applied sciences

MDPI

*Article*

# Mathematical Modelling Pneumatic Processes of Assessing the Technical Condition of Brake Systems in Passenger Cars

**Vasyl Ravlyuk [1], Juraj Gerlici [2], Alyona Lovska [2,*] and Yaroslav Derevianchuk [1]**

[1] Department of Wagon Engineering and Product Quality, Ukrainian State University of Railway Transport, Oboronnyi Val sq., 7, 61050 Kharkiv, Ukraine; ravvg@ukr.net (V.R.)

[2] Department of Transport and Handling Machines, Faculty of Mechanical Engineering, University of Žilina, Univerzitná 8215/1, 010 26 Žilina, Slovakia; juraj.gerlici@fstroj.uniza.sk

* Correspondence: alyona.lovska@fstroj.uniza.sk

## Abstract

The article addresses the assessment of the technical condition of pneumatic brake systems in passenger cars by means of mathematical modelling processes, taking into account the reduction in the flow capacity of throttle orifices. The existing approaches to describing processes in railway pneumatic systems were analysed, and the limitations in their application under operating conditions of passenger cars were identified. The mathematical model was applied to investigate pressure changes in the main components of a pneumatic actuator, taking into account the state of the throttle orifices, the parameters of the air distributor, and the supply line. The mathematical model specifically accounts for the effect of fouling of the throttle orifices by introducing the coefficient $k_z$, which makes it possible to investigate changes in the flow capacity of the pneumatic channels and their effect on pressure dynamics at different levels of fouling. The simulation results have shown that, as the fouling factor decreases, the flow capacity of the throttle orifices decreases and the duration of transient processes in the pneumatic system increases. By reducing the fouling factor to $k_z = 0.7$, the effective area of the throttle orifices is 70% of the nominal value, which is accompanied by an increase in the relative duration of the reservoir filling. Experimental tests were carried out on the test bench in the brake testing laboratory of the Department of Wagon Engineering and Product Quality at the Ukrainian State University of Railway Transport, during which the relationships between changes in pressure in the brake cylinder and the auxiliary reservoir were obtained. An analysis of the calculated dependencies has shown that a reduction in the flow capacity of the throttle orifices is accompanied by a slower rise in pressure in the pneumatic system. The quantitative validation of the accuracy of the proposed mathematical model requires further comparison of the calculated results with the results obtained during bench tests conducted under identical initial conditions. The obtained results can be used for the further development of algorithms for the automated monitoring of the technical condition of pneumatic brakes on passenger cars.

**Keywords:** algorithm; brake equipment; diagnostics; mathematical model; railway transport; rolling stock; technical condition

Academic Editor: Suchao Xie

Received: 29 October 2026
Revised: 26 November 2026
Accepted: 27 November 2026
Published: 29 November 2026

**Copyright:** © 2026 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license.

## 1. Introduction

The current stage in the development of rail transport is characterised by more stringent requirements for safety [1–3], reliability [4,5] and efficiency of rolling stock operation [6,7]. The increase in transport speeds and traffic intensity necessitate improve-

ments to the systems for managing and monitoring the technical condition of railway cars. Particular attention is paid to braking systems, as the braking stability and the passenger transport safety depend on their proper functioning. The traditional methods of technical inspection do not always provide sufficient accuracy when assessing the condition of pneumatic components; therefore, the introduction of modern diagnostic tools is of importance. One way of improving the effectiveness of diagnosis is the use of mathematical models, which can be used to predict changes in the parameters of a pneumatic system under different operating conditions.

Pneumatic braking systems in passenger cars are rather complex and comprise numerous components, such as reservoirs, valves, throttles, pipelines and connecting assemblies. The speed at which pressure is transferred, braking response time and smooth operation of the system depend on the proper functioning of each of the components. One of the key factors affecting the efficiency of the brake actuation system is the condition of the throttle orifices and lines in the pneumatic assembly, through which the flow of compressed air is regulated.

During operation, these components are exposed to dust, condensation, oil particles, rust and wear debris. Over time, this leads to blockages in the lines, a reduction in the cross-sectional area and changes in the flow characteristics. As a result, the stability of charging and discharging the reservoirs is deteriorating, the braking dynamics are altered, and in some cases individual system components may fail partially.

To analyse such processes, it is advisable to use mathematical models, which describe the interrelationship between the geometric parameters of the throttling elements, the flow characteristics and the system's response time. Consequently, the task of improving the diagnosis of the braking system on passenger rolling stock remains a pressing issue.

The issue of improving pneumatic braking systems for railway rolling stock is widely covered in publications on the modelling of dynamic processes, developing diagnostic methods and ensuring the reliability of braking equipment. The works studied lay the theoretical foundations for the development of modern pneumatic system models and the identification of the parameters that influence their operational efficiency.

A parallel computing scheme developed for modelling processes in pneumatic braking systems is described in [8]. The authors demonstrated that the use of parallel computing increases the speed of calculations while maintaining a high degree of accuracy in determining pressure changes in pipelines. The model takes into account the interaction between the individual components of the braking system, in particular the reservoirs and valves, which makes it possible to describe the system's actual transient operating conditions. The results obtained form the basis for the development of hydrodynamic models suitable for investigating the behaviour of brakes under various operating conditions. However, the study does not cover diagnostic tools for ensuring a higher level of monitoring the technical condition of the braking system on passenger vehicle while in motion.

A comprehensive approach to the online diagnosis of faults in the air brakes of freight trains is proposed in [9]. The methodology is based on a combination of experimental data and machine learning algorithms, which ensures a high level of accuracy in defect detection both for individual components and the entire wagon. The authors demonstrate that the use of the random forest algorithm enables a diagnostic accuracy of over 94%. Although the study focuses on freight trains, the proposed approach can be adapted for passenger car monitoring systems.

Mathematical modelling of emergency braking in the MATLAB LTDPoliTo environment is discussed in [10]. The model developed makes it possible to assess the influence

of the main design parameters of a rolling stock unit on the braking process without complex experiments. The authors note that the calculated data are in good conformity with the those prescribed in the UIC 544-1leaflet. However, the proposed approach to simulating the emergency braking process for European vehicles is difficult to apply for the analysis of the pneumatic brakes on passenger cars operated by Ukrzaliznytsia (Ukrainian Railways).

A new method for detecting early-stage faults in the braking system of an electric train is described in [11]. The authors apply the Variable Control Limit algorithm in conjunction with a description of the data domain, which enables defects to be detected online with high accuracy. This approach helps reduce the number of false alarms and improve the detection rate of faults in the braking system. The results obtained demonstrate the potential of such methods in on-board diagnostic systems for the pneumatic brakes of passenger cars. This analysis of the causes of faults in the braking system relates to electric trains operating on China's railways and does not correspond to the design and operational characteristics of the braking systems on passenger cars used in Ukraine.

A systematic review of modern fault detection and diagnosis methods used in pneumatic braking systems for motor vehicles is given in [12]. It covers both traditional approaches and intelligent algorithms utilising artificial neural networks and machine learning. The authors identify the main trends in the transition to autonomous maintenance systems. The findings of the study highlight the need to integrate hybrid models into the modern monitoring systems used for braking equipment on motor vehicles.

A hybrid model for real-time monitoring of the condition of pneumatic brakes on freight trains is given in [13]. The model combines simplified hydrodynamic equations with empirical data to determine the pressure in the brake cylinder. The proposed approach can be used for accurate prediction of changes in flow parameters and allows for the early detection of anomalies in the system's operation. The results obtained can be adapted to develop algorithms for diagnosing the condition of the pneumatic systems in passenger cars.

However, the mathematical models of pneumatic braking systems for locomotives and rail cars presented in [12,13] do not take into account the specific operating characteristics of the pneumatic brakes on passenger cars operated on Ukrainian Railways.

Study [14] focuses on the use of anti-sway devices with electro-pneumatic valves on passenger cars operated on the internal railways. The authors demonstrate the advantages of using systems to monitor the technical condition of braking equipment during operation and examine in detail the operation of electro-pneumatic assemblies that ensure enhanced braking stability for different operation speeds. However, the issue of developing automated diagnostic algorithms is not covered.

Study [15] presents a methodology for investigating the characteristics and performance of braking systems in passenger cars and freight wagons, based on a combination of theoretical and experimental methods. The author refines mathematical models of braking processes using computer simulation in Visual Basic for Applications (VBA) in Excel, which allows for a wide range of motion and braking parameters to be taken into account. This paper identifies empirical relationships between the variable parameters of pneumatic systems and refines the brake force ratio, friction, and wheel-to-rail adhesion. The proposed method makes it possible to predict the braking distance and the efficiency of the system, taking into account the track gradient, the train composition and the type of braking equipment. Particular attention is paid to improving the methods of experimental research on operational rolling stock. However, this study does not analyse the operation of the air distributors in the pneumatic braking system for an individual

rolling stock unit, which, in turn, affects the overall performance of the train's braking system.

A highly efficient model of a pneumatic braking system, designed for the use in real-time, is proposed in [16]. A distinctive feature of this approach is the use of simplified equations of gas dynamics while the physical processes remain physically consistent, which enables a significant reduction in the computational costs involved in modelling long trains. However, the study does not address the issue of changes in the characteristics of throttle elements during operation, which limits the model's applicability to diagnostic tasks.

A method for adaptive pressure control in electro-pneumatic brakes, taking temperature effects into account, is described in [17]. The Kalman filter-based state estimation algorithm ensures high accuracy in identifying system parameters under conditions of uncertainty. However, the proposed approach focuses primarily on control tasks and does not include the degradation processes associated with the fouling of the pneumatic system's lines.

An important contribution to the development of diagnostic methods is given in [18]. This approach is based on Bond graphs for modelling and monitoring, which takes into account uncertainties in parameters and diagnoses faults in complex dynamic systems. However, its application requires significant computational resources and involves a complex process of parameter identification.

Study [19] deals with a segmented electro-pneumatic braking system for long trains, which makes it possible to significantly reduce longitudinal dynamic loads and improve braking efficiency. However, the study focuses on the system level and does not include the effect of local defects in the components of the pneumatic system.

The latest research in the field of digital technologies is presented in [20], which describes the use of the digital twin concept to predict the technical condition of electro-pneumatic valves. This approach enables the degradation of components to be assessed in real time. However, the creation of a digital twin requires a reliable underlying mathematical model that takes into account the physical processes within the pneumatic system.

Contemporary research into pneumatic braking systems used for railway rolling stock is aimed at improving the accuracy of the mathematical description of transient processes and expanding the capabilities of technical diagnostics. Study [21] presents the modelling of the operation of pneumatic brakes on passenger trains and the experimental verification of the results obtained. Further development in this area can be associated with the use of pressure changes to detect faults in the braking equipment [22] and the application of neural network models to predict its operating parameters [23]. However, the effect of clogged throttle orifices in air distributors on the dynamics of pneumatic processes remains understudied, which makes it difficult to quantitatively assess their technical condition.

An analysis of literature [8–23] has shown that the issue of improving pneumatic braking systems for railway rolling stock is being actively studied is different countries including Ukraine. The main focus is on modelling processes in pneumatic assemblies, investigating pressure dynamics and developing fault diagnosis algorithms with modern data processing and machine learning methods.

However, the vast majority of studies focus on freight rolling stock, while the issue of assessing the technical condition of pneumatic systems in passenger cars, particularly air distributors, remains under-researched. This highlights the importance of further research aimed at developing an improved mathematical model of the pneumatic braking system, which will ensure greater reliability in monitoring the technical condition of

pneumatic systems and the safe operation of passenger cars.

The scientific and practical contribution of this work lies in the application and parameterization of the mathematical model of pneumatic processes to investigate the impact of clogging in the throttling elements of braking systems on passenger cars. The fouling factor $k_z$ can be used to qualify the flow characteristics of throttles by the level to which they are fouled. This can help to assess the technical condition of the air distributor by analysing the dynamics of pressure changes in the system's main components. The approach proposed enables a quantitative assessment of the effect of a reduction in the flow capacity of the throttle orifices on pressure change dynamics and provides a basis for the development of algorithms for diagnosing the technical condition of air distributors. However, determining the actual fouling and assessing the accuracy of the diagnosis require calibration.

The obtained results provide a basis for establishing diagnostic criteria to assess the technical condition of pneumatic brake systems in passenger cars.

The objective of the study is to apply the mathematical model to evaluate how a reduction in the flow capacity of throttle orifices affects pneumatic processes, and to determine the feasibility of using the derived relationships to assess the technical condition of the braking equipment on passenger cars.

To achieve the set objectives, the following tasks were solved:

- To analyse the current methods of mathematical modelling of pneumatic braking systems in passenger cars;
- To carry out simulation modelling to assess the impact of the technical condition of individual components on the overall performance of the pneumatic system;
- To analyse the conditions under which bench tests of air distributors are carried out and to determine the potential for using experimental data to validate the mathematical model;
- To develop recommendations for designing an algorithm for the automated diagnosis of the air distribution condition based on the results of mathematical modelling.

## 2. Materials and Methods

A typical mathematical model of a pneumatic actuation system includes the equations of motion for the actuator and the equations describing the pressure changes in the delivery chambers. A key element of this model is the description of the processes by which air escapes through the throttling elements, which determine the dynamics of how the reservoirs and chambers fill.

The model takes into account three main modes of the pneumatic system operation:

- Isothermal, when the air temperature remains constant;
- Adiabatic, which corresponds to the heat-insulated conditions of fast processes;
- Polytropic, which reflects the intermediate nature of heat exchange.

In most cases, an adiabatic process is assumed for practical calculations, as pressure changes rapidly in braking systems and heat exchange between the air and the pipe walls is negligible.

The change in pressure within the pneumatic system V is described by the following mathematical model:

$$\frac{dp}{dt} = \frac{kRT}{V}\left(G_1(t) - G_2(t)\right), \quad (1)$$

where $p$ is the chamber pressure; $G_1(t)$ and $G_2(t)$ are the mass flow rates of air entering and leaving through the throttles; $k$ is the adiabatic index; $R$ is the gas constant; $T$ is the

air temperature.

Equation (1) is derived from the laws of conservation of mass and energy for a pneumatic reservoir of constant volume, under the adiabatic approximation. Here, pressure is absolute, and the mass flow rates of air at the inlet and outlet are determined considering the direction of its flow. To simplify the mathematical description, the inlet air temperature is assumed to be equal to the air temperature inside the reservoir. Under these assumptions, the change in pressure is determined by the difference in mass flow rates, which ensures consistency in the mathematical description of the charging and discharging processes in the pneumatic system.

Thus, the accuracy of the pressure calculation directly depends on the correct determination of *G*.

Each unit of the passenger rolling stock is fitted with an automatic pneumatic brake (Figure 1), which is connected to the brake line (BL) 3 and has air distributor (AD) 2 to which additional braking devices are connected, namely brake cylinder (BC) 4 and auxiliary reservoir (AR) 1. The braking processes in the automatic brake system are achieved by a change in pressure within the brake line. When compressed air enters the brake line, the brake is applied and released; to ensure braking, the compressed air is released from the brake line (Figure 1a).

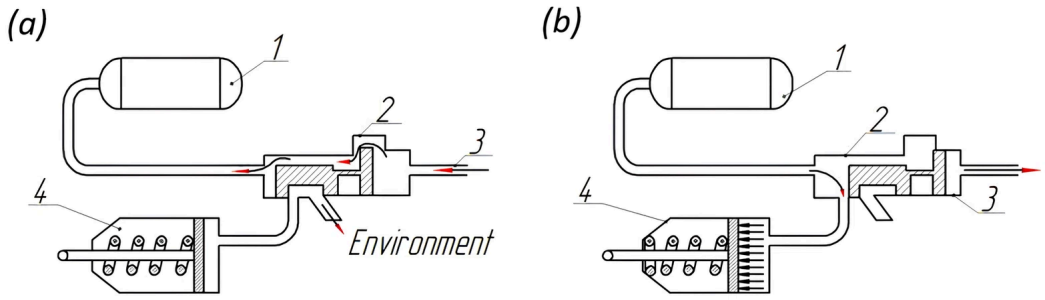



**Figure 1.** The diagram of the automatic brake operation: (**a**) when brakes are being charged; (**b**) when braking: 1—auxiliary reservoir (AR); 2—air distributor (AD); 3—brake line (BL); 4—brake cylinder (BC).

From BL 3, air flows through AD 2, the piston of which has three calibrated orifices with a diameter of 1.25 mm; it is then fed into AR 1. The brakes on each unit of the rolling stock are being applied. The air distributor connects BC 4 to the environment via an orifice with a cross-sectional area of 18 $mm^2$. Under the action of the spring, the piston rod moves to the right, the brake lever mechanism is activated and the brake shoes move away from the wheel surface, i.e., the brakes are being released.

The brakes on Ukrainian passenger cars with air distributor No. 292 are of the non-rigid type and provide brake insensitivity; that is, the brakes do not engage if the pressure in the brake pipe decreases slowly (insensitivity rate).

This allows the pressure in the brake line to decrease slowly. Overcharge is relieved as the pressure in the brake line decreases, when air from the auxiliary reservoir flows into the brake line. Here, there must be no pressure differential across the air distributor piston, which would cause it to move. The ranges of pressure reduction in the brake line for these air distributors during braking is more than 0.005 MPa per 1 s, and for insensitivity rate it is up to 0.05 MPa per 1 min.

To ensure braking (Figure 1b), compressed air from BL 3 is released into the environment; the brake line is partially or completely discharged. The air from the auxiliary reservoir does not have time to flow into the brake line through the three calibrated orifices with a diameter of 1.25 mm, which causes an increase in the pressure differential

across the air distributor piston and results in piston movement. Air distributor 2 is activated during braking, isolating BC 4 from the environment and connecting it to the auxiliary reservoir. The compressed air from the auxiliary reservoir enters the BC; the piston moves to the left and, via the brake lever mechanism, presses the brake shoes against the wheel tread (the braking process).

Based on Equation (1), a mathematical model of the operation for a serviceable air distributor No. 292 in the modes 'Charge and Release' and 'Train Position' has been proposed. This model can be used for the assessment of the technical condition of the air distributor based on an analysis of the dynamics of pressure changes in the main brake cylinder (MBC) and the auxiliary reservoir (Figure 2).

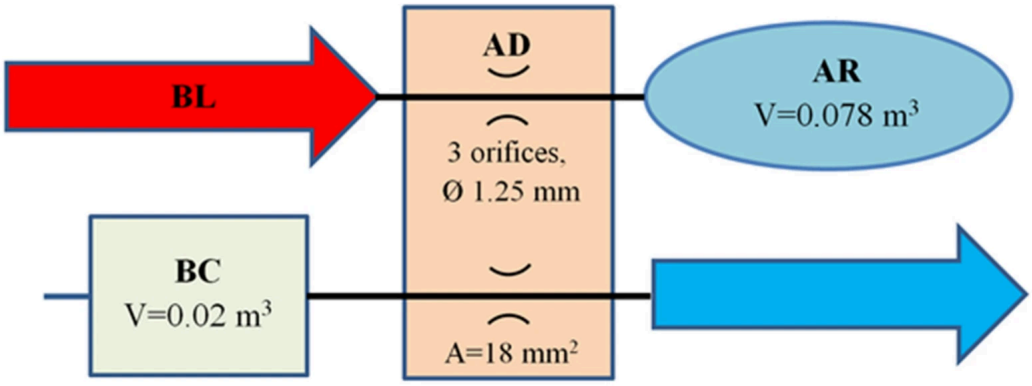



**Figure 2.** The functional diagram of an air distributor No. 292 in the modes 'Charge and Release' and 'Train Position'.

In the modes 'Charge and Release' and 'Train Position' there is an increase in pressure in the brake line. The brake line is connected through three calibrated orifices with a diameter of 1.25 mm in the air distributor with the auxiliary reservoir of a volume of $V = 0.078$ m$^3$. Simultaneously, through a cross-sectional orifice with an area of 18 mm$^2$, the brake cylinder with a volume of $V = 0.02$ m$^3$ is connected to the environment. According to the requirements, the time for reducing the pressure in the auxiliary reservoir ($P_{sr}$) to 0.14 MPa should be 9–12 s. When charging the pneumatic system, the pressure rise in the auxiliary reservoir is monitored to ensure it ranges from 0.40 to 0.45 MPa; according to the test procedure, this should take place over a period of 15–25 s [24].

An overcharge is eliminated during the transition from the mode 'Charge and Release' to the mode 'Train Position'; the pressure in the auxiliary reservoir and the main valve must decrease from 0.60 to 0.58 MPa in 100–120 s [14,15].

The air flow rate through the throttle is described by an equation with relative pressure before and after the throttle, cross-sectional area, gas temperature and flow coefficient. In general terms, this equation takes the form:

$$G = \begin{cases} C_d A p_1 \sqrt{\dfrac{k}{RT_1}\left(\dfrac{2}{k+1}\right)^{\frac{k+1}{k-1}}}, & \text{if } \dfrac{p_2}{p_1} \le 0.528 \\ C_d A p_1 \sqrt{\dfrac{2k}{(k-1)RT_1}\left[\left(\dfrac{p_2}{p_1}\right)^{\frac{2}{k}} - \left(\dfrac{p_2}{p_1}\right)^{\frac{k+1}{k}}\right]}, & \text{if } \dfrac{p_2}{p_1} > 0.528 \end{cases}, \quad (2)$$

where $G$ is the mass air flow rate; $C_d$ is the flow coefficient; $A$ is the nominal throttle orifice area; $p_1$ and $p_2$ are the pressure before and after the throttle; $T_1$ is the air temperature

before the throttle.

Under real conditions the flow coefficient $C_d$ is determined experimentally and depends on geometric parameters of the orifice, surface roughness, degree of fouling and gas flow regime. As the pressure drops after the throttle, the air flow rate increases to a certain critical value, after which it stabilises, as the flow velocity reaches the speed of sound. This is characteristic of a choked flow regime.

The mathematical model is based on the following assumptions and constraints:

- The flow of compressed air in a pneumatic system is regarded as a quasi-steady-state process;
- Pressure changes in the main components of the system are described using the adiabatic approximation;
- The air temperature within a single element is considered to be uniform;
- Air leakage due to loose connection is not taken into account;
- Flow coefficient $C_d$ is assumed to be constant for a given braking mode;
- The pipeline parameters (length, diameter, roughness) are assumed to be the same.

These assumptions make it possible to simplify the mathematical description of transient processes in the pneumatic system. Moreover, their impact on the accuracy of the calculations must be considered when comparing the simulation results with experimental data.

During operation, dust, moisture, grease and corrosion products accumulate in the throttle channels, leading to a reduction in their effective cross-sectional area and an increase in air flow resistance. To quantify this effect in the mathematical model, a fouling factor $k_z$ is used, which characterises the relative change in the flow capacity of the throttle orifice. Unlike the flow coefficient $C_d$, which considers the peculiarities of air passage through the opening, the coefficient $k_z$ is used to describe the change in its state due to fouling. Meanwhile, the cumulative effect of these coefficients is determined from the pressure variation curves; therefore, their independent assessment requires additional information on the throttle's geometric parameters and flow characteristics.

When eliminating the overcharge, the air flows from the auxiliary reservoir into the brake line through three calibrated orifices with a diameter of 1.25 mm. A reduction in their flow capacity due to fouling may affect the rate at which pressure is equalised and create conditions that alter the air distributor's operating mode. However, determining the conditions under which it switches to the braking mode requires a separate study of the main piston's movement and the operation of the valve components.

To take this effect into account, the fouling factor $k_z$ is introduced into the model, which reflects the relative reduction in the throttle's flow capacity:

$$k_z = \frac{A_z}{A}, \qquad 0 < k_z \leq 1. \tag{3}$$

Considering the fact, that the mass air flow rate through the throttles depends on the effective flow area—which decreases due to clogging—the mathematical model of the mass air flow rate in the braking system of a passenger car can be expressed as follows:

$$G = \begin{cases} k_z C_d A p_1 \sqrt{\frac{k}{RT}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}, & \text{if } \frac{p_2}{p_1} \le 0.528 \\ k_z C_d A p_1 \sqrt{\frac{2k}{(k-1)RT}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{k}} - \left(\frac{p_2}{p_1}\right)^{\frac{k+1}{k}}\right]}, & \text{if } \frac{p_2}{p_1} > 0.528 \end{cases}. \quad (4)$$

Equation (4) describes the mass flow rate of compressed air through a throttle orifice, considering the reduction in its effective cross-sectional area due to fouling. The coefficient $k_z$ characterizes the relative change in the throttle capacity, while the flow coefficient Cd includes the hydrodynamic characteristics of the air passing through the orifice. Within the accepted mathematical model, the coefficient $C_d$ is considered constant for a given flow regime, and the effect of fouling is considered by means of $k_z$. The mass flow rate obtained is subsequently used to determine the pressure change without re-multiplying by the fouling factor.

With a reduction in the fouling factor to $k_z = 0.7$, the effective area of the throttle orifices is 70% of the nominal area. This is accompanied by a reduction in flow capacity and an increase in the time taken to fill the auxiliary reservoir for the proposed mathematical model.

To determine the filling time for the auxiliary reservoir, the pressure change equation is used, which includes the mass flow rate of compressed air through the throttle orifices. Under the assumed adiabatic approximation, the absence of air leaks, and a constant temperature of the inlet flow, the filling time is determined by integrating the pressure change relationship from the initial to the final value:

$$t_3 = \frac{V}{kRT_1}\int_{p0}^{p_k} \frac{dp}{G(p;k_z)}, \quad (5)$$

where $t_3$ is the filling time for the auxiliary reservoir; $V$ is the reservoir volume; $k$ is the air adiabatic index; $T_1$ is the compressed air temperature at the inlet; $p_0$ and $p_k$ are the initial and final absolute pressures in the reservoir, respectively; G(p; $k_z$) is the mass flow rate of air, determined by dependence (4) with a one-time consideration of the fouling factor $k_z$.

Equation (5) applies to the process of filling the auxiliary reservoir, provided there is no air leakage from it and the inlet flow temperature remains constant. Here, the mass flow rate changes depending on the current pressure in the reservoir. Where air is entering and leaving simultaneously, the change in pressure must be determined using general Equation (6).

Dependency (4) enables the monitoring of pressure changes in the reservoir while the pneumatic system is being charged or discharged. It should be borne in mind, however, that the nature of the pressure change is determined not only by the reservoir's volume, but also by the temperature conditions and the mass flow rate of the air. As the process of filling the reservoir is non-stationary, the change in pressure is described by Equation (6), which takes into account the increase in air mass over a given period of time.

To determine the change in pressure in a reservoir of constant volume, the integral form of the equation derived from the laws of conservation of mass and energy is used. Under the adopted adiabatic approximation, the initial absolute pressure, the air temperature at the inlet and in the reservoir, and the mass flow rates of air at the inlet and outlet are considered.

$$p(t) = p_0 + \frac{kR}{V}\int_0^t [T_1 G_1(\tau) - T(\tau) G_2(\tau)]\, d\tau, \quad (6)$$

where $p_0$ is the initial absolute pressure; $t_0$ is the initial time point; $T_1$ is the air temperature at the inlet; $T(\tau)$ is the air temperature in the reservoir.

The mass flow rates $G_1(\tau)$ and $G_2(\tau)$ are determined considering the fouling factor $k_z$, so its re-inclusion in Equation (6) is not necessary.

To compare calculated and experimental data, the absolute pressure is defined as the sum of the excess pressure and atmospheric pressure.

The pressure change is determined numerically by integrating Equation (6) under the given initial conditions. At each stage of the calculation, the mass flow rate of air is determined using Equation (4), considering the current pressure and the fouling factor of the throttle orifices. This approach makes it possible to simulate the dynamics of the reservoir's filling and assess the effect of a reduction in the flow capacity of the throttles on the duration of the transient process.

The numerical calculations are carried out in Microsoft Excel. The obtained pressure values are used to build time dependencies for different values of the fouling factor of the throttle orifices. This enables comparing the nature of pressure changes and the duration of charging the auxiliary reservoir depending on the flow capacity of the pneumatic channels.

The numerical solution of Equation (6) under the assumed initial conditions is performed using the explicit Euler method. Moreover, the absolute pressure in the reservoir at the next time stage is determined by the following equation:

$$p_{i+1} = p_i + \frac{kR\,\Delta t}{V}\left[T_{1,i} G_{1,i} - T_i G_{2,i}\right], \quad (7)$$

where $p_i$ and $p_i + 1$ are the absolute pressures at the current and next calculation step, respectively; $\Delta t$ is the integration time step; $i$ is the calculation step number.

The indices $i$ for temperature and mass flow rate indicate their values at the corresponding step. The fouling factor $k_z$ is considered according to Equation (4) and is not entered again.

The numerical integration is performed with a time step of $\Delta t$ = 0.1 s. To check the convergence of the calculations, the obtained results are compared with the results for a reduced time step $\Delta t$ = 0.05 s.

Under real operating conditions, the parameters $G$, $T$ and $V$ can vary as a result of fluctuations in ambient temperature, pressure fluctuations in the pipeline and the effect of throttling elements. Therefore, to ensure the model is adequate, it is advisable to introduce correction factors that characterise heat losses, as well as the effect of internal flow resistance. Furthermore, the accuracy of the pressure calculation depends significantly on the choice of integration step, which determines the resolution of the calculation of changes in air mass in the reservoir.

To assess the convergence of the numerical solution, calculations are carried out to determine the time taken to fill the auxiliary reservoir using Euler's method, with time steps of 0.1 and 0.05 s. The results of the comparison are presented in Table 1.

**Table 1.** The verification of the convergence of the numerical solution for different values of the fouling coefficient.

| $k_z$ | Time for $\Delta t$ = 0.1, Sec | Time for $\Delta t$ = 0.05, Sec | Discrepancy, % |
|---|---|---|---|
| 1.0 | 12.866 | 12.871 | 0.039 |
| 0.9 | 14.297 | 14.302 | 0.035 |
| 0.7 | 18.385 | 18.390 | 0.027 |

The values are calculated for $V$ = 0.078 m$^3$, $C_d$ = 0.65, $T$ = 293.15 K, a line pressure of 0.52 MPa and an increase in overcharge in the reservoir from 0.40 to 0.45 MPa. The time at which the final pressure is reached is determined by linear interpolation within the last step.

The analysis of the results in Table 1 shows that the discrepancy between the charging times at different integration steps does not exceed 0.039%. This indicates the convergence of the numerical solution and justifies the use of a time step of 0.1 s within the accepted mathematical model.

## 3. Research and Discussion

The pressure dynamics resulting from the fouling of orifices, and pressure losses during transmission through orifices and channels, are reflected in a reduction in transmission efficiency.

In traditional mathematical models, changes in the flow capacity of orifices are considered by adjusting the effective cross-sectional area or the flow coefficient. The proposed approach involves the separate introduction of the fouling coefficient $k_z$, which allows investigating the impact of fouling on the dynamics of pressure changes without altering the nominal geometric parameters of the throttle. Meanwhile, the coefficient Cd is assumed to be constant for the given flow regime. This description simplifies the analysis of the impact of different levels of fouling and the comparison of the corresponding calculated dependencies. To generalize the impact of these factors, the effective flow transfer coefficient $k_e$ is used; it is determined by the following formula:

$$k_e = k_z \cdot C_d \quad (8)$$

To determine the fouling factor, it is necessary to first establish the flow characteristics of the throttle orifice in good working order. Given the known geometric parameters and the assumed value of the flow coefficient, a comparison of the experimental pressure variation curves with the calculated ones allows the relative reduction in flow capacity to be estimated. Meanwhile, it is impossible to independently determine $k_z$ and $C_d$ based solely on the pressure curves, since their effect on the mass flow rate is described by a common product. Therefore, assessing fouling requires first determining the characteristics of a properly functioning throttle and ensuring identical conditions for comparison.

To compare the proposed approach with the traditional description of air mass flow, the conditions of equal effective throttle orifice capacity are considered. In the traditional model, this parameter is determined by the product of the flow coefficient and the flow cross-sectional area, whereas in the proposed model, the fouling coefficient is additionally considered. Given the same total flow capacity, both approaches are mathematically equivalent, and the discrepancy between the corresponding calculated pressure values is 0%. Therefore, the introduction of a fouling factor does not in itself improve the accuracy of the prediction; however, it does allow the degree of fouling of the orifices to be specified separately and its effect on the dynamics of pneumatic processes to be investigated.

To quantify the fouling factor, it is proposed to calibrate the mathematical model us-

ing the experimental pressure-versus-time relationship. Meanwhile, the geometric parameters of the throttle orifice and the flow coefficient are predetermined for the serviceable condition and are assumed to be constant. The desired value of the fouling factor is determined under the condition of the minimum sum of squared deviations between the calculated and measured pressure values:

$$\hat{k}_z = \underset{0<k_z\le 1}{arg\ min} \sum_{i=1}^{n} \left[p_{e,i} - p_{c,i}(k_z)\right]^2, \quad (9)$$

where $\hat{k}_z$ is the estimated value of the fouling factor; $p_{e,i}$ and $p_{c\,i}$ are the experimental and calculated values of pressure at the $i$-th time point, respectively; $n$ is the number of comparison points.

Equation (9) defines a possible method for calibrating the model using an experimental pressure-versus-time curve. In the present study, the fouling factor is estimated based on the time taken to fill the auxiliary reservoir, whilst the application of the least squares method to the complete time series of pressure requires further processing of the experimental data.

This approach allows determining the fouling factor with a minimal deviation of the calculated curve from the experimental one under the same test conditions.

In the calculations provided, the value of the coefficient $k_z$ is set to study the effect of reducing the flow capacity of the throttle orifices on the pressure dynamics. Therefore, these values characterize the adopted calculated states, and not the directly measured degree of fouling of the air distributors under investigation. Finding the correspondence between the calculated coefficient and the actual condition of the throttle channels requires experimental calibration.

For the quantitative analysis, the calculated dependencies of pressure changes at different values of the fouling factor of the throttle orifices are used. The approximation of these dependencies by an exponential function shows that with a decrease in the throughput capacity of the throttle orifices, the time constant of the transient process increases from 6 to 10 s. Under the condition of inverse proportionality of the time constant, the calculated relative values of the latter are 1.000; 0.857 and 0.600 (Table 2). The difference between these estimates and the accepted value of the fouling factor indicates the need for separate calibration of the model parameters and verification of their compliance with the actual condition of the throttle orifices.

**Table 2.** The results of the assessment of transient process parameters based on pressure change curves.

| Preset Value of the Fouling Factor, $k_z$ | Time Constant, τ, Sec | Estimated Relative Throughput |
|---|---|---|
| 1.0 | 6.00 | 1.000 |
| 0.9 | 7.00 | 0.857 |
| 0.7 | 10.00 | 0.600 |

The value of the last column is determined by the ratio:

$$k_в = \frac{\tau_0}{\tau}, \quad (10)$$

where $k_в$ is the estimated relative capacity; $\tau_0$ is the time constant for the initial state, $\tau_0$ =

6 s.

To assess the discrepancies between the calculated pressure change relationships, the standard deviation is used; it is determined by the formula:

$$R_{MSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(p_{c,i} - p_{b,i}\right)^2}, \quad (11)$$

where $p_{b,i}$ is the pressure value for the initial state of the throttle orifices for $k_z = 1.0$.

For a quantitative comparison, the deviations of the pressure change curves from the initial relationship obtained in the absence of fouling are considered. According to the calculations, the standard deviation is 0.00580 MPa for the curve with a time constant of 7 s and 0.02059 MPa for the curve with a time constant of 10 s. This confirms that pressure differentials increase as the flow capacity of the throttle orifices decreases. With the same efficiency coefficient, the traditional and the proposed models are mathematically equivalent. Therefore, the advantage of introducing the fouling factor lies primarily in the possibility of analysing the impact of fouling separately, rather than in a guaranteed improvement in forecasting accuracy.

This approach includes both the changes in the geometric parameters of the throttle orifice due to fouling and the influence of the flow's hydrodynamic characteristics. In this case, the mass flow of air through the throttle can be expressed as:

$$G = k_e \cdot A \cdot f(p_1, p_2) \quad (12)$$

The presented relationship allows us to account the reduction in the flow capacity of throttle orifices during mathematical modelling.

The changes in pressure in the reservoir $p = f(t)$ at different values of the fouling factor $k_z$ were studied. When analysing the obtained results, it is necessary to distinguish between the charging time of the auxiliary reservoir and the brake response time. The first indicator characterizes the process of increasing the pressure in the reservoir, while the second depends on the operation of the air distributor and the charging of the brake cylinder. Therefore, a change in the charging time cannot be directly equated with a change in the brake response time without considering the corresponding operation modes of the pneumatic system. Figure 3 shows the dependencies of the effective area of the throttle orifice and the duration of the transient process on the fouling factor $k_z$. The obtained results indicate that as the throttle orifice capacity decreases, the duration of the corresponding process increases.

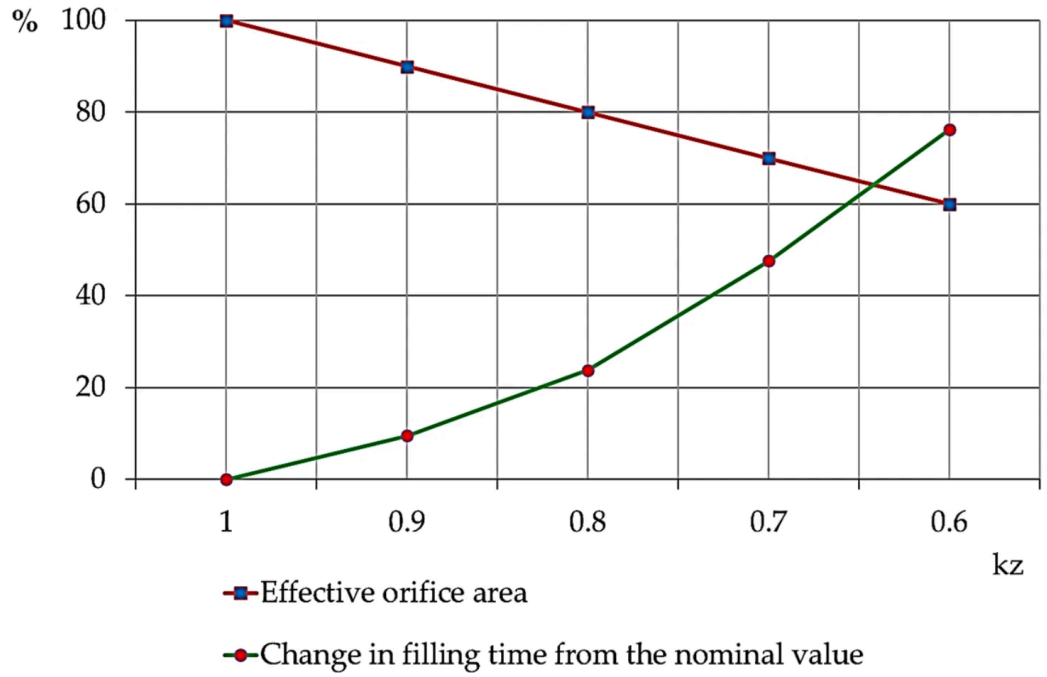



**Figure 3.** The dependence of the effective area of the throttle orifice and the relative increase in charging time on the fouling factor $k_z$.

The analysis of the dependencies shown in Figure 3 has revealed that as the fouling factor $k_z$ = 0.7 decreases, the effective area of the throttle orifices decreases, and the relative duration of reservoir filling increases. For $k_z$ = 0.7, the effective area is 70% of the nominal area, which is accompanied by a slowdown in the charging process. These results indicate that the flow capacity of the throttle orifices strongly influences pressure transient behaviour, confirming the importance of taking their technical condition into account when mathematically modelling a pneumatic system.

The reduced flow capacity of the throttle orifices can disrupt the established dynamics of pneumatic processes and affect the timing of brake application and release. The derived equations do not allow the conditions under which such faults occur to be determined directly, as this would require considering the specific characteristics of the air distributor valve switching and the operating modes of the braking system. Therefore, establishing a connection between the degree of fouling of the throttle orifices and specific brake malfunctions requires further research.

The performance of an air distributor No. 292 was tested on the test bench in the braking test laboratory of the Department of Wagon Engineering and Product Quality at UkrSURT (Figure 4). The air distributors that had been in service on passenger cars during the period between overhauls were installed on the test bench.

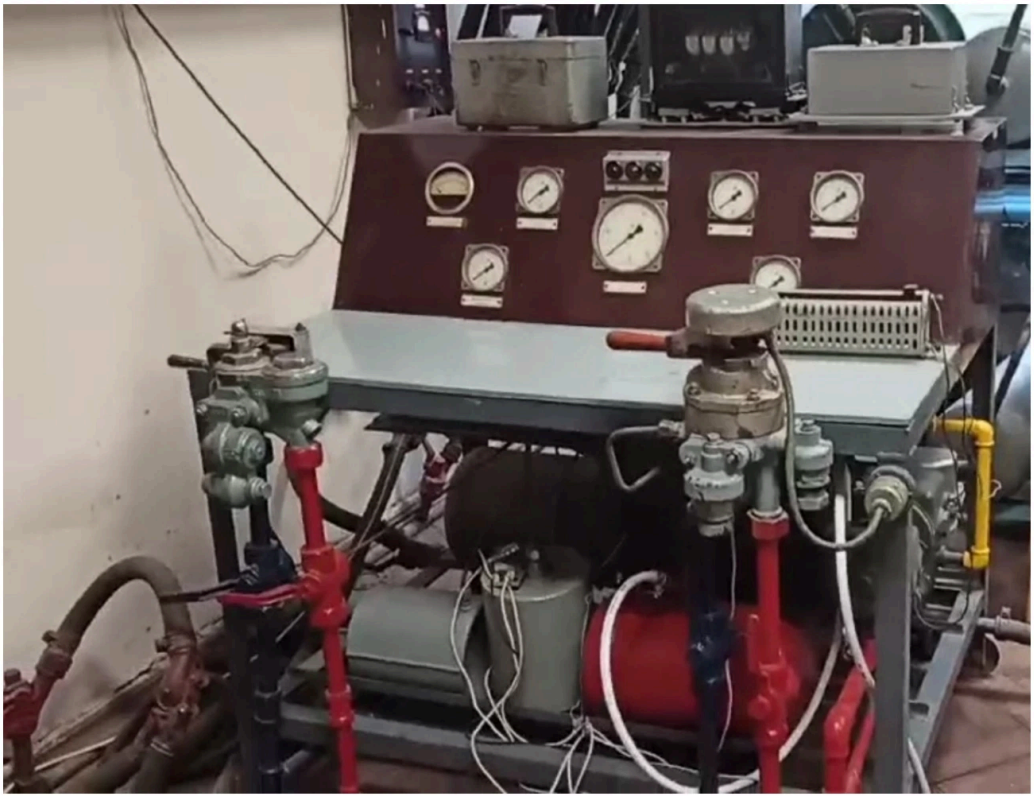


**Figure 4.** The test bench in the brake testing laboratory of the Department of Wagon Engineering and Product Quality of UkrSURT.

During the bench tests, the performance of air distributors No. 292 was evaluated based on the nature of pressure changes in the auxiliary reservoir and the brake cylinder. The focus was on the course of transient processes, as their duration depended on the flow capacity of the throttle orifices. The data obtained were used to compare with theoretical dependencies and to analyse the effect of the technical condition of the pneumatic components on pressure dynamics. The main characteristics of the equipment under study and the conditions for conducting bench tests are given in Table 3.

**Table 3.** The main characteristics of the equipment and conditions of bench tests.

| Parameter | Meaning or Characteristic |
|---|---|
| Type of equipment under study | Air distributor No. 292 |
| Test conditions | Stationary laboratory stand |
| Condition of the air distributors under study | Removed from passenger cars for repair |
| Regulatory document | Instruction TsT-TsV-0013 |
| Diameter of calibrated orifices | 1.25 mm (three pieces) |
| Controlled pressure range | 0.40–0.45 MPa |
| Pressure rise time in the auxiliary reservoir from 0.40 to 0.45 MPa according to the test procedure | 15–25 s |
| Total number of air distributors inspected | 519 |
| Number of repeat tests for a single air distributor | 3 |
| Accuracy of pressure measuring instruments | ±2.5% |
| Pressure signal recording frequency | 10 Hz (interval 0.1 s) |

The inspection of 519 air distributors and the calibration of the mathematical model constitute separate stages of the study. Data from three tests of a single air distributor (No. 292), presented in Table 4, were used for model calibration and for a benchmark comparison of the results. The tests were carried out in accordance with the instructions TsT-TsV-0013 [24,25]. When charging, the compressed air from the brake line flowed through three calibrated orifices with a diameter of 1.25 mm into the auxiliary reservoir.

**Table 4.** The results of repeated tests and comparison with the mathematical model.

| Test | Experimental Time, Sec | Estimated Time, Sec | Relative Deviation, % |
|---|---|---|---|
| 1 (calibration) | 19.0 | 19.0 | 0.00 |
| 2 (control) | 20.0 | 19.0 | 5.00 |
| 3 (control) | 21.0 | 19.0 | 9.52 |

The pressure rise time in the auxiliary reservoir from 0.40 to 0.45 MPa was monitored, which, according to the adopted procedure, was 15–25 s. This indicator characterizes the conditions of bench tests and cannot be directly compared with the time characteristics of the calculated dependencies unless the same initial conditions are considered.

In order to test the insensitivity rate of the air distributor No. 292, when the auxiliary reservoir has been fully charged, the pressure in the main reservoir should be reduced through an orifice with a 0.9 mm in diameter from the charging pressure to 0.45 MPa; while doing so, the auxiliary reservoir discharges through three calibrated orifices with 1.25 mm in diameter and the air distributor must not activate for braking.

During bench testing, changes in pressure in the auxiliary reservoir and the brake line were recorded. The pressure-dependent changes obtained during the bench tests were considered together with the results of mathematical modelling. Such a comparison allowed analysing the characteristics of transient processes; however, it did not provide grounds for unequivocally attributing the differences between the curves to the fouling of the throttle orifices, as other parameters of the pneumatic system might also influence the pressure dynamics.

According to the results of bench tests of air distributor No. 292 after operation, the rise time of the pressure the auxiliary reservoir from 0.40 to 0.45 MPa was 19.0 s. Based on the adopted parameters of the mathematical model and the flow rate coefficient $C_d$ = 0.65, the estimated value of the fouling factor was $k_z$ = 0.68. The value obtained represented a calculated estimate of the relative flow capacity of the throttle orifices, based on the assumed flow coefficient $C_d$ = 0.65 and was not the result of direct measurement of their actual flow cross-sectional area.

In the calculations, the consumption coefficient $C_d$ = 0.65 was assumed to be constant, which allowed estimating the fouling factor based on the experimental charging time. As both coefficients affected the effective flow capacity of the orifices, it was not possible to determine them independently based on just one time parameter. Therefore, the obtained value $k_z$ = 0.68 was considered as an estimate based on the accepted parameters of the mathematical model.

To assess the repeatability of the results, three tests of the same air distributor No. 292 were carried out under the same conditions. The first measurement was used to calibrate the mathematical model, whilst the next two were used to compare the calculated and experimental filling times for the auxiliary reservoir (Table 4).

Based on the results of three repeated tests, the average charging time was 20.0 s, the sample standard deviation was 1.0 s, the standard type A mean uncertainty was 0.58 s, and the coefficient of variation was 5.0%. The relative deviations of the estimated du-

ration from the two control measurements were 5.00% and 9.52%, and their average value was 7.26%. The obtained results characterize the repeatability of measurements of a single air distributor and the consistency of the estimated charging time with the control measurements. Nevertheless, the average relative deviation of 7.26% is not an estimate of the accuracy of reproducing the full-time dependence of pressure, for which an additional comparison of the calculated and experimental curves is required.

Sensors with an upper measurement limit of 0.7 MPa and a maximum permissible error of ±2.5% of the full scale were used to record the pressure. The corresponding limit of absolute error was ±0.0175 MPa. The signal recording frequency was 10 Hz, which provided a time interval of 0.1 s between successive readings. Given the controlled pressure increase from 0.40 to 0.45 MPa, the instrumental error of the sensors was considered a significant source of uncertainty in determining the charging duration.

To assess the impact of instrumental error on the measurement results, the characteristics of the pressure sensors used were considered. Assuming a uniform distribution of error, the standard uncertainty of pressure measurement is 0.0101 MPa. Its impact on the uncertainty of determining the charging time depended on the local rate of pressure change near the control values of 0.40 and 0.45 MPa. As the relevant experimental derivatives for the main series of tests have not been determined, a quantitative assessment of this component of uncertainty requires further analysis of the raw measurements. The propagation of pressure measurement uncertainty to the uncertainty in determining the charging duration and the fouling factor was not evaluated within the scope of the study.

To analyse the effect of the throttle orifice flow capacity on pressure dynamics, the calculated relationships based on an exponential approximation with different time constants were considered (Figure 5). The obtained curves allow tracking the change in the charging time of the pneumatic system, but they cannot be considered as the results of an experimental verification of the accuracy of the proposed mathematical model.

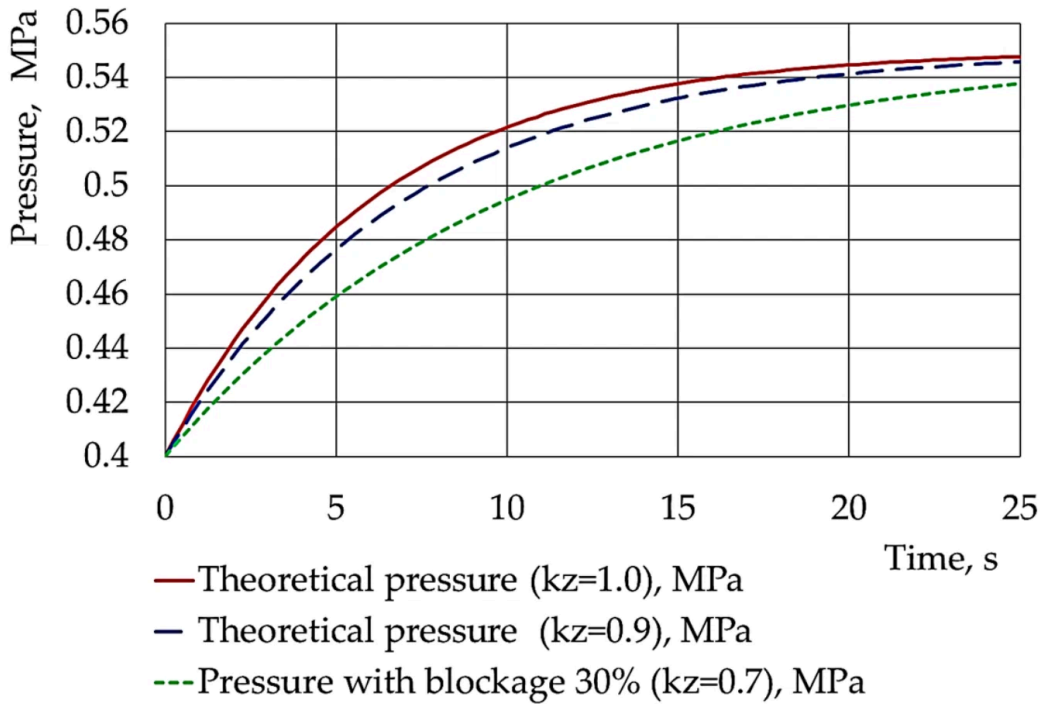



**Figure 5.** The calculated dependencies of pressure changes during the charging of a pneumatic system with air distributor No. 292 for different transient time constant values.

The analysis of the calculated dependencies (Figure 5) has shown that as the fouling factor decreases, the pressure rise in the pneumatic system slows down. The longest transient process time is observed at $k_z$ = 0.7, which corresponds to the lowest flow capacity of the throttle orifices at the states considered. The dependencies presented characterize the impact of the adopted parameters on pressure dynamics but are not the result of experimental confirmation of the mathematical accuracy of the model.

Importantly, the time characteristics shown in Figure 5 were obtained using the assumed parameters of the calculation equations; therefore, they do not directly reflect the standard 15–25 s filling time for the auxiliary reservoir during the bench tests. To make a quantitative comparison of these results, it is necessary to consider the actual air supply parameters, the initial conditions and the characteristics of the air distributor under investigation.

The bench tests of air distributors No. 292 were carried out in the brake testing laboratory of the Department for Railway Car Engineering and Product Quality at the Ukrainian State University of Railway Transport. The experimental data obtained characterise the operation of pneumatic equipment under laboratory conditions, whilst the calculated relationships (Figure 5) can be used to analyse the effect of the flow capacity of the throttle orifices on the duration of transient processes. The quantitative validation of the accuracy of a mathematical model requires a comparison of calculated and experimental results under identical initial conditions.

To quantitatively assess the accuracy with which a mathematical model reproduces changes in pressure, given comparable experimental data, it is advisable to use the relative error, which is determined by the following equation:

$$\varepsilon = \frac{\left| p_e - p_m \right|}{p_e} \cdot 100\,\%\,. \tag{13}$$

where $p_e$ is the experimental pressure value at the $i$-th time point; $p_m$ is the estimated pressure value at the same time point.

The analysis of the calculated dependencies has shown that the reduction in the flow capacity of the throttle orifices is accompanied by a slowdown in the pressure increase in the pneumatic system. However, the presented results do not allow determining the quantitative indicators of the mathematical model's accuracy without comparing with experimental data obtained under the same initial conditions.

The obtained results allow assessing the nature of the impact of fouling in the throttle orifice on the dynamics of pressure changes in the pneumatic system. Here, quantitative accuracy confirmation for the mathematical model requires additional comparison of calculated and experimental data under the same test conditions.

To assess the state of the throttle orifices, it is advisable to compare the actual and calculated dependencies of pressure changes during the charging of the pneumatic system. A decrease in the rate of pressure rise may indicate a deterioration in the capacity of the orifices due to fouling. However, such deviations may be caused by air leaks, temperature changes, or instability in the supply pressure. Therefore, for a reliable assessment of the state of throttle elements, it is necessary to consider the above factors and compare the results under the same test conditions.

The deviations in the operation of an air distributor can be assessed by using the time taken to reach the set pressure and the pattern of its variation during the transient process. Comparing these readings with those of a serviceable unit under identical test conditions makes it possible to identify any malfunctions in the pneumatic system. Meanwhile, the determination of the permissible limits of such deviations requires fur-

ther experimental justification.

The results obtained can be used in the development of automated systems for diagnosing pneumatic brake equipment of passenger cars. In particular, the analysis of the dynamics of pressure changes helps identify deviations from the design characteristics of a properly functioning system. However, to determine the causes of such deviations, it is also necessary to consider the tightness of the pneumatic connections, the air temperature, and the stability of the supply pressure. This creates preconditions for further development of methods for assessing the technical state of air distributors based on the results of measuring the parameters of transient processes.

Comparing the results of mathematical modelling with experimental data and the results of the previous studies [8–10] made it possible to analyse the nature of pressure changes in the pneumatic braking system. The relationships obtained reflect the influence of the throttle orifice flow capacity on the course of transient processes; however, quantitative confirmation of the model's accuracy requires further experimental verification. As noted in [9,10], the duration of charging and discharging in pneumatic braking systems depends to a large extent on the cross-sectional area of the throttle elements, the condition of the seals and the parameters of the main line. In this study, the influence of the flow capacity of the throttle orifices on the dynamics of pressure changes was quantified by introducing a fouling factor, which made it possible to analyse the transient processes under various calculated conditions of the pneumatic system.

A distinctive feature of the approach employed is the ability to separately analyze the impact of clogging in the throttle orifices on pressure dynamics, while keeping the nominal parameters of the pneumatic system constant. However, the introduction of a fouling factor does not automatically improve the accuracy of calculations compared with the traditional adjustment of the flow coefficient or the effective cross-sectional area of the orifice. Given identical effective throughput and other model parameters, these approaches are mathematically equivalent. Therefore, the scientific and practical contribution of this study lies in the application and parameterization of the mathematical model to evaluate the impact of reduced throughput in throttle orifices on the course of pneumatic processes.

An analysis of the numerical data used for Figure 5 has shown that the change in pressure for different values of the fouling factor is described by exponential relationships. For the curves considered at $k_z$ = 1.0, $k_z$ = 0.9 $k_z$ = 0.7, the values of the time constant are 6, 7 and 10 s, respectively. The results obtained indicate that the rate at which pressure rises slows down as the flow capacity of the throttle orifices decreases. At the same time, these dependencies do not allow for quantitative confirmation of the proposed model's advantage in terms of forecasting accuracy compared with traditional approaches.

Based on the results shown in Figure 3, it has been found that a decrease in the fouling factor to $k_z$ = 0.7 is accompanied by a reduction in the effective area of the throttle orifices to 70% of the nominal value and an increase in the relative reservoir filling. Study [11] deals with the possibilities of detecting initial malfunctions in the brake system of an electric train based on the results of the analysis of diagnostic data. In contrast to this approach, our work focuses primarily on the mathematical description of the effect of throttle orifice fouling on the behaviour of pneumatic processes. Unlike the approaches discussed in [12,13], in the proposed model the effect of fouling for the throttle orifices is represented by a separate coefficient, which can be used to analyse changes in pressure dynamics at different fouling levels. This approach makes it easy to assess the impact of the technical condition of the throttling elements on the performance of the pneumatic system; however, its advantages in terms of prediction accuracy require fur-

ther quantitative validation.

An analysis of the simulation results has shown that as the fouling factor decreases, the transient process time in the pneumatic system increases due to a reduction in the flow capacity of the throttle orifices, which affects the rate at which the pressure changes. Unlike the approach discussed in [14], the proposed one allows the impact of different levels of fouling on the operation of a pneumatic system to be quantified. At the same time, determining the flow regime of air through the throttle orifices requires further gas-dynamic analysis.

An analysis of the calculated pressure-change dependencies (Figure 5) has shown that a reduction in the flow capacity of the throttle orifices is accompanied by an increase in the transient process time. The results obtained illustrate the effect of the chosen values of the fouling factor on the charging dynamics of the pneumatic system. At the same time, these dependencies cannot be used to assess the accuracy of a mathematical model without comparing them with experimental data obtained under the relevant test conditions.

The results of the study have confirmed the findings given in [15], which emphasises the need to use analytical methods to assess the technical condition of pneumatic devices, rather than purely empirical approaches.

The reliability of diagnosing air distributors based on the transient process parameters has been determined by the ability to distinguish changes caused by fouling in the throttle orifices owing to other factors. The study has revealed theoretical dependencies between the flow capacity of the orifices and pressure dynamics; however, the fault detection accuracy and false alarm rates were not determined. Assessing them requires testing air distributors in good working order and comparing the diagnostic results with actual defects.

Some practical ways of improving the reliability of brake system diagnostics in passenger cars are confirmed in [26], in which the use of a real-time pneumatic process monitoring system makes it possible to increase the relevant value from 83.1 to 87.6% These results demonstrate the value of using pneumatic process parameters to assess the technical condition of braking equipment. However, the determination of similar parameters for the mathematical model proposed in this study requires further experimental verification.

Thus, the results of the study have made it possible to reveal the effect of a reduction in the flow capacity of the throttle orifices on the dynamics of pressure changes in the pneumatic system of a passenger car. Taking this effect into account in the mathematical model paves the way for the further development of methods for assessing the technical state of air distributors based on transient process parameters. The practical application of the proposed approach requires the calibration of the model and the experimental determination of diagnostic criteria for various operating states of pneumatic equipment.

## 4. Conclusions

A study has been carried out into the process of fouling throttle orifices and lines in the pneumatic assembly of a passenger car's braking system. It has made it possible to determine the effect of it on the performance of the pneumatic actuation system. A generalized mathematical model was used, which accounts air leakage through throttling elements, pressure variations in the supply lines, and the characteristics of adiabatic gas flow. A fouling correction factor has been incorporated into the model, which accounts for the reduction in the flow capacity of the orifices due to fouling or deformation, thereby improving the accuracy of the calculations compared with idealised conditions.

A fouling factor has been introduced into the mathematical model, which characterises the reduction in the flow capacity of the orifices due to fouling and enables the influence of the technical condition of the orifice elements on the dynamics of pressure changes to be assessed without adjusting their nominal geometric parameters.

The mathematical modelling has shown that a reduction in the flow capacity of the throttle orifices is accompanied by a time rise in the transient processes running in the pneumatic system. At $k_z$ = 0.7, the effective area of the orifices was 70% of the nominal area.

The calibration of the mathematical model and the verification comparison of the results were performed separately from the general inspection of 519 air distributors.

During three repeat bench tests of air distributor No. 292, the average duration of the pressure rise in the auxiliary reservoir from 0.40 to 0.45 MPa was 20.0 s, with a sample standard deviation of 1.0 s. The average relative deviation of the calculated duration from the two reference measurements was 7.26%. The indicated deviation characterizes only the consistency of the results regarding charging duration. The impact of pressure measurement uncertainty on the uncertainty in determining the charging duration and the clogging coefficient was not evaluated. The results obtained characterise the influence of the flow capacity of the throttle orifices on the charging process and the consistency between the model and repeat measurements of a single air distributor. Nevertheless, assessing the model's accuracy for other technical conditions and determining the conditions under which the brakes may actuate spontaneously requires further research.

Mathematical modelling has shown that a reduction in the flow capacity of the throttle orifices is accompanied by a slower rise in pressure within the pneumatic system. The dependencies obtained can be used to assess the effect of the chosen values of the fouling factor on the duration of transient processes. However, the quantitative confirmation of the model's accuracy and the reliability of the diagnosis requires further comparison of the calculated results with data from bench tests carried out under identical conditions.

This approach can be used for assessing the technical condition of pneumatic systems in passenger cars; it is based on the results of mathematical modelling of air flow processes through throttling elements. The results obtained provide a basis for the further development of algorithms for the automated diagnosis of pneumatic brakes on passenger cars, designed to detect malfunctions in the operation of air distributors based on transient process parameters.

**Author Contributions:** Conceptualization, V.R.; methodology, V.R. and J.G.; software, A.L. and Y.D.; validation, J.G. and Y.D.; formal analysis, V.R., A.L. and Y.D.; investigation, V.R. and Y.D.; resources, J.G., V.R. and Y.D.; data curation, J.G., A.L. and Y.D.; writing—original draft preparation, V.R. and A.L.; writing—review and editing, J.G. and V.R.; visualization, A.L. and Y.D.; supervision, J.G. and V.R.; project administration, J.G.; funding acquisition, J.G. All authors have read and agreed to the published version of the manuscript.

**Funding:** This publication was supported by the Cultural and Educational Grant Agency of the Ministry of Education of the Slovak Republic under the project KEGA 024ZU-4/2024: Deepening the knowledge of university students in the field of construction of means of transport by carrying out professional and scientific research activities in the field. This research was also supported by the Slovak Research and Development Agency of the Ministry of Education, Science, Research and Sport of the Slovak Republic VEGA 1/0602/26 "Research into phenomena arising in the braking process of railway rolling stock under simulated operating conditions on a flywheel brake stand.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** The data is contained within the article.

**Conflicts of Interest:** The authors declare no conflicts of interest.

## References


1. Dižo, J.; Harušinec, J.; Blatnický, M. Structural analysis of a modified freight wagon bogie frame. *MATEC Web Conf.* **2017**, *134*, 00010. [Google Scholar] [CrossRef]
2. Mikhailov, E.; Semenov, S.; Dižo, J.; Kravchenko, K. Research of possibilities of reducing the driving resistance of a railway vehicle by means of the wheel construction improvement. *Transp. Res. Procedia* **2019**, *4*, 831–838. [Google Scholar] [CrossRef]
3. Melnik, R.; Koziak, S.; Dižo, J.; Kuźmierowski, T.; Piotrowska, E. Feasibility study of a rail vehicle damper fault detection by artificial neural networks. *Eksploat. Niezawodn.* **2023**, *25*, 5. [Google Scholar] [CrossRef]
4. Dižo, J.; Blatnický, M.; Melnik, R.; Karľa, M. Improvement of Steerability and Driving Safety of an Electric Three-Wheeled Vehicle by a Design Modification of its Steering Mechanism. *LOGI–Sci. J. Transp. Logist.* **2022**, *13*, 49–60. [Google Scholar] [CrossRef]
5. Kochanov, V.; Píštěk, V.; Kondratiev, A.; Yuresko, T.; Kučera, P. Influence of geometric parameters of conical acrylic portholes on their stress-strain behavior. *Polymers* **2022**, *14*, 1041. [Google Scholar] [CrossRef] [PubMed]
6. Sapietová, A.; Ságа, M.; Novák, P.; Bednár, R.; Dižo, J. Design and application of multi-software platform for solving mechanical multi-body system problems. In *Mechatronics: Recent Technological and Scientific Advances*; Springer: Berlin/Heidelberg, Germany, 2011; pp. 345–354. [Google Scholar] [CrossRef]
7. Quoc, P.M.; Krzikalla, D.; Mesicek, J.; Petru, J.; Smiraus, J.; Sliva, A.; Poruba, Z. On Aluminum Honeycomb Impact Attenuator Designs for Formula Student Competitions. *Symmetry* **2020**, *12*, 1647. [Google Scholar] [CrossRef]
8. Wu, Q.; Cole, C.; Spiryagin, M.; Wang, Y.; Ma, W.; Wei, C. Railway air brake model and parallel computing scheme. *J. Comput. Nonlinear Dyn.* **2017**, *12*, 051017. [Google Scholar] [CrossRef]
9. Wang, Q.; Gao, T.; Tang, H.; Wang, Y.; Chen, Z.; Wang, J.; Wang, P.; He, Q. A feature engineering framework for online fault diagnosis of freight train air brakes. *Measurement* **2021**, *182*, 109672. [Google Scholar] [CrossRef]
10. Bosso, N.; Magelli, M.; Zampieri, N. A numerical method for the simulation of freight train emergency braking operations based on the UIC braked weight percentage. *Railw. Eng. Sci.* **2023**, *2*, 162–171. [Google Scholar] [CrossRef]
11. Sang, J.; Zhang, J.; Guo, T.; Zhou, D.; Chen, M.; Tai, X. Detection of incipient faults in EMU braking system based on data domain description and variable control limit. *Neurocomputing* **2020**, *383*, 348–358. [Google Scholar] [CrossRef]
12. Hou, Z.; Lee, C.K.M.; Lv, Y.; Keung, K.L. Fault detection and diagnosis of air brake system: A systematic review. *J. Manuf. Syst.* **2023**, *71*, 34–58. [Google Scholar] [CrossRef]
13. Galimberti, A.; Zanelli, F.; Tomasini, G.A. Hybrid Model for Freight Train Air Brake Condition Monitoring. *Appl. Sci.* **2024**, *14*, 11770. [Google Scholar] [CrossRef]
14. Fomin, O.V.; Turovets, D.A. Vykorystannia protyiuznykh prystroiv z elektropnevmatychnym klapanom ta system kontroliu yikh tekhnichnoho stanu na pasazhyrskykh vahonakh vnutrishnoho spoluchennia. *Podilskyi Visnyk Silske Hospod. Tekhnika Ekon.* **2023**, *1*, 104–109. [Google Scholar] [CrossRef]
15. Safronov, O.M. Rozvytok metodolohii doslidzhen kharakterystyk ta efektyvnosti halmivnykh system pasazhyrskykh ta vantazhnykh vahoniv: Dysertatsiia na zdobuttia naukovoho stupenia doktora tekhnychnykh nauk: 05.22.07. Kyiv, 2025, 348. Available online: https://uacademic.info/ua/document/0525U000326 (accessed on 29 October 2026).
16. Teodoro, I.P.; Ribeiro, D.F.; Botari, T.; Martins, T.S.; Santos, A.A. Fast simulation of railway pneumatic brake systems. *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit* **2019**, *233*, 1231–1245. [Google Scholar] [CrossRef]
17. Chen, B.; Zhang, R.; Huang, H.; Gao, K.; Yang, Y.; Du, R. Online parameter identification based predictive pressure control for train electro-pneumatic braking systems with thermal effect. *ISA Trans.* **2024**, *152*, 427–438. [Google Scholar] [CrossRef] [PubMed]
18. He, Y.; Zhao, Y.; Defoort, M.; Pecht, M. Fault diagnosis of locomotive electro-pneumatic brake through uncertain bond graph modeling and robust online monitoring. *Mech. Syst. Signal Process.* **2015**, *50–51*, 676–691. [Google Scholar] [CrossRef]
19. Wei, W.; Jiang, Y.; Zhang, Y.; Zhao, X.; Zhang, J. Study on a segmented electro-pneumatic braking system for heavy-haul trains. *Transp. Saf. Environ.* **2020**, *2*, 216–225. [Google Scholar] [CrossRef]
20. Xiong, L.; He, Y.; Chen, Y.; Lu, J.; Niu, G. Digital twin-based degradation prediction for train electro-pneumatic valve. *Reliab. Eng. Syst. Saf.* **2023**, *240*, 109627. [Google Scholar] [CrossRef]

21. Cantone, L.; Ottati, A. A simplified pneumatic model for air brake of passenger trains. *Railw. Eng. Sci.* **2023**, *31*, 145–152. [Google Scholar] [CrossRef]
22. Galimberti, A.; Zanelli, F.; Debattisti, N.; Mauri, M.; Tomasini, G. A novel fault injection and monitoring methodology for freight train air brakes: From simulation to field validation. *Int. J. Rail Transp.* **2026**, *14*, 737–767. [Google Scholar] [CrossRef]
23. Jo, S.C.; Byun, S.J.; Won, J.U.; Park, C.H. A Branch-Attention DeepONet for Brake Cylinder Pressure Prediction in a Railway Brake Operating Unit. *J. Korean Soc. Railw.* **2026**, *29*, 463–471. [Google Scholar] [CrossRef]
24. Ukrzaliznytsia. *Instructions for Repairing the Brake Equipment of Wagons; CV-CL-0013*; Approved by Order No. 022-CZ of 25 January 2005; SAM Publishing House: Kyiv, Ukraine, 2005. (In Ukrainian) [Google Scholar]
25. Ukrzaliznytsia. *Instructions for the Operation of Rolling Stock Brakes on the Railways of Ukraine; CT-CV-CL-0015*; Approved by Order No. 264-C of 28 October 1997; Poligrafservis: Kyiv, Ukraine, 2004. (In Ukrainian) [Google Scholar]
26. Ravlyuk, V.; Gerlici, J.; Lovska, A.; Derevianchuk, Y. The Test-Based Evaluation of the Diagnostic Accuracy of Brake Systems in Passenger Cars. *Commun. Sci. Lett. Univ. Žilina* **2026**, *28*, B127–B136. [Google Scholar] [CrossRef]

**Disclaimer/Publisher's Note:** The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.