

Article

Analytical Modeling of Geometrical Dot Gain Relationship Between AM and FM Halftone Screening Technologies

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Abstract

Geometrical dot gain represents a fundamental physical phenomenon influencing tonal reproduction in halftone printing, particularly in offset and flexographic processes. However, a formally defined analytical framework capable of determining the tonal conditions of equal geometrical dot gain, particularly for hybrid screening design and tonal consistency optimization, has not yet been clearly established. In this study, a geometrical analytical model is formulated to determine the transition points of equal geometrical dot gain between AM and FM screening. Two analytical approaches were applied. The first compares the total contour length of halftone elements in both screening technologies, while the second relates the AM dot diameter to predefined FM microdot sizes. Calculations were performed for eight AM screen rulings (120–340 lpi) and six FM microdot diameters (20–50 µm) under predefined geometrical conditions (2540 dpi output resolution and circular dot shape). The results indicate that transition points predominantly occur within the highlight tonal region and systematically shift toward higher tonal percentages with increasing screen ruling. Both analytical procedures, although conceptually different, yield identical results, confirming the internal consistency of the model. The analytically determined transition points provide a geometrically justified basis for defining switching zones in hybrid and XM screening systems, enabling improved tonal stability and more consistent screening transitions.

Keywords: geometrical dot gain; tone value increase; AM screening; FM screening; analytical model; transition points



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1. Introduction

Halftone reproduction remains a fundamental mechanism in modern printing technologies, where tonal values are simulated through the spatial modulation of microscopic halftone elements. The accuracy and stability of halftone dot reproduction directly determine tonal fidelity, print contrast, and overall image quality [1,2]. Among the most commonly used screening technologies are amplitude- (AM) and frequency-modulated (FM) screening, which differ in the geometrical distribution of halftone elements [3].

During the printing process, halftone dots undergo physical enlargement due to ink spreading, mechanical deformation, and ink–substrate interactions [4,5]. This phenomenon, known as physical or mechanical dot gain, represents one of the key variables

influencing tonal reproduction and print contrast [6]. Experimental studies have shown that physical dot gain depends on ink penetration into the substrate and on ink spreading properties [7–9], while significant differences in dot gain behavior have been observed between highlight, midtone, and shadow areas [10]. In addition, recent studies have demonstrated the potential of machine learning algorithms for modeling and predicting dot gain in flexographic color printing processes [11].

Existing studies predominantly approach dot gain through densitometric and colorimetric evaluation methods, focusing on tonal value increase measurement and perceptual quality assessment [12,13]. In addition, mathematical models have been developed primarily for dot gain compensation and process calibration [14–16], often relying on empirical fitting or parameters specific to printing conditions. While these approaches provide valuable tools for process control and quality optimization, they do not establish a generalized analytical framework that explicitly relates the geometrical properties of halftone elements across different screening technologies. This limitation motivates the development of a purely geometrical analytical model proposed in this study.

It is important to emphasize that dot gain is never fully compensated, as complete compensation would result in excessively bright, linear reproduction. Linear reproduction does not correspond to the characteristics of human visual perception, which exhibits a nonlinear, approximately logarithmic response to luminance and contrast variations [17]. For this reason, controlled dot gain plays an essential role in aligning technical reproduction with perceptual expectations, and standardized dot gain values are therefore fully acceptable in practical printing applications.

Numerous studies have compared dot gain behavior in AM and FM screening technologies. It has been reported that FM screening often exhibits higher dot gain in midtones, whereas in AM screening dot gain increases with increasing screen ruling [18–20]. Increasing the screen ruling increases the number of halftone dots per unit area, while in FM screening decreasing the micro-dot size increases their spatial density [21]. In both cases, the total contour length of halftone elements per unit area increases.

Although the experimental behavior of dot gain in AM and FM screening has been well documented, the literature lacks a formally defined analytical model capable of determining the exact tonal conditions under which geometrical dot gain becomes identical in both screening technologies. In particular, no analytical expression has been established for defining transition points based solely on geometrical equivalence of halftone elements.

This study is based on the theoretical premise that geometrical dot gain is proportional to the total contour length of halftone elements per unit area. On this basis, an analytical model was developed to determine the tonal values at which AM and FM screening technologies exhibit equal geometrical dot gain.

The main objective of this research is to develop an analytical model describing geometrical dot gain in AM and FM screening technologies. In this context, the study aims to:

- derive the transition conditions under which geometrical dot gain becomes identical in both screening methods;
- examine the model through two conceptually different analytical approaches;
- define the practical implications of the obtained transition points for screening selection and the design of hybrid (XM) screening technologies, as discussed in the literature [22,23].

The presented model focuses exclusively on geometrical dot gain under predefined and controlled geometrical assumptions. Optical dot gain, ink-substrate interaction, and process-dependent variables are intentionally excluded in order to isolate the geometrical component and establish a theoretical basis for further experimental studies in offset and flexographic printing systems. Existing mathematical models of dot gain are primarily

based on empirical or data-driven approaches, whereas the proposed model introduces a purely geometrical formulation. This gap motivates the development of the proposed geometrical analytical model. It should be noted that the model is not validated through experimental data within this study, as its primary purpose is to establish a theoretical framework for further experimental investigation and practical application, but is instead evaluated through analytical consistency and numerical agreement of the obtained results.

2. Materials and Methods

Since this study is based on analytical modeling rather than experimental procedures, Section 2 presents the theoretical assumptions, geometrical definitions, and analytical derivation procedures used in model development.

To establish a geometrically consistent comparison between AM and FM screening technologies, it is first necessary to formalize the theoretical principles governing geometrical dot enlargement. Since the objective of this study is to isolate and analytically describe the geometrical component of dot gain, this section defines the fundamental geometrical relationships that serve as the basis for model development.

2.1. Geometrical Dot Gain and Border Zone Theory

Geometrical dot gain represents the physical increase in the area of a halftone element caused by mechanical ink spreading and deformation during printing. Unlike optical dot gain, which results from light scattering within the substrate [24,25], geometrical dot gain directly modifies the effective printed area of the halftone dot.

According to the border zone theory, dot enlargement can be modeled as a circular ring of approximately constant width surrounding the nominal halftone element. Under stable printing conditions, the width of this border zone does not depend on the initial dot diameter, tonal percentage, or screening method [26]. Instead, the additional area generated by dot enlargement is proportional to the circumference of the halftone element.

If the border width is assumed to be constant, the increase in dot area becomes directly proportional to the circumference of the original halftone dot [27]. Consequently, the total geometrical dot gain per unit area is proportional to the cumulative contour length of all halftone elements within that area (Figure 1).

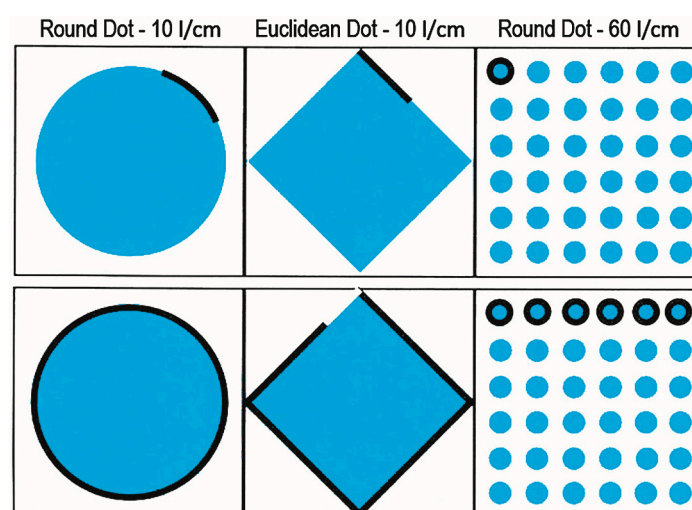


Figure 1. Halftone dot edge interaction at 50% dot area.

This proportionality forms the fundamental theoretical basis of the analytical model developed in this study. By comparing the total contour length per unit area in AM and

FM screening, it becomes possible to define conditions under which geometrical dot gain is equivalent in both technologies.

2.2. Amplitude-Modulated (AM) Screening

In amplitude-modulated (AM) screening, halftone elements are arranged on a regular grid defined by a constant screen ruling (lines per inch or lines per centimeter) [28]. Tonal modulation is achieved by varying the diameter of the halftone dot within a fixed halftone cell. Darker tonal values are represented by larger dots, while highlights are reproduced with smaller dots.

The number of halftone elements per unit area in AM screening is determined exclusively by the screen ruling. For a given tonal value, increasing the screen ruling results in a larger number of smaller halftone elements. Consequently, with increasing screen ruling, the total contour length per unit area also increases, which is directly associated with a potential increase in geometrical dot gain.

At fine screen rulings, reproduction in the midtone region is highly stable. However, in highlight and shadow areas, the smallest halftone elements may be lost. Loss of detail in shadow areas results in reduced print contrast, while in highlight regions it may lead to an abrupt tonal discontinuity known as the tone clipping effect [29]. This phenomenon is particularly pronounced at high screen rulings, where the minimum dot size may approach the dimension of a single imaging pixel (e.g., approximately 10 μm), representing a practical limitation of reproduction.

In practice, conventional AM screening is most commonly used up to approximately 200 lpi, whereas at higher screen rulings, hybrid solutions such as XM screening are applied, combining characteristics of AM and FM screening [30]. These transitional regions between different screening approaches further emphasize the need for a clear geometrical comparison criterion and an analytical definition of equivalence between AM and FM screening technologies.

2.3. Frequency-Modulated (FM) Screening

In frequency-modulated screening, tonal modulation is achieved by varying the number (frequency) of halftone elements, while their size remains constant. FM screening, also known as stochastic screening, overcomes many of the limitations of AM screening because it modulates the number or occurrence of dots rather than their size [31]. The microdots typically have a fixed diameter for a given implementation, and the tonal value is controlled by their spatial distribution density.

In practice, FM screening uses microdots with sizes in the range of approximately 10–60 μm , i.e., dimensions that are sustainable on the printing plate and stable during the printing process. Instead of placing halftone elements on a regular geometric grid, as in AM screening, FM screening distributes microdots according to the tonal value of the image. Although this distribution may appear random and unordered, it is in fact the result of controlled stochastic processes governed by algorithmic rules that regulate local dot density and spatial distribution [32]. The visual appearance of an FM screen closely resembles silver halide grains in photographic film, contributing to a perception of fine structure and smooth tonal transitions.

The relatively small dot size combined with a high spatial distribution results in a sharp image with a large amount of detail, enabling maximum utilization of high-resolution image data. The halftone element is not necessarily generated from a single pixel; it may also be defined by clustering individual pixels within matrices ranging from 4×4 to 16×16 pixels, which further enhances reproduction stability and tonal control [33].

For a given microdot diameter, the number of halftone elements required to achieve a specific tonal value is proportional to the desired dot area. Since the dot size does not change, the total contour length per unit area primarily depends on the density of halftone elements. Due to the large number of small microdots, particularly in highlight and midtone regions, the cumulative contour length per unit area can differ significantly from that of AM screening at the same tonal value.

This geometric distinction between a system with variable dot size (AM) and a system with variable dot density (FM) forms the basis for establishing analytical conditions of equivalence between the two screening technologies. By expressing the geometry of AM and FM screens through the total contour length per unit area, it is possible to establish a common analytical framework, enabling the derivation of transition points corresponding to equal geometric dot gain. The proposed analytical framework is applicable to halftone imaging systems based on AM, FM, and hybrid (XM) screening, particularly in high-resolution plate-making and printing processes such as offset and flexography.

3. Analytical Modeling of Geometrical Dot Gain

The objective of this study is to develop and analytically formulate a geometrical model capable of determining the tonal conditions at which geometrical dot gain in AM and FM screening technologies attains identical values. The proposed model is based on the proportional relationship between geometrical dot gain and the total contour length of halftone elements per unit area, as established in the previous section.

By systematically combining the fundamental screening parameters, the AM screen ruling and the diameter of the FM microdot, the model enables the calculation of dot percentages corresponding to the transition points of equal total contour lengths in both types of screening technologies.

In order to ensure controlled and reproducible geometrical conditions, two basic screening parameters are predefined within the model framework: a standard output resolution of 2540 dpi and a circular (round) dot shape. The circular dot shape was originally developed for flexographic printing applications, where dot coalescence occurs only after approximately 70% dot area, followed by the appearance of pillow shaped voids [34]. This geometrical behavior makes the circular dot particularly suitable for analytical modeling within the highlight tonal region.

The assumption of a fixed output resolution does not represent a limitation of the model but rather a reference condition for analytical consistency. Increasing the output resolution leads to a more accurate representation of halftone dots, particularly in highlight regions, where the dot geometry approaches an ideal circular form. Therefore, higher resolutions improve the geometrical fidelity of the model rather than constrain it.

Regarding dot shape, the model is formulated for circular geometries due to their analytical simplicity and relevance in flexographic applications. However, the proposed approach can be extended to other dot shapes by applying the corresponding geometrical expressions for contour length. In such cases, the transition points would be recalculated accordingly, while the underlying principle of contour-based equivalence remains unchanged.

Based on the defined geometrical assumptions, model evaluation was performed using eight commonly used AM screen rulings (120, 133, 150, 175, 210, 240, 280, and 340 lpi) and six representative FM micro-dot diameters (20, 25, 30, 36, 40, and 50 μm) were selected. These parameter ranges reflect typical industrial implementations and allow systematic comparison across coarse and fine screening conditions.

The analytical model is implemented through two conceptually different analytical derivation procedures. The first procedure establishes equivalence by comparing the total circumference of halftone elements in AM and FM screening. The second procedure derives

the nominal diameter of the AM halftone dot corresponding to a given tonal value and relates it to the FM micro-dot diameter. Together, these derivations constitute a unified geometrical model for defining transition points of equal geometrical dot gain. Although based on different geometrical formulations, both approaches describe the same underlying relationship and lead to analytically consistent results.

The following subsections present the mathematical formulation of each derivation procedure in detail.

3.1. Model Based on the Ratio of FM and AM Dot Circumference

The first implementation of the proposed geometrical model is based on the assumption that geometrical dot gain is proportional to the total contour length of halftone elements per unit area (Figure 2), as defined in the previous section. In accordance with this principle, the condition of geometrical equivalence between AM and FM screening is achieved when the total contour length per unit area (total circumference of halftone elements) in both systems attains the same value ($C_{FM} = C_{AM}$). The applicability of this strategy lies in the fact that it enables the determination of the dot percentage at which geometrical dot gain is identical for both screening technologies [35].

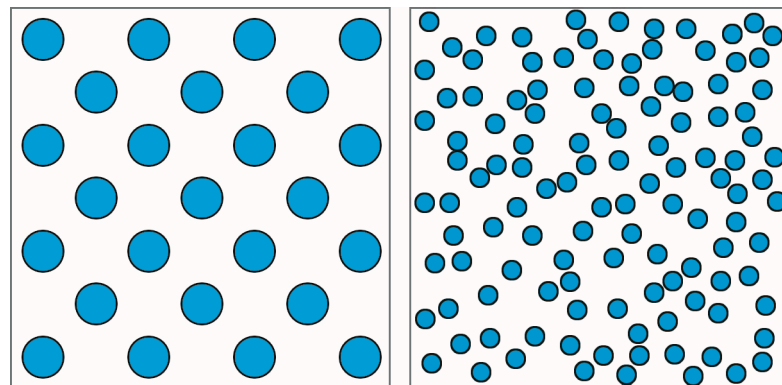


Figure 2. Schematic representation of geometrical dot gain in AM and FM screening at 20% dot area.

3.1.1. Geometrical Formulation for FM Screening

In FM screening, the microdot diameter remains constant, while the tonal value is achieved by varying the number of microdots per unit area. The number of microdots is proportional to the desired dot percentage, since higher tone values correspond to a greater spatial density of dots.

For the geometrical formulation of the model, it is first necessary to derive the expression for the total contour length (circumference) of halftone elements in FM screening, based on the area of a circular halftone element. If d_{FM} denotes the diameter of the microdot and n the number of halftone elements per unit area in FM screening, then the total area of halftone dots A_D is given by:

$$A_D = n \frac{\pi d_{FM}^2}{4} \quad (1)$$

The above formulation assumes an ideal uniform distribution of microdots within the observed area, which enables a direct analytical relationship between dot number and total area coverage. In practical FM screening, the distribution is governed by stochastic algorithms and may deviate from perfect uniformity at the local level. However, this assumption remains valid for describing average behavior.

From Formula (1), it follows:

$$n = \frac{4A_D}{\pi d_{FM}^2} \quad (2)$$

By using Formula (2), the circumference of all dots C_{FM} in the FM screen is:

$$C_{FM} = \pi d_{FM} n = \pi d_{FM} \frac{4A_D}{\pi d_{FM}^2} = \frac{4A_D}{d_{FM}} \quad (3)$$

3.1.2. Geometrical Formulation for AM Screening

In AM screening, the number of halftone elements per unit area is defined by the screen ruling (L), while the diameter of the halftone dot (d_{AM}) varies according to the tonal value. For a predefined screen ruling, the number of halftone cells remains constant, whereas the dot diameter is a function of the dot percentage.

In accordance with the formula used to calculate the total area of halftone dots in FM screening (1), the corresponding value in AM screening is given by:

$$A_D = \frac{\pi d_{AM}^2 L^2}{4} \quad (4)$$

From Formula (4), it follows that

$$d_{AM}^2 = \frac{4A_D}{\pi L^2} \quad \Rightarrow \quad d_{AM} = \frac{2}{L} \sqrt{\frac{A_D}{\pi}} \quad (5)$$

Thus, by substituting Equation (5) for the AM dot diameter, the total circumference of halftone dots per unit area in AM screening can be derived as follows:

$$C_{AM} = \pi d_{AM} L^2 = \pi L^2 \frac{2}{L} \sqrt{\frac{A_D}{\pi}} = 2L \sqrt{\pi A_D} \quad (6)$$

3.1.3. Condition of Equivalence and the Fundamental Model Equation

The derived expressions for the total circumference of halftone elements in FM (3) and AM (6) screening can now be placed into a ratio, since their relationship represents the key criterion for determining the tonal value at which geometrical dot gain is identical in both screening technologies. In other words, geometrical equivalence is achieved when the total contour length per unit area attains the same value in both the AM and FM systems.

The circumference of FM dots per unit area is denoted by C_{FM} , the circumference of AM dots by C_{AM} , the total area of halftone dots by A_D , the FM dot diameter by d_{FM} , and the screen ruling expressed in L/mm by L (L/mm). The formula for the ratio of the total circumference in FM and AM screening, applied in the analytical calculation, is given by:

$$\frac{C_{FM}}{C_{AM}} = \frac{\frac{4A_D}{d_{FM}}}{2L \sqrt{\pi A_D}} = \frac{2}{d_{FM} L} \sqrt{\frac{A_D}{\pi}} \quad (7)$$

In a situation where the ratio of the FM and AM dot circumference is equal to 1, it means that the circumference is equal, and thus the geometrical dot gain is equal. If the ratio is greater than 1, the circumference of the FM dots is greater, and thus the geometrical dot gain is greater and vice versa.

Therefore, the dot percentage where the dot gain is identical in both screening technologies is valid only when their ratio is equal to 1:

$$\frac{C_{FM}}{C_{AM}} = 1 \quad \Rightarrow \quad 1 = \frac{2}{d_{FM} L} \sqrt{\frac{A_D}{\pi}} \quad (8)$$

This condition is based on the assumption, derived from border zone theory, that geometrical dot gain is proportional to contour length. Furthermore, from Formula (8), A_D can be derived:

$$A_D = \frac{\pi}{4} d_{FM}^2 L^2 \quad (9)$$

The final obtained value is multiplied by 100 to be expressed as a dot percentage ($D_{\%}$):

$$D_{\%} = \frac{\pi}{4} d_{FM}^2 L^2 \cdot 100 \quad (10)$$

For the calculation of dot percentage at which geometrical dot gain is identical, the previously selected characteristic values of AM screen ruling (L) and FM microdot diameter (d_{FM}) are used, with all quantities expressed in millimeters.

If the diameter of a halftone element increases by Δd , the area of an individual halftone element increases by ΔA . Since, in FM screening, there are n elements of identical diameter d_{FM} , the total increase in dot area can be expressed as:

$$\begin{aligned} \sum_{i=1}^n \Delta A_i &= n \Delta A = n \left[(d_{FM} + \Delta d)^2 \frac{\pi}{4} - d_{FM}^2 \frac{\pi}{4} \right] \\ &= \frac{n \pi \Delta d}{4} [2d_{FM} + \Delta d] \end{aligned} \quad (11)$$

The expression in Equation (11) shows that the increase in dot area consists of both a term proportional to the contour length and a second-order term associated with Δd^2 , which is implicitly contained within the expanded form of $(d_{FM} + \Delta d)^2$. Under typical printing conditions, where the increase in dot diameter is small in relation to the nominal dot size ($\Delta d \ll d$), the quadratic term becomes negligible. Consequently, the geometrical dot gain can be approximated as being proportional to the contour length, which forms the basis of the proposed analytical model.

3.2. Model Based on the Calculation of AM Dot Diameter

The second method of the proposed geometrical model is based on calculating the nominal diameter of the AM halftone element and relating it to the predefined FM microdot diameter. Unlike the first approach, which relies on the equivalence of total contour length, this method evaluates geometrical dot gain based on the relationship between AM and FM dot sizes. Using the derived formula, the AM dot diameter is calculated for tonal values in the highlight region (1–10% dot area), allowing the tonal value at which geometrical dot gain is equal in both systems to be determined.

This approach enables a direct linkage between the AM screen ruling and the FM microdot diameter, providing an alternative analytical framework for determining transition points of equivalence, independent of the contour ratio based formulation.

3.2.1. Derivation of the Formula for Calculating the AM Halftone Dot Diameter

Figure 3 shows the structure of the halftone grid in AM screening, which serves as the basis for deriving the geometrical relationships required to determine the size of the halftone element. In AM screening, the number of halftone cells per unit area is defined by the screen ruling (L), while the tonal value is achieved by varying the diameter (d_{AM}) of the circular dot within each cell.

In the screening process, the smallest halftone element at an output resolution of 2540 dpi is defined by a single pixel, corresponding to an approximate dimension of 10 μm . This value represents the basic elementary unit from which larger halftone elements are formed.

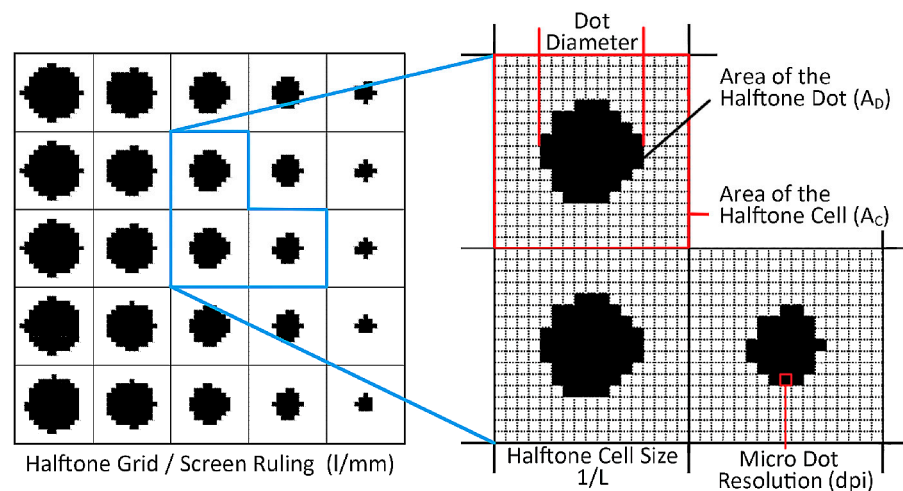


Figure 3. Halftone grid structure in the AM screening.

By increasing the number of pixels within a halftone cell, larger geometrical structures are created. Thus, a 2×2 pixel halftone element corresponds to approximately $20 \mu\text{m}$, while a 3×3 pixel element corresponds to approximately $30 \mu\text{m}$. Analogously, larger halftone element sizes are formed by combining a greater number of pixels (4×4 , 5×5 , etc.), with the physical dimension of the element increasing proportionally to the number of included pixels.

The derivation of the AM halftone element diameter (d_{AM}) starts from the definition of dot percentage $D_{\%}$ as the ratio between the area of the halftone dot (A_D) and the area of the halftone cell (A_C):

$$D_{\%} = \frac{A_D}{A_C} \cdot 100 \quad \Rightarrow \quad A_D = \frac{D_{\%}}{100} A_C \quad (12)$$

The areas of the halftone dot (A_D) and the halftone cell (A_C) are defined by the following geometrical expressions:

$$A_D = \left(\frac{d_{AM}}{2} \right)^2 \pi \quad (13)$$

$$A_C = \left(\frac{1}{L} \right)^2 \quad (14)$$

Inclusion in the initial version of Formula (12) results in the following:

$$\left(\frac{d_{AM}}{2} \right)^2 \pi = \frac{D_{\%}}{100} \left(\frac{1}{L} \right)^2 \quad (15)$$

The derivation assumes an idealized geometrical structure of the halftone cell, with uniform shape and dimensions. In practical printing conditions, deviations from this ideal geometry may occur; however, the assumption provides a consistent analytical basis for modeling the geometrical relationship.

After simplifying the expression, the final formula for the halftone dot diameter in millimeters is obtained. For presentation in micrometers, the resulting value is multiplied by 1000:

$$d_{AM} = \sqrt{\frac{D_{\%}}{25\pi L^2}} = \frac{1}{5L} \sqrt{\frac{D_{\%}}{\pi}} \cdot 1000 \quad (16)$$

The above formula is used to calculate the nominal diameter of the AM halftone element, i.e., the theoretical size of the halftone dot defined solely by the geometry of the halftone grid, without the influence of geometrical dot gain.

To convert the screen ruling expressed in lines per inch (LPI) into lines per millimeter (L/mm) at an output resolution of 2540 dpi, the following relationship is used:

$$L/\text{mm} = \frac{LPI}{25.4} \quad (17)$$

3.2.2. Matching the Size of the FM Micro Dot with the AM Dot Percentage

The dot percentage $D\%$, as a function of the halftone dot diameter d_{FM} and the screen ruling L is obtained from Equation (16):

$$D\% = \pi(5d_{FM} L)^2 \quad \Rightarrow \quad D\% = 25\pi(d_{FM} L)^2 \quad (18)$$

By applying this formula, it is possible to calculate the corresponding dot percentage in the AM system for predefined FM microdot diameters (20, 25, 30, 36, 40, and 50 μm). In this way, the size of the FM halftone dot is directly translated into an equivalent tonal value of the AM screen, thereby defining the transition point at which the circumference of the halftone dot in both systems is equal.

This procedure represents the final analytical step of the second model and enables a quantitative definition of equivalence between AM and FM screening based on the nominal size of the AM halftone elements. The calculation results obtained using Formula (18), i.e., the tonal values corresponding to the key transition points at which geometrical dot gain is identical.

4. Analytical Results and Interpretation

4.1. Nominal AM Halftone Dot Diameter in the Highlight Region

The nominal values of the AM halftone dot diameter in the highlight region (1–10% dot area) were calculated using formula (16). The calculations were performed for a standard output resolution of 2540 dpi and for selected screen rulings ranging from 120 to 340 lpi. The resulting theoretical dot diameter values are presented in Table 1. The values are obtained through analytical calculation based on the defined geometrical relations and therefore do not include experimental uncertainty or tolerance ranges.

Table 1. Nominal values of the AM halftone dot diameter (μm) for selected tone values.

Screen Ruling		Nominal Dot Diameter d_{AM} (μm)						
LPI	L/mm	1%	2%	3%	4%	5%	7%	10%
120	4.724	24	34	41	48	53	63	76
133	5.236	22	30	37	43	48	57	68
150	5.905	19	27	33	38	43	51	60
175	6.889	16	23	28	33	37	43	52
210	8.268	14	19	24	27	31	36	43
240	9.449	12	17	21	24	27	32	38
280	11.024	10	14	18	20	23	27	32
340	13.386	8	12	15	17	19	22	27

The nominal dot diameter values at selected tone values represent a key parameter for aligning the size of FM halftone elements with the AM system, indirectly through the dot

percentage. Based on these values, it is possible to identify the tonal value in which the geometrical dimensions of AM and FM elements correspond.

The highlighted value in Table 1 (1% dot area at a screen ruling of 340 lpi) illustrates a situation in which the nominal diameter of the AM halftone element becomes smaller than a single pixel at the defined output resolution. Since one pixel at 2540 dpi corresponds to approximately 10 μm , such a dot size cannot be physically reproduced and therefore cannot be reliably rendered in practice. Nevertheless, the reproduction of 1% dot area under these conditions may still be achieved by increasing the output resolution during screening. For example, the size of a single pixel is approximately 6 μm at 4000 dpi, about 4 μm at 5080 dpi, and approximately 10.6 μm at 2400 dpi. Increasing the output resolution therefore lowers the minimum reproducible dot size and improves the stability of very light tone reproduction.

Figure 4 illustrates the relationship between screen ruling and the nominal diameter of AM halftone dots in the highlight region. The graphical representation clearly shows the decrease in dot diameter with increasing screen ruling and with decreasing tone value.

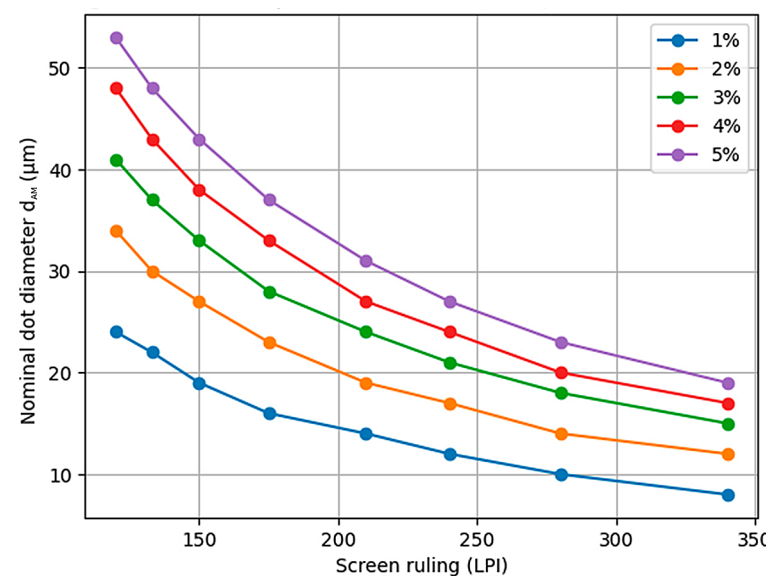


Figure 4. Relationship between screen ruling and nominal AM dot diameter in the highlight region.

The data derived from Table 1 indicate that an FM halftone element with a diameter of 20 μm is not smaller than the AM halftone element at 1% dot area for a screen ruling of 150 lpi, nor than the element at 2% dot area for 210 lpi, or 3% dot area for 240 lpi. This indicates that, in the region of very low tone values, the geometrical size of FM microdots can be comparable to or even larger than the nominal AM dots. Such relationships are particularly important when defining transition points of equivalence between the two screening technologies.

For the reproduction of the full tonal range at screen rulings of 175 lpi and higher, the prepress equipment must therefore be capable of reliably rendering a single pixel of approximately 10 μm . Otherwise, limitations in the reproduction of the lightest tonal values may occur, which may ultimately affect the accuracy of the analytical model as well as the actual tonal reproduction in printing.

4.2. Geometrical Relationship Between AM and FM Halftone Elements

The final Formulas (10) and (18) from both described analytical methods were used for the calculation in order to determine the dot percentage at which the geometrical dot gain in both halftone screening methods is identical. Since the dot percentage represents an

operational parameter in prepress workflows and tonal reproduction control, this approach enables a direct interpretation of the results in practical printing conditions. Predefined FM microdot diameters (20, 25, 30, 36, 40 and 50 μm) and selected screen rulings in the range from 120 to 340 lpi were used as input parameters. The corresponding results presented in Table 2 are derived analytically using the proposed model equations and were computed numerically to ensure consistency and accuracy. The equivalence of the results obtained using the two analytical approaches is confirmed by the identical numerical values presented in Table 2, which provide direct numerical verification of the stated relationship. This agreement follows from the fact that both approaches represent different formulations of the same underlying geometrical relationship.

Table 2. Dot percentage with an identical geometrical dot gain in the FM and AM screens.

Screen Ruling		Dot Percentage ($D\%$) for the Selected Sizes of FM Dots (%)					
LPI	L/mm	20 μm	25 μm	30 μm	36 μm	40 μm	50 μm
120	4.724	0.7	1.1	1.6	2.3	2.8	4.4
133	5.236	0.9	1.3	1.9	2.8	3.4	5.4
150	5.905	1.1	1.7	2.5	3.5	4.4	6.8
175	6.889	1.5	2.4	3.4	4.8	6.0	9.3
210	8.268	2.1	3.4	4.8	7.0	8.6	13.4
240	9.449	2.8	4.4	6.3	9.0	11.2	17.5
280	11.024	3.8	6.0	8.6	12.4	15.3	23.8
340	13.386	5.6	8.8	12.7	18.2	22.5	35.2

The results obtained by using both formulas are completely identical, which confirms the reliability of the research and the applicability of the described methods. The calculation results for the ratio $\frac{C_{FM}}{C_{AM}} = 1$ are shown indirectly through the dot percentage.

The transition point, a point with an identical geometrical dot gain, is shown as the dot percentage for all selected screen rulings. Its value increases with an increase in the screen ruling and with an increase in the size of the FM microdot, as shown in Figure 5, and based on the data from Table 2. Each observed FM halftone dot size (20–50 μm) appears across all eight analyzed screen rulings, but at different dot percentage. This clearly demonstrates that the same dot size corresponds to different tonal regions in screen systems with different rulings.

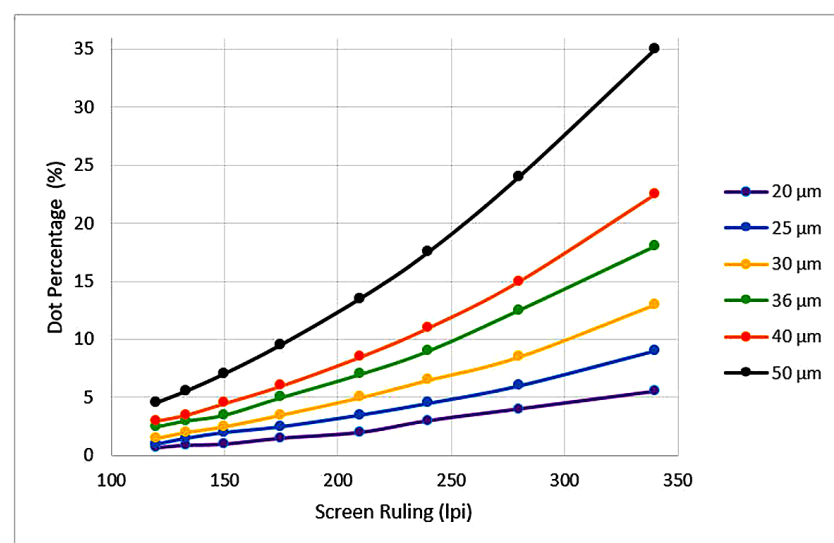


Figure 5. Transition points ($D\%$) as a function of screen ruling (L) and FM dot diameter (d_{FM}).

For the lowest selected screen ruling of 120 lpi, the dot percentage according to the FM microdot diameters from 20 to 50 μm ranges from 0.7% to 4.4%. In contrast, for the highest analyzed screen ruling of 340 lpi, this range reaches from 5.6% to 35.2%. With increasing screen ruling and FM microdot diameter, the curves in the graph exhibit a more pronounced nonlinear (exponential) character, while the tonal values shift from the highlight region toward the midtone range.

These results indicate that the practical applicability of the obtained transition points is primarily relevant for the highlight region, approximately up to 10% dot area, where the geometrical differences between AM and FM screening systems are most pronounced and where the analytical model provides the most significant contribution.

It is important to emphasize that the presented analytical methods do not determine the total dot gain occurring in the actual printing process. In addition to the type and screen ruling of the halftone, the total dot gain largely depends on the characteristics of the printing press, the dynamics of ink transfer, and the interaction between the ink and the printing substrate [36]. Its accurate determination in practice requires extensive experimental measurements, as well as considerable time and financial resources.

The proposed model therefore represents a geometrical basis for the comparison of halftone screening technologies, while real printing conditions introduce additional process variables that extend beyond the scope of this analytical approach.

4.3. Application of the Proposed Model in Hybrid (XM) Screening

XM screening represents a hybrid technology that combines AM screening in the mid-tones with FM screening in the highlight and shadow areas, with its key advantage being the ability to achieve an exceptionally smooth and continuous transition between these two screening methods. This smooth transition occurs at the transition point, defined as the moment when the AM dot size becomes equal to the FM dot size, ensuring continuity without visible artifacts in the screen structure. This point, expressed in dot percentage within the highlight region, also represents the smallest reproducible dot that a given printing system can reliably produce.

In this research, transition points were determined analytically, and the analysis included different FM dot sizes ranging from 20 to 50 μm , defined as predetermined minimum dot sizes. The results show a clear relationship between FM dot size, AM screen ruling (120–340 lpi), and the corresponding transition point, enabling the direct application of the model in defining XM screening under real printing conditions.

The analytical determination of transition points based on equal contour lengths represents a significant contribution to the design and optimization of hybrid and XM screening technologies. By providing a mathematically defined transition criterion based on the equivalence of geometrical dot gain, the proposed model introduces a structured approach to defining transition zones, reducing reliance on empirical adjustments and improving tonal consistency between different screening systems.

5. Conclusions

The analytical model is based on the principle that geometrical dot gain is proportional to the total contour length of halftone elements per unit area, where dot enlargement can be described as a circular ring of constant width.

The analytical evaluation identified a specific tonal value, predominantly within the highlight region, where geometrical dot gain in AM and FM screening coincides. These tonal values represent transition points that can be used to define switching zones between AM and FM screening in hybrid (XM) screening systems. Below these values, relatively

higher geometrical dot gain occurs in AM screening, whereas above them, greater dot gain is observed in FM screening.

By applying two conceptually different analytical approaches, a circumference-based model and a diameter-based model, identical results were obtained, as presented in Table 2. This agreement confirms the consistency and structural reliability of the proposed model. The analytically determined transition points provide a geometrically grounded and practically applicable criterion for integrating AM and FM screening, thereby reducing reliance on empirical adjustments in prepress for offset and flexographic printing. In doing so, the model establishes a formal analytical framework for systematically combining AM and FM screening within hybrid (XM) systems, directly addressing the identified research gap.

The analysis further confirmed that increasing the screen ruling results in a greater total contour length per unit area, leading to a higher geometrical dot gain for a given increment in dot diameter (Δd), consistent with established behavior in halftone printing systems.

The proposed model considers exclusively the geometrical component of dot gain and does not include optical or process-dependent factors, thereby providing a theoretical basis for further experimental research and integration into real printing conditions.

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Abbreviations

The following abbreviations are used in this manuscript:

AM	Amplitude-Modulated screening technology
FM	Frequency-Modulated screening technology
XM	Cross-Modulated screening technology

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