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Impact-Based Allocation of Common-Area Electricity Charges in Apartment Complexes

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Abstract

Under the single contract (SC) used in apartment complexes in the Republic of Korea, the total electricity consumption of an apartment complex is measured by a main electricity meter, and the residential high-voltage (HV) progressive tariff is applied to calculate the total electricity bill. The difference between the total electricity bill charged to the apartment complex and the sum of the electricity bills calculated for individual households constitutes the common-area electricity bill, which is generally allocated equally among households. However, because the residential progressive tariff is applied to the total electricity consumption, the common-area electricity bill depends not only on the electricity consumption of common-area facilities but also on the distribution of electricity consumption among individual households. Therefore, an allocation method that considers the impact of individual households on the common-area electricity bill is required. In this study, the impact value of each household on the common-area electricity bill is quantified by the change in the bill resulting from the inclusion of that household. Based on these impact values, proportional (PROP) and additive (ADD) allocation schemes are proposed, along with their nonnegative (NN) variants to prevent negative charges. A bounding method is also proposed to impose upper and lower bounds on the allocated charges. The proposed schemes were investigated using actual metering data collected from 13,335 households over 24 months, and their characteristics were analyzed and compared. The results show that PROP exhibits the largest changes in the allocated common-area electricity charges, whereas PROP-NN results in relatively smaller changes from the current uniform scheme. ADD and ADD-NN provide intermediate alternatives, with ADD generally showing larger changes than ADD-NN. Thus, PROP is suitable when the impact value is to be reflected strongly, whereas PROP-NN is preferable when relatively small changes in charges are desired. The bounding method is recommended for PROP and can be optionally applied to the other schemes.

Keywords: apartment complex; common-area electricity charge; impact-based allocation scheme; progressive tariff; single contract (SC)

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Academic Editor: Alessandro Lo Schiavo

Received: 11 October 2026
Revised: 1 November 2026
Accepted: 4 November 2026
Published: 5 November 2026

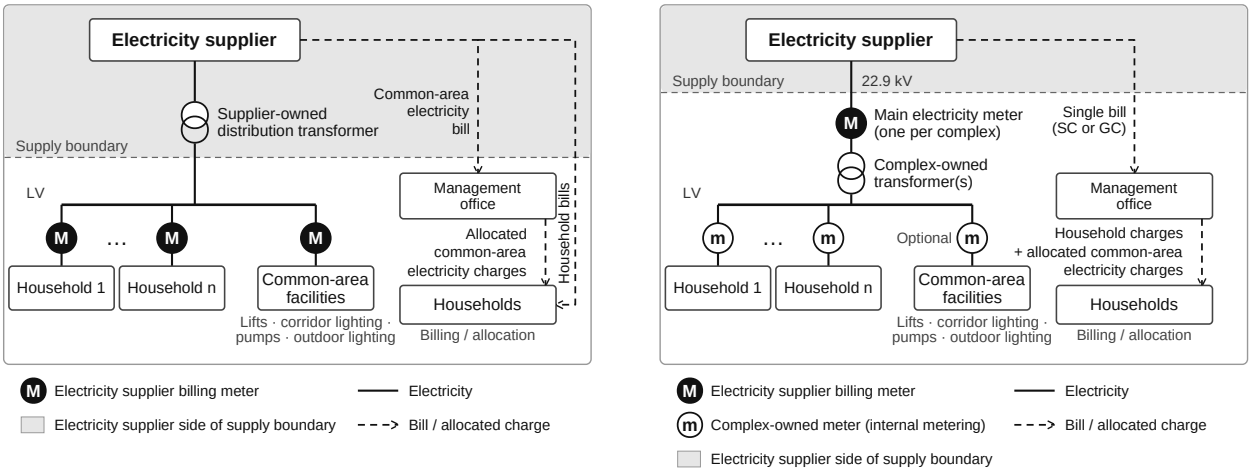
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1. Introduction

An apartment complex is a residential community where housing units for multiple households and common-area facilities for their shared use constitute a single management unit. Common-area facilities include equipment such as elevators, corridor lighting, and

water-supply pumps, which are installed for the shared use of multiple households within the apartment complex. The electricity billing schemes for an apartment complex can be classified into two types: a scheme in which the electricity supplier has an individual supply contract with each household in the apartment complex, and a scheme in which the electricity supplier has a single supply contract with the apartment complex as a whole.

The first type is referred to as an individual household contract, under which the electricity supplier has separate supply contracts with individual households in the apartment complex. Figure 1a illustrates an example of an individual household contract, in which separate electricity meters are installed for each household and for the common-area facilities. The electricity supplier bills each household directly for its electricity consumption, while the electricity consumed by the common-area facilities is billed to the management office.



(**a**) (**b**)

Figure 1. Electricity supply, metering points, contracts and billing in apartment complexes. (**a**) Individual household meters and supply contracts. (**b**) Single main electricity meter and joint household supply contract in Korea.

Individual household contracts are used in some states and by electricity suppliers in the United States [1–3]. These contracts can also be found in apartment buildings in Japan and France [4–6]. In Germany, electricity costs for lighting in common areas are allocated to households as a separate charge [7,8]. Relevant regulations in China also stipulate that electricity suppliers directly bill individual households [9,10]. The management office allocates the electricity charges for the common-area facilities among the households in accordance with the applicable contracts and management rules.

The second type is referred to as a joint household contract. Unlike the individual household contract, the total electricity consumption of the apartment complex is measured by a main electricity meter, a single supply contract is established and a single electricity bill is issued for the total consumption. The total electricity consumption in this case includes the electricity consumed by both the individual households and the common-area facilities. Therefore, under a joint household contract, since the total electricity consumption is billed as a single amount, an appropriate rule is required to allocate the electricity charges among the households.

This type of contract is currently used in apartment complexes in the Republic of Korea. Figure 1b illustrates an example of the joint household contract used in Korean apartment complexes. The electricity supplier supplies electricity at a high voltage (HV) of 22.9 kV through a main electricity meter to a transformer and on-site distribution lines that are owned and managed by the apartment complex. The electricity is then distributed at low voltage (LV) to individual households and common-area facilities [11,12]. The electricity bill

charged to the apartment complex as a whole is subsequently allocated among individual households by the management office based on the electricity consumption measured by the electricity meters installed for each household [13]. The electricity charges associated with the common-area facilities are allocated among the households according to a separate allocation method.

The joint household contract for apartment complexes in Korea can be divided into a general contract (GC) and a single contract (SC) according to the applicable electricity tariff structure [11,12,14]. Under GC, the electricity supplier calculates the household electricity bills by applying the residential LV rate to the electricity consumption of each household and the common-area electricity bill by applying the general-service HV rate to the electricity consumption of the common-area facilities. The total electricity bill for the apartment complex is then calculated as the sum of these charges. For the electricity supplier to calculate the electricity bill for each household, the management office of the apartment complex measures the electricity consumption of each household and reports the measurements to the electricity supplier.

In contrast, under SC, the electricity supplier does not separately account for household consumption and common-area electricity consumption. Instead, it measures the total electricity consumption using the main electricity meter and applies the residential HV rate to the total consumption to calculate the total electricity bill for the apartment complex. Therefore, the two contract types can result in different total electricity bills depending on the electricity consumption and its distribution among households and common-area facilities in a given apartment complex, and the complex can select the more favorable contract type.

As of 2022, 74.3% of apartment complexes in Korea had adopted SC [13]. Under SC, the total electricity bill charged to an apartment complex is calculated based on the total electricity consumption measured by the main electricity meter. The total consumption is first divided by the number of households to obtain the average household consumption, to which the residential HV progressive tariff is applied. The resulting charge is then multiplied by the number of households to obtain the total electricity bill. The electricity supplier charges this total amount to the apartment complex. The management office calculates the household electricity bill for each household by applying the same residential HV progressive tariff to its electricity consumption. The difference between the total electricity bill charged to the apartment complex by the electricity supplier and the sum of the household electricity bills constitutes the common-area electricity bill, which is shared by all households [12]. This common-area electricity bill is generally allocated equally among the households. However, the common-area electricity bill reflects not only the actual electricity consumption of common-area facilities but also the electricity consumption of individual households. In other words, even when the electricity consumption of common-area facilities remains unchanged, the total common-area electricity bill can vary depending on the distribution of household electricity consumption and the applicable progressive tariff tiers [15]. Consequently, the uniform allocation method fails to account for the impact of individual households on the common-area electricity bill and may result in an inequitable allocation among households. Therefore, an appropriate allocation method is needed to allocate the common-area electricity bill among households while considering the impact of each household on the total common-area electricity bill of the apartment complex.

Meanwhile, various methods have been studied to allocate costs or benefits arising from supply contracts or electrical equipment shared by a group of households, including renewable energy systems [16–18]. Related approaches include simple rules, such as uniform allocation and allocation in proportion to electricity consumption or floor area [3,8]; sharing coefficient methods that determine the proportion of electricity generated by a community

to be allocated to each member [19]; cooperative game-theoretic methods, such as the Shapley value and egalitarian nonseparable contribution (ENSC) [20–22]; and price-based methods that determine the internal price of electricity exchanged among members [23,24]. The allocation procedures and criteria used in these methods differ depending on the allocation target, which may be electricity, cost, or benefit. In addition, each allocation method can be evaluated according to various criteria, such as whether the sum of the allocated amounts equals the total amount to be allocated, the incentive for participation by individual members or subgroups within the community, the fairness of the resulting allocation, and the computational complexity of the method [16,18,24–26].

Among these allocation methods, the ENSC method allocates a common cost by considering the contribution of each member to the total cost [21]. The ENSC method has relatively low computational complexity [27] and can account for differences in the allocated costs among members by reflecting their marginal contributions in the allocation process. Under the ENSC method, the cost of the group excluding a particular member is first subtracted from the total cost of the group, and the resulting amount is allocated to that member as its separable cost. After the separable costs have been allocated to all members, their sum may differ from the total cost. This difference, referred to as the non-separable cost, is then allocated equally among all members. Thus, the ENSC method reflects differences in marginal contributions among members through the allocation of separable costs but does not reflect differences in member contributions when allocating the non-separable cost. In addition, the final allocation amount obtained using the ENSC method can be negative.

To practically allocate the common-area electricity bill, it is necessary to consider not only methods that reflect the contribution of each household but also the range of charges that can be imposed on individual households. If the allocation amount for a particular household is negative, the common-area electricity charge effectively reduces the household's electricity bill. Depending on the billing principles of an apartment complex, such a negative amount may not be permitted. In addition, upper and lower bounds can be imposed on the allocation amount for each household to limit the charges payable by individual households within a specified range. When negative allocation amounts are not allowed or upper and lower bounds are imposed on the allocation amount, any allocation amounts that fall outside these bounds need to be redistributed among the other households to preserve the total common-area electricity bill.

In this study, we propose methods for allocating the common-area electricity charges to individual households under SC in apartment complexes in Korea based on their contributions. First, an impact value is defined to represent the effect of including a particular household in the apartment complex on the common-area electricity bill. Specifically, the impact value of a particular household is defined as the difference between the common-area electricity bill with and without that household. The proposed allocation methods are based on either adding the impact value to or multiplying the impact value by a value associated with the common-area electricity bill and consist of four methods that differ according to whether negative allocation amounts are permitted. These methods include approaches that more directly reflect differences in the marginal contributions of individual households than the case of the existing ENSC-based allocation method, as well as approaches that prevent negative allocation amounts. In addition, methods that impose upper and lower bounds on allocation amounts for individual households while preserving the total common-area electricity bill are proposed.

To analyze the characteristics of the proposed methods, actual electricity consumption data collected over 24 months from 13,335 households in 31 apartment complexes are used. For each month, the impact value of each household is calculated based on household electricity consumption in each apartment complex, and the results obtained using the

four allocation methods are compared with those obtained using the commonly used uniform allocation method. Increases and decreases in the common-area electricity charges allocated to individual households relative to the current uniform allocation method, the distribution of these changes, and the characteristics of households experiencing relatively large changes in their charges are analyzed. In addition, these results are examined under different conditions regarding whether negative charges are permitted and whether upper and lower bounds are imposed on the charges. Through these analyses, the effects of different allocation methods and constraints on the allocation of common-area electricity charges under SC are examined. The findings provide empirical evidence of the practical applicability of allocation methods that account for household impact values. Furthermore, by demonstrating the distinct characteristics of the four proposed methods, this study provides practical insights that can help individual apartment complexes select an appropriate allocation method according to their specific circumstances and allocation objectives.

This paper is organized as follows. Section 2 briefly describes the electricity tariff applied to apartment complexes under SC. Section 3 analyzes the impact of household electricity consumption on the common-area electricity bill of apartment complexes. Section 4 quantifies the impact values of individual households on the common-area electricity bill and proposes impact-based allocation schemes. Section 5 presents and discusses the results obtained by applying the proposed schemes to actual metered data. Finally, the conclusion is presented in Section 6.

2. Electricity Bills of Households in Apartment Complexes Under SC

2.1. Bills by SC

Under SC, a household's electricity bill is determined based on the total electricity consumption of the apartment complex. Electricity is consumed in an apartment complex not only by household appliances within individual units but also by electrical facilities in common areas, such as elevators, lighting systems, and water-supply pumps. First, average electricity consumption per household, $\overline{W}$, is calculated by dividing the total energy consumption, including both household and common-area consumption, by the number of households, as

$$\overline{W} := \frac{\sum_{m=1}^{N} w_m + W_C}{N}, \tag{1}$$

where w_m, W_C, and N denote the electricity consumption of household m, common-area electricity consumption, and the number of households in the complex, respectively. Then, the average household charge is determined by applying the residential HV rate plan [14] to the average electricity consumption.

The residential HV rate plan employs a three-tier progressive rate system [14]. Let $f(w)$ denote the monthly charge under the residential HV rate plan for a monthly electricity consumption of w. Then,

$$f(w) := \begin{cases} B_1 + wR_1, & 0 \le w \le S_1, \\ B_2 + S_1R_1 + (w - S_1)R_2, & S_1 < w \le S_2, \\ B_3 + S_1R_1 + (S_2 - S_1)R_2 + (w - S_2)R_3, & w > S_2, \end{cases} \tag{2}$$

where B_1, B_2, and B_3 denote the base charges for the three tiers of the progressive tariff, while R_1, R_2, and R_3 represent the corresponding energy rates. The tier thresholds, S_1 and S_2, are set at 200 kWh and 400 kWh, respectively. During the summer months (July and August), these thresholds are increased to 300 kWh and 450 kWh, respectively.

On the other hand, during the winter season (December to February) and the summer season (July and August), when nationwide electricity demand is typically high, households

with high electricity consumption are subject to an additional higher rate. Specifically, households classified as super users, defined as those consuming more than 1000 kWh per month, are charged a special higher energy rate, denoted by R_s, for the portion of electricity consumption exceeding 1000 kWh. Therefore, for households with a monthly electricity consumption exceeding 1000 kWh, the monthly electricity bill during these periods is calculated as follows:

$$f(w) := B_3 + S_1R_1 + (S_2 - S_1)R_2 + (1000 - S_2)R_3 + (w - 1000)R_s, \quad w > 1000. \tag{3}$$

As of May 2026, the base charges and energy rates applied to SC are shown in Table 1, where KRW stands for the Korean Won. For reference, the corresponding values in USD (U.S. Dollar) are also provided in parentheses. USD 1 corresponds to KRW 1489.6 as of May 2026.

Table 1. Monthly HV rate for residential usage [28].

Base Rate (KRW)			**Usage Rate (KRW/kWh)**			
B_1	B_2	B_3	R_1	R_2	R_3	R_s
730 (0.490)	1260 (0.846)	6060 (4.068)	105.0 (0.071)	174.0 (0.117)	242.3 (0.163)	601.3 (0.404)

With SC, the electricity bill for the apartment complex, defined as B, is calculated as

$$B := Nf(\overline{W}). \tag{4}$$

The apartment management office pays this amount to Korea Electric Power Corporation (KEPCO) for the electricity consumption of the apartment complex and then allocates the electricity charges to individual households, taking into account both household consumption and the electricity consumption of common areas. The total bill for the apartment complex B is used to determine the common-area electricity bill. When the electricity consumption of household m is w_m, the corresponding household electricity charge is given by $f(w_m)$. Then, the total charge for household electricity consumption in the apartment complex, denoted by H, is given by

$$H := \sum_{m=1}^{N} f(w_m). \tag{5}$$

Then, the bill for the common-area consumption of the apartment complex, denoted by P, is determined by

$$P = B - H. \tag{6}$$

Each household shares the electricity bill for the common-area consumption. Let p_m denote the share of the common-area electricity bill allocated to household m. The allocated common-area electricity charges must satisfy the following relationship:

$$P = \sum_{m=1}^{N} p_m. \tag{7}$$

Then, the monthly electricity bill for household k, denoted by b_k, is calculated by

$$b_k := f(w_k) + p_k, \; k = 1, \ldots, N. \tag{8}$$

2.2. Current Billing for Common-Area Electricity Consumption Under SC

Currently, in most apartment complexes, the common-area electricity bill is allocated equally among all households, as this is the simplest and most straightforward method. Thus, the share of the common-area electricity bill given to household k, denoted by p^{UNI}, is calculated by

$$p^{\mathrm{UNI}} = \frac{P}{N}. \tag{9}$$

It is clear that the sum of the electricity bills charged to each household, b_k, is equal to the electricity bill for the apartment complex, B.

3. Analysis of the Impact of Household Electricity Consumption on Common-Area Electricity Bill Under SC

Assume that common-area electricity consumption and electricity usage of all households do not change, except for household k. Let Γ denote the set of all households, and let Γ_k denote a set excluding the household k. That is, $\Gamma := \{1, \cdots, N\}$ and $\Gamma_k := \{1, \cdots, k-1, k+1, \cdots, N\}$. From Equations (4)–(6), the common-area electricity bill is expressed as

$$P = Nf\left(\frac{w_k}{N} + \frac{(N-1)\overline{W}_k}{N}\right) - [f(w_k) + H_k], \tag{10}$$

where $\overline{W}_k := (\sum_{m\in\Gamma_k} w_m + W_C)/(N-1)$ and $H_k := \sum_{m\in\Gamma_k} f(w_m)$.

As assumed, the common-area electricity consumption and the electricity usage of all households except the household k remain unchanged. Therefore, $\overline{W}_k$ and H_k are fixed values. From Equation (10), it can be seen that the common-area electricity bill for the apartment complex varies as the electricity usage for household k changes. This means that the common-area electricity bill may increase or decrease solely due to a change in the electricity consumption of a household, even though the common-area electricity consumption remains unchanged.

The average electricity consumption of the apartment complex depends on the electric usage of individual households. When this average lies near the boundary of a progressive tariff tier, a change in the electricity consumption of a single household can significantly affect the total electricity charge of the complex. In addition, the characteristics of the variation in the common-area electricity bill are determined by the relationship between the progressive tariff tier corresponding to the average electricity consumption of the apartment complex and the tariff tier of household k. Specifically, when the two tariff tiers are the same, changes in the electricity consumption of the household do not affect the common-area electricity bill of the complex. However, when the tariff tier of the apartment complex is higher than that of the household, the value of P increases as the household's electricity consumption increases. In contrast, when the tariff tier of the apartment complex is lower than that of the household, the value of P decreases as the household's electricity consumption increases.

The impact of this phenomenon is illustrated through the following example. Consider an apartment complex with five households whose monthly electricity consumptions are {211, 221, 231, 241, 251} kWh. The common-area electricity consumption is set to 347 kWh, corresponding to 30% of the total household electricity consumption. Under this condition, an additional household k is introduced, and its monthly electricity consumption, w_k, is varied from 1 kWh to 1100 kWh. The resulting changes in the total electricity charge of the complex and the corresponding common-area electricity bill are then examined.

Figure 2a shows the total electricity charge of the apartment complex, B, and the sum of the individual household electricity charges, H, as functions of the electricity consumption of the additional household k, w_k. As w_k increases, the difference between B and H is

observed to increase, decrease, or remain unchanged. The difference between B and H is the common-area electricity bill of the apartment complex, P.

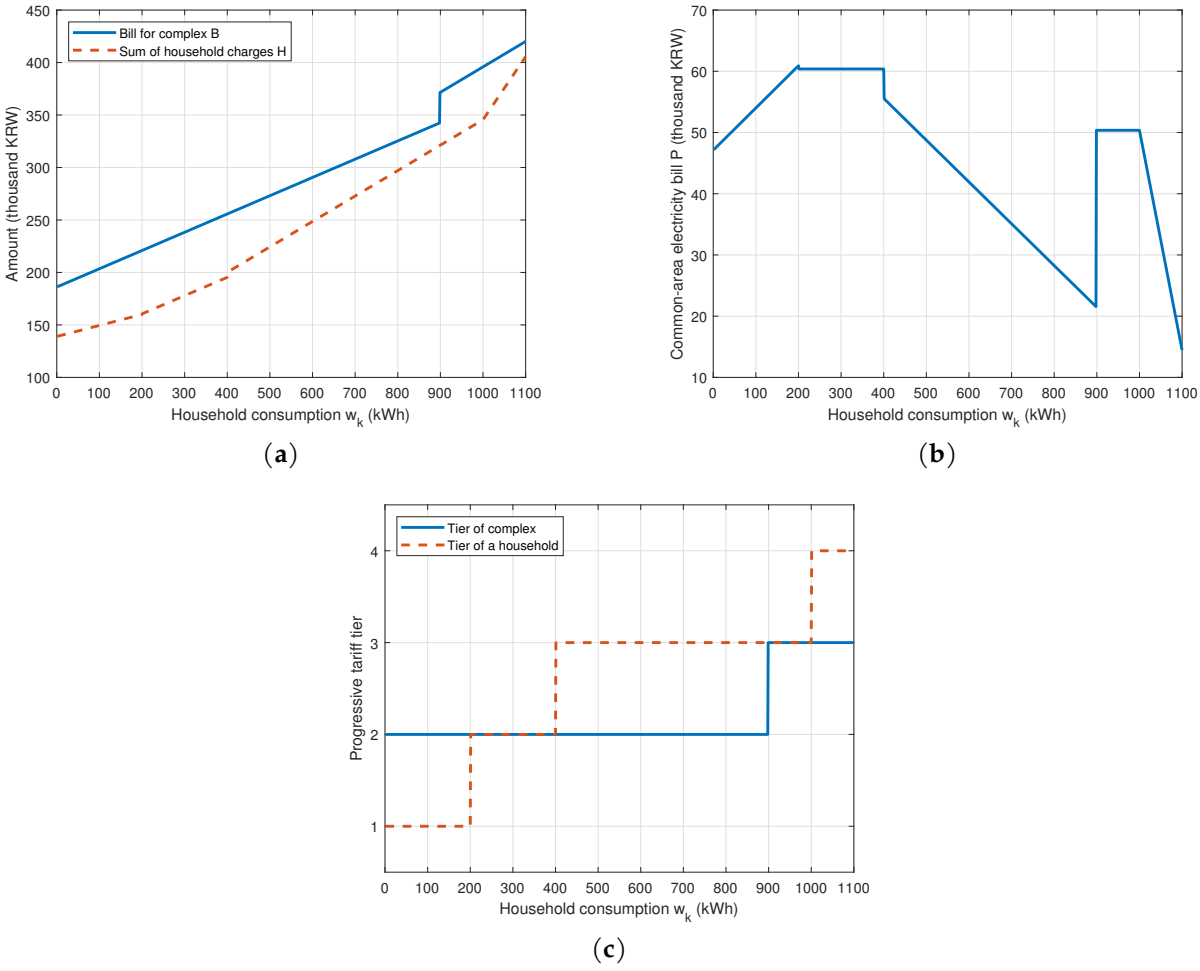


Figure 2. Changes in the total complex electricity bill and common-area electricity bill according to household electricity consumption. (**a**) Total complex electricity bill and sum of charges for household consumption. (**b**) Common-area electricity bill. (**c**) Household and apartment complex consumption tariff tiers.

Figure 2b shows the variation in the common-area electricity bill, P, as a function of w_k. The results indicate that, as w_k increases, the common-area electricity bill of the apartment complex may increase, decrease, or remain unchanged. This behavior is determined by the relationship between the progressive tariff tier corresponding to the average electricity consumption of the complex, $\overline{W}$, and that corresponding to the electricity consumption of the household, w_k.

The progressive tariff tiers corresponding to $\overline{W}$ and w_k in this example are illustrated in Figure 2c. When the progressive tariff tier corresponding to $\overline{W}$ is higher than that corresponding to w_k, an increase in w_k results in a smaller increase in the household electricity charge than in the total electricity charge of the apartment complex. Consequently, the common-area electricity bill increases as household electricity consumption increases. This case corresponds to the interval $0 < w_k \leq 200$. In the region where the progressive tariff tiers corresponding to $\overline{W}$ and w_k are the same, the rate of increase in the total electricity charge of the apartment complex is identical to that of the household electricity charge. Consequently, the common-area electricity bill remains unchanged as w_k varies. This region corresponds to $200 < w_k \leq 400$. In the next region, where the progressive tariff tier corresponding to $\overline{W}$ is lower than that corresponding to w_k, the household electricity charge

increases more rapidly than the total electricity charge of the apartment complex as w_k increases. Consequently, the common-area electricity bill gradually decreases. The interval $400 < w_k \leq 898$ corresponds to this case. For $898 < w_k \leq 1000$, w_k remains in the third tariff tier. Therefore, P would be expected to continue decreasing. However, due to the high w_k, the average electricity consumption of the complex, $\overline{W}$, also increases from the second tariff tier to the third tariff tier. As a result, both $\overline{W}$ and w_k belong to the same third tariff tier, and the common-area electricity bill remains unchanged regardless of increases in w_k. For $w_k > 1000$, both $\overline{W}$ and w_k still belong to the third tariff tier. However, the super-user tariff is applied to the household, resulting in a substantially higher electricity rate. Consequently, the household electricity charge increases much more rapidly as w_k increases, leading to a decrease in the common-area electricity bill in this region. It is noted that the common-area electricity bill must be shared among households, and currently, most apartment complexes adopt a uniform allocation scheme for these charges.

4. Impact-Based Allocation for Common-Area Electricity Bill

In this section, four schemes are proposed for allocating the common-area electricity bill for each household under SC by considering the impact of each household on the common-area electricity bill. First, the impact value of each household on the common-area electricity bill is quantified. Then, two impact-based allocation schemes are proposed: additive adjustment and proportional allocation based on the impact value. Two additional nonnegative variants are subsequently developed from these schemes.

In this study, the apartment complex is assumed to be sufficiently large such that the move-in or move-out of a single household does not affect the electricity consumption of the common areas. The common-area electricity bill for the entire apartment complex, P, is calculated as shown in Equation (10). Next, the common-area electricity bill of the complex after household k moves out, denoted by P_k, is calculated from

$$P_k := (N-1)f\left(\overline{W}_k\right) - H_k. \tag{11}$$

That is, P_k represents the common-area electricity bill when household k is excluded from the apartment complex. The difference between P and P_k quantifies the impact of household k on the common-area electricity bill, representing the effect of including a particular household k in the apartment complex. Therefore, the impact value of household k is defined as

$$\delta_k := P - P_k. \tag{12}$$

In the context of cooperative game theory, this impact value can be interpreted as the marginal contribution of household k. When $\delta_k > 0$, household k has a positive impact on the common-area electricity bill of the complex. In other words, when household k moves into the apartment complex, the common-area electricity bill increases even if there are no changes in the electricity consumption of the other households and the common area. On the contrary, when $\delta_k < 0$, household k has a negative impact on the common-area electricity bill of the complex. That is, when household k moves into the apartment complex, the common-area electricity bill decreases even though there are no changes in the electricity consumption of the other households and the common area. Therefore, δ_k represents the change in the common-area electricity bill caused by the inclusion of household k, which is used for the proposed impact-based allocation schemes.

There are various possible approaches to incorporating impact values into the allocation of common-area electricity charges, and identifying the most appropriate approach is a challenging task. From this perspective, this study considers two basic approaches to reflecting household impact values in charge allocation. The impact-based additive allocation scheme (ADD) reflects each household's contribution by adding its impact value

to the base common-area electricity charge allocated equally among households. In contrast, the impact-based proportional allocation scheme (PROP) allocates the total common-area electricity charge in proportion to each household's impact value, thereby reflecting the relative magnitude of each household's contribution in the allocation results.

In this study, fairness is considered in terms of the extent to which differences in household impacts on the total common-area electricity bill are reflected in the allocation of charges to individual households. These two approaches provide different ways of reflecting household impacts in charge allocation, and neither is necessarily optimal from all fairness perspectives. The choice between them may depend on the extent to which household impacts should be reflected and the allocation objectives of each apartment complex.

Meanwhile, allocation amounts determined based on household impact values can be negative. Such negative amounts can be regarded as bill credits and may be used to reduce individual households' electricity bills. In the experiments, negative allocation amounts occurred in approximately 4.37% of cases under the ADD scheme and 17.79% of cases under the PROP scheme. Whether negative allocation amounts are permitted may depend on the policies of individual apartment complexes. Accordingly, two additional schemes that impose nonnegativity constraints on the allocated charges are also proposed: the ADD scheme with nonnegative charges (ADD-NN) and the PROP scheme with nonnegative charges (PROP-NN).

4.1. Impact-Based Additive Allocation Scheme (ADD)

A positive value of δ_k indicates that the inclusion of household k increases the common-area electricity bill of the apartment complex, whereas a negative value indicates that it decreases the bill. To reflect this impact, the common-area electricity charge assigned to each household is obtained by adding the impact value, δ_k, to a base charge as

$$p_k = \delta_k + \beta. \tag{13}$$

Since $\sum_{k=1}^{N} p_k$ must be equal to P, we obtain the base charge β as follows:

$$\beta = \frac{P - \sum_{m=1}^{N} \delta_m}{N}. \tag{14}$$

By combining Equations (13) and (14), the common-area electricity charge for household k is obtained as

$$p_k^{\mathrm{ADD}} := \delta_k + \frac{P - \sum_{m=1}^{N} \delta_m}{N}. \tag{15}$$

Interestingly, the ADD scheme derived from the impact values is found to be mathematically equivalent to the ENSC allocation rule, with δ_k interpreted as the marginal contribution.

4.2. Impact-Based Additive Allocation Scheme with Nonnegative Charges (ADD-NN)

If δ_k is negative with large absolute value, the common-area electricity charge calculated using Equation (15), p_k^{ADD}, may be negative. To avoid negative common-area electricity charges for households, a minimum value for the common-area electricity charge can be established, and the following scheme is proposed.

Let the minimum common-area electricity charge be zero. Define Λ_0 as the set of households whose common-area electricity charges, p_k^{ADD}, are less than or equal to 0. That is,

$$\Lambda_0 := \{k \in \Gamma | p_k^{\mathrm{ADD}} \leq 0\}. \tag{16}$$

Then, for each household whose p_k^{ADD} value is negative, the common-area electricity charge is set to zero. Since the common-area electricity charge for such a household increases

from p_k^{ADD} to 0, the portion of the common-area electricity bill shared by households with positive common-area electricity charges is reduced by $|p_k^{\mathrm{ADD}}|$. Thus, the total reduced amount associated with the households in Λ_0 is computed as

$$\rho := \sum_{k \in \Lambda_0} |p_k^{\mathrm{ADD}}|. \tag{17}$$

We define the set of households with positive p_k^{ADD} as Λ', i.e., $\Lambda' := \{k \in \Gamma | p_k^{\mathrm{ADD}} > 0\}$. The value ρ is used to adjust the common-area electricity charges for households in Λ'. Let $|\Lambda'|$ denote the number of elements in Λ'. Then, ρ is divided by $|\Lambda'|$ and subtracted from the charge, which reduces the common-area electricity charges of the households in Λ'. The common-area electricity charge with a lower bound 0 for each household is given by

$$p_k^{\mathrm{ADD-NN}} := \begin{cases} p_k^{\mathrm{ADD}} - \frac{\rho}{|\Lambda'|}, & k \in \Lambda', \\ 0, & k \in \Lambda_0. \end{cases} \tag{18}$$

After some manipulation, it can be easily verified that $\sum_{k=1}^{N} p_k^{\mathrm{ADD-NN}} = P$ and that the non-decreasing relationship between the impact values and the allocated charges is preserved.

It is noted that a small positive value of $p_k^{\mathrm{ADD-NN}}$ may become negative after subtracting $\rho/|\Lambda'|$, particularly when $\rho/|\Lambda'|$ is large. In such cases, $p_k^{\mathrm{ADD-NN}}$ in Λ' is relabeled as p_k^{ADD}, and the set Λ' is redefined as the new set Γ. Equations (16)–(18) are then reapplied. This procedure is repeated iteratively until all the computed values become nonnegative. At each iteration, the sum of the allocated charges remains equal to P, and the non-decreasing relationship between the impact values and the allocated charges is preserved.

4.3. Impact-Based Proportional Allocation Scheme (PROP)

Another possible approach is to allocate p_k in proportion to δ_k. First, each impact value is multiplied by the total common-area electricity bill, and the resulting values are normalized such that their sum equals P. Accordingly, the charge allocated to household k is given by $P\delta_k / \sum_{m=1}^{N} \delta_m$. It should be noted that the mapping from δ_k to p_k must be monotonically non-decreasing. Specifically, if $\delta_l \geq \delta_m$, then $p_l \geq p_m$. Therefore, $P\delta_k / \sum_{m=1}^{N} \delta_m$ provides a valid allocation scheme only when $\sum_{m=1}^{N} \delta_m > 0$. If $\sum_{m=1}^{N} \delta_m < 0$, the ordering of δ_k and p_k is reversed. In this case, it is necessary to multiply the expression by -1 to preserve the monotonic relationship between δ_k and p_k. That is, the following expression can be used: $p_k = -P\delta_k / \sum_{m=1}^{N} \delta_m$. However, the resulting sum of the allocated charges is $-P$, rather than P. To satisfy the constraint that the total allocated charge equals P, a constant value of $2P/N$ is added to the charge for each household. Consequently, the final expression is given by

$$p_k^{\mathrm{PROP}} := \begin{cases} \frac{P\delta_k}{\sum_{m=1}^{N} \delta_m}, & \text{if } \sum_{m=1}^{N} \delta_m > 0, \\ -\frac{P\delta_k}{\sum_{m=1}^{N} \delta_m} + \frac{2P}{N}, & \text{if } \sum_{m=1}^{N} \delta_m < 0. \end{cases} \tag{19}$$

It can be readily verified that $\sum_{k=1}^{N} p_k^{\mathrm{PROP}} = P$. In addition, the case where $\sum_{m=1}^{N} \delta_m = 0$ should be addressed, since the calculation in Equation (19) is undefined. One possible approach is to introduce a positive number δ' for the denominator and use the modified formulation as $p_k^{\mathrm{PROP}} = P\delta_k/\delta' + P/N$. However, in the actual dataset collected from the 31 apartment complexes over a 24-month period, the minimum absolute value of $\sum_{m=1}^{N} \delta_m$ was approximately 1.21×10^4, while the average of the absolute values was approximately 4.26×10^5. This indicates that $\sum_{m=1}^{N} \delta_m$ remains sufficiently far from zero in practice.

4.4. Impact-Based Proportional Allocation Scheme with Nonnegative Charges (PROP-NN)

It should be noted that δ_k can take negative values. Since p_k^{PROP} is linearly related to δ_k, it may also become negative. Consequently, the charge assigned to a household for common-area electricity consumption can be a negative value. In this case, the electricity charge for

the household's own consumption is reduced due to a negative value of p_k^{PROP}. Such cases can either be allowed or excluded depending on the billing policy of the apartment complex. To exclude such cases, it is necessary to establish a minimum value for the common-area electricity charge for households.

First, a new impact value, d_k, is defined so that all impact values become nonnegative:

$$d_k := \delta_k - \min_{j \in \Gamma} \delta_j, \tag{20}$$

where $\min_{j\in\Gamma} \delta_j$ is the minimum value of δ_j in a given month. Then, the values of d_k are nonnegative. The proportional allocation scheme that ensures nonnegative common-area electricity charges for each household, referred to as PROP-NN, is given as

$$p_k^{\mathrm{PROP-NN}} := \frac{P d_k}{\sum_{m=1}^{N} d_m}. \tag{21}$$

It is straightforward to verify that $\sum p_k^{\mathrm{PROP-NN}} = P$ and that the non-decreasing relationship between the impact values and the allocated charges is preserved.

4.5. Application of Upper and Lower Bounds to Charges

It is noted that the total electricity charge of the apartment complex B in Equation (4) may change significantly when $f(\overline{W})$ changes abruptly. Specifically, when $\overline{W}$ is close to a threshold between progressive tariff tiers, the electricity consumption of a single household may shift $\overline{W}$ to a different progressive tariff tier, resulting in a substantial change in B. This is most likely to occur when the transition is between the second and third tariff tiers, where the difference in electricity charges is the largest. In such cases, the impact value of a particular household may become exceptionally large or small, leading to an unusually large difference from the household's current common-area electricity charge, although such instances occur rarely.

To alleviate the impact of such extreme cases, upper and lower bounds can be imposed on the monthly common-area electricity charge for each household. Let the upper and lower bounds be denoted by u and l, respectively, where $l < p^{\mathrm{UNI}} < u$. Define Λ'_u as the set of households whose common-area electricity charges are greater than or equal to u, and Λ_l as the set of households whose common-area electricity charges are less than or equal to l. In other words,

$$\Lambda'_u := \{k \in \Gamma | p_k \geq u\}, \tag{22}$$

$$\Lambda_l := \{k \in \Gamma | p_k \leq l\}, \tag{23}$$

and

$$\Lambda := \{k \in \Gamma | l < p_k < u\}. \tag{24}$$

The bounded common-area electricity charge for household k, denoted by q_k, is then given by

$$q_k := \begin{cases} u, & k \in \Lambda'_u, \\ l, & k \in \Lambda_l, \\ p_k + \epsilon, & k \in \Lambda, \end{cases} \tag{25}$$

where ϵ is a compensation term applied to the households whose charges are not bounded to maintain the total allocated common-area electricity charge equal to P. It is given by

$$\epsilon = \frac{\sum_{k\in\Lambda'_u}(p_k - u) + \sum_{k\in\Lambda_l}(p_k - l)}{|\Lambda|}. \tag{26}$$

It is noted that q_k may exceed the upper bound u after adding ϵ when p_k is slightly lower than u and the positive value of ϵ is large. Conversely, q_k may fall below the lower bound l

when p_k is just above the lower bound and the negative value of ϵ is large in magnitude. As a result, some values of q_k, obtained by updating p_k for households originally in Λ, may fall outside the bounds. In such cases, q_k in Λ is relabeled as p_k, and the set Λ is redefined as the new set Γ. Then, Equations (22)–(26) are applied iteratively until all the values of q_k lie within the bounds. This iterative procedure converges as the absolute value of ϵ in Equation (26) decreases monotonically.

5. Experimental Results and Discussion

5.1. Results and Discussions for Proposed Schemes in Each Apartment Complex

The proposed impact-based allocation schemes for common-area electricity bill were evaluated using actual electricity metering data collected from 13,335 households across 31 apartment complexes (Complex 1 to Complex 31), with household floor areas ranging from 31 m^2 to 134 m^2, over a 24-month period. According to the 2024 statistical data, apartments with a floor area less than or equal to 132 m^2 account for 92.8% of all apartment households in Korea [29]. Considering this statistical distribution, 31 apartment complexes were selected, focusing primarily on households within this floor-area range and covering a wide variety of floor areas and electricity consumption characteristics. This selection was intended to broadly capture the characteristics of households with diverse floor areas and electricity consumption levels. The number of households per apartment complex also varies considerably, ranging from 152 households (Complex 2) to 1426 households (Complex 11), with an average of 430 households per complex.

Separate preprocessing procedures, such as outlier removal, were not applied to the collected metering data, and no households were excluded during the experiments. Both the electricity metering data of individual households and the total electricity consumption reading from the main meter of the apartment complex were used in the experiments, and both are already collected for the existing billing process. Among the experimental results obtained from the 31 apartment complexes, Complex 10, which has 432 households and is close to the average of 430 households, is selected as an example, and its results are presented in Figure 3. One month from each season (January, March, August, and October) was selected for presentation in the figure. In Figure 3, the horizontal axis represents the household electricity consumption, and the vertical axis represents the corresponding common-area electricity charge assigned to each household. The common-area electricity charges obtained using the proposed schemes (ADD, ADD-NN, PROP, and PROP-NN) and the current UNI scheme are presented together for comparison.

Overall, the results obtained using the proposed schemes exhibit a trend similar to that shown in Figure 2b, with discontinuities or bends occurring at the thresholds of the progressive electricity tariff tiers. These thresholds are 300 and 450 kWh in summer, and 200 kWh and 400 kWh in the other seasons.

Let us compare the results obtained from each scheme. The PROP scheme exhibits the widest variation in the allocated common-area electricity charges among the four proposed schemes. Specifically, the results obtained with PROP exhibit a pattern similar to that of ADD but are spread over a considerably wider range. This tendency is consistently observed for all the seasons. In PROP, the impact value δ_k is multiplied by $P/\sum_{m=1}^{N}\delta_m$ when calculating the common-area electricity charge, as shown in Equation (19). Since the magnitude of $P/\sum_{m=1}^{N}\delta_m$ is relatively large, even a small change in δ_k can result in a large change in the common-area electricity charge allocated to household k. Both ADD and PROP allow positive and negative common-area electricity charges. A negative common-area electricity charge can be regarded as a bill credit and may be used to reduce individual households' electricity bills. Among the four proposed schemes, ADD exhibits the second-

largest variation in the common-area electricity charge with respect to household electricity consumption, following PROP.

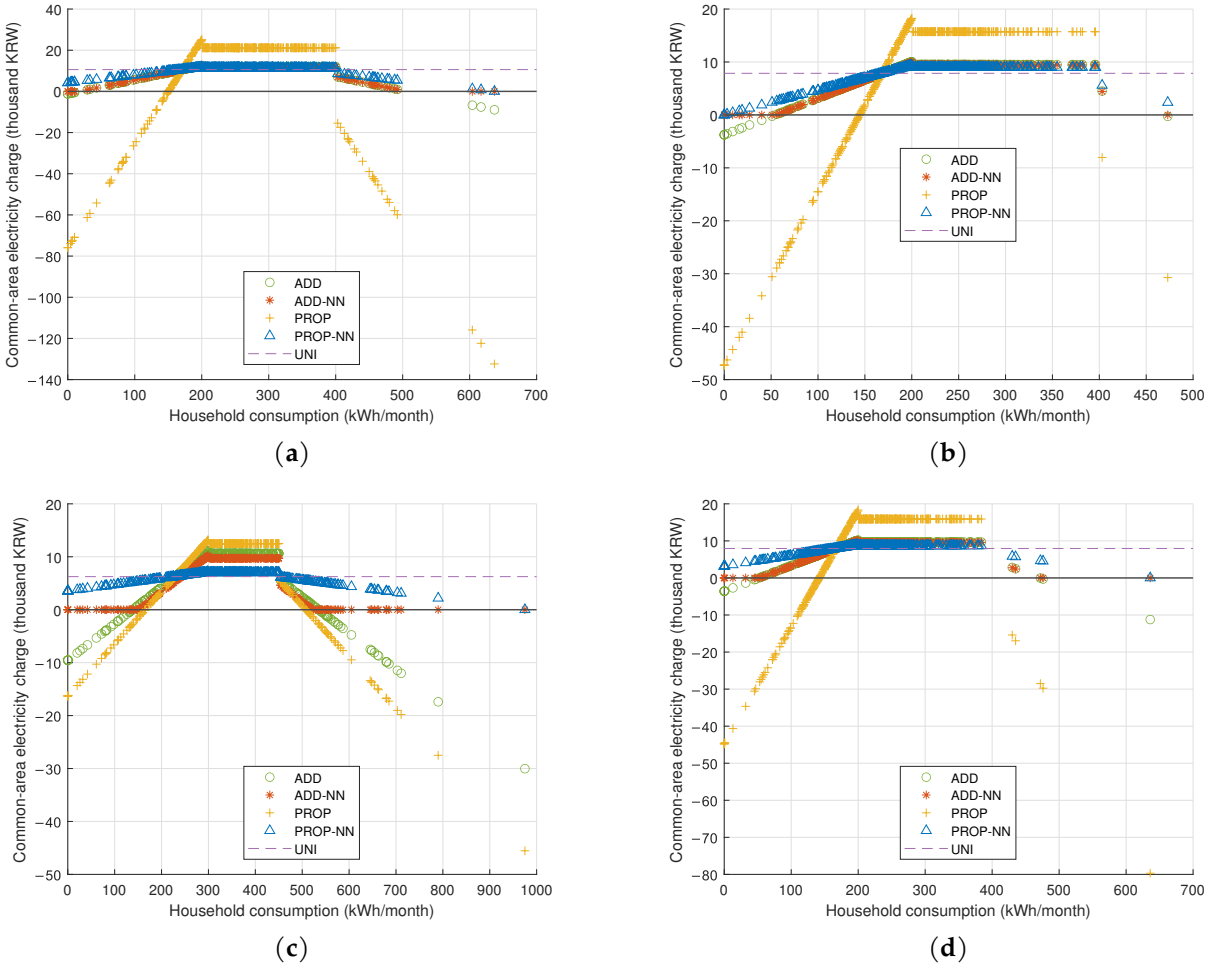


Figure 3. Common-area electricity charges for each household in Complex 10. (**a**) January. (**b**) March. (**c**) August. (**d**) October.

ADD-NN and PROP-NN allow only nonnegative common-area electricity charges and avoid negative charges, as designed. Furthermore, PROP-NN exhibits a relatively smaller variation in the allocated common-area electricity charges than ADD-NN. In PROP-NN, the sum of the d_k values is used as the denominator in calculating the common-area electricity charge, as shown in Equation (21). Since all d_k values are nonnegative, their sum over all households is a large positive value. Consequently, $P/\sum_{m=1}^{N} d_m$ is relatively small, resulting in smaller variations in the common-area electricity charges allocated to individual households, calculated as $d_k P/\sum_{m=1}^{N} d_m$.

On the other hand, the results obtained using ADD-NN exhibit the same overall shape as those obtained using ADD in the region where the charges are positive. However, the charge values obtained using ADD-NN are shifted slightly downward because all negative charges produced by ADD are set to zero. This adjustment reduces the positive common-area electricity charges so that the total allocated charge remains unchanged.

Similar trends were also observed in complexes with different household floor areas and sizes. Figure 4 presents the experimental results obtained in October for several apartment complexes selected to represent different household floor areas and complex sizes. Specifically, Complex 12 consists of 175 households with the smallest floor area (31 m^2), whereas Complex 29 consists of 396 households with relatively large floor areas (108–134

m^2). In terms of the number of households, Complex 2 is the smallest, with 152 households, while Complex 11 is the largest, with 1426 households. The household floor areas in Complexes 2 and 11 range from 60 to 85 m^2 and from 60 to 115 m^2, respectively. Overall, the results exhibit trends similar to those shown in Figure 3.

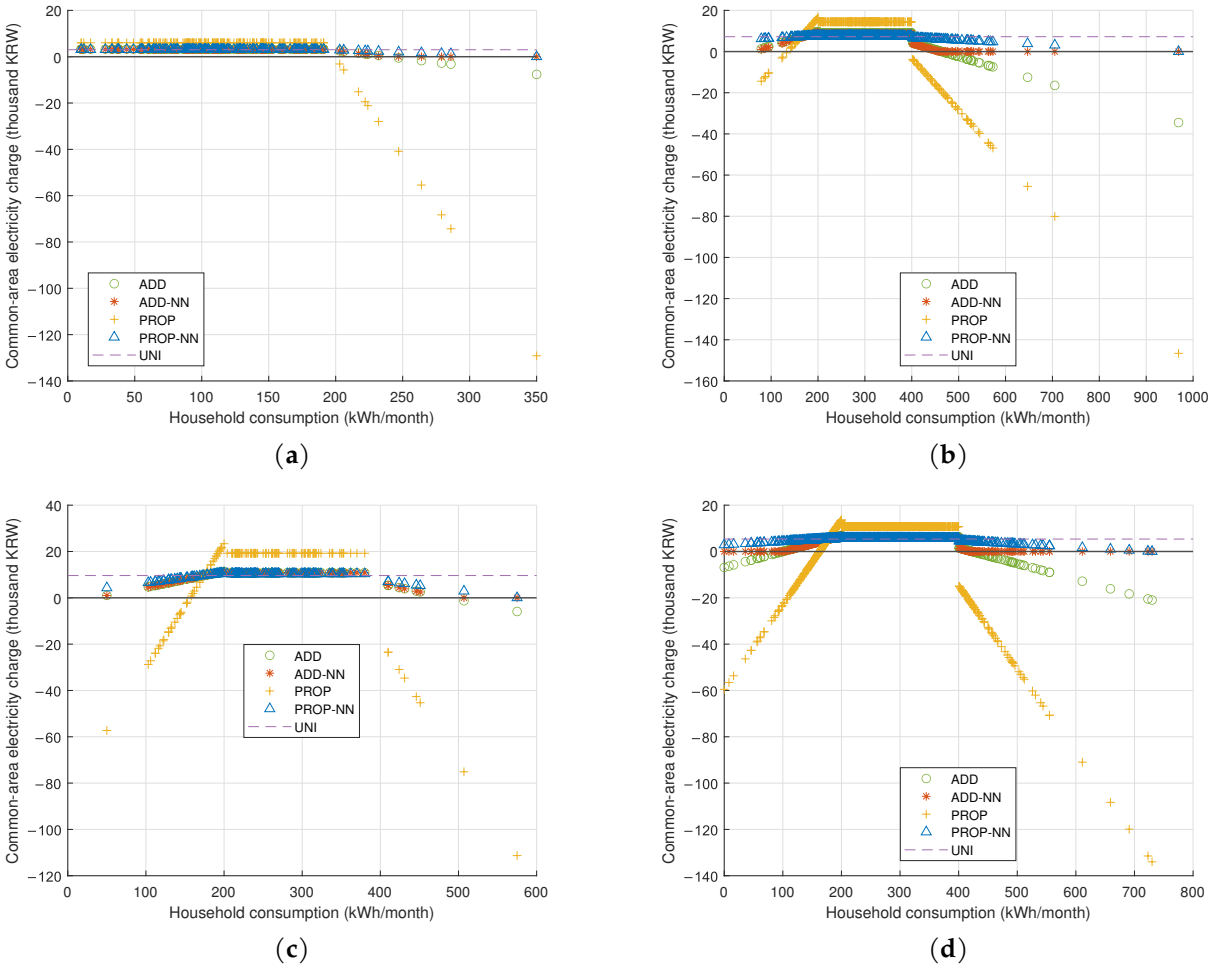


Figure 4. Common-area electricity charges for different apartment complexes in October. (**a**) Complex 12. (**b**) Complex 29. (**c**) Complex 2. (**d**) Complex 11.

The implementation of the proposed schemes involves calculations for household impact values and for allocating common-area electricity charges based on these values. Let $C(N)$ denote the number of arithmetic operations (additions and multiplications) required for an apartment complex consisting of N households. The estimated number of operations required to calculate the impact values is $C(N) \approx 9N$, indicating that the computational effort increases linearly with the number of households. The computational effort required for charge allocation is generally smaller than that required for calculating the impact values. Specifically, ADD requires approximately $2N$ operations, whereas PROP and PROP-NN require no more than $3N + 1$ operations. In the case of ADD-NN, additional iterative computations are required to set negative allocated charges to zero. Let λ denote the number of households with initially negative allocated charges and r the number of iterations. The corresponding number of operations can then be estimated as no more than $2(N + \lambda r) + 1$. These results indicate that the computational effort required for calculating the impact values and allocating charges generally increases linearly with the number of households. Therefore, the proposed schemes are not expected to impose a substantial computational burden, even when applied to large apartment complexes.

5.2. Analysis of the Experimental Results for All Apartment Complexes

To analyze the characteristics of the proposed allocation schemes in more detail, the common-area electricity charges calculated using the proposed schemes were compared with those obtained using the current UNI scheme for all 13,335 households. Because 24 months of metering data were collected for each household, a total of 320,040 monthly common-area electricity charges for individual households were calculated for each proposed allocation scheme. These monthly common-area electricity charges were then compared and analyzed based on their charge differences. The charge difference is defined as the value obtained by subtracting p^{UNI} from the charge calculated using the proposed allocation scheme.

To investigate the distribution characteristics of the charge differences for each proposed allocation scheme, the charge differences were obtained for all households, and histograms were constructed using these values. Figure 5 presents the resulting histograms for the four proposed allocation schemes. To facilitate a fair comparison, the same horizontal axis range was used for all histograms.

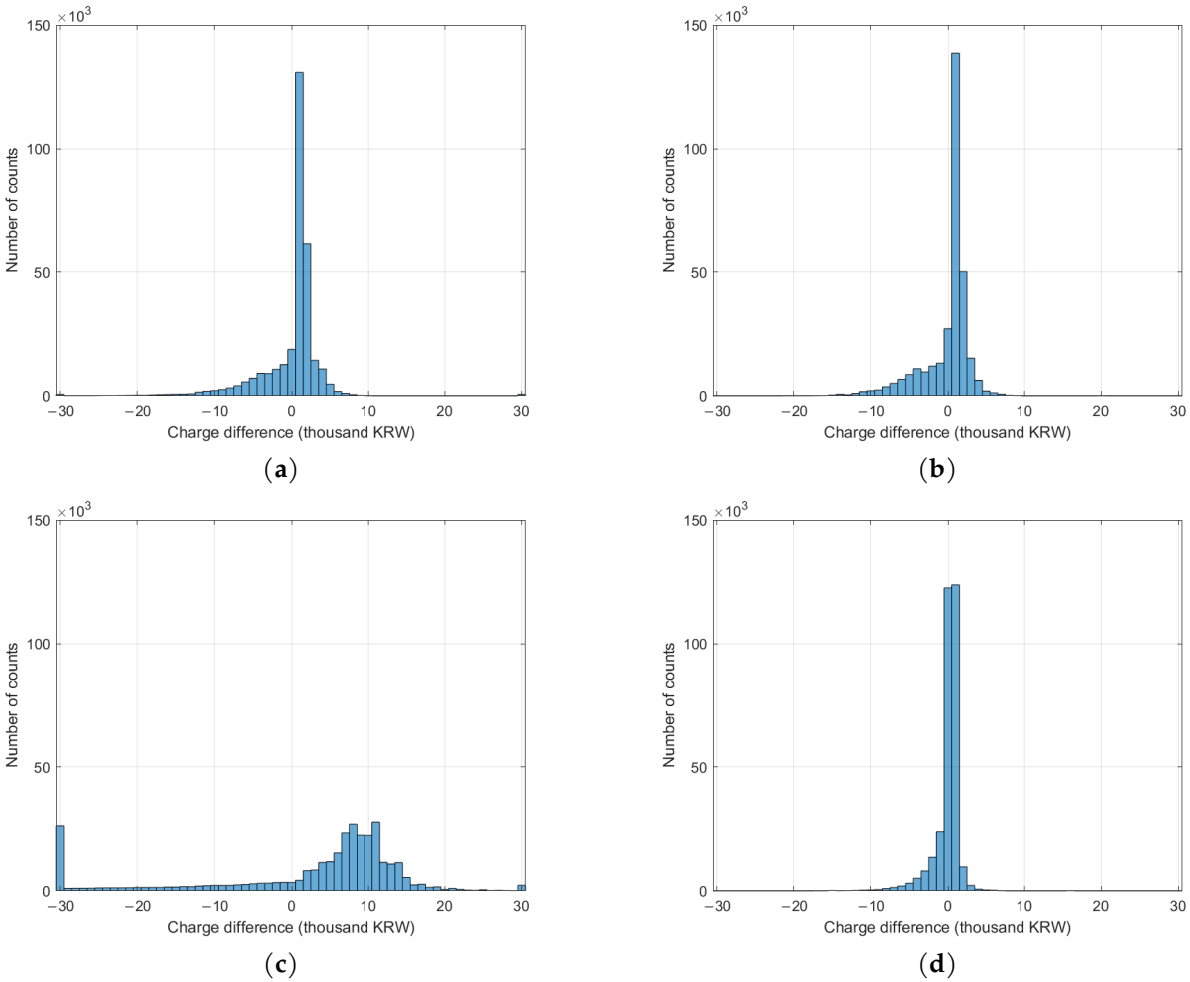


Figure 5. Histogram of the charge differences. (**a**) ADD. (**b**) ADD-NN. (**c**) PROP. (**d**) PROP-NN.

The ADD, ADD-NN, and PROP-NN schemes exhibit a peak near the center and are skewed to the left. However, in the case of PROP-NN, the peak is slightly broader, while the left tail is somewhat shorter than those of ADD and ADD-NN. In addition, the distribution of ADD in Figure 5a is slightly broader than that of ADD-NN because no lower-bound constraint is imposed.

For PROP, the distribution of charge differences spans a much wider range than those of the other schemes. Thus, when the entire range of charge differences is included in the

histogram, the central region of practical interest would be difficult to observe in detail because of the extreme values associated with only a small number of households. Therefore, values lying far outside the central region of the histogram were clipped for ease of observation. In Figure 5c, most of the values at the left end of the histogram are clipped values. Similarly, most of the values at the right end of the histogram also correspond to clipped values. Clipped values are rarely observed in Figure 5b,d for ADD-NN and PROP-NN, respectively. For ADD, only a few clipped values are observed at either end of the histogram in Figure 5a. In addition, it is observed that the histogram peak of PROP is shifted to the right compared with those for the other three schemes. This indicates that a relatively larger number of households experience larger increases in common-area electricity charges under PROP than under the other three schemes.

Compared with PROP, PROP-NN in Figure 5d exhibits a substantially narrower distribution, with the charge differences more concentrated around the center. The negative charge differences, representing reductions in charges compared with the current UNI scheme, are distributed over a relatively wider range, whereas the positive charge differences, representing increases in charges, are concentrated within a relatively narrow range.

For all four proposed schemes, the negative charge differences are distributed over a wider range than the positive charge differences, although they occur less frequently. This indicates that the proposed schemes produce positive charge differences more frequently, but their magnitudes are relatively small. In contrast, negative charge differences occur less frequently but tend to have larger absolute magnitudes. It should also be noted that, for every proposed scheme, the sum of all charge differences is zero. This is because the total monthly common-area electricity charge allocated to all households in an apartment complex should always be equal to the common-area electricity bill of the apartment complex, P, regardless of the allocation schemes.

5.3. Analysis of the Charge Differences from the Household Perspective

In this subsection, the charge difference, defined by the difference between the charges obtained using the proposed allocation scheme and the current UNI scheme, is analyzed from the household perspective. For each household, the charge difference varies from month to month depending on the monthly electricity consumption of the household and the apartment complex.

To observe the monthly variations in the charge differences for each household, a heatmap of the charge differences over the 24-month period is presented in Figure 6. Figure 6 shows the charge differences calculated using ADD-NN for 432 households in Complex 10. The horizontal axis represents individual households in the apartment complex, and the vertical axis represents the 24 consecutive months starting in July, arranged from top to bottom.

For ease of identifying increases and reductions in charge differences, positive charge differences are shown in red, whereas negative charge differences are shown in blue. The magnitude of the charge differences is represented by the color intensity. During the summer months of July and August, household electricity consumption is relatively high, resulting in wider ranges of both increases and decreases in charge differences. Consequently, larger variations in color intensity are observed during these months.

As shown in Figure 6, the charge differences for each household exhibit various monthly patterns. That is, a household experiencing a positive charge difference in one month may experience a negative charge difference in another month. This tendency is also observed for the other allocation schemes and apartment complexes. Therefore, in this study, the average charge difference over the 24-month period for each household is used to evaluate the influence of the proposed allocation schemes on individual households.

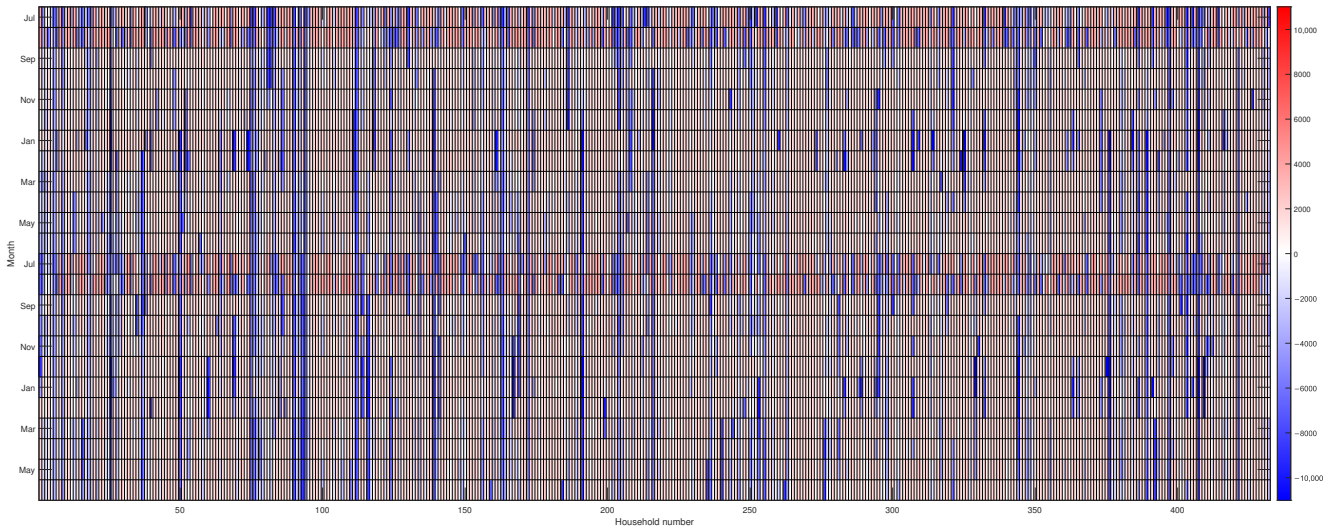


Figure 6. Heatmap of charge differences for individual households over 24 months.

To analyze the distribution of the 24-month average charge differences for each household, histograms of the average charge differences are presented in Figure 7. Compared with the results in Figure 5, the distributions in Figure 7 are more concentrated in a relatively narrower range. For ADD, ADD-NN, and PROP-NN schemes, the distributions of the average charge differences show that the negative region is relatively wider than the positive region. That is, positive charge differences are concentrated within a narrow range and occur for a larger number of households, whereas negative charge differences are distributed over a relatively wider range and occur for a smaller number of households.

For a more detailed analysis of the characteristics of the average charge differences, the positive and negative charge differences were separately collected, and their statistical characteristics were calculated. The results are summarized in Table 2. In Table 2, charge increase and decrease are defined relative to the current UNI scheme. Specifically, positive charge differences indicate charge increases, whereas negative charge differences indicate charge decreases relative to the current common-area electricity charge, p^{UNI}.

As shown in Table 2, the ratio of households with charge increases ranges from 65.2% to 69.5% depending on the proposed schemes, whereas the proportion of households with charge decreases ranges from 30.5% to 34.8%. Thus, the number of households with charge increases is more than twice that of households with charge decreases for all proposed allocation schemes except ADD-NN, for which the ratio of charge increases to charge decreases is approximately 1.87:1. However, the average charge increases range from 540 to 7446 KRW, whereas the average charge decreases range from −1129 to −16,980 KRW. This indicates that although a larger number of households experience charge increases, the magnitude of the increases for each household is relatively smaller than that of the charge decrease. This tendency is consistently observed for all four proposed schemes.

In terms of variability, the standard deviations (SDs) of the charge increases and decreases differ depending on the allocation scheme. The largest SD is observed for PROP, followed by ADD, ADD-NN, and PROP-NN. The SDs of the charge increases are smaller than those of the charge decreases for PROP-NN, ADD, and ADD-NN, whereas the corresponding SDs are similar for PROP.

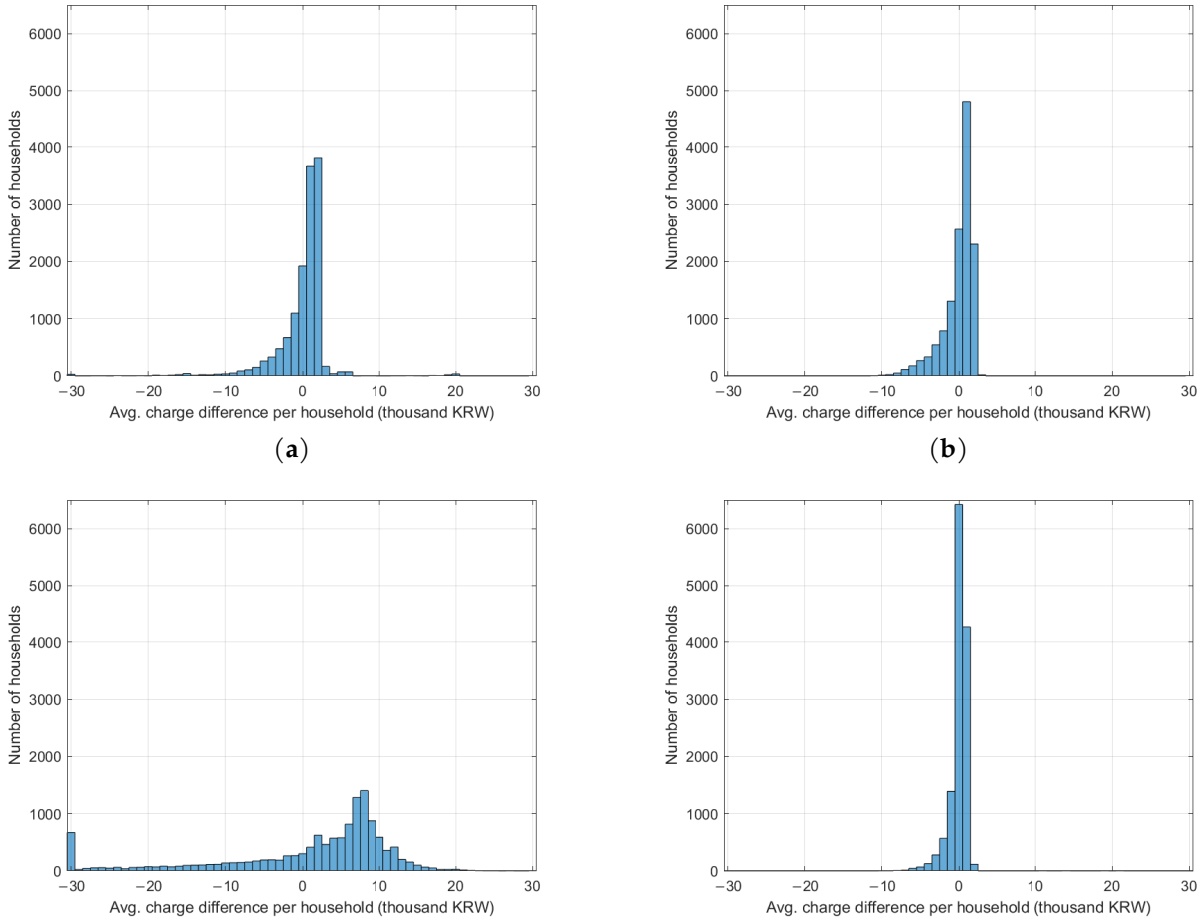


Figure 7. Histogram of the average charge differences for each household. (**a**) ADD. (**b**) ADD-NN. (**c**) PROP. (**d**) PROP-NN.

Table 2. Summary statistics of 24-month average charge differences for individual households.

Proposed Scheme	Ratio of Charge Increases (%)	Ratio of Charge Decreases (%)	Average Charge Increase (KRW)	Average Charge Decrease (KRW)	SD of Charge Increases (KRW)	SD of Charge Decreases (KRW)	Largest Charge Increase (KRW)	Largest Charge Decrease (KRW)
ADD	68.2	31.8	1590	−3415	2248	5839	74,343	−68,245
ADD-NN	65.2	34.8	1141	−2134	1406	2007	88,292	−11,493
PROP	69.5	30.5	7446	−16,980	19,909	19,718	1,856,300	−293,971
PROP-NN	67.6	32.4	540	−1129	516	1298	20,891	−10,088

Furthermore, the NN schemes exhibit smaller SDs for charge decreases than their corresponding non-NN schemes. This is because the NN schemes eliminate negative common-area electricity charges to zero, thereby reducing the variation in charge decreases. Similarly, the SDs of the charge increases are also smaller for the NN schemes because the magnitudes of the positive charge differences are reduced compared with those of the corresponding non-NN schemes.

Next, the largest values of charge increases and charge decreases are observed for the proposed allocation schemes. For PROP, both values are considerably larger in magnitude than those of the other three proposed schemes. In particular, the largest charge increase is exceptionally higher than those obtained using the other schemes. This exceptionally large value is observed for a specific household in Complex 4, primarily due to the exceptionally

large charge in July, resulting from an unusually large impact value and the characteristics of the PROP scheme. When this household was excluded from the apartment complex, the average electricity consumption of the complex was within the second tier of the progressive tariff. However, when this household was included, the average electricity consumption increased to the third tier of the progressive tariff. Consequently, the substantial difference between the second- and third-tier electricity rates, combined with the large number of households, led to a significant increase in the total electricity charge for the apartment complex. Moreover, in the PROP scheme, since the impact value was multiplied by the total common-area electricity bill of the apartment complex, the resulting common-area electricity charge allocated to the household became very large. Furthermore, in this case, there were no other households with comparable impact values to share the increased charge, resulting in the exceptionally large increase in the common-area electricity charge observed for this household. Such extreme variations can be effectively mitigated by applying the bounds introduced in Section 4.5. The experimental results obtained by applying the bounds are presented in the following subsection.

For reference, the ratios of various statistical measures of the charge differences obtained using the proposed schemes to the common-area electricity charge under the current UNI scheme are presented in Table 3. As shown in the table, the average charge increase ranges from 0.05 to 0.78 times that under the UNI scheme, depending on the proposed scheme. The average charge decrease ranges from 0.11 to 1.77 times that under the UNI scheme, and these values are considered acceptable from the perspective of individual households. However, the largest charge increase and decrease under the PROP scheme are excessively large, and it is therefore considered necessary to apply bounds, as discussed previously.

Table 3. Ratio to the common-area electricity charge under the current UNI scheme.

Proposed Scheme	Average Charge Increase	Average Charge Decrease	SD of Charge Increases	SD of Charge Decreases	Largest Charge Increase	Largest Charge Decrease
ADD	0.17	−0.37	0.21	0.66	8.72	−7.85
ADD-NN	0.12	−0.23	0.15	0.21	10.35	−1.00
PROP	0.78	−1.77	1.60	2.10	146.30	−30.21
PROP-NN	0.05	−0.11	0.05	0.12	2.45	−0.91

5.4. Analysis of the Charge Differences with Bounds Applied

This subsection presents the experimental results obtained by applying the proposed bounding method as an optional post-processing step to the four proposed common-area electricity charge allocation schemes. For each scheme, the upper and lower bounds introduced in Section 4.5 were applied to the calculated common-area electricity charges to obtain the final charges. Subsequently, for each month, the difference between the final common-area electricity charge and the current charge, denoted by p^{UNI}, was calculated for each household. The average charge difference over the 24-month period was then obtained for each household.

In the experiment, $u = 4p^{\mathrm{UNI}}$ was selected as the upper bound and $l = -u$ as the lower bound. The statistical characteristics after applying these bounds, such as the ratios of households experiencing charge increases and decreases, the corresponding averages and SDs, and largest increase and decrease, were calculated for each proposed scheme and are summarized in Table 4.

Table 4. Summary statistics of household average charge differences after applying bounds ($u = 4p^{\mathrm{UNI}}, l = -u$).

Proposed Scheme	Ratio of Charge Increases (%)	Ratio of Charge Decreases (%)	Average Charge Increase (KRW)	Average Charge Decrease (KRW)	SD of Charge Increases (KRW)	SD of Charge Decreases (KRW)	Largest Charge Increase (KRW)	Largest Charge Decrease (KRW)
ADD	67.6	32.4	1341	−2724	656	2849	4001	−34,092
ADD-NN	65.3	34.7	1121	−2136	574	2008	2879	−11,493
PROP	67.6	32.4	6206	−13,006	3304	12,160	22,277	−82,231
PROP-NN	67.7	32.3	538	−1129	364	1300	2167	−10,088

As shown in Table 4, the ratio of households with charge increases ranges from 65.3% to 67.7%, while the ratio with charge decreases ranges from 32.3% to 34.7%, depending on the allocation scheme. Compared with the results in Table 2, almost no difference is observed for ADD-NN and PROP-NN, whereas the ratios of households with charge increases decline slightly for ADD and PROP.

Regarding the average values, for ADD and PROP, the average charge increase becomes smaller, while the average charge decrease becomes larger after applying the bounds. Consequently, the difference between the average charge increase and average charge decrease is reduced, indicating that the overall variation in charge differences becomes smaller. In contrast, the average values for ADD-NN and PROP-NN remain almost unchanged.

A similar trend is observed in the SDs. For ADD and PROP, the SDs of the charge increases are reduced to 0.292 and 0.166 times those without bounds, respectively. Likewise, the SDs of the charge decreases are reduced to 0.488 and 0.617 times those without bounds, respectively. In contrast, almost no changes are observed in the SDs of either the charge increases or decreases for ADD-NN and PROP-NN. This is because the ADD and PROP schemes produce a relatively wide range of charge values, causing many values to be clipped by the bounds. In contrast, the distributions of ADD-NN and PROP-NN are much narrower, so very few charge values are affected when the bounds of $\pm 4p^{\mathrm{UNI}}$ are applied.

In terms of the largest charge increase and decrease, the exceptionally large values previously observed for PROP in Table 2 are not observed after applying the bounds. Specifically, the largest charge increase and decrease become 22,277 KRW and −82,231 KRW, respectively. For the other three schemes, the largest charge increase is also reduced after applying the bounds, whereas the change in the largest charge decrease is relatively small. In particular, the largest charge decrease for ADD-NN and PROP-NN remains almost unchanged. Furthermore, the average and SD of the charge decreases for these two schemes also remain almost unchanged. This suggests that the current bounds have little influence on the charge decrease results of these two schemes.

For reference, the ratios of various statistical measures of the charge differences obtained using the proposed schemes to the common-area electricity charge under the current UNI scheme are presented in Table 5. As shown in the table, the largest charge increase ranges from 0.22 to 1.53 times that under the UNI scheme, depending on the proposed scheme. The largest charge decrease ranges from 0.91 to 5.00 times that under the UNI scheme, and these values are also considered to be within an acceptable range from the perspective of individual households, as are the average charge increase and decrease values.

Table 5. Ratio to the common-area electricity charge under the current UNI scheme after applying bounds ($u = 4p^{UNI}, l = -u$).

Proposed Scheme	Average Charge Increase	Average Charge Decrease	SD of Charge Increases	SD of Charge Decreases	Largest Charge Increase	Largest Charge Decrease
ADD	0.15	−0.30	0.08	0.34	0.50	−3.80
ADD-NN	0.12	−0.23	0.06	0.21	0.34	−1.00
PROP	0.64	−1.34	0.26	1.17	1.53	−5.00
PROP-NN	0.05	−0.11	0.03	0.13	0.22	−0.91

Next, to examine the effect of the bound values, experiments were conducted by varying the bounds from $2p^{\mathrm{UNI}}$ to $8p^{\mathrm{UNI}}$. The results are presented in Figure 8. The horizontal axis represents the upper bound u expressed as a multiple of p^{UNI}, while the lower bound is set to $l = -u$. Figure 8a shows the average charge increases and decreases for the four proposed schemes. Except for PROP, the average values vary only slightly as the bound changes, indicating that relatively few households are affected by the bounds in the other three schemes. A similar tendency can be observed in the SDs shown in Figure 8b. For PROP, the SD changes considerably as the bound varies, particularly for charge decreases. This indicates that the PROP scheme produces a much wider range of charge differences and that large charge decreases are strongly affected by the bounds.

Figure 8c presents the largest charge increase and decrease as a function of the bound value. The PROP scheme exhibits a relatively large variation in the largest charge decrease as the bound value changes. However, the largest charge increase changes only slightly with the bound values. It should be noted that the largest charge increase and decrease are determined from the average charge increases and decreases over 24 months for each household. Consequently, the resulting curves are not linear functions of the bound value. On the other hand, the largest charge increases and decreases for the ADD-NN and PROP-NN schemes are almost unaffected by the bound values, indicating that these NN-based schemes inherently produce a relatively narrow range of common-area electricity charges.

Overall, the application of the proposed bounding method effectively suppresses extreme charge increases and decreases. In particular, the bounding method is effective for the PROP scheme. It suppresses rare but exceptionally large charge variations caused by the impact-based allocation mechanism while preserving the benefit of allocating common-area electricity charges according to each household's impact on the common-area electricity bill of the apartment complex. Therefore, the bounding method is recommended for use with the PROP scheme, whereas it can be considered optional for the ADD, ADD-NN, and PROP-NN schemes.

Finally, the characteristics of the charge results calculated using the four proposed schemes are summarized in Table 6. To facilitate comparison of the numerical characteristics of the results, they are classified into five relative levels. Specifically, based on the results obtained in Tables 2 and 4, values below 1000 are classified as very low, values from 1000 to less than 2000 as low, values from 2000 to less than 5000 as moderate, values from 5000 to less than 10,000 as high, and values of 10,000 or greater as very high. When the classification result is altered after applying the bounding method, the resulting classification is indicated in parentheses in the corresponding entry. The charge increases and decreases denote the differences obtained by subtracting the charge under the current uniform scheme from that under each proposed scheme. The variation in charge increases and decreases is represented by the corresponding standard deviations.

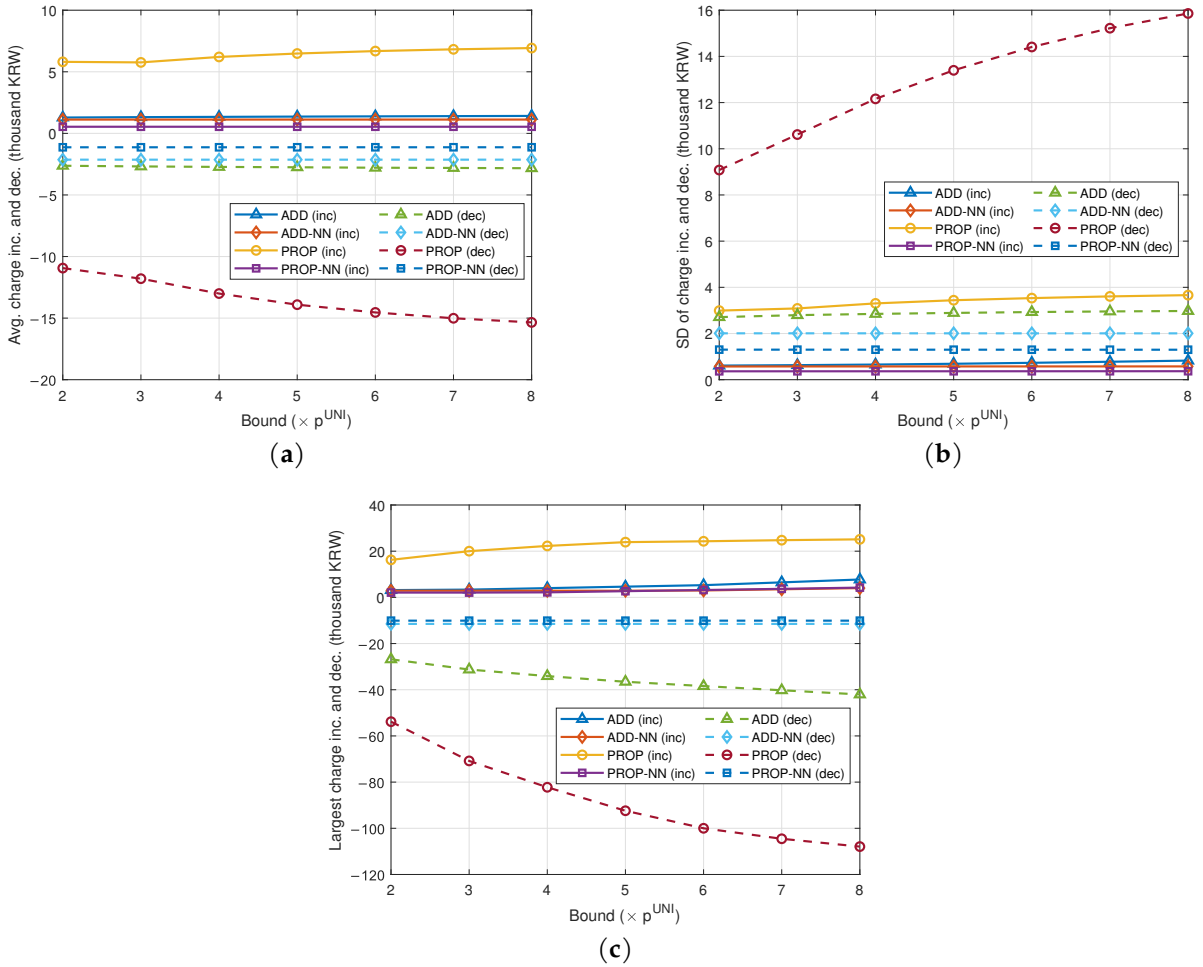


Figure 8. Variation of statistical measures with respect to the bounds. (**a**) Average of charge increase and decrease. (**b**) SD of charge increase and decrease. (**c**) Largest charge increase and decrease.

Table 6. Comparative summary of the proposed schemes. Levels in parentheses indicate the results after applying the bounding method, while unchanged levels are not shown.

Proposed Scheme	Average Charge Increase	Average Charge Decrease	Variation in Charge Increases	Variation in Charge Decreases	Negative Common-Area Electricity Charge	Bounding Method
ADD	Low	Moderate	Moderate (Very low)	High (Moderate)	Allowed	Optional
ADD-NN	Low	Moderate	Low (Very low)	Moderate	Not allowed	Optional
PROP	High	Very high	Very high (Moderate)	Very high	Allowed	Recommended
PROP-NN	Very low	Low	Very low	Low	Not allowed	Optional

As shown in Table 6, the PROP scheme consistently ranks highest among the four schemes across all aspects, whereas the PROP-NN scheme ranks lowest. Therefore, PROP is considered suitable when the impact value is to be reflected relatively strongly in the household common-area electricity charge, whereas PROP-NN is considered suitable when the impact value is to be reflected while keeping changes in the charge relatively small. The ADD and ADD-NN schemes exhibit intermediate characteristics between these two schemes, with ADD generally showing relatively higher values than ADD-NN. On the other hand, when negative common-area electricity charges are allowed, either ADD or

PROP can be selected. Conversely, when negative common-area electricity charges are not allowed, either ADD-NN or PROP-NN is more appropriate. In addition, the bounding method is recommended for use with the PROP scheme, while it can be applied optionally to the other three schemes depending on the requirements.

It should be noted that the analysis in this study is based on the Korean electricity tariff system, and the applicability of the proposed charge allocation schemes may vary depending on the characteristics of electricity tariff structures in other countries. In particular, the presence and structure of progressive tariffs, the relationship between electricity consumption and charges, and the methods used to calculate and allocate common-area electricity charges may affect household impact values and the resulting charge allocations. Nevertheless, the fundamental approach adopted in this study is to quantify the effects of individual household electricity consumption on the total common-area electricity bill and reflect these effects in the allocation of charges to individual households. This approach can also be applied to other tariff structures. However, the specific methods for calculating impact values and the resulting allocation outcomes would need to be reexamined to account for the characteristics of each tariff structure.

6. Conclusions

In this study, the impact value of each household on the common-area electricity bill is defined and calculated with reasonable computational complexity. The impact value of each household is quantified by the change in the apartment complex's common-area electricity bill resulting from the inclusion of that household. A positive value indicates an increase in the bill, whereas a negative value indicates a decrease, with its magnitude proportional to the amount of the change. Based on these impact values, four common-area electricity charge allocation schemes are proposed for apartment complexes adopting SC. Furthermore, a bounding method is proposed to impose upper and lower bounds on the common-area electricity charge allocated to each household. The proposed schemes were evaluated using actual metering data. The computational complexity of calculating household impact values and allocating common-area electricity charges increases linearly with the number of households. The proposed schemes can be applied to apartment complexes of various sizes and to households with different floor areas, using only the electricity metering data for individual households and the total electricity consumption reading from the main meter of an apartment complex. These data are already collected for the existing billing process. Therefore, the proposed schemes are applicable to a wide range of apartment complexes in Korea that adopt the SC system.

The common-area electricity charges calculated using the four proposed schemes were compared with those obtained using the current uniform allocation scheme and analyzed in terms of their differences. The PROP scheme showed the largest variation in charges, whereas PROP-NN showed the smallest. The ADD and ADD-NN schemes generally exhibited characteristics between those of PROP and PROP-NN. These trends were generally consistent across apartment complexes of different sizes and months. From the perspective of households, PROP most strongly reflects the impact value of each household in the allocated common-area electricity charge, whereas PROP-NN results in relatively smaller changes from the current uniform scheme. Therefore, PROP is suitable when the impact value is to be reflected strongly, while PROP-NN is preferable when relatively small changes in charges are desired. ADD and ADD-NN provide intermediate alternatives, with ADD reflecting the impact value more strongly than ADD-NN. When negative common-area electricity charges are not allowed, the nonnegative schemes, PROP-NN and ADD-NN, are appropriate. In addition, the bounding method is recommended for PROP and can be optionally applied to the other schemes. Finally, the average charge increase ranges

from 0.05 to 0.78 times the corresponding charge under the UNI scheme, depending on the proposed scheme. The average charge decrease ranges from 0.11 to 1.77 times the corresponding charge under the UNI scheme, and these charge changes are considered acceptable from the perspective of individual households.

Author Contributions: Y.M.C. developed the allocation schemes, conducted the analyses and simulations, and organized and refined the manuscript. D.S.K. conducted the analyses and simulations, and organized and refined the manuscript. B.J.C. collected the metering data, identified the issues related to the current allocation scheme, and organized and refined the manuscript. All authors have read and agreed to the published version of the manuscript.

Funding: The work of Beom Jin Chung was supported by the Korea Institute of Energy Technology Evaluation and Planning (KETEP) through the Ministry of Climate, Energy and Environment (MCEE) of the Republic of Korea under grants RS-2023-00236325 and RS-2024-00418486. The work of Young Mo Chung was financially supported by Hansung University.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The datasets presented in this article are not readily available due to privacy restrictions. Requests to access the datasets should be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

ADD	additive allocation scheme
ADD-NN	additive allocation scheme with nonnegative charges
ENSC	egalitarian nonseparable contribution
GC	general contract
HV	high voltage
KEPCO	Korea Electric Power Corporation
KRW	Korean Won
LV	low voltage
NN	nonnegative
PROP	proportional allocation scheme
PROP-NN	proportional allocation scheme with nonnegative charges
SC	single contract
SD	standard deviation
UNI	uniform
USD	U.S. Dollar

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