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# Comparative Analysis of Heuristic Algorithms Used for Solving a Production and Maintenance Planning Problem (PMPP)

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**Abstract:** In this work, we develop methods to assess the risk of profit–loss resulting from the choice of a computational method for solving a joint production and maintenance-planning problem. In fact, the optimal objective function is calculated via the use of algorithms and optimization methods. The use of these methods can have an impact on an event that can disrupt the optimal production and maintenance plan. To achieve our goals, we start with calculating the manufacturing system’s joint production and maintenance plans over a finite horizon using different methods. In the second part of the work, we propose analytical models to quantify the risk of profit–loss resulting from product returns and the integration of an imperfect maintenance policy. Numerical examples are conducted by adopting the different algorithms used. This study provides insights into the most efficient computational methods for the encountered problems. This research proposes new approaches to help and guide managers in the analysis and evaluation of their decisions.

**Keywords:** Lost profit–risk assessment; stochastic optimization; production system; preventive maintenance; product returns

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## 1. Introduction

A risk is described as a “hazard”, a chance of bad consequences, loss, or exposure to mischance (from the Concise Oxford English Dictionary, reported in [1]). Its assessment may be incorporated into the company’s planning and decision-making processes, encompassing profits, reliability, and other performance objectives [2].

The minimization of financial risks is one of the keys to sustainable business development [3]. Risk identification and assessment provided specific indications of managerial attention to achieve the performance objectives [2]. Indeed, Tuncel et al. [4] followed these indications using a timed Petri nets framework. Their main objective was to illustrate the challenges where coordination within supply chain networks brings to risk management. Furthermore, in his book, Zio [5] introduced the application of the Monte Carlo simulation method for the analysis of system reliability and risk. Failure mode and effects analysis (FMEA) is a risk assessment tool that reduces potential failures in systems, process, designs, or services [6]. Hubbard et al. [7] described a number of common scoring methods that are currently used to assess risk in a variety of different domains.

In our study, we propose methods in order to quantify the risk of certain strategies or events that can disrupt the system. In the literature, many researchers are interested in risk issues, and they have tried to classify types and propose tools to cope with some risks that can lead to bad consequences.

Our innovative contribution is to evaluate the impact of the use of optimization algorithms on certain events that may disrupt the calculated production and maintenance plan.

In fact, the relevance of the tools and algorithms used is never discussed in terms of impact on an incident that may occur, and that disrupts the optimal production and maintenance plan decided.

The paper proposes a new contribution to the literature, which proposes on the one hand analytical models to quantify the profit loss following an adopted strategy, and on the other hand the impact of the use of different optimization algorithms on the calculation of profit loss.

In recent years, researchers have tried to investigate the problem of choice regarding a maintenance policy that is adequate to their stochastic problems from a robustness and effectiveness point of view. Cherkaoui et al. [8] proposed mathematical cost models to assess jointly the economic performance and robustness of two maintenance policies. Caballé et al. [9] and Huynh et al. [10] also consider the same objectives. The purpose of their work was to help decision-makers choose the adequate maintenance policy that maximizes their profit as well as maintains a robust production system.

The integration of production and maintenance planning problems is a significant challenge for industrial companies, and hence has received much research attention in the last few decades. According to Alsyouf [11], the increased importance of maintenance activities is due to their impact on system availability, performance efficiency, product quality, etc. One of the first authors to have dealt with integrated production–maintenance strategies is Buzacott [12]. Rezg et al. [13] suggested a method for the joint optimization of preventive maintenance and inventory control in a production line that combined  $N$  machines, while Gharbi et al. [14] developed joint production and preventive maintenance policies incorporating inventory levels. Rezg et al. [15] proposed a model to determine a joint optimal inventory control policy and an age-based preventive maintenance policy for a randomly failing production system. Kenne and Nkeungoue [16] introduced another model for the same context. The work of Hajej et al. [17,18] considered the impact of the production plan on equipment degradation. Nodem et al. [19] evolved a method to find the optimal production, replacement/repair, and preventive maintenance policies for a deteriorated manufacturing system. Other researchers have had to adopt other aspects. Ben-Salem et al. [20] and Hajej et al. [21] considered an ecological aspect on the integration of production and maintenance planning problems of an unreliable manufacturing system that was subject to degradation. Otherwise, some researchers have used other types of maintenance to limit certain risks (unavailability of the manufacturing system, energy consumption, etc.). Selcuk [22] adopted predictive maintenance by proposing new trends and techniques to protect the studied system. Klos et al. [23] proposed a model of an intelligent maintenance management system in order to study the impact of the availability of manufacturing resources on the throughput and average lifespan of products. Renna [24] proposed the evaluation of the manufacturing system performance in dynamic conditions when different maintenance policies are implemented in a multi-machines manufacturing system controlled by multi-agent architecture. Table 1 summarizes the works that treat the problem of integrating production and maintenance, but use different approaches.

Researches in joint production and maintenance present a lack of knowledge and contribution. The area of integrated maintenance with production lot sizing is based on an optimization computation using operational research tools. The optimum is obtained via an objective function where the cost is to be optimized. Some works have proposed a multi-criterion optimization to take into account different functions to optimize under several constraints. Consequently, in our case, we propose to implement a risk analysis tool for the decisions taken following an optimization. It aims to evaluate the risks of a decision resulting from the results of an optimization. These optimization results are calculated by an analysis of the algorithms and tools used in the optimization step. Indeed, the risks assessment considers risks of a decision resulting from operational anomalies such as a failure that can exceed the maximum repair time and its impact on the robustness of the production and maintenance plan computation. Guiras et al. [25] introduced the risk assessment of the repair time of random failures following the optimization of a disassembly/assembly system.

The assessment of lost profit risk (LPR) in our work entails first establishing optimal production and maintenance plans, taking into account the impact of the production rate on the system’s deterioration. Indeed, we study the profit loss underlying the choice of an algorithm to find the optimal plans. We implement four different heuristic algorithms: the Nelder–Mead method, the differential evolution method (DE), the simulated annealing method (SA), and the random search method. Then, we determine the LPR for product returns and imperfect preventive maintenance actions.

The paper is structured as follows. The production and maintenance planning problems are described and mathematically formulated in Section 2. The four heuristic algorithms employed for these problems, along with a numerical example, are presented in Section 3. Next, we propose analytical models of lost profit risk and apply them to the above numerical example in Section 4. Finally, the last section presents our concluding remarks from the perspective of insights gained in this study, and our recommendations for future research.

**Table 1.** Approaches used in solving the integrated maintenance–production problem.

Used Approach	Authors
Simulation	Rezg et al. [13,15], Gharbi et al. [14]
Genetic algorithm	Benbouzid-Sitayeb et al. [26]
Ant colonies algorithms	Benbouzid-Sitayeb et al. [27]
Genetic algorithm	Belkaid et al. [28]
Linear stochastic optimal control approach	Hajej et al. [18]
Global approach	Hajej et al. [29]

## 2. Problem Description

Assume that a single machine ( $M$ ) is a single operation manufacturing system producing a single product with the view to satisfy a random demand over a finite horizon. We seek the combined production and maintenance plans, which minimize the holding, production, and maintenance costs. The production machine is subject to random failure, and the failure rate is increasing as a function of both time and the production rate. A minimal repair is performed at failure, and a periodic replacement is carried out periodically. According to this maintenance policy, preventive maintenance activities are supposed to be able to restore the machine to be “as good as new”.

To determine the optimal production plan, an analytical model is presented to satisfy the random demand. Secondly, depending on the production rate, we present the optimal preventive maintenance plan. The adoption of the optimal production plan as an input to the maintenance plan is justified by the influence of production rate variation on the failure rate of the machine. We denote the production problem by  $P$  and maintenance problem by  $M$ . The production and maintenance-planning problem (PMPP) is presented below.

### 2.1. Parameters

The following notations and assumptions used throughout this article are defined:

$\Delta t$	period length
$H$	period’s number in the planning horizon.
$H \cdot \Delta t$	length of the finite horizon plan.
$d(k)$	demand in period $k, k = \{0, 1, \dots, H\}$ .
$E[ ]$	mathematical expectation operator.
$\hat{d}(k)$	average demand in period $k = E[d(k)], k = \{0, 1, \dots, H\}$ .

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$S(k)$	level of the store at the end of period $k, k = \{0, 1, \dots, H\}$ .
$S(0)$	given initial inventory.
$\hat{S}(k)$	average inventory level in period $k = E[S(k)], k = \{0, 1, \dots, H\}$ .
$V(S(k))$	variance of storage $= E[S(k) - \hat{S}(k)]^2, k = \{0, 1, \dots, H\}$ .
$V_d(k)$	variance of demand $= E[d(k) - \hat{d}(k)]^2, k = \{0, 1, \dots, H\}$ .
$\varphi_{d,k}^{-1}(\alpha)$	inverse cumulative Gaussian distribution function with mean $\hat{d}(k)$ and finite variance $V_d(k)$ .
$U(k)$	production rate in each period $k, k = \{0, 1, \dots, H\}$ .
$U_{\max}$	maximum machine production rate.
$C_{pr}$	unit production cost.
$C_s$	holding cost, per unit of product per period.
$C_{pm}$	cost for a preventive maintenance action.
$C_{cm}$	cost for a corrective maintenance action.
$f(t)$	probability density function of time to failure.
$\lambda_n(t)$	nominal failure rate that occurs when the production rate is $U_{\max}$ .
$\lambda_k(t)$	failure rate of the machine during period $k, k = \{0, 1, \dots, H\}$ .
$N$	number of preventive maintenance actions over the finite horizon.
$A(U, N)$	average number of failures over the finite horizon $H$ .

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## 2.2. Formulation of the PMPP

### 2.2.1. Production Policy

A production system describes an integrated set of processes that allow us to produce a good service that meets the primary objective of the company, which is customer satisfaction. The coordination of different activities such as production and maintenance is necessary in order to achieve the profit maximization.

The aim of this subsection is to present the production plan that minimizes the production and holding costs. Assuming that the horizon is partitioned equally into  $H$  periods of length  $\Delta t$ . The demand  $d(k)$  is a random variable that follows a normal distribution. Let  $\{f_k(\cdot), k = 1, \dots, H\}$  represent the cost function of the production and storage. The following sequential stochastic linear programming problem provides an optimal production plan over the planning horizon.

$$\text{Min } E \left\{ \sum_{k=0}^{H-1} f_k(S(k), U(k)) + f_H(S(H)) \right\} \tag{1}$$

Subject to:

$$S(k + 1) = S(k) + U(k) - d(k) \quad k = 0, 1, \dots, H - 1 \tag{2}$$

$$P[S(k + 1) \geq 0] \geq \alpha \quad k = 0, 1, \dots, H - 1 \tag{3}$$

$$0 \leq U(k) \leq U_{\max} \quad k = 0, 1, \dots, H - 1 \tag{4}$$

The first constraint, (2), defines the inventory balance equation for each production period. The second constraint, (3), imposes a service level requirement in each period, which in turn induces a safety stock. Finally, constraints (4) define lower and upper bounds on the production rate during each period.

The following equation represents the expected production and holding costs for each period  $k$ . We use the quadratic form for our expression, which allows both an excess and shortage of inventory to be penalized [18].

$$f_k(S(k), U(k)) = C_s \times E[S(k)^2] + C_{pr} \times U(k)^2 \tag{5}$$

Hence, over the finite horizon  $H \cdot \Delta t$ , the total expected cost of production and inventory can be expressed as:

$$F(U) = \sum_{k=0}^H \{f_k(S(k), U(k))\} = C_s \times E[S(H)^2] + \sum_{k=0}^{H-1} [C_s \times E[S(k)^2] + C_{pr} \times U(k)^2] \quad (6)$$

Note that  $U(H)$  is not included in the cost formulation, because we do not consider the production order at the end of the horizon  $H.\Delta t$ . In practice, this constrained stochastic nonlinear programming problem is generally difficult to solve. Therefore, we transformed it into an equivalent deterministic problem, which becomes easier to solve [18]).

Note that  $V(S(0)) = 0$  and let  $V_d(k)$  be constant and equal to  $V_d$  for all of  $k$ . After simplifying, which is described in more detail in Hajej et al. [18], we obtain the following expression:

$$\text{Min } F(U) = C_s \cdot \hat{S}(H)^2 + \sum_{k=0}^{H-1} [C_s \cdot \hat{S}(k)^2 + C_{pr} \cdot U(k)^2] + C_s \cdot V_d \cdot \frac{H(H+1)}{2} \quad (7)$$

Subject to:

$$\hat{S}(k+1) = \hat{S}(k) + U(k) - \hat{d}(k) \quad k = \{0, 1, \dots, H-1\} \quad (8)$$

$$P[S(k+1) \geq 0] \geq \alpha \Rightarrow U(k) \geq \varphi_{d,k}^{-1}(\alpha)V_d(k) - S(k) + \hat{d}(k) \quad k = \{0, 1, \dots, H-1\} \quad (9)$$

$$0 \leq U(k) \leq U_{\max} \quad k = \{0, 1, \dots, H-1\} \quad (10)$$

$$F(U) = \sum_{k=0}^H \{f_k(S(k), U(k))\} = C_s \times E[S(H)^2] + \sum_{k=0}^{H-1} [C_s \times E[S(k)^2] + C_{pr} \times U(k)^2]$$

The transformation from stochastic to the deterministic problem is described as follows. We have:

$$E[d(k)] = \hat{d}(k)$$

and:

$$V_d(k) = \sigma_d^2 \geq 0 \quad \forall k$$

The storage variable  $S(k)$  is described statistically by the mean  $E[S(k)] = \hat{S}(k)$ .

And the variance  $E[(S(k) - \hat{S}(k))^2] = \text{Var}(S(k))$ .

Thus, the relation between  $\hat{S}(k)$  and  $\hat{S}(k+1)$  is defined by the following equation:

$$E[S(k+1)] = E[S(k) + U(k) - d(k)]$$

$$\Rightarrow \hat{S}(k+1) = \hat{S}(k) + U(k) - \hat{d}(k)$$

Indeed, this equation represents the average evolution of storage variable for each period  $k$ ,  $k \in \{1 \dots H-1\}$ .

Moreover,  $U(k)$  is essentially deterministic, since this variable no longer depends on the random variables  $d(k)$  and  $S(k)$ .

$$E[U(k)] = U(k)$$

with:

$$V(U(k)) = 0 \quad \forall k$$

By calculating the difference between:

$$S(k+1) - \hat{S}(k+1) = S(k) + U(k) - d(k) - \hat{S}(k) - U(k) + \hat{d}(k)$$

$$\Rightarrow S(k+1) - \hat{S}(k+1) = S(k) - \hat{S}(k) - (d(k) - \hat{d}(k))$$

$$\begin{aligned} \Rightarrow (S(k+1) - \hat{S}(k+1))^2 &= (S(k) - \hat{S}(k)) - (d(k) - \hat{d}(k))^2 \\ \Rightarrow E[(S(k+1) - \hat{S}(k+1))^2] &= E[(S(k) - \hat{S}(k)) - (d(k) - \hat{d}(k))^2] \\ \Rightarrow E[(S(k+1) - \hat{S}(k+1))^2] &= E[(S(k) - \hat{S}(k))^2 + (d(k) - \hat{d}(k))^2 - 2 \cdot (S(k) - \hat{S}(k)) \cdot (d(k) - \hat{d}(k))] \\ \Rightarrow E[(S(k+1) - \hat{S}(k+1))^2] &= E[(S(k) - \hat{S}(k))^2] + E[(d(k) - \hat{d}(k))^2] - 2 \cdot E[(S(k) - \hat{S}(k)) \cdot (d(k) - \hat{d}(k))] \end{aligned}$$

Since  $S(k)$  and  $d(k)$  are independent random variables, we can deduce that:

$$E[(S(k) - \hat{S}(k)) \cdot (d(k) - \hat{d}(k))] = E[(S(k) - \hat{S}(k))] \cdot E[(d(k) - \hat{d}(k))]$$

On the other hand, we can note that:

$$E[(S(k) - \hat{S}(k))] = E[S(k)] - E[\hat{S}(k)] = 0$$

$$E[d(k) - \hat{d}(k)] = E[d(k)] - E[\hat{d}(k)] = 0$$

Thus:

$$E[(S(k+1) - \hat{S}(k+1))^2] = E[(S(k) - \hat{S}(k))^2] + E[(d(k) - \hat{d}(k))^2]$$

Generally speaking,  $a_k$  is a random variable; we have:

$$E[a_k - \hat{a}_k]^2 = V_{a_k}(k) = E[a_k^2] - \hat{a}_k^2$$

Therefore, we have:

$$V_s(k+1) = V_s(k) + V_d(k) = V_s(k) + \sigma_d^2$$

We suppose that  $V_s(k=0) = 0$  and  $\sigma_d$  is constant and equal to  $\sigma_d$  for all of  $k$ .

We can deduce that  $V_s(k) = k \cdot \sigma_d^2$ .

Demonstration:

$$\text{For } k=0 \Rightarrow V_s(1) = V_s(0) + \sigma_d^2$$

$$k=1 \Rightarrow V_s(2) = V_s(1) + \sigma_d^2 = 2 \cdot \sigma_d^2$$

$$k=2 \Rightarrow V_s(3) = V_s(2) + \sigma_d^2 = 3 \cdot \sigma_d^2$$

$$\text{For } k \Rightarrow V_s(k) = k \cdot \sigma_d^2$$

According to the previous equation:

$$V_s(k+1) = V_s(k) + V_d(k)$$

$$\Rightarrow V_s(k+1) = k \cdot \sigma_d^2 + \sigma_d^2$$

$$\Rightarrow V_s(k+1) = (k+1) \cdot \sigma_d^2$$

Thus:

$$V_s(k) = k \cdot \sigma_d^2$$

$$E[(S(k) - \hat{S}(k))^2] = E[S(k)^2] - \hat{S}(k)^2$$

$$\Rightarrow E[S(k)^2] - \hat{S}(k)^2 = V_s(k) = k \cdot \sigma_d^2$$

So:

$$E[S(k)^2] = k \cdot \sigma_d^2 + \hat{S}(k)^2$$

We replace  $E[S(k)^2] = k \cdot \sigma_d^2 + \hat{S}(k)^2$  in  $F(U)$ , we obtain Equation (7):

$$F(U) = C_s \cdot \hat{S}(H)^2 + \sum_{k=0}^{H-1} [C_s \cdot \hat{S}(k)^2 + C_{pr} \cdot U(k)^2] + C_s \cdot V_d \cdot \frac{H(H+1)}{2}$$

For the constraint (9), we demonstrate it as follows:

We have:

$$\begin{aligned}
 S(k+1) &= S(k) + U(k) - d(k) \\
 \Rightarrow P[S(k+1) \geq 0] &\geq \alpha \\
 \Rightarrow P[S(k) + U(k) - d(k) \geq 0] &\geq \alpha \\
 \Rightarrow P[S(k) + U(k) \geq d(k)] &\geq \alpha \\
 \Rightarrow P[S(k) + U(k) - \hat{d}(k) \geq d(k) - \hat{d}(k)] &\geq \alpha \\
 \Rightarrow P\left[\frac{S(k)+U(k)-\hat{d}(k)}{V_d(k)} \geq \frac{d(k)-\hat{d}(k)}{V_d(k)}\right] &\geq \alpha
 \end{aligned}$$

We note that:

$X = \frac{d(k)-\hat{d}(k)}{V_d(k)}$ : A Gaussian random variable for the demand  $d(k)$ .

We suppose that,  $\varphi$  is the Gaussian demand distribution function, and  $f$  is the probability density function.

$$\varphi_{d,k}\left(\frac{S(k) + U(k) - \hat{d}(k)}{V_{d,k}}\right) \geq \alpha$$

$\varphi_{d,k}$  is strictly increasing, so we note that  $\varphi_{d,k}$ .

$$\begin{aligned}
 \Rightarrow \frac{S(k) + U(k) - \hat{d}(k)}{V_{d,k}} &\geq \varphi^{-1}(\alpha) \\
 \Rightarrow S(k) + U(k) - \hat{d}(k) &\geq V_{d,k} \cdot \varphi_{d,k}^{-1}(\alpha) \\
 \Rightarrow U(k) &\geq V_{d,k} \times \varphi^{-1}(\alpha) + \hat{d}(k) - S(k)
 \end{aligned}$$

So,  $P(S(k+1) \geq 0) \geq \alpha \Rightarrow (U(k) \geq V_{d,k} \cdot \varphi_{d,k}^{-1}(\alpha) + \hat{d}(k) - S(k)) \quad k = 0, 1, \dots, H - 1.$

### 2.2.2. Maintenance Strategy

The maintenance strategy aims to improve the availability of the production system by reducing the occurrence of failures in order to minimize the losses caused by the latter along with operating costs. This activity is becoming increasingly important to companies, and should thus be based on a clear strategy to reduce the associated costs while ensuring some level of equipment reliability. The optimization of maintenance strategies has been widely studied in the literature.

The stochastic nature of the system is due to the machine, which is subject to breakdowns and maintenance actions. Therefore, we consider a perfect periodic preventive maintenance scheme combined with minimal repair at failure [30].

The total maintenance actions cost can be summed up in the following analytic expression:

$$\varphi(U, N) = C_{pm} \times (N - 1) + C_{cm} \times A(U, N) \tag{11}$$

where  $N \in \{1, 2, 3, \dots\}$  and  $A(U, N)$  accords to the expected number of failures that occur during the horizon  $H$ .  $\Delta t$ , considering the influence of the production rate in each production period  $k$  on the failure rate of the machine,  $\lambda_k(t)$ . Recall that  $\lambda_n(t)$  = the failure rate for nominal conditions, which is equivalent to the failure rate under the maximum production rate during the horizon  $H$ .  $\Delta t$ ,

$$\lambda_k(t) = \lambda_{k-1}(\Delta t) + \frac{U(k)}{U_{\max}} \lambda_n(t) \quad \forall t \in [0, \Delta t] \tag{12}$$

So, over the horizon  $H$ .  $\Delta t$ , the average failure number is (see Hajej, et al. [18]):

$$A(U, N) = \sum_{j=0}^{N-1} \left[ \sum_{i=In(j \times \frac{H}{N\Delta t})+1}^{In((j+1) \times \frac{H}{N\Delta t})} \int_0^{\Delta t} \lambda_i(t) dt + \int_0^{(j+1) \times \frac{H}{N} - In((j+1) \times \frac{H}{N\Delta t}) \times \Delta t} \lambda_{In((j+1) \times \frac{H}{N\Delta t})+1}(t) dt + \int_0^{(In((j+1) \times \frac{H}{N\Delta t})+1) \times \Delta t - (j+1) \times \frac{H}{N}} \frac{U(In((j+1) \times \frac{H}{N\Delta t})+1)}{U_{max}} \times \lambda_n(t) dt \right] \tag{13}$$

In the next section, different algorithms are described for optimizing PMPP.

### 3. Optimization of the PMPP

Finding a “good” algorithm for a given optimization problem, which can be a knowledge and time-intensive activity, depends on the nature of the objective function, i.e., its behavior (continuity, differentiability, convexity), and the constraints characterizing the set of admissible solutions. One of the fundamental challenges in engineering design is that the multiplicity of local solutions has led to a major effort to develop global search algorithms. However, these often have a prohibitive computational cost when it comes to solving real-life problems. Optimization techniques and metaheuristics represent a different way to solve complex optimization problems and give good solutions within an acceptable period of time through the reduction of the explored space.

There are two major categories of solution methods for optimization problems: exact and approximation. Exact methods invariably yield optimal solutions, although the required computational effort is often excessive. On the other hand, approximation methods, which are also called heuristics, enable quickly achieving a suboptimal (but “good”) solution. This may be a suitable approach when dealing with an optimization problem exhibiting high complexity, a difficult structure, vast amounts of data, etc.

Many algorithms have been proposed in the literature to solve the joint production and maintenance problem. For instance, Hajej et al. [18] formulated the problem as a linear stochastic optimal control problem, while Hajej et al. [29] proposed a new optimization model. A joint genetic algorithm was proposed by Benbouzid-Sitayeb et al. [26] for the joint production and maintenance scheduling problem applied to a flow shop. For their part, Belkaid et al. (2013) proposed a genetic algorithm for the parallel machine-scheduling problem.

In this section, we will outline the numerical example of the problem studied and different algorithms for its solution, along with the latter’s execution times and relative performance.

#### 3.1. Numerical Example

We use the following numerical example in order to illustrate our proposed sequential approach for solving the joint production and maintenance optimization problem.

We consider a single machine  $M$  that has to satisfy, over a finite horizon  $H$ .  $\Delta t$ , a stochastic demand that follows a Gaussian law with a mean  $\hat{d}(k)$  and a variance  $V_d(k)$ . The number  $H$  of periods  $\Delta t$  is equal to 24, with  $\Delta t = 1$ . To satisfy the demand, we set a service level.

We consider that the deterioration of the production system follows a Weibull distribution with parameters  $\gamma$  and  $\beta$ . From Equation (13), we determined the average number of failures assuming that after each preventive maintenance action, the equipment is in the state “as good as new”.

The following data are used for the computations:

- $C_{pr} = 3$  monetary units (mu)/unit of product/period.
- $U_{max} = 15$  products.
- $\alpha = 0.95$ .
- $C_s = 5$  mu/unit of product/period.
- $S_0 = 20$ .
- $V_d(k) = 1.21$
- $C_{pm} = 212$  mu.

- $C_{cm} = 1000$  mu.
- $\Delta t = 1$ .
- $\lambda_0 = 0.2$ .
- $\gamma = 2$ .
- $\beta = 100$ .
- $H = 24 \cdot \Delta t$

Table 2 presents the average demand in each period.

**Table 2.** Random demands.

$k$	0	1	2	3	4	5	6	7	8
$d(k)$	15	17	15	15	15	14	16	14	16
$k$	9	10	11	12	13	14	15	16	17
$d(k)$	13	15	14	15	12	15	13	15	11
$k$	18	19	20	21	22	23			
$d(k)$	16	13	15	12	14	16			

### 3.2. Algorithms Used to Solve the PMPP in the Numerical Example

#### 3.2.1. Exact Method

First, we solve the PMPP with the solver FICO Xpress 8.0, which uses exact methods to determine an optimal solution. After computing, it provides four integer solutions. The best solution is optimal. It was able to solve the problem in polynomial time: 0.1 s

- ✓ Total cost = 22,281.25 mu.
- ✓ Optimal production plan (see Table 3).
- ✓ Optimal number of preventive maintenance actions =  $N^* = 3$ .

**Table 3.** Optimal production plan yielded by exact method.

$k$	0	1	2	3	4	5	6	7	8
$U(k)$	1	12	15	15	15	15	15	15	15
$k$	9	10	11	12	13	14	15	16	17
$U(k)$	13	15	14	15	12	15	13	15	12
$k$	18	19	20	21	22	23			
$U(k)$	15	13	15	12	14	15			

#### 3.2.2. Nelder–Mead Method

##### • Algorithm Description

For a function of  $n$  variables, the algorithm maintains a set of  $n + 1$  points forming the vertices of a polytope in  $n$ -dimensional space. This method is often termed the “simplex” method, which should not be confused with the well-known simplex method for linear programming. The algorithm performs a transformation sequence to decrease the function values of each vertex. At each iteration, new function values are calculated for several points and compared with the function values of the previous iteration that are at the vertices. This process is complete when the simplex becomes sufficiently weak in a certain direction, or when the function values are sufficiently close in one direction (provided that  $f$  is continuous). The Nelder–Mead simplex typically requires one or two evaluations of the function at each iteration. It uses only two types of transformation to constitute a new simplex in each iteration:

- Reflection: far from the worst peak (the one with the highest value of the function).

- Shrinkage: toward the best vertex (the one with the smallest function value) [31]. In these transformations, the angles between the edges of all of the simplexes remain constant throughout the iterations so that the working simplex can change size, but not shape.

The following steps briefly summarize the Nelder–Mead algorithm [32]

1. Initial Simplex: Make the initial set of points usually constructed by generating  $n + 1$  vertices.
2. Repeat the following tasks until the termination test is satisfied:
  - Calculate the transformation for termination test (reflection and shrinkage).
  - If the termination test is not satisfied, recalculate the simplex of resolution.
3. Return the best solution of the current simplex  $S$  and the value of the associated function.

• **Numerical Results**

When we apply the Nelder–Mead method to the above numerical example, we obtain the following results:

- ✓ Total Cost = 315483.7 mu.
- ✓ Optimal production plan (see Table 4).
- ✓ Optimal number of preventive maintenance actions =  $N^* = 2$ .

**Table 4.** Optimal production plan yielded by the Nelder–Mead method.

$k$	0	1	2	3	4	5	6	7	8
$U(k)$	0	15	0	15	15	15	0	15	15
$k$	9	10	11	12	13	14	15	16	17
$U(k)$	15	15	0	0	0	15	15	15	15
$k$	18	19	20	21	22	23			
$U(k)$	15	15	15	0	0	15			

3.2.3. Differential Evolution Method

• **Algorithm Description**

The remarkable performance of the differential evolution method (DE) as a global optimization algorithm for continuous minimization problems has been extensively explored [33]. Three decisive control parameters are involved in DE. First, there is the mutation control parameter, which is a real and constant factor that controls the amplification of the differential variation. The next parameter is the crossover control factor; it controls which parameter contributes to which trial vector parameter in the crossover operation. Lastly, the population size is the number of population members. The algorithm maintains a population of  $m$  points,  $\{x_1, x_2 \dots x_j \dots x_m\}$ , as a population for each generation. The initial vector population is randomly generated from the entire parameter space by adding normally distributed random deviations to the nominal solution ( $x_{nom}$  of the initial generation). The operation called “mutation” is when the DE generates new parameter vectors by combining the weighted difference between two population vectors and a third vector. The mutated vector’s parameters are then associated with the parameters of another predetermined vector. Parameter mixing is often referred to as crossover operation. The last operation is called selection; each population vector has to serve one as the target vector so that NP (Non-deterministic Polynomial) competitions take place in one generation [34].

The process is assumed to have converged if the difference between the best function values in the new and old populations, as well as the distance between the new best point and the old best point, are less than the tolerances provided by the accuracy goal and the precision goal. DE is an evolutionary algorithm; it belongs to a class that also includes genetic algorithms and strategies [35].

• **Numerical Results**

When we apply the differential evolution method to the numerical example, we obtain the following results:

- ✓ Total Cost = 23379.85 mu.
- ✓ Optimal production plan (see Table 5).
- ✓ Optimal number of preventive maintenance actions =  $N^* = 2$ .

**Table 5.** Optimal production plan yielded by the differential evolution (DE) method.

<i>k</i>	0	1	2	3	4	5	6	7	8
<i>U(k)</i>	14	10	10	13	12	15	15	14	15
<i>k</i>	9	10	11	12	13	14	15	16	17
<i>U(k)</i>	15	14	15	14	12	14	14	14	14
<i>k</i>	18	19	20	21	22	23			
<i>U(k)</i>	13	15	14	14	13	14			

3.2.4. Simulated Annealing Method

• **Algorithm Description**

The idea of the algorithm is to perform a movement according to a probability distribution that depends on the quality of the various neighbors, with the best neighbors having a higher probability and the worst having a lower probability. A parameter (T) called the temperature is used. When T is high, all of the neighbors have approximately the same probability of being accepted. At low T, a movement, which degrades the cost function, has a low probability of being selected. A  $T = 0$ , no degradation of the cost function is accepted [36].

The following steps describe the simulated annealing algorithm:

1. Generate an initial solution  $S_0$  of S with  $S: =S_0$ . Set an initial temperature  $T = T_0$ .
2. Generate a random solution in the neighborhood of the current solution.
3. Compare the two solutions according to the criterion of metropolis.
4. Repeat 2 and 3 until the statistical stability is reached.
5. Decrease the temperature, and repeat until the system is frozen.

• **Numerical Results**

When we apply the simulated annealing method to the numerical example, we obtain the following results:

- ✓ Total Cost = 31,820.01 mu.
- ✓ Optimal production plan (see Table 6).
- ✓ Optimal number of preventive maintenance actions =  $N^* = 2$ .

**Table 6.** Optimal production plan yielded by the simulated annealing (SA) method.

<i>k</i>	0	1	2	3	4	5	6	7	8
<i>U(k)</i>	12	15	15	13	12	15	14	15	15
<i>k</i>	9	10	11	12	13	14	15	16	17
<i>U(k)</i>	15	15	9	13	15	9	15	13	15
<i>k</i>	18	19	20	21	22	23			
<i>U(k)</i>	15	15	15	15	2	15			

### 3.2.5. Random Search Method

- **AlgorithmDescription**

The last method used, random search, was first proposed by Anderson [37] and later by Rastrigin [38] and Karnopp [39]. Let  $f : R^n \rightarrow R$  be the fitness or cost function, and  $S$  be a subset of  $R^n$ . Let  $x \in S$  designate a position or candidate solution in the search space [40]. The random search method is described by the following steps:

1. Initialize  $x$  with a random position in the search space. Set  $k = 0$ .
2. Until a termination criterion is met (e.g., number of iterations performed, or adequate fitness reached), repeat the following:
  - Sample a new position  $x_k$  from the hypersphere of a given radius surrounding the current position  $x$ .
  - If  $f(x_k) < f(x)$ , then move to the new position by setting  $x = x_k$ .

- **Numerical Results**

When we apply the random search method to the numerical example, we obtain the following results:

- ✓ Total Cost = 386,743.62 mu.
- ✓ Optimal production plan (see Table 7).
- ✓ Optimal number of preventive maintenance actions =  $N^* = 2$ .

**Table 7.** Optimal production plan yielded by the random search method.

$k$	0	1	2	3	4	5	6	7	8
$U(k)$	12	12	11	5	11	4	13	10	10
$k$	9	10	11	12	13	14	15	16	17
$U(k)$	11	1	11	15	10	13	11	4	13
$k$	18	19	20	21	22	23			
$U(k)$	2	10	13	3	10	4			

Table 8 summarizes the results obtained for each method presented in this section. We calculate the relative gap between the best solution and the solution found from each used method. We note the relative gap as follow:

$$GAP = \frac{\text{HeuristicSolution} - \text{OptimalSolution}}{\text{OptimalSolution}}$$

**Table 8.** Comparative summary of the performance of the used methods.

Method	Total Cost (mu)	N*	Execution Time (s)	GAP
Exact Method	22,281.25	3	0.01	0
Nelder–Mead	315,483.7	2	12.93	13.16
Differential Evolution	23,379.85	2	9.18	0.046
Simulated Annealing	31,820.01	2	5.72	0.43
Random Search	386,743.62	2	404.67	16.36

Based on the results obtained (Table 7), using a medium data size, we observe that the differential evolution method provides the best production plan and the best number of preventive maintenance

actions compared with the other algorithms (simulated annealing, random search, and Nelder–Mead) in order to obtain the best cost with a minimal execution time that gives the minimal GAP. Therefore, our results echo what many types of research have observed. In fact, Lagarias [41] mentioned that Nelder–Mead is not a true global optimization algorithm, although, in practice, it gives good results for problems that do not have many local minima. Regarding the random search method, its significant inconvenience is that the execution time increases exponentially with the problem size [42]. On the other hand, the differential evolution algorithm is applied in solving complex optimization problems according to its effectiveness and efficiency and robustness in a wide variety of applications. The importance of DE is its ability to handle non-differentiable, nonlinear, and multimodal objective functions; it mostly converges to the optimal solution, and uses few control parameters. For simulated annealing, its main advantage is the ability to get out of a local minimum, based on an acceptance probability related to an exponential function, called transformation Gibbs-Boltzmann [43].

Based on this numerical example, we can conclude that if we need to increase the data size, the exact method may not lead to optimal solutions. Thus, we adopt the differential evolution method for our problem, since it gives the minimal GAP.

In the next section, we relax our problem by adding other constraints and another hypothesis. We tried to calculate the risk of loss profit using the results found by the different methods used in this section.

#### 4. Risk Assessment Study

Profit maximization is considered to be a main desired objective in the manufacturing sector. Moreover, since companies and their environments evolve dynamically over time, the inherent risk under which they must operate also changes [2]. In this section, we will tackle a production and maintenance optimization problem from a financial risk point of view. Our model considers a contract between a single operation manufacturing system and a customer, which calls for the former satisfying the latter's random demand for a single part type over a finite planning horizon. Anytime demand cannot be satisfied; a financial loss ensues corresponding to the risk of non-payment by the customer. We herein carry out an assessment of this risk, which is classified as low, medium, or high, and stems from different factors such as machine failures, time to repair, inventory shortages, transportation delays, etc. We will discuss risk assessment for two kinds of events, namely in regard to (i) the returned products, and (ii) the quality of preventive maintenance actions.

##### 4.1. Lost Profit Risk for Returned Products

Product returns give rise to high costs associated with inventory, transportation, handling, and warehousing. According to Shear et al. [44], inbound handling costs alone can reach \$50 per item, possibly triple outbound shipping costs. Hence, the management of product returns, often involving a high degree of uncertainty in terms of the return period and quantity of returned products, can be a competitive differentiator [45].

This subsection deals with the financial risk associated with the returned product, namely its impact on the loss of profit that is computed using each of the computational methods presented previously. There exists a risk where the product sold in period  $k$  comes back at period  $k + a$ ,  $a = 0, 1$ , or 2 (for the purpose of this specific case study). Let the returned quantity in production period  $k$  be denoted  $\delta(k) = d(k - a) \rho$ , where  $\rho$  is a random variable that follows a Bernoulli distribution,  $\rho \in [0, 1]$ .

Let be  $LRP_{RP}$  the lost profit risk of a returned product for the horizon  $H$  based on the computation method presented previously. We can assess LPR as the difference between the revenue generated by selling the optimal product quantity during the finite horizon,  $\sum_{k=0}^{H-1} U(k)$ , and the costs incurred as a result of the product being returned.

The assessment of LPR can be computed as follows, where  $g$  = unit sale price of the product.

The revenue:

$$G = g \times \left[ \sum_{k=0}^{H-1} U^*(k) + S_0 \right] \tag{14}$$

The quantity of returned products:

$$R_p = \sum_{k-a=0}^H \delta(k) \tag{15}$$

The loss:

$$L = g \times R_p \tag{16}$$

$$LRP_{RP} = \frac{loss}{gain} = \frac{L}{G} \tag{17}$$

#### 4.2. Lost Profit Risk for Imperfect Preventive Maintenance

In this section, we consider the case of machine failures and their subsequent repairs that cause lost production and perhaps lost sales during downtimes. Production reliability depends on the maintenance strategies; in other words, the quality of both corrective and preventive maintenance actions is essential for improving system availability.

In this section, we deal with lost profit risk in our assessment in the context of imperfect preventive maintenance. We assume that each preventive maintenance (PM) action restores the machine to between the “as good as new” state and the “as bad as old” state.

The failure rate  $\lambda(t)$  can be written as:

$$\lambda(t^\pm) = (1 - \alpha) \cdot \lambda(t) \tag{18}$$

where  $\alpha$  is a random variable that follows a Bernoulli distribution,  $\alpha \in [0, 1]$ .

Thus, based on Equation (12), the failure rate in the case of imperfect maintenance is expressed as follows:

$$\lambda_k(t) = \lambda_{k-1}(\Delta t) \left( 1 - \left[ \frac{k-1}{\left( \left\lfloor \frac{k-2}{T} \right\rfloor + 1 \right) T} \right] \right) + \frac{U_k}{U_{\max}} \lambda_n(t) + \left[ \frac{k-1}{\left( \left\lfloor \frac{k-2}{T} \right\rfloor + 1 \right) T} \right] \cdot (1 - \alpha) \cdot \lambda_T(\Delta t) \tag{19}$$

Let  $LRP_{PM}$  be the risk assessment for lost profit due to imperfect preventive maintenance based on the computation method that was presented previously.

Consider that  $\bar{A}(U, N)$  is the average number of failures during the horizon  $H$ , and  $B$  is the total downtime of the machine over the horizon. We moreover note that for each failure, we need  $b$  time units to repair the machine:

$$B = \bar{A}(U, N) \cdot b \tag{20}$$

With:

$$\bar{A}(U, N) = \sum_{i=0}^{N-1} \left[ \sum_{k=In(i \times \frac{T}{\Delta t})+1}^{In((i+1) \times \frac{T}{\Delta t})} \int_0^{\Delta t} \lambda_k(t) dt \right] + \sum_{k=N \times T}^{H \cdot \Delta t} \int_0^{\Delta t} \lambda_k(t) dt \tag{21}$$

Thus, the production quantity during  $(H - B)$  is equal to:

$$C = \frac{\sum_k U^*(k) \times (H - B)}{H} \tag{22}$$

Consequently, the loss of production is:

$$D = \sum_{k=0}^{H-1} U^*(k) - C \tag{23}$$

In this case, we can express the risk by the following equation:

$$LRP_{PM} = \frac{D}{\sum_{k=1}^{H-1} U^*(k)} \tag{24}$$

### 4.3. Comparative Analysis of Lost Profit Risk

The tables below present the comparative results for the key quantities derived in the foregoing subsection using the numerical example and different algorithms introduced in Section 3. For the calculation of returned product quantity, we take  $a = 1$ .  $\Delta t$ ,  $g = 4$  mu/product, and the probability distribution of  $\rho$  in Table 9.

**Table 9.** The probability distribution of  $\rho$  during the finite horizon.

$k$	0	1	2	3	4	5	6	7	8
$\rho_k$	0.51	0.47	0.67	0.021	0.996	0.039	0.456	0.722	0.41
$k$	9	10	11	12	13	14	15	16	17
$\rho_k$	0.271	0.259	0.045	0.562	0.50	0.31	0.129	0.064	0.33
$k$	18	19	20	21	22	23			
$\rho_k$	0.086	0.352	0.873	0.082	0.432	0.075			

Therefore, the quantity of returned product is  $R_p = 118$  products. Thus,  $L = 472$  mu.

For the lost profit risk of imperfect preventive maintenance, we take  $b = \frac{\Delta t}{4}$  and  $\alpha = 0.99$ .

The aim of calculating the lost profit risk of some uncertainties is to evaluate the economic risk and its impact on the system planning, and help decision-makers consider risk factors that have a big influence on the system disruption. In our case study, the risk of having returned products at each period gives a high risk (between 50–66% for all of the production plans with different computational methods) (Table 10), which means that if we take into consideration a quantity returned in each period, it will generate an important loss compared to the gain. This motivates decision-makers to find a solution to reduce these returns, or incorporate this uncertainty into the production planning.

For the imperfect maintenance LPR, we take a non-negligible period for failure machine reparation. We determine that if we take a period for reparation into consideration, the LPR is between 4.82% for our example (Table 11). This result shows that it will have a risk percentage of lost production quantity during the working horizon. The influence of these percentages vary from one decision-maker to another; some find that this loss is important compared to the gain, which they bring to find solutions that reduce these risks.

**Table 10.** Comparative financial losses using different computational methods for returned products.

	Nelder–Mead Method	Differential Evolution Method	Simulated Annealing Method	Random Search Method
$G$	1040	1388	1368	956
$LRP_{RP}$	45.38%	34%	34.5%	50.6%

**Table 11.** Comparative financial losses using different computational methods for imperfect preventive maintenance.

	Nelder–Mead Method	Differential Evolution Method	Simulated Annealing Method	Random Search Method
$C$	228,903	311,872	307,101	208,881
$D$	11.097	151,285	14.8988	10.1195
$LRP_{PM}$	4.62%	4.62%	4.62%	4.62%

## 5. Conclusions

In order to resolve the often-conflicting objectives of system reliability and profit maximization, an organization should establish appropriate maintenance guidelines that take into consideration costs associated with both production activities and equipment failures, the latter of which include e.g., costs due to lost production. In fact, there is no profit without risk. Therefore, decision-makers always try to study the effectiveness of their adopted strategies, and want to know how their decisions lead to a loss of profit. Companies seek to generate more profit, without having to regret their decision.

In this context, this research can serve as a decision support to quantify the profit loss following decisions made. Furthermore, in real life, decision-makers may involve this research in their work strategies. Each company has a system that helps them make predictions before the manufacturing process. The proposed models can be integrated into the employed systems, taking into account the impact of the computational methods used. The work proposes a set of mathematical tools that help evaluate decision-making in a random and risky environment, and can guide decision-makers in their choices. Therefore, it is important to have approaches and tools to help managers make judicious and efficient decisions in the face of this type of uncertainty.

The proposed approach aims to analyze the risk of profit–loss following a decision made on the manufacturing system under uncertainties. A single operation manufacturing system is considered, which produces a single product in order to satisfy a random multi-period demand over a finite horizon  $H$ . A mathematical model is presented to formulate the model. The first objective is to find the optimal production and maintenance plans with different optimization algorithms to deduce among the most appropriate computational algorithm for our problem. Based on the results of each method, the second objective of this study is to propose analytical models that evaluate the financial losses for some work strategies by using the optimal production and maintenance plans found by the presented methods. The goal is to evaluate the impact of the results of these computational methods on the calculation of profit loss for the two cases presented in the paper. The first case studied is the integration of an imperfect preventive maintenance policy. The second is the integration of a returned product strategy in random production periods. The implemented numerical examples show that the risk of profit–loss differs from one use of computational method to another. This shows that the use of an algorithm can influence profit and cause loss of profit.

Our work is a new contribution, which treats the impact of optimization algorithms on decisions made following incidents that may disrupt the systems. However, it is not exempt from limitations. In fact, in the literature, several approaches can solve the problem type proposed in the paper. However, this work deals with only four approximate methods, which are compared with the solution found by an exact method. The choice of these algorithms is based on the most used ones in such types of problems, and thus were easily adapted to our analytical model.

For future research, we will study the impact of the decisions made by decision-makers on various industrial problems. We will also study the impact of the decision on the optimization of assembly and disassembly systems, under different uncertainties.

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