

Article

Magnetohydrodynamic Nanofluid Natural Convection in a Cavity under Thermal Radiation and Shape Factor of Nanoparticles Impacts: A Numerical Study Using CVFEM

Ali J. Chamkha ^{1,2}, A.S. Dogonchi ^{3,*} and D.D. Ganji ⁴

¹ Mechanical Engineering Department, Prince Sultan Endowment for Energy and Environment, Prince Mohammad Bin Fahd University, Al-Khobar 31952, Saudi Arabia; achamkha@pmu.edu.sa

² RAK Research and Innovation Center, American University of Ras Al Khaimah, P.O. Box 10021, Ras Al Khaimah, UAE

³ Department of Mechanical Engineering, Aliabad Katoul Branch, Islamic Azad University, Aliabad Katoul, Iran

⁴ Mechanical Engineering Department, Babol Noshirvani University of Technology, Babol 484, Iran; mirgang@nit.ac.ir

* Correspondence: a.s.dogonchi@stu.nit.ac.ir

Received: 8 November 2018; Accepted: 23 November 2018; Published: 26 November 2018



Abstract: In this study, the natural convection of a magnetohydrodynamic nanofluid in an enclosure under the effects of thermal radiation and the shape factor of nanoparticles was analyzed numerically using the control-volume-based finite element method (CVFEM). Columns, spheres, and lamina are examples of the nanoparticle shapes used in the investigation. The study of nanofluid flow and heat transfer was accomplished with an extensive range of nanofluid volume fractions, radiation parameters, Hartmann numbers, Rayleigh numbers, and nanoparticle shape factors. Also, the correlation between the average Nusselt number and the influencing parameters of the current study was determined. The findings demonstrate that laminar nanoparticles have a more notable impact on the average and local Nusselt numbers than the other nanoparticle shapes.

Keywords: nanofluid; CVFEM; natural convection; shape factor of nanoparticles; thermal radiation; Nusselt number

1. Introduction

The inherently moderate thermal conductivity of regular fluids, such as water, oil, etc., causes a deterioration in the performance of engineering systems. To overcome this barrier, researchers add metallic nanoscale particles—owing to the fact that they have a higher thermal conductivity than regular fluids—to these fluids. This combination is called a nanofluid. Nanofluids have higher thermal conductivity in comparison with regular fluids. Most recently, a vast number of studies have been carried out concerning heat transfer and the flow of nanofluids [1–24]. Entropy generation on a magnetohydrodynamic nanofluid toward a stagnation-point flow over a penetrable expanding wall was investigated by Bhatti and Rashidi [1]. They observed the increasing velocity of the fluid due to the pronounced effects of the magnetic field and the porosity parameter. Moreover, they noted that the entropy generation number increased with increasing Brinkman number and Reynolds number. Entropy generation for a non-Newtonian Eyring–Powell nanofluid through a penetrable expanding wall was analyzed by Bhatti et al. [2]. They deduced that the velocity profile rises owing to the high impact of the suction parameter. Moreover, they concluded that Brownian motion and thermophoresis

raised the temperature profile remarkably. The impacts of the flow of an electromagnetohydrodynamic nanofluid over a Riga sheet were analyzed by Abbas et al. [3]. The convective Poiseuille boundary-layer flow of an ethylene glycol (C₂H₆O₂)-based nanofluid with suspended aluminum oxide (Al₂O₃) nanoparticles through a porous wavy channel was investigated by Zeeshan et al. [4]. In the presence of thermal radiation, the heat transfer and flow of a magnetohydrodynamic Go-water nanofluid between parallel surfaces were examined by Dogonchi et al. [5]. They determined that the Nusselt number and temperature profile both increased as the radiation parameter decreased and the solid volume fraction increased. Dogonchi and Ganji [6] examined nanofluid flow, mass, and heat transfer between non-parallel surfaces subject to a magnetic field. They concluded that concentration and temperature profiles along with the Nusselt number go up as the Schmidt number rises.

Recently, several researchers have considered the natural convection flow of a pure fluid and nanofluid in enclosures [25–31] due to the extensive scientific and engineering applications. Heat transfer and fluid flow in a wavy enclosure subject to a porous medium and nanoparticles were explored by Sheremet et al. [25]. They illustrated that an increasing Rayleigh number causes a secondary vortex and temperature stratification core in the left bottom corner and center of the cavity, respectively. In a wavy enclosure containing nanoparticles and a porous medium, natural convection and the influence of thermal dispersion was studied by Sheremet et al. [26]. They determined that the heat transfer was enhanced with the Rayleigh number, undulation number, and dispersion parameter. Sheremet et al. [27] analyzed entropy generation and nanofluid flow in an enclosure. The outcomes indicate that a higher wavenumber leads to greater mean entropy generation. Numerical and experimental investigations on natural convection in a cavity were conducted by Mahmoudinezhad et al. [28].

This concise review illustrates that it is still worthwhile to allocate more effort to the study of natural convection in enclosures using a combination of metallic nanoscale particles and normal fluids. Therefore, this work analyzed magnetohydrodynamic nanofluid natural convection in an enclosure and the impact of the shape factor of the nanoparticles and thermal radiation using control-volume finite element method (CVFEM) and also obtained a correlation with the average Nusselt number. The flow and heat transfer effects of diverse control parameters were explored.

2. Problem Description

The natural convection of a magnetohydrodynamic nanofluid in an enclosure subject to the effects of thermal radiation and the shape factor of nanoparticles is analyzed. A sample grid distribution and a physical model of this work are displayed in Figure 1. We assume the nanofluid to be Newtonian and incompressible, while we assume that the flow is laminar. Under the above-mentioned suppositions and the Boussinesq approximation, the energy, momentum, and continuity equations for the current problem can be defined as:

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0 \quad (1)$$

$$v \frac{\partial u}{\partial y} + u \frac{\partial u}{\partial x} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2} \right) \quad (2)$$

$$v \frac{\partial v}{\partial y} + u \frac{\partial v}{\partial x} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}} \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} \right) + \beta_{nf} g (T - T_c) - \frac{\sigma_f B_0^2}{\rho_{nf}} v \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho C_p)_{nf}} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - \frac{1}{(\rho C_p)_{nf}} \left(\frac{\partial q_{x,rad.}}{\partial y} + \frac{\partial q_{y,rad.}}{\partial x} \right) \quad (4)$$

In these equations v , u , B_0 , P , σ_f , T , and q_{rad} denote the velocity in the y -direction, velocity in the x -direction, magnetic field, pressure, electric conductivity, and temperature, respectively, with the last variable representing radiative heat flux.

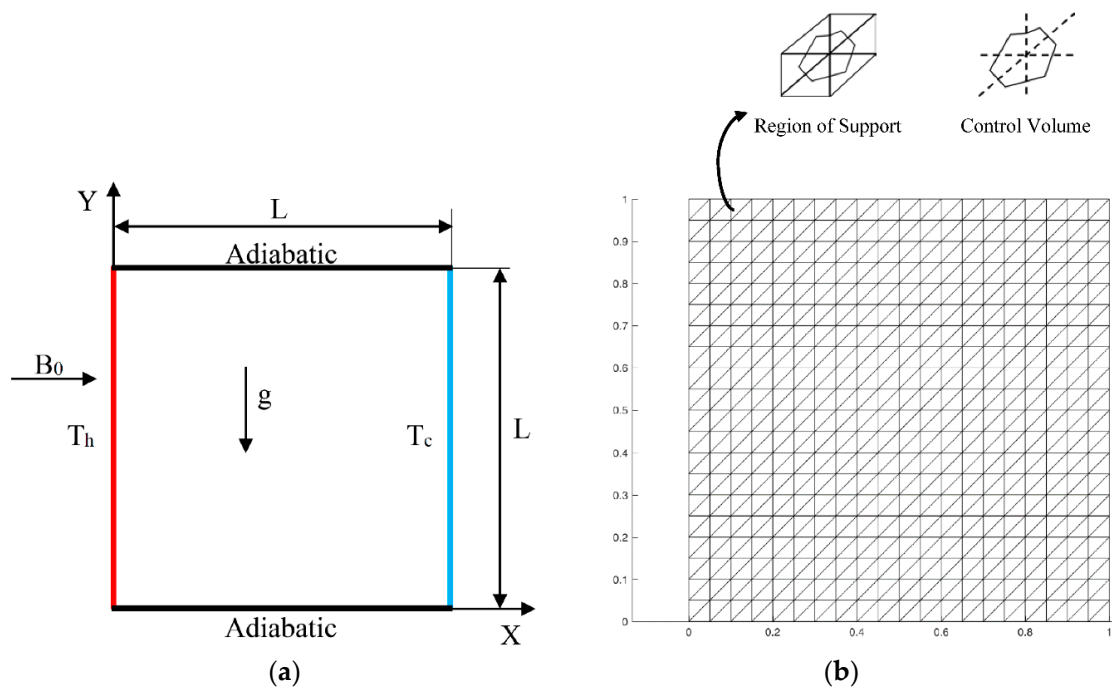


Figure 1. (a) Physical model and coordinate system (b) grid distribution.

For radiation, by applying the Rosseland approximation, we have:

$$q_{x,rad.} = -\left(4\sigma^*/3k_{nf}^*\right)\frac{\partial T^4}{\partial x}, \quad q_{y,rad.} = -\left(4\sigma^*/3k_{nf}^*\right)\frac{\partial T^4}{\partial y} \quad (5)$$

$$T^4 \cong -3T_\infty^4 + 4T_\infty^3 T$$

So, Equation (4) has been altered to

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho C_p)_{nf}} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{16\sigma^* T_\infty^3}{3k_{nf}^* (\rho C_p)_{nf}} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (6)$$

ρ_{nf} , μ_{nf} , $(\rho C_p)_{nf}$ are defined as follows:

$$\rho_{nf} = \phi \rho_s + (1 - \phi) \rho_f, \quad \mu_{nf} = (1 - \phi)^{-2.5} \mu_f, \quad (\rho C_p)_{nf} = \phi (\rho C_p)_s + (1 - \phi) (\rho C_p)_f \quad (7)$$

Considering the influences of nanoparticle shape, the nanofluid's thermal conductivity (k_{nf}) is characterized as follows:

$$\frac{k_{nf}}{k_f} = \frac{(m - 1)\phi(k_s - k_f) + k_s - (1 - m)k_f}{k_s - (1 - m)k_f - \phi(k_s - k_f)} \quad (8)$$

The shape factor is represented here as m . The shape factor for diverse particle shapes is illustrated in Table 1 [32]. Further, the thermophysical features of the nanofluid are listed in Table 2 [15].

Table 1. Shape factor values for various nanoparticle shapes [32].

Particle Shapes	Sphere	Column	Lamina
m	3	6.3698	16.1576

Table 2. Thermophysical properties of water and nanoparticle [15].

	$\rho(\text{kg/m}^3)$	$C_p(\text{J/kgK})$	$k(\text{W/mK})$
Copper (Cu)	8933	385	401
Pure water	997.1	4179	0.613

The vorticity and stream function are stated as:

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}, \omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad (9)$$

We define the following dimensionless variables:

$$Y = \frac{y}{L}, X = \frac{x}{L}, \Omega = \frac{\omega L^2}{\alpha_f}, \Psi = \frac{\psi}{\alpha_f}, V = \frac{uL}{\alpha_f}, U = \frac{vL}{\alpha_f}, \theta = \frac{T - T_c}{T_h - T_c} \quad (10)$$

Subject to Equation (14), the governing equations are changed to the following non-dimensional forms:

$$\frac{\partial \Omega}{\partial X} \frac{\partial \Psi}{\partial Y} - \frac{\partial \Psi}{\partial X} \frac{\partial \Omega}{\partial Y} = \frac{\mu_{nf}}{\mu_f} \frac{\rho_f}{\rho_{nf}} \text{Pr} \left(\frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} \right) + \frac{\beta_{nf}}{\beta_f} \text{RaPr} \frac{\partial \theta}{\partial X} - \frac{\rho_f}{\rho_{nf}} \text{Ha}^2 \text{Pr} \frac{\partial V}{\partial X} \quad (11)$$

$$\frac{\partial \Psi}{\partial Y} \frac{\partial \theta}{\partial X} - \frac{\partial \Psi}{\partial X} \frac{\partial \theta}{\partial Y} = \frac{k_{nf}}{k_f} \frac{(\rho C_p)_f}{(\rho C_p)_{nf}} \left(1 + \frac{4}{3} N \right) \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (12)$$

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -\Omega \quad (13)$$

Subject to the boundary conditions:

$$\begin{aligned} \theta &= 1 && \text{left wall} \\ \theta &= 0 && \text{right wall} \\ \frac{\partial \theta}{\partial Y} &= 0 && \text{top and bottom walls} \\ \Psi &= 0 && \text{all walls} \end{aligned} \quad (14)$$

In these equations, Pr, Ra, N, and Ha denote the Prandtl number, Rayleigh number, radiation parameter, and Hartmann number, respectively, which are defined as:

$$\begin{aligned} \text{Pr} &= \nu_f / \alpha_f, \text{Ra} = g \beta_f (T_h - T_c) L^3 / \alpha_f \nu_f, N = 4 \sigma^* T_\infty^* / k_{nf} k_{nf}^* \\ \text{Ha} &= B_0 L \sqrt{\sigma_f / \mu_f} \end{aligned}$$

Along the hot wall, the local and average Nusselt number can be determined by:

$$Nu_{loc.} = \frac{k_{nf}}{k_f} \left(1 + \frac{4}{3} N \right) \frac{\partial \theta}{\partial X}, Nu_{ave.} = \int_0^1 Nu_{loc.} dY \quad (15)$$

3. Numerical Solution and Validation

We solved Equations (11)–(13) subject to the conditions in Equation (14) via the control-volume-based finite element method (CVFEM) [33–36]. To ensure mesh independence, Nu_{ave} was obtained for diverse mesh sizes. Table 3 indicates that a grid of 141×141 should be applied to the present problem. Further, we measured our FORTRAN code outcomes against other works in the literature to prove the numerical solution under study [37,38]. They are in excellent accordance, as is exhibited in Table 4.

Table 3. Impact of grid size on Nu_{avg} when $Ra = 10^5$, $Ha = 50$, $\phi = 0.04$, $N = 4$ and $m = 16.1576$.

Grid Dimension (X × Y)	Nu_{avg}
61 × 61	8.892393
81 × 81	8.848909
101 × 101	8.821577
121 × 121	8.802795
141 × 141	8.789614
161 × 161	8.784837

Table 4. Comparison between present results and other works for Nu_{avg} .

Ra	Present Work	Khanafer et al. [37]	De Vahl Davis [38]
10^3	1.1307	1.118	1.118
10^4	2.2674	2.245	2.243
10^5	4.5851	4.522	4.519
10^6	8.8341	8.826	8.799

4. Results and Discussion

In this research, a numerical study was carried out to analyze the influences of the Hartmann number ($Ha = 0$ –50), Rayleigh number ($Ra = 10^3$, 10^4 , and 10^5), radiation parameter ($N = 1$ –3), volume fraction of the nanofluid ($\phi = 0$ –0.04), and shape factor ($m = 3$, 6.3698, and 16.1576) on magnetohydrodynamic nanofluid natural convection in a cavity. The influences of thermal radiation and the shape factor of nanoparticles were determined.

The impacts of the Ra and Ha on isotherms and streamlines are illustrated in Figures 2 and 3, respectively. As shown by the increasing Ra , the stream function's maximum absolute value increases; hence, the flow moves more quickly in the cavity. This phenomenon is due to the intensity of the natural convection, which has a direct relationship with the stream function's maximum absolute value. On the other hand, a higher Ra leads to a distortion of the isotherms and consequently causes the vertical stratification of isotherms, particularly at lower Ha and higher Ra . That is, the thermal boundary layer develops along two side walls. This is because, at a higher Ra and a lower Ha , the prevailing mode of heat transfer is convection. Moreover, the power of natural convection and, consequently, the stream function's maximum absolute value both diminish as Ha goes up. In addition, as Ha goes up, the central streamline expands, and the core's temperature stratification diminishes. So, the two walls' thermal boundary layers will disappear. In fact, in this case, the isotherms are parallel to each other. Since an increasing Ha leads to a decreased buoyancy force, the conduction mode of heat transfer dominates over the convection mode, especially at higher Ha .

Figure 4 demonstrates the influences of N and ϕ on the local and average Nusselt number. Based on Equation (15), the multiplication of three terms, i.e., (k_{nf}/k_f) , $((4/3)N + 1)$, and $\partial\theta/\partial X$, will create a non-dimensional number called the local Nusselt number. By increasing ϕ , the growth of the ratio (k_{nf}/k_f) will overcome a smaller $\partial\theta/\partial X$ for a specified N , so the local Nusselt number will rise as ϕ is increased. Conversely, by increasing N , the increased $((4/3)N + 1)$ term will counteract the reduced value of $\partial\theta/\partial X$ for a given ϕ , which will result in a higher Nu_{loc} . However, the average Nu number behaves similarly to the local Nu number.

Figures 5 and 6 illustrate the impacts of Ra , Ha , and m (sphere, column, and lamina) on the local and average Nusselt number, respectively. According to Equation (8) and Table 1, for a specified nanofluid (for example, Cu –water), as the shape factor increases, (k_{nf}/k_f) also increases. For instance, for a Cu –water nanofluid with $\phi = 2\%$, the spherical nanoparticle ($m = 3$) results in the lowest thermal conductivity rate ($k_{nf}/k_f = 1.0609$), while the laminar nanoparticle ($m = 16.1576$) yields the highest thermal conductivity rate ($k_{nf}/k_f = 1.3216$) of the studied various nanoparticle shapes in this work. Hence, based on Equation (15), by increasing the shape factor, the increased ratio (k_{nf}/k_f) will overcome the reduction in $\partial\theta/\partial X$ for a specified ϕ , so the local Nusselt number increases with an increased

shape factor. One can easily surmise, then, that both the local and average Nusselt numbers increase with increasing Ra and decreasing Ha. These fluctuations of Nu_{ave} and Nu_{loc} with Ha and Ra can be explained as follows: lower values of Ra lead to a weakened buoyancy force in the system. So, in this case, the conduction mode of heat transfer governs the system. However, the convection mode dominates over the conduction mode at higher Ra because of the increased buoyancy force. Furthermore, introducing a magnetic field can lessen the power of the buoyancy force in the cavity. So, at higher Ha, the conduction mode will govern the system.

Finally, we defined the relationship for the average Nusselt number and the influencing parameters in the current study. The relationship is defined as follows:

$$\begin{aligned}
 Nu_{ave} = & 2.18015 + 5.79302 \times 10^{-5} \times Ra - 0.067176 \times Ha \\
 & + 1.37548 \times N + 6.56049 \times \phi - 1.34977 \times 10^{-6} \times Ra \times Ha \\
 & + 6.23829 \times 10^{-6} \times Ra \times N - 0.024939 \times Ha \times N \\
 & + 1.49480 \times 10^{-3} \times Ha^2
 \end{aligned} \tag{16}$$

This correlation's R-squared equals 0.9927. The above correlation is valid for $Ha = 10\text{--}30$, $\phi = 0.01\text{--}0.03$, $N = 1\text{--}3$, and $m = 3$.

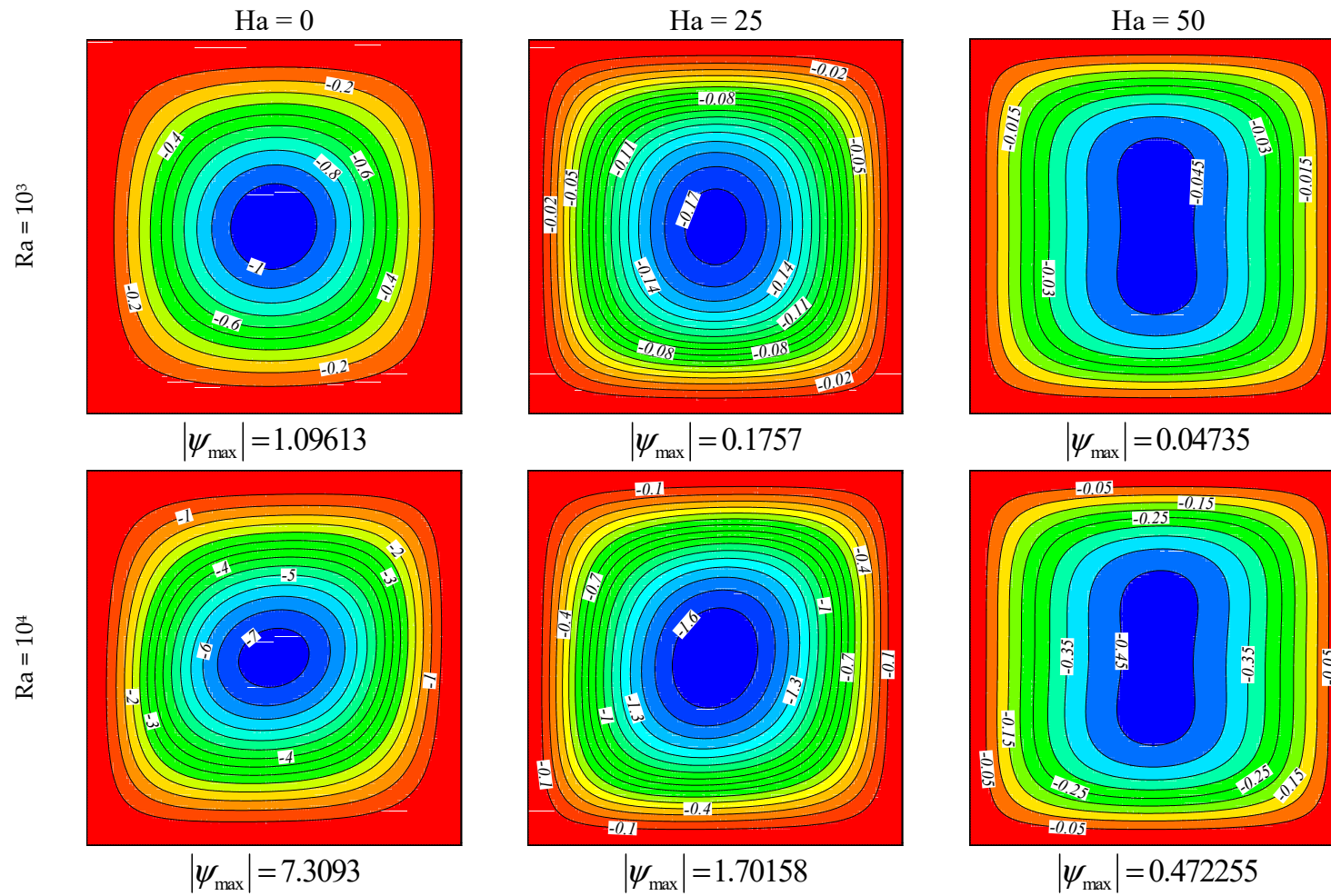


Figure 2. Cont.

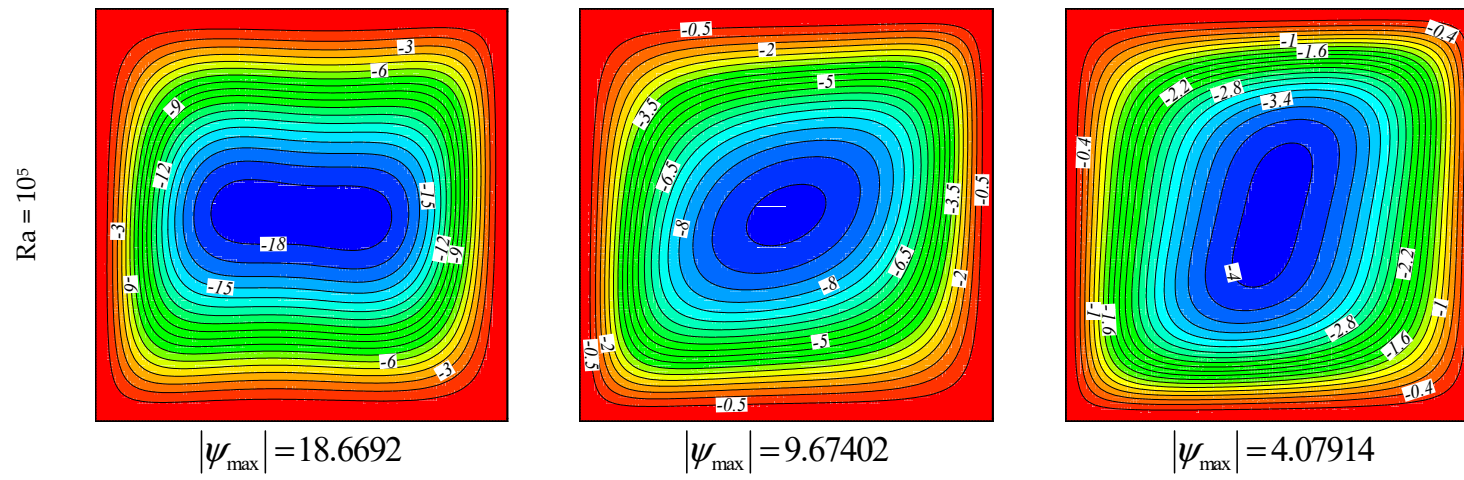


Figure 2. Streamline contours for different values of Rayleigh and Hartmann numbers when $N = 1$, $m = 3$, and $\phi = 2\%$.

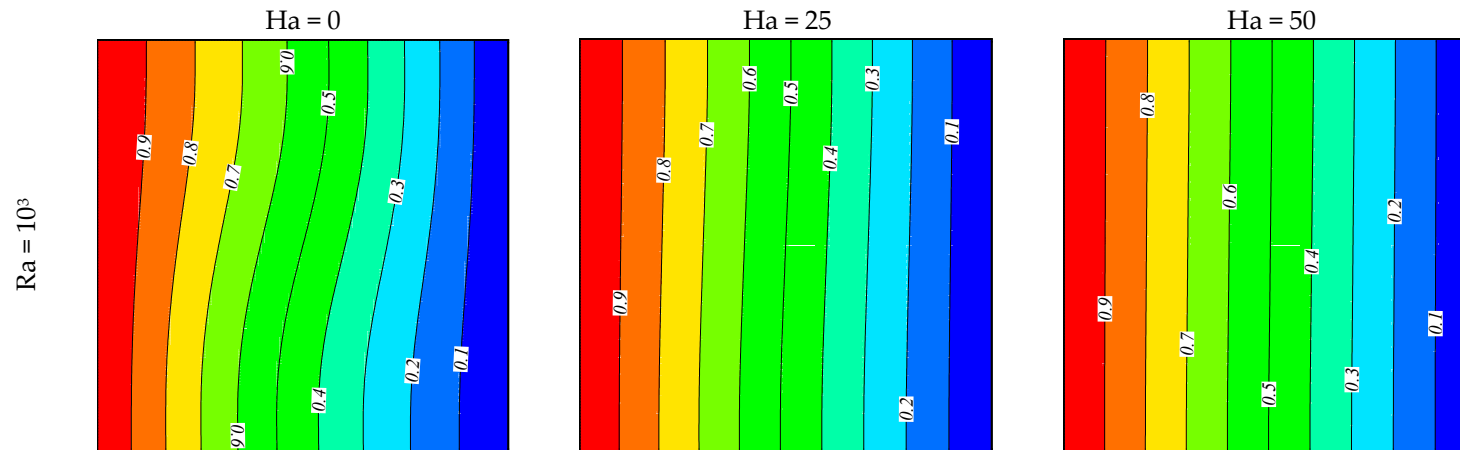


Figure 3. Cont.

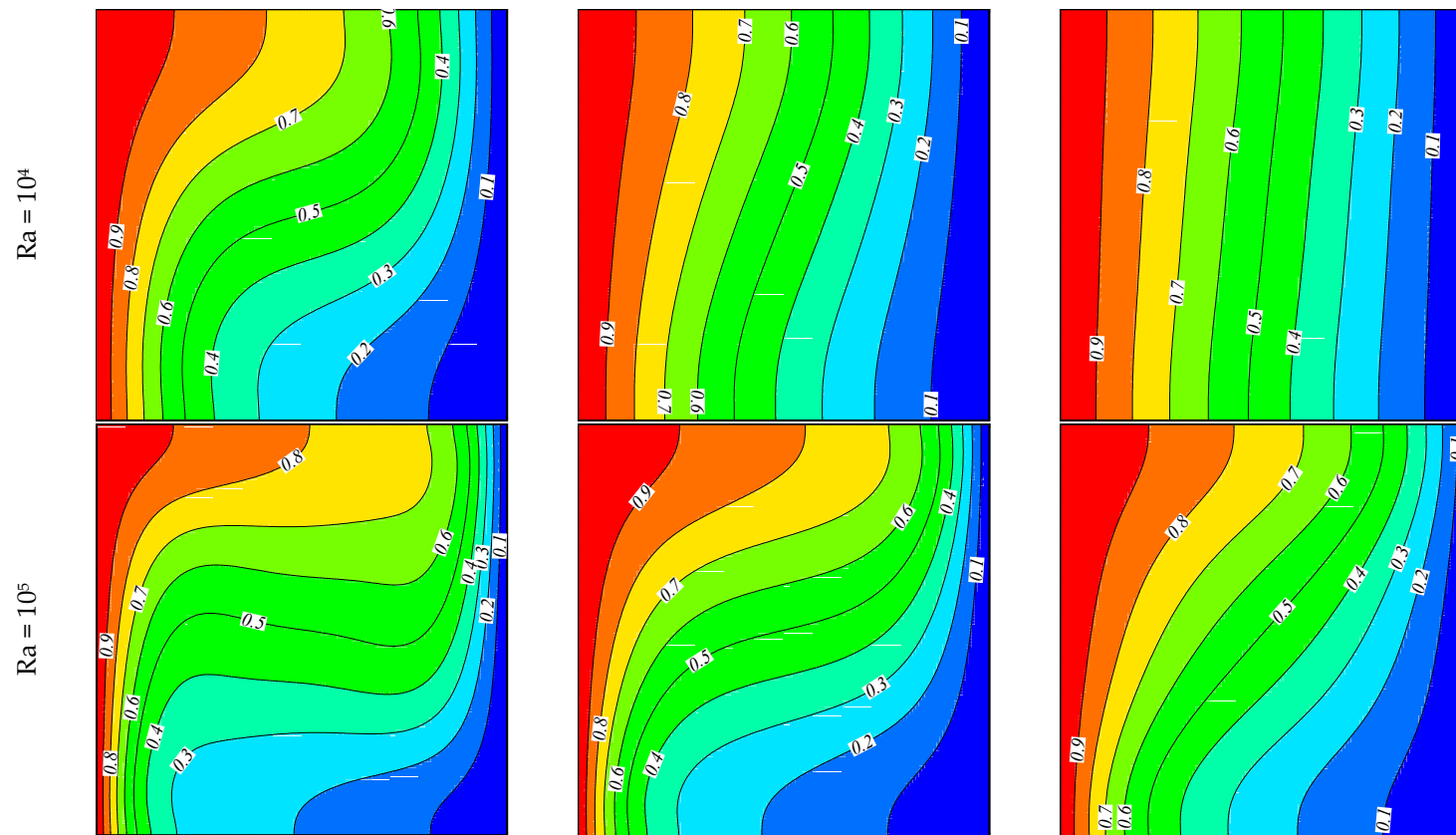


Figure 3. Isotherm contours for different values of Rayleigh and Hartmann numbers when $N = 1$, $m = 3$, and $\phi = 2\%$.

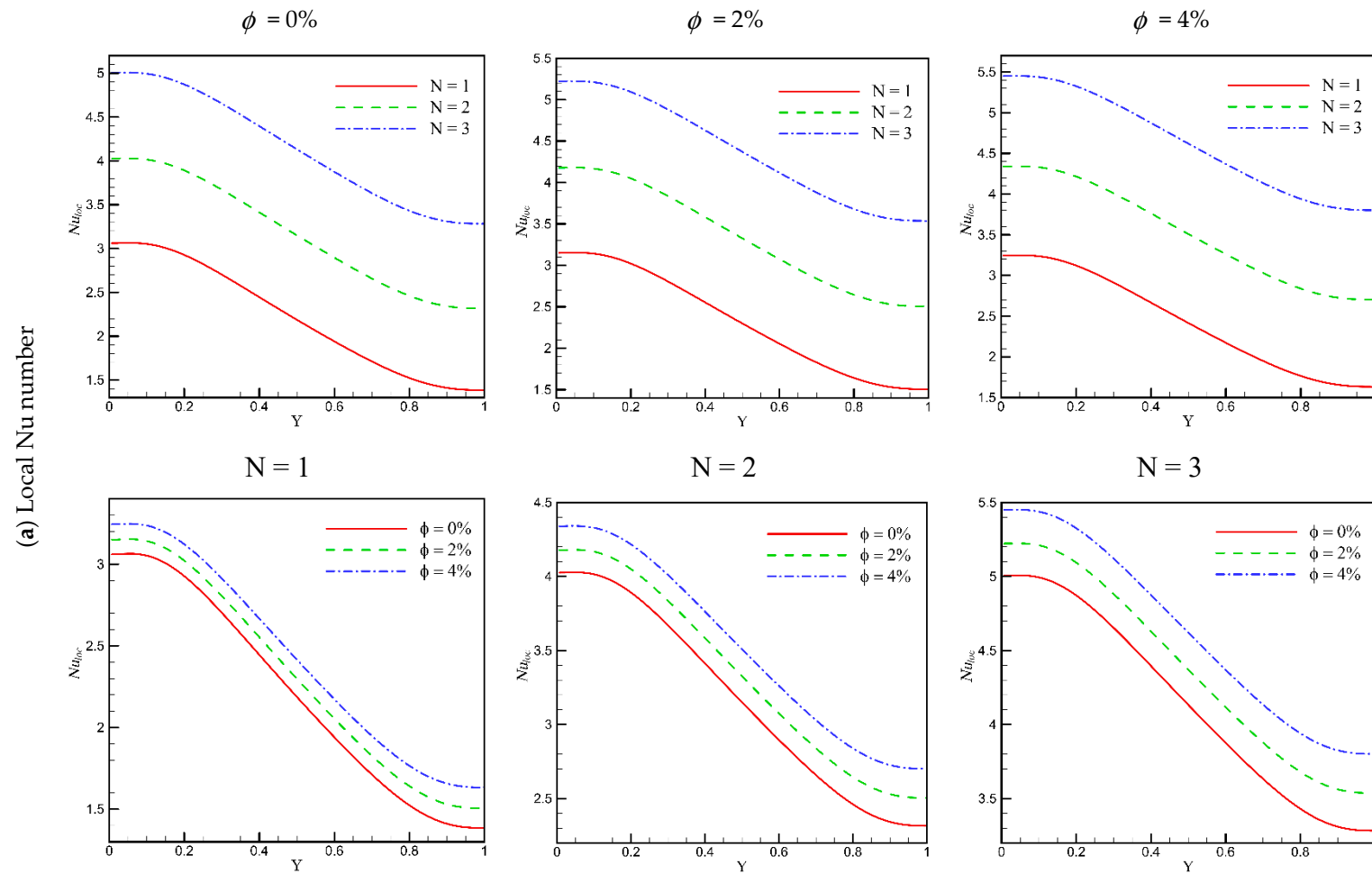


Figure 4. Cont.

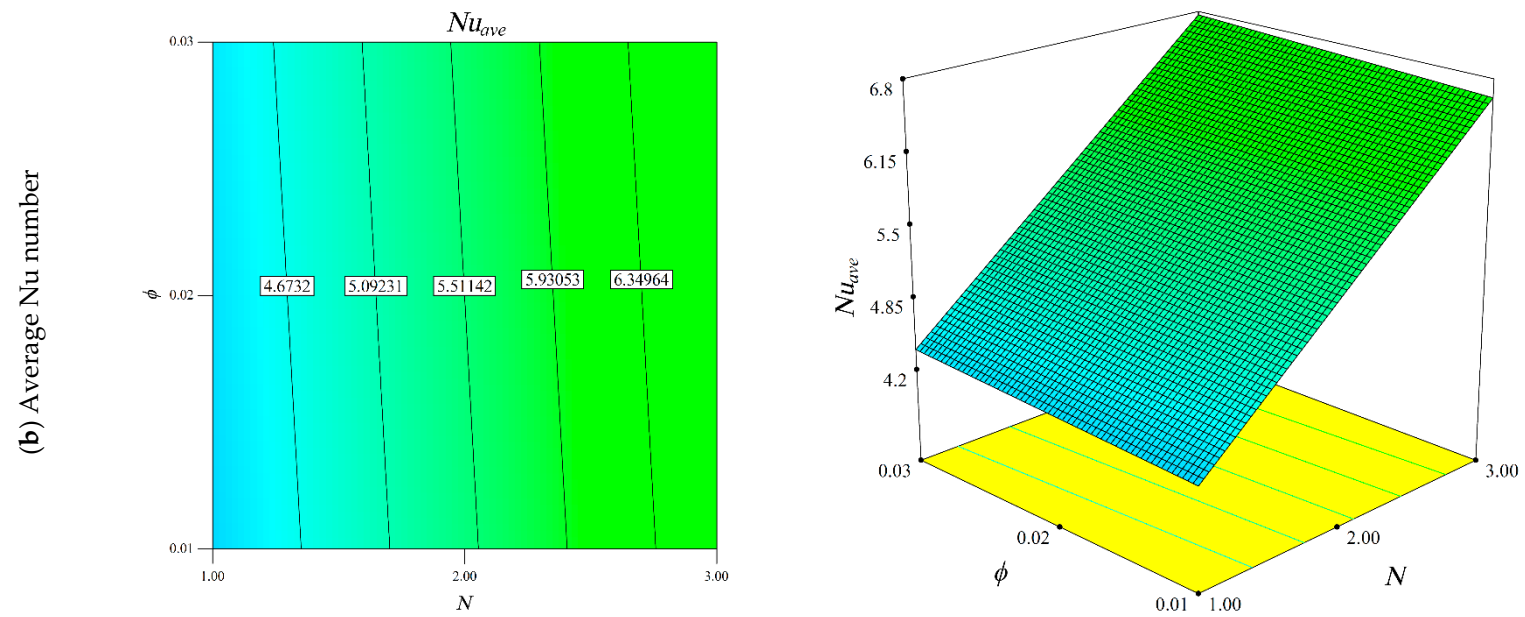


Figure 4. (a) Local Nusselt (Nu) numbers for different values of volume fractions of the nanofluid and radiation parameters when $Ra = 10^4$, $Ha = 25$, and $m = 3$, (b) average Nu number for different values of volume fractions of the nanofluid and radiation parameters.

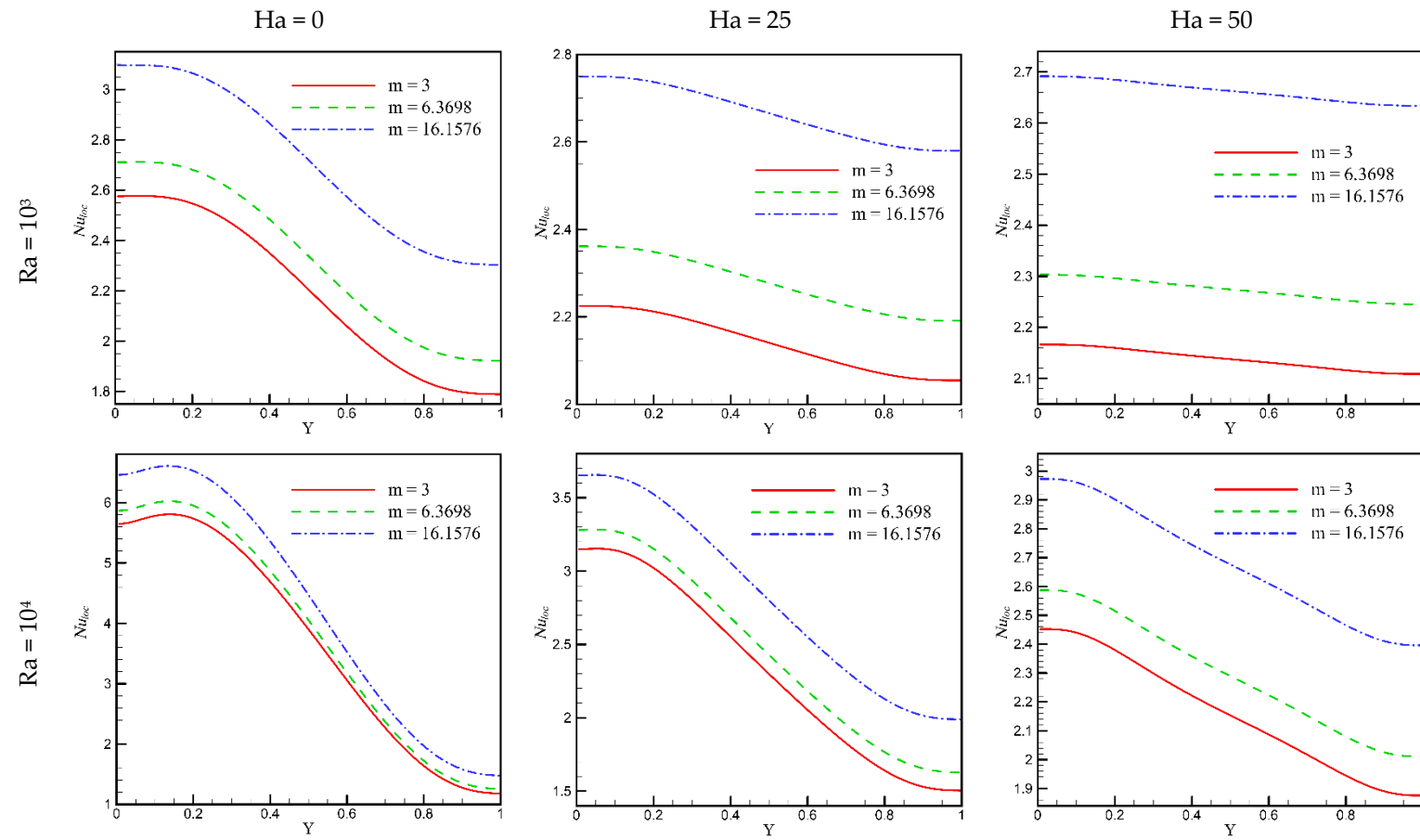


Figure 5. Cont.

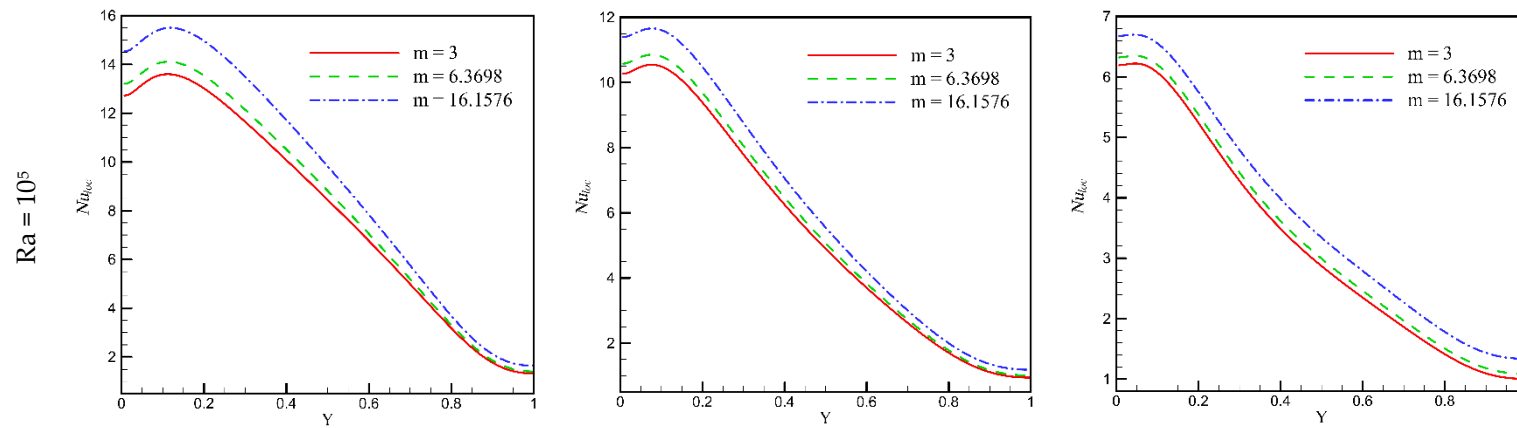


Figure 5. Local Nu number for different values of Rayleigh and Hartmann numbers and shape factors when $N = 1$ and $\phi = 2\%$.

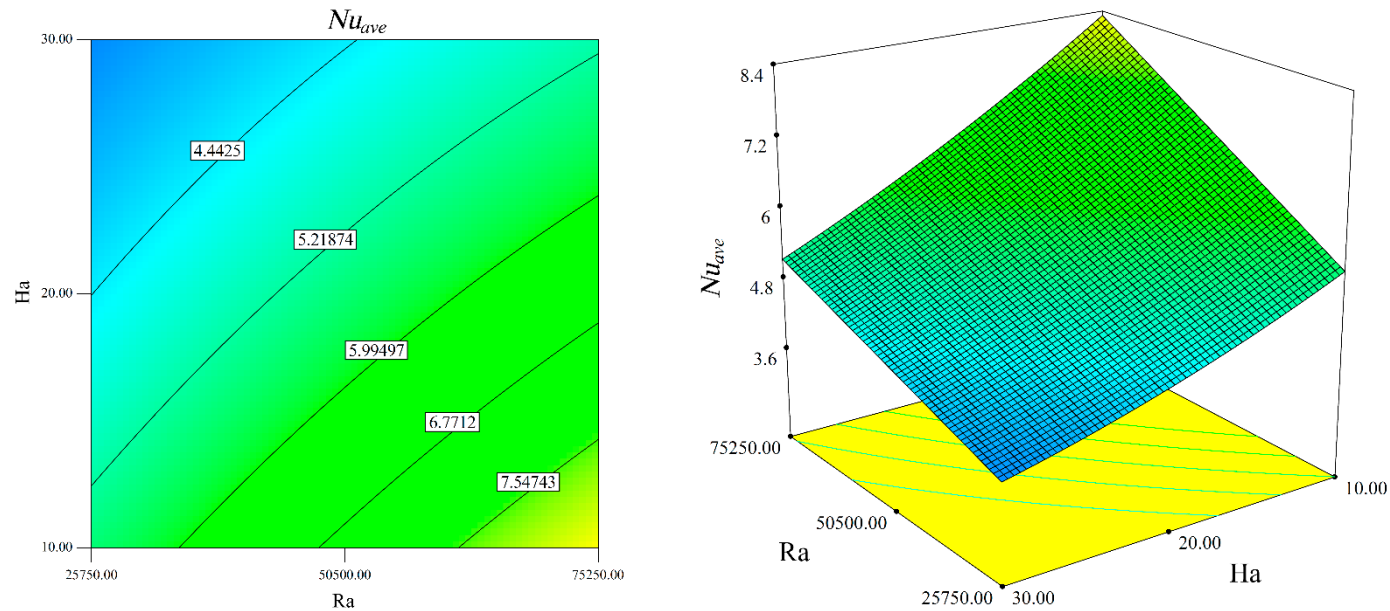


Figure 6. Average Nu number for different values of Rayleigh and Hartmann numbers and shape factors when $N = 2$, $\phi = 2\%$, and $m = 3$.

5. Conclusions

The natural convection of a magnetohydrodynamic nanofluid in an enclosure subject to the effects of thermal radiation and the shape factor of nanoparticles was examined numerically using CVFEM. The investigation of nanofluid heat transfer and flow was conducted using an extensive range of Rayleigh numbers, radiation parameters, nanofluid volume fractions, Hartmann numbers, and nanoparticle shape factors. The outcomes indicate that the local and average Nusselt numbers increase with increasing Rayleigh number, radiation parameter, and nanofluid volume fraction, while they decrease with decreasing Hartmann number. Moreover, the laminar nanoparticle has a higher local Nusselt number than the other nanoparticle shapes. Also, in this study, a correlation for the average Nusselt number and the other factors studied was obtained with an R-squared of 0.9927.

Author Contributions: A.J.C.: Proposed the problem, developed the governing equation and helped significantly in the writing and proofreading of the paper. A.S.D.: Programmed the governing equations and obtained the numerical results. D.D.G.: Helped in the write-up of the paper.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflicts of interest.

Nomenclature

Ha	Hartmann number	Greek symbols	
Pr	Prandtl number	β	Thermal expansion coefficient (K^{-1})
B_0	Strength of magnetic field	ρ	Density ($kg\ m^{-3}$)
C	Specific heat ($J\ kg^{-1}\ K^{-1}$)	μ	Dynamic viscosity ($kg\ m^{-1}\ s^{-1}$)
T	Temperature (K)	ν	Kinematic viscosity ($m^2\ s^{-1}$)
$Nu_{loc.}$	Local Nusselt number	σ	Electrical conductivity ($\Omega^{-1}\ m^{-1}$)
$Nu_{ave.}$	Average Nusselt number	ϕ	Nanoparticles volume fraction
k	Thermal conductivity ($W\ m^{-1}\ K^{-1}$)	σ^*	Stefan–Boltzmann constant
u, v	Velocity components in x and y directions, respectively ($m\ s^{-1}$)	Subscripts	
Ra	Rayleigh number	f	Base fluid
m	Shape factor of nanoparticles	nf	Nanofluid
grad.	Radiative heat flux	s	Solid nanoparticles
N	Radiation parameter		
p	Pressure term		
k_{nf}^*	Mean absorption coefficient		

References

1. Bhatti, M.M.; Rashidi, M.M. Numerical simulation of entropy generation on MHD nanofluid towards a stagnation point flow over a stretching surface. *Int. J. Appl. Comput. Math.* **2017**, *3*, 2275–2289. [\[CrossRef\]](#)
2. Bhatti, M.M.; Abbas, T.; Rashidi, M.M. Entropy generation as a practical tool of optimisation for non-Newtonian nanofluid flow through a permeable stretching surface using SLM. *J. Comput. Des. Eng.* **2017**, *4*, 21–28. [\[CrossRef\]](#)
3. Abbas, T.; Hayat, T.; Ayub, M.; Bhatti, M.M.; Alsaedi, A. Electromagnetohydrodynamic nanofluid flow past a porous Riga plate containing gyrotactic microorganism. *Neural Comput. Appl.* **2017**. [\[CrossRef\]](#)
4. Zeeshan, A.; Shehzad, N.; Ellahi, R.; Sultan Alamri, Z. Convective Poiseuille flow of Al_2O_3 -EG nanofluid in a porous wavy channel with thermal radiation. *Neural Comput. Appl.* **2017**. [\[CrossRef\]](#)
5. Dogonchi, A.S.; Alizadeh, M.; Ganji, D.D. Investigation of MHD Go-water nanofluid flow and heat transfer in a porous channel in the presence of thermal radiation effect. *Adv. Powder Technol.* **2017**, *28*, 1815–1825. [\[CrossRef\]](#)
6. Dogonchi, A.S.; Ganji, D.D. Analytical Solution and heat transfer of two-phase nanofluid flow between Non-parallel walls considering Joule heating effect. *Powder Technol.* **2017**, *318*, 390–400. [\[CrossRef\]](#)

7. Dogonchi, A.S.; Divsalar, K.; Ganji, D.D. Flow and heat transfer of MHD nanofluid between parallel plates in the presence of thermal radiation. *Comput. Methods Appl. Mech. Eng.* **2016**, *310*, 58–76. [[CrossRef](#)]
8. Waqas, M.; Hayat, T.; Alsaedi, A. A theoretical analysis of SWCNT–MWCNT and H₂O nanofluids considering Darcy–Forchheimer relation. *Appl. Nanosci.* **2018**. [[CrossRef](#)]
9. Hayat, T.; Sajjad, R.; Alsaedi, A.; Muhammad, T.; Ellahi, R. On squeezed flow of couple stress nanofluid between two parallel plates. *Results Phys.* **2017**, *7*, 553–561. [[CrossRef](#)]
10. Ellahi, R.; Hassan, M.; Zeeshan, A.; Khan, A.A. The shape effects of nanoparticles suspended in HFE-7100 over wedge with entropy generation and mixed convection. *Appl. Nanosci.* **2016**, *6*, 641–651. [[CrossRef](#)]
11. Dogonchi, A.S.; Chamkha, A.J.; Seyyedi, S.M.; Ganji, D.D. Radiative nanofluid flow and heat transfer between parallel disks with penetrable and stretchable walls considering Cattaneo–Christov heat flux model. *Heat Transf. Asian Res.* **2018**, *47*, 735–753. [[CrossRef](#)]
12. Bhatti, M.M.; Zeeshan, A.; Ellahi, R. Endoscope analysis on peristaltic blood flow of Sisko fluid with Titanium magneto-nanoparticles. *Comput. Biol. Med.* **2016**, *78*, 29–41. [[CrossRef](#)] [[PubMed](#)]
13. Dogonchi, A.S.; Ganji, D.D. Effects of Cattaneo–Christov heat flux on buoyancy MHD nanofluid flow and heat transfer over a stretching sheet in the presence of Joule heating and thermal radiation impacts. *Indian J. Phys.* **2018**, *92*, 757–766. [[CrossRef](#)]
14. Ellahi, R.; Tariq, M.H.; Hassan, M.; Vafai, K. On boundary layer nano-ferroliquid flow under the influence of low oscillating stretchable rotating disk. *J. Mol. Liq.* **2017**, *229*, 339–345. [[CrossRef](#)]
15. Alizadeh, M.; Dogonchi, A.S.; Ganji, D.D. Micropolar nanofluid flow and heat transfer between penetrable walls in the presence of thermal radiation and magnetic field. *Case Stud. Therm. Eng.* **2018**, *12*, 319–332. [[CrossRef](#)]
16. Rashidi, M.M.; Ganesh, N.V.; Hakeem, A.A.; Ganga, B. Buoyancy effect on MHD flow of nanofluid over a stretching sheet in the presence of thermal radiation. *J. Mol. Liq.* **2014**, *198*, 234–238. [[CrossRef](#)]
17. Dogonchi, A.S.; Ganji, D.D. Study of nanofluid flow and heat transfer between non-parallel stretching walls considering Brownian motion. *J. Taiwan Inst. Chem. Eng.* **2016**, *69*, 1–3. [[CrossRef](#)]
18. Dogonchi, A.S.; Ganji, D.D.; Makinde, O.D. Impact of Stretching and Penetration of Walls on Nanofluid Flow and Heat Transfer in a Rotating System. *Defect Diffus. Forum* **2018**, *387*, 37–50. [[CrossRef](#)]
19. Ghalambaz, M.; Doostani, A.; Izadpanahi, E.; Chamkha, A.J. Phase-change heat transfer in a cavity heated from below: The effect of utilizing single or hybrid nanoparticles as additives. *J. Taiwan Inst. Chem. Eng.* **2017**, *72*, 104–115. [[CrossRef](#)]
20. Dogonchi, A.S.; Ganji, D.D. Thermal radiation effect on the nano-fluid buoyancy flow and heat transfer over a stretching sheet considering Brownian motion. *J. Mol. Liq.* **2016**, *223*, 521–527. [[CrossRef](#)]
21. Dogonchi, A.S.; Ganji, D.D. Investigation of MHD nanofluid flow and heat transfer in a stretching/shrinking convergent/divergent channel considering thermal radiation. *J. Mol. Liq.* **2016**, *220*, 592–603. [[CrossRef](#)]
22. Rashidi, M.M.; Nasiri, M.; Shadloo, M.S.; Yang, Z. Entropy Generation in a Circular Tube Heat Exchanger Using Nanofluids: Effects of Different Modeling Approaches. *Heat Transf. Eng.* **2017**, *38*, 853–866. [[CrossRef](#)]
23. Dogonchi, A.S.; Hashim, Heat transfer by natural convection of Fe₃O₄-water nanofluid in an annulus between a wavy circular cylinder and a rhombus. *Int. J. Heat Mass Transf.* **2019**, *130*, 320–332. [[CrossRef](#)]
24. Dogonchi, A.S.; Ganji, D.D. Impact of Cattaneo–Christov heat flux on MHD nanofluid flow and heat transfer between parallel plates considering thermal radiation effect. *J. Taiwan Inst. Chem. Eng.* **2017**. [[CrossRef](#)]
25. Sheremet, M.A.; Cimpean, D.S.; Pop, I. Free convection in a partially heated wavy porous cavity filled with a nanofluid under the effects of Brownian diffusion and thermophoresis. *Appl. Therm. Eng.* **2017**, *113*, 413–418. [[CrossRef](#)]
26. Sheremet, M.A.; Revnic, C.; Pop, I. Free convection in a porous wavy cavity filled with a nanofluid using Buongiorno’s mathematical model with thermal dispersion effect. *Appl. Math. Comput.* **2017**, *299*, 1–15. [[CrossRef](#)]
27. Sheremet, M.A.; Grosan, T.; Pop, I. Natural convection and entropy generation in a square cavity with variable temperature side walls filled with a nanofluid: Buongiorno’s mathematical model. *Entropy* **2017**, *19*, 337. [[CrossRef](#)]
28. Mahmoudinezhad, S.; Rezaia, A.; Yousefi, T.; Shadloo, M.S.; Rosendahl, L.A. Adiabatic partition effect on natural convection heat transfer inside a square cavity: Experimental and numerical studies. *Heat Mass Transf.* **2018**, *54*, 291. [[CrossRef](#)]

29. Dogonchi, A.S.; Ismael, M.; Chamkha, A.J.; Ganji, D.D. Numerical analysis of natural convection of Cu-water nanofluid filling triangular cavity with semi-circular bottom wall. *J. Therm. Anal. Calorim.* **2018**. [[CrossRef](#)]
30. Sheremet, M.A.; Grosan, T.; Pop, I. Free convection in a square cavity filled with a porous medium saturated by nanofluid using Tiwari and Das' nanofluid model. *Transp. Porous Med.* **2015**, *106*, 595–610. [[CrossRef](#)]
31. Seyyedi, S.M.; Sahebi, N.; Dogonchi, A.S.; Hashemi-Tilehnoee, M. Numerical and experimental analysis of a rectangular single-phase natural circulation loop with asymmetric heater position. *Int. J. Heat Mass Transf.* **2019**, *130*, 1343–1357. [[CrossRef](#)]
32. Xu, X.; Chen, S. Cattaneo-Christov heat flux model for heat transfer of Marangoni boundary layer flow in a copper-water nanofluid. *Heat Transf.-Asian Res.* **2017**. [[CrossRef](#)]
33. Dogonchi, A.S.; Chamkha, A.J.; Ganji, D.D. A numerical investigation of magneto-hydrodynamic natural convection of Cu-water nanofluid in a wavy cavity using CVFEM. *J. Therm. Anal. Calorim.* **2018**. [[CrossRef](#)]
34. Dogonchi, A.S.; Sheremet, M.A.; Ganji, D.D.; Pop, I. Free convection of copper-water nanofluid in a porous gap between hot rectangular cylinder and cold circular cylinder. *J. Therm. Anal. Calorim.* **2018**. [[CrossRef](#)]
35. Dogonchi, A.S.; Sheremet, M.A.; Pop, I.; Ganji, D.D. MHD natural convection of Cu/H₂O nanofluid in a horizontal semi-cylinder with a local triangular heater. *Int. J. Numer. Methods Heat Fluid Flow* **2018**. [[CrossRef](#)]
36. Dogonchi, A.S.; Selimefendigil, F.; Ganji, D.D. Magneto-hydrodynamic natural convection of CuO-water nanofluid in complex shaped enclosure considering various nanoparticle shapes. *Int. J. Numer. Methods Heat Fluid Flow* **2018**. [[CrossRef](#)]
37. Khanafer, K.; Vafai, K.; Lightstone, M. Buoyancy-driven heat transfer enhancement in a two dimensional enclosure utilizing nanofluids. *Int. J. Heat Mass Transf.* **2003**, *46*, 3639–3653. [[CrossRef](#)]
38. De Vahl Davis, G. Natural convection of air in a square cavity, a benchmark numerical solution. *Int. J. Numer. Methods Fluids* **1962**, *3*, 249–264. [[CrossRef](#)]



© 2018 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>).