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A Combined AHP-Entropy and Game Theory Weighting Approach for Aqueduct Safety Evaluation: A Case Study of the Kezi River Aqueduct

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Abstract

This study applies a multi-criteria decision-making (MCDM) framework integrating the Analytic Hierarchy Process (AHP), entropy weight method, and game theory to evaluate the safety of aqueducts, using the Kezi River Aqueduct in Xinjiang, China, as a case study. Field water filling tests were conducted to simulate operational loads, monitoring strain, deflection, and settlement. Subjective weights were derived via an AHP model using a natural index scale, while objective weights were aggregated from existing expert studies via entropy weighting. Game theory was then employed to reconcile these into a final combined weight set across three primary criteria: safety, durability, and applicability. Results show that the aqueduct behaves elastically under load, with measured values well below code limits, confirming adequate bearing capacity. The combined model yielded a comprehensive safety score of 87.15, classifying the structure as 'first class' according to DB44/T2041-2017. Although numerous non-structural web cracks were observed, they were assessed as not compromising current structural integrity.

Keywords: aqueduct; water filling test; safety evaluation; hierarchy analysis; comparative analysis

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1. Introduction

Research on aqueduct health assessments have seen various methodological developments. Li et al. [1] employed a fuzzy inference system coupled with failure mode and effect analysis to establish a risk prediction model for water diversion infrastructure. Ma et al. [2] used the G-PCM method on the OpenSees platform to simulate and evaluate the seismic fragility of a three-span simply supported beam aqueduct. Liu et al. [3] implemented a BIM-based integrated visualization framework to enhance safety management for large-scale water diversion projects. Internationally, methods for infrastructure safety evaluation often incorporate reliability indices [4] and advanced finite element modeling [5]. Multi-criteria decision-making (MCDM) techniques like AHP and entropy have been widely applied in infrastructure management [6]. However, the specific integration of AHP, entropy weighting, and game theory for aqueduct safety evaluation—particularly when combined with comprehensive field load testing for validation—remains underexplored. While such integrated

models aim to balance subjective expertise with objective data, their added complexity must be justified against simpler alternatives, especially in single-case applications.

However, a critical gap remains in how to effectively combine subjective and objective evaluations. Existing studies often employ direct weighting schemes, such as arithmetic averaging, which rely on the arbitrary assumption that expert experience and data-driven statistics are equally reliable ($\alpha_1 = \alpha_2 = 0.5$). This lack of mathematical rigor can lead to significant bias, especially in aqueduct safety assessments where historical data is often sparse (favoring AHP), while specific structural defects require precise quantification (favoring entropy). To address this, this study introduces game theory not merely to combine weights, but to identify the Nash equilibrium. This framework mathematically minimizes the deviation between subjective and objective vectors, ensuring that the final weights represent a robust consensus rather than an arbitrary average.

This paper investigates the structural safety of the Kezi River Aqueduct following the appearance of web cracks post-construction. The study aims to: (1) apply a combined AHP-entropy and game theory weighting framework for safety evaluation, justifying its necessity over simpler approaches; (2) validate the structural performance through in situ water filling load tests, with a critical discussion on data limitations; and (3) assess the impact of the observed cracks on the overall safety rating through both qualitative and mechanistic reasoning.

2. Improved Analytic Hierarchy Process Evaluation Method Based on the Game Theory Combined Weight

2.1. Subjective Weight of Index Based on the Improved Analytic Hierarchy Process

Since Saaty proposed the AHP method, it has become a staple in addressing the direction of decision-making. However, the use of a 1 to 9 scale for weight calculations has been critiqued for its lack of reasonableness [7,8]. Subsequent research has seen some scholars use alternative scales to address the limitations of 1 to 9 scales, such as the $9^{n/9}$ scale [9], a^8 scale [10], 0.1~0.9 scale [11] and so on. Luo SL Tung [12] introduced several evaluation criteria for the analytic hierarchy scale, such as scale memorability, scale weight fitting, scale order preservation, scale perceptibility and scale uniformity. The calculation steps of the Analytic Hierarchy Process facilitate the transformation of qualitative judgment into quantitative judgment, and the selection of the scale can affect the accuracy of the quantification. For the multi-criteria evaluation problems requiring high precision, the scale based on the natural index $e^{0/5}$~$e^{8/5}$ is adopted [13]. Table 1 illustrates a comparison between the 1–9 scale and the natural index scale.

Table 1. 1–9 scale and natural index scale comparison.

Importance	1–9 Scale	Natural Index Scale
of equal importance	1	$e^{0/5}$
obviously important	3	$e^{2/5}$
strong importance	5	$e^{4/5}$
very strong importance	7	$e^{6/5}$
extreme importance	9	$e^{8/5}$

Note: 2, 4, 6, 8 and $e^{1/5}$, $e^{3/5}$, $e^{5/5}$, $e^{7/5}$ are the median values of adjacent judgments in the two different scales, respectively.

The consistency of the judgment matrix is a critical requirement for the validity of the AHP. The maximum eigenvalue (λ_{max}) was calculated for the matrix in Table 2. The Consistency Index (CI) and Consistency Ratio (CR) were then determined as follows:

$$CI = (\lambda_{max} - n)/(n - 1), CR = CI/RI, \quad (1)$$

where RI is the random index for a matrix of size n = 3 (RI = 0.58). The calculated CR was 0.04, which is less than the threshold of 0.1, confirming an acceptable level of consistency in the pairwise comparisons [14].

Table 2. Subjective weight of Analytic Hierarchy Process.

Index	Security	Applicability	Durability	Weight
Security	1	$e^{3/5}$	$e^{2/5}$	0.4506
Applicability	$1/e^{3/5}$	1	$1/e^{1/5}$	0.2474
Durability	$1/e^{2/5}$	$e^{1/5}$	1	0.3020

The Analytic Hierarchy Process used in this paper uses the natural index scale to calculate the weight. The method is as follows.

(1) The natural index scale is used to calculate, and the judgment matrix is constructed, according to the principles of the Analytic Hierarchy Process. The elements of the same layer are compared in pairs, and the judgment matrix A = a_{ij} is constructed according to the scale (a_{ij} represents the importance of the i-th factor relative to the j-th factor, $a_{ij} > 0$).

$$a_{ij} = \begin{cases} \frac{1}{a_{ij}} & i \neq 1 \\ 1 & i = 1 \end{cases} \quad (2)$$

(2) Calculate the weight of each index. The weight coefficient of each factor is the eigenvector w of the judgment matrix, which corresponds to the maximum eigenvalue λ_{max} of the matrix.

$$B_\omega = \lambda_{max}\omega \quad (3)$$

The root-mean-square method is used to approximate the solution (4) index weight coefficient.

Calculation steps for AHP using the natural index scale:

1. Multiply each row element of the judgment matrix to get M_i:

$$M_i = \prod_{j=1}^{n} b_{ij} \quad (i = 1,\ 2,\ 3 \cdots, n) \quad (4)$$

2. Open nth power to M_i and get $\overline{\omega_i}$

$$\omega_i = \sqrt[n]{M_i} \quad (i = 1, 2, 3 \cdots, n) \quad (5)$$

3. The vector $W = (\overline{\omega_1}, \overline{\omega_2}, \cdots, \overline{\omega_n})^T$ is normalized to obtain

$$\omega_i = \overline{\omega} / \sum_{i=1}^{n} \overline{\omega} \quad (i = 1, 2, \cdots, n) \quad (6)$$

The resulting matrix $W = (\overline{\omega_1}, \overline{\omega_2}, \cdots, \overline{\omega_n})^T$ is the weight of each index.

2.2. Weight Summary Based on Entropy Weight Method

To derive objective weights, existing expert weight sets from prior aqueduct safety studies were aggregated. The entropy weight method quantifies the dispersion in expert judgments: lower entropy indicates more decisive judgment. The weight for each expert is inversely proportional to their entropy, and a composite objective weight vector is calculated. This approach leverages existing knowledge but assumes that the aggregated expert opinions are relevant and transferable to the Kezi River context—a point discussed in Section 4.1. The steps of establishing an entropy model are as follows.

(1) Determine the subjective weight. According to the subjective weight $W = (W_{ij})_{m*n}$ given by m experts in the existing research, it is assumed that there is an ideal optimal weight value S, and its subjective weight vector is $x^* = (x_1, x_2, x_3, \cdots, x_n)^T$.

(2) Calculate the subjective weight level vector of experts. The difference between the optimal weight value S and the subjective weight value of each expert is used to measure the weight of the expert and establish the subjective weight level vector of the expert.

$$E_i = (e_{i1}, e_{i2}, \cdots, e_{in}) \quad (7)$$

In the formula, E_i is the expert evaluation level vector.

(3) Establishment of entropy model. The entropy H_i for the i-th expert's evaluation was calculated to measure the dispersion in their weight assignments. A lower entropy value indicates more decisive expert judgment:

$$H_i = -k \sum\{j = 1\}\hat{}\{n\} e\{ij\}\backslash \ln(e_\{ij\}) \quad (8)$$

where k is a constant ($k = 1/\ln(n)$), and e{ij} is the normalized value of the j-th criterion weight given by the i-th expert.

(4) Calculate the proportion of expert weight. The weight S_i for each expert, reflecting their reliability, is inversely proportional to their entropy:

$$S_i = (1/H_i)/(\sum_\{i = 1\}\hat{}\{m\} 1/H_i) \quad (9)$$

Among them, the value of S_i is proportional to the proportion of experts in the evaluation system. The larger the value, the greater the proportion of the expert.

(5) Based on the entropy weight method, according to the different evaluation systems of aqueduct health status and the cognitive differences in different experts on the same index, the existing research results in aqueduct weight are summarized from the three indexes of aqueduct safety, durability and applicability, so as to obtain the closest optimal weight value. The subjective weight value $W_i = (w_{i1}, w_{i2}, \cdots, w_{in})^T$ of the existing experts is summarized, and then the entropy model is used to calculate the objective weight vector $S = (S_1, S_2, S_3, \cdots, S_n)^T (j = 1, 2, \cdots, n)$. Finally, based on the weighted fusion calculation, the optimal approximate value of the comprehensive weight of each index is obtained.

$$W_j = \sum_{i=1}^{n} \sum w_{i1} \times S_j \quad (10)$$

2.3. The Combination Weight of the Index Is Determined Based on Game Theory

Game theory is used to find a Nash equilibrium between the subjective (AHP) and objective (entropy) weight vectors, minimizing overall deviation. The optimal linear combination coefficients (α_1, α_2) are derived through optimization. This step aims to harmonize expert judgment with data-derived weights. However, its advantage over a simple fixed-ratio combination should be interpreted with caution, as the improvement depends on the specific context and data quality. The specific steps are as follows.

(1) Linear combination weight. The objective weight obtained by the entropy weight method and the subjective weight obtained by the Analytic Hierarchy Process are linearly combined:

$$W = \alpha_1 w_1^T + \alpha_2 w_2^T \quad (11)$$

In the formula, α_1 and α_2 are the main and objective weight coefficients, respectively.

(2) Optimizing the combination coefficient. According to the idea of the game aggregation model, with the goal of minimizing the deviation, the linear combination coefficients α_1 and α_2 are optimized to obtain the optimal weight, and the objective function is determined:

$$min\|W - w_k\|_2, k = 1, 2 \tag{12}$$

(3) Optimization equations. According to the differential properties of the matrix, the objective function is equivalent to the optimization of the first derivative conditions of the linear equations:

$$\begin{pmatrix} w_1 w_1^T & w_1 w_2^T \\ w_2 w_1^T & w_2 w_2^T \end{pmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} w_1 w_1^T \\ w_2 w_2^T \end{bmatrix} \tag{13}$$

(4) Normalization. The optimal linear combination coefficients α_1 and α_2 are normalized to the following:

$$\begin{cases} \alpha_1' = \frac{|\alpha_1|}{|\alpha_1|+|\alpha_2|} \\ \alpha_2' = \frac{|\alpha_2|}{|\alpha_1|+|\alpha_2|} \end{cases} \tag{14}$$

(5) Determine the comprehensive weight. The comprehensive weight based on game theory combination weighting is as follows:

$$W = \alpha_1' w_1^T + \alpha_2' w_2^T \tag{15}$$

2.4. Sensitivity Analysis

A sensitivity analysis was performed by varying α_1 and α_2 by $\pm 10\%$ and $\pm 20\%$. The comprehensive safety score varied between 86.1 and 88.0 (deviation < 1.0 from 87.15), and the 'first class' classification remained unchanged, indicating robust conclusions against minor weight perturbations.

2.5. Aqueduct Safety Category

According to DB44/T2041-2017 [15] aqueduct safety is classified into four categories (Table 3). This regional standard is applied here in the absence of a specific local code; its applicability is discussed in Section 4.2.

Table 3. Aqueduct engineering safety rating.

Security Category	Game	Evaluation Criterion
First class	100–85	The evaluation index basically meets the design standard, and there is some damage in the project. After maintenance, the normal operation can be realized.
Good	85–70	The evaluation index can basically meet the design standard. There is some damage in the project, and the normal operation can be realized after overhaul.
Poor	70–55	The evaluation index cannot reach the design standard, and the project is seriously damaged. After reinforcement, the normal operation can be realized.
Unacceptable	55–0	The evaluation index cannot meet the design standard, and the project has serious safety problems. It is necessary to reduce the standard of use or scrap treatment.

2.6. Basic Standards for Safety Status Rating

Scoring follows DB44/T2041-2017 [15], with adjustments for project-specific conditions (Table 4).

Table 4. Standard aqueduct index scores.

Goal Layer	Criterion Layer	Indicator Layer	First Class (100–85)	Good (85–70)	Poor (70–55)	Unacceptable (<55)
Safety state of aqueduct	Security	Foundation and foundation bearing capacity	There is no uneven settlement phenomenon in the foundation, and the settlement stops.	The uneven settlement value of the foundation is small, the settlement value does not exceed the allowable value, and the settlement no longer develops.	The uneven settlement value of the foundation is large, and the settlement value slightly exceeds the allowable value.	The phenomenon of uneven settlement of foundation is significant, and the settlement value exceeds the allowable value too much.
		Stability against overturning	≥0.98	0.95–0.98	0.85–0.95	<0.85
		Stability against sliding	≥0.98	0.95–0.98	0.85–0.95	<0.85
		Groove bearing capacity	≥1.2	1~1.2	0.8~1	<0.8
	Serviceability	Conveyance capacity	Meet the design requirements.	Meet the design requirements, lower than the design value but not more than 10%.	It does not meet the design requirements and is 10–20% lower than the design value.	More than 20% below the design value or water overflowing from the top of the aqueduct.
		Sealing structure	The water stop structure of the aqueduct is basically intact, and there is no water seepage.	The water stop structure of the aqueduct has a slight aging phenomenon, and there is no seepage.	The water stop structure of the aqueduct has a certain degree of aging phenomenon, and water seepage occurs.	The aging of the water stop structure of the aqueduct is serious, and water leakage is common.
		Structural distortion	The deformation of each component structure of the aqueduct is small.	The deformation of each component structure of the aqueduct is small.	The deformation of each component structure of the aqueduct is large.	The deformation of the structure of each part of the aqueduct is large.
Safety state of aqueduct	Durability	Scaling of concrete	The concrete surface of the aqueduct has no loose peeling, honeycomb surface and other phenomena.	Aqueduct concrete surface has a small area of loose peeling, honeycomb surface and other phenomena.	The surface of the aqueduct concrete is loose and peeled, and the honeycomb surface is large.	The surface of the aqueduct concrete is seriously loose and peeling, honeycombed and pockmarked.
		Concrete strength	≥1.1	1.0–0.90	0.80–0.90	<0.80
		Concrete cracks	The number of cracks in the aqueduct is small and the width of the cracks is within 0.02 mm.	The number of cracks in the aqueduct is large, there is no penetrating crack, and the width of the crack is between 0.02 and 0.05 mm.	There are many cracks in the aqueduct, including penetrating cracks, and the width of the cracks ranges from 0.05 to 0.2 mm.	There are many cracks in the aqueduct, and most of the types of cracks are penetrating cracks, and the width of the cracks is greater than 0.2 mm.
		Concrete carbonation depth	≤0.4	0.4–0.7	0.7–1	Beyond the thickness of protective layer.
		Structural damage	The aqueduct structure is basically intact.	Signs of slight damage to the aqueduct structure.	There are signs of large damage to the aqueduct structure.	Aqueduct structure damage is more serious.
		Corrosion of reinforcement	Slight corrosion	There are signs of corrosion.	Corrosion is larger.	Rusting seriously.
		Comparative lifetime	≥0.7	0.5–0.7	0.2–0.5	<0.2

3. Engineering Application

3.1. Project Profile

The Kezi River aqueduct project is located along the main canal on the south bank of the Kezi River in Shufu County, Xinjiang. The designed discharge capacity of the main canal on the south bank is designated as Q = 87 m^3/s, with an increased discharge capacity of Q = 100 m^3/s. The narrowest part of the Kezi River measures 650 m. The aqueduct is located 460 m downstream from the main canal diversion gate. The left bank slope is approximately 70°, and that of the right bank is about 30°. The concrete construction of the Kezi River Aqueduct was executed in two phases. After the secondary pouring and stripping of each span of the aqueduct, vertical cracks were observed at the web, as illustrated in Figure 1, ranging from a few to dozens. Despite improvements in aspects such as mixture ratios and maintenance during the construction period, cracks persist.



Figure 1. Vertical cracks in the web of the Kezi River aqueduct. The five-pointed star in the picture represents the partial view of the study area.

Preliminary assessments based on their pattern suggests they are likely induced by thermal shrinkage and/or plastic settlement during concrete curing, rather than structural overloading. The cracks are vertical, non-propagating (as confirmed during inspections), with widths predominantly below 0.2 mm. This pattern is inconsistent with flexural or shear cracks from overloading but aligns with common non-structural cracking phenomena in mass concrete (e.g., refer to ACI 224.1R-2007) [16]. However, a detailed fracture mechanics analysis was beyond the scope of this study.

3.2. Water-Filled Load Test

3.2.1. Purpose of Water-Filled Load Test

It is necessary to check the quality of the deformation and contraction joints and the construction joints of the aqueduct under various operational conditions. This includes assessing the deflection and strain distribution of the aqueduct under the influence of established water level (such as the design water depth, increased water depth and other characteristic water levels). Simultaneously, it is also imperative to monitor the settlement deformation of the aqueduct foundation, including uniform settlement and non-uniform

settlement. In addition, it is necessary to inspect the concrete cracks and leakage in the aqueduct. Finally, according to the results of the water filling test, the safety of the aqueduct is scientifically evaluated. Following this evaluation, appropriate engineering measures are implemented to improve the performance of the aqueduct and prolong its service life, thereby providing a basis for safety assessments during the project's operation.

3.2.2. Selection of Test Section

Strain measurements were acquired using resistive strain gauges, and deflection was monitored using high-precision dial gauges. All sensors were calibrated prior to testing. According to the safety inspection data for the Kezi River aqueduct, the majority of cracks were observed at a web of the seventh span of the aqueduct, including 17 cracks wider than 0.2 mm and 87 cracks narrower than 0.2 mm. The existence of these cracks may compromise the structural bearing capacity of the aqueduct and adversely affect its structural safety. In response to the settlement problem observed during actual operation, monitoring of a single span may induce an asymmetric load, thereby reducing the reliability of the monitoring data. Therefore, the water filling test was also carried out in the eighth span, applying a symmetrical load to the pier between the seventh and eighth spans to ensure that the monitoring results more accurately reflected the actual operation situation. The main monitoring instruments were arranged at the seventh span. Figure 2 displays the Kezi River Aqueduct Load Test Figure.

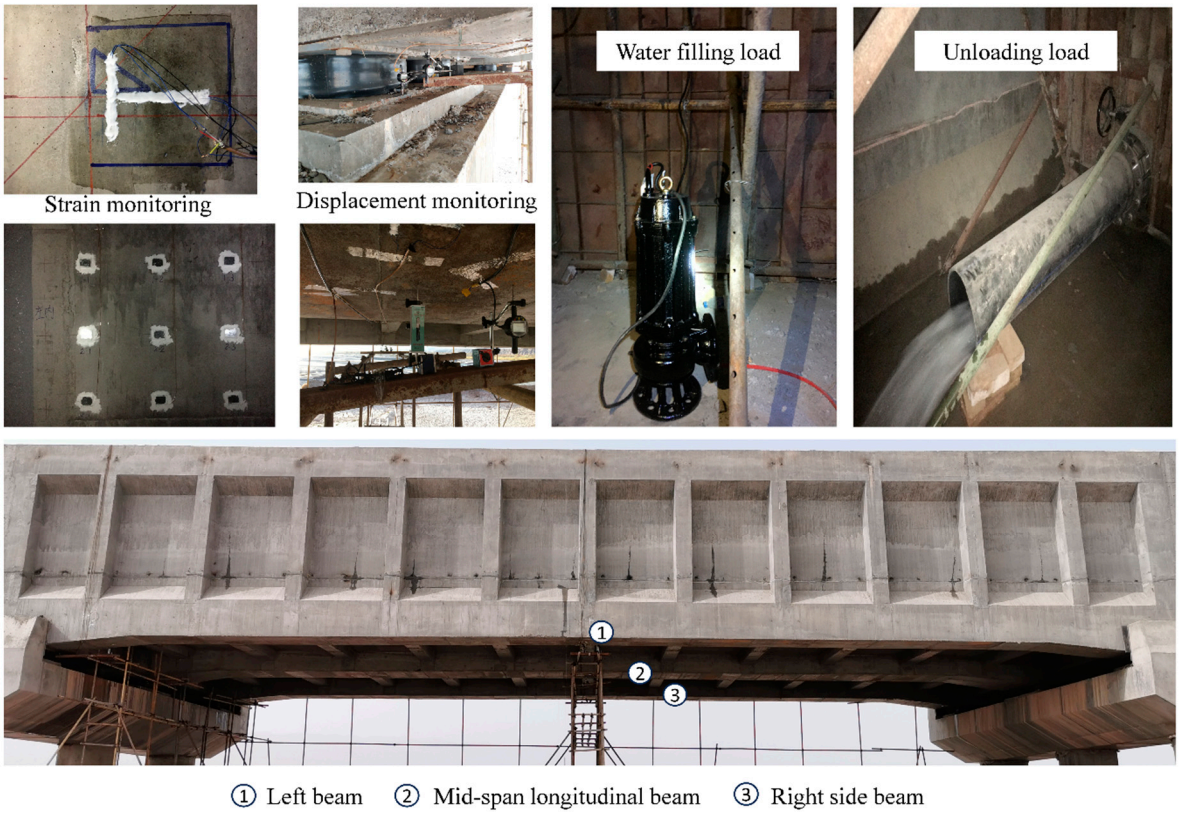


Figure 2. Schematic diagram of instrument layout for the Kezi River Aqueduct load test.

3.2.3. Loading and Unloading of Water Filling Test

This loading test adopted a six-stage loading process, where the design water depth is denoted as H, the increased water depth as H plus, and the full tank water depth as H full. The stages of loading were as follows: 1/4*H, 1/2*H, 3/4* H, H, H plus and full tank H

full for six-stage loading. After the completion of water loading at each stage, the water level was kept static for 30 min before the monitoring data was recorded.

The unloading test was divided into six stages: full tank H full, H plus, H, 3/4*H, 1/2*H, 1/4*H and emptying. Similarly to the water filling stages, each subsequent stage began after the water level had remained static for 30 min, during which the changes under each condition were recorded.

3.3. Safety Monitoring Results and Analysis

3.3.1. Safety Categories and Evaluation Criteria

(1) **Pier settlement**. The time-history curve (Figure 3) showed increasing settlement with water depth, but no significant differential settlement, indicating satisfactory foundation performance.

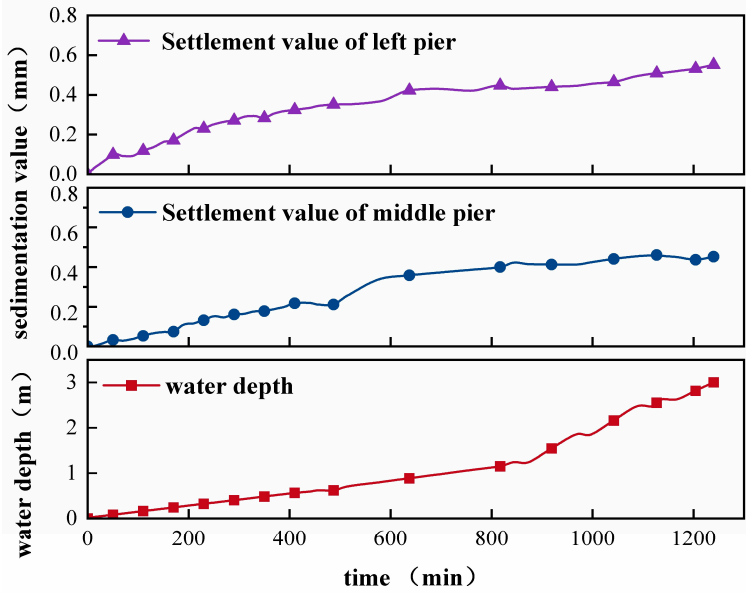


Figure 3. Time-history curve of pier settlement.

(2) **Mid-span deflection**. Deflection increased linearly with load (Figure 4), indicating elastic behavior. Maximum deflection was 3.40 mm.

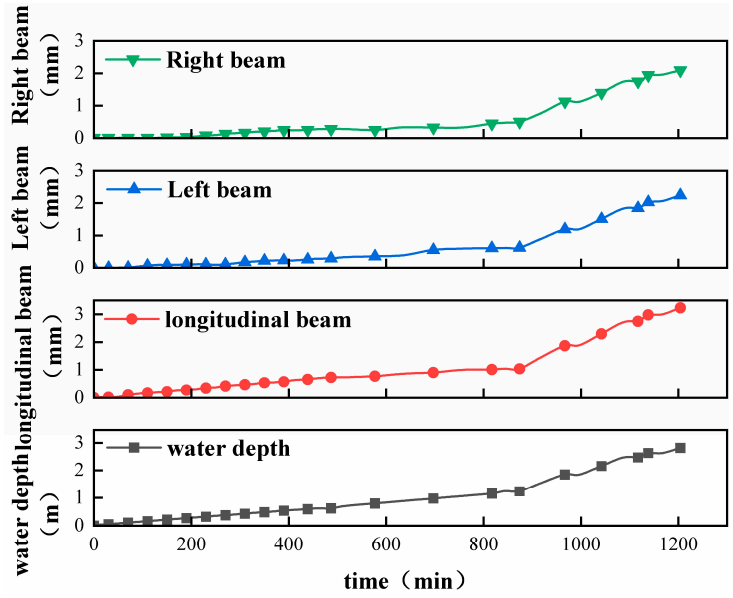


Figure 4. Deflection time-history curve of each beam in mid-span.

(3) **Strain monitoring**. Strain increased linearly with water depth and recovered partially upon unloading (Figure 5), confirming elastic response.

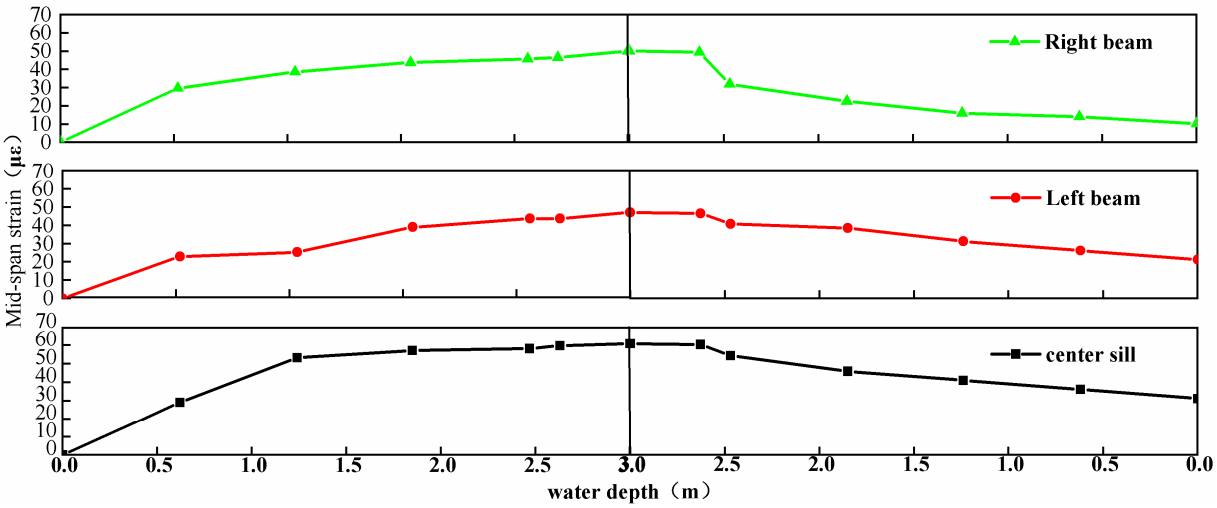


Figure 5. Water depth and mid-span strain curve.

3.3.2. Analysis of Bearing Capacity of Aqueduct Structure

(1) **Deflection analysis**. Allowable deflection per SL191-2008 [17] is 50 mm. Measured max. deflection was 3.40 mm, well within limits.

(2) **Relative residual strain analysis**. Relative residual strain S′~ p~ was 10.4% (<20% per JTG/TJ2l-2011) [7]. It should be noted that this result is based on a single load cycle; repeated cycles would be needed to assess long-term cumulative effects and measurement variability. The calculation formula is as follows:

$$S'_p = \frac{S_p}{S_t} \times 100\% \quad (16)$$

In the formula: S_p is the residual strain measured after unloading, S'_p is the relative residual strain, and S_t is the maximum strain measured under load.

The residual strain of the Kezi River Aqueduct, recorded after unloading from the point of maximum strain, measured only 6.32 um. At the apex of the water level during the water-filled load test, the mid-span strain observed amounts up to 60.89 um. According to the specification formula, the relative residual strain calculates to 10.4%, which meets the specification. This value remains well within the 20% threshold, indicating that the deformation of the aqueduct box girder maintains a robust elastic working state. It is explicitly noted that this measurement was derived from a single load cycle; the test was not repeated multiple times due to field constraints. Therefore, classical statistical measures of uncertainty (e.g., standard deviation from repeated trials) are not available. The result should be interpreted as a point estimate within this specific test context.

(3) **Comparative Analysis**: Table 5 shows similar aqueducts also performed well within limits.

Table 5. Statistics of water load test results of some aqueduct structures in China.

Project Name	Water Depth (m)	Span (m)	Mid-Span Deflection (mm)	Specification Allowable Value (mm)
Diaohe Aqueduct	5.75	40	3.91	67
South-to-North Water Diversion Aqueduct	7.05	30	3.34	50

Table 5. *Cont.*

Project Name	Water Depth (m)	Span (m)	Mid-Span Deflection (mm)	Specification Allowable Value (mm)
Jiji River Aqueduct	5.63	30	2.60	50
Tuanhe River Aqueduct	1.70	24	6.40	67
Shahe Aqueduct	6.05	30	0.85	50
Shierli River Aqueduct	6.35	30	3.34	50
Kezi River Aqueduct	3.00	30	3.40	50

4. Comprehensive Aqueduct Safety Assessment

4.1. Determine the Weight of Aqueduct Safety Evaluation

(1) **AHP subjective weights**. Using the natural index scale, weights for safety, applicability, and durability were 0.4506, 0.2474, and 0.3020, respectively (Table 2).

(2) **Entropy objective weights**. Aggregating seven prior studies (Table 6 and Figure 6) yielded objective weights of 0.5380, 0.1920, and 0.2600 for safety, applicability, and durability, respectively. Safety received the highest weight, consistent with its priority in most evaluations. However, synthesizing weights from studies with different contexts and standards introduces uncertainty regarding their direct applicability to this case.

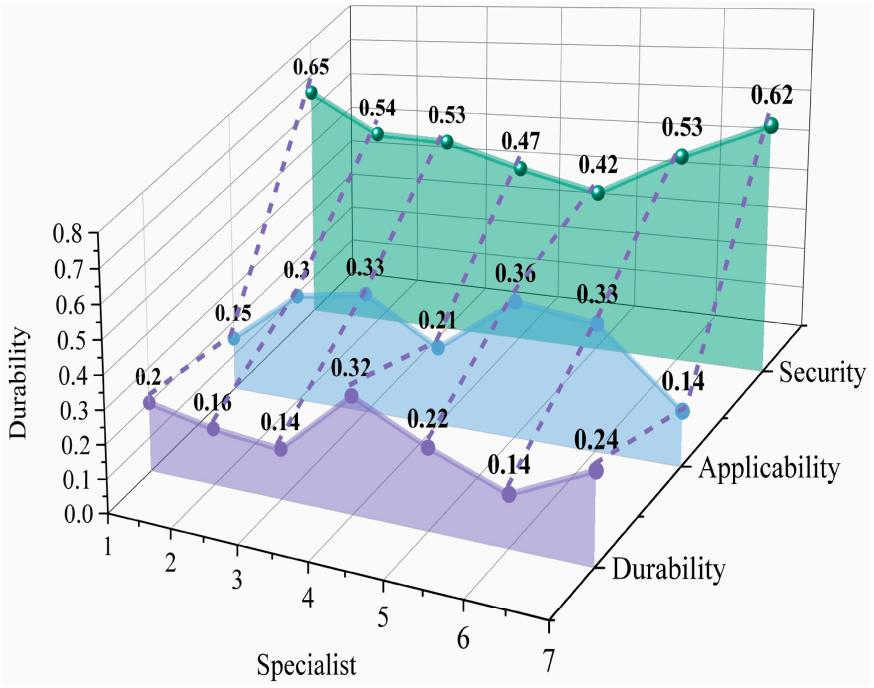


Figure 6. Objective weights derived from the entropy method for the three primary criteria.

Table 6. Summary weight of entropy weight method.

Specialist	Security	Applicability	Durability	Source
1	0.65	0.15	0.2	Comprehensive evaluation of large-scale aqueduct safety based on dynamic weight-cloud model [18]

Table 6. *Cont.*

Specialist	Security	Applicability	Durability	Source
2	0.54	0.3	0.16	Study on comprehensive safety evaluation of aqueduct project in Ningxia Yellow River irrigation area [19]
3	0.53	0.33	0.14	Fuzzy comprehensive evaluation of aqueduct health status [19,20]
4	0.47	0.21	0.32	AHP aqueduct safety comprehensive evaluation method based on group decision-making [21]
5	0.42	0.36	0.22	Comprehensive evaluation of aqueduct safety based on improved FCE [22]
6	0.53	0.33	0.14	Comprehensive evaluation of aqueduct health status based on SQL-Server database [23]
7	0.62	0.14	0.24	Study on structural safety assessment of Shangwu Aqueduct [24]
Entropy weight method	0.5380	0.1920	0.2600	

From Table 6 and Figure 6, it is evident that the weight of safety indicators peaks at 0.5380, aligning with the general trend of expert evaluations, thus underscoring safety as the paramount criterion in aqueduct safety assessments. The weight assigned to durability is 0.2600, ranking it second, indicating its substantial impact on the safety evaluation of aqueducts. Although the weight of applicability is relatively lower at 0.1920, it retains significant relevance. Through the entropy weight method, the data variability of each index is quantitatively analyzed, enabling an objective determination of weights. This approach minimizes the influence of subjective human factors, ensuring a more scientific and equitable distribution of weights.

(3) **Game theory combined weights**. The combined weights were 0.5177 (safety), 0.2089 (applicability), and 0.2734 (durability), with $\alpha_1 = 0.7394$ and $\alpha_2 = 0.2606$.

4.2. Safety Assessment of Aqueduct

The combined weights (Figure 7) emphasize safety as the most critical criterion. Scoring each indicator against Table 5 and aggregating yields a total score of 87.15, classifying the aqueduct as 'first class'. It is acknowledged that the use of DB44/T2041-2017 [15], a regional standard from Guangdong Province, is based on the absence of a local standard and similar evaluation principles. Nevertheless, climatic and operational differences may affect the direct transferability of threshold values. Within durability, concrete cracking scored low due to the observed web cracks. Although currently assessed as non-structural, these cracks warrant monitoring and repair (e.g., epoxy injection and surface sealing) to ensure long-term durability.

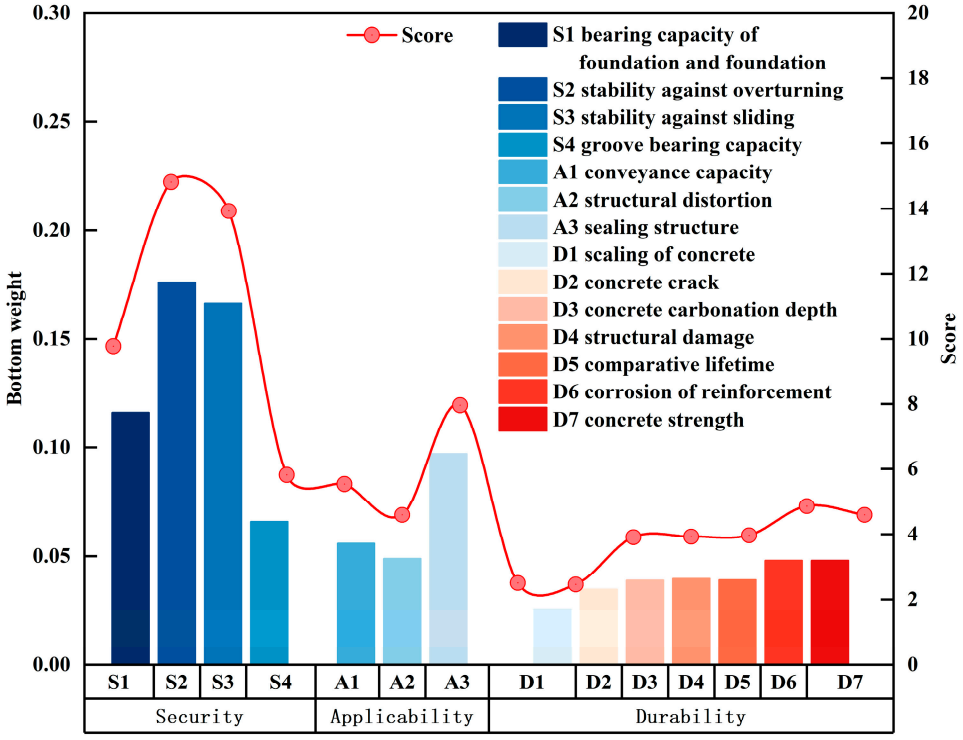


Figure 7. Summary of weight of each evaluation index of the Kezi River Aqueduct.

4.3. Results Validation

Field tests confirmed that the concrete strength mostly meets design; carbonation depth is minimal; and max deflection (3.4 mm) and relative residual strain (10.4%) are within limits. These results align with the model's 'first class' conclusion, supporting the aqueduct's current structural adequacy.

4.4. Comparative Analysis and Methodological Validation

To validate the necessity and superiority of the game theory-based weighting model, a quantitative comparative analysis was conducted against three simplified approaches: the single subjective method (AHP), the single objective method (entropy), and the direct arithmetic mean method (assuming equal reliability, i.e., $\alpha_1 = \alpha_2 = 0.5$). The weight distributions and final evaluation results are summarized in Table 7.

Table 7. Comparison of weights and evaluation results under different methods.

Method	Weight Coefficient (α)	Weight: Safety (WS)	Weight: Applicability (WA)	Weight: Durability (WD)	Final Score	Safety Level
AHP (Subjective Only)	$\alpha = 1.000$	0.5105	0.2149	0.2782	87.42	I
Entropy (Objective Only)	$\alpha = 1.000$	0.538	0.1920	0.2600	86.46	I
Arithmetic Mean	$\alpha_1 = \alpha_2 = 0.5$	0.5242	0.2035	0.2691	86.94	I
Game Theory (Proposed)	$\alpha_1 = 0.7394$, $\alpha_2 = 0.2606$	0.5177	0.2089	0.2734	87.15	I

4.4.1. Analysis of Weight Deviation

As observed in Table 7, although all methods classify the aqueduct as 'first class' (level I), the internal distribution of risk weights varies significantly, which affects maintenance prioritization. The entropy method assigned the lowest weight to 'durability' (0.2600) solely based on data discreteness. However, given the observed cracks in the aqueduct web (as detailed in Section 3.3), relying purely on data distribution would underestimate the structural durability risks. Conversely, the AHP method assigned a higher weight (0.2782)

to durability, reflecting the experts' engineering judgment regarding potential hazards. The arithmetic mean method ($\alpha = 0.5$) produced a durability weight of 0.2691. By arbitrarily assuming a '50/50' balance, this simple method diluted the critical expert's warning regarding structural defects. In contrast, the game theory method optimized the coefficients ($\alpha_1 = 0.7394$), mathematically proving that for this specific case, the expert diagnostic model (AHP) holds higher reliability than the available data distribution. Consequently, the game theory weight for durability (0.2734) effectively retains the focus on structural safety while still incorporating necessary corrections from the monitoring data.

4.4.2. Quantitative Verification of Added Value

To address whether the game-theoretic combination offers added value beyond a direct weighted scheme, we calculated the deviation indicator (D) based on the objective function in Equation (12). This indicator represents the Euclidean distance (inconsistency) between the combined weights and the source information.

Deviation of arithmetic mean (D_{mean}): when applying the simple average ($\alpha_1 = \alpha_2 = 0.5$), the calculated deviation is 0.0394.

Deviation of game theory (D_{GT}): with the optimized coefficients ($\alpha_1 = 0.7394$, $\alpha_2 = 0.2606$), the deviation is reduced to 0.0104.

The comparison demonstrates that the game theory optimization reduced the information deviation by approximately 73.6% compared to the arithmetic mean method. This significant reduction ($D_{GT} \ll D_{mean}$) serves as concrete evidence that the proposed framework is not merely a complex calculation, but a necessary optimization to minimize information loss and maximize the credibility of the safety assessment.

5. Conclusions

This study integrated field load testing with an AHP-entropy-game theory model to evaluate the Kezi River Aqueduct.

(1) Water filling tests confirmed elastic structural behavior with all key indicators below code limits. Observed web cracks were deemed non-structural based on pattern and global response, though detailed fracture analysis was not conducted.
(2) The combined model produced a safety score of 87.15 ('first class'), effectively merging subjective and objective weighting.
(3) The primary limitation is the single-case validation, which restricts generalizability. The methodological complexity, while thorough, may be disproportionate for a single case without comparative demonstration against simpler models. Future work should apply this framework to diverse aqueducts to test its robustness, incorporate repeated load cycles and statistical uncertainty analysis, and integrate mechanical crack assessment or finite element modeling for deeper durability evaluation.

In summary, the framework provides a structured, quantifiable approach for aqueduct condition assessment, as demonstrated in this case, but its broader utility requires further validation and potential simplification for routine application.

Furthermore, the comparative analysis confirmed that the game theory-based approach offers substantive added value over simpler weighting schemes. By optimizing the combination coefficients, the proposed framework reduced the information deviation by 73.6% compared to the arithmetic mean method, validating its robustness in balancing subjective expert experience with objective monitoring data.

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