applied sciences

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A Risk-Based Universal Calibration Interval Model Using Monte Carlo Simulation

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Abstract

Sustainable manufacturing requires modern intelligent approaches to monitoring products of the manufacturing process. An integral part of intelligent manufacturing is the measurement of geometric parameters of products, which allows diagnosing the state of the manufacturing process, optimizing it and predicting its further development. For these reasons, it is necessary to monitor the condition of measuring instruments, as decision-making is based on the data provided by them. Calibration intervals of measuring instruments are commonly defined using fixed time-based rules that are not explicitly linked to measurement uncertainty growth or conformity risk. This practice may lead to either unnecessary recalibration or an increased probability of using out-of-tolerance instruments. In this study, a Monte Carlo-based methodology for determining recalibration intervals is proposed, in which recalibration decisions are derived from the probabilistic evolution of measurement error over time. Measurement uncertainty is modeled as a time-dependent stochastic process combining calibration uncertainty, drift behavior, and repeatability. Monte Carlo simulation is used to propagate uncertainty and to estimate both the expanded uncertainty and the probability that the measurement error exceeds the maximum permissible error (MPE). The recalibration interval is defined as the earliest time at which this probability exceeds a predefined acceptable risk threshold. A numerical experiment using realistic synthetic data representative of a typical dimensional measuring instrument demonstrates that probability-based and uncertainty-based criteria may lead to substantially different recalibration intervals. The results confirm that risk-informed recalibration intervals provide a more transparent and metrologically justified alternative to fixed schedules while remaining fully compatible with ISO/IEC 17025 and GUM principles. The proposed approach is instrument-agnostic and readily applicable in calibration laboratories and industrial measurement systems.

Keywords: Monte Carlo simulation; measurement uncertainty; recalibration interval; conformity risk; dimensional metrology; ISO/IEC 17025; GUM

Check for updates

Academic Editor: Radu Godina

Received: 20 January 2026
Revised: 5 February 2026
Accepted: 23 February 2026
Published: 26 February 2026

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1. Introduction

In intelligent and sustainable manufacturing systems, reliable measurement data are a critical enabler for process control, quality assurance, and data-driven decision-making. As manufacturing increasingly relies on digital twins, adaptive control, and predictive analytics, the validity of measurement results must be ensured not only at discrete calibration events but continuously over time. Traditional fixed recalibration intervals do not reflect the actual evolution of measurement uncertainty and may therefore undermine

both operational efficiency and resource sustainability. Risk-informed calibration strategies, grounded in probabilistic modeling of measurement behavior, support intelligent manufacturing by enabling data-driven recalibration decisions and support sustainable manufacturing by reducing unnecessary recalibration activities, downtime, and material waste. In this context, the present work contributes a metrologically sound framework that aligns calibration interval determination with uncertainty propagation, conformity risk, and modern manufacturing paradigms.

Calibration is a fundamental element of metrological traceability and measurement quality assurance in industrial, scientific, and regulatory measurement systems. According to ISO/IEC 17025 and the International Vocabulary of Metrology (VIM), calibration establishes a documented relationship between the indication of a measuring instrument and corresponding reference values realized by measurement standards, together with the associated measurement uncertainty [1–8]. Calibration therefore represents a validated state of metrological conformity at a specific point in time. However, it does not guarantee the long-term stability of the instrument's metrological characteristics. During routine operation, measuring instruments are subject to mechanical wear, environmental influences, ageing of components, and handling effects, which may lead to gradual drift, increased variability, or abrupt deviations from reference values [9–12].

In industrial practice, recalibration intervals are still predominantly defined using fixed, time-based rules, such as annual or biennial calibration cycles. These intervals are often determined by internal quality procedures, contractual requirements, or regulatory expectations rather than by an objective analysis of instrument behaviour and performance history [6–8,13]. Although fixed intervals are straightforward to implement, they are increasingly recognized as suboptimal from both metrological and economic perspectives. Excessively short intervals result in unnecessary calibration costs and instrument downtime, whereas overly long intervals increase the probability of using an out-of-tolerance instrument, potentially compromising product quality, safety, or regulatory compliance [14–16].

International metrological guidelines explicitly recommend that calibration intervals should be established on the basis of technical evidence rather than arbitrary schedules. ILAC-G24/OIML D10 and NCSLI Recommended Practice RP-1 emphasize the use of historical calibration data, uncertainty evaluation, and risk-based considerations when defining or adjusting recalibration intervals [6,7]. Similar principles are reflected in EURAMET guidance documents and ISO 10012, which underline that measurement management systems must account for instrument stability, drift, and intended use [8,17]. Despite these recommendations, no universally accepted computational method for calibration interval determination is prescribed, leaving laboratories and industrial users to select or develop their own approaches [18].

Early research on calibration interval determination focused on deterministic drift models and classical statistical techniques. Cox and Harris demonstrated that measurement drift can be treated as a time-dependent contribution to measurement uncertainty, which increases between calibrations and directly affects conformity assessment [9]. Subsequent work by Cox and Siebert showed that neglecting drift effects may lead to systematic underestimation of long-term uncertainty, particularly for instruments with gradual degradation [10]. Related studies in mass and dimensional calibration confirmed that uncertainty budgets constructed for a single calibration event often fail to represent the temporal evolution of measurement performance [11,19].

The development of probabilistic approaches marked a significant advancement in this field. Monte Carlo methods were introduced into metrology as a practical numerical alternative to analytical uncertainty propagation and were later formalized within the GUM framework through Supplement 1 and related documents [3,4,14–17]. These meth-

ods allow direct propagation of probability distributions through complex or non-linear measurement models and are particularly well suited for modelling uncertainty growth over time. As demonstrated in recent applied studies, Monte Carlo simulation enables explicit representation of drift, environmental variability, and non-Gaussian behaviour, making it an effective tool for calibration interval analysis [12,20–22].

Bayesian statistical methods provide an additional framework for uncertainty evaluation by enabling the combination of prior knowledge with new calibration data. Elster and Toman compared Bayesian uncertainty analysis with the Monte Carlo approach recommended in GUM Supplement 1 and showed that Bayesian inference can offer advantages when calibration histories are limited or irregular [23]. Possolo and Bodnar further demonstrated that approximate Bayesian methods yield consistent uncertainty estimates while maintaining computational efficiency [24]. Recent work by Possolo emphasized the role of probabilistic reasoning in redefining measurement uncertainty as a decision-support quantity rather than a purely descriptive parameter [25]. These concepts are particularly relevant for calibration interval determination, where decisions are inherently risk-based.

Calibration interval determination is also closely related to reliability engineering and maintenance optimization. Concepts originating from condition-based maintenance and reliability analysis have been extensively applied in engineering systems to balance maintenance costs against failure risk [18,20,26–29]. Similar principles have been adapted to metrology, where recalibration can be treated as a preventive action triggered when the predicted probability of nonconformance exceeds an acceptable limit [20,30]. However, many reliability-based approaches assume large datasets or well-defined failure distributions, which are not always available for routine measuring instruments used in industrial practice.

Dimensional measuring instruments represent a particularly important class of devices due to their widespread use in manufacturing, inspection, and quality control. International standards such as ISO 13385-1 specify technical requirements and maximum permissible errors for vernier calipers but do not provide explicit guidance on recalibration interval determination beyond general traceability requirements [31,32]. Studies in engineering and industrial metrology indicate that for many dimensional instruments, drift is typically gradual and predictable rather than abrupt, suggesting that fixed calibration intervals may be unnecessarily conservative in some applications while insufficiently protective in others [12,18,28–33].

Recent research has increasingly focused on uncertainty growth as a quantitative criterion for recalibration decisions. Burrows demonstrated that Monte Carlo simulation can be applied to assess calibration strategies and to derive recalibration intervals based on predefined uncertainty or risk thresholds [14]. Tarabini investigated uncertainty behaviour in dimensional measurements under industrial conditions and confirmed that uncertainty-based criteria provide a more consistent basis for recalibration decisions than simple pass/fail evaluation at discrete calibration events [21]. Pendrill and Forbes further emphasized the importance of integrating uncertainty evaluation with conformity assessment to ensure transparent and defensible decision-making in accordance with ISO 14253-1 [29,32,33].

Despite significant progress, several challenges remain unresolved. Many proposed methods are tailored to specific instruments or measurement tasks, limiting their general applicability. Furthermore, the integration of drift modelling, uncertainty propagation, and conformity risk into a single framework that is both standards-compliant and practical for routine industrial use remains limited [2,17,34,35]. In particular, there is a need for universal methodologies applicable across different classes of measuring instruments while remaining compatible with ISO/IEC 17025, GUM principles, and accreditation requirements.

Despite extensive research on calibration interval determination, several important limitations remain unresolved. First, many existing approaches are developed for specific instrument types or measurement tasks, which restricts their general applicability in industrial and laboratory practice [6–8,22]. Second, deterministic or semi-empirical drift models are often used without explicit probabilistic uncertainty propagation, leading to recalibration intervals that are not directly linked to quantified measurement risk [9,10,15]. Third, conformity assessment is frequently treated as a binary decision at discrete calibration events, while the probability of nonconformance between calibrations is rarely evaluated explicitly [32–40]. Finally, although international guidelines recommend evidence-based calibration interval determination, no universally accepted computational framework exists that simultaneously integrates drift modeling, uncertainty propagation, and risk-based decision rules in a standards-compliant manner [3,6,36,40]. These limitations motivate the development of a universal probabilistic methodology for recalibration interval determination, as proposed in this work.

Based on the literature review and experimental motivation, the following research criteria are defined:

Criterion 1: Development of a universal, uncertainty-based framework for determining recalibration intervals applicable to a broad range of measuring instruments.

Criterion 2: Explicit modeling of measurement drift and uncertainty growth using Monte Carlo simulation.

Criterion 3: Definition of recalibration decision rules based on acceptable risk of exceeding the maximum permissible error.

Criterion 4: Practical applicability of the method using limited calibration history data.

The objective of this study is to develop and validate a Monte Carlo-based methodology for determining recalibration intervals that explicitly accounts for measurement uncertainty, drift behaviour, and conformity risk. The proposed approach is instrument-agnostic and relies on generic metrological inputs such as calibration results, uncertainty components, and MPE specifications, while remaining fully compatible with ISO/IEC 17025, GUM, and related international standards [1–4,34–42].

The main contributions of this work are as follows:

(1) a unified probabilistic framework combining uncertainty propagation, drift modeling, and recalibration decision-making using Monte Carlo simulation;
(2) a risk-informed alternative to fixed time-based calibration intervals aligned with international metrological guidance; and
(3) a practical demonstration of the method using a representative dimensional measuring instrument.

The remainder of this paper is organized as follows. Section 2 presents the theoretical framework for uncertainty growth and Monte Carlo-based recalibration interval determination. Section 3 describes the experimental data and representative measuring instrument. Section 4 discusses the results and compares the proposed approach with conventional fixed-interval strategies. Section 5 concludes the paper and outlines directions for future research.

See the end of the document for further details on references.

2. Drift-Based Error Model

Measurement drift represents a time-dependent change in the indication of a measuring instrument under nominally constant operating conditions and constitutes one of the primary mechanisms leading to degradation of metrological performance between successive calibrations. Within the framework of ISO/IEC 17025 and the International Vocabulary of Metrology, calibration establishes traceability and quantifies measurement uncertainty at

a specific point in time, but it does not ensure the long-term stability of the instrument [1–5]. As a result, drift must be explicitly considered when assessing measurement reliability over extended periods of use.

The assumption of a zero-mean drift rate adopted in this study represents a neutral baseline modeling choice, particularly suitable when long-term calibration data are limited or when no statistically significant systematic trend can be identified. In practical industrial scenarios, however, certain measuring instruments may exhibit systematic drift with a non-zero mean, caused by wear mechanisms, ageing effects, or persistent environmental bias. Such behavior can be readily accommodated within the proposed Monte Carlo framework by introducing a non-zero expected value of the drift distribution or by applying time-dependent drift models. Importantly, the presence of systematic drift affects the numerical value of the recalibration interval but does not alter the underlying probabilistic structure of uncertainty propagation or the risk-based recalibration decision principle.

Let the indication of a measuring instrument at time t be denoted as $y(t)$. A general measurement model incorporating drift can be expressed as

$$y(t) = x + \delta_0 + \delta_d(t) + \varepsilon, \tag{1}$$

where x is the true value of the measurand, δ_0 is the residual systematic error remaining after calibration, $\delta_d(t)$ is the drift-induced error evolving over time, and ε represents random effects such as repeatability, environmental fluctuations, and short-term noise. At the time of calibration $t = 0$, the expected value of $\delta_d(0)$ is assumed to be zero within the calibration uncertainty. However, as emphasized in metrological studies, this assumption is no longer valid for $t > 0$, where drift contributes to increasing uncertainty and affects conformity decisions [9].

In practical applications, drift is rarely deterministic. Even when an apparent trend is observed, its rate and direction are influenced by environmental conditions, usage intensity, ageing processes, and maintenance actions. Consequently, drift is more appropriately modelled as a stochastic process rather than as a fixed deterministic function [10,24]. A commonly adopted representation expresses drift as a combination of a mean trend and a random component,

$$\delta_d(t) = \mu_d \cdot t + \omega(t), \tag{2}$$

where μ_d denotes the mean drift rate and $\omega(t)$ is a zero-mean stochastic process describing random deviations from the trend. When historical calibration data are limited, it is often reasonable to model $\omega(t)$ as a Gaussian process with variance increasing as a function of time, reflecting growing uncertainty about the instrument state [19].

According to the GUM and related guidance documents, the combined standard uncertainty at time t is obtained by propagating all relevant uncertainty contributions, including those associated with drift [2–4]. If u_0 denotes the combined standard uncertainty at the time of calibration and $u_d(t)$ represents the standard uncertainty associated with drift, the combined uncertainty can be written as

$$u_c(t) = \sqrt{u_0^2 + u_d^2(t)}, \tag{3}$$

This formulation highlights a key limitation of conventional uncertainty budgets, which are often constructed for a single calibration event and implicitly assumed to remain valid over the entire calibration interval. As noted by Cox and Harris, neglecting time-dependent uncertainty contributions leads to systematic underestimation of long-term uncertainty and may result in incorrect conformity assessment outcomes [9]. Similar conclusions have been drawn in engineering metrology and uncertainty analysis, where temporal degradation of instrument performance has been identified as a dominant factor affecting measurement reliability [14,32].

Analytical propagation of uncertainty becomes increasingly difficult when drift models are non-linear, non-Gaussian, or based on sparse data. In such cases, Monte Carlo simulation provides a robust numerical alternative, as formally recommended by GUM Supplement 1 and subsequent extensions [3,4,11–17,38]. In the Monte Carlo framework, each input quantity in the measurement model is represented by a probability distribution, and repeated random sampling is used to generate an empirical distribution of the measurement result at time t. For each simulation run i, the measurement result is computed as

$$y_i(t) = x_i + \delta_{0,i} + \delta_{d,i}(t) + \varepsilon_i, \tag{4}$$

where the random variables are sampled according to their assigned distributions. Repeating this process for a sufficiently large number of iterations yields a numerical approximation of the probability distribution of $y(t)$, from which uncertainty intervals and risk metrics can be directly evaluated without assuming normality or linearity.

Monte Carlo-based uncertainty propagation is particularly well suited for calibration interval determination, as it allows explicit modelling of uncertainty growth over time and direct evaluation of the probability that an instrument exceeds its specified limits. Let MPE denote the maximum permissible error defined by the relevant standard or manufacturer specification. Within a risk-based framework, recalibration can be triggered when the probability of exceeding the MPE reaches a predefined threshold,

$$P(|y(t) - x| > \mathrm{MPE}) \geq \alpha, \tag{5}$$

where α represents the maximum acceptable risk of nonconformance. This probabilistic decision rule is fully consistent with ISO 14253-1 and established principles of conformity assessment and guard banding [33,34,36].

From a conformity assessment perspective, the proposed probability-based recalibration criterion can be interpreted as a dynamic decision rule analogous to guard banding concepts used in ISO 14253-1. Instead of applying a static acceptance or rejection decision at discrete calibration events, the method continuously evaluates the predicted probability of nonconformity between calibrations. Recalibration is triggered when this probability exceeds the predefined acceptable risk level, thereby ensuring that conformity decisions remain aligned with quantified measurement risk throughout the calibration interval. This interpretation directly connects Monte Carlo-based uncertainty propagation with practical decision-making processes used in accredited calibration and inspection activities.

The use of Monte Carlo simulation for calibration strategy assessment has been demonstrated in several applied studies. Burrows showed that Monte Carlo methods enable quantitative comparison of alternative calibration strategies and support uncertainty-based interval optimization [14]. Tarabini confirmed that uncertainty stability in dimensional measurements provides a more reliable basis for recalibration decisions than simple pass/fail evaluation at discrete calibration events [21]. These findings align with broader research in reliability engineering and maintenance optimization, where recalibration is treated as a preventive action analogous to condition-based maintenance, initiated when the predicted risk exceeds an acceptable level [20–26,29,36].

The drift-based error model presented in this work is intentionally instrument-agnostic. It relies on generic metrological inputs—calibration uncertainty, drift estimates, and MPE—that are available for most measuring instruments operating under ISO/IEC 17025-compliant systems [1,6,32]. As such, the model can be applied to a wide range of instruments, including dimensional measuring devices such as vernier calipers defined in ISO 13385-1, as well as other measurement systems where drift-driven uncertainty growth is a dominant factor [21]. This generality makes the approach suitable for routine industrial use while remaining fully consistent with international standards and accreditation requirements.

3. Monte Carlo-Based Recalibration Interval Determination

In industrial and laboratory practice, recalibration intervals for measuring instruments are commonly defined using fixed, time-based rules specified by standards, accreditation bodies, or internal quality management systems. For dimensional-measuring instruments such as vernier calipers, typical recalibration intervals ranging from 12 to 24 months, depending on usage intensity, required accuracy and historical stability. These intervals are often adopted as default values within ISO/IEC 17025-compliant laboratories, even though the standards themselves do not prescribe mandatory calibration periods.

Such time-based intervals are primarily administrative in nature and are rarely derived from quantitative models of measurement uncertainty growth or conformity risk. Consequently, instruments may be recalibrated either too frequently, leading to unnecessary cost and downtime, or too late, increasing the probability of producing nonconforming measurement results.

The Monte Carlo-based approach proposed in this work directly addresses this limitation by linking the recalibration decision to the statistical evolution of measurement error and the associated probability of exceeding the maximum permissible error. Instead of assuming a fixed calibration period, the method evaluates recalibration necessity as a function of time-dependent uncertainty and explicitly defined risk criteria.

To demonstrate the practical implications of this approach, a numerical experiment was performed using synthetic but realistic data representative of a typical dimensional measuring instrument. The following sections present the input parameters, simulation results, and a direct comparison between normative time-based intervals and the recalibration interval obtained using the proposed probabilistic framework.

Table 1 presents the complete set of input parameters used in the Monte Carlo simulation. The table is intentionally introduced at the beginning of this chapter to clearly separate model assumptions from numerical results and to ensure traceability of the simulation outcomes. Such an approach is consistent with best practices in uncertainty analysis and numerical experimentation recommended in metrological literature.

Table 1. Input parameters used in the Monte Carlo simulation for recalibration interval determination.

Parameter	Symbol	Distribution	Mean Value	Standard Deviation	Unit	Description
Initial calibration error	e_0	Normal	0	2	µm	Residual error after calibration
Drift rate	d	Normal	0	0.5	µm/month	Long-term systematic drift
Repeatability error	ε	Normal	0	1	µm	Short-term random variability
Maximum permissible error	MPE	Deterministic	±20	—	µm	Instrument tolerance limit
Coverage factor	k	—	2	—	—	Expanded uncertainty
Acceptable risk threshold	P_{crit}	—	5	—	%	Probability of exceeding MPE
Number of Monte Carlo runs	N	—	100,000	—	—	Simulation sample size
Time step	Δt	—	1	—	month	Discretization interval
Maximum simulation time	t_{max}	—	60	—	month	Simulation horizon

The Monte Carlo input parameters listed in Table 1 were selected to represent realistic operating conditions of a typical dimensional measuring instrument under controlled laboratory use. The assumed standard deviation of the drift rate (0.5 µm/month) corresponds to gradual long-term drift magnitudes commonly reported for mechanical dimensional instruments and calibration artifacts in industrial practice [6,9,22]. The repeatability standard deviation of 1 µm reflects short-term variability associated with operator influence, contact conditions, and instrument resolution, as reported in dimensional metrology studies and calibration guidelines [1,28,35]. The residual calibration uncertainty was chosen in accor-

dance with typical capabilities of ISO/IEC 17025-accredited calibration laboratories and GUM-consistent uncertainty evaluations [1–3,40]. These parameters were intentionally selected as representative rather than instrument-specific in order to demonstrate the general applicability of the proposed Monte Carlo-based recalibration interval methodology.

The selected parameter values were chosen to represent a mid-range dimensional measuring instrument, such as a vernier caliper, operating under controlled laboratory conditions. The initial calibration error e_0 is modeled as a normal distribution with zero mean, reflecting the assumption of unbiased calibration, and a standard deviation of 2 µm, corresponding to typical residual errors observed after calibration of manual dimensional instruments. The repeatability component ε is also assumed to be normally distributed with a standard deviation of 1 µm, which reflects short-term variability due to operator influence, contact conditions, and reading resolution.

The drift rate d represents long-term systematic changes caused by wear, ageing, and environmental effects. It is modeled as a normally distributed random variable with zero mean and a standard deviation of 0.5 µm/month, which corresponds to gradual and predominantly random drift behavior reported for dimensional instruments subjected to stable operating conditions. The normal distribution assumption is consistent with both GUM principles and previous studies on long-term instrument stability.

The maximum permissible error is treated as a deterministic parameter defined by the relevant product or instrument standard. In this study, a value of ± 20 µm was selected, which is representative of typical MPE limits specified for vernier calipers according to ISO 13385-1. The coverage factor $k = 2$ is adopted for expanded uncertainty evaluation, corresponding to an approximate 95% level of confidence under the assumption of normality.

In addition to uncertainty-based criteria, a probabilistic decision rule is introduced through the acceptable risk threshold P_{crit}. A value of 5% was selected, reflecting commonly accepted risk levels in conformity assessment and quality control practice. This choice is consistent with risk-based decision-making approaches recommended in ISO 14253-1 and related conformity assessment guidance. The Monte Carlo sample size of 10^5 runs was chosen to ensure stable estimation of tail probabilities, which are critical for recalibration decisions based on the probability of exceeding the MPE.

The measurement error at time t after calibration is modeled as

$$e(t) = e_0 + dt + \varepsilon, \tag{6}$$

where each term is treated as a statistically independent random variable. This model explicitly captures both short-term variability and long-term systematic effects and provides a suitable basis for analyzing uncertainty growth between calibration events. The simulation was performed over a maximum time horizon of 60 months with a time step of one month, allowing the long-term evolution of uncertainty and risk to be investigated.

For each Monte Carlo realization, the measurement error was evaluated at each time step following calibration. The resulting ensemble of error values was used to estimate the probability distribution of the measurement error as a function of time. From these distributions, two complementary criteria were evaluated: the expanded uncertainty $U(t)$ and the probability $P(|e(t)| > \text{MPE})$ that the error exceeds the maximum permissible error.

Figure 1 illustrates the time evolution of the probability of exceeding the MPE. Immediately after calibration, the probability of nonconformity is negligible, confirming the effectiveness of the calibration process. As time progresses, the uncertainty associated with the drift term accumulates, leading to a monotonic increase in the probability of exceeding the tolerance limit. The intersection of this curve with the predefined acceptable risk

threshold of 5% defines the recommended recalibration interval according to the proposed probabilistic criterion.

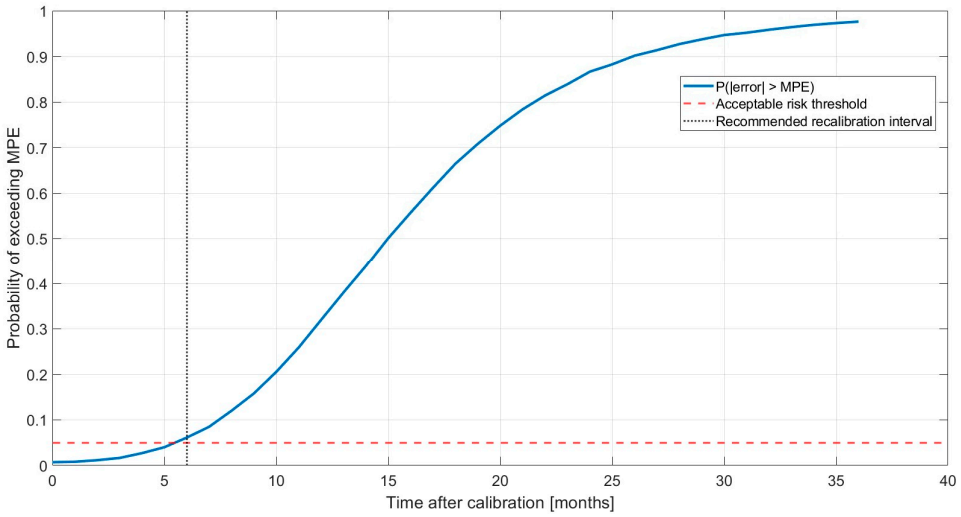

Figure 1. Probability-based recalibration criterion. The figure shows the evolution of the probability $P(|e(t)| > \text{MPE})$ that the absolute measurement error exceeds the MPE as a function of time after calibration. The blue curve represents the Monte Carlo-estimated probability obtained from 10^5 simulation runs. The horizontal dashed line denotes the acceptable risk threshold (5%), consistent with risk-based conformity assessment principles. The vertical dotted line indicates the recommended recalibration interval, defined as the earliest time at which the probability exceeds the acceptable risk level.

The probability-based recalibration interval obtained from Figure 1 can be directly compared with commonly applied normative calibration intervals. While a fixed interval of 12 months is frequently adopted in practice, the Monte Carlo simulation indicates that the acceptable risk threshold of 5% is exceeded after approximately 6 months. This result implies that, under the assumed drift and uncertainty conditions, a conventional annual calibration interval would allow a significant probability of nonconforming measurements during the second half of the calibration period.

Conversely, for instruments exhibiting lower drift rates or improved stability, the proposed method may justify calibration intervals exceeding one year without compromising measurement reliability. This demonstrates that the Monte Carlo-based approach does not inherently lead to shorter calibration intervals but instead adapts the interval length to the actual statistical behavior of the instrument.

Figure 2 presents the corresponding evolution of the expanded measurement uncertainty $U(t)$ evaluated with a coverage factor of $k = 2$. The expanded uncertainty increases approximately linearly with time due to the increasing contribution of drift uncertainty. In this example, the expanded uncertainty remains within the MPE limits for a longer period than suggested by the probability-based criterion. This discrepancy highlights an important limitation of relying solely on expanded uncertainty when determining recalibration intervals.

A direct comparison of Figures 1 and 2 demonstrates that uncertainty-based and probability-based criteria may lead to substantially different recalibration intervals. While the expanded uncertainty reflects the central dispersion of the measurement error distribution, the probability-based criterion explicitly accounts for the distribution tails, which govern the risk of nonconforming measurements. Consequently, the probability-based approach provides a more conservative and risk-informed basis for recalibration decisions.

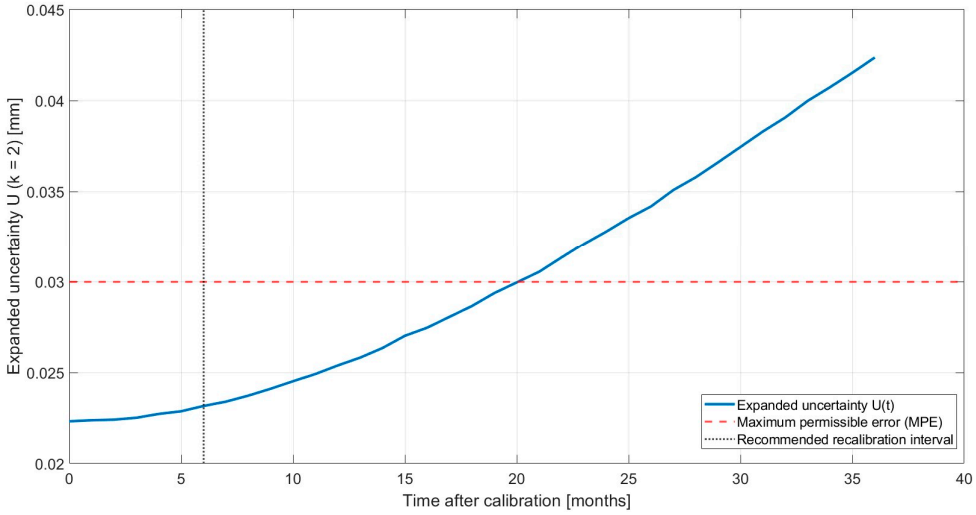

Figure 2. Growth of expanded measurement uncertainty. The figure presents the time evolution of the expanded uncertainty $U(t)$ with coverage factor $k = 2$. The dashed horizontal line corresponds to the MPE limit. The vertical dotted line marks the recommended recalibration interval obtained from the probability-based criterion. Although the expanded uncertainty remains below the MPE for a longer period, the probability-based approach identifies a shorter, more conservative recalibration interval.

Although the proposed Monte Carlo framework is presented in a generic form, its application to a specific measuring instrument requires appropriate selection of a limited number of input parameters. Instrument-specific parameters include the expected drift rate and its variability, repeatability or short-term noise, and the maximum permissible error defined by the relevant standard or manufacturer specification. Application-specific parameters may include the intended calibration interval range, acceptable conformity risk, and environmental stability assumptions. In contrast, the structure of the uncertainty propagation, drift modeling, and risk-based decision rule remains unchanged. This separation allows the framework to be readily adapted to different classes of measuring instruments, such as dimensional gauges, coordinate measuring machines, or optical measurement systems, while maintaining compliance with ISO/IEC 17025 and GUM principles [1,2].

The numerical experiment presented in this study is based on synthetic data. This choice was made intentionally, as the primary objective of the work is to establish and analyze a general recalibration framework rather than to characterize a specific measuring instrument. The synthetic input parameters were selected to reflect realistic industrial conditions and typical calibration practice, including uncertainty levels, drift variability, repeatability, and maximum permissible error specifications commonly encountered in dimensional metrology [1,2,6].

The use of synthetic data enables systematic investigation of uncertainty growth and conformity risk under controlled conditions, while avoiding biases related to a particular instrument or limited historical datasets. Nevertheless, validation of the proposed methodology using long-term calibration histories of real instruments represents an important next step. Future work will focus on applying the framework to experimental calibration data in order to assess parameter identification, model adaptation, and long-term predictive performance. The complete MATLAB R2024b (The MathWorks, Inc., Natick, MA, USA) implementation of the Monte Carlo simulation corresponding to Algorithm 1 is provided in Appendix A.

Algorithm 1. Monte Carlo-based recalibration interval determination

Step 1: Define input parameters, including initial calibration error, standard uncertainty components, drift model parameters, repeatability, maximum permissible error (MPE), acceptable conformity risk, and simulation time horizon.

Step 2: Specify probability distributions for all stochastic input variables and assume statistical independence unless stated otherwise.

Step 3: Generate a large number of Monte Carlo samples for each input parameter according to the defined distributions.

Step 4: For each simulation time step, propagate the input distributions through the measurement model, including the time-dependent drift contribution.

Step 5: Compute the combined standard uncertainty and expanded uncertainty at each time step from the simulated output distributions.

Step 6: Evaluate conformity by calculating the probability that the simulated measurement error exceeds the MPE at each time step.

Step 7: Determine the recalibration interval as the time at which either the expanded uncertainty exceeds a predefined limit or the conformity risk reaches the acceptable threshold.

Step 8: Repeat the simulation to assess the stability and robustness of the estimated recalibration interval.

4. Discussion

The results obtained in this study demonstrate that calibration interval determination based on Monte Carlo simulation provides a fundamentally different and more informative basis for decision-making than conventional fixed time-based approaches. By explicitly modelling measurement drift, uncertainty growth, and conformity risk, the proposed framework enables recalibration intervals to be derived from the actual statistical behaviour of the measuring instrument rather than from administrative or historical conventions.

A key finding of the numerical experiment is the divergence between normative calibration intervals commonly applied in industrial and laboratory practice and the recalibration interval obtained using a risk-based probabilistic criterion. For dimensional measuring instruments such as vernier calipers, recalibration intervals of 12 or 24 months are frequently adopted as default values within ISO/IEC 17025-compliant systems, despite the absence of mandatory interval prescriptions in international standards. The Monte Carlo simulation results show that, under realistic assumptions regarding drift and repeatability, the probability of exceeding the maximum permissible error (MPE) may surpass an acceptable risk threshold significantly earlier than the nominal calibration date. This observation confirms concerns raised in earlier studies that fixed intervals may not adequately control measurement risk over time, particularly for instruments subject to gradual but non-negligible drift.

The probability-based recalibration criterion employed in this work offers a direct and transparent link between uncertainty evaluation and conformity assessment. Unlike traditional approaches that rely solely on expanded uncertainty evaluated at discrete calibration events, the proposed method explicitly considers the full probability distribution of the measurement error and its evolution between calibrations. As illustrated by the comparison between the probability-of-nonconformity curve and the expanded uncertainty growth, the two criteria may lead to substantially different recalibration intervals. While the expanded uncertainty may remain within the MPE limits for an extended period, the probability of exceeding the tolerance can already reach unacceptable levels due to the influence of distribution tails. This finding supports previous work by Pendrill [33] and

Forbes [29], who emphasized that conformity decisions should be based on risk rather than on uncertainty magnitude alone.

From a metrological perspective, the results highlight the limitations of treating calibration as a purely episodic activity. Calibration establishes traceability and quantifies uncertainty at a specific point in time, but it does not ensure sustained conformity throughout the calibration interval. The drift-based error model used in this study captures the temporal degradation of instrument performance and demonstrates that uncertainty must be regarded as a time-dependent quantity. This interpretation aligns with the conceptual framework proposed by Cox and Harris [9], as well as with the recommendations of ILAC-G24/OIML D10 and NCSLI RP-1 [6,7], which explicitly call for evidence-based calibration interval determination using historical data and uncertainty analysis.

The Monte Carlo approach adopted here offers several practical advantages over analytical uncertainty propagation and deterministic drift models. First, it allows nonlinear and non-Gaussian effects to be incorporated without simplifying assumptions, which is particularly relevant when calibration histories are short or when drift behaviour is only partially characterized. Second, the method provides direct access to risk metrics, such as the probability of exceeding the MPE, which are not readily available from classical GUM-based uncertainty budgets. Third, the simulation framework is flexible and instrument-agnostic, requiring only generic metrological inputs that are routinely available in calibration laboratories.

The choice of synthetic yet realistic input parameters in the numerical experiment was motivated by the need to demonstrate the method independently of a specific instrument dataset. The selected values for drift rate, repeatability, and MPE correspond well to typical characteristics of manual dimensional measuring instruments operating under controlled laboratory conditions. Although the exact numerical results are parameter-dependent, the qualitative behaviour observed in the simulations is robust and reflects general principles of uncertainty growth and risk accumulation. Importantly, the method does not inherently lead to shorter calibration intervals; instruments exhibiting lower drift rates or improved stability may justify extended intervals while maintaining acceptable risk levels. This adaptive behaviour represents a clear advantage over fixed-interval strategies.

In real industrial environments, measurement drift is affected by multiple external factors, including usage intensity, mechanical loading, environmental fluctuations such as temperature and humidity, and maintenance or handling practices. These factors may lead to non-stationary drift behavior and changes in uncertainty growth over time. In the present study, such influences are implicitly aggregated within the drift and repeatability parameters in order to maintain model transparency and generality. However, the proposed Monte Carlo framework readily allows the explicit inclusion of usage- or environment-dependent drift models by introducing time-varying parameters or conditional probability distributions. Incorporating such effects represents a natural extension of the present work and is particularly relevant for instruments operating under highly variable conditions.

The proposed framework is also closely related to concepts from reliability engineering and condition-based maintenance, where preventive actions are triggered when the predicted probability of failure exceeds a predefined limit. By treating recalibration as a preventive measure against measurement nonconformance, the Monte Carlo-based approach provides a natural bridge between metrology and reliability analysis. However, unlike many reliability models that require large datasets or well-defined failure distributions, the present method remains applicable under typical calibration laboratory conditions, where data availability is often limited.

Despite its advantages, the proposed approach has certain limitations that must be acknowledged. The accuracy of the recalibration interval estimate depends on the validity

of the assumed drift and uncertainty models. In practice, drift behaviour may be influenced by factors such as usage intensity, environmental variability, maintenance actions, and operator effects, which are not explicitly modelled in the present study. Furthermore, the assumption of statistical independence between uncertainty components may not always hold, particularly for complex measurement systems. These limitations suggest that the method should be applied with appropriate engineering judgement and supported by periodic review as additional calibration data become available.

In the present study, Gaussian distributions were assigned to the random input variables representing calibration uncertainty, repeatability, and drift variability. This choice follows established metrological practice and the recommendations of the Guide to the Expression of Uncertainty in Measurement, where normal distributions are commonly used when uncertainty components arise from the combination of multiple independent effects or when only limited statistical information is available.

For repeatability and calibration uncertainty, the assumption of normality is well justified by the central limit effect and by extensive empirical evidence from calibration laboratories. For drift-related parameters, a zero-mean normal distribution was selected as a conservative and symmetric model in the absence of long-term calibration data indicating systematic bias.

It should be emphasized that the Monte Carlo framework adopted in this work is fully distribution-agnostic. If experimental data or expert knowledge indicate non-Gaussian behavior, such as bounded, skewed, or heavy-tailed distributions, these can be directly incorporated into the simulation without modification of the underlying methodology. Future work will focus on validating alternative distribution models using long-term calibration histories of real instruments. The recalibration interval obtained using the proposed Monte Carlo-based framework depends on a set of modelling assumptions, including the statistical characterization of drift, repeatability, and the acceptable risk threshold. In this study, these parameters were selected to represent realistic operating conditions of a typical dimensional measuring instrument and to illustrate the methodology rather than to prescribe universal numerical values. Importantly, the proposed framework is not tied to specific parameter choices and remains fully adaptable to different instrument types, operating environments, and risk policies.

The acceptable risk threshold of 5% used in this work reflects commonly adopted limits in conformity assessment and quality assurance practice and is consistent with decision-making principles described in ISO 14253-1 and related guidance documents. Nevertheless, this value should be regarded as an adjustable decision parameter rather than a fixed requirement. Lower risk thresholds will lead to more conservative recalibration intervals, while higher thresholds may justify extended intervals for stable instruments. The methodology explicitly supports such customization by directly linking recalibration decisions to quantified conformity risk.

Although a formal numerical sensitivity analysis was not performed, the sensitivity of the proposed Monte Carlo-based recalibration framework can be directly inferred from the structure of the uncertainty propagation model. The recalibration interval is primarily governed by the rate of uncertainty growth over time and by the selected conformity risk criterion.

Among the input parameters, the drift rate and its associated variability represent the most influential factors. An increase in either the mean drift rate or its standard deviation leads to faster growth of the simulated error distribution, resulting in an earlier exceedance of the maximum permissible error and, consequently, a shorter recalibration interval. This behavior reflects the dominant role of time-dependent systematic effects in long-term instrument stability.

In contrast, parameters such as repeatability and initial calibration uncertainty mainly affect the initial width of the error distribution but have a comparatively smaller impact on the slope of uncertainty growth. Variations in these parameters shift the recalibration interval moderately but do not fundamentally change the qualitative behavior of the model.

The recalibration interval is also sensitive to the selected maximum permissible error and acceptable conformity risk. Tighter MPE limits or lower acceptable risk thresholds naturally result in shorter recalibration intervals, whereas more permissive criteria extend the usable period between calibrations. This trade-off explicitly reflects the balance between measurement risk and calibration effort and is a key advantage of the proposed probabilistic framework.

Overall, the sensitivity characteristics of the model are physically intuitive and consistent with metrological experience. The dominance of drift-related parameters confirms that the framework captures the essential mechanisms governing long-term measurement reliability, while the moderate sensitivity to secondary uncertainty components indicates robust behavior with respect to typical parameter variations. Similarly, the assumed probabilistic model of drift influences the rate of uncertainty growth and, consequently, the estimated recalibration interval. While a normal distribution with zero mean and constant variance growth was adopted in this study due to limited historical data, alternative drift models—such as non-zero mean drift, time-dependent variance, or bounded distributions—can be readily implemented within the same Monte Carlo framework. Changes in model assumptions affect the numerical value of the recalibration interval but do not alter the underlying principle of risk-based decision-making. This robustness highlights the general applicability of the proposed approach and its suitability for evidence-based calibration interval determination under varying practical conditions.

In the proposed Monte Carlo model, all uncertainty components are assumed to be statistically independent. This assumption is consistent with common metrological practice and with the principles outlined in the Guide to the Expression of Uncertainty in Measurement, where independence is adopted in the absence of reliable information on correlation.

The individual components considered in this study—instrument drift, short-term repeatability, and calibration uncertainty—originate from different physical mechanisms and time scales. Drift is associated with long-term aging and environmental effects, repeatability reflects short-term random variability, and calibration uncertainty is linked to reference standards and calibration procedures. Under typical industrial conditions, these effects are generated by largely independent processes, justifying the independence assumption as a reasonable first-order approximation.

It should be noted, however, that correlations may exist in specific applications, for example when environmental conditions simultaneously influence drift and repeatability. The Monte Carlo framework used in this work is fully capable of incorporating correlated input variables through appropriate covariance structures if such information becomes available. Investigation of correlated uncertainty models and their impact on recalibration intervals represents a relevant direction for future research.

The numerical experiment presented in this study is based on synthetic data designed to represent realistic drift and repeatability behavior observed in industrial measuring instruments. This choice enables controlled parameter studies, transparency of assumptions, and full reproducibility of the results. It should be noted, however, that validation using long-term calibration data from real instruments would provide additional confidence in the practical applicability of the proposed method. Such datasets could be directly incorporated into the Monte Carlo framework by estimating drift and uncertainty parameters from

historical calibration records. This extension is planned as part of future work, particularly in collaboration with industrial and calibration laboratories.

Future work should focus on extending the proposed framework to incorporate adaptive updating of model parameters using Bayesian inference as new calibration results are obtained. Such an approach would allow recalibration intervals to be continuously refined based on accumulated evidence, further reducing both measurement risk and unnecessary calibration effort. Experimental validation using long-term calibration histories of real instruments, including different classes of dimensional and non-dimensional measuring devices, would also strengthen the general applicability of the method. Additionally, integration of the Monte Carlo-based interval determination into laboratory information management systems (LIMS) could facilitate practical implementation in accredited calibration laboratories.

Overall, the results presented in this study confirm that Monte Carlo-based recalibration interval determination provides a scientifically sound, standards-compliant, and practically feasible alternative to conventional fixed time-based strategies. By explicitly linking uncertainty growth, drift behaviour, and conformity risk, the proposed method supports more transparent and defensible calibration decisions and contributes to the ongoing transition from prescriptive to risk-based measurement management.

5. Conclusions

In this work, a Monte Carlo-based methodology for determining recalibration intervals of measuring instruments was systematically developed and demonstrated. The study addressed a fundamental limitation of conventional calibration practice, where recalibration intervals are commonly defined using fixed time-based rules that are not explicitly linked to measurement uncertainty growth or conformity risk. By modeling measurement error as a time-dependent stochastic process and propagating uncertainty through Monte Carlo simulation, the proposed approach enables recalibration decisions to be derived directly from quantitative risk criteria.

The main novelty and contributions of this article can be summarized as follows:

(a) Development of an uncertainty- and risk-based recalibration framework. A unified probabilistic framework was developed that links calibration uncertainty, drift behavior, and conformity assessment within a single Monte Carlo simulation model. The study demonstrated that recalibration intervals can be determined as the time at which the probability of exceeding the maximum permissible error (MPE) reaches a predefined acceptable risk level. This approach provides a scientifically justified alternative to fixed recalibration intervals and is fully compatible with ISO/IEC 17025 and GUM principles.

(b) Quantitative demonstration of uncertainty growth and conformity risk. Using realistic synthetic data representative of a typical dimensional measuring instrument, the study quantitatively illustrated how uncertainty and conformity risk evolve between calibration events. The results showed that expanded uncertainty and probability-based criteria may lead to substantially different recalibration intervals, highlighting the importance of explicitly accounting for distribution tails when controlling measurement risk.

(c) Practical applicability and instrument-agnostic implementation. The proposed methodology relies exclusively on generic metrological inputs, such as calibration results, uncertainty components, drift estimates, and MPE specifications, which are routinely available in calibration laboratories. As a result, the method is instrument-agnostic and can be readily applied to a wide range of measuring devices without requiring modifications to existing calibration procedures or standards.

The numerical experiment confirmed that instruments with identical nominal specifications may exhibit significantly different recalibration needs depending on drift variability and uncertainty growth. This finding emphasizes that fixed recalibration intervals may be either overly conservative or insufficiently protective when instrument-specific behavior is ignored.

Although the present study was based on synthetic data, the selected parameters were chosen to reflect realistic industrial conditions and typical calibration practice. Validation using long-term calibration histories of real instruments, as well as extensions to include systematic drift components and environmental influences, represent important directions for future research.

Overall, the proposed Monte Carlo-based framework provides a transparent, flexible, and traceable tool for optimizing recalibration intervals based on quantified risk, supporting more efficient and reliable metrological control in both industrial and laboratory environments.

Future research will focus on experimental validation using long-term calibration data from real measuring instruments, extension of the model to non-linear and non-stationary drift behavior, and integration of Bayesian updating to enable adaptive recalibration intervals. Further work may also investigate the economic impact of risk-based recalibration strategies and their implementation within laboratory information management systems.

In conclusion, this work provides a validated methodological framework for recalibration interval determination based on Monte Carlo uncertainty propagation and conformity risk assessment. The proposed approach supports calibration laboratories, industrial users, and metrology institutions in achieving more reliable, transparent, and economically efficient measurement management while remaining fully compliant with international standards.

Author Contributions: Conceptualization, D.M.; Software, D.M.; Validation, D.M.; Formal analysis, D.M.; Investigation, D.M., T.K. and M.K.; Resources, T.K.; Data curation, T.K.; Writing—review and editing, M.K.; Supervision, T.K. and M.K.; Project administration, M.K. All authors have read and agreed to the published version of the manuscript.

Funding: The authors would like to thank the Slovak Grant Agency—project VEGA 1/0191/24, and project KEGA 004TUKE-4/2024.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A. MATLAB Implementation of the Monte Carlo Simulation

This appendix presents the MATLAB implementation corresponding to the Monte Carlo-based recalibration framework described in Algorithm 1. The code was developed to ensure full reproducibility of the numerical experiment and can be readily adapted to other measuring instruments by modifying the input parameters.

Appendix A.1. Overview of the Simulation Structure

The Monte Carlo simulation follows the uncertainty propagation principles defined in the GUM Supplement 1 (JCGM 101) and consists of the following stages:

1. Definition of metrological input parameters and simulation settings;
2. Random sampling of uncertainty components and drift parameters;

3. Time-dependent propagation of measurement error;
4. Evaluation of expanded uncertainty and conformity risk;
5. Determination of the recalibration interval based on predefined decision criteria.

All random variables are sampled independently, and a sufficiently large number of Monte Carlo trials is used to ensure numerical stability of the estimated probability distributions.

Appendix A.2. Definition of Input Parameters

The simulation begins with the definition of the main input parameters, including uncertainty components, drift characteristics, and decision thresholds:

```
% Monte Carlo settings
N_MC = 1 × 10^5; % Number of Monte Carlo samples
t_max = 36; % Maximum simulation time [months]
t = 0:1:t_max; % Time vector [months]

% Instrument parameters
MPE = 5; % Maximum permissible error [μm]
risk_limit = 0.05; % Acceptable conformity risk

% Uncertainty components (standard uncertainties)
u_cal = 1.0; % Calibration uncertainty [μm]
u_rep = 1.0; % Repeatability [μm]

% Drift model parameters
mu_drift = 0.0; % Mean drift rate [μm/month]
sigma_drift = 0.5; % Standard deviation of drift rate [μm/month]
```

Appendix A.3. Monte Carlo Sampling and Drift Propagation

For each Monte Carlo trial, random realizations of the calibration error, repeatability, and drift rate are generated. The total measurement error is then propagated over time:

```
% Random sampling
cal_error = u_cal * randn(N_MC,1);
rep_error = u_rep * randn(N_MC,1);
drift_rate = mu_drift + sigma_drift * randn(N_MC,1);

% Preallocation
error_t = zeros(N_MC, length(t));

% Time-dependent error propagation
for k = 1:length(t)
    error_t(:,k) = cal_error + rep_error + drift_rate * t(k);
end
```

Formulation allows both random variability and systematic drift accumulation to be represented within a unified probabilistic framework.

Appendix A.4. Uncertainty Evaluation and Conformity Assessment

At each time step, the simulated error distribution is used to estimate the combined standard uncertainty, expanded uncertainty, and probability of exceeding the MPE:

```
U = zeros(1,length(t));
risk = zeros(1,length(t));

for k = 1:length(t)
    u_c = std(error_t(:,k));
    U(k) = 2 * u_c; % Expanded uncertainty (k = 2)
    risk(k) = mean(abs(error_t(:,k)) > MPE);
end
```

The conformity risk is defined as the probability that the absolute measurement error exceeds the specified MPE.

Appendix A.5. Determination of the Recalibration Interval

The recalibration interval is identified as the first time instant at which the conformity risk exceeds the predefined acceptable level:

```
idx = find(risk >= risk_limit, 1, 'first');

if ~isempty(idx)
    recal_interval = t(idx);
else
    recal_interval = NaN; % Risk limit not reached within simulation horizon
end
```

This criterion provides a risk-based alternative to fixed recalibration intervals and allows direct control of conformity risk over time.

Appendix A.6. Adaptability of the Code

The presented MATLAB code is instrument-agnostic and can be adapted to other measuring devices by modifying:

- the uncertainty components,
- the drift model parameters,
- the MPE value,
- and the acceptable risk threshold.

As such, the implementation serves as a generic computational template for uncertainty- and risk-based recalibration interval determination in accordance with ISO/IEC 17025 and GUM principles.

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