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Inversion of Thermal Conductivity in Two-Dimensional Unsteady-State Heat Transfer System Based on Finite Difference Method and Artificial Bee Colony

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Abstract: Based on the finite difference method and the artificial bee colony algorithm, the thermal conductivity in the two-dimensional unsteady-state heat transfer system is deduced. An improved artificial bee colony algorithm (IABCA), that artificial bee colony algorithm (ABCA) coupled with calculated deviation feedback, is proposed to overcome the shortcomings of insufficient local exploitation capacity and slow convergence rate in the late stage of the artificial bee colony algorithm (ABCA). For the forward problems, the finite difference method (FDM) is used to calculate the required temperature value of a discrete point; for the inverse problems, the IABCA is applied to minimize the objective function. In the inversion problem, the effects of colony size, number of measuring points, and the existence of measurement errors on the results are studied, and the inversion convergence rate of IABCA and ABCA is compared. The results demonstrate that the methods adopted in this paper had good effectiveness and accuracy even if colony sizes differ and measurement errors exist; and that IABCA has a more efficient convergence rate than ABCA.

Keywords: thermal conductivity; heat transfer; artificial bee colony algorithm; inverse heat conduction problem

1. Introduction

The inverse heat conduction problem is a typical inversion problem. It is based on the local temperature and other known parameters of a heat transfer system surface or interior that are used to inversely solve some unknown characteristic parameters of the system; examples are geometric boundary, thermal conductivity of materials, thermal boundary conditions (heat flux, surface heat transfer coefficient, temperature distribution), and historical temperature. Inverse heat conduction problems have been widely applied with great success in materials science, geology and environmental science, biological engineering, mechanical engineering, power engineering, construction engineering, aerospace engineering, metallurgical engineering, and other fields [1–32].

The inversion of thermal conductivity as one branch of inverse heat conduction problems has been studied by a significant number of scholars with different optimized algorithms. The optimized algorithms mentioned are mainly divided into gradient-based and non-gradient-based optimization algorithms. The former includes the conjugate gradient method (CGM), the steepest descent method (SDM), and the Levenberg–Marquardt (LM) algorithm, while the latter includes particle swarm optimization (PSO), the genetic algorithm (GA), the artificial neural network algorithm (ANNA), and the ant colony algorithm (ACA).

The CGM, SDM, and LM are currently used in more extensive inversion algorithms and have been used in inverse heat conduction problems. Wang et al. studied a two-dimensional unsteady heat conduction system according to the temperature measured on the side of the target to be measured with the method of decentralized fuzzy inference [1]. Yu solved the problem of inversion of the thermal conductivity of inhomogeneous materials with the complex variable differentiation method [19]. Sawaf et al. solved the problem of inversion of thermal conductivity and specific heat capacity of materials based on the least square function and LM [20]. A highly accurate result of thermal diffusivity of composite materials, such as agar hydrogel and glycerol, has been inverted by Ukrainczyk with the LM method [21]. Using CGM, Huang et al. inverted 1D and 2D transient thermal conductivity, which could be accurately estimated in noisy conditions [22,23]. Zhou et al. verified the stability and effectiveness of thermal conductivity inversion of a two-dimensional transient heat conduction system analyzed with CGM [24]. Mohamed et al. [32] used the recursion relations associated with Padé digital filters, efficiently solving real-time direct and inverse heat conduction problems.

However, the gradient-based optimization algorithm is a form of local search algorithm, and is not without drawbacks, namely, that it easily settles on a locally optimal solution. The inversion result depends crucially on the initial guess. More importantly, the inverse problem of heat transfer is a typical ill-posed problem. When the temperature measurement information is not complete or there is a large measurement error, the inversion results obtained based on the gradient optimization method may deteriorate. In addition, when the number of inversion points is larger, the calculation of the gradient matrix is difficult and time-consuming, which directly affects the engineering application of the gradient-based optimization algorithm [2].

A non-gradient optimization algorithm is usually a global search algorithm, which has good adaptability and overcomes the difficulty of the gradient optimization algorithm easily settling on a local minimum. It does not involve the calculation of a Jacobian partial derivative matrix in the inversion process. Many scholars have used a non-gradient optimization algorithm in inverse thermal conductivity problems. Ardakani and Khodadad studied thermal conductivity and irregular inclusive geometry in a two-dimensional medium using PSO [27]. Samarjeet et al. solved the problem of inversion of the thermal conductivity of composites using artificial neural nets [28]. The inverse thermal conductivity of materials was solved by Tang et al. and Zhao et al. with a genetic algorithm [29,30]. However, this kind of method often has a high computation cost in the search process and has a slow convergence rate in the latter stage, which limits its application in the inverse heat conduction problem. In particular, when the measurement information is incomplete or there is a large measurement error, the inversion results obtained according to such intelligent optimization methods will also have some gaps, and the inversion method lacks the necessary means of re-alignment [2].

The main differences between gradient-based and non-gradient-based optimization methods are as follows. The gradient optimization method searches for the minimum value of the objective function along its gradient or conjugate direction and can find the optimal solution quickly. However, it can settle on a local extreme value and is highly affected by the initial value. Non-gradient optimization starts from any solution, searches in the whole solution space with a certain probability according to a certain mechanism, does not use the gradient of the objective function in inversion, and does not involve the calculation problem of the Jacobian partial derivative matrix. However, the search process of non-gradient optimization is time-consuming, and the calculation cost is higher than that of the gradient optimization algorithm, especially when the parameters to be inverted have obvious spatial distribution characteristics. Thus, the disadvantages of the non-gradient optimization algorithm are significant.

The artificial bee colony algorithm (ABCA) is a kind of non-gradient-based optimization algorithm, which is more simple and flexible compared to other colony-based algorithms [33–35] and is mostly applied to the treatment of constrained optimization and integer programming, such as in issues of transportation, reaction-diffusion, and generalized assignment [36–50]. E. Hetmaniok et al. [38,39] solved the inversion of the heat transfer coefficient and studied the inverse problem of binary alloy

solidification in a casting mold using ABCA. Few uses of ABCA for inverse thermal conductivity have been reported.

The present work addresses the developments of the ABCA for estimating thermal conductivity in the transient heat conduction problem. According to the definite spatial distribution characteristics of the information measured and to be inverted, we take the difference of the measured information and the calculated information of the measuring points of inversion iteration as the input information of the classical control mechanism, and introduce it to the ABCA optimized process to derive the improved artificial bee colony algorithm (IABCA), which is adapted to two-dimensional unsteady-state thermal conductivity inversion. This overcomes the shortcomings of insufficient local mining capacity, the high computational cost in the search process, and slow convergence rate in the latter stage. The convergence of IABCA and traditional ABCA is compared with numerical examples. The influence of colony size, number of measurement points, and measurement error on the inversion result is also studied and the true thermal conductivity is obtained.

2. Forward Problem Description

The two-dimensional unsteady-state heat transfer system is shown in Figure 1, in which Ω is the finite plate $L \times L \times H$, $L \gg H$, and the initial temperature is T_0 . The boundary conditions are that Γ_1 is heated with constant heat flux q_1 , the lower surface Γ_2 is adiabatic, and M points of temperature measurement are evenly distributed between Γ_1 and Γ_2 .

Figure 1. Two-dimensional unsteady-state heat transfer model.

The governing equations are:

$$\lambda \frac{\partial^2 T}{\partial x^2} + \lambda \frac{\partial^2 T}{\partial y^2} = \rho c \frac{\partial T}{\partial \tau}, \quad x, y \in \Omega, \tau > \tau_0 \quad (1)$$

The boundary conditions are:

$$-\lambda \frac{\partial T}{\partial y} = q_1, \quad y \in \Gamma_1 \quad (2a)$$

$$-\lambda \frac{\partial T}{\partial y} = 0, \quad y \in \Gamma_2 \quad (2b)$$

The initial condition is:

$$T(y, \tau_0) = T_0, \quad x, y \in \Omega \quad (3)$$

where c , ρ and λ are, respectively, the specific heat, mass density, and thermal conductivity; τ and (x, y) denote, respectively, the time and spatial location, where T defines the distribution of temperature; and q_1 is heat flux.

The finite difference method (FDM) is used to solve the forward problem of Equations (1)–(3).

3. The Inverse Problem

3.1. Objective Function of Inverse Problem

This is a two-dimensional unsteady-state inverse thermal conductivity problem, using temperature measurement of inner points to solve the unknown of thermal conductivity. The solution of the

inverse problem can be mathematically transformed into the minimum of the following objective function, namely:

$$J(\lambda) = \sum_{\tau=1}^{N_t} \sum_{i=1}^M (T(i, \tau) - T^*(i, \tau))^2 < \varepsilon \quad (4)$$

where $T^*(i, \tau)$ is the measured temperature of the internal measured points, $T(i, \tau)$ is the calculated temperature by FDM, and ε is a smaller positive number.

3.2. Artificial Bee Colony Algorithm

In ABCA optimization, each food source represents one possible solution to the optimization problem, and the fitness values represent the solution quality. In detail, the ABCA can be listed in the form of the following steps.

1. The algorithm is initialized: SN is the number of employed bees (=the number of onlooker bees); D is the dimension of the solution; $limit$ is the number of corrective flights around λ_i ; and MCN is the maximum number of iterations.

2. Initial solutions of thermal conductivity λ_i ($i = 1, \dots, SN$) are randomly generated using Equation (5):

$$\lambda_i = lb + (ub - lb) \cdot rand(0, 1) \quad i = 1, 2, \dots, SN \quad (5)$$

where ub, lb are the upper and lower limit of λ_i , respectively.

3. The FDM is used to solve the positive forward problem of Equations (1)–(3) with λ_i , then $J(\lambda_i)$ is calculated. If the stop criterion is satisfied, the inversion of iteration will be finished, otherwise, go to step 4.

4. Each employed bee searching for a new solution according to Equation (6):

$$\lambda_{it+1,i} = \lambda_{it,i} + r_i \cdot (\lambda_{it,i} - \lambda_{it,j}) \quad (6)$$

where it is iteration number, $r_i \in [-1, 1]$ is random data, and $i \neq j$ and belongs to $1-SN$.

If $J(\lambda_{it+1,i}) < J(\lambda_{it,i})$ then $\lambda_i = \lambda_{it+1,i}$ (greedy algorithm).

In the ABCA, after $limit$ times of cycling, if one solution λ_i has not been improved, it will be given up and the employed bee will become a scout bee, and generate a new solution according to:

$$\lambda_i = \lambda_{min} + (\lambda_{max} - \lambda_{min}) \cdot rand(0, 1) \quad (7)$$

λ_{min} is the current minimum solution, λ_{max} is the maximum solution.

5. The fitness value and probabilities are derived as follows. The fitness value of the food source corresponds to its richness (quality) and its calculation form is:

$$fit\lambda_i = \begin{cases} \frac{1}{1+J(\lambda_i)} & J(\lambda_i) \geq 0 \\ 1 + abs(J(\lambda_i)) & J(\lambda_i) < 0 \end{cases} \quad (8)$$

The probabilities P_i are defined as:

$$P_i = prob(\lambda_i) = \frac{fit\lambda_i}{\sum_{n=1}^{SN} fit\lambda_n} \quad (9)$$

6. Each onlooker chooses one of the sources λ_i , $i = 1, \dots, SN$, with probability P_i . One source can be chosen by a group of onlooker bees. Then, each onlooker bee explores the chosen λ_i and modifies it according to the procedure described in step 4.

7. The choice of $\Delta T(\lambda_{it+1,i}^{best})$ of $\lambda_{it+1,i}$ is determined by the bee colony. If $\Delta T(\lambda_{it+1,i}^{best})$ is better than the choice from the previous iteration, it is accepted as λ^{best} .

8. If $J(\lambda^{best}) < \varepsilon$ or if $it+1=MCN$ then iterations are complete, else $it=it+1$, and steps 4–8 are repeated until the stop criterion is satisfied.

3.3. Improved Artificial Bee Colony Algorithm

The measurement and inverted information in the heat conduction system have certain spatial distribution characteristics [1]. Therefore, the inverse heat conduction problem can also be regarded as a kind of feedback control problem; that is, a closed-loop system, where the observation results (measured temperature) of the system are the setting value, the parameters to be solved are the controlled object, and the calculated temperature of the positive problem is the sensor measurement information. The control model is shown in Figure 2. ΔT is obtained according to Equation (10); if $\Delta T > 0$, in order to eliminate or reduce the deviation, it is necessary to increase λ ; if $\Delta T < 0$, to eliminate or reduce the deviation, it is necessary to decrease λ .

$$\Delta T = T^* - T \quad (10)$$

Figure 2. Feedback control algorithm model.

An improved ABCA in which feedback control and the ABCA are effectively combined is proposed to overcome the shortcomings of insufficient local mining capacity and high computational cost in the search process. Thus, we add step 7.1 to the application of the ABCA outlined above; that is, the optimal solution is updated according to Equation (10), the objective function is calculated, and the cyclic optimal solution is then selected.

$$\lambda'_{it+1} = \lambda_{it+1}^{best} + k_p \cdot \Delta T(\lambda_{it+1}^{best}) \quad (11)$$

where k_p is the feedback factor, and $\Delta T(\lambda_{it+1}^{best})$ is the difference of the calculated and measured temperature. If $J(\lambda'_{it+1}) < J(\lambda_{it+1}^{best})$ then $\lambda_{it+1}^{best} = \lambda'_{it+1}$.

The IABCA not only retains the advantages of ABCA, but also improves the local mining ability of the algorithm.

3.4. Inverse Problem Solving Process

The process for solving the inverse problem is as follows:

- (1) Randomly generate the initial value of thermal conductivity using Equation (5);
- (2) Solve the forward problem, calculate the objective function, and calculate the probability of each employed bee;
- (3) According to Equation (6), employed bees search for the solution. After the forward problem is solved, calculate the value of the objective function and probability, and determine if it needs to be updated by the greedy algorithm;
- (4) The onlooker bees search for the solution according to Equation (6). After the forward problem is solved, calculate the value of the objective function.
- (5) According to Equation (7), the scout bee searches for the solution. After the forward problem is solved, update the value of the objective function and probability. Select the optimal and update the optimal solution using Equation (11), then determine if it needs to be updated by the greedy algorithm.
- (6) Stop iteration once the stop criterion or the maximum number of iterations is satisfied, otherwise, return to step 2.

4. Numerical Experiment and Analysis

The model size was $350\text{ mm} \times 350\text{ mm} \times 30\text{ mm}$, the thermal conductivity was $\lambda = 12\text{ W}/(\text{m}\cdot\text{K})$, $c = 500\text{ KJ}/(\text{Kg}\cdot\text{K})$, $\rho = 7930\text{ Kg}/(\text{m}^3)$, $q_1 = 9500\text{ J}/(\text{m}^2\cdot\text{s})$, the lower surface was adiabatic, $T_0 = 25\text{ }^\circ\text{C}$. From top to bottom, the temperature measuring points $M = 6$ were set evenly.

4.1. Contrast of Convergence Rate of IABCA and ABCA

ABCA and IABCA's colony size was set as 6, the maximum iteration cycles were 25, and the other parameters were the same. When the inversion satisfied the stop criterion of iteration, the iteration times of the two methods were compared and analyzed.

The ABCA convergence results are shown in Figure 3a–c, where the horizontal axis shows the iteration cycle count and the vertical axis shows the best solution of each iteration.

Figure 3. Artificial bee colony algorithm (ABCA) inversion convergence curve. (a) First inversion; (b) Second inversion; (c) Third inversion.

The ABCA needed at least 18 iterations until the stop criterion was reached.

The IABCA inversion results are shown in Figure 4a–c, where the horizontal axis shows the iteration cycle count and the vertical axis shows the best solution of each iteration.

Figure 4. Improved artificial bee colony algorithm (IABCA) inversion convergence curve. (a) First inversion; (b) Second inversion; (c) Third inversion.

The experiments show that the colony size was the same, and the IABCA satisfied the stop criterion after no more than five iterations, while the ABCA needed no less than 18. The iteration count of the IABCA was obviously lower than that of the ABCA, so the IABCA can effectively enhance the inversion convergence rate.

4.2. Impact of Colony Size

The number of temperature measurement points was taken as $M = 6$ and the colony sizes were $N = 4, 6, 8$. The maximum number of iterations was seven and each case was inverted 10 times. The results are shown in Figures 5–7.

(a) Inversion results when the colony size $N = 4$ (Figure 5).

Figure 5. The inverse convergence curve of IABC colony, $N = 4$.

(b) Inversion results when the colony size $N = 6$ (Figure 6).

Figure 6. The inverse convergence curve of IABC colony, $N = 6$.

(c) Inversion results when the colony size $N = 8$ (Figure 7).

Figure 7. The inverse convergence curve of IABC colony, $N = 8$.

Experiments show that when the number of colony size was 4, 6, or 8, the exact solution can be reached. The larger the colony size, the faster the iteration converged, with more calculation cost. The colony size of 6 will be used later in this study.

4.3. Impact of the Number of Measuring Points

The number of temperature measurement points was taken as $M = 2, 4, 6$. The maximum number of iterations was 25, and the temperature at the measurement point and the corresponding thermal conductivity were obtained by inversion. The inversion result is shown in Table 1.

Table 1. The value of λ and the relative error.

Test Points	Calculated Value of λ Relative Error (%)		Calculated Value of λ Relative Error (%)	
	Measurement Error = 0		Measurement Error = $\pm 0.1\%$	
2	11.9610	−0.33	11.9490	−0.43
4	12.0049	0.04	11.9700	−0.25
6	12.0000	0.00	11.9871	−0.11

At the measurement point $M = 2, 4, 6$, λ was calculated by the least square method, and the relative error is shown in Table 1. Results are as follows:

- When the measurement error was 0: When $M = 2$, the relative error of λ was -0.33% ; when $M = 4$, the relative error of λ was 0.04% ; and when $M = 6$, the relative error of λ was 0.00% .
- When the measurement error was $\pm 0.1\%$: When $M = 2$, the relative error of λ was -0.43% ; when $M = 4$, the relative error of λ was -0.25% ; and when $M = 6$, the relative error of λ was -0.11% .

Thus, it is shown that by increasing the number of measuring points, the relative error decreases and the inversion accuracy improves.

4.4. Impact of Measurement Error

To assess the impact of measurement error, the number of measuring points was set as $M = 6$ and fine groups of measurement error of $0\%, \pm 0.3\%, \pm 0.5\%, \pm 1\%$, and $\pm 2\%$ were used. The values of λ were calculated by the least square method; the relative error is shown in Table 2. Results are as follows:

- when measurement error was 0% , the average relative error of λ was 0.00% ;
- when measurement error was $\pm 0.3\%$, the average relative error of λ was 0.50% ;
- when measurement error was $\pm 0.5\%$, the average relative error of λ was 0.84% ;
- when measurement error was $\pm 1.0\%$, the average relative error of λ was -1.40% ;
- when measurement error was $\pm 2.0\%$, the average relative error of λ was -1.94% .

It can be seen that there were some measurement errors under the premise of a large amount of measurement data, and the inversion results were still satisfactory. However, the larger the measurement error, the more distorted the inversion results.

Table 2. The value of λ and the relative error.

Measurement Error (%)	Calculated Value of λ	Relative Error (%)
0	12.0000	0.00
± 0.3	12.0600	0.50
± 0.5	12.1005	0.84
± 1.0	11.8316	−1.40
± 2.0	11.7669	−1.94

4.5. Impact of the Relative Placement

The number of measuring points was set as $M = 2$, and measurement error was set equal to $\pm 0.1\%$, and the relative placement between two consecutive points was set as 4.3 mm, 8.6 mm, and 12.9 mm.

The relative error is shown in Table 3. When the relative placement was 4.3 mm, the average relative error of λ was -0.43% ; when relative placement was 8.6 mm, the average relative error of λ was -0.55% ; and when the relative placement was 12.9 mm, the average relative error of λ was -0.31% . The results show that the inversion results are insensitive to the location interval.

Table 3. Impact of the relative placement.

Relative Placement (mm)	Calculated Value of λ	Relative Error (%)
4.3	11.9490	-0.43
8.6	11.9337	-0.55
12.9	11.9635	-0.31

5. Conclusions

In this paper, the finite difference method is applied to the solution of the forward problem, and the IABCA is applied to the optimal objective function, in the inversion of thermal conductivity in a two-dimensional unsteady-state heat transfer system. The effects of colony size, number of measurement points, and measurement errors on the results are analyzed and discussed. The inverse results show that when there is measurement error, inversion results can also be satisfactory. Through the calculation and analysis of examples, the accuracy and stability of the thermal conductivity inversion algorithm are verified.

The conclusions are as follows:

1. In the application of the ABCA to the inversion of thermal conductivity of a two-dimensional unsteady-state heat transfer model in this paper, an improved ABCA method is proposed to overcome the ABCA shortcomings of less local exploitation capacity and slow late convergence rate. This study verifies that the efficiency of IABCA's inversion convergence is higher than that of ABCA with numerical calculated examples.
2. This paper also studies the impacts of colony size, number of measurement points, and measurement errors on the inversion result with numerical examples. Instance calculation and analysis prove that the method applied in this paper has good effectiveness and accuracy even if measurement error exists. The comparison indicates the influence of error on the inversion solution can be minimized effectively using this method. Effectiveness and stability of the IABCA for unsteady thermal conductivity inversion are verified.

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References

1. Wang, S.; Zhang, L.; Sun, X.; Jia, H. Inversion of Thermal Conductivity in Two-Dimensional Unsteady-State Heat Transfer System Based on Boundary Element Method and Decentralized Fuzzy Inference. *Complexity* **2018**, *2018*, 8783946. [[CrossRef](#)]
2. Wang, S.; Jia, H.; Sun, X. Two-Dimensional Steady Boundary Shape Inversion of CGM-SPSO Algorithm on Temperature information. *Adv. Mater. Sci. Eng.* **2017**, *2017*, 2461498. [[CrossRef](#)]

3. Fan, C.; Sun, F.; Yang, L. A simple method for inverse estimation of surface temperature distribution on a flat plate. *Inverse Probl. Sci. Eng.* **2009**, *17*, 885–899. [[CrossRef](#)]
4. Cui, M.; Gao, X.; Liu, Y. Inversion of Temperature-dependent Thermal Conductivity Based on Transient Inverse Heat Conduction Problem. *Proc. CSEE* **2012**, *32*, 82–87.
5. Wang, S.; Zhang, L.; Sun, X. Solution to Two-Dimensional Steady Inverse Heat Transfer Problems with Interior Heat Source Based on the Conjugate Gradient Method. *Math. Probl. Eng.* **2017**, *2017*, 1–9. [[CrossRef](#)]
6. Baranov, V.L.; Zasyad'ko, A.A.; Frolov, G.A. Integro-differential method of solving the inverse coefficient heat conduction problem. *J. Eng. Phys. Thermophys.* **2010**, *83*, 60–71. [[CrossRef](#)]
7. Sriram, S.B.; Sravan, S.; Gnanasekaran, N. Numerical Estimation of Heat Flux and Convective Heat Transfer Coefficient in a One Dimensional Rectangular Plate by Levenberg-Marquardt Method. *Indian J. Sci. Technol.* **2016**, *9*. [[CrossRef](#)]
8. Ruan, Z.H.; Yuan, Y.; Chen, Q.X.; Zhang, C.X.; Shuai, Y.; Tan, H.P. A new multi-function global particle swarm optimization. *Appl. Soft Comput.* **2016**, *49*, 279–291. [[CrossRef](#)]
9. Kosaka, M.; Monde, M. Simultaneous measurement of thermal diffusivity and thermal conductivity by means of inverse solution for one-dimensional heat conduction (anisotropic thermal properties of CFRP for FCEV). *Int. J. Thermophys.* **2015**, *36*, 2590–2598. [[CrossRef](#)]
10. Czél, B.; Gróf, G. Genetic algorithm-based method for determination of temperature-dependent thermophysical properties. *Int. J. Thermophys.* **2009**, *30*, 1975–1991. [[CrossRef](#)]
11. Mohebbi, F.; Sellier, M.; Rabczuk, T. Estimation of linearly temperature-dependent thermal conductivity using an inverse analysis. *Int. J. Therm. Sci.* **2017**, *117*, 68–76. [[CrossRef](#)]
12. Bozzoli, F.; Mocerino, A.; Rainieri, S.; Vocale, P. Inverse heat transfer modeling applied to the estimation of the apparent thermal conductivity of an intumescent fire retardant paint. *Exp. Therm. Fluid Sci.* **2017**, *90*, 143–152. [[CrossRef](#)]
13. Peng, C. Simultaneous determination of thickness, thermal conductivity and porosity in textile material design. *J. Inverse Ill-Posed Probl.* **2016**, *24*, 59–66.
14. Huang, S.J. An Ant Colony Optimization Algorithm Suitable for Searching Heat Source Location in IHCP. *J. Eng. Thermophys.* **2013**, *34*, 694–697.
15. Mahmud, K.; Mohsen, D.A. Inclusion Identification by Inverse Application of Boundary Element Method, Genetic Algorithm and Conjugate Gradient Method. *Am. J. Appl. Sci.* **2008**, *5*, 1158–1166.
16. D'Haeyer, S.; Johansson, B.T.; Slodička, M. Reconstruction of a Spacewise-Dependent Heat Source in a Time-Dependent Heat Diffusion Process. *IMA J. Appl. Math.* **2014**, *79*, 33–53. [[CrossRef](#)]
17. Chen, H.T.; Wu, X.Y. Investigation of Heat Transfer Coefficient in Two-Dimensional Transient Inverse Heat Conduction Problems Using the Hybrid Inverse Scheme. *Int. J. Numer. Methods Eng.* **2008**, *73*, 107–122. [[CrossRef](#)]
18. Zhu, L.; Wang, G.; Chen, H. Estimating Steady Multi-variables Inverse Heat Conduction Problem by Using Conjugate Gradient Method. *Proc. CSEE* **2011**, *31*, 58–61.
19. Yu, X. Inverse Analysis of Thermal Conductivities in Non-Homogeneous Heat Conductions Using Boundary Element Method. Master's Thesis, Dalian University of Technology, Dalian, China, 2013.
20. Sawaf, B.; Ozisik, M.N. An inverse analysis to estimate linearly temperature dependent thermal conductivity components and heat capacity of an orthotropic medium. *Int. J. Heat Mass Transf.* **1995**, *38*, 3005–3010. [[CrossRef](#)]
21. Ukrainczyk, N. Thermal diffusivity estimation using numerical inverse solution for 1D heat conduction. *Int. J. Heat Mass Transf.* **2009**, *52*, 5675–5681. [[CrossRef](#)]
22. Huang, C.H.; Yan, J.Y. An inverse problem in simultaneously measuring temperature-dependent thermal conductivity and heat capacity. *Int. J. Heat Mass Transf.* **1995**, *38*, 3433–3441. [[CrossRef](#)]
23. Huang, C.H.; Chin, S.C. A two-dimensional inverse problem in imaging the thermal conductivity of a non-homogeneous medium. *Int. J. Heat Mass Transf.* **2000**, *43*, 4061–4071. [[CrossRef](#)]
24. Zhou, H.; Xu, X. Identification of Temperature-Dependent Thermal Conductivity for 2-D transient Heat Conduction Problem. *Appl. Math. Mech.* **2014**, *12*, 1341–1351.
25. Cui, M.; Zhu, Q.H.; Gao, X.W. A modified conjugate gradient method for transient nonlinear inverse heat conduction problems: A case study for identifying temperature-dependent thermal conductivities. *Heat Transf.-Trans. ASME* **2014**, *136*, 091301. [[CrossRef](#)]

26. Cao, K.; Lesnic, D.; Colaco, M.J. Determination of thermal conductivity of inhomogeneous orthotropic materials from temperature measurements. *Inverse Probl. Sci. Eng.* **2019**, *27*, 1372–1398. [[CrossRef](#)]
27. Ardakani, M.D.; Khodadad, M. Identification of thermal conductivity and the shape of an inclusion using the boundary elements method and the particle swarm optimization algorithm. *Inverse Probl. Sci. Eng.* **2009**, *17*, 855–870. [[CrossRef](#)]
28. Chanda, S.; Yenni, G.R.; Ambirajan, A.; Balaji, C.; Venkateshan, S.P. An Inverse Analysis for Estimation of Thermal Conductivity of Orthotropic Composite Medium Using Artificial Neural Network. In Proceedings of the 21st National and 10th ISHMT-ASME Heat and Mass Transfer Conference, IIT Madras, India, 27–30 December 2011.
29. Tang, Z.H.; Qian, G.H.; Qian, W.Q. Estimation of temperature-dependent function of thermal conductivity for a material. *Chin. J. Comput. Mech.* **2011**, *28*, 377–382.
30. Zhao, J.; Fu, Z.; Jia, X.; Cai, Y. Inverse determination of thermal conductivity in lumber based on genetic algorithms. *Holzforschung* **2016**, *70*, 235–241. [[CrossRef](#)]
31. Lei, C. Research on Inversion Algorithms of the Thermo Physical Properties. Master's Thesis, Harbin Institute of Technology, Harbin, China, 2015.
32. Lotfi, M.; Mezrigui, L.; Heyd, R. Study of heat conduction through a self-heated composite cylinder by Laplace transfer functions. *Appl. Math. Model.* **2016**, *40*, 10360–10376. [[CrossRef](#)]
33. Karaboga, D. *An Idea Based on Honey Bee Swarm for Numerical Optimization, Technical Report-TR06*; Erciyes University, Engineering Faculty, Computer Engineering Department: Kayseri, Turkey, 2005.
34. Karaboga, D. A powerful and efficient algorithm for numerical function optimization: Artificial bee colony (abc) algorithm. *Glob. Optim.* **2007**, *39*, 459–471. [[CrossRef](#)]
35. Karaboga, D.; Akay, B. A comparative study of artificial bee colony algorithm. *Appl. Math. Comput.* **2009**, *214*, 108–132.
36. Zhang, W. Research on Artificial Bee Colony Based Hybrid Optimization Algorithms and Application. Ph.D. Thesis, Zhejiang University, Hangzhou, China, 2013.
37. Omars, S.; Senthilnath, J.; Khandelwal, R. Artificial bee colony (abc) for multi-objective design optimization of composite structures. *Appl. Soft Comput.* **2011**, *11*, 489–499.
38. Hetmaniok, E. Inverse problem for the solidification of binary alloy in the casting mould solved by using the bee optimization algorithm. *Heat Mass Transf.* **2016**, *52*, 1369–1379. [[CrossRef](#)]
39. Hetmaniok, E.; Słota, D.; Zielonka, A. Identification of the Heat Transfer Coefficient in the Inverse Stefan Problem by Using the ABC Algorithm. *Arch. Foundry Eng.* **2012**, *12*, 27–32.
40. Wong, L.-P.; Low, M.Y.H.; Chong, C.S. A Bee Colony Optimization Algorithm to Job Shop Scheduling. Ph.D. Thesis, Nanyang Technological University, Singapore, 2007.
41. Pham, D.T.; Castellani, M. The bees algorithm: Modeling foraging behaviour to solve continuous optimization problems. *Proc. Inst. Mech. Eng. Part C* **2009**, *223*, 2919–2938. [[CrossRef](#)]
42. Hemamalini, S.; Simon, S.P. Artificial bee colony algorithm for economic load dispatch problem with non-smooth cost functions. *Electr. Power Compon. Syst.* **2010**, *38*, 786–803. [[CrossRef](#)]
43. Singh, A. An Artificial Bee Colony Algorithm for the Leaf-constrained Minimum Spanning Tree Problem. *Appl. Soft Comput.* **2009**, *9*, 625–631. [[CrossRef](#)]
44. Bahamish, H.; Abdullah, R.; Salam, R. Protein Tertiary Structure Prediction Using Artificial Bee Colony Algorithm. In Proceedings of the Third Asia International Conference on Modelling & Simulation, Bali, Indonesia, 25–29 May 2009; pp. 258–263.
45. Karaboga, N. A New Design Method Based on Artificial Bee Colony Algorithm for Digital IIR filters. *J. Frankl. Inst.* **2009**, *346*, 328–348. [[CrossRef](#)]
46. Rao, R.S.; Narasimham, S.; Ramalingaraju, M. Optimization of Distribution Network Configuration for Loss Reduction Using Artificial Bee Colony Algorithm. *Int. J. Electr. Power Energy Syst. Eng. (IJEPESE)* **2008**, *1*, 709–715.
47. Bi, X.; Wang, Y. An improved artificial bee colony algorithm. In Proceedings of the 2011 3rd International Conference on Computer Research and Development (ICCRD 2011), Shanghai, China, 11–13 March 2011; pp. 174–179.
48. Yu, X.; Zhan, D.; Nie, L. An artificial bee colony algorithm for resource-constrained project scheduling problem with spatial resource. *J. Comput. Inf. Syst.* **2012**, *8*, 6723–6732.

49. Zhang, C.; Zhen, J.; Zhou, Y. Two modified artificial bee colony algorithms inspired by grenade explosion method. *Neuro Comput.* **2015**, *151*, 1198–1207. [[CrossRef](#)]
50. Zhang, W.; Wang, N.; Yang, S. Hybrid artificial bee colony algorithm for parameter estimation of proton exchange membrane fuel cell. *Int. J. Hydrogen Energy* **2013**, *38*, 5796–5806. [[CrossRef](#)]

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