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Inducing Strong Non-Linearities in a Phonon Trapping Quartz Bulk Acoustic Wave Resonator Coupled to a Superconducting Quantum Interference Device

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Abstract: A quartz Bulk Acoustic Wave resonator is designed to coherently trap phonons in such a way that they are well confined and immune to suspension losses so they exhibit extremely high acoustic Q -factors at low temperature, with $Q \times f$ products of order 10^{18} Hz. In this work we couple such a resonator to a Superconducting Quantum Interference Device (SQUID) amplifier and investigate effects in the strong signal regime. Both parallel and series connection topologies of the system are investigated. The study reveals significant non-Duffing response that is associated with the nonlinear characteristics of Josephson junctions. The nonlinearity provides quasi-periodic structure of the spectrum in both incident power and frequency. The result gives an insight into the open loop behaviour of a future Cryogenic Quartz Oscillator in the strong signal regime.

Keywords: BAW Resonator; SQUID; phonon trapping; low noise oscillator; non-linear coupling

1. Introduction

In the past ultra-stable low phase noise photonic frequency sources have been implemented for a range of applications including very high precision probes of fundamental physics, high precision oscillators as local oscillators for atomic clocks and advanced radar systems [1–4]. The best results at microwave and radio frequencies to date have been those derived from sapphire oscillators at room [3,4] and cryogenic temperatures [1,2,5] as well as lasers locked to optical cavities down converted through a frequency comb [6–10]. For example, such oscillators have been exploited to demonstrate possible violation of the Lorentz symmetry in the photon sector [1]. Though the photonic devices have extraordinary stability, they can be used to test only certain range of physical phenomena, and devices where the resonance conditions depend upon mechanical motion of matter can test totally different sectors including frequencies of the bulk elastic waves, which are sensitive to the photon-, electron-, proton- and the neutron-sector. Moreover, recently new oscillator systems with rival performance of pure photonic systems have been demonstrated involving high- Q frequency stable phonon systems. Such systems include but are not limited to, phonon lasers [11], optomechanical systems [12–14] and phonon induced Brillouin scattering devices [15–19].

Traditionally, the most stable oscillators based on acoustic resonance have been realised from technology based on Bulk Acoustic Wave (BAW) resonators, which trap phonons in a cavity in a similar way to a Fabry Perot cavity. These techniques have been perfected for decades allowing

precision room temperature oscillators and related devices, culminating in $Q \times f$ products as high 2×10^{13} Hz and oscillator frequency stability reaching below 10^{-13} between 1 and 10 s of integration time [20]. Only recently has this technology has been extended to cryogenic temperatures attaining $Q \times f$ products as high 2×10^{18} Hz. Thus, such cryogenic BAW systems shows great potential for use in applications that require precision control, measurement, and sensing at the quantum limit. This is mainly due to the relatively high mechanical frequencies and extremely high Q -factors achievable at cryogenic temperatures ($Q \approx 10^{10}$) [21] for frequencies ranging from a MHz to tens of GHz, beyond the capability of any other competing technology compared in Aspelmeier et al. and others [22–25]. Such high- Q devices with nonlinear properties are very valuable in many areas of physics. This includes quantum metrological applications where nonlinearities are employed to perform quantum state preparation [26]. In regards to creating low noise oscillators, nonlinear processes are used to create a limit cycle in feedback frequency to maintain stable output power.

The further improvement of BAW oscillators can only be achieved by cooling the resonators and reducing the resonator flicker phase self-noise, since this is the dominant noise source both at cryogenic and room temperature. The influence on the frequency stability of high-performance quartz oscillators on time scales of order 1–50 s is well-documented and it has been observed that the flicker self-noise decreases with decreasing power of the incident signal, and our recent results confirm that the resonators are thermal noise limited, and flicker-free without the carrier [27]. Thus, the noise in the quartz oscillator is dependent on power, with the white noise floor decreasing with power, while the flicker noise increases. The best quartz typically has frequency instabilities of better than 10^{-13} limited by flicker fluctuations. However at cryogenic temperatures the white noise floor is reduced by 40 dB, allowing a much lower oscillator power and a large reduction of the flicker noise so the increased Q -factor may be exploited. Assuming the typical phase noise of -130 dBc/Hz at 1 Hz Fourier frequency limited by the resonator self noise, the increase in Q -factor at 4K should see frequency instabilities as low as 2×10^{-16} [28], and if this can be pushed down further due to power optimisations it is strongly feasible to push the stability into the 10^{-17} regime. The other advantage at low temperatures, is the significantly reduced temperature coefficient, in the standard quartz BAW, the coefficient is annulled at around 1 K in operating temperature [29]. However, this value may be raised by varying the cut angle with respect to the anisotropy of quartz [30].

Although, in general all devices are nonlinear in nature, the feasibility to achieve nonlinear regimes are usually limited for a variety of reasons. For example, these regimes usually require considerably high amounts of incident power that may also cause nonlinear effects in auxiliary components, induce noise processes such as flicker noise, and cause the heating. So, in order to reduce this threshold of nonlinearity, it is required to reduce the system losses and increase nonlinear interactions. These requirement, in particular for BAW devices, usually contradict each other leading to trade offs. Another possible solutions is to utilise ultra high Quality factor resonators and couple them to a nonlinear superconducting circuit with a low threshold power. The former can thus provide very narrow spectral lines, while the latter adds the strong nonlinearities required at low powers. Thus, our goal to achieve better performance is to couple BAWs at low temperatures [27] to SQUID amplifiers [31] and operate them with low power oscillations of order -80 dBm. To further increase the power output, a chain of regular cryogenic ultra-low noise amplifiers will be used allowing us to generate reasonable power of order 10 dBm, with fractional frequency instabilities of 10^{-16} or better. In this work we make the first step towards this goal and show the large signal characteristics of a SQUID coupled to a BAW resonator. Similar SQUID-mechanical resonator systems have been used in the past as Gravity Wave detectors [32–34] and more recently as quantum hybrid systems [35,36] but only in the non-driven or weakly driven regimes.

2. Results: Towards a Cryogenic Quartz Oscillator

One of the major requirements for feedback oscillators is an existence of a nonlinear element used to create a limit cycle by transferring the main tone energy to higher harmonics. Despite the long

history of classical feedback oscillators, the latter problem has recently emerged again for designing of CQO. Typically a CQO operating around 7 K has an extremely high power dependence with a power coefficient of frequency of order 1 Hz μ W at -30 dBm of input power, about 1000 times higher than a room temperature oscillator [37]. A solution is to utilise an ultra high Quality factor resonator coupled to a nonlinear superconducting circuit to lower the threshold power. The BAW resonator can thus provide very narrow spectral lines, while the superconducting circuit adds the strong nonlinearities required at low powers. In this work we utilize a Superconducting Quantum Interference Device (SQUID) circuit [31] coupled to a high Q BAW Cavity [27].

The connection topologies involve a SQUID input coil, a two electrode BAW resonator and an external signal source. The parallel connection is realised when all three devices share the same voltage (see Figure 1A), and the series connection, when the three devices form a single current contour (see Figure 1B). These topologies may be understood as loaded feedback QCO with an open-loop. In both cases, the SQUID-BAW system is cooled down to 3.8 K with a conventional pulse-tube cryocooler. The acoustic resonator is an SC-cut [38] BVA (electrodeless) [39] quartz BAW device (1 mm thick, 30 mm diameter), whereas the SQUID is a commercial Niobium amplifier for which the current bias is applied across the loop. The SQUID amplifier does not have additional noise cancellation, and its flux bias is static. Bias parameters are tuned to maximise gain in the small signal case that is estimated to be 1.2 MOhm in terms of transimpedance relating input current and the output voltage.

Both devices have been used for Nyquist noise measurements at liquid helium temperatures [27]. The signal is fed through a long coaxial line ending at a -60 dB cold attenuator. The output signal of the cold part of the DC SQUID amplifier is retrieved via a micro-coaxial line, which is connected to and read out by a room temperature amplifier. All the data is acquired by a Vector Network Analyser locked to a Hydrogen maser providing extra frequency stability over long averaging times. Long averaging times are required to keep the measurement bandwidth as low as 3–10 Hz in order to avoid system ringing due to very high Quality factors.

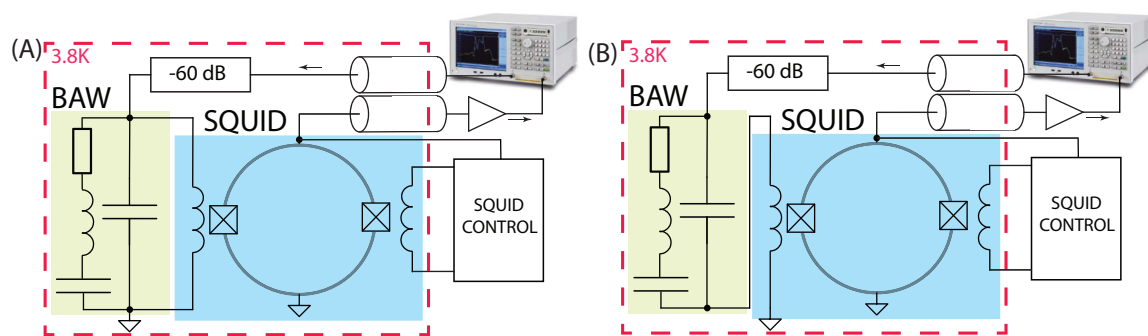


Figure 1. Experimental setups representing two ways to connect a Bulk Acoustic Wave (BAW) resonator and a Superconducting Quantum Interference Device (SQUID) amplifier: (A) parallel connection, (B) series connection.

The BAW resonator equivalent model may be represented by an equivalent circuit comprising a number of motional branches and a shunt capacitance. Each motional branch is a series connection of resistive, capacitive and inductive components. In this work we limit the investigation to a few low order modes that fall into the frequency range of the SQUID amplifier. The nonlinear response of mechanical resonators can be usually approximated by the Duffing model [40–43] arising from the nonlinear elastic terms of the constitutive equations or thermoelectroelasticity [44]. This holds true for the ultra-high Quality factor cryogenic BAW resonators under investigation in this work [24,45], with the exception of resonators which are not swept of impurities and as a result have a large amount [46].

The amplifier is a standard DC Niobium SQUID device with an nominal input inductance of 400 nH and the transfer coefficient of about $300 \mu\text{V}/\phi_0$. The experiment was repeated with two identical amplifiers with similar results. For each experiment the SQUID bias was kept in such a way that the system works on the linear part of its characteristic.

Figure 2 shows the amplitude and phase response of the parallel system as a function of the incident power and frequency of excitation in the vicinity of the 3rd overtone of the C bulk acoustic mode (4.993027 MHz), with the periodicity of the power dependence apparent and demonstrated further for three values of the detuning signal. These dependencies can not be described by the Duffing model used to describe the response of bare mechanical resonators and requires full trigonometric function representation. It is seen that in both frequency and power the system response is best described by chirp functions. The applied powers are too low to induce the resonator own nonlinearity, and the same type of response is observed for all low frequency BAW modes with the nonlinearity becoming apparent at different power levels depending on the mode Quality factor and motional resistance (which describes the electromechanical coupling). For the 3rd overtone of the C bulk acoustic mode, the Quality factor is 4.9×10^7 as found from Nyqvist noise spectrum [27].

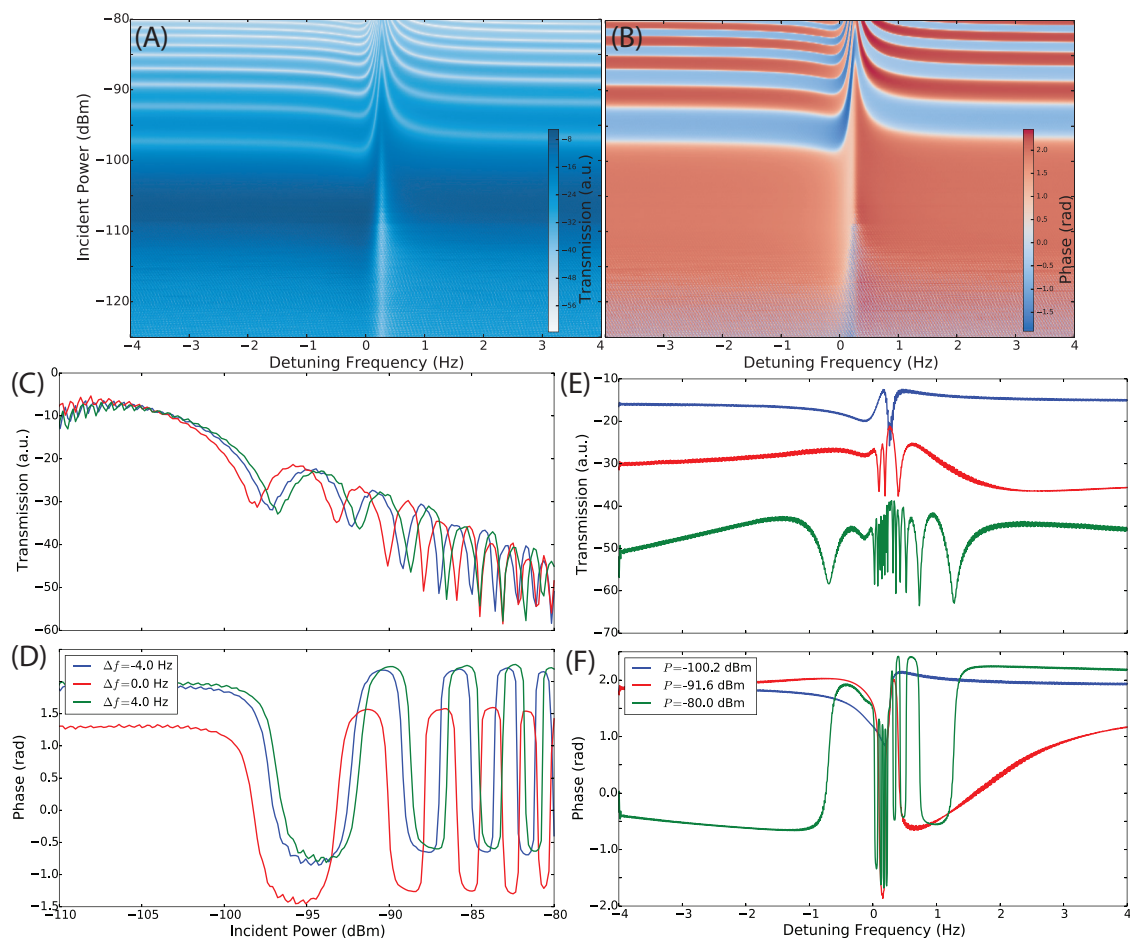


Figure 2. Top: Amplitude (A) and Phase (B) response of the parallel system as a function of the incident power in the vicinity of the 3rd overtone of the C bulk acoustic mode (4.993027 MHz). Bottom Left: Amplitude (C) and Phase (D) response as a function of power for three specific values of the detuning frequency from the data represented on the left. Bottom Right: Amplitude (E) and Phase (F) response for three specific values of the incident power from the data represented on the left.

Figure 3 shows the amplitude and phase response of the series system as a function of the incident power and frequency of excitation in the vicinity of the 5th overtone of the A (quasi-longitudinal) mode

(15.732444 MHz). In addition to quasi-periodic structure of the spectrum, the system demonstrates symmetry around the resonance frequency in the linear regime: mirror symmetry for the magnitude and diagonal symmetry for the phase. Also, similar to the parallel connection results, the intrinsic BAW nonlinearity is not observed. For this mode, the Quality factor is 27×10^7 obtained from Nyqvist noise spectrum [27].

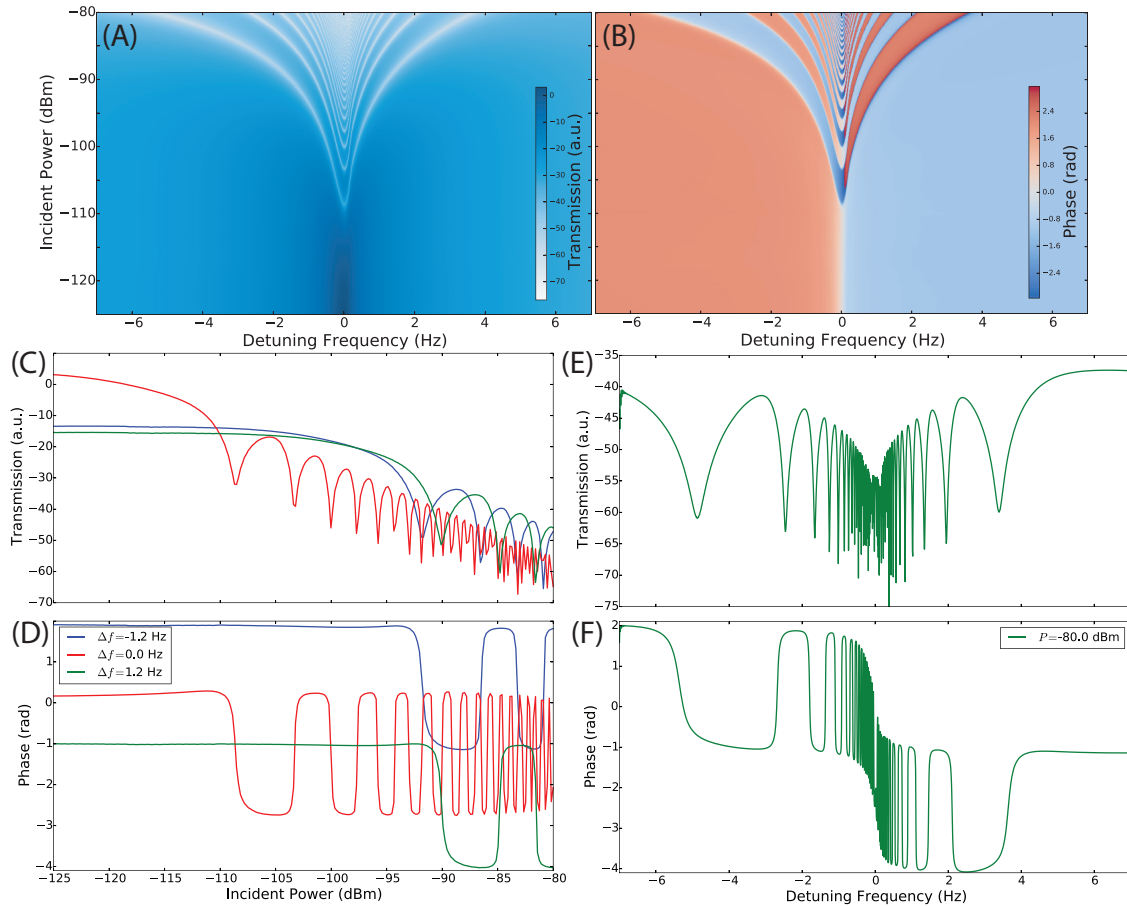


Figure 3. Top: Amplitude (A) and Phase (B) response of the series system as a function of the incident power in the vicinity of the 5th overtones of the A (quasi-longitudinal) mode (15.732444 MHz). Bottom Left: Amplitude (C) and Phase (D) response as a function of power for three specific values of the detuning frequency from the data represented on the left. Bottom Right: Amplitude (E) and Phase (F) response for three specific values of the incident power from the data represented on the left.

The system Hamiltonian (in the units with $\hbar = 1$) can be written based on the equations of motion:

$$H = \omega_m a^\dagger a + \omega_J \sum_i \left(\frac{q_i^2}{2} + \frac{\delta_i^2}{2} - \xi \cos \delta_i \right) - \omega_J \delta_1 \delta_2 - \omega_J (\phi_+ \delta_1 + \phi_- \delta_2) - g(a^\dagger + a)(\delta_1 - \delta_2) \quad (1)$$

where a ($a^\dagger$) is an annihilation (creation) operator for an acoustic mode, ω_m is an angular frequency of the mechanical mode, q_i and ϕ_i are conjugate variables for two SQUID branches, $\omega_J = 1/\sqrt{C_J L_J}$ is the Josephson junction plasma frequency, ϕ_+ and ϕ_- are properly scaled biasing current and flux, $\xi = \frac{I_0}{\phi_0} L_J$. The charge terms containing q_i may be removed since the effect of shunting capacitances is negligible at the working frequencies. It is also worth noting that none of the higher order modes of the resonator is an integer or rational multiple of the modes of interest. The deviation from being integer multiples of the fundamental mode for quartz resonators is due to the piezoelectric effect.

Nevertheless, direct simulation of this system is associated with certain difficulties. For example, low dimensional nonlinear physical models are often treated asymptotically using perturbation techniques, which utilize only one higher harmonic [47,48]. Though this technique is able to give adequate approximations for weakly nonlinear systems, in contrast systems with strong nonlinearities require complex numerical techniques such as the Harmonic Balance approach. The Harmonic Balance approach is a powerful technique to simulate the steady-state response of nonlinear systems in the frequency domain [49,50]. Generally, this numerical approach splits a system into a linear and nonlinear parts and requires representation of each variable in a series of harmonics. Whereas the linear part represents the dynamical response at each frequency, the nonlinear part mixes the harmonic components. As a result the system is represented by a set of nonlinear algebraic equations that are solved numerically. The more advanced versions of this methods have been implemented for solving nonlinear circuit problems [51,52]. These types of software for designing nonlinear electrical circuits may be used for further numerical system analysis and design of future related devices.

3. Discussion

Understanding of the nonlinearity of a SQUID amplifier coupled to a high-Q resonator is important for designing of a future Cryogenic Quartz Oscillators. The experiments presented in this work present results for open loop response of possible CQO topologies. Being a necessary condition for any feedback oscillator to create a limit cycle, the degree of nonlinearity controls the phase noise budget trade off. The strong low power nonlinearity keeps the oscillator circulating power low, which will typically reducing the flicker noise (the main limit in room temperature oscillators) but increasing the effect of thermal fluctuations due to the weaker signal to noise ratio of the acoustic resonance frequency determination. Weaker nonlinearity needs higher circulating power for saturation, thus minimising the thermal noise but inducing excess flicker noise [53]. For CQO, thermal noise is naturally reduced by orders of magnitude by keeping the temperature low, so the SQUID-BAW system gives a way to reduce the oscillator power significantly to keep flicker noise as low as possible. Thus, this work is an enabling step towards producing high frequency stability [54] beyond the limit achieved at room temperature devices [20] required for some tests of fundamental physics, e.g., the Lorentz invariance in the neutron sector [28].

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References

1. Nagel, M.; Parker, S.R.; Kovalchuk, E.V.; Stanwix, P.L.; Hartnett, J.G.; Ivanov, E.N.; Peters, A.; Tobar, M.E. Direct terrestrial test of Lorentz symmetry in electrodynamics to 10^{-18} . *Nat. Commun.* **2015**, *6*, 8174.
2. Abgrall, M.; Guéna, J.; Lours, M.; Santarelli, G.; Tobar, M.E.; Bize, S.; Grop, S.; Dubois, B.; Fluhr, C.; Giordano, V. High-Stability Comparison of Atomic Fountains Using Two Different Cryogenic Oscillators. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **2016**, *63*, 1198–1203.
3. Ivanov, E.N.; Tobar, M.E. Low phase-noise microwave oscillators with interferometric signal processing. *IEEE Trans. Microw. Theory Tech.* **2006**, *54*, 3284–3294.
4. Ivanov, E.N.; Tobar, M.E. Low phase-noise sapphire crystal microwave oscillators: Current status. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **2009**, *56*, 263–269.
5. Locke, C.R.; Ivanov, E.N.; Hartnett, J.G.; Stanwix, P.L.; Tobar, M.E. Invited Article: Design techniques and noise properties of ultrastable cryogenically cooled sapphire-dielectric resonator oscillators. *Rev. Sci. Instrum.* **2008**, *79*, 051301, doi:10.1063/1.2919944.
6. Millo, J.; Abgrall, M.; Lours, M.; English, E.M.L.; Jiang, H.; Guéna, J.; Clairon, A.; Tobar, M.E.; Bize, S.; Coq, Y.L.; et al. Ultralow noise microwave generation with fiber-based optical frequency comb and application to atomic fountain clock. *Appl. Phys. Lett.* **2009**, *94*, 141105, doi:10.1063/1.3112574.

7. Fortier, T.M.; Rolland, A.; Quinlan, F.; Baynes, F.N.; Metcalf, A.J.; Hati, A.; Ludlow, A.D.; Hinkley, N.; Shimizu, M.; Ishibashi, T.; et al. Optically referenced broadband electronic synthesizer with 15 digits of resolution. *Laser Photonics Rev.* **2016**, *10*, 780–790.
8. Baynes, F.N.; Quinlan, F.; Fortier, T.M.; Zhou, Q.; Beling, A.; Campbell, J.C.; Diddams, S.A. Attosecond timing in optical-to-electrical conversion. *Optica* **2015**, *2*, 141–146.
9. Diddams, S.A. The evolving optical frequency comb. *J. Opt. Soc. Am. B* **2010**, *27*, B51–B62.
10. Fortier, T.M.; Kirchner, M.S.; Quinlan, F.; Taylor, J.; Bergquist, J.C.; Rosenband, T.; Lemke, N.; Ludlow, A.; Jiang, Y.; Oates, C.W.; et al. Generation of ultrastable microwaves via optical frequency division. *Nat. Photonics* **2011**, *5*, 425.
11. Grudinin, I.S.; Lee, H.; Painter, O.; Vahala, K.J. Phonon Laser Action in a Tunable Two-Level System. *Phys. Rev. Lett.* **2010**, *104*, 083901.
12. Luan, X.; Huang, Y.; Li, Y.; McMillan, J.F.; Zheng, J.; Huang, S.W.; Hsieh, P.C.; Gu, T.; Wang, D.; Hati, A.; et al. An integrated low phase noise radiation-pressure-driven optomechanical oscillator chipset. *Sci. Rep.* **2014**, *4*, 6842.
13. Seok, H.; Wright, E.M.; Meystre, P. Dynamic stabilization of an optomechanical oscillator. *Phys. Rev. A* **2014**, *90*, 043840.
14. Hossein-Zadeh, M.; Vahala, K.J. Observation of injection locking in an optomechanical rf oscillator. *Appl. Phys. Lett.* **2008**, *93*, 191115.
15. Li, J.; Yi, X.; Lee, H.; Diddams, S.A.; Vahala, K.J. Electro-optical frequency division and stable microwave synthesis. *Science* **2014**, *345*, 309–313.
16. Tomes, M.; Carmon, T. Photonic Micro-Electromechanical Systems Vibrating at X-band (11-GHz) Rates. *Phys. Rev. Lett.* **2009**, *102*, 113601.
17. Grudinin, I.S.; Matsko, A.B.; Maleki, L. Brillouin Lasing with a CaF Whispering Gallery Mode Resonator. *Phys. Rev. Lett.* **2009**, *102*, 043902.
18. Lee, H.; Chen, T.; Li, J.; Yang, K.Y.; Jeon, S.; Painter, O.; Vahala, K.J. Chemically etched ultrahigh-Q wedge-resonator on a silicon chip. *Nat. Photonics* **2012**, *6*, 369–373.
19. Li, J.; Lee, H.; Chen, T.; Vahala, K.J. Characterization of a high coherence, Brillouin microcavity laser on silicon. *Opt. Express* **2012**, *20*, 20170–20180.
20. Salzenstein, P.; Kuna, A.; Sojdr, L.; Chauvin, J. Significant step in ultra-high stability quartz crystal oscillators. *Electron. Lett.* **2010**, *46*, 1433–1434.
21. Galliou, S.; Goryachev, M.; Bourquin, R.; Abbe, P.; Aubry, J.; Tobar, M. Extremely Low Loss Phonon-Trapping Cryogenic Acoustic Cavities for Future Physical Experiments. *Nat. Sci. Rep.* **2013**, *3*, 2132.
22. Aspelmeyer, M.; Kippenberg, T.J.; Marquardt, F. Cavity optomechanics. *Rev. Mod. Phys.* **2014**, *86*, 1391–1452.
23. Renninger, W.H.; Kharel, P.; Behunin, R.O.; Rakich, P.T. Bulk crystalline optomechanics. *arXiv* **2017**, arXiv:1703.08231.
24. Goryachev, M.; Creedon, D.; Galliou, S.; Tobar, M. Observation of Rayleigh phonon scattering through excitation of extremely high overtones in low-loss cryogenic acoustic cavities for hybrid quantum systems. *Phys. Rev. Lett.* **2013**, *111*, 085502.
25. Teufel, J.D.; Li, D.; Allman, M.S.; Cicak, K.; Sirois, A.J.; Whittaker, J.D.; Simmonds, R.W. Circuit cavity electromechanics in the strong-coupling regime. *Nature* **2011**, *471*, 204.
26. Hofheinz, M.; Wang, H.; Ansmann, M.; Bialczak, R.C.; Lucero, E.; Neeley, M.; O’Connell, A.D.; Sank, D.; Wenner, J.; Martinis, J.M.; et al. Synthesizing arbitrary quantum states in a superconducting resonator. *Nature* **2009**, *459*, 546–549.
27. Goryachev, M.; Ivanov, E.N.; van Kann, F.; Galliou, S.; Tobar, M.E. Observation of the fundamental Nyquist noise limit in an ultra-high Q-factor cryogenic bulk acoustic wave cavity. *Appl. Phys. Lett.* **2014**, *105*, 153505.
28. Lo, A.; Haslinger, P.; Mizrachi, E.; Anderegg, L.; Müller, H.; Hohensee, M.; Goryachev, M.; Tobar, M.E. Acoustic Tests of Lorentz Symmetry Using Quartz Oscillators. *Phys. Rev. X* **2016**, *6*, 011018.
29. Goryachev, M.; Creedon, D.; Ivanov, E.; Galliou, S.; Bourquin, R.; Tobar, M. Extremely low-loss acoustic phonons in a quartz bulk acoustic wave resonator at millikelvin temperature. *Appl. Phys. Lett.* **2012**, *100*, 243504.
30. Bon, J.; Galliou, S.; Bourquin, R. Temperature coefficients of crystalline-quartz elastic constants over the cryogenic range [4 K, 15 K]. *Appl. Phys. Lett.* **2016**, *109*, 231902, doi:10.1063/1.4971335.

31. Clarke, J.; Braginski, A. *The SQUID Handbook: Fundamentals and Technology of SQUIDS and SQUID Systems*; Wiley-VCH: Weinheim, Germany, 2004.
32. Xu, B.; Hamilton, W.O. Analysis of dc SQUID readout for a gravity wave detector using an inductive transducer. *J. Appl. Phys.* **1985**, *57*, 1729–1733.
33. Pleikies, J.; Usenko, O.; Kuit, K.; Flokstra, J.; De Waard, A.; Frossati, G. SQUID Developments for the Gravitational Wave Antenna MiniGRAIL. *IEEE Trans. Appl. Supercond.* **2007**, *17*, 764–767.
34. Goryachev, M.; Tobar, M.E. Gravitational wave detection with high frequency phonon trapping acoustic cavities. *Phys. Rev. D* **2014**, *90*, 102005.
35. O'Connell, A.D.; Hofheinz, M.; Ansmann, M.; Bialczak, R.C.; Lenander, M.; Lucero, E.; Neeley, M.; Sank, D.; Wang, H.; Weides, M.; et al. Quantum ground state and single-phonon control of a mechanical resonator. *Nature* **2010**, *464*, 697–703.
36. Woolley, M.J.; Emzir, M.F.; Milburn, G.J.; Jerger, M.; Goryachev, M.; Tobar, M.E.; Fedorov, A. Quartz-superconductor quantum electromechanical system. *Phys. Rev. B* **2016**, *93*, 224518.
37. Ivanov, E.N.; Goryachev, M.; Galliou, S.; Tobar, M.E. Pound Stabilized Cryogenic Bulk Acoustic Wave Resonator-Oscillator. In Proceedings of the 2018 IEEE Int'l Frequency Control Symposium (2018 IFCS), Lake Tahoe, CA, USA, 21–24 May 2018.
38. ANSI/IEEE. *IEEE Standard on Piezoelectricity*; Std 176-1987; Institute of Electrical and Electronics Engineers: New York, NY, USA, 1988.
39. Besson, R.; Peier, U. Further Advances on BVA Quartz Resonators. In Proceedings of the 34th Annual Symposium on Frequency Control, Philadelphia, PA, USA, 28–30 May 1980; pp. 175–182.
40. Gagnepain, J. Non Linear Mechanisms in Quartz Crystal Resonators. Ph.D. Thesis, University of Franche Comté, Besancon, France, 1972.
41. Gagnepain, J.; Besson, R. *Physical Acoustics*; Chapter Nonlinear Effects in Piezoelectric Quartz Crystals; Elsevier: New York, NY, USA, 2012; Volume 11, pp. 245–288.
42. Tiersten, H.; Stevens, D. An analysis of nonlinear resonance in contoured-quartz crystal resonators. *J. Acoust. Soc. Am.* **1986**, *80*, 1122–1132.
43. Nosek, J. Drive level dependence of the resonant frequency in BAW quartz resonators and his modeling. *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **1999**, *46*, 823–829.
44. Tiersten, H. On the nonlinear equations of thermoelectroelasticity. *Int. J. Eng. Sci.* **1971**, *9*, 587–604.
45. Goryachev, M.; Galliou, S.; Imbaud, J.; Bourquin, R.; Abbé, P. Recent investigations on BAW resonators at cryogenic temperatures. In Proceedings of the EFTF & IEEE IFCS Joint Meeting, San Francisco, CA, USA, 2–5 May 2011.
46. Goryachev, M.; Farr, W.; Ivanov, E.; Tobar, M. Anomalously strong nonlinearity of unswept quartz acoustic cavities at liquid helium temperatures. *J. Appl. Phys.* **2013**, *114*, 094506.
47. Krylov, N.; Bogolyubov, N. *Introduction to Non-Linear Mechanics*; Princeton University Press: Princeton, NJ, USA, 1947.
48. Nayfeh, A.; Mook, D. *Nonlinear Oscillations*; John Wiley and Sons: New York, NY, USA, 1979.
49. Nakhla, M.; Vlach, J. A piecewise harmonic balance technique for determination of periodic response of nonlinear systems. *IEEE Trans. Circuits Syst.* **1976**, *23*, 85–91.
50. Gilmore, R.J.; Steer, M.B. Nonlinear circuit analysis using the method of harmonic balance—A review of the art. Part I. Introductory concepts. *Int. J. Microw. Millim.-Wave Comput.-Aided Eng.* **1991**, *1*, 22–37.
51. Agilent Technologies, Inc. Harmonic Balance Simulation. 2011. Available online: http://edadownload.software.keysight.com/eedl/ads/2011_01/pdf/cktsimhb.pdf (accessed on 10 April 2018).
52. *Simulation and Analysis Guide*, AWR Design Environment v13.03 Edition; NI Company: El Segundo, CA, USA, 2017.
53. Gagnepain, J.J.; Oliver, M.; Walls, F.L. Excess Noise in Quartz Crystal Resonators. In Proceedings of the 37th Annual Symposium on Frequency Control, Philadelphia, PA, USA, 1–3 June 1983; pp. 218–225.
54. Goryachev, M.; Galliou, S.; Imbaud, J.; Abbé, P. Advances in development of quartz crystal oscillators at liquid helium temperatures. *Cryogenics* **2013**, *57*, 104–112.

