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# Entropy Generation in MHD Eyring–Powell Fluid Flow over an Unsteady Oscillatory Porous Stretching Surface under the Impact of Thermal Radiation and Heat Source/Sink

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Abstract: In this article, we have briefly examined the entropy generation in magnetohydrodynamic (MHD) Eyring–Powell fluid over an unsteady oscillating porous stretching sheet. The impact of thermal radiation and heat source/sink are taken in this investigation. The impact of embedded parameters on velocity function, temperature function, entropy generation rate, and Bejan number are deliberated through graphs, and discussed as well. By studying the entropy generation in magnetohydrodynamic Eyring–Powell fluid over an unsteady oscillating porous stretching sheet, the entropy generation rate is reduced with escalation in porosity, thermal radiation, and magnetic parameters, while increased with the escalation in Reynolds number. Also, the Bejan number is increased with the escalation in porosity and magnetic parameter, while increased with the escalation in thermal radiation parameter. The impact of skin fraction coefficient and local Nusselt number are discussed through tables. The partial differential equations are converted to ordinary differential equation with the help of similarity variables. The homotopy analysis method (HAM) is used for the solution of the problem. The results of this investigation agree, satisfactorily, with past studies.

**Keywords:** entropy; MHD; Eyring–Powell fluid; thermal radiation; porosity; oscillatory stretched sheet; HAM

# 1. Introduction

A non-Newtonian Fluid has exclusive features: it illustrates both the properties of liquid and solid, as the relationship between the shear stress and the shear rate becomes non-linear. In everyday life, industries, and technologies non-Newtonian fluids are used frequently. Non-Newtonian fluid flow problems in different dimensions, through a porous stretching sheet with heat transfer and magnetohydrodynamic effects, have plentiful and inclusive applications in several engineering and industrial sectors. They include heat exchanger design, glass blowing melt spinning, production of glass fibers, fiber and wire coating, industrialization of rubber and plastic sheets, etc. Eyring–Powell



fluid model is an interesting model of non-Newtonian fluids. The Eyring–Powell fluid model has much importance, but the most important feature of this model is that it diminishes to Newtonian behavior for high and low shear stress. Hayat et al. [1] investigate the Eyring–Powell fluid flow over a stretching surface with convective boundary condition. Patel et al. [2] numerically studied Eyring–Powell fluids. Dawar et al. [3] analytically examined an Eyring–Powell fluid under the influence of thermal radiation and a heat source/sink. The boundary layer and heat transfer of Eyring–Powell fluid over a continuously moving surface has been investigated by Jalil et al. [4], who observed many different problems. Over an inclined stretching sheet, Hayat et al. [5] examined the unsteady flow of Eyring–Powell fluid. Fluids, including Newtonian fluids and non-Newtonian fluids, are widely used in petroleum engineering, fuel-cell industry, and food industry. The fractal model for water flow through unsaturated porous rocks has been examined by Xiao et al. [6]. The perforation erosion impact on practical hydraulic fracturing has been studied by Gongbo Long and Guanshui Xu [7]. The analytical model for the transverse permeability of gas diffusion has proposed by Liang et al. [8]. The fractal model for relative permeability of gas diffusion in proton exchange membrane under the influence of pressure has been proposed by Xiao et al. [9]. Other related studies can be seen in [10,11].

Entropy is a disorder of a system and surrounding, for example, spin movements, molecular vibration and friction, kinetic energy, displacements of molecules, and others, due to which a loss of useful heat occurs and, thus, heat cannot transmit fully into work. Due to these additional movements, chaos in a system and its surroundings are created. For this, microscopic chaos results in macroscopic level chaos, which occurs because of some unnecessary irreversibilities. For example, mixing of fluids, electric resistance, unstained expansion, friction, chemical reaction, inelastic deformation of solids and unnecessary heat transmission in finite temperature difference. The entropy generation was originally formulated by Bejan [10]. Over an unsteady stretching surface, Sarojamma et al. [11] have examined the entropy generation on a thin film flow. Along with an inclined permeable surface, Soomro et al. [12] have recently examined, numerically, the entropy generation in MHD water-based CNTs. Mansour et al. [13] have been examined the entropy generation rate in a laminar viscous flow in a circular flow, and have deliberated that the entropy generation rate is high near the wall than that of the center of the pipe. For the stagnation point, Rashidi et al. [14] have been examined the flow in a porous medium through entropy generation. The related study about entropy generation can be seen in [15-17]. The flow of nanoparticles in a rotating system through entropy generation has been examined by Hayat et al. [18]. The flow of nanofluids with spherical heat source/sink through entropy generation has been examined by Nouri et al. [19]. Over a stretching surface, the flow of Jeffery nanofluids through entropy generation has been examined by Dalir et al. [20]. The unsteady squeezing flow of viscous fluid through entropy generation has been examined by Ahmed et al. [21]. Ishaq et al. [22] have studied the thin film flow of nanofluid under the influence of entropy generation over a time-dependent spreading surface. Other related important studies about entropy generation can be seen in [23–25]. Recently Shah et al. [25] investigated radiative Darcy–Forchheimer flow carbon nanotubes with microstructure and inertial characteristics. Shah et al. [26,27] have studied the Hall effect on micropolar nanofluid flow with radiative heat and mass transfer analysis. Khan et al. [28] have investigated Darcy–Forchheimer flow of micropolar nanofluid non-uniform heat generation/absorption.

Sheikholeslami [29–33] analyzed nanofluids and their applications using magnetic fields and porous media.

The aim of this work is to examine the entropy generation on MHD Eyring–Powell flow over an unsteady oscillatory stretching sheet under the impact of thermal radiation and a heat source/sink. The impact of different imbedding parameters are sketched through graphs and discussed. The analytical result for velocities and temperature profiles are obtained using the HAM technique [34–41]. In Section 2, the mathematical modeling of the problem, entropy generation, Bejan number, and solution by HAM, is presented. In Section 3, the results and discussion of the imbedded parameters are presented. The theme of this work is presented in Section 4.

## 2. Mathematical Modeling

Consider a two-dimensional (2-D) incompressible boundary layer flow of Eyring–Powell fluid over an oscillatory stretching sheet concurring with plane y. In this Cartesian coordinate system,  $\bar{x}$  is parallel to the stretching sheet, and  $\bar{y}$  is perpendicular to the stretching sheet. The stretching sheet is kept porous, and the flow is supposed in an unsteady state. The magnetic field is applied in the y direction. It is assumed that  $T_w > T_{\infty}$ , where  $T_w$  is the surface temperature and  $T_{\infty}$  is the temperature as the distance from the surface tends to infinity. The physical model of the problem is shown in Figure 1.



Figure 1. Geometrical representation of the problem.

The above-stated problem satisfies all the conditions [29].

$$\frac{\partial u}{\partial \overline{x}} + \frac{\partial v}{\partial \overline{y}} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial \overline{x}} + v \frac{\partial u}{\partial \overline{y}} = \left( v + \frac{1}{\rho \alpha C} \right) \frac{\partial^2 u}{\partial \overline{y}^2} - \frac{1}{2\rho \alpha C^3} \left( \frac{\partial u}{\partial \overline{y}} \right)^2 \frac{\partial^2 u}{\partial \overline{y}^2} - \left( \frac{v}{k} + \sigma B_0^2 \right) u(t), \tag{2}$$

$$\rho C_p \left( \frac{\partial \mathbf{T}}{\partial t} + u \frac{\partial \mathbf{T}}{\partial \overline{x}} + v \frac{\partial \mathbf{T}}{\partial \overline{y}} \right) = k \left( \frac{\partial^2 \mathbf{T}}{\partial \overline{y}^2} \right) - \left( \frac{\partial q_r}{\partial \overline{y}} \right) + \frac{Q_r}{\rho C_p} (\mathbf{T} - \mathbf{T}_{\infty}). \tag{3}$$

In Equations (1)–(3), u and v are the velocity components in the direction of  $\overline{x}$  and  $\overline{y}$ , respectively. Also, v is the kinematic viscosity,  $\sigma$  is the electrical conductivity,  $\alpha$  and C are the fluid materials,  $\rho$  is the density,  $C_p$  is the specific heat, k is the thermal conductivity,  $Q_r$  is the heat source, and  $q_r$  is the radiative heat flux.  $q_r$  is defined as

$$q_r = -\frac{4\sigma *}{3k*} \frac{\partial T^4}{\partial \overline{y}},\tag{4}$$

where  $\sigma * (\text{Stefan-Boltzmann constant})$  and k \* (absorption coefficient). Expending T<sup>4</sup> by Taylor series expansion, we obtained [28]

$$T^{4} = T_{\infty}^{4} + 4T_{\infty}^{3}(T - T_{\infty}) + 6T_{\infty}^{2}(T - T_{\infty})^{2} + \dots$$
(5)

Neglecting higher terms from Equation (5) and substituting in Equation (3), we have

$$\rho C_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial \overline{x}} + v \frac{\partial T}{\partial \overline{y}} \right) = \left( k + \frac{16\sigma * T_\infty^3}{3k*} \right) \frac{\partial^2 T}{\partial \overline{y}^2} + \frac{Q_r}{\rho C_p} (T - T_\infty).$$
(6)

With the following boundary conditions

$$u = U_w = c\overline{x}\sin\xi t, \ v = 0, \ T = T_w \text{ at } \overline{y} = 0, \ t > 0, u \to 0, \ T \to T_\infty \text{ at } y \to \infty,$$
(7)

we introduce the following dimensionless variables for the non-dimensionalization of the flow problem.

$$y = \left(\frac{c}{\nu}\right)^{\frac{1}{2}} \overline{y}, \ \tau = t\xi, \ u = c\overline{x}f_y(y,\tau), \ v = -(\nu c)^{\frac{1}{2}}f(y,\tau), \ g(y,\tau) = \frac{T - T_{\infty}}{T_w - T_{\infty}}.$$
(8)

An influential tool in fluid mechanics is the thought of dimensional analysis and scaling laws; by looking at the physical properties present in a system, we may estimate their size and hence which, for example, might be neglected. In some cases, the system may not have a fixed natural length scale (timescale) while the solution depends on space (time). It is then necessary to construct a length scale (timescale) using space (time) and the other dimensional quantities. In a study of partial differential equations, particularly fluid dynamics, similarity variables is a form of solution which is similar to itself if the independent and dependent variables are appropriately scaled.

In the observation of above defined dimensionless variables, Equation (1) satisfied (2) and (6), and can be written as

$$(1+P)f''' - Df'' - (f')^{2} + ff'' - \lambda P(f'')^{2}f''' - (\beta + M)f' = 0,$$
(9)

$$\left(1+\frac{4}{3}K_r\right)g'' + \Pr\left(fg' - Dg'\right) - \varepsilon g' = 0, \tag{10}$$

with the following boundary conditions

$$f'(0,\tau) = \sin\tau, \ f(0,\tau) = 0, \ g(0,\tau) = 1, \ f'(\infty,\tau) = 0, \ g(\infty,\tau) = 0.$$
(11)

In the above equations,  $P = \frac{1}{\mu\alpha C}$  and  $\lambda = \frac{\overline{x}^2 c^3}{2\nu C^2}$  are the fluid parameters,  $\beta = \frac{\nu}{kc}$  is the porosity parameter,  $D = \frac{\xi}{c}$  depicts the oscillating frequency,  $M = \frac{\sigma B_0}{c}$  is the magnetic field,  $\varepsilon = \frac{\nu Q_r}{kc\rho C_p}$  is the heat source/sink,  $\Pr = \frac{\mu C_p}{k}$  is the Prandtl number, and  $K_r = \frac{4\sigma * T_{\infty}^3 T}{kk*}$  is the radiation parameter. Equation (11) is subject to the constraints  $\lambda P << 1$ .

#### 2.1. Physical Quantities of Interest

For engineering interest, the skin fraction coefficient  $C_f$  and local Nusselt number Nu is defined as

$$C_f = \frac{\tau_w}{\rho U_w^2}, \ Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \ q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}.$$
 (12)

In observation of Equation (9), Equation (12) can be written as

$$\operatorname{Re}_{x}^{\frac{1}{2}}Cf_{x} = (1+P)f'' - \frac{P}{3}\alpha(f''(0)), \quad \operatorname{Re}_{x}^{\frac{1}{2}}Nu_{x} = -\left(1 + \frac{4}{3}K_{r}\right)g'(0).$$
(13)

#### 2.2. Entropy Generation and Bejan Number

For the above-stated problem, the local entropy generation rate can be defined as [10]

$$N_G = \frac{k}{T_{\infty}^2} \left( 1 + \frac{4}{3} \frac{4\sigma * T^3}{k * k_f} \right) \left\{ \left( \frac{\partial T}{\partial y} \right)^2 \right\} + \frac{1}{T_{\infty}} \left( \frac{\nu}{k} + \sigma B_0^2 \right) u^2.$$
(14)

Executing Equation (8), the above equation becomes

$$N_G = \operatorname{Re}\left(1 + \frac{4}{3}K_r\right)g^{\prime 2} + \frac{\operatorname{Re}}{\mho}B_r(\beta + M)f^{\prime 2},\tag{15}$$

where  $\text{Re} = \text{T}_{\infty}^2 L^2$  is the Reynolds number,  $B_r = \frac{U_w^2}{k(T_{\infty} - T_w)}$  is the Brinkman number, and  $\mho = \frac{T_{\infty} - T_w}{T_w}$  is the dimensionless temperature.

The Bejan number is defined as

$$Be = \frac{\left(1 + \frac{4}{3}K_r\right){g'}^2}{\left(1 + \frac{4}{3}K_r\right){g'}^2 + \frac{\text{Re}}{\mho}B_r(\beta + M){f'}^2}.$$
(16)

#### 2.3. Solution by HAM

Homotopy analysis method was introduced by Liao [30–32] for the first time. He used one of the elementary ideas of topology, called homotopy, to derive this method. He used two homotopic functions in the derivation of this technique. The functions are called homotopic function when one of them can be continuously distorted into another. Assume that  $Z_1$ ,  $Z_2$  are two functions which are continuous, and X, Y are two topological spaces where  $Z_1 \& Z_2$  map from X to Y, then  $Z_1$  is said to be homotopic to  $Z_2$  if there is a continuous function  $\Re$ ,

$$\Re: X \times [0, 1] \to Y. \tag{17}$$

Thus,  $x \in X$ , then

$$\Re[x, 0] = Z_1(x) \text{ and } \Re[x, 1] = Z_2(x).$$
 (18)

The mapping  $\Re$  is called homotopic.

In order to solve Equations (9) and (10) with the boundary conditions Equation (11), we use HAM [38–41] with the succeeding process.

The initial suppositions are chosen as follows:

$$f_0(\varphi) = 1 - e^{-\varphi} \sin \varphi, \ g_0(\varphi) = e^{-\varphi}.$$
 (19)

The linear operators are taken as  $L_f$  and  $L_g$ :

$$L_f(f) = f''' - f', \ L_g(g) = g'' - g,$$
(20)

which have the following succeeding properties:

$$L_f(n_1 + n_2 e^{-\varphi} + n_3 e^{\varphi}) = 0, \ L_g(n_4 e^{-\varphi} + n_5 e^{\varphi}) = 0, \tag{21}$$

where  $n_i(i = 1 - 5)$  are constants.

The resultant non-linear operators,  $NL_f$  and  $NL_g$ , are specified as

$$NL_{f}[f(\varphi; \mho)] = (1+P)\frac{\partial^{3}f(\varphi; \mho)}{\partial \varphi^{3}} - D\frac{\partial^{2}f(\varphi; \mho)}{\partial \varphi^{2}} - \left(\frac{\partial f(\varphi; \mho)}{\partial \varphi}\right)^{2} + f(\varphi; \mho)\frac{\partial^{2}f(\varphi; \mho)}{\partial \varphi^{2}} - \lambda P\left(\frac{\partial^{2}f(\varphi; \mho)}{\partial \varphi^{2}}\right)^{2} - (\beta+M)\frac{\partial f(\varphi; \mho)}{\partial \varphi}$$
(22)

$$NL_{g}[g(\varphi; \mho)] = \left(1 + \frac{4}{3}K_{r}\right)\frac{\partial^{2}g(\varphi; \mho)}{\partial\varphi^{2}} + \Pr\left(f(\varphi; \mho)\frac{\partial g(\varphi; \mho)}{\partial\varphi} - D\frac{\partial g(\varphi; \mho)}{\partial\varphi}\right) - \varepsilon g(\varphi; \mho).$$
(23)

The zero-order problems from Equations (9) and (10) are

$$(1 - \Omega)L_f[f(\varphi; \mho) - f_0(\varphi)] = \Omega h_f NL_f[f(\varphi; \mho)], (1 - \Omega)L_g[g(\varphi; \mho) - g_0(\varphi)] = \Omega h_g NL_g[f(\varphi; \mho), g(\varphi; \mho)]$$
(24)

The equivalent boundary conditions are

$$\begin{aligned} f(\varphi;\Omega)|_{\varphi=0} &= 0, \ \left. \frac{\partial f(\varphi;\Omega)}{\partial \varphi} \right|_{\varphi=0} = \sin \tau, \ \left. \frac{\partial f(\varphi;\Omega)}{\partial \varphi} \right|_{\varphi\to\infty} = 0. \\ g(\varphi;\Omega)|_{\varphi=0} &= 1, \ g(\varphi;\Omega)|_{\varphi\to\infty} = 0, \end{aligned}$$
(25)

When  $\Omega = 0$  and  $\Omega = 1$ , we have

$$f(\varphi; 1) = f(\varphi) \text{ and } g(\varphi; 1) = g(\varphi).$$
 (26)

Expanding  $f(\varphi; \Omega)$  and  $g(\varphi; \Omega)$  by Taylor series,

$$f(\varphi; \Omega) = f_0(\varphi) + \sum_{q=1}^{\infty} f_q(\varphi) \Omega^q$$
  

$$g(\varphi; \Omega) = g_0(\varphi) + \sum_{q=1}^{\infty} g_q(\varphi) \Omega^q$$
(27)

where

$$f_q(\varphi) = \frac{1}{q!} \frac{\partial f(\varphi; \Omega)}{\partial \varphi} \bigg|_{\Omega=0} \text{ and } g_q(\varphi) = \frac{1}{q!} \frac{\partial g(\varphi; \Omega)}{\partial \varphi} \bigg|_{\Omega=0}.$$
 (28)

Setting  $\Omega = 1$  in (28), we obtain

$$f(\varphi) = f_0(\varphi) + \sum_{q=1}^{\infty} f_q(\varphi),$$
  

$$g(\varphi) = g_0(\varphi) + \sum_{q=1}^{\infty} g_q(\varphi).$$
(29)

The *q*th-order problem satisfies the following:

$$L_f [f_q(\varphi) - \chi_q f_{q-1}(\varphi)] = h_f U_q^f(\varphi)$$

$$L_g [g_q(\varphi) - \chi_q g_{q-1}(\varphi)] = h_g U_q^g(\varphi)$$
(30)

with the conditions

$$f_q(0) = f'_q(0) = f'_q(\infty) = 0, g_q(0) = g_q(\infty) = 0.$$
(31)

Here,

$$U_{q}^{f}(\varphi) = (1+P)f_{q-1}^{'''} - Df_{q-1}^{''} - \sum_{k=0}^{q-1} f_{q-1-k}^{\prime} f_{k}^{\prime} + \sum_{k=0}^{q-1} f_{q-1-k} f_{k}^{''} - \lambda P \sum_{k=0}^{q-1} f_{q-1-k}^{''} \sum_{j=0}^{k} f_{k-j}^{''} f_{j}^{'''} - (\beta+M)f_{q-1}^{\prime},$$
(32)

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$$U_{q}^{g}(\varphi) = (1+K_{r})g_{q-1}'' + \Pr\left[\sum_{k=0}^{q-1}F_{q-1-k}g_{k}' - Dg_{q-1}'\right] - \varepsilon g_{q-1}.$$
(33)

where

$$\chi_q = \begin{cases} 0, \text{ if } \Omega \leq 1\\ 1, \text{ if } \Omega > 1 \end{cases}$$
(34)

the overall homotopic series solutions in general form are specified as

$$f_{i}(\eta) = \hat{f}_{i}(\eta) + n_{1} + n_{2}e^{-\varphi}y + n_{3}e^{\varphi}y, g_{i}(\eta) = \hat{g}_{i}(\eta) + n_{4}e^{-\varphi} + n_{5}e^{\varphi}y$$
(35)

# 2.4. HAM Convergence

Whenever we calculate the series solution of the velocity function and temperature function by using HAM, the parameters  $h_f$  and  $h_g$ , which are called assisting parameters, appear. The function of these parameters is to adjust the convergence of these solutions. At the 5th order approximation, the *h*-curves of f''(0) and g'(0) are plotted in Figures 2 and 3, respectively. The convergence region of the velocity function is  $-0.8 \le h_f \le -0.1$ , and the convergence region of the temperature function is  $-1.0 \le h_g \le -0.5$ .



**Figure 3.**  $\hbar$ -curve for g'(0).

## 3. Results and Discussion

#### 3.1. Tables Discussion

Tables 1 and 2 depict the influence of various parameters on skin fraction coefficient  $Cf_x$  and local Nusselt number  $Nu_x$ . These tables express the best agreement with our previous study. The impacts of emerging parameters on skin fraction coefficient are presented in Table 1. From the tabulated values, we see that the skin fraction coefficient reduces with the escalation in  $\beta$ , P, and  $\lambda$ . The impacts of emerging parameters on local Nusselt number is presented in Table 2. The local Nusselt number reduces with the escalation in  $\varepsilon$  and  $K_r$ .

**Table 1.** Numerical values for skin fraction coefficient  $Cf_x$ , when D = 0.5,  $\alpha = 1.0$ , M = 0.008, at time instant  $\tau = \frac{\pi}{2}$ .

β	Р	λ	Previous Result Ref. [3].	Present Study
0.5			-1.29447	-1.29876
0.7			-1.39927	-1.40331
0.9			-1.50053	-1.50452
1.1	1.0		-1.59913	-1.60303
	1.3		-1.68426	-1.68834
	1.5		-1.73747	-1.74168
	1.7	0.5	-1.78810	-1.79324
		0.6	-1.81316	-1.81795
		0.7	-1.84069	-1.84533

**Table 2.** Numerical values for heat flux  $Nu_x$ , when D = 0.5,  $\alpha = 1$ , 0.1, M = 0.008, at time instant  $\tau = \frac{\pi}{2}$ .

Pr	ε	K <sub>r</sub>	Previous Results, Ref [3].	Present Study
1.0			1.82770	1.82748
1.2			1.80574	1.80200
1.4			1.78404	1.77763
1.6	2.5		1.76259	1.75229
	2.6		1.79740	1.79024
	2.7		1.83133	1.82557
	2.8	0.3	1.85965	1.85951
		0.4	1.97571	1.97496
		0.5	2.08693	2.08544

#### 3.2. Graphical Discussion

In this section, we have discussed the influences of different embedded parameters and dimensionless numbers on velocity function  $f'(\varphi)$ , temperature function  $g(\varphi)$ , entropy generation rate  $N_G$ , and Bejan number *Be*. These embedded parameters and dimensionless numbers are oscillating frequency *D*, porosity parameter  $\beta$ , fluid parameters *P* and  $\lambda$ , magnetic parameter *M*, heat source/sink  $\varepsilon$ , Prandtl number Pr, rotation parameter  $K_r$ , and Reynolds number Re. To comprehend the influence of these parameters and dimensionless numbers, Figures 4–19 are schemed.

Figure 4 displays the influence of oscillating frequency *D* on velocity function  $f'(\varphi)$ . It is observed that augmented values of *D* increased the flow motion. Actually, the higher value of oscillating frequency *D* increases the kinetic energy of fluid molecules which result in increases in the flow motion. The impact of porosity parameter  $\beta$  on velocity function  $f'(\varphi)$  is shown in Figure 5, which has a dominating effect on the flow motion. Generally, the porosity creates resistance in the flow path, and declines the velocity of the flow motion. In fact, growing values of  $\beta$  show the large number of

porous spaces, which create resistance in the flow path and reduce overall fluid motion. Basically, with the increase, the number of holes in the porous plates are increased. The nanoliquid particle aspect hurdles in, flowing over these holes. Hence, it is obvious that the increasing values of  $\beta$  reduce the velocity function  $f'(\varphi)$ .



**Figure 4.** Influence of *D* on  $f'(\varphi)$ , when P = 0.1,  $\lambda = 0.3$ ,  $\beta = 0.4$ , M = 0.5, sin  $\tau = 0.6$ .



**Figure 5.** Influence of  $\beta$  on  $f'(\phi)$ , when P = 0.1, D = 0.2,  $\lambda = 0.3$ , M = 0.5,  $\sin \tau = 0.6$ .

Figures 6 and 7 depict the influence of fluid parameters P and  $\lambda$  on velocity function  $f'(\varphi)$ , respectively. It is evident that the higher values of fluid parameters raise the velocity profile. Figure 8 shows the impact of magnetic field M on velocity function  $f'(\varphi)$ . Lorentz force theory says that the magnetic field has a reverse effect on velocity function. Hence, the higher values of M reduce  $f'(\varphi)$ . This important effect of M on velocity profile  $f'(\varphi)$  is because of the fact that the increases in the M movements, or the friction force, is named the Lorentz force. It has the affinity to reduce the fluid velocity in the boundary sheet. Figure 9 shows the impact of heat source/sink  $\varepsilon$  on temperature function  $g(\varphi)$ . Generally, the heat source/sink performs like a heat generator, which releases heat to the flow of fluid. Therefore, the enhancement in  $\varepsilon$  improves the temperature field  $g(\varphi)$ . In addition, this helps to grow the thickness of the boundary layer. Figure 10 demonstrates the impact of oscillating frequency D on temperature function  $g(\varphi)$ . Generally, the high oscillating frequency reduces the temperature function much more. Hence, the increasing values of D reduce  $g(\varphi)$ . Figure 11 demonstrates the impact of radiation parameter  $K_r$  on temperature function  $g(\varphi)$ . Thermal radiation has a leading role

in heat transmission when the coefficient of convection heat transmission is small. The enhancement in  $K_r$  improves  $g(\varphi)$ . Actually, when we increase thermal radiation parameter Rd, then it is apparent that it enhances the temperature in the boundary layer area in the fluid layer. This increase leads to a drop in the rate of cooling for nanofluid flow. Figure 12 shows the impact of Prandtl number Pr on temperature function  $g(\varphi)$ . Physically, the nanofluids have a large thermal diffusivity with small Pr, but this effect is reversed for higher Pr, therefore, the temperature of liquid shows a decreasing behavior. Physically, the fluids having a small number of Pr have a larger thermal diffusivity, and this effect is opposite for higher Prandtl numbers. Due to this fact, a large value of Pr causes the thermal boundary layer to drop. Figures 13 and 14 demonstrate the influence of porosity parameter  $\beta$  on the entropy generation rate  $N_G$  and Bejan number *Be*, respectively. From these figures, we observe that the porosity parameter  $\beta$  has a reversed impact on  $N_G$  and Be. Figures 15 and 16 demonstrate the impact of magnetic parameter M on generation rate  $N_G$  and Bejan number Be, respectively. Here, M has a reversed impact on  $N_G$  and Be. That is, the enhancement in magnetic parameter reduces  $N_G$  and Be. Figures 17 and 18 are plotted to describe the impact of radiation parameter  $K_r$  on generation rate  $N_G$ and Bejan number Be, respectively. From Figure 17, we observed that the increase in  $K_r$  reduces  $N_G$ , while Figure 18 depicts the reverse impact on Be. Figure 19 demonstrates the impact of (Re) on  $N_G$ . From this figure, we observed that the increasing Reynolds number increases the entropy generation.



**Figure 6.** Influence of  $\lambda$  on  $f'(\varphi)$ , when P = 0.1, D = 0.2,  $\beta = 0.4$ , M = 0.5, sin  $\tau = 0.6$ .



**Figure 7.** Influence of *P* on  $f'(\varphi)$ , when D = 0.2,  $\lambda = 0.3$ ,  $\beta = 0.4$ , M = 0.5,  $\sin \tau = 0.6$ .



**Figure 8.** Influence of *M* on  $f'(\varphi)$ , when P = 0.1, D = 0.2,  $\lambda = 0.3$ ,  $\beta = 0.4$ , sin  $\tau = 0.6$ .



**Figure 9.** Influence of  $\varepsilon$  on  $g(\varphi)$ , when  $K_r = 0.1$ , Pr = 0.2, D = 0.3.



**Figure 10.** Influence of *D* on  $g(\varphi)$ , when  $K_r = 0.1$ , Pr = 0.2,  $\varepsilon = 0.4$ .



**Figure 11.** Influence of  $K_r$  on  $g(\varphi)$ , when Pr = 0.2, D = 0.3,  $\varepsilon = 0.4$ .



**Figure 12.** Influence of Pr on  $g(\varphi)$ , when  $K_r = 0.1$ , D = 0.3,  $\varepsilon = 0.4$ .



**Figure 13.** Influence of  $\beta$  on  $N_G$ , when Re = 0.1,  $K_r = 0.2$ ,  $\mho = 0.3$ ,  $B_r = 0.4$ , M = 0.6.



**Figure 14.** Influence of  $\beta$  on *Be*, when  $K_r = 0.2$ ,  $\Im = 0.3$ ,  $B_r = 0.4$ , M = 0.6.



**Figure 15.** Influence of *M* on *N*<sub>*G*</sub>, when Re = 0.1,  $K_r = 0.2$ ,  $\mho = 0.3$ ,  $B_r = 0.4$ ,  $\beta = 0.5$ .



**Figure 16.** Influence of *M* on *Be*, when  $K_r = 0.2$ ,  $\mho = 0.3$ ,  $B_r = 0.4$ ,  $\beta = 0.5$ .



**Figure 17.** Influence of  $K_r$  on  $N_G$ , when Re = 0.1,  $\mho = 0.3$ ,  $B_r = 0.4$ ,  $\beta = 0.5$ , M = 0.6.



**Figure 18.** Influence of  $K_r$  on Be, when  $\mho = 0.3$ ,  $B_r = 0.4$ ,  $\beta = 0.5$ , M = 0.6.



**Figure 19.** Influence of Re on  $N_G$ , when  $K_r = 0.2$ ,  $\Im = 0.3$ ,  $B_r = 0.4$ ,  $\beta = 0.5$ , M = 0.6.

# 4. Conclusions

In this article, we investigated the entropy generation on MHD Eyring–Powell fluid over an unsteady oscillatory porous stretching sheet. The impact of thermal radiation and heat source/sink is taken into account. Also, this article is compared with one from our previous study, and found to agree satisfactorily. The concluding remarks of this study are listed below:

- The velocity function reduces with the enhancement in magnetic field and porosity parameter, and escalates with the enhancement in oscillating frequency and fluid parameters.
- The temperature function reduces with the enhancement in oscillating frequency and Prandtl number, and escalates with the enhancement in heat source/sink and radiation parameter.
- The entropy generation rate reduces with the escalation in porosity parameter, thermal radiation, magnetic field, and escalates with the enhancement in Reynolds number.
- The Bejan number reduces with the enhancement in porosity parameter and magnetic field, and increases with the enhancement in thermal radiation parameter.

**Author Contributions:** S.O.A. and A.D. modelled the problem and with physical sketch. Z.S. and W.K. introduced the similarity transformation and transformed the modeled problem into dimensionless form. M.I. and S.I. solved the problem via homotopy analysis method (HAM). I.K. and S.O.A. computed the results numerically. All the authors equally contributed in writing and revising the manuscript.

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# Nomenclature

В	magnetic field $(NmA^{-1})$
B <sub>r</sub>	Brinkman number
$C_p$	specific heat $\left(\frac{J}{kgK}\right)$
C <sub>f</sub>	skin friction coefficient
Ď	oscillating frequency (Hz)
k	thermal conductivity ( $Wm^{-1}K^{-1}$ )
k*	absorption coefficient
K <sub>r</sub>	radiation parameter
Μ	magnetic parameter
$Nu_x$	Nusselt number
Р	fluid parameter
Pr	Prandtl number
Qr	heat source
$q_r$	heat flux $(Wm^{-2})$
Т	fluid temperature (K)
$T_{\infty}$	infinity temperature (K)
$T_w$	surface temperature (K)
и, v	velocity components $(ms^{-1})$
$\overline{x}, \overline{y}$	coordinates
Χ,Υ	topological spaces
$Z_1, Z_2$	homotopic functions
<b>Greek Letters</b>	
α	fluid material
ν	kinematic viscosity,
ρ	fluid density $\left( \text{Kgm}^{-3} \right)$
σ	electrical conductivity of fluid $\left( Sm^{-1} \right)$

- $\sigma^*$  Stefan–Boltzmann constant h assisting parameter
- λ fluid parameter
- $\beta$  porosity parameter
- $\varepsilon$  heat source/sink
- ΰ dimensionless temperature

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