



Article Sub-Terahertz Computer Generated Hologram with Two Image Planes

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Abstract: An advanced optical structure such as a synthetic hologram (also called a computer-generated hologram) is designed for sub-terahertz radiation. The detailed design process is carried out using the ping-pong method, which is based on the modified iterative Gerchberg–Saxton algorithm. The novelty lies in designing and manufacturing a single hologram structure creating two different images at two distances. The hologram area is small in relation to the wavelength used (the largest hologram dimension is equivalent to around 57 wavelengths). Thus, it consists of a small amount of coded information, but despite this fact, the reconstruction is successful. Moreover, one of the reconstructed images is larger than the hologram area. Good accordance between numerical simulations and experimental evaluation was obtained.

Keywords: far infrared or terahertz optics; computer-generated holography; optical processing; THz optical systems design

1. Introduction

Relatively long wavelengths corresponding to the terahertz (THz) range of electromagnetic radiation make them safe and allow penetrating different substances, like plastics, polyethylenes, polystyrenes [1], polyamides [2], wood, concrete, foams [3], papers [4,5], and other materials [6]. These features opens up many possible applications like non-destructive testing [7,8], non-invasive quality control [9], security screening [10], and many more [11]. Technologies based on THz radiation are quickly evolving, and that is why recent years have brought vast improvements and many new ideas in this field [12–15].

The main drawback of THz radiation is the fact that it is strongly attenuated by water vapor, and thus, efficient communication using these waves is limited to particular frequencies, often named propagation windows [16]. Nevertheless, in the case of outdoor applications THz waves exhibit significantly lower losses in comparison with visible light while propagating in conditions of fog [17,18]. Thus, using this range of radiation for communication or automotive systems becomes promising. Taking into account the transferring of THz data or signals, we should focus on multiple-input multiple-output systems (MIMO) [19], which can redirect energy in the desired way. Some effort was put into creating a THz MIMO channel and link that enabled higher transmission speed than a single-channel system [20]. Moreover, designing a graphene-based reconfigurable antenna that can be programmed dynamically allowed forming a MIMO system with higher spectral efficiency [21]. Considerations related to the increasing number of antennas were discussed in the case of massive MIMO systems [22] and their design schemes. Growing interest in transferring data using multiple-input and multiple-output systems makes this research direction desirable. Using optical

structures to combine and divide input and output signals may also be an interesting application, especially for indoor wireless data transferring.

Moreover, using optical diffractive structures (like synthetic holograms) may allow focusing electromagnetic radiation in more than a single plane (which increases the amount of accessible spatial dimensions for signal processing, the main idea of MIMO technique according to [23]). Other optical methods for the implementation of optical MIMO may be carried out by introducing dispersive and modal multiplexing (widely described in [24]).

Diffractive structures can be realized for example using computer-generated holographic design. Examples of THz holograms reproducing simple images have been already proposed [25]. However, it has to be indicated that in that paper, only a single image was obtained from one phase plate (the main goal was to prove that a commercially-available 3D printer can be used to produce such complicated structures). Here, the presented paper further develops the topic of terahertz holograms by introducing multiple image planes with different images. Such an approach, however, is not without disadvantages: images have to be simple, and overlapping of images may cause some sort of ghosting or shadowing effect on further image planes.

Terahertz holographic structures have been used for the generation of vortices [26] and Bessel beams [27] or to focus radiation in a matrix of spots [28]. However, in the aforementioned cases, relatively more expensive methods of fabrication were used in comparison with the 3D printing technique (here, selective laser sintering (SLS)) and did not allow redirecting the energy for example at different distances.

Up to now, the possibility to project in only one image plane limits the application in scanning systems or MIMO devices; thus, using more reconstructed image planes becomes desirable, especially using a single-hologram structure. All previously-mentioned holograms have proven that in the current state-of-the-art, both of these features (multiple image plane coming from one holographic structure) are the novelty of our solution for THz optical structures.

In this article, we propose a single computer-generated hologram reconstructing two images at two different planes. Such a hologram is designed as a phase structure using an iterative algorithm, increasing its efficiency. The image size is larger than the hologram. Moreover, the hologram area is small in comparison to the design wavelength (DWL). In order to provide the reader with a simple comparison: the presented hologram's largest dimension is equivalent to around tens of DWLs (less than 60), whereas in the case of visible light holograms having a size of for example 6 cm would be equal to half of a million DWLs.

Such a hologram is a proof of concept, enabling the design of holographic structures, for example for MIMO systems. Moreover, it may be also used to display markers; in motorization to detect distance from objects in bad visibility conditions.

2. Results

In order to obtain good quality image reconstruction, firstly, we should carry out a proper design process. Thus, in this article, we decided to use an iterative method of calculating hologram phase distribution based on the Gerchberg–Saxton algorithm [29], but using more planes and propagation between them instead of Fourier transforms, called the ping-pong algorithm [30]. It performed iterative propagations between two object and one hologram planes with phase preservation and forcing desired amplitude distributions. In image planes, amplitude distributions were forced to have the shape of the designed images, and in hologram plane, the amplitude was unified (set to one); see Figure 1. In such a way, a kind of synthetic Fresnel hologram could be obtained, previously described for visible light [31]. In the original Gerchberg–Saxton algorithm, the iterations were realized only between two planes (one hologram and one image), and fast-Fourier transform (FFT) was used. Here, we created an iteration process between three planes with the assumption that the light field was propagated between planes using the modified convolution method [32], making our solution closer to the ping-pong algorithm. The number of iterations was limited to 25, which was based on the convergence of algorithm used [33].



For clarity, the simplified pseudocode is provided (see Algorithm 1).

Figure 1. Designing a synthetic hologram: numerical simulation scheme. *Prop* indicates propagation at particular distance.

| Alg | orithm 1 Modified Gerchberg–Saxton algorithm. | |
|-----|---|---|
| 1: | procedure MODIFIEDGS | |
| 2: | initial steps: | |
| 3: | illumination \leftarrow planewave | |
| 4: | $amp \leftarrow Object2$ | ▷ plane with "square" |
| 5: | OffAxisPropagation(-150) | ▷ argument in mm |
| 6: | $amp \leftarrow Object1$ | ▷ plane with "cross" |
| 7: | OFFAXISPROPAGATION(-150) | |
| 8: | $amp \leftarrow 1$ | ▷ amplitude equalization; phase not changed |
| 9: | loop through planes: | |
| 10: | for i = 0; i<= 20; ++i do | |
| 11: | OFFAXISPROPAGATION(150) | |
| 12: | $amp \leftarrow Object1$ | ▷ plane with "cross" |
| 13: | OFFAXISPROPAGATION(150) | |
| 14: | $amp \leftarrow Object2$ | ▷ plane with "square" |
| 15. | OFFAXISPROPAGATION(-150) | |
| 16: | $amn \leftarrow Object1$ | ▷ plane with "cross" |
| 10. | | |
| 17: | OFFAXISPROPAGATION(-150) | |
| 18: | $amp \leftarrow 1$ | ⊳ hologram plane |
| 19: | end: | |
| 20: | SAVE(phase) | ⊳ save hologram |

Moreover, we decided to use the off-axis approach due to the assumed relatively small size of the hologram of 100 mm and the large image size at close distances. Thus, choosing two images—a cross

having a size of 150 mm and a square having a size of 60 mm—distant from the hologram plane at $z_1 = 150$ mm and $z_1 + z_2 = 300$ mm, respectively, violates the assumption of the paraxial approach (Figure 2). Therefore, the numerical description was conducted in the off-axis regime [34].



Figure 2. Schemes of the experimental setups and its photograph with marked paths of the beam and two image planes. The detector was moving on an XYZ stage.

All design procedures concerning diffractive optics (here, our hologram) by definition assumed using perfectly temporally-coherent illumination with a plane wave. For this reason, we designed the hologram structure in such a regime. Nevertheless, we were able to conduct additional simulations taking into account different shapes and divergences of the experimental beam.

Due to a lack of a priori information about the shape of the beam in the experimental setup, first, simulations were performed for plane wave illumination, and also, the hologram structure was manufactured for such assumptions. Later, to obtain simulations corresponding to the experimental data, we carried out additional simulations taking into account different shapes of the beam based on data obtained from the optical setup (Figure 3).

The first setup (A) consisted of an emitter, a hologram placed 790 mm after, and a detector (both based on frequency multipliers and Schottky diodes), as shown in Figure 2a. Next, we used an additional lens, having both a diameter and focal length equal to 300 mm, which was placed 600 mm after the emitter, and then, 100 mm after the lens was the hologram structure, as shown in Figure 2b. Such setups introduced scaling of the reconstructed images and caused the change of distances where these images were created, which is summarized and compared in Table 1. However, Setup B was

used to assure the wavefront was close to a plane wave illumination of the hologram. As can be seen, the distances were close to the designed ones, and the magnification was close to one.

The x and y cross-sections of the beam illuminating the hologram were measured in the experimental setup for two cases illustrated in Figure 2. Registered data enabled determining the overall shape of the incident beam and allowed approximating it with a Gaussian wave having a particular intensity and divergence. The full width half maxima of beams were calculated for fitted curves (Table 1), together with their maximal values. The fitted curves were analyzed due to the fact that measured data were characterized by oscillations resulting from additional interferences. The radiation emitted by Schottky diodes with frequency multipliers was highly coherent, and thus, many spurious interferences appeared (as shown in the inset in Figure 3).



Figure 3. X and Y cross-sections of the measured illuminating beam in the case of Setup A.

Table 1. Incident beam parameters (full width at half maximum (FWHM)) for A and B setups. Reconstructed image parameters: distance from the hologram to a particular image and magnifications of the images formed.

| Setup | FWHM x/y (mm) | z ₁ (mm) | z_1+z_2 (mm) | M_1 | M_2 |
|---------|---------------|---------------------|----------------|-------|-------|
| Setup A | 102/114 | 190 | 480 | 1.27 | 1.65 |
| Setup B | 93/96 | 140 | 270 | 0.95 | 0.93 |

3. Methods

The set of simulation parameters was imposed mainly by the available apparatus and materials. The design wavelength (DWL) was set to 1760 µm and was dictated by the accessible source and detector. The frequency multiplier from Virginia Diodes Inc. (at a working range of 140–220 GHz) was used as a source of radiation (the maximum of spectral power distribution fell at around 1760 µm). Detection was carried out also with a Schottky diode from Virginia Diodes Inc. (with a working range of 140–220 GHz). The detector was attached to the set of motorized stages, allowing scanning in all three directions (a span of 0–15 cm). Furthermore, a relatively long wavelength was chosen to ensure a high quality manufacturing of the hologram structure (shown in Figure 4) and to assure low attenuation. Additionally, for this wavelength, the refractive index of polyamide 12 (PA12) was established (with a Teraview TPS Spectra 3000 spectrometer under conditions of low humidity, not exceeding 0.5%) as equal to 1.59 (while the absorption coefficient $\alpha = 1$ cm⁻¹). From the optical point of view and accounting for the wide range of highly-absorptive materials for THz radiation,

such a value of absorption coefficient is relatively low. PA12 is a material often used for 3D printers, which together with the high resolution of the 3D printer (compared to DWL), allowed printing the simulated hologram. The available 3D printer imposed a sampling of 39 μ m (we assumed a voxel size of 117 μ m to assure good quality of printing and the possibility to print structures with a thickness varying from 0 to almost 3 mm). Such a value was sufficient due to the millimeter-size antenna used during the experiment and was necessary only to create a proper file for manufacturing. In order to reduce the neighbor effect, to weaken the impact of boundaries in the simulations carried out, and to fit the designed image, the calculation matrix size was established to be 8192 px by 8192 px. It has to be noted that with the image sampling of 117 μ m and the real size of the hologram equal to 10 cm, its size in pixels was equal to 855 px by 855 px. All simulations were carried out using LightSword software (based on the modified convolution propagation method) in version 6.4.9. Structures were printed with the EOS P 396 printer. It is a selective laser sintering printer providing a layer thickness of 0.12 mm (for PA12) and a standard accuracy of $\pm 0.3\%$ with a lower limit at ± 0.3 mm.

The sizes of the images obtained (Figure 5) in comparison with the hologram size (Figure 4) were bigger. Here, it should be underlined that we can test whether reconstruction from a relatively small hologram (approximately 57 wavelengths) will provide enough information to obtain a reasonable quality image.



Figure 4. Manufactured hologram (**left**) and its phase distribution (**right**), where black color denotes the 0π phase shift and white 2π .



Figure 5. Presentation of objects (**upper**) and the results of the simulation for plane wave illumination (**lower**) for both image planes. Red squares denote the size of the hologram.

Figure 5 illustrates the amplitude distributions of the designed objects. Then, the images obtained as a result of simulations, at first, with plane wave illumination assumed, are shown in the lower row of Figure 5. In all distributions, a hologram area is marked with a red square.

Additionally, a set of simulations for a larger number of objects was carried out. We have demonstrated that using such simulation parameters, we can create a hologram consisting of 4 different simple objects.

4. Discussion

Due to the high coherence of the source and properties of terahertz radiation, adding any unnecessary object to the optical path resulted in undesirable interferences. In order to resolve this problem, some simplifications in the experimental system were done. For this reason, the classical collimator was not used in this setup.

The prepared structure was tested in two setups. The first one (Setup A in Figure 2) consisted only of the hologram illuminated directly by the source. Due to the size of the image and the long registration time for the second setup, the hologram was illuminated with the beam close to the plane wave (lens f = 300 mm d = 300 mm placed 600 mm from the source). Scanned positions were chosen after performing simulations of reconstruction in such a setup. To reduce the stray beams, an additional isolator was necessary.

Incredible correspondence between simulations and obtained experimental measurements stood out and is shown in Figure 6. In the case of Setup A, the images were larger than designed, and thus, the whole area shown has a size of 21 mm. All presented images have the same size, and due to the fact that experimental scans had smaller dimensions, the additional white square were added to keep proper magnification. Experimental results allowed clearly identifying the features of defined objects despite a large amount of interferences (particularly visible in Figure 4), usually present in such THz setups. The presence of the parallel image plane is visible, although it does not disturb the image significantly. It should be noted that reconstruction from such a small hologram area introduced a large depth of focus due to the small aperture size. Moreover, the size of speckles was larger in such case.



Figure 6. Comparison of the simulation with realistic illumination and experimental results for reconstructing two images from a single hologram. Represented values for the experimental evaluation are normalized.

5. Conclusions

We designed and simulated a single computer-generated Fresnel hologram for a sub-terahertz band (0.17 THz). The novel solution was to use a single hologram containing information about two images placed at two separate planes. the larger size of the image in relation to the hologram area was crucial also, which was in the range of dozens of wavelengths (less than 60).

To design such an element, we used the ping-pong algorithm, a modified version of the Gerchberg–Saxton algorithm. Furthermore, additional simulations showed the possibility of encoding even a larger amount of planes in a single-phase plate; in this case, four. We presented experimental data proving that a relatively small area of sub-THz hologram (having a size of about 57 wavelengths) contains enough information to obtain fine features of the two chosen binary objects. The presented concept can be potentially used in MIMO telecommunication systems or even in the transport industry as markers for distance detection in reduced visibility conditions.

Author Contributions: A.S. was responsible for the total conceptualization, methodology, and supervision of the experiments, A.S. and M.S. wrote, reviewed, and edited the original draft; M.S. and I.D. performed numerical simulations and data analysis, and prepared files for manufacturing the hologram structure, A.S., M.S., and P.Z. performed experiments and evaluated the experimental data.

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