	7.22 d	1.30 b	1.51 e	17.66 e	16.27 d
2023	W1	A	7543 a	2.34 bc	8.38 b	1.39 a	2.31 a	25.74 a	27.63 a
		B	7321 a	2.34 bc	8.13 c	1.37 a	2.17 b	23.77 b	26.91 a
		C	7106 b	2.11 c	7.90 d	1.37 a	2.03 c	21.33 c	22.61 c
	W2	A	6853 bc	2.72 a	9.14 a	1.21 b	2.65 a	24.83 a	25.82 b
		B	6465 bc	2.59 b	8.62 b	1.25 b	2.09 c	22.37 b	24.64 b
		C	6120 c	2.54 b	8.16 c	1.23 b	1.88 d	19.77 c	20.10 c
	W3	A	4980 d	2.44 b	8.30 b	1.36 a	1.95 c	20.85 c	18.61 d
		B	4630 d	2.46 b	7.72 d	1.24 b	1.76 d	18.61 d	19.22 e
		C	4570 d	2.39 bc	7.62 d	1.23 b	1.63 e	16.33 e	16.78 f
2024	W1	A	7768 a	2.41 a	8.09 b	1.34 a	2.25 a	25.42 a	28.46 a
		B	7533 a	2.21 b	7.84 c	1.32 a	2.09 b	24.11 a	25.77 b
		C	7205 a	2.07 c	7.51 c	1.35 a	1.88 c	22.07 b	22.13 c
	W2	A	7120 ab	2.52 a	9.12 a	1.35 a	2.31 a	24.77 a	26.55 a
		B	6689 b	2.14 b	8.57 b	1.24 b	1.97 bc	22.51 b	25.41 b
		C	6225 bc	1.95 c	7.98 c	1.24 b	1.74 c	20.03 c	19.34 d
	W3	A	5020 c	2.31 a	8.36 b	1.25 b	1.88 c	21.66 b	19.84 d
		B	4550 d	2.05 c	7.58 c	1.23 b	1.61 d	17.36 d	19.84 d
		C	4330 d	1.89 c	7.21 c	1.23 b	1.46 e	16.02 e	16.27 e

Note: Different letters following the values in the same column indicate significant differences among the various treatments ($p < 0.05$), and the same applies below.

3.2. Sensitivity Analysis of Crop Model Parameters

In order to elucidate and analyze the intrinsic correlations of the model parameters and their influence on the model output, we conducted a comprehensive analysis of its input parameters. The correlation analysis (Figure 3) revealed significant interrelationships among the parameters, most notably a strong positive and synergistic relationship

between the temperature factor (TFI), the flowering parameter (FLWR), and the maximum photosynthetic rate (A_{MAX}). Meanwhile, there was a significant negative correlation between potassium nutritional status (K_{status}) and ecological onset temperature (TBASE). At the same time, we performed a global sensitivity analysis to quantify the effect of each parameter on the model variance (Figure 4). The results clearly showed that maximum photosynthetic rate (A_{MAX}) and water co-factor (α) were moderately influential parameters, which mainly stemmed from the effect of the regulation between water and potassium during crop growth, rather than from crop-independent contributions. Potassium nutritional status (K_{status}) and soil water factor (SWF) were the main drivers of the differences in model outputs, which stemmed from the significant dissimilarity between their total order indices and first-order indices.

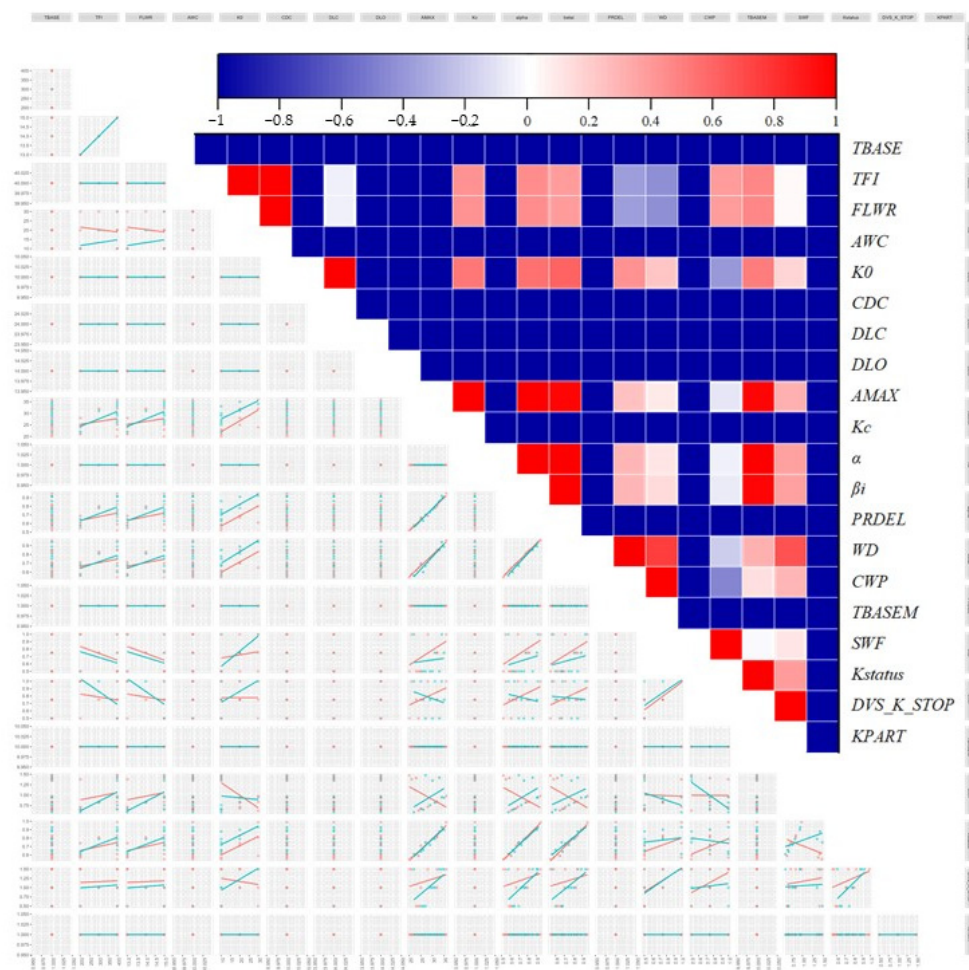


Figure 3. Parameter correlation analysis. Note: The lower left content expresses the regression relationship between the parameters and the upper right content expresses the correlation relationship between the parameters. The red color indicates a positive correlation between the parameters and vice versa.

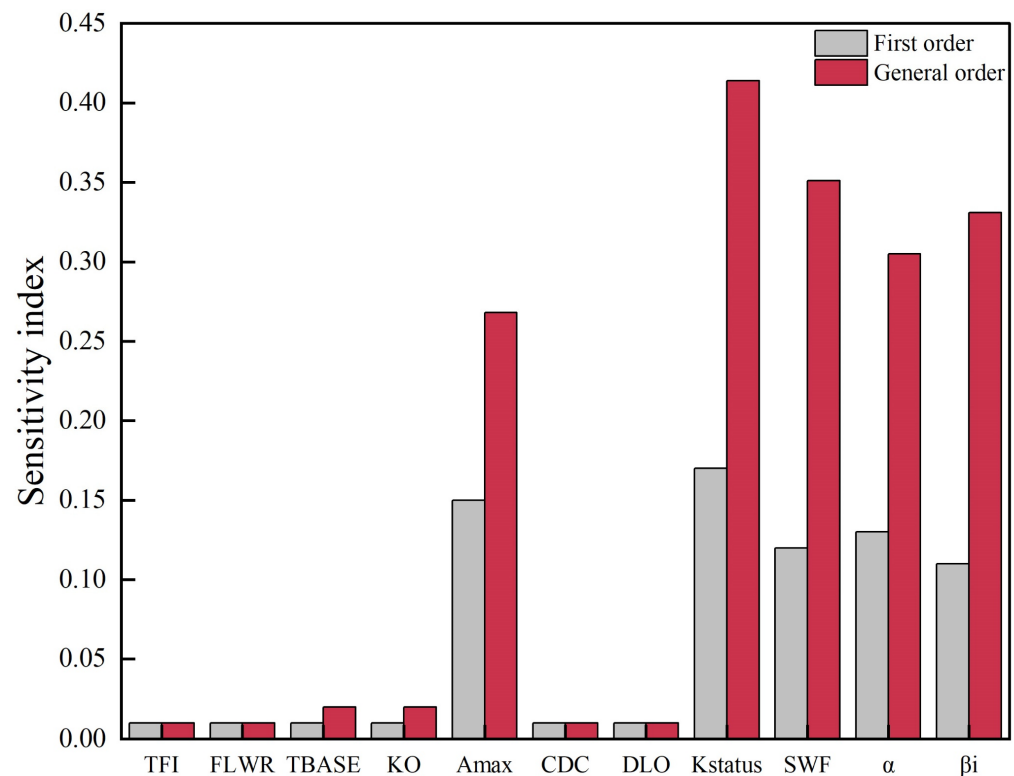


Figure 4. Parameter sensitivity results.

3.3. The Contribution Degree of Crop Model Parameters to Yield

To clarify the causal relationship between different water and potassium measures on crop yield, we constructed a segmented structural equation model (P-SEM) based on the WOFOST framework, focusing on key sensitive parameters including A_{MAX} , K_{status} , and SWF. The results showed that photosynthesis played the role of a central hub, which regulated the transformation of inputs to outputs and showed a strong direct correlation with final yields. The main pathway affecting this core hub was by water–potassium synergism: potash fertilization directly controlled plant potassium content (K_{status}), which in turn regulated the maximum photosynthetic rate (A_{MAX}) as a major determinant of crop growth and development (0.669–0.681). At the same time, irrigation controlled the soil water factor (0.717 and 0.728). The structural equation models for the two-year experiment yielded R^2 values of 0.85 and 0.77, respectively (Figure 5). K_{status} and SWF acted as the key factors with the most significant direct positive effects on photosynthesis (path coefficients, 0.734–0.753 and 0.717–0.728, respectively). Taken together, this revealed a strong synergistic relationship between these two factors, and their interaction significantly modulated the overall photosynthetic efficiency. This interaction was dynamic and enhanced by 34–52% ($p < 0.05$) in 2023 compared with 2022. Overall, these findings suggested that crop production followed a well defined “environmentally driven → physiological transformation → yield response” cascade. The effects of water and potassium application were not direct, but were critically regulated through this physiological network, and the results indicated that a moderate water–fertilizer balance was found to increase the efficiency of photosynthesis to yield conversion by 12%.

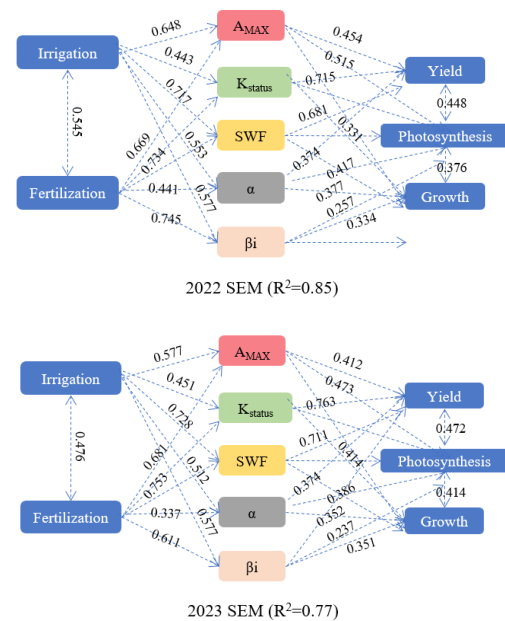


Figure 5. Parameter path contribution analysis for 2022–2023.

3.4. Calibration and Verification of WOFOST Model Parameters

The WOFOST model parameters were calibrated and subsequently validated using leaf area index data (Figure 6) under different treatment conditions between 2022 and 2024. As shown in Figure 6, the measured and simulated values of the leaf area index under different water–potassium treatments were in good agreement, with a coefficient of determination $R^2 \geq 0.73$, an NRMSE ranging from 4.6% to 24.9%, and EF and d values ranging from 0.73 to 0.96 and 0.90 to 0.99, respectively. This indicated that the WOFOST model could simulate the dynamic process of the leaf area index of date palm in arid zones.

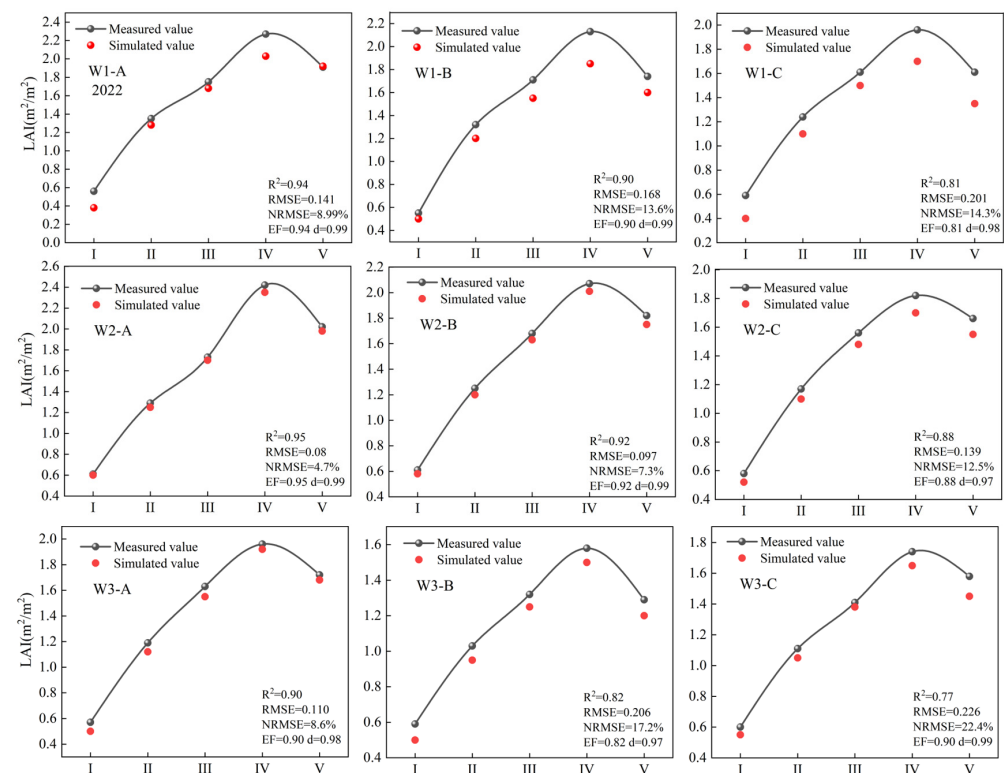


Figure 6. Cont.

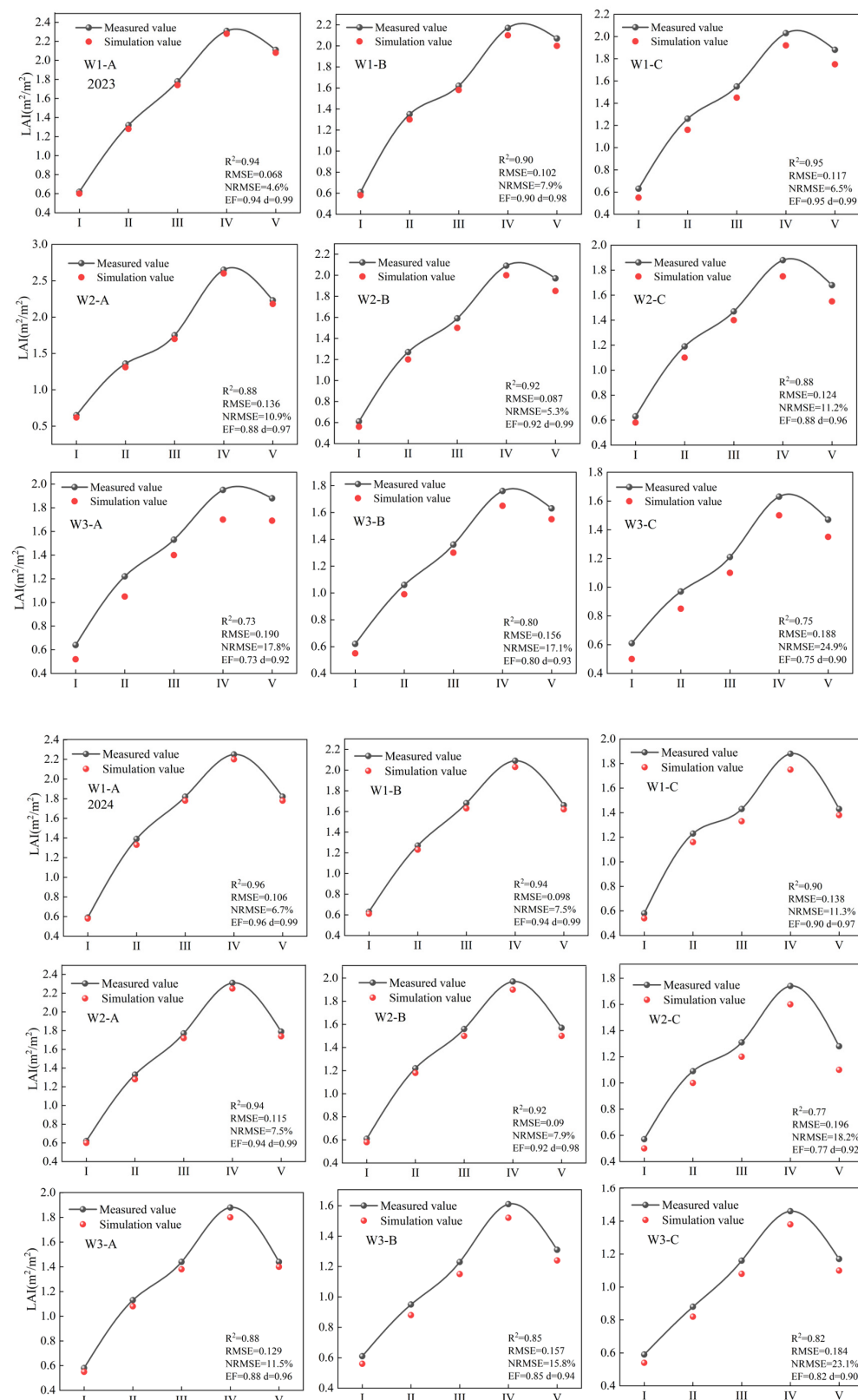


Figure 6. Measured and simulated values of jujube tree leaf area from 2022 to 2024.

In this study, the performance of the WOFOST model showed a decreasing trend under severe water deficit and potassium deficiency conditions, with coefficients of determination (R^2) ranging from 0.73 to 0.90. Such a situation may be due to the fact that under low-potassium conditions under a severe deficit, water and potassium allocation to various organs show irreversible damage, which increases the sensitivity of the crop to drought

signals (ABA), and thus contributes to the early presentation of emergence of resistance mechanisms in the crop. The model's consistency was improved by incorporating water and potassium factors. Specifically, under a mild water deficit, the introduction of the water stress factor (SWF) significantly improved the consistency between simulated and observed values, with R^2 increasing from 0.77 to 0.95. Similarly, when the growth of the date palm was mainly limited by potassium supply, the integration of the potassium nutritional index and potassium partitioning coefficient significantly improved the prediction accuracy of the model, with R^2 increasing from 0.73 to 0.95. In summary, water and potassium have key interactive effects on date palm growth. The optimized WOFOST model showed good consistency between the simulated and observed data, thus demonstrating that the calibrated WOFOST model can dynamically simulate the key processes of date palm growth, development, and yield formation.

3.5. Adaptability Test of the WOFOST Model

The results comparing the model-simulated and measured yields from 2022 to 2024 are presented in Figure 7. The first row shows the yield change curves under different treatments for each year, and the second row shows the corresponding scatter plots of the simulated versus measured values (Figure 7). The results showed that the model reproduced the observed yield trends well in all three years. It was found that the highest yields (7356–7543 kg/ha) were obtained under the W1-A treatments and the lowest yields (4330–4570 kg/ha) under the W3-C treatments from 2022 to 2023, reflecting the significant differences in yields among the different water–potassium combinations. The simulation results in both years were in high agreement with the measured yields: the model's coefficient of determination (R^2) was 0.942, the consistency index (d) was 0.921, and the Nash–Sutcliffe efficiency factor (EF) was 0.883 in 2022, while $R^2 = 0.952$, $d = 0.881$, and $EF = 0.921$ in 2023. These results indicated that the model had good reproducibility and robustness. A further analysis of the normalized root mean square error (NRMSE) for the different treatments revealed that the NRMSE at different potassium levels was only 4.1% under fully irrigated and mildly stressed conditions, whereas it rose to 12.2% under severe water stress conditions. It is possible that this occurs due to irreversible damage to the date palm at this time, which results in a downward trend in crop simulation. In the 2024 simulation, the R^2 was 0.96, the RMSE decreased to 133.2 kg/ha, the NRMSE declined to 4.7%, and EF and d reached 0.92 and 0.95, respectively. Furthermore, the linear fit plots of the simulated and measured values showed that the data from the three years had a high linear correlation; the slope of the fitted line was close to 1, and the data points were distributed within the confidence and prediction intervals. The simulation accuracies for the high-yield and low-yield treatments were particularly high. The fitting performance of the model under the high-water–potassium treatment was better than that under the severe deficit treatment, indicating that there is still room for the optimization of the water response mechanism.

In conclusion, the WOFOST model significantly improved simulation accuracy by introducing a potassium nutrient index and water stress factors. This enhancement can provide effective support for yield prediction and accurate decision-making in date palm plantations in southern Xinjiang under different water and potassium management scenarios.

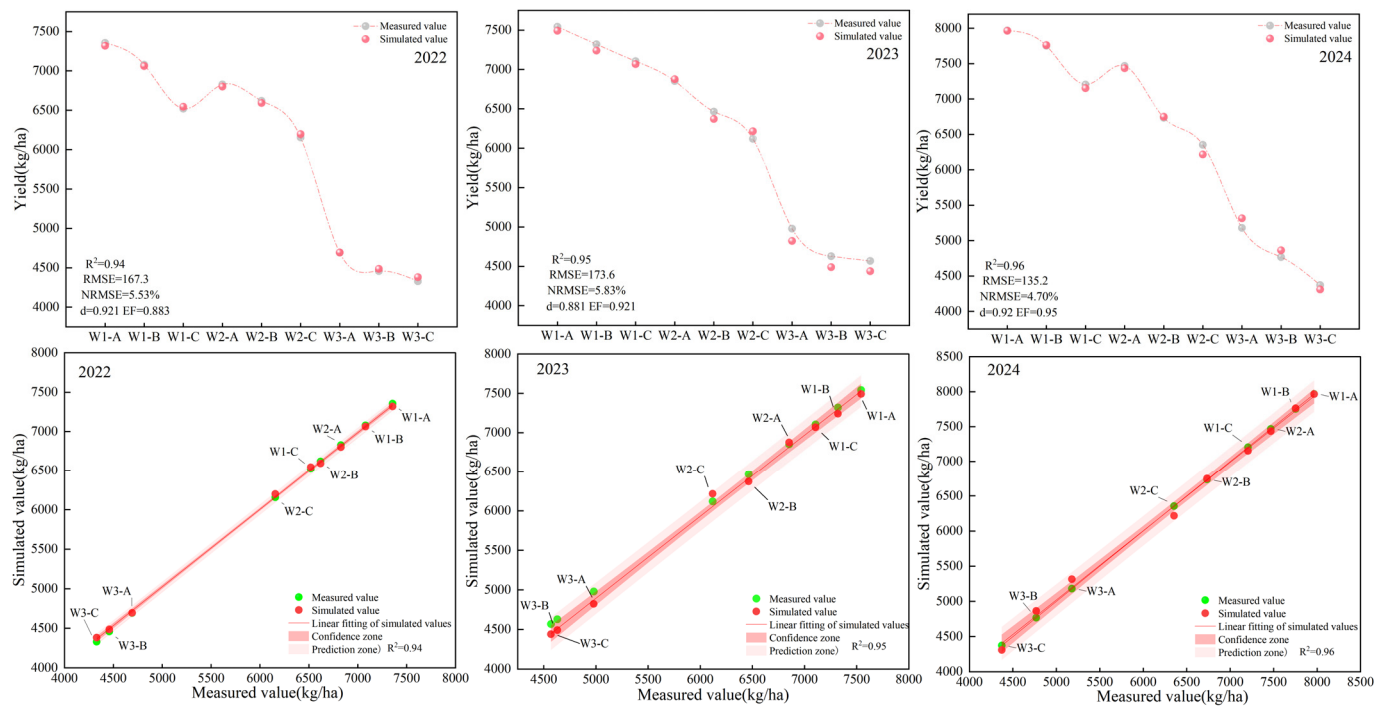


Figure 7. Measured and simulated production values from 2022 to 2024.

3.6. Optimize Irrigation System Decision-Making Based on the WOFOST Model

To determine the optimal irrigation regime for jujube in arid areas, different combinations of irrigation quotas and potassium application treatments were evaluated. The water–potassium combination scenarios were simulated using the WOFOST model. A dual-objective optimization model, based on the NSGA-II algorithm, was established to balance irrigation quotas and potassium application rates. This model was solved using MATLAB (2022b) to generate the Pareto front. The algorithm parameters were set as follows: an initial population size of 50, a maximum of 300 iterations, a crossover probability of 0.8, and a mutation probability of 0.05. This process yielded a set of optimal solutions to inform local irrigation system decisions. Figure 8 shows the crop yields under different combinations of irrigation rates (50% to 120% of ET_c) and potassium applications (100–300 kg/ha). The results revealed that yields did not increase linearly with increasing water and fertilizer inputs but instead showed a significant nonlinear interaction. The optimal synergistic effect was achieved with an irrigation level of 110% ET_c and a potassium application of 320 kg/ha, at which point the crop yield peaked. However, any deviation from these optimal practices resulted in a significant yield reduction. Higher yields could not be achieved by simply increasing irrigation under either low- or high-fertilizer conditions. Similarly, under conditions of water stress or water redundancy, the yield-enhancing effect of increased fertilization was significantly limited, and yield suppression even occurred under extreme conditions. The response of crop water productivity to water and fertilizer management was also analyzed. The response surface for CWP showed both similarities to and differences from that for yield. The peak CWP occurred within a narrower irrigation range than that of the high-yield area. This indicated that CWP had already begun to decrease at the irrigation levels that produced the highest yield. Specifically, CWP decreased sharply when irrigation levels exceeded 110% ET_c , indicating that excess water input was not efficiently converted into biomass, resulting in water wastage. This highlights the trade-off between yield maximization and efficiency optimization.

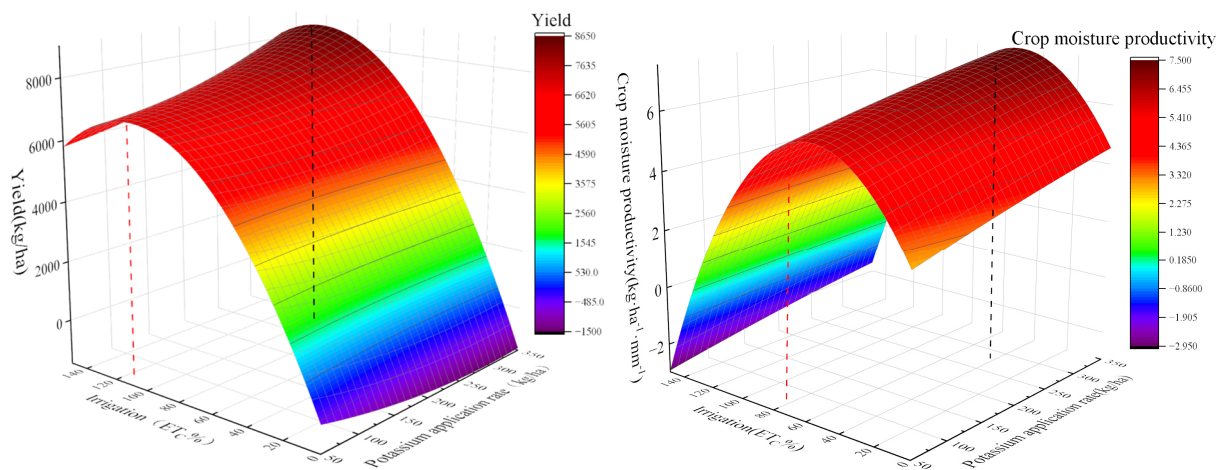


Figure 8. Decision-making on irrigation system for jujube trees in arid areas. Note: The red line corresponds to the highest peak value and the black color is the value of the preferred solution.

In summary, these results suggest a clear “optimal synergistic window” for water and fertilizer management. Beyond this window, excessive inputs not only fail to sustainably increase yields but also lead to a significant decrease in resource use efficiency. Based on the analysis, the recommended optimal equilibrium solution for achieving relatively stable yields and high crop water productivity was an irrigation level of 80% ET_c and a potassium application of 300 kg/ha.

4. Discussion

4.1. Effects of Different Synergistic Effects of Water and Potassium on the Growth Indicators of Jujube Trees

Drought stress, as the most destructive major environmental factor constraining crop growth and development, is often accompanied by the depletion of potash fertilizers, which also exacerbates negative crop regulatory mechanisms. Potash, as an activator of various enzymes in crops, is a well known stress reliever that alleviates the negative consequences of stress by participating in crop water regulation and enhancing photosynthesis and metabolism. As shown in Table 4, there were significant differences ($p < 0.05$) among treatments in crop yield, water use efficiency, and other growth indicators. The leaf area index (LAI) and photosynthetic rate of date palm decreased with the reduction in irrigation and potash application. Water use efficiency (WUE) and crop water productivity (CWP) showed a single-peak response; they first increased under mild water deficit conditions and then decreased sharply with an increasing deficit. Severe drought stress inhibited the growth of aboveground nutrient organs and reduced the leaf area index [30,31], an observation consistent with the findings of numerous previous studies [32,33]. Existing studies have shown that water deficit triggers an increase in biochemical indicators such as malondialdehyde, superoxide dismutase, and peroxidase, leading to a high accumulation of reactive oxygen species in the cytosol, which can damage cell membranes [34]. A moderate water deficit can rapidly increase the levels of antioxidant enzymes (e.g., superoxide dismutase and peroxidase) in plant organs and help them to resist stress [35].

The present study showed that potassium supply was able to mitigate the adverse effects of water deficit in date palm. This may be achieved by improving photosynthetic performance, which in turn helps to counteract the physiological decline induced by water stress. This explanation is consistent with previous findings that photosynthesis and yield are affected when crops experience water stress [36,37]. In summary, when the crop was treated with a mild deficit, potash fertilizer prioritized the transport and accumulation

mechanisms that affect the development of crop branches and stem thickness, thereby impacting crop yield and quality. When the crop suffered from a severe water deficit, its reproductive organs sustained irreversible damage, thus affecting fruit yield and quality by reducing crop growth indices and photosynthetic products. Under these conditions, potassium had no significant effect on crop stress regulation.

4.2. Parameter Sensitivity Analysis and Verification of WOFOST Model

It is well known that the growth process and yield of crops simulated by WOFOST are affected by a variety of parametric factors [38]. Among them, sensitivity analysis is a key tool that can be used to filter out the parameters that have a strong influence on the results from numerous inputs, thus reducing the model calibration effort, decreasing uncertainty, and improving the efficiency and accuracy of the whole process [39]. In this study, we used the EFAST method to assess the sensitivity indices of more than 20 WOFOST parameters on date palm growth. The results showed that for yield-related outputs (Figure 4), the most sensitive parameters were maximum photosynthesis rate ($A_{\max}$), potassium nutrient status (K_{status}), water stress factor (SWF), the synergistic water–potassium coefficient for photosynthesis (α), and potassium partition coefficients (β_i). Moisture limitation was found to have a significant effect on growth-related metrics, particularly the transport and storage of photosynthetic products, water stress, and the potassium nutritional index itself. This indicates that different levels of water and potassium during the growth of the date palm had a significant effect on crop yield. The accumulation of assimilates and transported products through date palm photosynthesis improves crop growth and development, thus increasing yield. This finding is in line with previous studies [40]. The sensitivity of the leaf area index and crop transpiration coefficient under specific moisture and potassium conditions was found to be insignificant in this experiment. This is likely attributed to the relatively stable meteorological conditions in the study area, a finding that emphasizes the importance of performing sensitivity analyses in specific environments before applying models to different environments [41].

While parameter sensitivity analyses deepen the knowledge of the parameters within the model on the final simulation effect, Piecewise Structural Equation Modeling (P-SEM) elucidates the effects of different parameters on the date palm growth process in a visual and quantitative manner. Our P-SEM analysis showed that SWF not only affected yield directly by limiting photosynthesis but also indirectly influenced yield formation through its effects on intermediate physiological processes (Figure 5), such as crop development and water uptake efficiency. At the same time, the model quantified how the potassium index (K_{status}) contributes to yield formation by increasing photosynthetic efficiency and improving the partitioning of photosynthates to the fruit. This reveals a key mechanism: water and potassium improve overall productivity and efficiency by indirectly regulating these intermediate growth processes [42].

Through the optimization and parameterization of the WOFOST model, as shown in Figure 6, compared with the results before optimization, the R^2 and d values increased and the RMSE decreased after optimization. This indicates that by calibrating the model parameters according to the sensitivity analyses and optimization results, we can more accurately simulate the changes in the growth and yield of jujube under different water and potassium regulations. The WOFOST model simulations of leaf area index (LAI) yielded normalized root mean square errors (NRMSEs) that ranged from 4.6% to 23.1%, and model efficiency (EF) and d values that ranged from 0.75 to 0.95 and 0.90 to 0.99, respectively. The simulated LAIs were lower than the observed values in most of the treatments between 2022 and 2024, a trend that is consistent with the results of a previous study [39]. This discrepancy may be due to the fact that, although our model was optimized

for water and potassium dynamics, it did not account for key horticultural practices such as the regular pruning of unproductive nutrient branches. Such management reduces competition for resources among organs in the field, a dynamic that may still be present in the model's simulations, leading to underestimation. This observation is similar to previous findings [43]. In summary, a comprehensive comparison of simulated and observed values of leaf area index and yield from 2022 to 2024, evaluated using a range of statistical indicators (R^2 , RMSE, NRMSE, d, and EF), confirms the ability of the improved WOFOST model to accurately simulate the growth dynamics and eventual yields of date palm trees in this arid region.

4.3. Decision-Making on Irrigation Systems in Southern Xinjiang Based on Crop Models

Scientific irrigation and fertilization regimes aim to optimize irrigation and fertilizer utilization to promote high-quality crop yield and efficiency [44]. The present study effectively circumvents the complexity, time constraints, and high cost of implementing such a wide range of treatments in a field trial by simulating yields and crop water productivity under different combinations of irrigation and potash fertilizer application rates. A previous study found that for an apple cropping system, magneto-electric water irrigation combined with 675 mm of irrigation and a 520 kg/ha fertilizer application rate was the most effective combination [45]. For date palm cropping systems in arid areas, irrigation at 90% of the full requirement is recommended, which makes it possible to achieve a balance between yield and water savings for dates [46].

In previous studies, numerous scholars have simulated crop growth and development using crop models, which often aimed to simply maximize a single variable or yield [47,48]. Some studies have used the AquaCrop model to construct a crop growth model, optimize it using a multi-objective genetic algorithm (NSGA-II) supplemented by the effectiveness coefficient method, and make decisions based on the optimization results [49]. The optimization of crop yield and quality by genetic algorithms confirms their reliability and superiority in solving crop water and fertilizer management problems [50]. This approach helps to develop efficient irrigation systems in drought-prone areas with limited water resources. Therefore, in this study, the WOFOST model was combined with a multi-objective optimization algorithm to optimize irrigation strategies for date palm in arid regions, balancing competing agronomic objectives. As shown in Figure 8, an irrigation quota of 80% ET_c was recommended in combination with a potash application rate of 300 kg per hectare. This solution was determined to be the optimal balancing strategy, and the optimization achieved the multiple goals of improved crop water productivity, fertilizer savings, and increased yields. This provides an effective and fast way to make customized and efficient irrigation management decisions for productive date palm cultivation in the region.

5. Conclusions

This study quantitatively evaluated the impact of varying water and potassium treatments on the growth and yield of jujube trees. Our results demonstrate that under a mild water deficit, potassium application effectively alleviates stress by enhancing photosynthetic activity, which subsequently improves water use efficiency. In contrast, under a severe water deficit, crops appeared to experience irreversible stress, where the beneficial regulatory effects of potassium became negligible. To mechanistically explain these findings, we integrated this synergistic water–potassium response into the WOFOST crop model. The model's most sensitive parameters were first identified using the Extended Fourier Amplitude Sensitivity Test (EFAST) and then optimized. Piecewise Structural Equation Modeling (P-SEM) was subsequently employed to quantify the contribution

of these parameters, successfully delineating their direct and indirect pathways to yield formation. This analytical approach revealed the core physiological mechanism: water and potassium co-regulate crop productivity by modulating photosynthetic efficiency and optimizing the allocation of photosynthates to the economic organs (i.e., fruit). This provides a foundational understanding of crop responses under the dual limitations of water and nutrient availability. Translating these model-driven insights into a practical management strategy, our multi-objective optimization analysis identified an irrigation regime of 80% ET_c coupled with a potassium application of 300 kg/ha. This specific regime is recommended as it strikes an optimal balance, ensuring high yields while maximizing crop water productivity and thus achieving significant water conservation. Ultimately, this work offers a scientifically grounded and optimized management strategy for sustainable jujube cultivation in the arid environment of southern Xinjiang.

Author Contributions: Writing—review and editing, S.S. and P.A.; investigation, M.H. and Z.M.; supervision, P.A., Y.M. and M.H. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by Pengrui Ai of the “Autonomous Region Youth Science Foundation”, grant number 2023D01B28, “The Central Guidance on Local Science and Technology Development Fund”, grant number ZYYD2023A10, and the “Talent Development Fund of the Autonomous Region’s Tianchi Talents Introduction Program”; Yingjie Ma of the National Natural Science Foundation of China, grant number 52069027; Xinjiang Uygur Autonomous Region Talent Development Fund “Tianshan Leading Talent” Program, grant number 2024TSYCLJ0013; Shihao Sun the Xinjiang Water Conservancy Engineering Safety and Water Disaster Prevention Key Laboratory 2024 Research Project, grant number ZDSYS-YJS-2024-20.

Data Availability Statement: The data presented in this study are available on request from the corresponding author. The data are not publicly available due to the policies and confidentiality agreements adhered to in the laboratory; we regretfully cannot provide the raw data.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Zhang, W.; Liu, D.; Chen, X.; Zhou, X.; Ding, Y.; Zhang, J. Driving factors for decoupling water resources ecological footprint and economic growth in water-deficient cities dominated by agriculture. *Water Sci. Technol.* **2023**, *88*, 1246–1268. [[CrossRef](#)] [[PubMed](#)]
2. Bai, S.; Abulizi, A.; Mamitim, Y.; Wang, J.; Yuan, L.; Zhang, X.; Yu, T.; Akbar, A.; Shen, F. Coordination analysis and evaluation of population, water resources, economy, and ecosystem coupling in the Tuha region of China. *Sci. Rep.* **2024**, *14*, 17517. [[CrossRef](#)]
3. Lin, B.; Ge, J. Carbon sinks and output of China’s forestry sector: An ecological economic development perspective. *Sci. Total. Environ.* **2019**, *655*, 1169–1180. [[CrossRef](#)] [[PubMed](#)]
4. Wang, Q.; Wang, H.; Liu, Q.; Zhu, T.; Xu, H.; Ren, H.; Yang, R.; Wu, L.; Zhang, Q.; Ke, J.; et al. Moderate alternate wetting and drying irrigation enhances drought-resistance abilities by improving structural mesophyll conductance of water-saving and drought-resistant rice under severe drought. *Physiol. Plant.* **2024**, *176*, e14518. [[CrossRef](#)] [[PubMed](#)]
5. Li, Y.; Zhou, X.; Zhao, K.; Liu, J.; Chen, G.; Zhang, Y.; Ma, J.; Sun, N.; Li, X. Cultivation and morphology of jujube (*Ziziphus jujuba* Mill.) in the Qi River Basin of Northern China during the Neolithic Period. *Sci. Rep.* **2024**, *14*, 2305.
6. Shen, H.; Wang, J.; Lyu, B. A study on factors influencing Chinese farmers’ willingness to input in Jujube planting. *Heliyon* **2023**, *9*, e21470. [[CrossRef](#)] [[PubMed](#)]
7. Liu, J.; Wang, D.; Yan, X.; Jia, L.; Chen, N.; Liu, J.; Zhao, P.; Zhou, L.; Cao, Q. Effect of nitrogen, phosphorus and potassium fertilization management on soil properties and leaf traits and yield of *Sapindus mukorossi*. *Front. Plant Sci.* **2024**, *15*, 1300683. [[CrossRef](#)]
8. Zhang, X.; Wang, J.; Bai, X.; An, S.; Zheng, Q.; Tang, Z.; Zhi, J. Water, nitrogen, and phosphorus coupling improves gray jujube fruit quality and yield. *Open Life Sci.* **2024**, *19*, 20220863. [[CrossRef](#)]
9. Wang, C.; He, W.; Kang, L.; Yu, S.; Wu, A.; Wu, W.; Rutherford, S. Two-dimensional fruit quality factors and soil nutrients reveals more favorable topographic plantation of Xinjiang jujubes in China. *PLoS ONE* **2019**, *14*, e0222567. [[CrossRef](#)]
10. Tang, L.; Tang, J.; Huang, K.; Huang, X. Potassium fertilizer promotes the thin-shelled Tartary buckwheat yield by delaying senescence and promoting grain filling. *Front. Plant Sci.* **2024**, *15*, 1385548. [[CrossRef](#)]

11. Yang, M.; Lu, J.; Liu, M.; Lu, Y.; Yang, H. Leaf Nutrient Resorption in Lucerne Decreases with Relief of Relative Soil Nutrient Limitation under Phosphorus and Potassium Fertilization with Irrigation. *Sci. Rep.* **2020**, *10*, 10525. [[CrossRef](#)]
12. Li, T.; Gao, Z.; Bai, X.; Yu, S.; An, S.; Zheng, Q.; Tang, Z.; Zhi, J. Exploring the coupling mode of water and fertilizer for improving growth, fruit quality, and yield of the pear in the arid region. *Open Life Sci.* **2024**, *19*, 20220911. [[CrossRef](#)]
13. Meng, X.; Liu, X.; Duan, H.; Hu, Z.; Wang, M. Research on Yield Prediction Model Driven by Mechanism and Data Fusion. *Sensors* **2025**, *25*, 1946. [[CrossRef](#)]
14. Leng, G.; Hall, J.W. Predicting spatial and temporal variability in crop yields: An inter-comparison of machine learning, regression and process-based models. *Environ. Res. Lett.* **2020**, *15*, 044027. [[CrossRef](#)]
15. Zhi, J.-H.; Zhang, D.; Huang, W.-J.; Chen, Q.-L.; Wang, J.-J.; Zheng, Q.-Q. Effects of compound growth regulators on the anatomy of Jujube Leaf and Fruit. *Saudi J. Biol. Sci.* **2020**, *27*, 3660–3668. [[CrossRef](#)]
16. Mason, N.W.H.; Palmer, D.J.; Romera, A.; Waugh, D.; Mudge, P.L. Combining field experiments and predictive models to assess potential for increased plant diversity to climate-proof intensive agriculture. *Ecol. Evol.* **2017**, *7*, 4907–4918. [[CrossRef](#)] [[PubMed](#)]
17. Tan, Y.; Cheng, E.; Feng, X.; Zhao, B.; Chen, J.; Xie, Q.; Peng, H.; Li, C.; Lu, C.; Li, Y.; et al. Application of APSIM model in winter wheat growth monitoring. *Front. Plant Sci.* **2024**, *15*, 1500103. [[CrossRef](#)] [[PubMed](#)]
18. Li, G.; Chen, W.; Cui, Y.; Wang, H.; Chi, Y. The adaptability and irrigation constraints analysis of the WOFOST model for grain production in the Songhua River Basin. *J. Sci. Food Agric.* **2024**, *104*, 7996–8007. [[CrossRef](#)] [[PubMed](#)]
19. Tang, R.; Supit, I.; Hutjes, R.; Zhang, F.; Wang, X.; Chen, X.J.; Zhang, F.; Chen, X. Modelling growth of chili pepper (*Capsicum annuum* L.) with the WOFOST model. *Agric. Syst.* **2023**, *209*, 103688. [[CrossRef](#)]
20. Yinfang, S.; Zhaoyang, W.; Cheng, H.; Puhon, Z. Yield estimation of *Lycium barbarum* L. based on the WOFOST model. *Ecol. Model.* **2022**, *473*, 110146.
21. Krishnan, P.; Maity, P.P.; Kundu, M. Sensitivity analysis of cultivar parameters to simulate wheat crop growth and yield under moisture and temperature stress conditions. *Heliyon* **2021**, *7*, e07602. [[CrossRef](#)]
22. Moreno-Cadena, L.P.; Hoogenboom, G.; Fisher, M.J.; Ramirez-Villegas, J.; Prager, S.D.; Lopez-Lavalle, L.A.B.; Pypers, P.; de Tafur, M.S.M.; Wallach, D.; Muñoz-Carpena, R.; et al. Importance of genetic parameters and uncertainty of MANIHOT, a new mechanistic cassava simulation model. *Eur. J. Agron.* **2020**, *115*, 126031. [[CrossRef](#)]
23. Gao, S.; He, P.; Gong, X.; Li, H.; Liu, Y.; Wang, Q.; Wang, X.; He, A.; Fu, Y. Sensitivity and uncertainty analysis of wheat cultivar parameters of the DSSAT model under different water and N treatments. *Sci. Rep.* **2025**, *15*, 10441. [[CrossRef](#)]
24. Casadebaig, P.; Zheng, B.; Chapman, S.; Huth, N.; Faivre, R.; Chenu, K.; Wu, R. Assessment of the Potential Impacts of Wheat Plant Traits across Environments by Combining Crop Modeling and Global Sensitivity Analysis. *PLoS ONE* **2016**, *11*, e0146385. [[CrossRef](#)]
25. Najjar, A.; Hamadeh, A.; Krause, S.; Schepky, A.; Edginton, A. Global sensitivity analysis of Open Systems Pharmacology Suite physiologically based pharmacokinetic models. *CPT Pharmacomet. Syst. Pharmacol.* **2024**, *13*, 2052–2067. [[CrossRef](#)] [[PubMed](#)]
26. Dela, A.; Shtylla, B.; de Pillis, L. Multi-method global sensitivity analysis of mathematical models. *J. Theor. Biol.* **2022**, *546*, 111159. [[CrossRef](#)]
27. Safikhani, H. Sensitivity analysis of the effective centrifugal pump parameters using the EFAST method. *Sci. Iran.* **2019**, *26*, 421–427. [[CrossRef](#)]
28. Folberth, C.; Elliott, J.; Müller, C.; Balkovič, J.; Chryssanthacopoulos, J.; Izaurralde, R.C.; Jones, C.D.; Khabarov, N.; Liu, W.; Reddy, A.; et al. Parameterization-induced uncertainties and impacts of crop management harmonization in a global gridded crop model ensemble. *PLoS ONE* **2019**, *14*, e0221862. [[CrossRef](#)] [[PubMed](#)]
29. Kim, M.; Sung, K. Assessment of causality between climate variables and production for whole crop maize using structural equation modeling. *J. Anim. Sci. Technol.* **2021**, *63*, 339–353. [[CrossRef](#)]
30. Hong, M.; Zhu, H.; Mo, H.; Zhao, J.; Ma, Y. The water consumption rule of jujube under different emitter flow rate and irrigation quota. *Agric. Res. Arid. Areas* **2014**, *32*, 72–77.
31. Ai, P.; Ma, Y.; Hai, Y. Influence of jujube/cotton intercropping on soil temperature and crop evapotranspiration in an arid area. *Agric. Water Manag.* **2021**, *256*, 107118. [[CrossRef](#)]
32. Zhong, Y.; Huang, Z.; Hao, K.; Zhu, S.; Fei, L.; Zeng, J.; Dai, Z.; Wang, Y. Effects of water deficit on fruit quality and water productivity of citrus under plastic film mulching in Western Hubei, China. *Front. Plant Sci.* **2025**, *16*, 1498798. [[CrossRef](#)] [[PubMed](#)]
33. Jing, Y.; Pei, T.; Zhang, S.; Li, C.; Zhan, M.; Li, C.; Gong, X.; Mao, K.; Liu, C.; Ma, F. Overexpression of FERONIA receptor kinase MdMRLK2 regulates lignin accumulation and enhances water use efficiency in apple under long-term water deficit condition. *Plant J.* **2024**, *119*, 2638–2653. [[CrossRef](#)] [[PubMed](#)]
34. Caliskan, S.; Ozkaya, I.; Caliskan, M.E.; Arslan, M. The effects of nitrogen and iron fertilization on growth, yield and fertilizer use efficiency of soybean in a Mediterranean-type soil. *Field Crops Res.* **2008**, *108*, 126–132. [[CrossRef](#)]
35. Farooq, M.; Wahid, A.; Kobayashi, N.S.M.A.; Fujita, D.B.S.M.A.; Basra, S.M. Plant drought stress: Effects, mechanisms and management. *Agron. Sus-Tainable Dev.* **2009**, *29*, 185–212. [[CrossRef](#)]

36. Candar, S.; Seçkin, G.U.; Kizildeniz, T.; Korkutal, İ.; Bahar, E. Variations of Chlorophyll, Proline, and Absciscic Acid (ABA) Contents in Grapevines (*Vitis vinifera* L.) Under Water Deficit Conditions. *Erwerbs-Obstbau* **2023**, *65*, 1965–1977. [[CrossRef](#)]
37. Tognetti, R.; Longobucco, A.; Miglietta, F.; Raschi, A. Water relations, stomatal response and transpiration of *Quercus pubescens* trees during summer in a Mediterranean carbon dioxide spring. *Tree Physiol.* **1999**, *19*, 261–270. [[CrossRef](#)]
38. Li, X.; Tan, J.; Li, H.; Wang, L.; Niu, G.; Wang, X. Sensitivity Analysis of the WOFOST Crop Model Parameters Using the EFAST Method and Verification of Its Adaptability in the Yellow River Irrigation Area, Northwest China. *Agronomy* **2023**, *13*, 2294. [[CrossRef](#)]
39. Vanuytrecht, E.; Raes, D.; Willems, P. Global sensitivity analysis of yield output from the water productivity model AquaCrop. *Environ. Model. Softw.* **2014**, *51*, 323–332. [[CrossRef](#)]
40. Hu, W.; Ren, T.; Meng, F.; Cong, R.; Li, X.; White, P.J.; Lu, J. Leaf photosynthetic capacity is regulated by the interaction of nitrogen and potassium through coordination of CO₂ diffusion and carboxylation. *Physiol. Plant.* **2019**, *167*, 418–432. [[CrossRef](#)]
41. Liu, J.; Liu, Z.; Zhu, A.-X.; Shen, F.; Lei, Q.; Duan, Z. Global sensitivity analysis of the APSIM-Oryza rice growth model under different environmental conditions. *Sci. Total. Environ.* **2019**, *651*, 953–968. [[CrossRef](#)]
42. Zhu, L.; Sun, Y.; Wang, R.; Zeng, J.; Li, J.; Huang, M.; Wang, M.; Shen, Q.; Guo, S. Applied potassium negates osmotic stress impacts on plant physiological processes: A meta-analysis. *Hortic. Res.* **2024**, *12*, uhae318. [[CrossRef](#)]
43. Bažok, R. Integrated Pest Management of Field Crops. *Agriculture* **2022**, *12*, 425. [[CrossRef](#)]
44. Ji, Y.; Ma, J.; Ma, L.; Xu, W.; Ji, Y.; Wang, L.; Feng, Y.; Feng, Z. Combination of water-saving irrigation and controlled-release fertilizer application reduced gaseous nitrogen loss in single-crop paddy soil. *J. Environ. Manag.* **2025**, *377*, 124695. [[CrossRef](#)]
45. Duan, X.; Wang, Q.; Mu, W.; Wei, X. Optimization of irrigation and fertilization of apples under magnetoelectric water irrigation in extremely arid areas. *Front. Plant Sci.* **2024**, *15*, 1356338. [[CrossRef](#)]
46. Bai, T.; Zhang, N.; Chen, Y.; Mercatoris, B. Assessing the Performance of the WOFOST Model in Simulating Jujube Fruit Tree Growth under Different Irrigation Regimes. *Sustainability* **2019**, *11*, 1466. [[CrossRef](#)]
47. Bai, Y.; Zhang, F.; Ma, X.; Ma, H.; Liu, Q. Enhancing crop yield predictions under drought: Integrating Accumulated Drought Degree Days into the WOFOST model. *Ecol. Model.* **2025**, *508*, 111224. [[CrossRef](#)]
48. Singh, A.P.; Yerudkar, A.; Liuzza, D.; Liu, Y.; Glielmo, L. An Optimal Decision Support System Based on Crop Dynamic Model for N-Fertilizer Treatment. *Sensors* **2022**, *22*, 7613. [[CrossRef](#)]
49. Song, J.; Li, J.; Yang, Q.H.; Mao, X.M.; Yang, J.; Wang, K. Multi-objective optimization and its application on irrigation scheduling based on AquaCrop and NSGA-II. *Water Resour. Hydropower Eng.* **2018**, *49*, 1284–1295.
50. Qu, F.; Zhang, J.; Wang, J.Z.; Ma, X.Q.; Gao, Z.X.; Liu, D.N.; Hu, X.H. Optimization of nutrient solution formula for soilless cucumber cultivation based on genetic algorithm. *Trans. Chin. Soc. Agric. Eng.* **2021**, *37*, 96–104.

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.