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The Experimental Realization of an Acoustic Cloak in Air with a Meta-Composite Shell

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In circular transformation layer, the generalized acoustic wave equation [1] is $\nabla\cdot(\boldsymbol{\rho}^{-1}\nabla p)+\frac{w^2}{\lambda}p=0$. The acoustic wave equation in cylindrical coordinate (r, θ, z) can be written as follows [1]:

$$\frac{1}{r}\frac{\partial}{\partial r}(\frac{r}{\rho_r}\frac{\partial p}{\partial r})+\frac{1}{r^2}\frac{\partial}{\partial\theta}(\frac{1}{\rho_\theta}\frac{\partial p}{\partial\theta})+\frac{1}{\rho_z}\frac{\partial^2 p}{\partial z^2}+\frac{k_0^2}{\lambda}p=0 \tag{S1}$$

Where λ and ρ are fluid bulk modulus and mass density relative to background medium λ_0 and ρ_0. $k_0^2=\omega^2\rho_0/\lambda_0$ is a material parameter of background material. If we consider the coordinate transformation both in r and z directions, the transformation can explained as $r'=f(r)$, $\theta'=\theta$, and $z'=g(z)=\delta\cdot z$. Function $f(r)$ is the coordinate transform function in radial direction. Parameter δ is the scale factor (real constant) in the vertical z direction. The difference identities are $\frac{\partial}{\partial r}=\left(\frac{df}{dr}\right)\frac{\partial}{\partial r'}$ and $\frac{\partial^2}{\partial z^2}=(\delta)^2\frac{\partial^2}{\partial z'^2}$, respectively. After substituting the identities into equation (S1), the acoustic equation of transformed domain is as follows

$$\frac{1}{r}\left(\frac{df}{dr}\right)\frac{\partial}{\partial r'}\left\{\frac{r}{\rho_r}\left[\left(\frac{df}{dr}\right)\frac{\partial p}{\partial r'}\right]\right\}+\frac{1}{r^2\rho_\theta}\frac{\partial^2 p}{\partial\theta'^2}+\frac{(\delta)^2}{\rho_z}\frac{\partial^2 p}{\partial z'^2}+\frac{k_0^2}{\lambda}p=0 \tag{S2}$$

If the region of transformed space equals to the background medium, there will be no scattering. The governing field is obtained upon setting $\lambda=1$, $\rho_r=\rho_\theta=\rho_z=1$ [1], the acoustic govern equation of background medium in equation (S1) changes to below

$$\frac{1}{r'}\frac{\partial}{\partial r'}(r'\frac{\partial p}{\partial r'})+\frac{1}{r'^2}\frac{\partial^2 p}{\partial\theta'^2}+\frac{\partial^2 p}{\partial z'^2}+k_0^2 p=0 \tag{S3}$$

Here the primed coordinates (r',θ',z') is hypothetical ones, simply used for distinction with the physical (original) coordinates. Multiply both sides of equation (S2) by λ and base on the equivalence between equations (S2) and (S3), we can have four constrain equations as below

$$\frac{\lambda}{r}\left(\frac{df}{dr}\right)=\frac{1}{r'},\quad \frac{r}{\rho_r}\left(\frac{df}{dr}\right)=r',\quad \frac{\lambda}{r^2\rho_\theta}=\frac{1}{r'^2},\quad \frac{\lambda(\delta)^2}{\rho_z}=1 \tag{S4}$$

From the relation $\frac{\lambda}{r}\left(\frac{df}{dr}\right)=\frac{1}{r^{/}}$, and use the relation $r^{/}=f(r)$, the bulk modulus can be explained below

$$\lambda=\frac{r}{f}\left(\frac{df}{dr}\right)^{-1} \tag{S5}$$

Secondly, from $\frac{r}{\rho_r}\frac{df}{dr}=r^{/}$ and the coordinate transformation relations $r^{/}=f(r)$ the material density in radial direction is as follows:

$$\rho_r=\frac{r}{f}\frac{df}{dr} \tag{S6}$$

From constrain equation $\frac{\lambda}{r^2\rho_\theta}=\frac{1}{r^{/2}}$ and $r^{/}=f(r)$, It have $\rho_\theta=\frac{\lambda f^2}{r^2}$. By substituted equation (S5) into above equation, the material density in θ direction is as follows

$$\rho_\theta=\frac{f}{r}\left(\frac{df}{dr}\right)^{-1} \tag{S7}$$

Finally from the constrain relation $\frac{\lambda(\delta)^2}{\rho_z}=1$, the material density in z direction can be written as $\rho_z=\lambda(\delta)^2$. After replaced the bulk modulus as $\lambda=\frac{r}{f}\left(\frac{df}{dr}\right)^{-1}$ from equation (S5), we can have the relation below

$$\rho_z=(\delta)^2\frac{r}{f}\left(\frac{df}{dr}\right)^{-1} \tag{S8}$$

Finally, the cloak shell material parameters of both radial and vertical directions coordinate transformation can be written as

$$\boldsymbol{\rho}=\begin{bmatrix}\frac{r}{f}\frac{df}{dr} & 0 & 0\\ 0 & \frac{f}{r}\left(\frac{df}{dr}\right)^{-1} & 0\\ 0 & 0 & (\delta)^2\frac{r}{f}\left(\frac{df}{dr}\right)^{-1}\end{bmatrix} \tag{S9}$$

$$\lambda=\frac{r}{f}\left(\frac{df}{dr}\right)^{-1} \tag{S10}$$

References

1. Chen, T.Y.; Tsai, Y.-L. A derivation for the acoustic material parameters in transformation domains. *J. Sound Vib.* **2013**, *332*, 766–779.