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A Novel Approach to Design Flood Hydrograph Plotting in Water Management

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Featured Application

This research, conducted jointly by the Faculty of Hydroengineering and Water Resources Management within the Technical University of Civil Engineering of Bucharest and the Faculty of Civil Engineering at Politehnica University Timisoara, establishes a simplified and mathematically robust framework for defining the design flood hydrograph. It serves as a graphical representation intended for integration into future hydraulic structure design standards, and the revision of the Romanian hydrological standard. By optimizing rational functions, including the Cadariu function, and integrating cubic spline interpolation, the study offers engineers a standardized, practical approach for the accurate representation of flood hydrograph shapes across diverse sites.

Abstract

This study proposes a unified analytical framework for plotting design flood hydrographs (DFH) based on four characteristic parameters: total duration (T_t), time to peak (T_p), flood volume (W), and peak discharge (Q_p) associated with specific exceedance probabilities. The objective is to improve the mathematical representation of hydrograph shapes for engineering applications and future updates of hydrological design standards. Leveraging a dataset of 150 representative cross-sections of Romanian rivers, the research employs normalized axes to facilitate a dimensionless comparative analysis. This study presents three modeling approaches: the Refined Rational Function (RRF), which significantly enhances the framework initially developed by Radu Cadariu; a [4/4] Padé approximant, transitioning from a second-degree to a fourth-degree rational framework and featuring a depressed quartic numerator and a fourth-degree denominator for superior degrees of freedom; and the plotting of non-parametric hydrographs via natural cubic spline interpolation. The results demonstrate that the RRF method extends the admissible range of the shape coefficient γ beyond the traditional interval 0.15–0.50, enabling representation of hydrographs with γ values up to 0.99. The [4/4] Padé approximant provides improved flexibility for asymmetric and multimodal hydrographs, while the natural cubic spline interpolation method ensures accurate reconstruction of atypical hydrographs with volume conservation errors below 3%. These methods offer a unified, objective framework, ensuring high accuracy and adaptability across diverse Romanian hydrological regimes.

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1. Introduction

In water resources engineering, design flood hydrographs are widely applied in designing hydraulic structures such as dams, levees, and weirs, and for effective flood risk management. They are also used to calibrate hydrological models for flood prediction and warning systems, as well as for basin management and planning [1].

In Romania, the design flood hydrograph (single peak) is a mandatory input for sizing hydraulic structures and for flood risk management. The design flood hydrograph represents flow as a function of time, based on four known parameters provided by the National Institute of Hydrology and Water Management (INHGA): total flood duration (T_t), time to peak (t_p), flood volume (W), and peak discharge (Q_p) [2,3].

Estimating the Design Flood Hydrograph (DFH) for both gauged and ungauged watersheds is a fundamental challenge in water resources engineering, critical for the safe and economical design of hydraulic structures. The DFH represents the discharge as a function of time (hydrograph) resulting from a design storm of a specific probability (e.g., the T = 100-year flood, which has a 1% annual exceedance probability-AEP). Two distinct approaches for its estimation are the Synthetic Unit Hydrograph (SUH) method and the regional flood frequency analysis, which employs regional synthesis relationships [4–6].

Synthetic Unit Hydrograph (SUH) methods (such as the SCS, Snyder, or Clark methods) are commonly applied for estimating the DFH in small, unmonitored (ungauged) hydrographic basins (watersheds). These methods rely on empirical coefficients or relationships (e.g., time-to-peak) that are regionalized, meaning they are derived from calibration on gauged catchments within a specific, hydrologically homogeneous region [6–8].

The fundamental concept, the Unit Hydrograph (UH), is a linear and time-invariant response function representing the direct runoff resulting from one unit of effective rainfall (rainfall excess) falling uniformly over the catchment for a specified duration. The temporal and spatial variability of natural rainfall excess is significant, especially during extreme events, which does not comply with the UH's uniform assumption and introduces a major source of error in predicting the DFH [9]. Consequently, applying standard SUH methods with generalized parameters can introduce large uncertainties and become unreliable in watersheds like Romania, which exhibits highly variable orography, soil, and hydrographic network characteristics [10,11].

Instead of modeling the physical process, INHGA analyzes the observed outputs, namely the actual flood hydrographs recorded at hydrometric stations. The characteristic elements of a design flood hydrograph are determined, namely peak discharge (Q_p), volume (W), time to peak (T_p), and total flood duration (T_t), based on regional synthesis relationships, usually regression equations [2,10].

These relationships are derived from the statistical and regional processing of the largest floods recorded at hydrometric stations in the analyzed region.

Characteristics of DFH for single-peak flood events are typically derived from long-term hydrometric records (ideally 25 years or more) at gauged stations. These derived characteristics are then used to estimate DFHs for both ungauged and gauged basins by applying regional synthesis relationships and hydrological analogy.

For determining the Design Maximum Discharge ($Q_{max,p\%}$), it is standard practice to employ Regional Flood Frequency Analysis (RFFA). This method involves direct synthesis (or regional regression) based on the maximum observed discharges within the hydrologically homogeneous region [12].

For the volume, the total flood duration and the time to peak of the design flood hydrograph, correlations can be established between these time parameters and key basin characteristics, such as the river length (L_r), the average slope of the river (I_r), the average slope of the basin (IB), and the catchment area (F_{BH}) [5,10].

Total flood duration was observed to be nonlinearly dependent on parameters like Lr/IB or the ratio of the basin's average elevation (H_m) to the logarithmic value of catchment area. Time to peak (t_p) is nonlinearly dependent on Lr/Ir and, for small watersheds, simply on the river length (L_r). Flood volume is primarily dependent on the catchment area and the basin's average elevation. For small watersheds, a good correlation with the river length (L_r) has also been established [10]. While hydromorphometric regionalization remains essential for defining the potential runoff response, in the future it must be coupled with climate-dynamic regionalization to accurately predict flood evolution in a non-stationary environment [13]. This methodology is preferred because flood processes, especially extreme events, are inherently non-linear and non-stationary, a reality that the linear UH concept cannot accurately model [14]. The INHGA methodology effectively eliminates the highly uncertain intermediate step of estimating a DFH and integrating it with a UH, instead using the regionalized relationships to directly estimate the characteristic DFH elements for a specific return period.

In this approach, the baseflow of the flood (Q_b) is typically considered zero for very small catchments (or watersheds) or equal to an estimated value (e.g., twice the multi-annual average discharge or other value) for larger catchments [10,15].

To define the shape of the DFH, a series of standardized mathematical models (e.g., the Γ-function or other established parametric models like those referenced in earlier Romanian hydrology practice [10]. The specific model and its parameters are chosen to minimize the deviation between the flows calculated by the model and the flows corresponding to the statistically derived characteristic elements [10].

The INHGA statistical and regional approach offers a more robust methodology for DFH estimation for extreme events. By grounding its synthesis relationships directly in the statistical frequency of observed large flood characteristics, it implicitly accounts for the dominant hydrological conditions and non-linearities that the standard SUH methods explicitly ignore or simplify. The INHGA does not provide a full hydrograph shape, the shape must be plotted using characteristic elements, namely peak discharge (Q_p), volume (W), time to peak (T_p), and total flood duration (T_t).

Although the Romanian technical standard NP 129-2011 provides a general framework for estimating design floods, several limitations affect its applicability in modern hydrological engineering. The standard primarily defines characteristic flood parameters but does not provide a unified analytical formulation for the hydrograph shape, leaving the graphical representation dependent on empirical approximations or simplified interpolation techniques.

Furthermore, traditional analytical formulations used in Romanian practice impose restrictive conditions on the admissible range of the flood shape coefficient γ and on the ratio between time to peak and total flood duration. As a result, certain hydrographs ob-served in regulated basins or large catchments cannot be reproduced accurately within the limits of classical functions.

Another limitation arises from the increasing variability of hydrological regimes caused by climate change and anthropogenic modifications of river basins, which pro-duce

atypical hydrograph shapes that are insufficiently represented by traditional para-metric formulations. These limitations highlight the need for flexible mathematical tools capable of reproducing a broader spectrum of hydrograph geometries while maintaining compatibility with engineering standards.

There is a national approach based on international literature [10,16–20], for reproducing design hydrographs based on known characteristic elements (Q_p, T_t, T_p, γ). This approach relies either on density functions derived from three-parameter distributions (which inherently account for skewness) or on functions used to approximate the rising limb (ascending branch) and the recession limb (retreating branch). However, these methods have shortcomings, which will be detailed below.

Recent international research has significantly advanced the fitting of design flood hydrographs through both parametric and non-parametric approaches. Modern parametric formulations include generalized probability density functions, flexible gamma-type distributions, and higher-order rational approximations capable of reproducing asymmetric flood shapes under non-stationary climatic conditions [21,22]. Non-parametric approaches increasingly rely on spline-based interpolation techniques and data-driven reconstruction methods that preserve integral hydrological characteristics such as peak discharge timing and flood volume [23,24].

Recent developments also include machine-learning-assisted hydrograph generation methods, where artificial neural networks, Gaussian process regression and hybrid stochastic–deterministic models improve hydrograph prediction accuracy in data-scarce environments [25]. These approaches allow flexible adaptation to non-linear basin responses and climate variability, providing improved robustness compared to classical synthetic unit hydrograph techniques.

Despite these advances, many modern methods require extensive calibration datasets or introduce increased computational complexity, limiting their applicability in engineering practice where standardized reproducibility and transparency are required. Therefore, a methodological gap persists between highly flexible mathematical models and simplified engineering tools suitable for regulatory implementation.

According to national regulations, the methods used to plot DFHs must be aligned with the available synthetic data. Furthermore, these methods should be straightforward or, at least, easily implementable within standard software applications. A prime example is the Cadariu formula, which was developed as a MathCAD application by this article's lead author and has since become a standard tool for experts and designers in hydrotechnical construction and water management.

The present study addresses this gap by proposing a unified analytical framework that preserves mathematical flexibility while maintaining the simplicity required for standardization in hydraulic design codes.

The application of density functions often necessitates complex calculations, frequently reducing to the solution of a system of nonlinear integral equations. This involves a set of two or more equations that must be satisfied concurrently, where at least one equation is an integral equation, and the system is nonlinear.

The practice of using two-function hydrographs to approximate the rising limb (ascending branch) and the recession limb is often dismissed in Romania as a 'flood that pricks your finger' (meaning an artificial or unnatural phenomenon). Consequently, this approach is excluded from the start, as it is considered a remnant characteristic of the Soviet hydrology period [10].

Based on research conducted by the Technical University of Civil Engineering Bucharest and Politechnica University Timisoara, three methods were selected for the graphical representation of flood design hydrographs. The Cadariu method and the cubic

spline interpolation method, championed by the Faculty of Hydraulic Engineering and Water Resources Management for over 20 years, have demonstrated remarkable longevity. Their reliability is evidenced by their consistent application in technical expert reports for hydraulic structures, dam and hydroelectric plant operating regulations, and documentation for water management permits. Furthermore, through joint research, the two universities have refined a comprehensive methodology intended to serve as the future national standard for design flood hydrograph representation.

These research findings and novelty elements are synthesized in Table 1, which establishes the foundation for a unitary methodology for the graphic representation (plot-ting) of the design flood hydrograph.

Table 1. Synthesis of novelty elements and methodological approaches for the DFH plotting.

Method	Mathematical Framework	Novelty Elements
Refined Rational Function	Second-degree Rational Function (Classic Cadariu)	Optimization and refinement of the classical Cadariu method by extending its applicability beyond original constraints for singular theoretical floods, specifically by surpassing the traditional limits for shape coefficients (0.15 to 0.50) and total duration to time-to-peak ratios of 2 to 6.5.
Padé Approximant [4/4]	Higher-order Rational Function (4th-degree polynomials)	Transition from a second-degree to a fourth-degree rational framework, featuring a depressed quartic numerator and a fourth-degree denominator for superior degrees of freedom. The model coefficients are rigorously determined through the Levenberg–Marquardt nonlinear optimization algorithm, enabling high-fidelity curve fitting that captures intricate discharge variations.
Natural Cubic Spline	Piecewise Cubic Polynomial Interpolation	Implementation of natural boundary conditions (second derivatives are zero at endpoints) for hydrograph reconstruction [1]

[1] Provides a high-fidelity alternative for atypical hydrographs and regulated catchments where standard functions may fail.

The first method utilizes a refined rational function (RF), which was initially proposed by Cadariu and subsequently refined by the authors of this article. This function is currently the standard for calculating the design flood hydrograph in most technical documentation (e.g., expertise reports, authorizations) throughout Romania. Its implementation is primarily achieved through computer applications developed by the authors [3].

The second method involves the use of a Padé approximant of ordinal [4/4], whose coefficients are obtained by the Levenberg–Marquardt nonlinear optimization method. This function represents a much more complex generalization of the classical Cadariu function (which is of degree 2). The use of degree 4 polynomials allows for the modeling of much more flexible curves, being able to capture finer variations in the flood hydrograph. This specific configuration, characterized by a depressed quartic numerator and a fourth-degree polynomial denominator, allows for a superior degree of freedom in curve fitting. By moving beyond lower-order approximations, this methodology effectively captures the intricate non-linearities and subtle discharge variations inherent in complex flood events.

The third method employs cubic spline interpolation for the graphic representation. Specifically, it uses the natural cubic spline, which inherently satisfies natural boundary conditions (i.e., the second derivatives are zero at the endpoints).

The objective of this research is to propose a methodology for integration into future regulatory frameworks and to disseminate advanced mathematical models to the hydrotechnical engineering community. This approach integrates the authors' practical expertise in hydraulic design and their experience in coordinating undergraduate and postgraduate professional development programs.

2. Materials and Methods

2.1. Conceptual and Analytical Framework of the Novel Approach

In water resources engineering, the Annual Exceedance Probabilities (AEPs) for design maximum discharges are typically prescribed by national regulations. For these standardized probabilities, the Design Flood Hydrograph (DFH) must be obtained. Table 2 summarizes the specific AEP provisions applicable to the Romanian regulatory context [26–28].

Table 2. Annual exceedance probabilities in FFA.

The Importance Class of the Construction	The Annual Exceedance Probability, %	
	Normal Operating Conditions	Special Operating Conditions (Dams Only)
I	0.1	0.01
II	1	0.1
III	2	0.5
IV	5	1 *
V (provisionally constructions)	10	3 *
Other important values	**The annual exceedance probability %**	
Significant Flood Event **	20%	
Bankfull discharge/Channel-Forming Discharge (Order no. 2.115/2021)	50%	
Channel-Forming Discharge (Order no 326 12-03-2007, repealed) ***	80%	

Note. * Based on a technical and economic justification approved together with the investment, verification under special operating conditions may be waived. ** According to INHGA, a significant flood is defined as an event where the peak discharge has an exceedance probability of 20% or less, recorded across a river sector containing at least three water-balance nodes. It should be noted, however, that other official INHGA documents may specify a more restrictive threshold of 10% for the same classification. *** Reference to this superseded regulation remains essential, as the conveyance capacity of most minor channels in Romania was historically delineated based on maximum discharges associated with an 80% Annual Exceedance Probability (AEP). Furthermore, this threshold continues to be integrated into specific dam operation rules to define the maximum downstream channel capacity. Adherence to this value is critical for constraining controlled releases, particularly during the functional testing of bottom outlets.

Plotting flood hydrographs is a complex task because they must be represented by mathematical functions whose parameters are calculated to respect the known characteristics of a flood hydrograph. This research, based on an analysis of one hundred fifty representative cross-sections across Romania's diverse zones, introduces a pragmatic approach, useful to the hydrotechnical field.

The dataset used in this research includes 150 representative river cross-sections distributed across Romania's main hydrological regions, covering mountain, sub-mountain, plateau, and lowland basins (Figure 1).

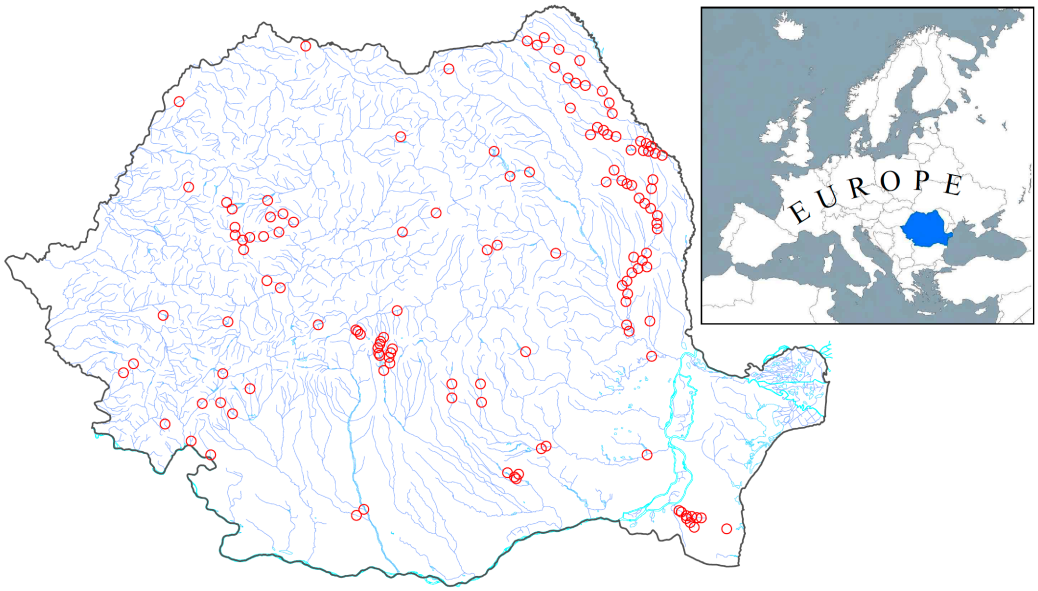


Figure 1. Reference locations in Romania used for DFH analysis.

The data used to validate the flood routing methods were selected to ensure a sample with diverse physiographic characteristics. This sample represents all hydrographic regions in Romania, spanning altitudes from mountainous headwaters at 2050 m down to the Black Sea level, thereby covering mountain, hill, plain, and coastal areas to ensure geographical applicability.

Furthermore, by encompassing basins across all orders of magnitude (ranging from 1 km^2 to 576,230 km^2) regional representativeness was achieved. This bestows the method with a universal character, enabling the generation of design hydrographs for both small catchments and large river basins.

Data completeness is guaranteed by selecting monitoring stations with a record length of at least 30 years. This longitudinal data allows for the accurate modeling of the rising limb and the overall hydrograph shape. Hydrological data for both natural and modified flow regimes are officially verified and provided by the INHGA. This rigorous selection process transforms the software application (originally developed in MathCAD Prime 11.0.0.0) from a simple computational tool into a reliable design standard for hydrotechnical infrastructure.

A reliable hydrograph can be drawn graphically with the help of mathematical functions. However, there is no single solution, as a hydrograph can have various shapes for a given set of data. This aspect leads to the need for additional criteria and, ultimately, to the choice of suitable mathematical functions.

If we analyze the graphical representation of a DFH, for example, a p% AEP, it is defined by its key characteristics: total flood duration (T_t), time to peak (t_p), flood volume (W), and peak discharge (Q_p shown to Figure 2). The INHGA provides the design flood volume indirectly by providing the flood shape coefficient (γ). This coefficient is defined as the ratio of the flood volume (W) to the product of the peak discharge (Q_p) and the total flood duration (T_t).

$$\gamma = \frac{W}{Q_p\, T_t} = \frac{\int_0^{T_t} Q(t)dt}{Q_p\, T_t} = \frac{(Q_p - Q_b)\, T_t \int_0^1 q(\tau)d\tau + Q_b\, T_t}{Q_p\, T_t} = \frac{Q_p - Q_b}{Q_p}\int_0^1 q(\tau)d\tau + \frac{Q_b}{Q_p} \quad (1)$$

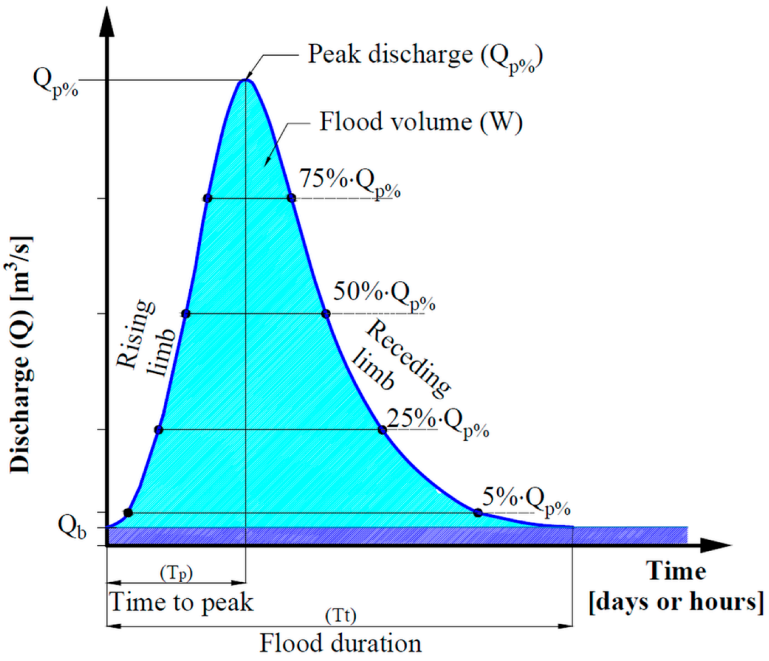


Figure 2. The design flood hydrograph.

For flood hydrographs where the base flow is assumed to be zero, the shape coefficient corresponds to the integral of the dimensionless discharge function $q(\tau)$.

The DFH is typically derived from regional synthesis relationships developed using observed floods that generally have a higher exceedance probability than the design event, usually in the range of 5–20% (10%) AEP. Consequently, for a given gauging station, the expected sample size of significant flood events (k) available for analysis is determined by the record length (n) and the selected threshold exceedance probability (P_s)—typically set at 10% or 20%. This stochastic relationship is formally expressed as follows:

$$k = (n + 1)\, P_s \tag{2}$$

In practice, a continuous recording period of 25–30 years yields a limited sample size for hydrograph analysis. Specifically, this duration provides approximately 5–6 significant flood events suitable for preliminary processing when adopting an exceedance probability threshold of P_s = 20%. This sample size further diminishes to only 3 events when a more restrictive threshold of P_s = 10% is applied. However, current practice shows that, to obtain truly representative floods, only those with peak flows corresponding to an exceedance probability of less than 10% AEP are typically retained. As a result, the number of usable floods is often reduced to around three.

For this reason, when dealing with short data series, the average-parameter method (also known as the mean hydrograph method) is recommended for determining design flood parameters. This method assumes that key hydrograph characteristics—total flood duration (T_t), time to peak (T_p or rise time), and shape coefficient (γ)—are independent of the peak discharge (Q_p) and its exceedance probability. This assumption allows the development of regional relationships for T_t, T_p, and γ [10,29,30].

The assumption of independence between T_t, T_p, γ and Q_p represents a practical simplification commonly adopted in regional hydrological synthesis. Empirical analyses of Romanian hydrometric records indicate that the variability of hydrograph shape descriptors is significantly smaller than the variability of peak discharge magnitude. This allows the geometric characteristics of the hydrograph to be estimated using regional relationships while peak discharge is determined through frequency analysis. The assumption is there-

fore consistent with engineering practice and ensures methodological compatibility with regulatory hydrological studies developed by INHGA.

More advanced statistical techniques, such as parameter estimation using multivariate distribution functions or copulas, are recommended only for gauging stations with long records (typically more than 50 years) and very low data uncertainty. These methods are mathematically complex and can produce unreliable results when applied to limited or uncertain datasets [29].

In the methodology proposed by the authors, the design flood hydrograph is derived using the Synthetic Mean Hydrograph Method. This approach relies on the simplifying assumption that the characteristic parameters—total flood duration (T_t), time to peak (T_p), and shape coefficient (γ)—are independent of the peak flood discharge (Q_p). Although a simplification, this assumption is necessary to ensure practical applicability and compatibility with existing design standards. The DFH also features several other important hydrologic and regulatory reference points, including the base flow (Q_b) and the proposed interpolation points for hydrograph shape definition, typically set at 75%, 50%, 25%, and 5% of the peak discharge (Q_p). Q_b is the sustained streamflow from groundwater and subsurface runoff that persists throughout a flood event and is conventionally considered constant in Romanian practice The final value (typically 5% of the peak discharge, Q_p) is selected to ensure the numerical stability of the interpolation function and the conservation of the DFH volume. This threshold is physically grounded in the Romanian hydrological context, where the channel-forming discharge is conventionally associated with an 80% AEP. For a Pearson Type III distribution characterized by a coefficient of variation $C_v = 2$ and a skewness coefficient $C_s = 2C_v$, the 5% fraction of the $Q_{1\%}$ magnitude aligns with this 80% AEP threshold.

This research proposes and implements three distinct methods for graphically representing the design flood hydrograph.

2.2. Refined Rational Function

The plotting of design flood hydrographs is a complex task, as they must be represented by mathematical functions whose parameters are calculated to satisfy the known characteristics of a given flood event (or design flood).

The rational function provides the essential analytical foundation, ensuring that the hydrograph generated adheres rigorously to the hydrological reality defined by the DFH, compared to other functions that alter the total flood duration (T_t) through the asymptotic trend on the abscissa. By adjusting the function's parameters, engineers can ensure both the rising and recession limbs conform to hydrographic basin characteristics while mathematically guaranteeing the flood characteristics. Cadariu's function, a rational function whose numerator is a quadratic polynomial without a constant term (a depressed quadratic) and whose denominator is also a polynomial of degree two, is central to this approach. Cadariu's rational function proves inappropriate for the full range of possible values for the flood hydrograph shape coefficient (γ) and for the ratio of the total flood duration (T_t) to the time to peak (T_p). Specifically, Cadariu's function is only valid for a restricted range, with the shape coefficient typically between $0.15 \leq \gamma \leq 0.50$ and the time ratio between $2 \leq T_t/T_p \leq 6.5$ [31]. This limitation stems from the specific normalization method used for the hydrograph, specifically, the division of the time axis by the time to peak and from insufficient research into the symmetry of the two-dimensional function that governs the parameter A in Cadariu's equation. For instance, Cadariu's function could not be successfully applied to the design floods stipulated in the regulatory standard NP 129-2011, a constraint that effectively rendered it obsolete. Figure 3 illustrates that shape

coefficient values for flood hydrographs in Romania, derived from hydrological studies and regulatory standards, are above 0.50.

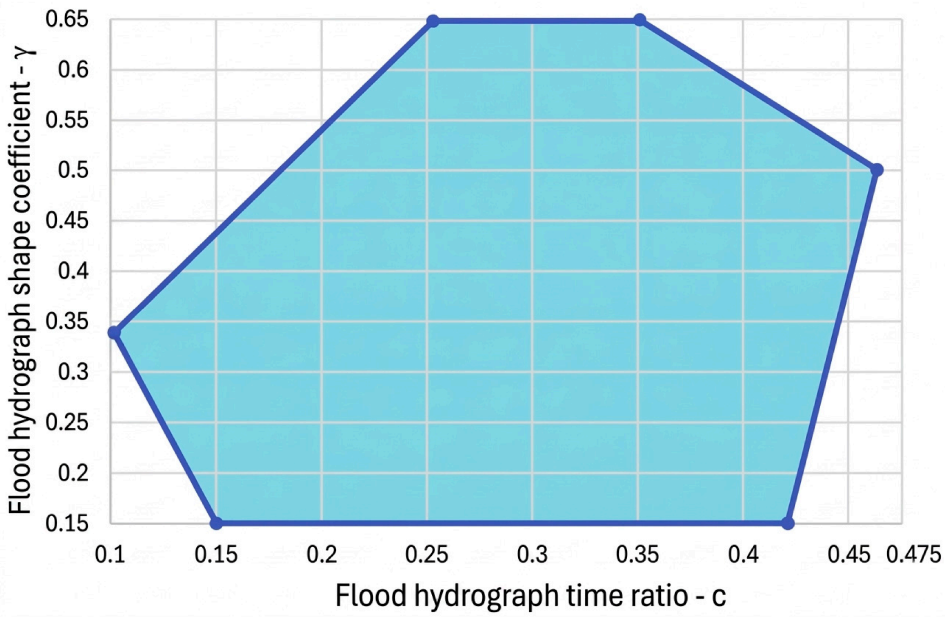


Figure 3. Variation range of the shape coefficient (γ) relative to the time ratio (c) for DFH.

The underlying assumption in the current research is that the design flood should be normalized to values ranging from 0 to 1, both on the abscissa (duration/time) and on the ordinate (discharge/flow rate). This normalization approach enables the shape coefficient to span the entire range from 0 to 1, and, crucially, permits the ratio between the time to peak and the total flood duration to also range between 0 and 1 ($0 < T_p/T_t < 1$).

The dimensionless form of the DFH, Refined Rational Function (RRF), derived from Cadariu's formula and expressed as a rational function of the single parameter A, is:

$$q_R(\tau) = \frac{(1-\tau)\tau}{A\tau^2 + B\tau + C} = \frac{(1-\tau)\tau}{A\tau^2 + (1-2c(A+1))\tau + (A+1)c^2} \tag{3}$$

where $\tau \in (0, 1)$ and $c = T_p/T_t$.

Thus, the shape coefficient is defined by the integral $\gamma = \int_0^1 q_R(\tau)\, d\tau$, yielding the following expression for $\gamma(A, c)$:

$$\begin{aligned}\gamma(A,c) = {} & \tfrac{A+1}{A^2}(1-2c)\ln\left(\tfrac{1-c}{c}\right) - \tfrac{1}{A} + \tfrac{A+1}{A^2}\frac{c(1-c)(A+2)-\frac{1}{2}}{\sqrt{c(1-c)(A+1)-\frac{1}{4}}} \\ & \times\left[\tan^{-1}\left(\frac{A-c(A+1)+\frac{1}{2}}{\sqrt{c(1-c)(A+1)-\frac{1}{4}}}\right) - \tan^{-1}\left(\frac{\frac{1}{2}-c(A+1)}{\sqrt{c(1-c)(A+1)-\frac{1}{4}}}\right)\right]\end{aligned} \tag{4}$$

A two-dimensional contour plot of the shape coefficient γ is presented as a function of the parameters A and c in Figure 4. The horizontal axis corresponds to the variable A, while the vertical axis represents variable c. The contour lines, representing constant values of the function $\gamma(A, c)$, are shown across distinct ranges of A: $[-1.0, 0]$, $[0, 3]$, $[1.0, 10.0]$, and $[10.0, 200.0]$.

The function exhibits symmetry relative to the line $c = 0.5$ for any value of A, satisfying the condition:

$$\gamma(A, 0.5 - d) = \gamma(A, 0.5 + d),\ \forall d \in (0, 0.5). \tag{5}$$

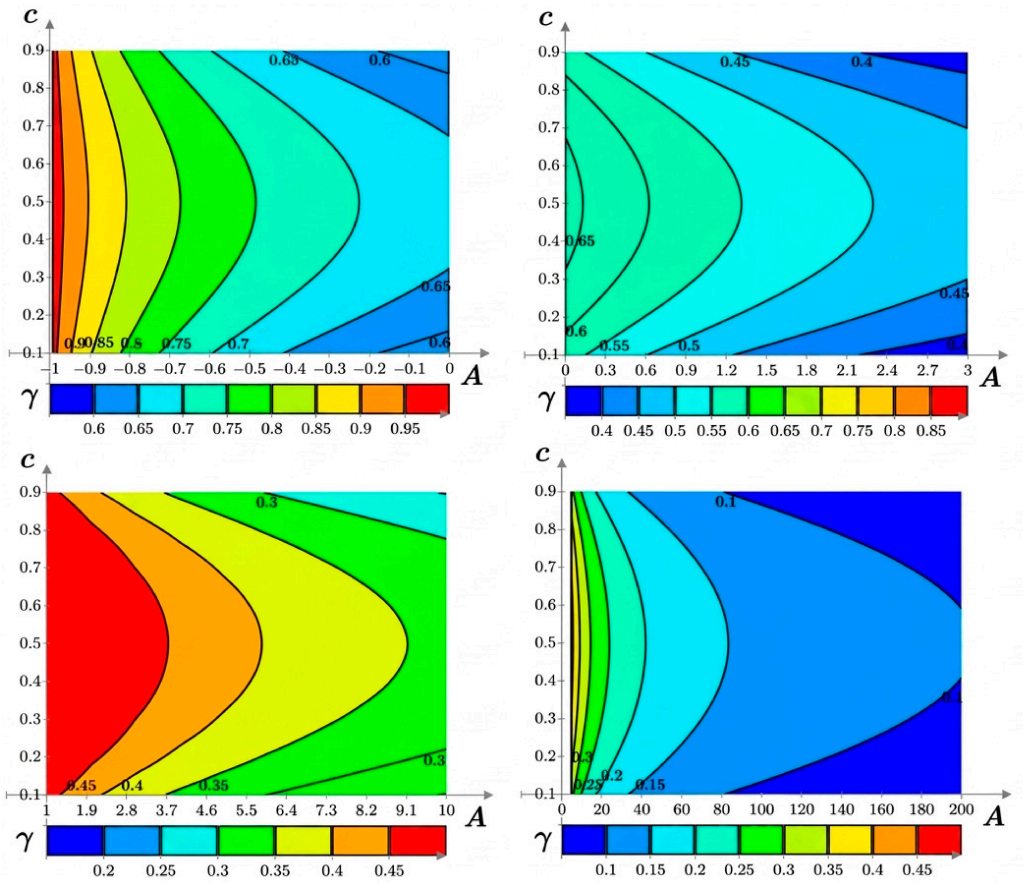


Figure 4. Two-dimensional contour plot of the shape coefficient γ.

The proposed approximation function $A_a(\gamma, c)$ is a bivariate polynomial regression in the variables $\log(\gamma)$ and c, designed to approximate the parameter A. This form was specifically selected to capture the quasi-parabolic symmetry around $c = 0.5$ by employing second-order polynomials in c, whose coefficients are sixth-degree polynomials in $\log(\gamma)$:

$$\begin{aligned} A_a(\gamma, c) = & \sum_{i=0}^{6} \left(\left(\sum_{j=0}^{2} C_{i,j}\, c^j \right) \log(\gamma)^i \right) \\ & = \left(-1.00295 + 0.42589\, c - 0.42589\, c^2\right) \\ & + \left(-0.47381 + 11.22654\, c - 11.22654\, c^2\right) \log(\gamma) \\ & + \left(4.3119 + 216.9812\, c - 216.9812\, c^2\right) \log(\gamma)^2 \\ & + \left(-1.02772 + 707.36397\, c - 710.57453\, c^2\right) \log(\gamma)^3 \\ & + \left(6.33308 + 2158.42912\, c - 2149.42049\, c^2\right) \log(\gamma)^4 \\ & + \left(3.01414 + 2322.44524\, c - 2318.37684\, c^2\right) \log(\gamma)^5 \\ & + \left(1.80002 + 1443.14658\, c - 1440.54208\, c^2\right) \log(\gamma)^6, \end{aligned} \tag{6}$$

where $C_{i,j}$ are the regression coefficients.

This function ensures the desired symmetry properties while providing sufficient flexibility to accurately fit the data.

To determine the durations $\tau(q)$ during which the discharge equals 75%, 50%, 25%, and 5% of the peak discharge (Q_p), two separate relationships were developed: one for the rising limb and one for the falling (recession) limb of the design flood hydrograph.

$$\begin{bmatrix} \tau_i \\ \tau_d \end{bmatrix} = \begin{bmatrix} \frac{1+[(A+1)\ c-1]\ q-\sqrt{(q-1)(c-1)[4\ c\ q(A+1)+(q-1)]}}{2(A\ q+1)} \\ \frac{1+[(A+1)\ c-1]\ q-\sqrt{(q-1)(c-1)[4\ c\ q(A+1)+(q-1)]}}{2(A\ q+1)} \end{bmatrix} \tag{7}$$

These relationships are essential for quickly determining the characteristic durations of peak flows within a flood hydrograph. Such data supports operational decision-making for dams, such as managing bottom outlets to control reservoir releases while ensuring downstream safety.

2.3. Higher-Order Rational Function

Central to this framework is the classical Cadariu function, a second-degree rational function comprising a depressed quadratic numerator and a quadratic denominator. Building upon this foundation, the study introduces a significant methodological advancement: the [4/4] order Padé approximant. This higher-order rational function represents a sophisticated transition from degree 2 to degree 4, featuring a depressed quartic numerator and a fourth-degree polynomial denominator [32]. This increased complexity provides superior degrees of freedom, allowing the model to capture intricate non-linearities and subtle discharge variations in complex flood events that lower-order models might overlook [33]. The coefficients for this advanced configuration are rigorously determined via Levenberg–Marquardt nonlinear optimization [34].

$$q_P(\tau) = \frac{\alpha_1\tau + \alpha_2\tau^2 + \alpha_3\tau^3 + \alpha_4\tau^4}{1 + \alpha_5\tau + \alpha_6\tau^2 + \alpha_7\tau^3 + \alpha_8\tau^4} \tag{8}$$

The [4/4] order Padé Approximant is a rational function that offers greater flexibility in modeling design flood hydrographs, making it highly adaptable for reproducing realistic flood events. There are two primary approaches to determining the coefficients of this rational function.

The first method involves decomposing the flood components and solving a system of equations based on that analysis. This method involves solving a system of nonlinear equations based on the fundamental characteristics of the design flood. It allows for the modeling of bimodal hydrographs, which typically occur due to delayed basin responses or the synchronization of flood peaks from different tributaries. In this method, conditions are established by decomposing complex, multi-peak events into individual single-peak components. By defining the specific parameters for each constituent single-peak hydrograph, the overall rational function can be precisely calibrated to represent the full event.

The following system of equations describes the practical application of the first method, specifically the decomposition of flood hydrograph components. The objective of this system is to determine the unknown coefficients (a_i, i = 1..8) of the rational function, ensuring the resulting curve strictly adheres to the physical characteristics of, for instance, a bimodal flood (Figure 5).

$$\gamma = \int_0^1 q_P(\tau)d\tau \tag{9}$$

$$\gamma_{csr} = \int_0^{c_s} q_P(\tau)d\tau \tag{10}$$

$$\gamma_{csd} = \int_{c_s}^{\tau_s} q_P(\tau)d\tau \tag{11}$$

$$\gamma_{cr} = \int_{\tau_s}^{c} q_P(\tau)d\tau \tag{12}$$

$$\gamma_{cd} = \int_{c}^{1} q_P(\tau)d\tau \tag{13}$$

$$\frac{d}{d\tau}q_P(\tau)\Big|_{\tau=c_s} = 0, \quad q_P(c_s) = q_{c_s} \tag{14}$$

$$\frac{d}{d\tau} q_P(\tau)\Big|_{\tau=c} = 0, \quad q_P(c) = 1 \tag{15}$$

$$q_P(1) = 0 \tag{16}$$

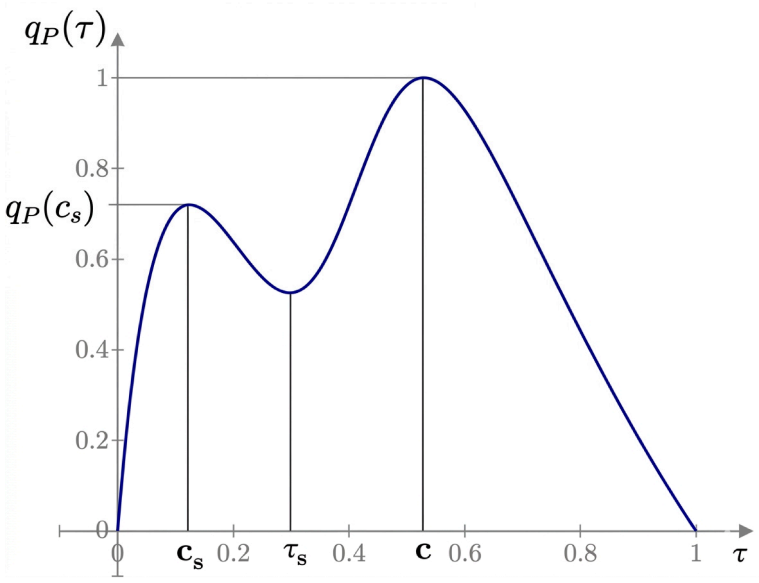


Figure 5. Bimodal design flood hydrograph (using a [4/4] order Padé approximant).

Equations (9)–(13) define the runoff volumes, where the integral of the discharge over time represents the cumulative volume for specific hydrograph segments corresponding to the various phases of the bimodal event. Equations (14) and (15) impose geometric constraints to ensure the structural integrity of the hydrograph; specifically, they define the peaks. Setting the derivatives to zero identifies the critical points (local maxima) at non-dimensional times c and c_s, respectively. The final condition (16) ensures that at the end of the normalized duration ($\tau = 1$), the discharge returns to zero, marking the ending of the flood.

By imposing these boundary conditions, integrating segment-specific volumes and precisely positioning the peaks via first-order derivatives, a nonlinear system of equations is established. Solving this system yields the coefficients for the [4/4] Padé rational function, providing the necessary mathematical flexibility to accurately replicate the complex, double-peaked profile of the observed flood.

Figure 5 illustrates the application of the [4/4]-order Padé approximant in modeling a bimodal flood hydrograph for the Câinelui river at the Vârtoapele hydrometric station ($Q_{1\%}$ = 82 m^3/s, Q_s = 67 m^3/s, T_t = 5.4 days, T_s = 1.25 days, T_p = 3.25 days, F_{bh} = 535 km^2, L_r = 106 km). As shown, the rational function $q_P(\tau)$ successfully captures the complex flow variations characteristic of multi-peak flood events, demonstrating the model's ability to represent delayed basin responses or the synchronization of flood peaks from different tributaries. The normalized flow values range from 0 to 1.0, with the bimodal nature clearly visible through two distinct maxima separated by an intermediate trough.

The second method approximates the function by discretizing the flood hydrograph.

Specifically, using the mean hydrograph method, it solves a nonlinear system of equations that best fits the defined data points.

Instead of decomposing the flood into analytical components (as in the first method), this approach works directly with observed or synthetic data points from the flood hydrograph. The continuous discharge function $q(\tau)$ is represented by a series of discrete measurements (t_j, q_j). This discretization transforms the problem into a curve-fitting

method, where the [4/4] Padé approximant must pass near these data points. Additional constraints are applied for the shape coefficient, rise time, and total flood duration.

Since the system is typically overdetermined (more data points than unknowns), the method uses nonlinear least squares optimization (such as the Levenberg–Marquardt algorithm mentioned earlier) to minimize:

$$S = \sum_{j=1}^{m} \left(q_j - q_P(\tau_j)\right)^2 = min \quad (17)$$

Additional constraints are incorporated to ensure the physical consistency of the solution. Specifically, these involve controlling the shape coefficient and the peak discharge, maintaining methodological alignment with the previously described approach.

In practical implementation, the Padé coefficients are obtained using iterative nonlinear optimization, the Levenberg–Marquardt algorithm, starting from initial approximations derived from the RRF model. The algorithm updates the coefficient vector until convergence is achieved according to the selected tolerance criteria.

For numerical stability and reproducibility of the optimization procedure, the Levenberg–Marquardt algorithm was implemented using standardized parameter settings. Initial coefficient values were selected based on lower-order rational approximations to ensure convergence stability. The termination criteria included both a maximum iteration number of 2000 and a convergence tolerance of 10^{-6} for the residual norm.

The fitting accuracy was evaluated using the Root Mean Square Error (RMSE) and the relative error of flood volume, ensuring that both local and integral characteristics of the hydrograph are preserved. These settings ensure reproducibility of the optimization results and allow consistent implementation in engineering software. For example, for the Câinelui River at the Vârtoapele hydrometric station, the following coefficients for the [4/4]-order Padé approximant are obtained: α_{1-8} = (16.242, −100.82, 215.81, −131.231, 2.605, −9.131, −11.406, 49.996).

And this method validates the superior flexibility of higher-order rational functions in reproducing realistic flood events that exhibit nonlinear behavior, features that lower-order models would endeavor to capture accurately. The smooth transitions between peaks and the gradual recession limb further demonstrate the effectiveness of the adopted relationship in maintaining physical consistency while providing the degrees of freedom necessary for modeling complex hydrographs. It should be noted that this method is also useful for the graphical representation of unimodal flood hydrographs with atypical shapes.

2.4. Natural Cubic Spline

The employment of Spline Interpolation functions offers a numerically precise and highly adaptable methodology for the accurate plotting of the atypical design flood hydrograph. These are usually the results of a modified hydrologic regime, such as the propagation or transit of a flood hydrograph due to hydraulic structures (or water-resource infrastructure). Unlike predetermined analytical functions, the spline method does not impose a rigid mathematical shape but rather constructs a curve that passes precisely through all the predefined points of the Design Flood Hydrograph. This characteristic ensures a rigorous adherence to the hydrological reality established by the design data, without introducing distortions related to asymptotic behavior or fixed mathematical constraints. By utilizing cubic splines, engineers can achieve smoothness and continuity across the entire curve, accurately reflecting the dynamic transitions between the rising and recession limbs of the single-peak flood hydrograph.

Adopting the spline interpolation approach within standardized computer applications offers a significant advantage for consistency and verification. Since the spline

function is uniquely determined by the input coordinates (the design hydrograph points), any engineer using the same set of input parameters in the standardized software will generate an identical design flood hydrograph.

A core challenge lies in the accurate representation of flood events within the modified hydrological regime, a condition prevalent across much of Romania due to extensive river basin management, including large-scale water diversions, dams, water intakes, and dike systems. Given that the Romanian technical standard NP 129-2011 mandates the use of non-parametric methods for determining maximum flow floods (denoted as p%), this research proposes to define the design flood hydrograph using data points for interpolation with spline curves. The resulting graphical representation, Figure 6, of the non-parametric design flood hydrograph requires an efficient approach to data provision. This is achieved by representing the hydrograph using a minimum set of hydrologically relevant points that ensure the accurate capture of the hydrograph's curvature across its essential phases (peak, body, rising tail, and recession tail). For the plotting of non-parametric hydrographs via interpolation, times of rise of the maximum discharge percentiles, namely 75%, 50%, 25%, and 5%, and the characteristic durations W75%, W50%, W25%, and W5% have been adopted. These descriptors serve as reliable indicators to characterize the flood hydrograph shape. The descriptor $Wq\%$ represents the total duration during which the flood discharge exceeds the q-th percentile of the maximum flow [35,36].

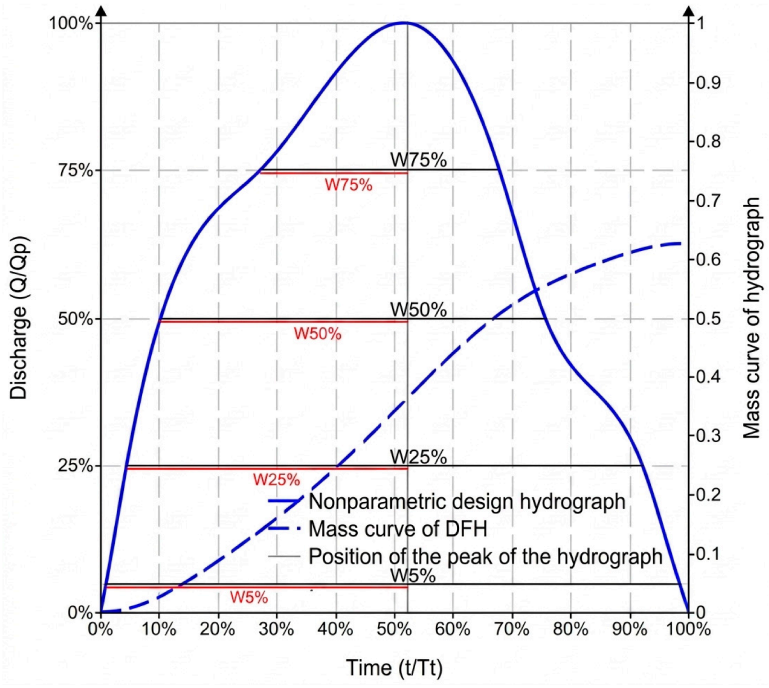


Figure 6. Example nonparametric design hydrograph.

The selection of the 75%, 50%, 25%, and 5% percentiles as control points is a strategic decision designed to capture the geometric profile of a flood hydrograph using a minimal dataset.

These specific thresholds enable the efficient discretization of the hydrograph throughout its entire temporal evolution. The 75% and 50% zones are established benchmarks in international literature for defining the "peak region" or the crest of the flood. Conversely, the 25% and 5% percentiles are included to ensure an accurate representation of the lower sections, specifically the extended recession limb (the "flood tail").

The use of 75%, 50%, and 25% is not only intuitive, following the logic of quartiles, but is also validated by established international research [30,31]. Furthermore, since the flood

volume is the integral of the area under the discharge curve, the inclusion of the 5% control point is critical; it ensures that the hydrograph's duration (width) is not underestimated. This preserves the total flood volume, establishing the MathCAD implementation as a reliable standard for safety-critical hydraulic infrastructure [3].

This methodology successfully balances the "universal" need for simplicity, allowing for easy software integration, with the "technical" requirement for accuracy. The result is a synthetic hydrograph that is both computationally accessible and hydrologically rigorous. This approach simplifies the hydrograph representation proposed in the NP 129-2011 standard by ensuring accurate interpolation, without significantly compromising a crucial flood parameter: the volume.

For the purpose of representing the Non-Parametric Design Flood Hydrograph (DFH), a discrete set of control points (t_i,Q_i) is extracted from the empirically derived non-parametric hydrograph. This set includes the maximum discharge ($Q_{max\ p\%}$), the start and end points of the hydrograph, and intermediate points defined by the percentile descriptors Wq%.

The discharge $Q(t)$ is defined by a Cubic Spline function $S(t)$, which is a composite function formed by piecewise third-degree polynomial segments, $S_i(t)$, defined over each subinterval $[t_i, t_{i+1}]$. The method ensures a high degree of smoothness by imposing C^1 (first derivative) and C^2 (second derivative) continuity at all internal nodes [37,38].

$$S_i(t) = a_i \cdot (t - t_i)^3 + b_i \cdot (t - t_i)^2 + c_i \cdot (t - t_i) + d_i \tag{18}$$

$$\frac{d^2}{dt^2} S_i(t) = \frac{d^2}{dt^2} S_{i+1}(t) \tag{19}$$

$$\frac{d}{dt} S_i(t) = \frac{d}{d} S_{i+1}(t) \tag{20}$$

Importantly, the specific implementation uses Natural Cubic Spline interpolation, which is defined by the boundary condition [39]:

$$\frac{d^2}{dt^2} S_{start} = 0 \quad \frac{d^2}{dt^2} S_{end} = 0 \tag{21}$$

This condition, that the second derivative is zero at the flood's beginning and end, reflects the physical reality that the curvature of the hydrograph is negligible near the base flow, where the rate of change of the hydrograph slope approaches zero.

The use of spline interpolation provides significant hydrological advantages, one of the most important aspects is the conservation of the total flood volume (V). Once the function $Q(t) \equiv S(t)$ is mathematically defined, the volume can be precisely calculated by integrating the function over the duration of the flood.

The careful choice of Wq% control points ensures that the resulting spline function closely approximates the area under the original non-parametric hydrograph, preserving this crucial flood parameter.

Due to its inherent C^2 smoothness property, cubic spline interpolation can efficiently handle the complex and asymmetric shapes often resulting from a modified hydrological regime (e.g., basins affected by dams or embankments). This offers a superior graphical representation compared to rigid analytical functions.

3. Results

The DFH, characterized by hydrological parameters such as rise time, total duration, peak discharge, and flood volume, can be constructed using the RRF method. This approach accommodates any shape coefficient and any ratio of total duration to time-to-peak.

Extending beyond traditional analytical limits was necessary due to the constraints of the NP 129-2011 norm, which lacks the required flexibility for modeling complex flood wave profiles. As illustrated in Figure 7, the RRF method enables the generation of flood waves across an unrestricted range of the parameters that define the DFH.

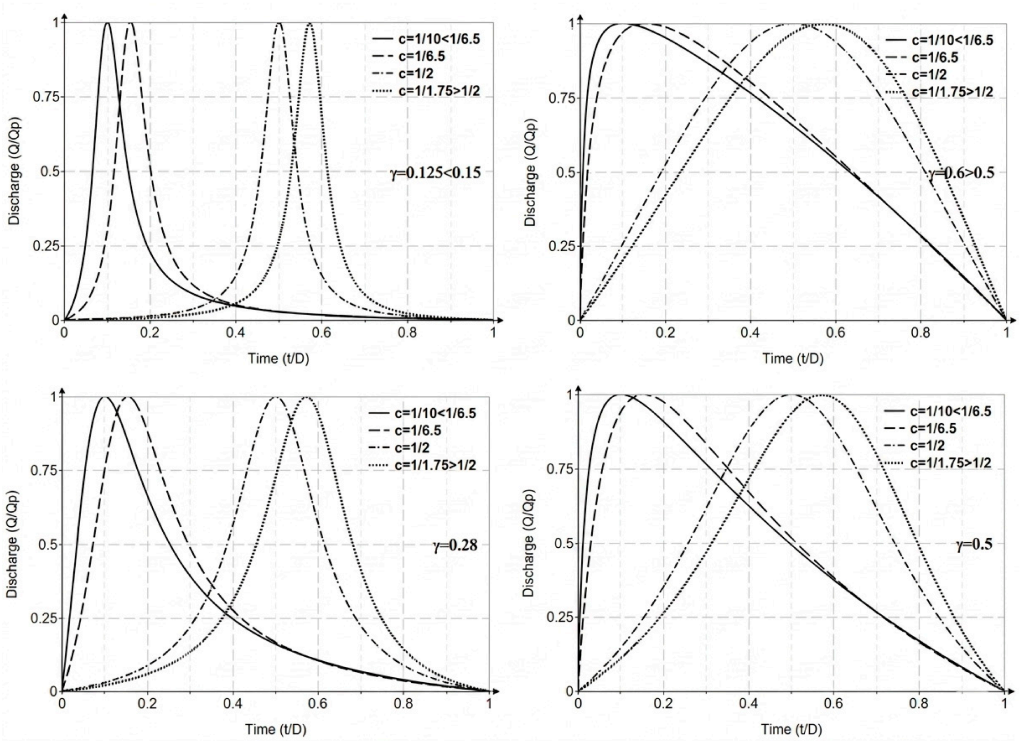


Figure 7. The design flood hydrographs plotted with the RRF method.

The efficiency of the RRF method in plotting DFH is demonstrated by its capacity to process a broad range of hydrological parameters, including rise time (T_p), total duration (T_t), peak discharge ($Q_{1\%}$), and flood volume. Deviating from the constraints of the NP 129-2011 norm, the proposed method provides the flexibility required to model complex flood wave profiles, regardless of the shape coefficient (gamma) or the peak-to-total duration ratio (c). Table 3 summarizes these parameters for various river sections in Romania, highlighting the significant variability of the shape index g, which ranges from 0.15 to 0.60.

Table 3. Characteristic flood parameters for various river sections in Romania.

Cadastral Code	River	F_{BH}	H_m	L_r	$Q_{1\%}$	T_t	c = T_p/T_t	γ
		[km^2]	[m a.s.l.]	[km]	[m^3/s]	[hr]		
XI-1	Ialomița	264	1356	32	293	20	0.15	0.18
VIII-1.176	Caracal	112	123	21	69.4	48	0.333	0.19
X-1	Capra	2.2	2036	1.5	23	18.5	0.25	0.2
XII-1.82	Buzău	680	950	59	1077	96	0.125	0.26
V-1.10	Gladna	173	282	34	118	89	0.146	0.3
XII-1.69.22	Uz	160	1074	50	470	95	0.4	0.35
XII-1.78	Bârlad	7220	211	207	400	215	0.15	0.44
XIII-1.15	Jijia	5535	155	227	500	265	0.25	0.45
II-2	Crasna	1518	268	91	424	100	0.28	0.48
XI-1	Ialomița	10,100	345	415	629	410	0.46	0.5
XIV-1.44	Dunărea	576,230	350	1075	16,350	336	0.36	0.6

Table 4 details the specific combinations of shape coefficients (γ) and dimensionless peak-to-total duration ratios (c) used to calibrate the RRF model, along with their corresponding computed parameters (A) that define the diverse hydrograph profiles illustrated in Figure 7.

Table 4. Representative parameters used in the application of the RRF method.

Shape Coefficients (γ)	Peak Time/Total Duration Ratio (c)	Parameters A	Observations
0.125	1/10	53.88	Exceeds Cadariu
0.125	1/6.5	73.20	Exceeds Cadariu
0.125	1/2	136.25	Exceeds Cadariu
0.125	1/1.75	133.46	Exceeds Cadariu
0.28	1/10	7.23	Exceeds Cadariu
0.28	1/6.5	9.51	
0.28	1/2	17.63	
0.28	1/1.75	17.25	Exceeds Cadariu
0.5	1/10	0.607	Exceeds Cadariu
0.5	1/6.5	0.952	
0.5	1/2	2.295	
0.5	1/1.75	2.228	Exceeds Cadariu
0.6	1/10	−0.177	Exceeds Cadariu
0.6	1/6.5	−0.017	Exceeds Cadariu
0.6	1/2	0.626	Exceeds Cadariu
0.6	1/1.75	0.593	Exceeds Cadariu

Notably, parameter A exhibits negative values for shape coefficients (γ) exceeding 0.5 and relative peak times (c) below 1/6.5, thereby demonstrating that the operational limits of the conventional Cadariu rational function have been transcended. Consequently, the dimensionless design flood hydrograph can be efficiently reconstructed using the final expression of Equation (3), which depends exclusively on this single parameter.

The transition of parameter A into the negative domain for $\gamma > 0.5$ suggests a fundamental attenuation of the flood hydrograph, which the original Cadariu function was mathematically unable to accommodate. This extension demonstrates that the RRF method is sufficiently robust to model hydrographs characterized by significant damping or prolonged recession limbs, features commonly observed in large basins or highly regulated catchments.

In the Refined Rational Function (RRF), parameter A represents the principal shape parameter controlling the global geometry of the dimensionless flood hydrograph. The value of A governs the distribution of discharge over time by regulating the relative balance between the rising limb and the recession limb of the hydrograph.

The coefficient γ describes the degree of "fullness" of the hydrograph, expressing how much of the theoretical rectangular volume ($Q_p{}^*T_t$) is effectively occupied by the flood wave. Consequently, parameter A acts as the modelling control variable that determines how the flood volume is distributed around the peak discharge.

Variation in parameter A produces systematic changes in hydrograph geometry. When A approaches zero, the hydrograph becomes more peaked and triangular, indicating a rapid rise and rapid recession of flow. This configuration corresponds to relatively small values of the shape coefficient ($\gamma \approx 0.15$–0.3) and is typical of basins characterized by low storage capacity, steep slopes, or highly impermeable surfaces. Such hydrographs are commonly associated with flash floods, where the flood wave propagates rapidly and the recession limb is short.

As parameter A decreases and takes negative values, the hydrograph becomes progressively flatter and more rectangular in shape. Negative values of A correspond to situations

where the flood volume is relatively large compared to the peak discharge, producing shape coefficients γ greater than 0.5. From a hydrological perspective, this behavior indicates strong attenuation processes within the basin–river system. The discharge remains close to the peak value for a longer duration, and the recession limb becomes more gradual. This type of hydrograph is typical for large basins, low-gradient rivers, floodplains, or regulated catchments influenced by reservoirs, wetlands, or hydraulic structures.

Hydrographs characterized by negative A values typically require larger storage volumes for reservoirs because the flood wave maintains high discharge values for extended durations. In contrast, hydrographs associated with A values close to zero require design solutions focused primarily on peak discharge capacity, such as spillway sizing or culvert capacity verification.

In ungauged basins, parameters A and c may be regionalized using physiographic descriptors such as basin area, slope, land use, and storage index. In this context, negative values of A are expected in basins with significant storage capacity or regulated flow regimes. The ability of the RRF formulation to represent negative A values therefore significantly extends the applicability of the classical Cadariu model, allowing the representation of hydrographs that were previously outside the admissible mathematical domain.

Overall, the introduction of parameter A as a continuous descriptor of hydrograph geometry provides a physically interpretable link between basin characteristics and flood wave shape. The possibility of negative values allows the model to represent sustained flood events with large volumes, improving the realism of design hydrographs used in hydraulic engineering applications.

The stability of the RRF method across these expanded parameter ranges facilitates its integration into automated hydrological modeling software, where manual adjustment of traditional rational functions would otherwise fail. This approach accommodates any shape coefficient and any ratio of total duration to time-to-peak. Extending beyond traditional analytical limits was necessary due to the constraints of the NP 129-2011 standard, which requires additional flexibility for modeling complex flood wave profiles. As illustrated in Figure 6, the RRF method enables the generation of flood waves across an unrestricted range of the parameters defining the DFH.

To illustrate the application of the RRF method, DFH for a cross-section of the Brescioara River in the Olt River basin is presented. The hydrograph corresponds to an annual exceedance probability of 1% ($Q_{1\%}$ = 102 m^3/s), with a total duration of 35 h and a time-to-peak of 9 h. The base flow is considered to be twice the multiannual average flow ($Q_b = 2 \times Q_m = 2 \times 0.694$ $m^3/s = 1.39$ m^3/s).

The graphical representation, Figure 8, of the dimensional hydrograph is obtained by converting the dimensionless flood hydrograph using the following relationship (Equation (22)):

$$Q(t) = (Q_p - Q_b)q(\tau) + Q_b, \; where \; \tau = \frac{t}{T_t}, \; t = 0..T_t \tag{22}$$

The Design Flood Hydrograph (DFH) obtained via the [4/4]-order Padé approximant closely matches the NP 129-2011 standard (Figure 9). This rational function provides a simplified representation while preserving essential flood characteristics, making it highly effective for water management simulations and modeling. For example, the peak flood flow for the Ialomița River at the Țăndărei gauging station is presented, with a 1% annual exceedance probability.

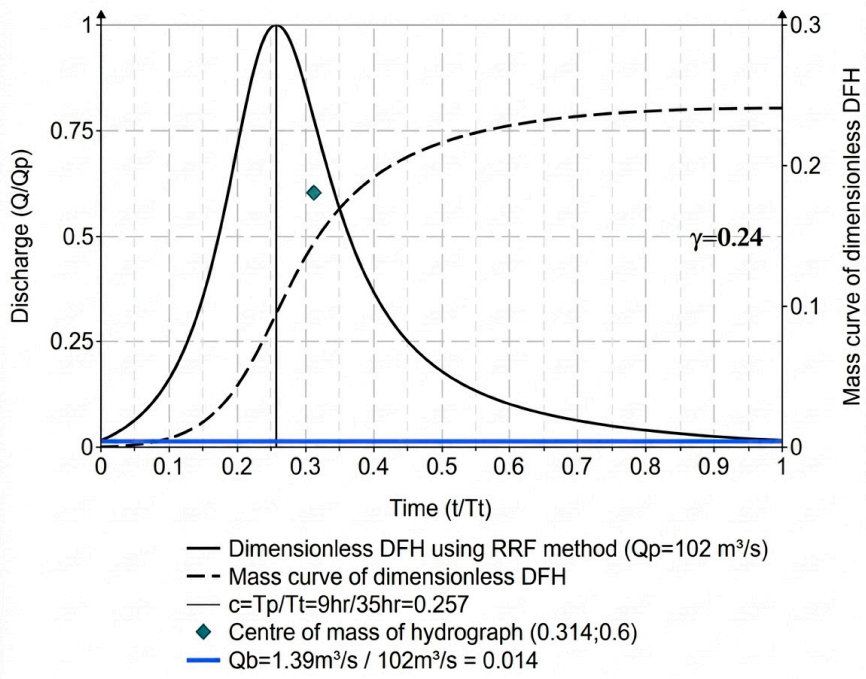


Figure 8. Example of design flood hydrographs plotted with the RRF method.

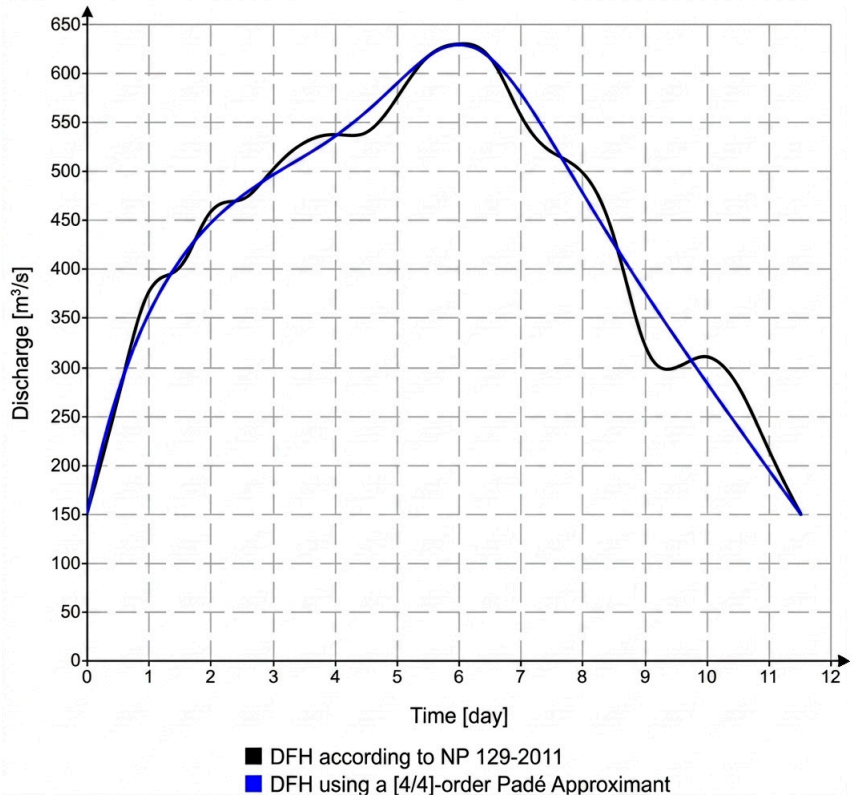


Figure 9. The design flood hydrographs plotted with the [4/4] Padé approximants (Higher-order Rational Function).

The eight coefficients defining the higher-order rational function (a [4/4]-order Padé approximant) for the dimensionless Design Flood Hydrograph (corresponding to Equation (8)) are 7.338, −32.612, 51.713, −26.44, 0.916, −12.665, 17.855, and 3.104. To reconstruct the design flood hydrographs, the following parameters are also provided: the base flow (Q_b = 150 m^3/s), the peak flow ($Q_{1\%}$ = 629 m^3/s), and the total flood duration

(T_t = 11.5 days). It should be noted that plotting the DFH using this method requires more data points compared to the Refined Rational Function (RRF) method, which only necessitates parameter A, the ratio between the time to peak (c), and the total duration (T_t).

This example further illustrates the versatility of the [4/4] Padé approximant for dimensionless design flood hydrographs (DFH), extending to shape coefficients well beyond the standard range ($0.15 \leq \gamma \leq 0.50$) of the Cadariu relation, a method promoted by the first author for over 25 years (Equations (23) and (24)).

$$\gamma_q = \int_0^1 q_P(\tau)d\tau = 0.604 \tag{23}$$

$$\gamma = \frac{\int_0^{T_t} Q(t)dt}{Q_p\ T_t} = 0.7 \tag{24}$$

Ultimately, the integration of [4/4] Padé approximants into hydrological practice facilitates a more flexible and precise modeling of design floods, essential for optimizing water management strategies and hydraulic structure safety.

Unlike rational functions, which impose a rigid mathematical shape and may exhibit restrictive asymptotic behavior, Natural Cubic Spline Interpolation (NCSI) is constructed from piecewise third-degree polynomials that pass precisely through all predefined control points, as illustrated in Figure 10.

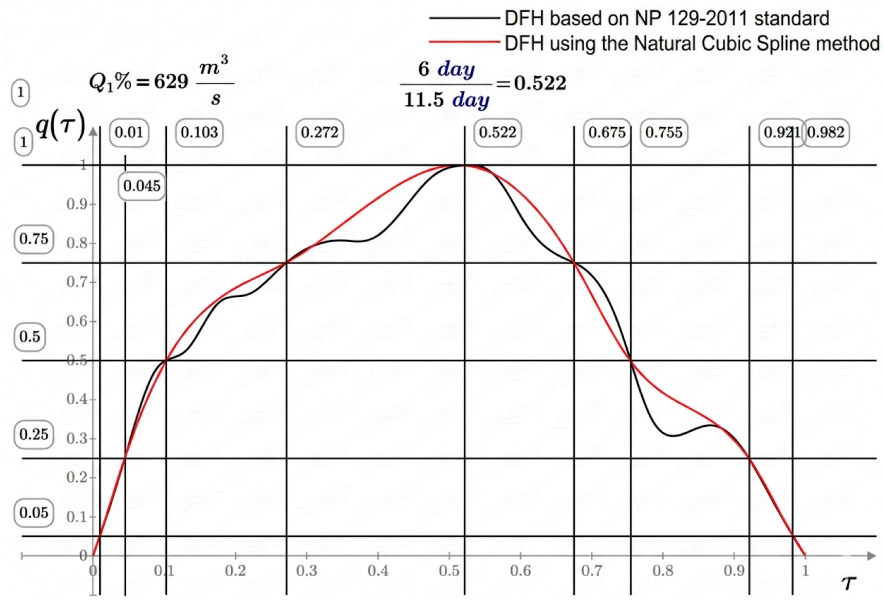


Figure 10. The design flood hydrographs plotted with the NCSI method.

The fundamental advantage of the spline approach lies in its data-driven nature, ensuring rigorous adherence to actual hydrological data without the distortions often introduced by fixed analytical constraints. The method is highly adaptable to atypical hydrograph shapes resulting from human intervention, such as dams, diversions, or dikes, where standard analytical curves frequently fail to fit the profile. One of the most critical advantages is the precise conservation of the total flood volume (W). Since the function $Q(t)$ is mathematically defined for each sub-interval, the volume can be exactly calculated via integration. Furthermore, the spline function is uniquely determined by the input coordinates ($Q_{W\%}$: 5% Q_p, 25% Q_p, 50% Q_p, 75% Q_p, and Q_p). This eliminates subjectivity, as any engineer using the same set of parameters in standardized software will generate an identical hydrograph. While highly accurate, the spline method necessitates a larger,

discrete set of coordinates compared to other methods to capture the essential phases of the flood hydrograph.

Figure 11 illustrates an example of the DFH based on NP 129-2011 (hereafter referred to as the standard hydrograph) for the Țăndărei hydrometric station, where three distinct plotting methods were evaluated. The RRF method, while offering the simplest geometric form, exhibits significant deviations in both the evolution of the mass curve and the slopes of the standard hydrograph. Although it preserves the total flood volume, it yields a root mean square error (RMSE) of 1.764. This high error value renders the RRF method inadequate for this specific application; it is included here primarily for illustrative purposes. The NCSI method provides a superior approximation of the rising and recession limbs. However, the resulting shape coefficient is γ = 0.622 compared to the actual value of 0.604, representing a 3% error. The RMSE for this method is significantly lower, at 0.038. The [4/4] Padé approximant method, in this context, proves most effective for representing complex hydrographs. It preserves the synthetic characteristics of the flood while accurately approximating the limbs. Its RMSE of 0.043 is comparable to that of the NCSI. Furthermore, the Padé method easily accommodates bimodal design flood hydrographs. For highly complex hydrographs, such as those under modified regimes, interpolation methods remain the most appropriate choice. Ultimately, these three methods should be applied on a case-by-case basis. Selecting the most suitable approach is an engineering decision in the design of hydraulic structures, guided by specific project requirements and the type of available data.

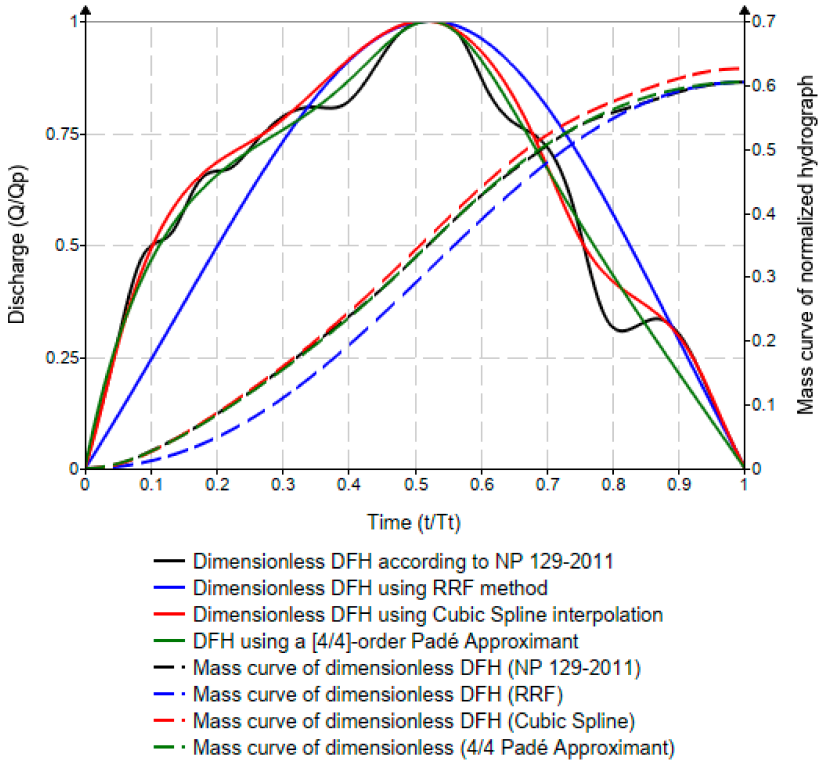


Figure 11. Comparison of dimensionless hydrographs and their corresponding mass curves.

Model performance was evaluated using dimensionless error indicators derived from normalized hydrograph coordinates. The average deviation between analytical and reconstructed hydrographs was found to be less than 5% for the RRF method and less than 3% for the Padé approximant. The spline method demonstrated minimal volume devia-tion due to exact interpolation through control points. These results confirm the numerical robustness of the proposed modelling framework.

4. Discussion

The results presented in this study demonstrate that the application of advanced mathematical methods for representing the DFH, such as the RRF, higher-order [4/4] Padé approximants and natural cubic spline interpolation represent a significant evolution over the Cadariu method, which relied on a rational function that remained mathematically underutilized.

These results demonstrate that the flexibility of higher-order rational functions allows improved representation of asymmetric hydrographs associated with storage effects and basin regulation. The extension of the admissible range of γ reflects the diversity of flood wave attenuation processes observed in large and modified catchments.

A primary critical observation concerns the versatility of these function-approximation methods, which allow for adaptation to shape coefficients (γ) that exceed the classical $0.15 \leq \gamma \leq 0.50$ range established by the Cadariu relation. This adaptability is essential in the context of current climate change, where hydrological regimes are becoming increasingly variable and atypical. The case study of the Ialomița River (Tăndărei station) clearly illustrates how higher-order rational functions preserve essential flood characteristics, providing a simplified yet robust representation for water management simulations.

A fundamental aspect addressed in this research is the simultaneous conservation of the total flood volume (W) alongside other critical hydrograph parameters.

Regarding natural cubic spline interpolation, its superiority lies in its data-driven nature. Unlike rigid analytical functions, cubic splines eliminate asymptotic distortions and ensure a precise passage through all predefined control points. This method proves indispensable for modeling multimodal hydrographs or those originating from anthropogenically modified basins (affected by dams, diversions, or embankments). The C^2 continuity (continuity of both slope and curvature) is not merely a mathematical advantage; it reflects the physical reality of flood wave propagation, where transitions between the rising and falling limbs must be smooth to remain hydraulically plausible. Furthermore, the spline method ensures high precision in calculating the integral flow, a vital parameter for designing dams and dykes. However, it must be noted that the spline approach requires a denser set of input data (percentile durations, W% and flows $Q_{W\%}$) compared to methods based strictly on analytical functions. While the natural cubic spline interpolation method inherently guarantees a zero root mean square error (RMSE) at predefined control points (e.g., 75%, 50%, 25%, and 5% of Q_p), this does not necessarily ensure a faithful representation of the hydrograph's behavior between these nodes. However, the synthetic mean hydrograph method used in Romania simplifies hydrograph shapes such that these deviations remain minimal. This method excels at modeling key synthetic flood parameters by linking peak discharges and durations with total flood volume, even for complex multimodal hydrographs.

Several limitations must be acknowledged. The dataset is limited to Romanian hydrological conditions. Uncertainty associated with parameter estimation was not explicitly quantified and represents an important direction for future research.

Despite their advantages, each proposed method presents specific limitations. The [4/4] Padé approximant requires solving nonlinear optimization problems, which increases computational complexity. The cubic spline interpolation method depends strongly on the quality and availability of control points, which may limit its applicability in data-scarce regions.

Future hydrological design methodologies must consider the increasing influence of non-stationary climatic conditions, including changing precipitation patterns and increased frequency of extreme flood events. The mathematical flexibility of the proposed framework enables adaptation to evolving hydrograph characteristics, supporting future updates of hydraulic design standards.

5. Conclusions

This research establishes a modern and versatile mathematical framework for delineating design flood hydrographs, effectively overcoming the inherent limitations of current regulations (NP 129-2011).

The study demonstrates that the core objectives—mathematical flexibility, representation accuracy, volume conservation, and practical applicability—have been decisively met.

The RRF method successfully supersedes the constraints of the classical Cadariu model, a standard promoted by the author for over 25 years.

The [4/4] Padé approximant has proven to be a formidable tool for representing dimensionless hydrographs; its ability to accommodate a vast spectrum of flood wave profiles—many of which fall entirely outside the reach of traditional analytical methods, marks a significant technical advancement.

For complex flood hydrograph modeling, natural cubic spline interpolation provides the most faithful representation.

Central to all proposed methods is a critical emphasis on volume conservation and the preservation of key flood characteristics. This ensures a direct, unwavering correlation between DFH parameters and geometric representation, a factor vital to the structural safety of hydraulic works.

Practically, the implementation of this methodology reduces design uncertainty and computational overhead, allowing for the optimized dimensioning of hydraulic structures, which directly enhances infrastructure resilience and reduces long-term construction and maintenance costs.

This research proves that only by adopting these high-order mathematical tools can the systemic geometric rigidity and volume inaccuracies of obsolete approaches be decisively overcome, establishing a new, superior benchmark for the future of hydrological design and water management safety.

The developed methodology is specifically optimized for integration into engineering software. This framework enables any practitioner to generate identical, verifiable results from the same input parameters, thereby eliminating the risks associated with arbitrary interpretation.

Compared to classical analytical formulations used in Romanian practice, the proposed methods adjust any type of design flood hydrograph while improving flood volume conservation accuracy to within 1–3% for complex flood hydrographs. These improvements support the integration of the methodology into future revisions of NP 129-2011 and related hydrotechnical standards.

In conclusion, these techniques not only redefine the accuracy of graphical representations but also provide a superior foundation for flood risk assessment and the optimization of water management infrastructure in an increasingly volatile hydrological environment.

Future research should focus on uncertainty quantification of hydrograph parameters, validation using extended international datasets, and integration of stochastic rainfall-runoff modeling approaches. The proposed framework provides a flexible basis for further development of standardized computational tools for hydraulic engineering design.

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Abbreviations

The following abbreviations are used in this manuscript:

DFH	Design Flood Hydrograph
INHGA	National Institute of Hydrology and Water Management
SUH	Synthetic Unit Hydrograph
UH	Unit Hydrograph
RFFA	Regional Flood Frequency Analysis
RMSE	Root Mean Square Error
AEP	Annual Exceedance Probabilities
RRF	Refined Rational Function
NCSI	Natural Cubic Spline Interpolation

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