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Incomplete Similarity Error Analysis Method of Steel Frame Structure Based on Correlation

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Featured Application: The similar error calculation method proposed in this paper can be applied to the error analysis of various scale model tests.

Abstract: In most scale model tests of steel frame structures, researchers have usually paid more attention to the outcome of one single experiment rather than to the similitude error between the scale model and the prototype. As a result, only the experimental data of the scale model are obtained rather than the relationship between the scale model and the prototype, which greatly limits scale model tests in the investigation and is also a significant waste of resources. In addition, the effect of the geometric incomplete similarity in steel joints and its contribution to similitude error of the entire frame has rarely been evaluated in the past. This paper is based on similarity theory, investigating the effect of three incomplete geometric similarity factors on the similitude error between a scale model and a prototype, which are the geometric similarity of steel components, the stiffness of steel joints and the stiffness of the column base. Global sensitivity is found by calculation. Furthermore, this paper presents the correlation of the three factors mentioned based on structural sensitivity analysis by use of BP (Back Propagation) algorithm. A method to investigate the similitude error between the scale model and prototype is proposed that can correct similitude errors in the experimental results.

Keywords: structural model test; incomplete geometric similarity; error analysis; sensitivity analysis; correlation of parameters

1. Introduction

Similarity theory is one of the basic theories in the structural scaled down model test and plays a vital role in engineering-oriented design [1]. However, this theory is facing the challenges of complicated theoretical derivation, identification of the number of similarity factors and a lack of research into showing the numerical error between the scale model and the prototype [2]. In addition, the effect of the incomplete geometric similarity in steel joints and its contribution to similitude error of the entire steel frame has rarely been evaluated until now. In test data analysis, prior research has not thoroughly investigated the similitude problem, most likely leading to errors in the experimental results [3]. This paper investigates the effect of three incomplete geometric similarity factors on the similitude error between the scale model and the prototype, which are the geometric similarity of steel components, the stiffness of steel joints and the stiffness of the column base and the contribution of each to the similitude error within the whole structure. The remaining similarity factors demonstrate complete similarity.

First of all, much is known about the influence of the geometric similarity of steel components. Pan Jianrong et al. studied the scaling laws in a model test of steel beam-to-column connections and, demonstrated that geometric similarity is the main cause of simulation errors [4]; however, this study was only concerned with the geometric similarity of steel joints. In the entire frame, prior studies have failed to evaluate the result when many parts of the scaled down frame are dissimilar to those of the prototype. Wang Zhan et al. studied the semi-rigid characteristic of steel beam-to-column connections, which play a vital role in the whole frame [5]. Thus, the similitude of the semi-rigid characteristic of steel beam-to-column connections is also an important aspect of overall similarity. However, little is known about the mechanisms. In the same way, the similarity of the stiffness of the column base, also has an important influence on the overall similarity of the structure [6].

This paper is based on similarity theory—by adding the semi-rigid characteristics of both the beam-to-column connections and the column base to the derivation of similar conditions, the similarity conditions of semi-rigid steel frame are developed. The three incomplete geometric similarity factors are thoroughly examined to obtain the sensitivity of each factor. Sensitivity analysis based on BP (Back Propagation) algorithm shows that each factor has different influence on overall similarity. The purpose of this paper is to study the effects of three incomplete similar factors on the overall incomplete similarity error of steel frame structures and its mechanism. This paper presents a method to evaluate the similitude error of a scaled down model, which can correct errors in experimental results.

2. Methods

2.1. Derivation of Completely Similar Conditions for Semi-rigid Steel Frames

In the study of scaled down model tests, little research has been conducted to investigate the influence of the semi-rigid characteristic of the joint [7]. In this section the Buckingham π theorem and dimensional analysis are used to develop the similarity relations in the semi-rigid frame [8]. A few dimensionless numbers are obtained through the derivation, which are the necessary conditions of complete similarity in the scaled down model frame [9].

The relevant system parameters are listed in the first place to suit the requirement of the derivation of similarity criteria [10–13]. The number of parameters should be exact to ensure the accuracy of the similarity relation. Dynamic parameters are also included for future study of dynamic similitude law [14–16]. The relevant parameters are listed as follows:

L: geometric dimension.	γ: poisson ratio.
g: acceleration of gravity.	ρ: density.
F: static load.	I: moment of inertia.
a: prompting acceleration.	σ: stress.
K: joint stiffness of semi-rigid frame.	x: displacement.
K_r: joint stiffness of semi-rigid column base.	f: frequency.
E: elastic modulus.	a_r: response acceleration.

The primary dimensions selected were (F, L, T). Representing the force (F), length (L) and time (T). The number of listed parameters is 14 in total; the number of restrictions is equal to the number of primary dimensions. Table 1 shows all the relevant system parameters used in the case study.

Table 1. Basic dimension of related parameters.

Basic Dimension	***L***	***g***	***F***	***K***	***E***	γ	***I***
F	0	0	1	1	1	0	1
L	1	1	0	1	−2	0	2
T	0	−2	0	0	0	0	1
Basic Dimension	***Kr***	ρ	***f***	***a***	***ar***	σ	***X***
F	1	1	0	0	0	1	0
L	1	−4	0	1	1	−2	1
T	0	2	−1	−2	−2	0	0

According to the second law of similitude theory (Buckingham π theorem), if there is a physically meaningful equation involving a certain number n of physical variables, then the original equation can be rewritten in terms of a set of p = n − k dimensionless parameters constructed from the original variables.

$$f(\pi_1, \pi_2, \cdots, \pi_{n-k}) = 0 \tag{1}$$

Geometric dimension L, acceleration of gravity g, static load F, these three parameters are chosen as the basic parameters with independent dimension. So the 14 relevant system parameters can be transformed into certain function of 11 dimensionless parameters. The function can be derived as followed:

According to the principle of dimensional homogeneity, the dimensions of both sides of the equations should be the same. Assuming the power exponent as x_1, x_2, x_3 and x_4, the equation representing K (joint stiffness of semi-rigid frame) in the form of basic parameters are expressed as followed:

$$L^{x_1} \cdot \left(LT^{-2}\right)^{x_2} \cdot F^{x_3} = (F \cdot L)^{x_4} \tag{2}$$

Classify the dimensions in the form of basic parameters,

$$L^{x_1+x_2} \cdot T^{-2x_2} \cdot F^{x_3} = F^{x_4} \cdot L^{x_4} \tag{3}$$

Make the power exponent of both sides equal,

$$\begin{cases} x_3 = x_4 \\ x_1 + x_2 = x_4 \\ -2 \cdot x_2 = 0 \end{cases} \tag{4}$$

Solve the equations,

$$x_2 = 0 \tag{5}$$

$$x_1 = x_3 = x_4 \tag{6}$$

Assuming x_1 = 1, thus,

$$L \cdot F = K \tag{7}$$

Finally, the first dimensionless parameter is obtained:

$$\pi_1 = \frac{K}{L \cdot F} \tag{8}$$

In the same way, the left 10 dimensionless parameters can be derived and shown as followed:

$$\begin{gathered}\pi_2 = \frac{L^2 \cdot E}{F}, \pi_3 = \gamma, \pi_4 = \frac{g \cdot I^2}{L^5 \cdot F^2}, \pi_5 = \frac{K_r}{L \cdot F}, \\ \pi_6 = \frac{L^3 \cdot g \cdot \rho}{F}, \pi_7 = \frac{g}{L \cdot f^2}, \pi_8 = \frac{g}{a}, \pi_9 = \frac{g}{a_r}, \\ \pi_{10} = \frac{F}{\sigma \cdot L^2}, \pi_{11} = \frac{X}{L}\end{gathered} \tag{9}$$

If the scaled down model is completely similar to the prototype, then all the parameters should satisfy the equations below,

$$\begin{gathered}K_m = S_K \cdot K_p, E_m = S_E \cdot E_p, \\ \gamma_m = S_\gamma \cdot \gamma_p, I_m = S_\gamma \cdot I_p, K_{r_m} = S_{K_r} \cdot K_{r_p}, \\ \rho_m = S_p \cdot \rho_p, f_m = S_f \cdot f_p, a_m = S_a \cdot a_p, \\ a_{\gamma_m} = S_{a_r} \cdot a_{\gamma_p}, \sigma_m = S_\sigma \cdot \sigma_p, X_m = S_X \cdot X_p\end{gathered} \tag{10}$$

In the equations, S_K, S_E, S_γ, S_γ, S_{Kr}, S_ρ, S_f, S_a, S_{ar}, S_σ, S_X, are the similarity ratios between the model and the prototype. K_m, E_m, γ_m, γ_m, K_{rm}, ρ_m, f_m, a_m, a_{rm}, σ_m, X_m, are the parameters of the model, K_p, E_p, γ_p, γ_p, K_{rp}, ρ_p, f_p, a_p, a_{rp}, σ_p, X_p, are the parameters of the prototype.

According to the first law of similitude theory, the dimensionless parameters are equal in all the models of one prototype,11 similarity ratio equations can be derived, thus,

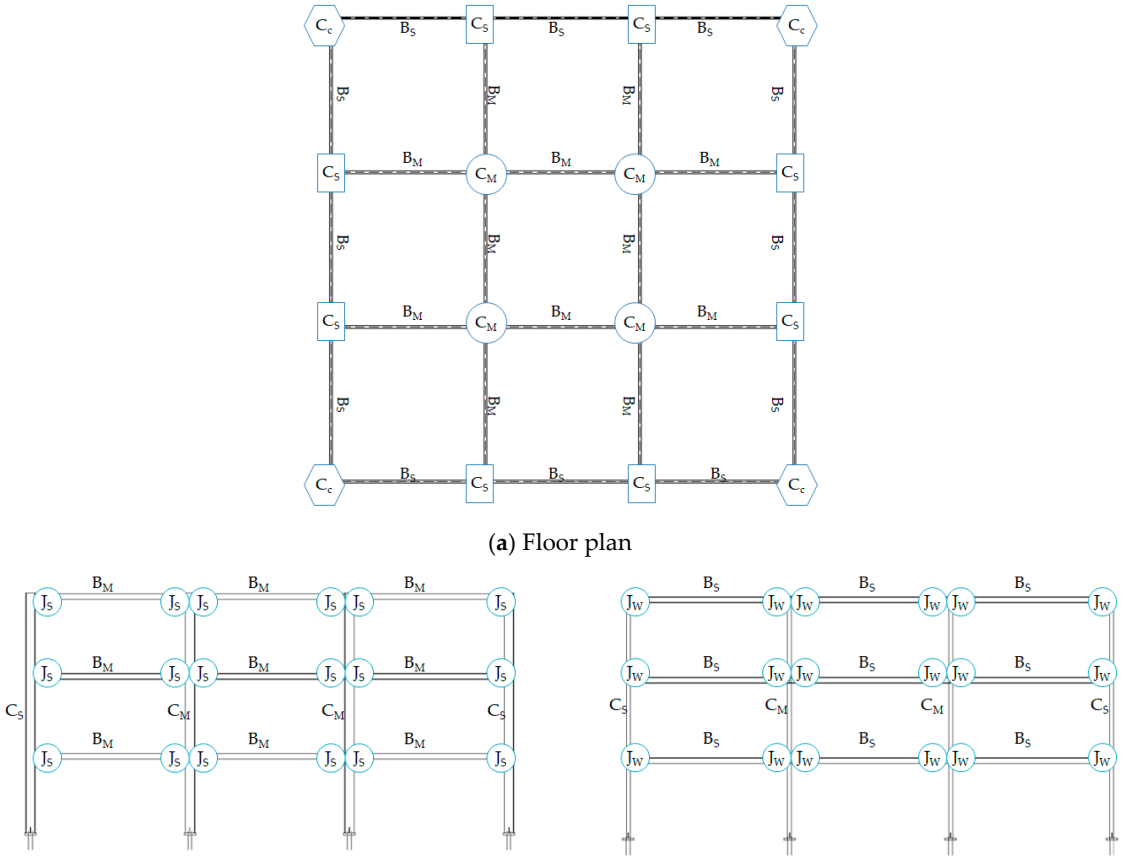
$$\begin{gathered}\frac{S_K}{S_L \cdot S_F} = 1, \frac{{S_L}^2 \cdot S_E}{S_F} = 1, S_\gamma = 1, \\ \frac{S_g \cdot {S_I}^2}{{S_L}^5 \cdot {S_F}^2} = 1, \frac{S_{K_r}}{S_L \cdot S_F} = 1, \frac{{S_L}^3 \cdot S_g \cdot S_\rho}{S_F} = 1, \\ \frac{S_F}{S_\sigma \cdot {S_L}^2} = 1, \frac{S_X}{S_L} = 1\end{gathered} \tag{11}$$

These 11 similarity ratio equations will be the basic relations in a completely similar steel frame structure, under consideration of the semi-rigid characteristic of the joint.

2.2. Semi-rigid Steel Frame Similarity Ratio

Based on the necessary conditions of complete similarity in the scaled down model frame derived in the first part, in this part a steel frame of three stories and three span is proposed as the research object, which has a span length of 8400 mm and a story height of 4200 mm. The frame was simulated by setting the fixed boundary condition at the end of each column base. The horizontal loads applied at the top of the columns were 100 kN. All the beam-column joints are End-Plate connection joints. The structure's grouping of frame members and joints is shown in Figure 1. Sensitivity analyses are premised on the basis of the grouping.

Assuming the geometry scaled ratio of the frame is 1:4, according to basic similarity relations derived in the first part, all the other parameters' scaled ratios can be derived and listed in Table 2. The frame's component specification of the scaled down model and the prototype is listed in Table 3. This paper mainly discusses the effect of the joint stiffness to the frame but not the joint itself so a theoretical stiffness of the joint is proposed, which is shown in Table 4, on the basis of long-term theoretical analysis and experiments in our research group.



(**a**) Floor plan

(**b**) 1-1 Sectional view.

(**c**) 2-2 Sectional view

Figure 1. Semi-rigid frame prototype schematic. (B_M: Middle beam; B_S: Side beam; J_S: Strong axis; J_W: Weak axis).

Table 2. Similarity ratio of each factor.

Factor	Similarity Ratio	Factor	Similarity Ratio
S_L	1:4	S_F	2:1
S_F	1:16	S_X	1:4
S_K	1:64	S_{ar}	1:1
S_E	1:1	S_σ	1:1
S_g	1:1	S_{Kr}	1:64
S_γ	1:1	S_a	1:1
S_ρ	4:1		

Table 3. Component modulus of completely similar.

Prototype	Component Modulus (mm)	Model	Component Modulus of Completely Similar (mm)
B_S	HN300 × 150 × 6.5 × 9	B_S	75 × 37.5 × 1.625 × 2.25
B_M	HN400 × 150 × 8 × 13	B_M	100 × 37.5 × 2 × 3.25
C_C	HN500 × 200 × 10 × 16	C_C	125 × 50 × 2.5 × 4
C_S	HN400 × 200 × 5 × 13	C_S	100 × 50 × 1.25 × 3.25
C_M	HN450 × 200 × 9 × 14	C_M	112.5 × 50 × 2.25 × 3.5

Note: BS: Side beam; BM: Middle beam; C_S: Side column; C_C: Corner column; C_M: Middle column.

Table 4. Joint stiffness of completely similar.

Prototype	Joint Stiffness ($N \cdot m \cdot rad^{-1}$)	Model	Joint Stiffness of Completely Similar ($N \cdot m \cdot rad^{-1}$)
J_S	$4 \times e^7$	J_S	$6.25 \times e^5$
J_W	$2.4 \times e^7$	J_W	$3.75 \times e^5$
J_{SC}	$2 \times e^8$	J_{SC}	$3.125 \times e^6$
J_{WC}	$1 \times e^8$	J_{WC}	$1.5625 \times e^6$

Note: J_S: Strong axis; J_W: Weak axis; J_{SC}: Strong axis of column base; J_{WC}: Weak axis of column base.

3. Results

The experimental results of the model and the prototype must satisfy the similitude ratio under the condition of complete similarity. Conversely, incomplete similitude error is the difference between them under the condition of incomplete similarity. This part takes the three-span semi-rigid steel frame as an example, aiming to obtain the similitude sensitivity (the contribution to the similarity degree of the model) of every factor. This part adopts the finite element software of ANSYS 14.5 (2012, ANSYS, Inc., Canonsburg, PA, USA), the beams and columns of the frame are simulated by the beam188 element, the semi-rigid characteristic of beam-to-column connections is simulated by the matrix27 element. The steel members and connections are sorted in Figure 2.

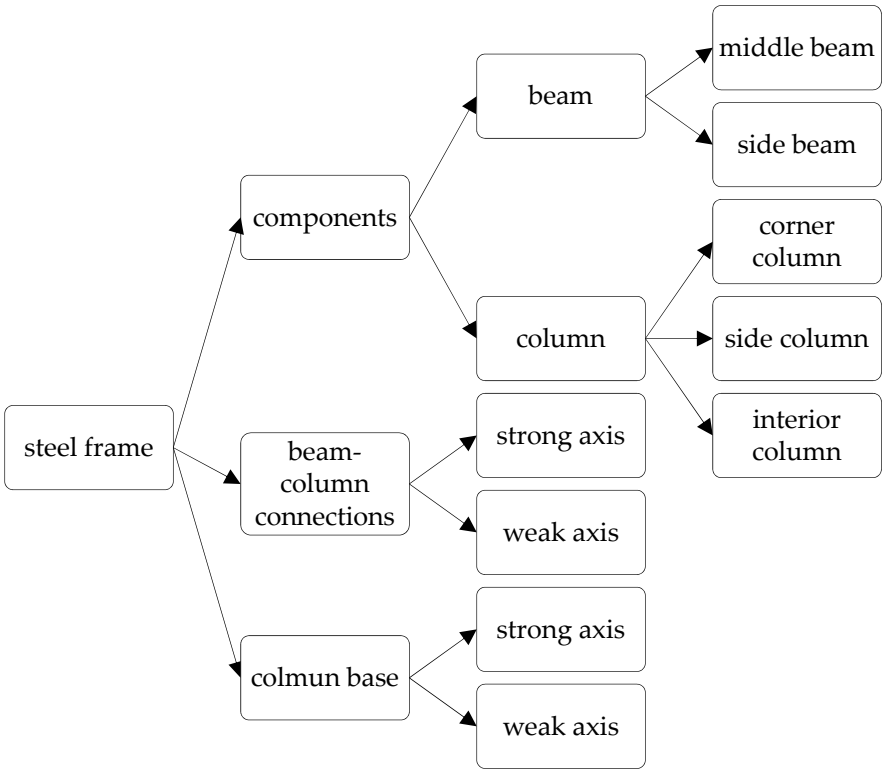


Figure 2. Schematic diagram of component and joint grouping.

3.1. Influence of Modular Modulus of Beam-column Components

The geometric incomplete similarity of members in steel frame has important significance in engineering, for instance, if the plate thickness of the prototype is 15 mm, the similarity ratio is 1:4, on this premise the model of complete similarity should be 3.75 mm. In fact, a plate of this thickness does not exist, so the complete similarity on the premise of 1:4 similarity ratio is impossible. Therefore, it is necessary to investigate the similitude error caused by the geometric incomplete similarity of steel components (beams and columns). This part is based on the common assigning groups in engineering,

dividing the frame into four groups, which are the edge beams, the middle beams, the edge columns, the middle columns. Global sensitivity is calculated on the basis of the grouping.

This paper adopts the method of sensitivity analysis based on the loop nesting of MATLAB (2016, The MathWorks, Inc., Natick, MA, USA) and ANSYS. A model database is established in MATLAB, including model information such as beam height, beam width, plate thickness and joint stiffness, each set of data in MATLAB represents a model in ANSYS. Then the database is introduced into ANSYS to carry out the analysis calculation. After the calculation is completed, the results are returned to MATLAB for statistical analysis and the the resulting cyclic calculation method can realize automated sensitivity analysis, achieving a sensitivity analysis of extremely large amounts of data. A group of 2950 models was calculated; the variation range of all parameters is listed in Table 5.

Table 5. Parameter variation range of sensitivity calculation.

Components	Modulus (mm)	Changing Part	Variation Range (mm)	Components	Modulus (mm)	Changing Part	Variation Range (mm)
B_S	75 × 37.5 × 1.625 × 2.25	H	10~210	C_S	100 × 50 × 1.25 × 3.25	H	10~210
		W	10~210			W	10~210
		T_W	1~15			T_W	1~15
		T_F	1~15			T_F	1~15
B_M	100 × 37.5 × 2 × 3.25	H	10~210	C_M	112.5 × 50 × 2.25 × 3.5	H	10~210
		W	10~210			W	10~210
		T_W	1~15			T_W	1~15
		T_F	1~15			T_F	1~15
C_C	125 × 50 × 2.5 × 4	H	10~210	J_S	$6.25 \times e^5$	J_S	$1 \times e^5$~$2 \times e^7$
		W	10~210	J_W	$3.75 \times e^5$	J_W	$1 \times e^5$~$2 \times e^7$
		T_W	1~15	J_{SC}	$3.125 \times e^6$	J_{SC}	$1 \times e^5$~$2 \times e^7$
		T_F	1~15	J_{WC}	$1.5625 \times e^6$	J_{WC}	$1 \times e^5$~$2 \times e^7$

Note: The unit of stiffness is $N{\cdot}m{\cdot}rad^{-1}$. H: Height; W: Width; T_W: Thickness of web; T_F: Thickness of flange.

The variation ranges of the steel components' height and width should keep consistent for comparison purposes, as should the other parameters (the thickness of flange and web, the joint stiffness). Limiting conditions were added into MATLAB to reject the unqualified component groups, as a result the calculation will accord with engineering practice.

Figures 3–14 show the results of sensitivity calculation; every group of components' height and width were shown in the same figure, as did the thickness of flange and web. Figures 3, 5, 7, 9 and 11 show the sensitivity calculation results of height and width. Figures 4, 6, 8, 10 and 12 show the sensitivity calculation results of flange and web's thickness. The results show that keeping the other parameters (force, *E*, etc.) under the condition of complete similarity, the target variable has a violent change when the side beams' height varies within 50 mm, which indicates the height of side beams have a bigger influence in the situation. When the corner columns' height varies within 100 mm, the target variable is 220~175 mm, when the side columns' height varies within 50 mm, the target variable is 310~180 mm. The middle beams' height and width, the thickness of all the plates, have little influence on the value of the target variable.

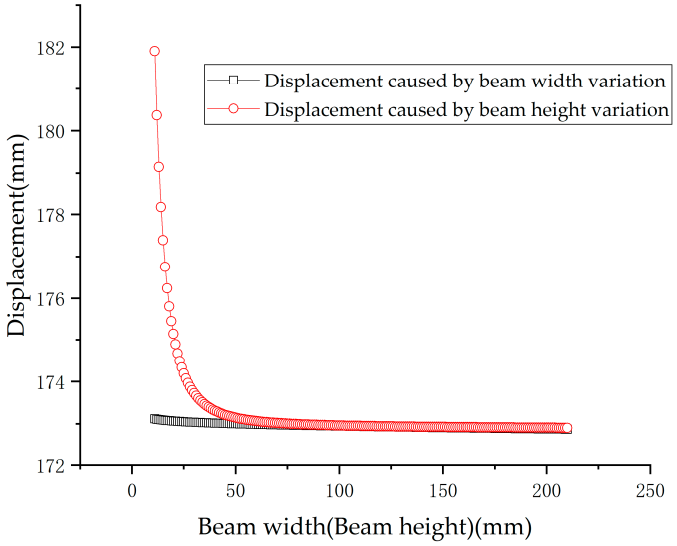


Figure 3. Sensitivity curve of B_S height and width.

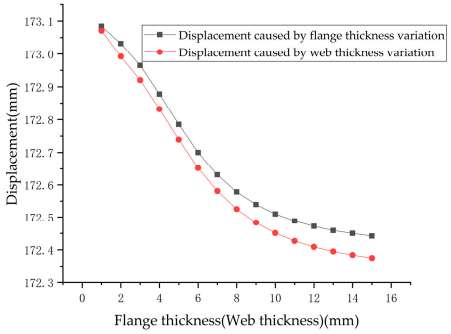


Figure 4. Sensitivity curve of B_S plate thickness.

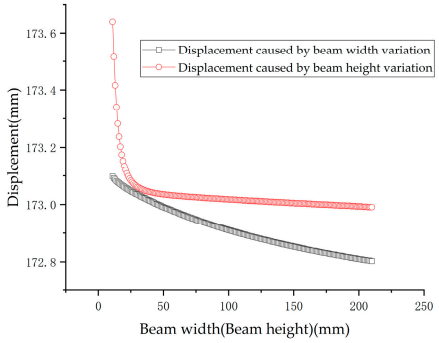


Figure 5. Sensitivity curve of B_M height and width.

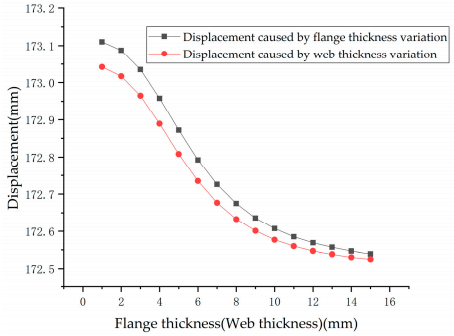


Figure 6. Sensitivity curve of B_M plate thickness.

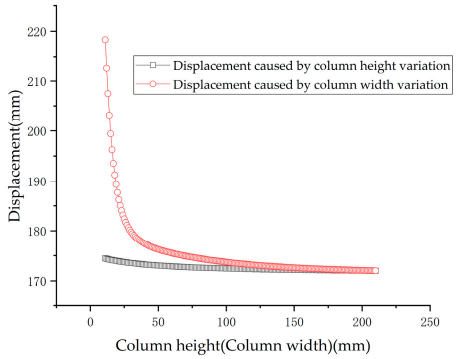


Figure 7. Sensitivity curve of C_C height and width.

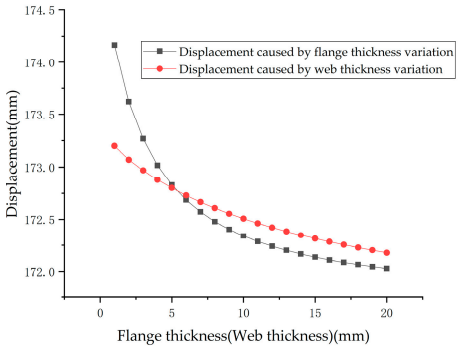


Figure 8. Sensitivity curve of C_C plate thickness.

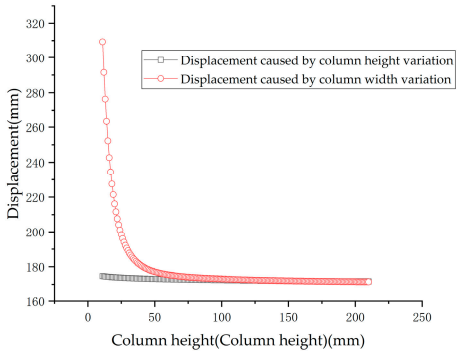


Figure 9. Sensitivity curve of C_S height and width.

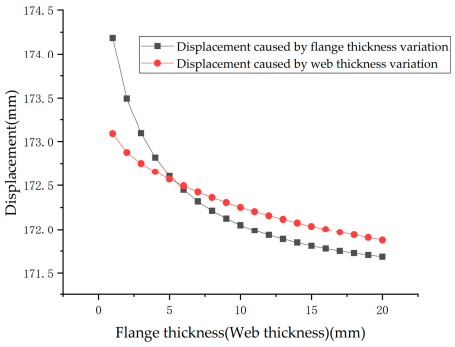


Figure 10. Sensitivity curve of C_S plate thickness.

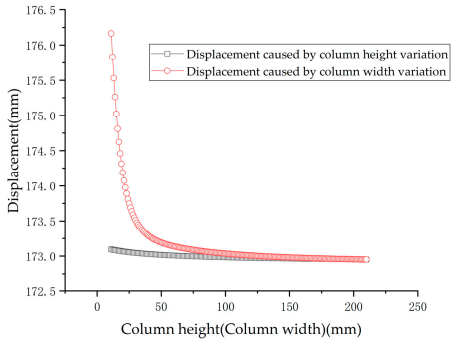


Figure 11. Sensitivity curve of C_M height and width.

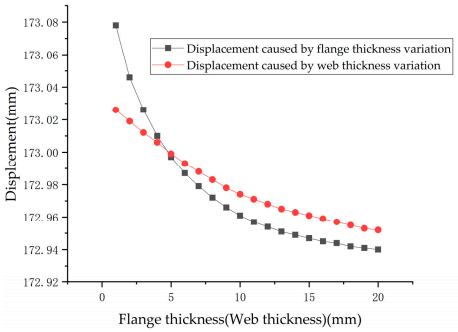


Figure 12. Sensitivity curve of C_M plate thickness.

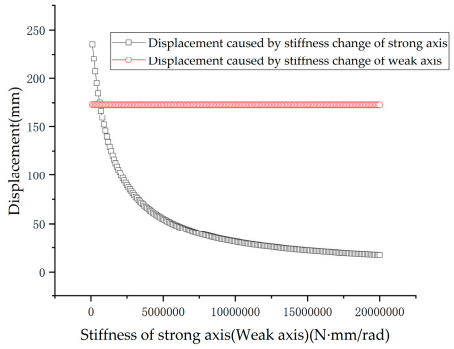


Figure 13. Sensitivity curve of J_S/J_W stiffness.

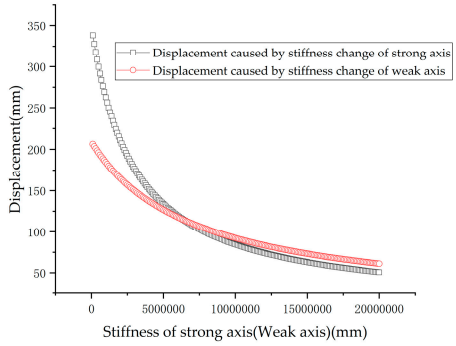


Figure 14. Sensitivity curve of J_{SC}/J_{WC} stiffness.

3.2. Influence of Stiffness of Semi-rigid Joints of Beams and Columns

Keeping the other parameters (force, *E*, etc.) under the condition of complete similarity, the joint stiffness of the frame is varied as listed in Figure 5; in this way the influence of joint stiffness can be fully examined. Figure 13 shows the sensitivity analysis result for the joint stiffness of both strong axis and weak axis. In Figure 13, the joint stiffness of the strong axis has a great influence on the target variable. The target variable is 230~25 mm, the two extreme cases can be considered as fully rigid or pinned. The joint stiffness of the weak axis has little influence on the target variable; in Figure 13 it is mainly a straight line.

3.3. Influence of Joint Stiffness on Column Boundary Conditions

As the only boundary condition, the semi-rigid characteristic of the column base also has great influence on the whole steel structure. Figure 14 shows the sensitivity analysis result for the column base joint stiffness of both strong axis and weak axis. Different to the beam-to-column joint, both the strong axis and the weak axis of the column base have a great influence on the target variable, especially the strong axis; the target variable is 350~50 mm, the weak axis is 210~70 mm.

3.4. Summary

Define similitude error as:

$$\text{Similitude error} = \frac{\text{Calculation results} - \text{Completely similar results}}{\text{Completely similar results}} \times 100\%$$

According to the result of the sensitivity calculation, the upper and lower limits are shown in Table 6.

Table 6. Similar error and Sensitivity coefficient.

Components	Changing Part	Similar Error %	Sensitivity Coefficient	Components	Changing Part	Similar Error %	Sensitivity Coefficient
B_S	H	−0.056~5.146	0.028	C_S	H	−0.925~78.765	0.055
	W	−0.076~0.067	0.019		W	−0.746~0.948	0.017
	T_W	−0.361~0.041	0.023		T_W	−0.650~0.052	0.023
	T_F	−0.323~0.049	0.021		T_F	−0.761~0.684	0.017
B_M	H	−0.005~0.370	0.023	C_M	H	−0.028~1.829	0.021
	W	−0.114~0.056	0.016		W	−0.028~0.055	0.021
	T_W	−0.275~0.024	0.025		T_W	−0.028~0.015	0.018
	T_F	−0.266~0.064	0.019		T_F	−0.035~0.045	0.011
C_C	H	−0.721~26.190	0.019		J_S	−89.964~35.982	0.346
	W	−0.592~0.813	0.022		J_W	−0.010~0.042	0.019
	T_W	−0.475~0.116	0.013		J_{SC}	−70.730~95.447	0.116
	T_F	−0.562~0.673	0.022		J_{WC}	−64.752~19.188	0.089

Note: H: Height; W: Width; T_W: Thickness of web; T_F: Thickness of flange.

Figure 15 shows the standardized sensitivity coefficients of every factor, as it is shown, particular steel components' geometric characteristics are not the critical factor in the similitude error analysis calculation because of the diversity of the steel components. The components' height has a bigger influence than width and thickness. The most important factors are beam-to-column joint stiffness and column base stiffness. It shows that the contribution to uncomplete similitude error of semi-rigid characteristic cannot be ignored, joint stiffness plays an important role in the uncomplete similitude error calculation of the model.

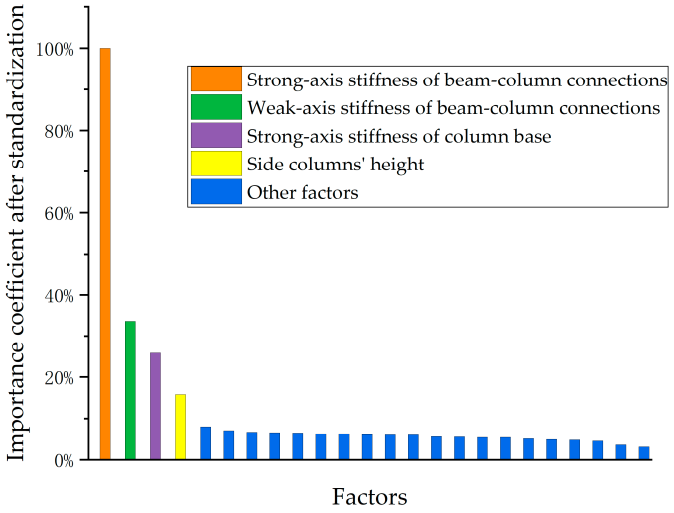


Figure 15. Importance coefficient after standardization.

4. Discussion

4.1. Relevant Effects of Various Factors

As was shown in the previous section, as every similitude factor changes, the similitude error differs. However, in practice during the experiment, similitude error is usually caused by many factors together. When similitude errors of various factors exist simultaneously, the overall error of the model is not simply superimposed on the error of each factor but is the result of their interaction. The change of each factor and the final similarity error can be regarded as a complex mapping relationship, which is difficult to express with mathematical function relations [17]. In this section, the artificial intelligence neural network algorithm is used to simulate the map.

The system composed of similar factors and target variables can be obviously regarded as a complex nonlinear system. The system may have very complicated system state equations but it is difficult to express with mathematical equations. In this case, a BP (Back Propagation) neural network is used to simulate the nonlinear system. The nonlinear function fitting algorithm based on the BP neural network includes BP neural network construction, training and prediction. The details are as follows: Firstly, the structure of the neural network is determined according to the characteristics of the nonlinear function, then the neural network is trained by using a large amount of data and finally, the trained network is used to predict the output.

BP neural network simulation requires a large number of raw data models; this section uses the MATLAB+ANSYS cycle calculation method in Section 3, randomly establishes 50,000 sets of finite element models according to the Monte Carlo method and imposes constraints on MATLAB to delete the model data that does not meet the engineering requirements and obtains the 46,582 groups of the qualified model. The 46,582 models are cyclically calculated to obtain the original training data of the neural network.

According to the experience of the research group, the neural network adopts a double hidden layer and the number of hidden layer nodes is nine. The neural network was trained by using the 46,582 sets of data obtained.

Using the trained neural network for data prediction, taking 180 sets of finite element calculation data, the initial conditions were input to obtain the neural network prediction results as shown in Figure 16, where the abscissa is 180 sets of input data conditions and the ordinate is the trained neural network prediction errors.

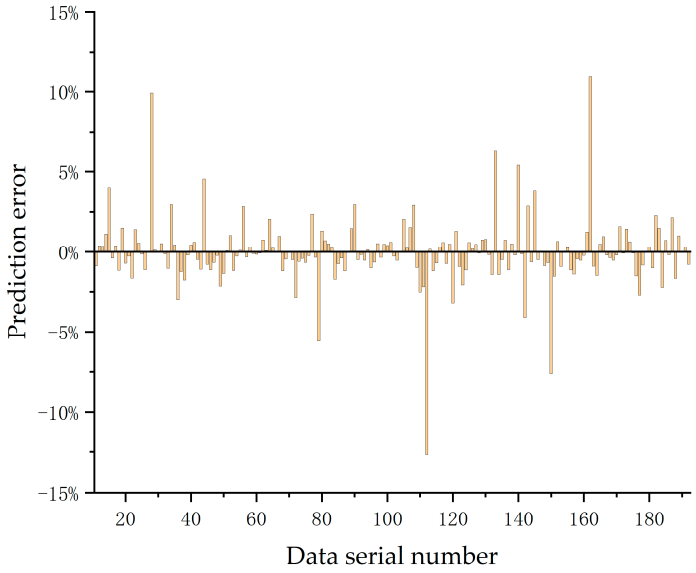


Figure 16. Prediction error.

As can be seen from Figure 16, the maximum error of the few points (about 4–5 groups) in the prediction data is about 10%, the vast majority of data errors are less than 5%, the average error is –0.03% and the standard deviation is 0.0213. The median is −0.13%, which can be considered to meet the accuracy requirements of the project.

4.2. Prediction of Similarity Error of Semi-rigid Steel Frame

This section uses the trained neural network in Section 4 to predict the similarity error of the steel frame and test the prediction accuracy of the neural network. Take the same three-layer three-span steel frame structure, the basic parameters of the structure are shown in Table 7.

Table 7. Similar error of each component.

Components	Parameters	Value	Similar Error	Components	Parameters	Value	Similar Error
B_S	H	100	25%	C_S	H	100	0%
	W	50	25%		W	50	0%
	T_W	5	67.5%		T_W	5	75%
	T_F	7	67.86%		T_F	7	53.57%
B_M	H	100	0%	C_M	H	125	10%
	W	50	25%		W	60	16.67%
	T_W	5	60%		T_W	6	62.5%
	T_F	7	53.57%		T_F	8	56.25%
C_C	H	125	0%		J_S	$4 \times e^5$	-56.25%
	W	60	16.67%		J_W	$1 \times e^5$	-275%
	T_W	6	58.33%		J_{SC}	$3 \times e^6$	-4.17%
	T_F	8	50%		J_{WC}	$1 \times e^6$	-56.25%

Note: H: Height. W: Width. T_W: Thickness of web. T_F: Thickness of flange.

Two types of commonly used H-beams with beam heights of 100 and 125 were chosen. From former analysis, it can be seen that the plate thickness contributes less to the similarity error, so the plate thickness difference is allowed to be larger and more in line with engineering practice. According to the three factors that have a great influence on the sensitivity analysis—which are the columns' height, the strong-axis stiffness of beam-column connections and the stiffness of column base—the similarity error of beams' height is 25% and the similarity error of strong-axis stiffness of beam-column connections is 56.25%. The similarity error of weak-axis stiffness of column base is 56.25%. After modeling and

calculation using ANSYS, the exact displacement of the structure is 212.6 mm. The calculation shows that the error of the structure and the completely similar result is 22.89%. Then, through the neural network obtained from the previous section, it can be seen whether the error can be accurately predicted.

The structural parameters in this example are input into the trained neural network and the predicted target displacement is 205.75 mm. The calculation shows that the error between the predicted result and the completely similar result is 18.93% and the difference between the predicted similarity error and the calculated complete similar error is calculated, which is 3.96% within the accuracy allowed by the project. Therefore, we have obtained a set of calculation methods that can predict the structurally incomplete similarity error and the accuracy is within the controllable range.

5. Conclusions

In this paper, the joint stiffness of semi-rigid steel frame is taken as the entry point and the joint stiffness is introduced in the similar condition derivation of steel frame to make the similar conditions more perfect. The overall geometric similarity of steel members, the semi-rigidity similarity of beam-column connections and of the column base are calculated respectively. The global correlation sensitivity calculation of the similarity leads to the following conclusions:

(1) In the completely similar conditions of semi-rigid steel frame, the joint stiffness and the geometric similarity ratio are in accordance with the square relationship.

(2) After the components and nodes are grouped according to the common classification of engineering, the results of sensitivity calculation show that the stiffness characteristics of the strong-axis beam-column connections have an important influence (more than 90%) on the similarity error of the semi-rigid structure. The influence of the stiffness of the weak-axis beam-column connections and the plate thickness can be neglected (less than 5%).

(3) Random sampling calculation based on Monte Carlo method to obtain artificial intelligence neural network mapping is a feasible incomplete similarity error prediction method. When the training data volume is large enough, the accuracy requirement can be guaranteed (less than 5% error). This method is also applicable to the similarity error prediction of other structures.

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