Article

Analysis of Basic Failure Scenarios of a Truss Tower in a Probabilistic Approach

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Featured Application: Failures and collapses of steel truss towers may be caused by various factors, which often occur simultaneously. The evaluation of truss tower reliability when single elements fail constitutes an important research area. It may happen that the failure of a single element of the structure leads to a redistribution of internal forces that results in the exceedance of the capacity of subsequent bars. This brings about the occurrence of a mechanism, i.e., the structure failure. In this paper, the impact of the failure of individual elements on the overall reliability of the tower is analyzed. The literature lacks research using a system approach to complex truss structures. This article describes the original procedure for assessing the reliability of truss towers after the failure of a single element. This method can be used to assess the safety of truss towers with damaged elements.

Abstract: This paper concerns the system reliability analysis of steel truss towers. Due to failures of towers, the assessment of their reliability seems to be a very important problem. In the analysis, two cases are examined: when the buckling coefficient is a deterministic value and when it is a random variable. The impact of failures of single elements on the structure reliability was investigated. Calculations of the standard deviation of the capacity and reliability indexes were made using author-developed programs in the Mathematica environment.

Keywords: system analysis; steel truss tower; failure scenarios; reliability index

1. Introduction

Truss towers are multi-element, self-supporting structures, rigidly fixed to the base. Due to many factors such as large height, relatively low material consumption, and simple erection process, steel truss towers are the most obvious choice for a tower structure. With respect to their use, a number of types can be identified [1], namely, antenna towers, architectural towers, transmission towers, chimney towers, light towers, viewing towers, or water towers.

The experience shows steel truss tower failures may be attributed to different reasons. Such issues were explored in many studies [2–5]. The truss performance in a potentially dangerous condition may result in the structural collapse of the whole tower. Examples of failures of truss towers were discussed in study [6]. Failures and collapses of steel truss towers may be caused by various factors, which often occur simultaneously. The most common causes are related to errors in design, such as inappropriate determination of external loads or wrong loading diagram of the structure. Flaws in erection include, among others, faulty workmanship of connections, the weakening of individual bars, attachment of additional gear (unaccounted for in the design). A common cause of failures of towers with main legs made from closed sections is water freezing in them, which leads to the bursting of sections. In such cases, the indirect cause lies in careless workmanship of welded connections, which allows water penetration into the section. Another cause of failure may be the action of abnormal wind or snow loads,

or both loads combined. Because of a large range of load variation, the design assumptions sometimes do not account for abnormal natural phenomena like vehement storms, whirlwinds, or hurricanes. Another important failure-contributing factor is the corrosion of steel sections and connections, which leads to their reduced capacity. The action of corrosion depends, to a large extent, on the provision of adequate anticorrosion protection and the severity of the corrosion environment. Other factors that can lead to the failure of truss towers include non-uniform settlement of the ground underneath main leg foundations, mining-induced damages, failures of connections, and vehicles crashing into the structure.

Taking into account the threats mentioned above, it is clear that computer simulations of truss towers deformations and failures represent a relevant research focus [7–10]. Full-scale investigations provide equally important findings [8,9]. An important issue is also proper condition evaluation of existing truss structures [5,11], which aims at the identification of a potentially dangerous state. In case a failure of the structure element that might result in the collapse is found, it is necessary to design proper strengthening of the structure to ensure its safe performance [2,4]. The subject of failure scenarios includes work on progressive failure, in which local failure of a structural component can cause failure of the whole structure [12,13].

The reliability analysis of truss structures has been investigated in many studies, e.g., in [14–17], where various methods of estimating the reliability index have been proposed. The evaluation of truss tower reliability when individual single elements fail also constitutes an important research area. In study [18], the author emphasises the need for investigations into the failure of statically indeterminate structures. It may happen that a failure of a single element of the structure leads to a redistribution of internal forces that results in the exceedance of the capacity of subsequent bars, bringing about the occurrence of a mechanism, i.e., the structure failure. In this paper, the impact of the failure of individual elements on the overall reliability of the tower is analyzed.

2. Materials and Methods

There are many methods to assess the reliability of a structure. In the paper, the system approach is used for reliability analysis. In this case, structural characteristics of the truss structure are taken into account, i.e., interaction of the structural elements in the load-carrying system [19–24]. In the reliability theory, there are three models: serial, parallel, and mixed (parallel-serial and serial-parallel) systems. To identify the reliability model, the kinematically admissible failure mechanisms (KAFMs) should be specified. Each KAFM contains the minimal critical sets of elements (MCSE), which have a particular number of tshe causative elements (l).

The serial system consists of k minimal critical sets of elements (k MCSE), each of which has one causative element ($l = 1$). The serial system (Figure 1a) is operable if all elements are operable. Reliability R of the serial system is calculated from the formula

$$R = \prod_{i=1}^k R_i \quad (1)$$

where R_i —reliability of a single element and k —number of the system elements.

In the case of a parallel system, there is one minimal critical set of elements (1 MCSE) and it consists of k causative elements ($l = k$). The parallel system (Figure 1b) is operable if at least one element is operable. Reliability R of the parallel system is calculated from the formula

$$R = 1 - \prod_{i=1}^k (1 - R_i) \quad (2)$$

For more complex truss structures forming mixed systems, the reliability is evaluated by assuming actual connections and interactions between the elements. To identify the reliability model, it is

necessary to analyze how the elements are coupled, i.e., what relations hold between them. Mixed systems consist of parallel-serial and serial-parallel arrangements.

The parallel-serial system (Figure 1c) consists of m MCSEs, each of which includes k causative elements ($l = k$). Reliability R of the parallel-serial system is calculated from the formula

$$R = 1 - \prod_{j=1}^m \left[1 - \prod_{i=1}^k (1 - R_{ji}) \right] \quad (3)$$

For the last serial-parallel system (Figure 1d), the number of MCSEs is $m \cdot k$ and each MCSE consists of k causative elements ($l = k$). Reliability R of the serial-parallel system is calculated from the formula

$$R = 1 - \prod_{j=1}^m \left[1 - \prod_{i=1}^k (1 - R_{ji}) \right] \quad (4)$$

To carry out the spectral analysis of the linear stiffness matrix (identify the reliability model) and to calculate the reliability index, the program developed by the authors in the Mathematica environment was applied.

Figure 1. Types of reliability models: (a) serial system, (b) parallel system, (c) parallel-serial system, and (d) serial-parallel system.

To determine the reliability of the system, it is necessary to calculate the reliability of a single element. The procedure that leads to the determination of the reliability of a single element R_i was presented, for example, in [21,22]. The first step is to determine the effect of action E_i , and capacity N_i . The next step is to determine the safety margin Z_i . It is a difference between the effect of action E_i and capacity N_i . To compute reliability of the single element, it is necessary to determine expected value $\mu_{Zi} = \mu_{Ni} - \mu_{Ei}$ and standard deviation $\sigma_{Zi} = \sqrt{\sigma_{Ei}^2 + \sigma_{Ni}^2}$ of the safety margin. A subsequent step is computing the reliability index for a single element $\beta_i = \mu_{Zi} / \sigma_{Zi}$ and probability of the element failure $P_{fi} = \Phi(-\beta_i)$, where Φ is the Laplace function. Finally, the reliability of a single element is estimated from the formula $R_i = 1 - P_{fi}$. To compute the reliability of the system R , it is necessary to identify the reliability model. After that, the probability of failure of whole structure $P_f = 1 - R$ and reliability index of whole structure $\beta = -\Phi^{-1}(P_f)$ are computed.

In this paper, the algorithm described above was used to assess the reliability of a steel truss tower. The influence of randomness is taken into account in the capacity and the effect of actions. As in publication [22], both values have a normal distribution. The coefficient of variation for the effect of actions is equal to 6%. In this paper, two cases of load capacity description are considered. In the

first case, coefficient of variation for the capacity is equal to 10%. In the second case, it was taken into account that the capacity is a function of several random variables (area of cross section, yield strength, modulus of elasticity, minimum moment of inertia, and length of element). Each random variable describing the capacity of an element may have a different value of the coefficient of variation. This causes difficulties in calculating the standard deviation of the capacity, which can be solved using the following approximation [25,26]:

$$\sigma \approx \sqrt{\sum_{i=1}^m \left(\frac{\partial f}{\partial X_i} \right)^2 \sigma_{X_i}^2} \quad (5)$$

where f —is the function of uncorrelated variables, X_i —single random variable, and m —number of variables.

The values of variation coefficients from Table 1 were used in the calculation of the standard deviation of the buckling coefficient and capacity. Values of coefficients of variation are defined on the basis of the following papers [27,28]. Calculations of these values for each types of elements (compression and tension) were carried out in the Mathematica program.

Table 1. Coefficients of variation for random variables describing the capacity of elements.

Random Variable	Coefficient of Variation
area of cross section (A)	6%
yield strength (f_y)	8%
modulus of elasticity (E)	5%
minimum moment of inertia (J_{min})	6%
length of element (L)	6%

3. Results

The article presents an analysis of the reliability truss tower with the number of repeating sections $lm = 6$ and topology shown in Figure 2. Towers are designed from S235 steel with the yield strength $f_y = 235$ MPa. The structure is designed according to the standards [29–31]. The reliability index is calculated according to the assumption of parallel-serial connections. This assumption is more appropriate than the assumption of serial connections. It is due to taking into account the real performance of the structure. The study investigated the effect of removing each individual element on the reliability structure. The first step, however, involved the assessment of the reliability of the structure in its original state and number of elements being le . This analysis includes the following:

1. Input data (static scheme, load on the structure for the dominant combination, and capacity N_i of individual elements)—shown in Figure 2;
2. Spectral analysis of linear stiffness matrix K_L (determination the number of KAFMs and MCSEs)—shown in Figure 3;
3. Determination the effect of actions E_i of individual elements— μ_{Ei} (Table 2);
4. Checking the effort of individual elements— μ_{Ei}/μ_{Ni} (Table 2);
5. Calculation of the reliability index for elements R_i and assessment of the structure reliability.

In the case when there is no failure mechanism, the reliability assessment consists in comparing the reliability index of the structure with the required value of the reliability index. The required value of the reliability index was chosen for the RC2 reliability class (50 years reference period). In accordance with the standard [29], “Three reliability classes (RC1, RC2, and RC3) may be associated with the three consequence classes CC1, CC2, and CC3”. The tower’s construction was classified as a structure with the consequence class CC2. This means that “Medium consequence for loss of human life and economic, social, or environmental consequences are considerable”. The required value for the reliability class RC2 is $\beta_{EC} = 3.8$. The reliability index for parallel-serial system is 4.471. The structure is sufficiently safe, because the reliability index β is greater than 3.8.

Figure 2. Static scheme of the studied truss tower: (a) topology and dimensions, (b) vertical load, and (c) horizontal load.

Figure 3. All of 300 kinematically admissible failure mechanisms for the studied truss tower.

Table 2. Initial reliability analysis.

Element	Profile	Expected Value of the Effect of Actions of the ith Element μ_{Ei} [kN]	Standard Deviation of the Effect of Action σ_{Ei} (6%) [kN]	Expected Value of the Capacity of the ith Element μ_{Ni} [kN]	Standard Deviation of the Capacity σ_{Ni} [kN]	Effort of the ith Element μ_{Ei}/μ_{Ni} [%]	Probability of the Failure of the ith Element P_{fi} [-]	Reliability of the ith Element R_i [-]
1	L60 × 6	25.979	1.55874	162.385	16.23850	16.0	0.0000000000	1.0000000000
2	L40 × 4	0.678	0.04068	72.380	7.23800	0.9	0.0000000000	1.0000000000
3	L60 × 6	35.021	2.10126	64.304	8.68104	54.5	0.0005217330	0.9994782670
4	L45 × 6	6.393	0.38358	119.615	11.96150	5.3	0.0000000000	1.0000000000
5	L45 × 6	9.163	0.54978	15.430	2.51509	59.4	0.0074607216	0.9925392784
6	L60 × 6	16.699	1.00194	162.385	16.23850	10.3	0.0000000000	1.0000000000
7	L40 × 4	0.319	0.01914	72.380	7.23800	0.4	0.0000000000	1.0000000000
8	L60 × 6	24.301	1.45806	64.304	8.68104	37.8	0.0000027541	0.9999972459
9	L45 × 6	5.376	0.32256	119.615	11.96150	4.5	0.0000000000	1.0000000000
10	L45 × 6	7.352	0.44112	15.430	2.51509	47.6	0.0007793554	0.9992206446
11	L60 × 6	9.620	0.57720	162.385	16.23850	5.9	0.0000000000	1.0000000000
12	L40 × 4	0.127	0.00762	72.380	7.23800	0.2	0.0000000000	1.0000000000
13	L60 × 6	15.380	0.92280	64.304	8.68104	23.9	0.000000105	0.999999895
14	L45 × 6	4.073	0.24438	119.615	11.96150	3.4	0.0000000000	1.0000000000
15	L45 × 6	5.827	0.34962	15.430	2.51509	37.8	0.0000778524	0.9999221476
16	L60 × 6	4.507	0.27042	162.385	16.23850	2.8	0.0000000000	1.0000000000
17	L40 × 4	0.084	0.00504	14.400	2.26080	0.6	0.0000000001	0.9999999999
18	L60 × 6	8.493	0.50958	64.304	8.68104	13.2	0.0000000001	0.9999999999
19	L45 × 6	2.819	0.16914	119.615	11.96150	2.4	0.0000000000	1.0000000000
20	L45 × 6	4.252	0.25512	15.430	2.51509	27.6	0.0000048967	0.9999951033
21	L60 × 6	1.410	0.08460	162.385	16.23850	0.9	0.0000000000	1.0000000000
22	L40 × 4	0.347	0.02082	14.400	2.26080	2.4	0.0000000003	0.9999999997
23	L60 × 6	3.590	0.21540	64.304	8.68104	5.6	0.0000000000	1.0000000000
24	L45 × 6	1.542	0.09252	119.615	11.96150	1.3	0.0000000000	1.0000000000
25	L45 × 6	2.701	0.16206	15.430	2.51509	17.5	0.0000002202	0.9999997798
26	L60 × 6	0.243	0.01458	162.385	21.92198	0.1	0.0000000000	1.0000000000
27	L40 × 4	0.257	0.01542	14.400	2.26080	1.8	0.0000000002	0.9999999998
28	L60 × 6	0.757	0.04542	64.304	8.68104	1.2	0.0000000000	1.0000000000
29	L45 × 6	0.363	0.02178	119.615	11.96150	0.3	0.0000000000	1.0000000000
30	L45 × 6	1.051	0.06306	15.430	2.51509	6.8	0.0000000055	0.9999999945

The next step is to analyze the effect of removing each individual element of the truss tower. The algorithm for the reliability analysis of truss structure with number of elements $le - 1$ is shown together with an example of the effect of element No. 1 removal, as follows:

1. Input data (new static scheme, load on the structure for the dominant combination, and capacity N_i of individual elements)—shown in Figure 4;
2. Spectral analysis of linear stiffness matrix K_L for new static scheme (determination of the new number of KAFMs and MCSEs);
3. Determination of the effect of actions E_i for individual elements under new static scheme— μ_{Ei} (Table 3);
4. Checking the effort of individual elements for the new static scheme— μ_{Ei}/μ_{Ni} (Table 3).

Figure 4. The new static scheme after the removal of element No. 1: (a) topology and dimensions, (b) vertical load, and (c) horizontal load.

Table 3. Reliability analysis after the removal of element No. 1.

Element	Profile	Expected Value of the Effect of Actions of the ith Element μ_{Ei} [kN]	Standard Deviation of the Effect of Action σ_{Ei} (6%) [kN]	Expected Value of the Capacity of the ith Element μ_{Ni} [kN]	Standard Deviation of the Capacity σ_{Ni} [kN]	Effort of the ith Element μ_{Ei}/μ_{Ni} [%]	Probability of the Failure of the ith Element P_{fi} [-]	Reliability of the ith Element R_i [-]
-	-	-	-	-	-	-	-	-
2	L40 × 4	21.078	1.26468	14.400	2.26080	146.4	failure of the element	
3	L60 × 6	61.000	3.66000	64.304	8.68104	94.9	0.3629051144	0.6370948856
4	L45 × 6	43.134	2.58804	119.615	11.96150	36.1	0.0000000002	0.9999999998
5	L45 × 6	27.577	1.65462	119.615	11.96150	23.1	0.0000000000	1.0000000000
6	L60 × 6	20.922	1.25532	162.385	16.23850	12.9	0.0000000000	1.0000000000
7	L40 × 4	3.855	0.23130	72.380	7.23800	5.3	0.0000000000	1.0000000000
8	L60 × 6	20.078	1.20468	64.304	8.68104	31.2	0.0000002253	0.9999997747
9	L45 × 6	0.596	0.03576	15.430	2.51509	3.9	0.0000000018	0.9999999982
10	L45 × 6	13.324	0.79944	15.430	2.51509	86.4	0.2124343328	0.7875656672
11	L60 × 6	8.934	0.53604	162.385	16.23850	5.5	0.0000000000	1.0000000000
12	L40 × 4	0.448	0.02688	14.400	2.26080	3.1	0.0000000003	0.9999999997
13	L60 × 6	16.066	0.96396	64.304	8.68104	25.0	0.000000167	0.999999833
14	L45 × 6	5.044	0.30264	119.615	11.96150	4.2	0.0000000000	1.0000000000
15	L45 × 6	4.856	0.29136	15.430	2.51509	31.5	0.0000148149	0.9999851851
16	L60 × 6	4.618	0.27708	162.385	16.23850	2.8	0.0000000000	1.0000000000
17	L40 × 4	0.010	0.00060	72.380	7.23800	0.0	0.0000000000	1.0000000000
18	L60 × 6	8.382	0.50292	64.304	8.68104	13.0	0.0000000001	0.9999999999
19	L45 × 6	2.661	0.15966	119.615	11.96150	2.2	0.0000000000	1.0000000000
20	L45 × 6	4.410	0.26460	15.430	2.51509	28.6	0.0000065777	0.9999934223
21	L60 × 6	1.392	0.08352	162.385	16.23850	0.9	0.0000000000	1.0000000000
22	L40 × 4	0.362	0.02172	14.400	2.26080	2.5	0.0000000003	0.9999999997
23	L60 × 6	3.608	0.21648	64.304	8.68104	5.6	0.0000000000	1.0000000000
24	L45 × 6	1.567	0.09402	119.615	11.96150	1.3	0.0000000000	1.0000000000
25	L45 × 6	2.675	0.16050	15.430	2.51509	17.3	0.0000002084	0.9999997916
26	L60 × 6	0.246	0.01476	162.385	16.23850	0.2	0.0000000000	1.0000000000
27	L40 × 4	0.254	0.01524	14.400	2.26080	1.8	0.0000000002	0.9999999998
28	L60 × 6	0.754	0.04524	64.304	8.68104	1.2	0.0000000000	1.0000000000
29	L45 × 6	0.359	0.02154	119.615	11.96150	0.3	0.0000000000	1.0000000000
30	L45 × 6	1.055	0.06330	15.430	2.51509	6.8	0.0000000055	0.9999999945

The removal of element No.1 from the structure results in exceeding the capacity of element No. 2. However, it does not cause a failure mechanism. Therefore, it is necessary to check the reliability of the system after removing elements No. 1 and No. 2:

1. Input data (new static scheme, load on individual elements E_i for the dominant combination, and load capacity of N_i individual elements)—shown in Figure 5;
2. Spectral analysis of the linear stiffness matrix K_L for new static scheme (determination of the number of KAFMs and MCSEs);
3. Determination of the effect of actions E_i for individual elements under new static scheme— μ_{Ei} (Table 4);
4. Checking the effort of individual elements for new static scheme— μ_{Ei}/μ_{Ni} (Table 4).

It turns out that the consequence of the removal of the elements No. 1 and No. 2 is exceeding the capacity in elements No. 9 and No. 10. This is the cause of the failure mechanism of the structure. The same analysis was carried out for all thirty cases when single elements were removed from the tower. The results are shown in Tables 5 and 6.

Figure 5. New static scheme after the removal of the elements No. 1 and No. 2: (a) topology and dimensions, (b) vertical load, and (c) horizontal load.

Table 4. Reliability analysis after the removal of elements No. 1 and No. 2.

Element	Profile	Expected Value of the Effect of Actions of the ith Element μ_{Ei} [kN]	Standard Deviation of the Effect of Action σ_{Ei} (6%) [kN]	Expected Value of the Capacity of the ith Element μ_{Ni} [kN]	Standard Deviation of the Capacity σ_{Ni} [kN]	Effort of the ith Element μ_{Ei}/μ_{Ni} [%]	Probability of the Failure of the ith Element P_{fi} [-]	Reliability of the ith Element R_i [-]
-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-
3	L60 × 6	61.000	3.66000	64.304	8.68104	94.9	0.3629051144	0.6370948856
4	L45 × 6	43.134	2.58804	119.615	11.96150	36.1	0.0000000002	0.9999999998
5	L45 × 6	27.577	1.65462	119.615	11.96150	23.1	0.0000000000	1.0000000000
6	L60 × 6	42.000	2.52000	162.385	16.23850	25.9	0.0000000000	1.0000000000
7	L40 × 4	21.507	1.29042	72.380	7.23800	29.7	0.0000000000	1.0000000000
8	L60 × 6	1.000	0.06000	162.385	16.23850	0.6	0.0000000000	1.0000000000
9	L45 × 6	30.406	1.82436	15.430	2.51509	197.1	failure of the element	
10	L45 × 6	43.134	2.58804	15.430	2.51509	279.5	failure of the element	
11	L60 × 6	5.507	0.33042	162.385	16.23850	3.4	0.0000000000	1.0000000000
12	L40 × 4	3.318	0.19908	14.400	2.26080	23.0	0.0000005227	0.9999994773
13	L60 × 6	19.493	1.16958	64.304	8.68104	30.3	0.0000001563	0.9999998437
14	L45 × 6	9.889	0.59334	119.615	11.96150	8.3	0.0000000000	1.0000000000
15	L45 × 6	0.010	0.00060	15.430	2.51509	0.1	0.0000000004	0.9999999996
16	L60 × 6	5.175	0.31050	162.385	16.23850	3.2	0.0000000000	1.0000000000
17	L40 × 4	0.476	0.02856	72.380	7.23800	0.7	0.0000000000	1.0000000000
18	L60 × 6	7.825	0.46950	64.304	8.68104	12.2	0.0000000000	1.0000000000
19	L45 × 6	1.874	0.11244	119.615	11.96150	1.6	0.0000000000	1.0000000000
20	L45 × 6	5.197	0.31182	15.430	2.51509	33.7	0.0000269857	0.9999730143
21	L60 × 6	1.301	0.07806	162.385	16.23850	0.8	0.0000000000	1.0000000000
22	L40 × 4	0.438	0.02628	14.400	2.26080	3.0	0.0000000003	0.9999999997
23	L60 × 6	3.699	0.22194	64.304	8.68104	5.8	0.0000000000	1.0000000000
24	L45 × 6	1.695	0.10170	119.615	11.96150	1.4	0.0000000000	1.0000000000
25	L45 × 6	2.547	0.15282	15.430	2.51509	16.5	0.0000001587	0.9999998413
26	L60 × 6	0.260	0.01560	162.385	16.23850	0.2	0.0000000000	1.0000000000
27	L40 × 4	0.240	0.01440	14.400	2.26080	1.7	0.0000000002	0.9999999998
28	L60 × 6	0.740	0.04440	64.304	8.68104	1.2	0.0000000000	1.0000000000
29	L45 × 6	0.339	0.02034	119.615	11.96150	0.3	0.0000000000	1.0000000000
30	L45 × 6	1.075	0.06450	15.430	2.51509	7.0	0.0000000058	0.9999999942

Table 5. The effect of the removal of individual elements of a truss tower when the buckling coefficient is a deterministic value.

No. of Removed Element	No. of Tower Segment	β	Effect of the Element Removal
1	1	-	Removal of element No. 1 results in exceeding the capacity in the element No. 2 → removal of the next element (No. 2) results in exceeding the capacity in elements No. 9 and No. 10 → the result is a mechanism
2		6.299	remov. el. No. 2 does not result in the mech.; β is at a high enough level
3		-	remov. el. No. 3 results in exceeding the cap. in el. No. 4 and No. 5 → the result is a mech.
4		-	remov. el. No. 4 results in exceeding the cap. in el. No. 5 → the result is a mech.
5		3.307	remov. el. No. 5 does not result in the mech.; β is not at high enough level
6	2	1.936	remov. el. No. 6 does not result in the mech.; β is not at high enough level
7		6.098	remov. el. No. 7 does not result in the mech.; β is at high enough level
8		-	remov. el. No. 8 results in exceeding the cap. in el. No. 9 and No. 10 → the result is a mech.
9		1.569	remov. el. No. 9 does not result in the mech.; β is not at high enough level
10		5.206	remov. el. No. 10 does not result in the mech.; β is at high enough level
11	3	5.848	remov. el. No. 11 does not result in the mech.; β is at high enough level
12		6.114	remov. el. No. 12 does not result in the mech.; β is at high enough level
13		-	remov. el. No. 13 results in exceeding the cap. in el. No. 14 and No. 15 → the result is a mech.
14		3.345	remov. el. No. 14 does not result in the mech.; β is not at high enough level
15		6.174	remov. el. No. 15 does not result in the mech.; β is at high enough level
16	4	6.102	remov. el. No. 16 does not result in the mech.; β is at high enough level
17		6.113	remov. el. No. 17 does not result in the mech.; β is at high enough level
18		-	remov. el. No. 18 results in exceeding the cap. in el. No. 20 → the result is a mech.
19		5.223	remov. el. No. 19 does not result in the mech.; β is at high enough level
20		6.106	remov. el. No. 20 does not result in the mech.; β is at high enough level
21	5	6.114	remov. el. No. 21 does not result in the mech.; β is at high enough level
22		6.113	remov. el. No. 22 does not result in the mech.; β is at high enough level
23		4.747	remov. el. No. 23 does not result in the mech.; β is at high enough level
24		6.113	remov. el. No. 24 does not result in the mech.; β is at high enough level
25		6.114	remov. el. No. 25 does not result in the mech.; β is at high enough level
26	6	6.113	remov. el. No. 26 does not result in the mech.; β is at high enough level
27		6.113	remov. el. No. 27 does not result in the mech.; β is at high enough level
28		6.113	remov. el. No. 28 does not result in the mech.; β is at a high enough level
29		6.113	remov. el. No. 29 does not result in the mech.; β is at a high enough level
30		6.113	remov. el. No. 30 does not result in the mech.; β is at a high enough level

Table 6. The effect of the removal of individual elements of a truss tower when the buckling coefficient is a random variable.

No. of Removed Element	No. of Tower Segment	β	Effect of the Element Removal
1	1	-	Removal of element No. 1 results in exceeding the capacity in element No.2 $\rightarrow$ removal of the next element (No. 2) results in exceeding the capacity in elements No.9 and No.10 $\rightarrow$ the result is a mechanism
2		4.441	remov. el. No. 2 does not result in the mech.; β is at high enough level
3		-	remov. el. No. 3 results in exceeding the cap. in el. No.4 and No.5 $\rightarrow$ the result is a mech.
4		-	remov. el. No. 4 results in exceeding the cap. in el. No.5 $\rightarrow$ the result is a mech.
5		2.525	remov. el. No. 5 does not result in the mech.; β is not at high enough level
6	2	1.484	remov. el. No. 6 does not result in the mech.; β is not at high enough level
7		4.462	remov. el. No. 7 does not result in the mech.; β is at a high enough level
8		-	remov. el. No. 8 results in exceeding the cap. in el. No.9 and No.10 $\rightarrow$ the result is a mech.
9		1.028	remov. el. No. 9 does not result in the mech.; β is not at high enough level
10		3.876	remov. el. No. 10 does not result in the mech.; β is at high enough level
11	3	4.015	remov. el. No. 11 does not result in the mech.; β is at high enough level
12		4.471	remov. el. No. 12 does not result in the mech.; β is at high enough level
13		-	remov. el. No. 13 results in exceeding the cap. in el. No.14 and No.15 $\rightarrow$ the result is a mech.
14		2.140	remov. el. No. 14 does not result in the mech.; β is not at high enough level
15		4.493	remov. el. No. 15 does not result in the mech.; β is at high enough level
16	4	4.465	remov. el. No. 16 does not result in the mech.; β is at high enough level
17		4.471	remov. el. No. 17 does not result in the mech.; β is at high enough level
18		-	remov. el. No. 18 results in exceeding the cap. in el. No. 20 $\rightarrow$ the result is a mech.
19		3.275	remov. el. No.19 does not result in the mech.; β is not at high enough level
20		4.467	remov. el. No. 20 does not result in the mech.; β is at high enough level
21	5	4.471	remov. el. No. 21 does not result in the mech.; β is at high enough level
22		4.471	remov. el. No. 22 does not result in the mech.; β is at high enough level
23		2.990	remov. el. No. 23 does not result in the mech.; β is not at high enough level
24		4.296	remov. el. No. 24 does not result in the mech.; β is at high enough level
25		4.471	remov. el. No. 25 does not result in the mech.; β is at high enough level
26	6	4.471	remov. el. No. 26 does not result in the mech.; β is at high enough level
27		4.471	remov. el. No. 27 does not result in the mech.; β is at high enough level
28		4.467	remov. el. No. 28 does not result in the mech.; β is at high enough level
29		4.470	remov. el. No. 29 does not result in the mech.; β is at high enough level
30		4.471	remov. el. No. 30 does not result in the mech.; β is at high enough level

Based on the results presented in Tables 5 and 6, it can be concluded that the description of the buckling coefficient has a large influence on the reliability. When the buckling coefficient is treated as a probabilistic variable, the results are lower. Therefore, these results are treated as representative and compared in the form of graphs (Figures 6–10). The results refer to the required value of the reliability index for the reliability class RC2 ($\beta_{EC} = 3.8$). The graphs in Figures 6–10 show the data from Table 6. For each of the figures on the left, the removed elements are underlined. Figure 6 shows the results for removed left main legs (elements no. 1, 6, 11, 16, 21, and 26). Removal of item no. 1 causes failure of the whole structure. Removal of item no. 6 causes failure of the whole structure. Removal of element number 6 causes the reliability index is insufficient ($\beta = 1.484 < \beta_{EC} = 3.8$). Removal of elements no. 11, 16, 21, and 26 does not cause the reliability index to drop below the required level. The graphs on the following figures (Figures 7–10) should be interpreted as shown in Figure 6.

Figure 6. Reliability index when the buckling coefficient is a random variable for removed elements 1, 6, 11, 16, 21, and 26 (left main legs).

Figure 7. Reliability index when the buckling coefficient is a random variable for removed elements 2, 7, 12, 17, 22, and 27 (horizontal bracings).

Figure 8. Reliability index when the buckling coefficient is a random variable for removed elements 3, 8, 13, 18, 23, and 28 (right main legs).

Figure 9. Reliability index when the buckling coefficient is a random variable for removed elements 4, 9, 14, 19, 24, and 29 (diagonal bracings).

Figure 10. Reliability index when the buckling coefficient is a random variable for removed elements 5, 10, 15, 20, 25, and 30 (diagonal bracings).

4. Discussion

The aim of the paper was to analyze the reliability of the steel truss tower after the failure of individual elements. The analyses carried out for the study led to some valid conclusions.

An important issue is the impact of the description of the buckling coefficient (probabilistic or deterministic) on the results of the reliability index. The difference in the results of the reliability index β is up to approximately 37%. Coefficients of variation of random variables, on which the buckling

coefficient depends, play a key role. Therefore, determining the appropriate values of variability coefficients is an important issue in future studies.

The failure of individual elements of a truss tower can bring about different effects. In the case of the analyzed tower, the removal of six individual elements leads to the failure of the whole structure. For a buckling coefficient being a probabilistic value, six cases of a single element removal result in the insufficient value of the reliability index β (below 3.8).

The analysis of the impact of removing individual elements of the truss tower makes it possible to determine elements that can be additionally strengthened due to their major significance for safety. For the analyzed case, they are all elements whose removal causes the failure of the whole structure. It should be emphasized that the results obtained cannot be treated as a reference, because the reliability analysis should be carried out individually for each structure.

Analysis of failure scenarios of truss structures is a highly time-consuming task. In order to study more complex cases, it is necessary to develop an additional program that would allow automation of calculations.

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