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A Safety-Condition Assessment Framework for Adjacent Deep Excavations Incorporating Nonlinear Indicator Coupling and Neighboring Influence

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Abstract

Adjacent deep excavations may interact through stress redistribution, deformation propagation, structural interaction, and groundwater variation. Existing risk assessment methods mainly focus on individual indicators or single excavations, which may overlook nonlinear indicator interactions and neighboring-excavation influence. This study proposes an extensible framework that combines response-based indicator scoring, expert-derived weights, nonlinear indicator interactions, and a distance-based neighboring-excavation contribution. Its output is a composite safety-condition index on a conventional 0–100 scale, with higher values indicating more adverse conditions under the adopted assessment rules. Indicators are organized into deformation, structural resistance, and hydrological domains. Heterogeneous indicators are converted into dimensionless indicator scores using the adopted scoring criteria. Indicator weights and a nonlinear interaction model are incorporated to aggregate scores and include interaction contributions for selected indicator pairs. A distance-based attenuation operator is further introduced to represent modeled neighboring-excavation contributions to the index. The framework is illustrated using a representative twin-excavation configuration, with weights derived from expert judgment, interaction coefficients estimated from numerical degradation scenarios, and attenuation parameters informed by published excavation cases. The example demonstrates the calculation procedure, scenario comparison, identification of governing support levels and sensitivity effects of selected parameters on the calculated index.

Keywords: adjacent deep excavations; safety-condition assessment; nonlinear indicator interaction; neighboring-excavation influence; composite condition index

Academic Editor: Paulo Santos

Received: 30 July 2026

Revised: 10 October 2026

Accepted: 15 October 2026

Published: 19 October 2026

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1. Introduction

With the continuous development of urban underground space, deep excavations are increasingly implemented in dense urban environments. In many metropolitan areas, subway stations, underground commercial complexes, municipal facilities, and high-rise building basements are frequently constructed in close proximity. Under such conditions, excavation safety is influenced both by the evolution of individual risk indicators within

each excavation and by interactions among neighboring excavation systems. At the individual excavation level, conventional risk assessment approaches often evaluate monitoring indicators independently, which may neglect the nonlinear interactions among structural response, deformation behavior, and support conditions. For example, abnormal support behavior may modify the restraint conditions of the excavation system and amplify deformation responses beyond the contribution of individual indicators.

At the group level, adjacent excavations may further interact through stress redistribution, ground deformation propagation, and changes in the surrounding soil environment. Consequently, a local deterioration in one excavation may be transmitted through the intervening soil and influence the safety state of adjacent excavations. Therefore, risk assessment for adjacent deep excavations should move beyond the isolated-reference condition indices of individual excavations and explicitly consider both nonlinear interactions among multiple risk factors and spatial risk transmission between excavation units.

For excavations in rock or soil–rock composite ground, rock-mass tremors associated with blasting, excavation-induced seismicity, or sudden brittle failure may introduce transient dynamic loads that differ from conventional quasi-static excavation loading. Under such conditions, support selection should consider not only static bearing capacity but also ductility, anchorage performance, deformation compatibility, redundancy, and energy-absorption capacity [1]. Although rock-mass tremors are not explicitly considered in the present soft-clay application, vibration- or dynamic-response indicators can be incorporated into the proposed extensible framework when required.

Existing studies on excavation risk assessment can be broadly grouped into three categories: risk identification and indicator system development, traditional risk assessment methods, and numerical simulation or data-driven prediction methods. Risk identification and indicator-system studies provide the basis for structured risk assessment. Tah and Carr [2] proposed a project risk knowledge management framework to support the structured representation and reuse of risk information in construction projects. Choi et al. [3] developed a systematic risk assessment methodology for underground construction projects, including risk checklists, risk information tables, and structured risk identification procedures. Fault tree analysis has also been widely used to decompose complex accident mechanisms into hierarchical risk events and identify critical causal paths in deep excavation projects [4–7]. With the development of digital construction, knowledge-management-based and BIM-assisted approaches have been introduced to improve the efficiency and consistency of safety risk identification [8]. These studies improve the completeness and operability of risk identification. However, many existing indicator systems are still organized according to construction components, work activities, or risk sources, rather than according to physical response mechanisms. As a result, disaster-inducing inputs, such as surcharge, rainfall, and construction sequence deviations, may be mixed with response indicators represented by wall displacement, ground settlement, and support–force imbalance, thereby weakening the causal consistency and physical interpretability of the assessment results.

Traditional risk assessment methods have been widely applied to deep excavation safety evaluation. The Analytic Hierarchy Process provides a practical way to transform expert judgment into quantitative weights [9], and has been combined with fuzzy comprehensive evaluation, gray theory, and matter-element extension methods to address uncertainty and ambiguity in engineering judgment [10–14]. Bayesian networks and dynamic Bayesian networks have also been introduced to represent probabilistic dependence among risk events and update risk probabilities using monitoring information or expert knowledge [15–18]. These methods improve the quantitative and probabilistic representation of excavation risk. Nevertheless, most of them are still based on linear weighting, subjective prior probability, or predefined conditional probability structures.

They are effective for organizing expert knowledge and uncertainty, but they are limited in explicitly quantifying nonlinear amplification among coupled physical responses, such as support–force imbalance, retaining-wall deformation, ground settlement, and ground-water variation.

Numerical simulation and data-driven methods have further improved the analysis and prediction of excavation-induced responses. Finite element and finite difference methods have been widely used to simulate soil–structure interaction, retaining-wall deformation, basal stability, and support internal forces during excavation [19–23]. These methods provide physical insight into the spatial distribution of stress, displacement, and structural response. Meanwhile, machine learning methods, including support vector machines, random forests, Bayesian optimization, and long short-term memory networks, have been explored for deformation prediction and construction risk assessment [24–27]. Data-driven models can capture complex statistical relationships and improve prediction accuracy when sufficient data are available. However, numerical simulation results are often used as mechanical response outputs rather than being transformed into a unified condition score, while purely data-driven methods may require large training datasets and often lack physical interpretability. These limitations are particularly challenging in geotechnical risk assessment, where failure events are usually rare, highly consequential, and strongly affected by site-specific geological conditions.

Recent studies have begun to consider coupling effects among risk factors. For example, N – K models, dynamic Bayesian networks, complex networks, and coupling-risk analysis have been used to describe interdependence among multiple risk events in underground construction and adjacent excavation systems [26,28,29]. In the broader risk-analysis field, the importance of interaction, uncertainty, and system-level risk has also been emphasized by Kaplan and Garrick [30] and Aven [31]. These studies suggest that engineering risk should be interpreted as a coupled process involving individual risk-factor magnitudes and their interactions, rather than as the simple aggregation of isolated indicators. However, most existing coupling-risk models remain probability-based or network-based, and the associated coupling coefficients are rarely parameterized using physical response increments derived from numerical simulation or monitoring data. In addition, the spatial transmission of risk between adjacent excavations is often simplified, although excavation spacing, intervening soil conditions, and source-risk intensity can significantly affect the safety state of a target excavation.

These studies leave a need for an assessment framework that integrates response-based indicators, selected nonlinear interactions, and neighboring-excavation influence within a common, updateable calculation procedure.

To address these limitations, this study proposes a physics-informed and extensible safety-condition assessment framework for adjacent deep excavations. The main contributions of this study are summarized as follows:

- (1) A physics-informed model to assess the safety condition of a single excavation is developed by organizing heterogeneous indicators into deformation, structural resistance, and hydrological response domains. The indicators are aggregated through indicator weighting and expressed on a common 0–100 condition-score scale. Scenario-based nonlinear coupling coefficients are further introduced for selected indicator pairs to represent additional interaction contributions to the condition index beyond linear aggregation.
- (2) A neighboring-influence operator is proposed to extend single-excavation assessment to condition assessment considering neighboring influence. Each excavation can be regarded as either a source or a target excavation, and the neighboring-excavation contribution is represented through an updateable distance-based attenuation function.

- (3) The proposed framework is implemented through an illustrative twin-excavation case study. The influence of nonlinear coupling and modeled neighboring influence is demonstrated, and sensitivity analyses are performed to examine the effects and limitations of key updateable parameters.

2. Safety-Condition Assessment Framework for Single and Adjacent Excavations

2.1. Framework of the Proposed Study

The proposed unified and extensible framework aims to assess the safety condition of individual excavations and the composite condition index of adjacent excavations. Figure 1 presents four sequential components of the framework workflow: (1) a physics-informed indicator system, (2) a condition-index calculation model with updateable parameters, (3) parameter instantiation based on project-specific information, and (4) illustrative application and sensitivity analysis.

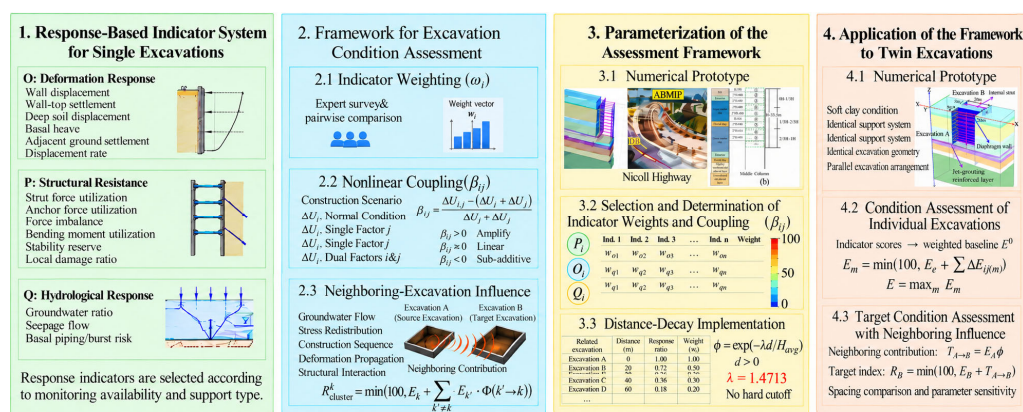


Figure 1. The proposed multi-level safety-condition assessment framework for adjacent deep excavations.

The first component establishes a physics-informed indicator system for characterizing excavation safety conditions. The heterogeneous response indicators are organized into three physical domains, including deformation response, structural resistance, and hydrological response. Instead of defining a universal fixed indicator set, the proposed framework allows for the flexible selection of active indicators according to excavation support systems, geological and hydrological conditions, monitoring availability, and project-specific risk concerns. Indicators that are unavailable or unsuitable for a specific excavation can be excluded, and the remaining indicator weights can be adjusted accordingly.

The second component establishes the condition-index calculation model by introducing three updateable parameter groups. Indicator weights ω are used to represent the relative importance of different response indicators. Nonlinear interaction coefficients β are introduced to capture additional contributions to the condition index caused by concurrent deterioration of multiple indicators beyond linear aggregation. Furthermore, a neighboring-influence operator with spatial attenuation parameters λ is established to describe the modeled influence of a source excavation on neighboring target indices to adjacent target excavations. These parameters enable the transformation from individual response indicators to an inherent excavation condition index and subsequently to a composite condition index.

The third component focuses on the parameterization of the proposed framework for practical implementation. Since the proposed framework is designed as an extensible

structure rather than a fixed parameter system, the values of ω , β , and λ should be determined according to project-specific information. In this study, indicator weights are determined based on expert-based weighting, nonlinear interaction coefficients are obtained from representative numerical scenarios, and spatial attenuation parameters are derived from case-history-informed influence characteristics. Due to limited site-specific information, the transfer operator is implemented using excavation spacing as the dominant factor, while other influencing factors, including geological variability, groundwater connectivity, and construction-stage interaction, are retained as potential extensions.

The fourth component demonstrates the application workflow of the proposed framework through an illustrative twin-excavation case. The calculated isolated-reference condition indices of individual excavations is first evaluated considering indicator contributions and nonlinear interactions. Subsequently, neighboring-excavation contributions from neighboring excavations are incorporated to obtain the composite condition index. Sensitivity analyses are further conducted to investigate the influence of the three updateable parameter groups and to discuss the applicability and limitations of the proposed framework.

2.2. Response-Based Indicator System and Indicator Scoring

In this study, the response-based assessment indicators are organized into three physics-informed domains according to their dominant response mechanisms: deformation response, structural resistance, and hydrological response. The deformation response characterizes stiffness degradation in the coupled soil-structure system. It integrates retaining-structure deformation and ground settlement into a unified deformation-response domain, thereby capturing the coordinated deformation of the excavation and its surrounding ground. The structural resistance characterizes the degradation of structural resistance and the potential insufficiency of support components. The hydrological response characterizes the seepage-induced instability associated with the groundwater flow.

In practical applications, the active assessment indicators should be jointly selected and reviewed by a multidisciplinary project team, typically including geotechnical engineers, excavation and support designers, construction engineers or site managers, and monitoring personnel. Their involvement ensures that the selected indicators reflect geological conditions, structural design requirements, construction-stage characteristics, and the availability and reliability of monitoring information.

These indicators in the three physics-informed responses are summarized based on design-code limits, monitoring-control criteria, published excavation performance records, and conservative engineering judgment. Table 1 lists the indicators with mathematical formulations and physical parameters for the evaluation and description.

Table 1. Assessment indicator system.

Category	Evaluation Indicator	Description of Parameters
Deformation response subdomain	O_1 Maximum horizontal displacement of retaining wall $\varepsilon_h = \frac{\delta_h}{H}, H < 10 \text{ m}; \quad \varepsilon_h = \delta_h, H \geq 10 \text{ m}$	δ_h Maximum absolute cumulative X-direction displacement over the full height and length of the selected retaining-wall face; H excavation design depth
	O_2 Vertical displacement at the top of retaining wall: $\varepsilon_v = \frac{\delta_v}{H}, H < 10 \text{ m}; \quad \varepsilon_v = \delta_v, H \geq 10 \text{ m}$	δ_v Measured maximum vertical displacement; H excavation design depth
	O_3 Relative displacement of deep soil: $\varepsilon_{soil} = \frac{\delta_{soil}}{H}, H < 10 \text{ m}; \quad \varepsilon_{soil} = \delta_{soil}, H \geq 10 \text{ m};$	δ_{soil} Measured maximum deep soil displacement; H excavation design depth
	O_4 Relative basal heave: $\varepsilon_{heave} = \delta_{heave}$	ε_{heave} Measured basal heave

Structural resistance domain	O ₅ Relative settlement of adjacent soil: $\varepsilon_{set} = \frac{\delta_{set}}{H} \cdot k$	k: Environmental distance sensitivity coefficient; H excavation design depth
	O ₆ Relative tilt rate of retaining structure: $\varepsilon_{tilt} = \Delta\theta$	$\Delta\theta$: Relative rotation angle between top and bottom
	O ₇ Relative structural displacement rate: $\varepsilon_{rate} = \frac{v_{cur}}{C_{rate}}$	v_{cur} Current displacement rate, C_{rate} Alarm rate threshold
	P ₁ Strut axial force utilization ratio: $\eta_{strut} = \frac{ N_{strut} }{N_{strut,d}}$	N_{strut} measured or calculated axial force of the strut; $N_{strut,d}$ design control value or design resistance of the strut axial force
	P ₂ Anchor tension utilization ratio: $\eta_{anchor} = \frac{ T_{anchor} }{T_{anchor,d}}$	T_{anchor} measured or calculated anchor tension; $T_{anchor,d}$ design control value of the anchor tension
	P ₃ Strut–force imbalance coefficient of the strut system: $\eta_{diff} = \frac{2 N_i - N_j }{N_i + N_j}$	N_i, N_j axial force utilization ratios of two adjacent struts
	P ₄ Structural bending moment utilization ratio: $\eta_{moment} = \frac{ M_{max} }{M_d}$	M_{max} maximum bending moment of the retaining structure; M_d design bending resistance of the retaining structure
	P ₅ Overall stability safety reserve consumption coefficient: $\eta_{stab} = \frac{K_{req} S_{over}}{R_{resist}}$	S_{over} overturning effect, R_{resist} resisting effect against overturning or sliding, K_{req} required safety factor specified by the code
	P ₆ Local structural damage ratio: $\eta_{dam} = \frac{L_{dam}}{L_{span}}$	L_{dam} length affected by local damage, L_{span} span or effective length of the member
	Q ₁ Relative change rate of groundwater level: $\mu_{draw} = \frac{V_{draw}}{C_{limit}}$	V_{draw} Water level drawdown rate, C_{limit} Specification limit
Hydrological response subdomain	Q ₂ Critical ratio of seepage flow: $\mu_{leak} = \frac{Q_{seep}}{C_{piping}}$	Q_{seep} Measured seepage, C_{piping} Critical piping flow
	Q ₃ Basal burst stability: $\mu_{burst} = \frac{P_{water}}{C_{weight}}$	P_{water} Confined water pressure, C_{weight} Overburden soil weight

For O_1 and O_2 , the indicator input is the relative displacement δ/H when $H < 10$ m and absolute displacement δ in millimeters for $H \geq 10$ m, where H denotes the design excavation depth. The 10 m reference depth is adopted with reference to the excavation-depth classification in Clause 3.3.2 and Table 3.3.2 of GB 50911–2013 [32]; its use as the switching depth is a modeling choice in this study. Relative thresholds are obtained by dividing the corresponding absolute thresholds by 10,000 mm and are expressed as percentages in Table 8, ensuring consistent classifications and scores at $H = 10$ m. These depth-dependent inputs are subsequently mapped onto the dimensionless 0–100 score scale.

2.3. Safety-Condition Assessment of a Single Excavation with Nonlinear Indicator Interaction

For excavation k and assessment unit m , indicator scores $x_i^{k,m}$ are combined on the 0–100 scale. Equation (1) defines the baseline weighted condition index $E_{k,m}^0$ as the weighted sum.

$$E_{k,m}^0 = \sum_{i=1}^n \omega_i x_i^{k,m} \quad (1)$$

Here, $x_i^{k,m}$ is the score of active indicator i , ω_i is the normalized weight, and n is the total number of active indicators.

A linear weighted aggregation does not represent non-additive contributions from interactions between indicators. For example, support–force imbalance may induce local stiffness degradation, which can further amplify retaining-wall deformation and adjacent ground settlement. The additional score contribution assigned to the interaction of indicators i and j is defined as $\Delta E_{ij}^{k,m}$ and calculated by Equation (2).

$$\Delta E_{ij}^{k,m} = \max\left(0, \beta_{ij}^k \cdot \sqrt{\omega_i \omega_j} \cdot \frac{x_i^{k,m} x_j^{k,m}}{100}\right) \quad (2)$$

The parameter β_{ij}^k is an interaction parameter between indicators i and j . A positive value of β_{ij}^k indicates super-additivity in the response measure used in Equation (7); values near zero indicate approximate additivity, and negative values indicate sub-additivity. In Equation (2), negative coefficients contribute zero and do not reduce the baseline weighted score. This conservative scoring convention does not imply that the corresponding mechanical response is below the baseline or that the calculated index is an upper bound on actual engineering risk.

In this framework, β_{ij}^k should be interpreted as an updateable interaction parameter rather than a universal constant. Its value can be parameterized using project-specific monitoring data, numerical simulation results, engineering judgment, or case-history information. The present study demonstrates this parameterization only for selected deformation-structure interaction scenarios in the illustrative implementation described in Section 3.

The term $\sqrt{\omega_i \omega_j}$ represents the symmetric joint importance of the paired indicators, while $x_i^{k,m} x_j^{k,m}$ represents their joint score activation in the adopted semi-empirical model. Under an assumed bilinear interaction form, requiring the interaction contribution to vanish whenever either score is zero eliminates the constant and single-score terms, leaving only their product. Dividing each score by 100, multiplying the normalized scores, and rescaling the product by 100 yields the score-product factor in Equation (2).

Equation (3) adds the contributions for pairs in the active interaction set C_β to the baseline index $E_{k,m}^0$ and caps the result at 100.

$$E_{k,m} = \min\left(100, E_{k,m}^0 + \sum_{(i,j) \in C_\beta} \Delta E_{ij}^{k,m}\right) \quad (3)$$

In Equation (3), m denotes an assessment unit, defined as a support level in the present case study. The scoring inputs O_1 and O_2 , and $P_3(m)$ are obtained from the same reference assessment state before additional structural deactivation or stiffness reduction. O_1 and O_2 are shared wall-level inputs, whereas $P_3(m)$ characterizes the force imbalance of the support pair at level m . The interaction coefficients are estimated separately from the prescribed numerical operations in Section 3.3. The resulting score represents a support-level assessment of the reference state under the adopted interaction parameterization.

$$E_k = \max_{m \in M}(E_{k,m}) \quad (4)$$

Equation (4) selects the maximum support-level assessment score as the controlling score for the assessed scenario. The corresponding support level is identified as the controlling level under the adopted assessment rules. Scores are not summed across support levels.

Level V covers indicator scores in $[80, 100]$, as determined by the continuous function in Equation (6). Entry into Level V does not automatically set the composite index to 100; aggregation and capping follow Equations (1)–(4).

2.4. Safety-Condition Assessment of a Target Excavation Considering Neighboring Influence

Equation (5), combines the target excavation's inherent index E_k with contributions $E_{k'} \cdot \Phi_{(k' \rightarrow k)}$ from neighboring source excavations k' . The operator $\Phi_{(k' \rightarrow k)}$ provides an updateable representation of neighboring influence; the source and target indices are calculated using Section 2.3.

$$R_{cluster}^{(k)} = \min \left(100, E_k + \sum_{k' \neq k} E_{k'} \cdot \Phi_{(k' \rightarrow k)} \right) \quad (5)$$

In Equation (5), the upper limit of 100 is retained to ensure consistency with the 0–100 condition-score scale defined in Section 2.2. The distance-weighted source contribution and its additive incorporation into the target score are index-aggregation assumptions for scenario comparison, rather than a mechanically derived or independently validated transfer law. The inherent indices of both the source and target excavations are defined using isolated-reference responses that exclude neighboring-excavation influence. This influence is introduced only through the transfer term. Coupled twin-excavation responses are not used as the inherent-index inputs to this additive formulation, thereby avoiding double counting.

3. Parameterization of the Proposed Framework

The framework is parameterized through indicator weights ω , interaction coefficients β , and the spatial attenuation coefficient λ .

The numerical prototype supplies responses for estimating β and is compared with the numerical results of Zheng et al. [31]. Weights are determined by expert-based AHP, β by comparing paired and single-operation response increments, and λ by reference-distance and residual-weight assumptions informed by published soft-clay excavation cases.

3.1. Numerical Prototype Development

A representative soft-clay deep excavation, based on the Nicoll Highway Circle Line C824 excavation case in Singapore, was selected as the prototype for the numerical analysis. The maximum design excavation depth was 33.5 m. The benchmark analysis entered the excavation phase with a prescribed depth of 33.5 m. The adopted analysis could not continue tracking the large deformation associated with excavation collapse, and the calculation was terminated without reaching equilibrium in that phase. The recorded terminal response was used for comparison with the collapse-related stopped-run response of Zheng et al. [31]. The interaction-parameter analyses instead used the equilibrated state at an excavation depth of 29.5 m as their reference. The retaining system consisted of diaphragm walls extending to a depth of 44.5 m and nine levels of internal steel struts.

A three-dimensional model was established using PLAXIS 3D to represent the excavation geometry, soil stratification, and retaining system. The computational domain measured 90 m $\times$ 8 m $\times$ 73 m in the transverse (x), longitudinal (y), and vertical (z) directions, respectively, representing an 8 m long longitudinal segment. As shown in Figure 2a, the transverse domain comprised an 80 m wide retained-soil region extending from the diaphragm wall to the external lateral boundary and a 10 m wide excavation-side region extending from the wall to the opposite lateral boundary. The reference excavation depth for interaction-parameter estimation was 29.5 m, and the diaphragm wall extended to a depth of 44.5 m. The model geometry, soil profile, and support layout are shown in Figure 2. In Figure 2b, H-350, H-400, and H-414 denote H-shaped steel strut section types, and 2 $\times$ H-400 denotes two H-400 steel struts at the same support level.

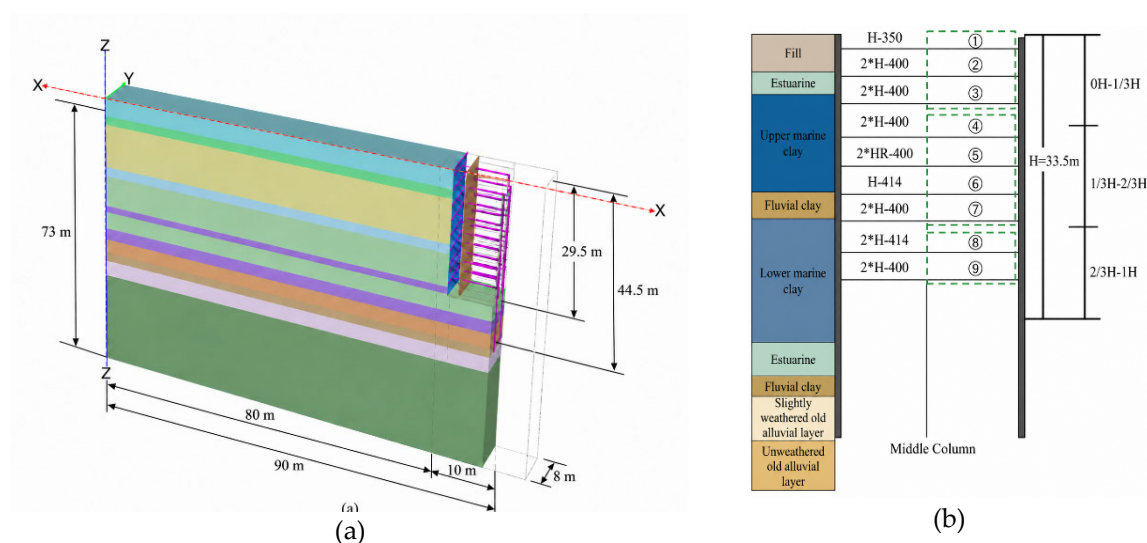


Figure 2. Numerical prototype used for interaction-parameter estimation: (a) three-dimensional numerical model of the excavation system; (b) stratigraphic profile and support layout of the prototype excavation.

The baseline mesh contained 5004 elements and 9824 nodes. Horizontal displacement was restrained at the lateral boundaries, the bottom boundary was fixed, and the top surface was free. Before excavation, initial geostatic stresses were generated using the k_0 procedure, with $(k_0 = 1 - \sin \varphi')$, where (φ') is the effective friction angle of the corresponding soil layer listed in Table 2. No separate soil-structure interface elements were introduced, and perfect bonding was assumed between the soil and diaphragm wall. Excavation and strut installation were simulated sequentially under undrained conditions. Groundwater effects were not considered. Deterioration scenarios were applied after the target excavation stage reached equilibrium, following the procedures described in Section 3.3.

Table 2. Soil parameters for each layer.

Soil Layer	Depth (m)	γ (kN m ⁻³)	φ' (°)	c' (kPa)	E_{50}^{ref} (MPa)	E_{oed}^{ref} (MPa)	E_{ur}^{ref} (MPa)	G_0^{ref} (MPa)	ν_{ur}
Fill	−4.5	19	30	0	10.0	6.7	50.0	186.8	0.26
Estuarine	−6.5	15	18	0	6.0	4.0	30.0	97.6	0.3
Upper marine clay	−17.0	16	22	0	8.0	5.3	40.0	153.2	0.28
Fluvial clay	−19.5	19	22	0	8.0	5.3	40.0	130.9	0.28
Lower marine clay	−34.5	16	24	0	16.0	10.7	80.0	285.6	0.28
Estuarine	−39.5	15	18	0	25.0	16.7	125.0	423.1	0.28
Fluvial clay	−41.5	19	22	0	10.4	6.9	51.7	192.3	0.25
Slightly weathered old alluvial layer	−45.5	20	32	5	46.5	31.0	232.5	807.55	0.25
Unweathered old alluvial layer	−73.0	20	33	5	61.0	40.7	305.0	1079.95	0.30

The soil layers were simulated using the Hardening Soil model with small-strain stiffness, except for the jet-grouting pile reinforcement layer, which was represented by a linear elastic model. The small-strain reference shear modulus and unloading-reloading Poisson's ratios, ν_{ur} , are listed in Table 2. A reference pressure of $p^{ref} = 100$ kPa was adopted for all HSsmall soil layers. The characteristic shear strain, $\gamma_{0.7}$, was 1.0×10^{-4} for all HSsmall

soil layers. These values are model-specific inputs rather than universal values for Singapore soils. The diaphragm wall and steel struts were modeled as linear elastic structural elements. Excavation was simulated in stages following the prescribed construction sequence, with undrained conditions adopted to represent the short-term response of soft clay. The soil and structural parameters used in the numerical model are summarized in Tables 2 and Table 3, respectively.

Table 3. Physical and mechanical structural parameters.

Diaphragm Wall		Strut: Q235 Steel, $E_{eff} = 120 \text{ GPa}$, $\nu = 0.2$			
E (kPa)	3.25×10^7	Strut Type	H-350	H-400	H-414
ν	0.2	$I_x \text{ (cm}^4\text{)}$	39 800	66 600	92 800
Thickness (m)	0.8	$I_y \text{ (cm}^4\text{)}$	13 600	22 400	31 000
$\gamma \text{ (kN m}^{-3}\text{)}$	25	$A \text{ (cm}^2\text{)}$	171.9	218.7	295.4

Following Zheng et al. [33], the baseline steel-strut modulus was 120 GPa, or 60% of the reference value of 200 GPa. This baseline effective-stiffness assumption is distinct from the subsequent deactivation of an individual strut used to represent complete loss of its load-carrying function in the interaction-parameter estimation scenarios. Figure 3 illustrates the numerical displacement and bending-moment distributions. Table 4 compares the recorded benchmark wall responses with the numerical results of Zheng et al. [33]. The displacement columns identify observation depths along the wall, not excavation depths. The 83.86 mm value is the horizontal displacement at a wall observation depth of 29.5 m in the last output of the non-converged 33.5 m excavation phase. The reference numerical analysis of Zheng et al. [33] likewise provides a stopped-run response associated with excavation collapse. The comparison therefore concerns recorded instability responses at calculation termination, rather than converged equilibrium responses at the full prescribed excavation depth. The 83.86 mm terminal value is not the wall-face maximum of the equilibrated 29.5 m reference state used in Section 3.3. The comparison shows a deformation peak in the middle-to-lower wall and alternating positive and negative bending-moment zones.

Table 4. Comparison of recorded wall responses associated with excavation instability with the numerical results of Zheng et al. [33].

	Maximum Moment ($\text{N} \cdot \text{m} \cdot \text{m}^{-1}$)		Horizontal Displacements at Wall Observation Depths (mm)		
	Positive	Negative	8.5 m	22.5 m	29.5 m
Proposed model	1.72×10^6	-3.78×10^6	18.59	55.24	83.86
Zheng's model (Zheng et al. [33])	2.29×10^6	-3.45×10^6	20	60	90
Relative difference	24.89%	9.57%	7.05%	7.93%	6.82%

Note: Displacement columns refer to observation depths along the wall. The 83.86 mm value is from the last output of the non-converged 33.5 m excavation phase. The comparison with the collapse-related terminal response of Zheng et al. [33] describes recorded instability responses; the relative differences do not establish equilibrium-response accuracy.

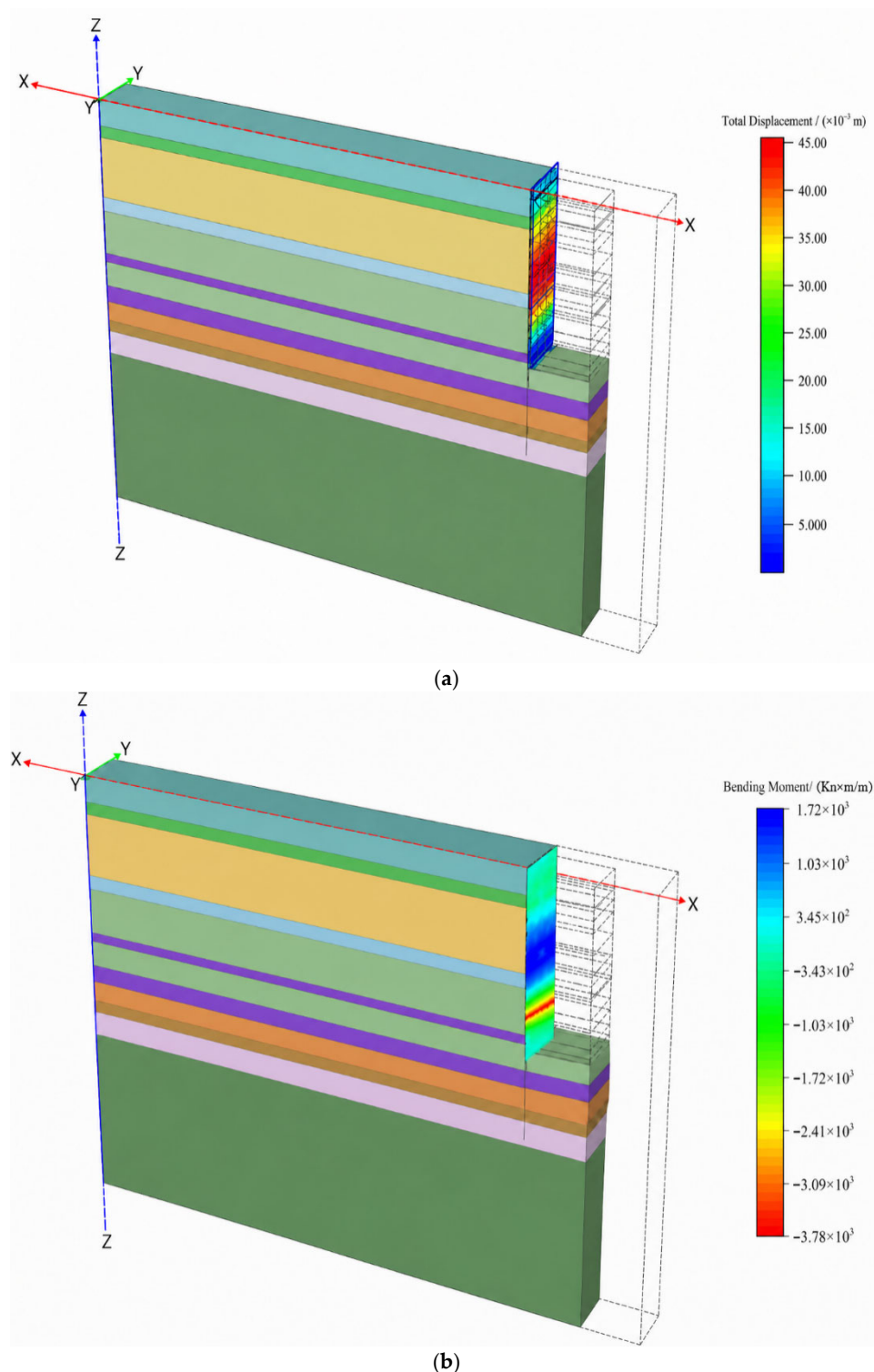


Figure 3. Numerical responses of the prototype excavation: (a) total displacement distribution of the excavation system; (b) bending moment distribution of the diaphragm wall.

The maximum positive bending moments predicted by the present model and the model of Zheng et al. [33] were 1720 and 2290 kN·m/m, respectively, giving a relative difference of 24.89%. This difference concerns a local internal-force response and is not directly equivalent to an error of the same magnitude in the subsequent assessment results, because the maximum positive bending moment is not a direct input to the three active indicators or the displacement-increment calculation of the interaction coefficient. The numerical results are interpreted within the scope of the selected model and scenarios.

3.2. Expert-Based Determination of Indicator Weights

The framework uses expert-derived weights to express relative importance within the deformation (*O*), structural resistance (*P*), and hydrological (*Q*) subdomains. Pairwise comparisons determine within-subdomain weights, while a separate point-allocation question determines subdomain importance.

Individual consistency was checked before aggregation using $CI = (\lambda_{max} - n)/(n - 1)$ and $CR = CI/RI$. The *RI* values were 1.32, 1.26, and 0.58 for matrix orders of seven, six, and three, respectively. A response set was retained for AHP aggregation only if all three matrices satisfied $CR < 0.1$. Fifteen of the 20 sets met this criterion. Corresponding judgments from these 15 sets were combined using an equally weighted geometric mean. Local weights were obtained from the normalized principal right eigenvector of each aggregate matrix, and consistency was checked again. The aggregate *CR* values were 0.010882, 0.010256, and 0.000093 for *O*, *P*, and *Q*. Between-expert variability was assessed using the sample standard deviation of individual local weights across all 20 respondents. Comparing all-20 and retained-15 aggregates showed maximum absolute local-weight changes of 0.003936, 0.007208, and 0.003306, respectively, with unchanged rankings within each subdomain.

The initial aggregates, expert variability, and screening results were presented to the expert panel for discussion. All 20 valid point allocations were averaged and divided by 100, giving initial subdomain weights of 0.4555, 0.3485, and 0.1960. This module was processed separately from the AHP comparisons. Following review, the panel finalized the within-subdomain matrices and agreed on subdomain weights of 0.45, 0.35, and 0.20 for *O*, *P*, and *Q*.

Local weights were recalculated from the final matrices, all of which satisfied $CR < 0.1$. The global weight W_{dj} was calculated as $W_{dj} = \alpha_d \cdot w_{dj}$, where w_{dj} is the local weight of indicator *j* within subdomain *d* and α_d is the corresponding subdomain weight. Local weights sum to one within each subdomain. Global weights sum to 0.45 within *O*, 0.35 within *P*, and 0.20 within *Q*, and to one across the complete hierarchy before display rounding. Tables 5–7 distinguish local and global weights and report the final matrices and consistency results.

Table 5. Weights of assessment indicators in the displacement and deformation subdomain.

Indicator	<i>O</i> ₁	<i>O</i> ₂	<i>O</i> ₃	<i>O</i> ₄	<i>O</i> ₅	<i>O</i> ₆	<i>O</i> ₇	<i>LW</i> _{<i>Oj</i>}	<i>W</i> _{<i>Oj</i>}
<i>O</i> ₁	1	3	2	2	1	9	4	0.2763	0.1243
<i>O</i> ₂	1/3	1	1/2	1	1/2	3	2	0.1057	0.0476
<i>O</i> ₃	1/2	2	1	1	1	5	2	0.1641	0.0738
<i>O</i> ₄	1/2	1	1	1	1/2	4	2	0.1284	0.0578
<i>O</i> ₅	1	2	1	2	1	7	4	0.2290	0.1031
<i>O</i> ₆	1/9	1/3	1/5	1/4	1/7	1	1/2	0.0322	0.0145
<i>O</i> ₇	1/4	1/2	1/2	1/2	1/4	2	1	0.0642	0.0289
$\lambda_{max} = 7.073, CI = 0.012, RI = 1.32, CR = 0.009 < 0.1$. Meet consistency requirements.									

Table 6. Weights of assessment indicators in the structural resistance subdomain.

Indicator	<i>P</i> ₁	<i>P</i> ₂	<i>P</i> ₃	<i>P</i> ₄	<i>P</i> ₅	<i>P</i> ₆	<i>LW</i> _{<i>Pj</i>}	<i>W</i> _{<i>Pj</i>}
<i>P</i> ₁	1	1	2	1	1	4	0.2154	0.0754
<i>P</i> ₂	1	1	1	1	1/2	2	0.1551	0.0543
<i>P</i> ₃	1/2	1	1	1/2	1/2	2	0.1205	0.0422
<i>P</i> ₄	1	1	2	1	1	3	0.2051	0.0718
<i>P</i> ₅	1	2	2	1	1	4	0.2409	0.0843
<i>P</i> ₆	1/4	1/2	1/2	1/3	1/4	1	0.0630	0.0221

$\lambda_{max} = 6.079$, $CI = 0.016$, $RI = 1.26$, $CR = 0.013 < 0.1$. Meet consistency requirements.

Table 7. Weights of assessment indicators in the hydrological response subdomain.

Indicator	Q_1	Q_2	Q_3	LW_{Q_j}	W_{Q_j}
Q_1	1	5	3	0.6483	0.1297
Q_2	1/5	1	1/2	0.1220	0.0244
Q_3	1/3	2	1	0.2297	0.0459

$\lambda_{max} = 3.003$, $CI = 0.0019$, $RI = 0.58$, $CR = 0.003 < 0.1$. Meet consistency requirements.

For the illustrative assessment, the active set was $S = \{O_1, O_2, P_3\}$. Their global weights sum to 0.214081 before display rounding. Each selected global weight was divided by this sum, giving normalized weights of 0.58, 0.22, and 0.20 for O_1 , O_2 , and P_3 , respectively. The same indicator set, normalized weights, and indicator-to-score mappings were used across all compared scenarios, with the weights applied consistently to both direct and interaction contributions.

Table 8 summarizes the classification criteria and corresponding 0–100 condition-score ranges for the three active indicators: strut axial-force imbalance (P_3), maximum horizontal displacement of the retaining wall (O_1), and wall vertical displacement (O_2). The depth branches and units are consistent with Table 1. The five score bands have equal widths of 20 points, although their physical input intervals differ. Displacement thresholds are informed by project monitoring-control criteria, while the force-imbalance thresholds are the adopted modeling assumptions for the illustrative assessment.

Table 8. Classification criteria and condition-score ranges for active indicators P_3 , O_1 , and O_2 .

Score Interval (Level)	P_3	O_1 Maximum Horizontal Displacement of Retaining Wall		O_2 Vertical Displacement at Wall	
	Strut Axial-Force Imbalance	$H \geq 10$ m	$H < 10$ m	$H \geq 10$ m	$H < 10$ m
	η_{diff}	δ_h (mm)	δ_h/H	δ_v (mm)	δ_v/H
[0, 20) (I)	[0, 0.20)	[0, 21)	[0, 0.210%)	[0, 7)	[0, 0.070%)
[20, 40) (II)	[0.20, 0.30)	[21, 25.5)	[0.210%, 0.255%)	[7, 8.5)	[0.070%, 0.085%)
[40, 60) (III)	[0.30, 0.40)	[25.5, 30)	[0.255%, 0.300%)	[8.5, 10)	[0.085%, 0.100%)
[60, 80) (IV)	[0.40, 0.60)	[30, 50)	[0.300%, 0.500%)	[10, 25)	[0.100%, 0.250%)
[80, 100] (V)	[0.60, $+\infty$)	[50, $+\infty$)	[0.500%, $+\infty$)	[25, $+\infty$)	[0.250%, $+\infty$)

The indicator values in Table 8 are mapped onto a dimensionless 0–100 condition score using the following continuous piecewise-linear function in Equation (6). Linear interpolation is applied between adjacent classification thresholds. Within Level V, the scoring slope of Level IV is retained until the score reaches the upper limit of 100.

$$s(x) = \begin{cases} 0, & x < 0, \\ 20k + 20 \frac{x - x_k}{x_{k+1} - x_k}, & x_k \leq x < x_{k+1}, \quad k = 0, 1, 2, 3, \\ \min \left\{ 100, 80 + 20 \frac{x - x_4}{x_4 - x_3} \right\}, & x \geq x_4. \end{cases} \quad (6)$$

Here, x is the indicator input and $s(x)$ is the dimensionless condition score. The lower anchor is $x_0 = 0$, and x_1 through x_4 are the increasing entry thresholds of Levels II through V listed in Table 8, expressed in the same units as the applicable input. The score is zero at $x = 0$. The score intervals are [0, 20), [20, 40), [40, 60), [60, 80), and [80, 100], with internal boundaries assigned to the higher level. Within Level V, the score increases from 80 at x_4

by retaining the Level IV slope, reaches 100 at $x = 2x_4 - x_3$, and remains capped thereafter. This continuous extension and saturation rule is a modeling assumption.

3.3. Scenario-Based Estimation of the Indicator Interaction Parameter

The interaction parameter β was estimated from the numerical prototype by comparing the displacement increment under paired degradation with the sum of the corresponding single-operation increments.

The estimation was restricted to the active interaction set C_β defined in Section 2.3, involving strut–force imbalance (P_3), maximum horizontal displacement of the retaining wall (O_1), and relative vertical displacement at the wall top (O_2). These are assessment indicators, whereas the model inputs are the structural degradation operations specified in Table 9. Each operation may affect multiple indicators. The notation $P_3(m)$ identifies the support level m associated with strut degradation; it does not denote a prescribed force–imbalance value. The strut–column and strut–wall combinations are associated with the $P_3(m)$ – O_1 and $P_3(m)$ – O_2 assessment terms, respectively, while the column–wall combination is associated with O_1 – O_2 . These associations constitute the scenario-to-indicator mapping adopted for this demonstration.

Table 9. Numerical degradation scenarios used to estimate β_{ij} .

Scenario	Model Operation and Degradation Magnitude	Affected Component or Region
Baseline	Follow the prescribed construction sequence without additional degradation; structural properties are given in Table 3.	Entire model
Strut degradation associated with $P_3(m)$	Deactivate one strut at level m to represent complete loss of its load-carrying function. The companion strut remains active with unchanged material and sectional properties.	Affected struts at level m
Column deactivation associated with O_1	Deactivate all elements along the full length of one specified central column.	Central column
Wall stiffness reduction associated with O_2	Reduce E from 3.25×10^7 kN/m ² to 5×10^3 kN/m ² , with the cross-sectional geometry unchanged.	Wall region
Strut–column $P_3(m)$ – O_1	Simultaneously apply the individual-strut deactivation and central-column deactivation, using the same components as in the corresponding single-operation scenarios.	Same affected struts and column
Strut–wall $P_3(m)$ – O_2	Simultaneously apply the specified individual-strut deactivation and wall stiffness reduction, using the same components and wall-stiffness reduction as in the corresponding single-operation scenarios.	Same affected struts and wall region
Column–wall O_1 – O_2	Simultaneously apply column deactivation and wall stiffness reduction, using the same locations and magnitudes as in the single-operation scenarios.	Same column and wall region

Separate individual-strut deactivation scenarios were analyzed for $m = 2, 3, 4, 5, 7, 8$, and 9. At each investigated level, one specified strut was deactivated to represent complete loss of its load-carrying function, while the companion strut remained active. Each scenario started independently from the same equilibrated baseline at an excavation depth of 29.5 m, with all nine support levels installed; strut deactivations were not accumulated across levels.

The strut–column and strut–wall combinations retain the P - m 01 and P - m 02 identifiers in Table 10, respectively. Each paired scenario applies the same individual-strut

deactivation as its constituent single-operation scenario, together with the prescribed column deactivation or wall stiffness reduction. The computational domain and boundary conditions remain unchanged, and responses are extracted after equilibrium is reached following the prescribed operations.

Table 10. Response-increment-based estimation of representative interaction parameters.

Scenario	U_0 (mm)	U_i (mm)	U_j (mm)	$U_{i,j}$ (mm)	ΔU_j (mm)	ΔU_i (mm)	$\Delta U_{i,j}$ (mm)	$\beta_{i,j}$	Interpretation
P-201	32.75	33.73	34.42	33.2	1.67	0.98	0.45	−0.83	Sub-additive
P-301		32.96	34.42	33.25	1.67	0.21	0.5	−0.73	Sub-additive
P-401		33.2	34.42	34.91	1.67	0.45	2.16	0.02	Super-additive
P-501		33.19	34.42	38.31	1.67	0.44	5.56	1.64	Super-additive
P-701		33.34	34.42	37.6	1.67	0.59	4.85	1.15	Super-additive
P-801		32.97	34.42	36.49	1.67	0.22	3.74	0.98	Super-additive
P-901		33.69	34.42	34.16	1.67	0.94	1.41	−0.46	Sub-additive
O1-O2		40.02	34.42	40.09	1.67	7.27	7.34	−0.18	Sub-additive
P-202		33.73	40.02	40.03	7.27	0.98	7.28	−0.12	Sub-additive
P-302		32.96	40.02	39.83	7.27	0.21	7.08	−0.05	Sub-additive
P-402		33.2	40.02	40.79	7.27	0.45	8.04	0.04	Super-additive
P-502		33.19	40.02	41.18	7.27	0.44	8.43	0.09	Super-additive
P-702		33.34	40.02	43.94	7.27	0.59	11.19	0.42	Super-additive
P-802		32.97	40.02	40.95	7.27	0.22	8.2	0.09	Super-additive
P-902		33.69	40.02	40.1	7.27	0.94	7.35	−0.10	Sub-additive

Note: $U_0 = 32.75$ mm is the maximum absolute cumulative X-direction displacement over the full height and length of the selected wall face in the equilibrated 29.5 m excavation reference state, with all nine strut levels installed. All single-operation and paired-operation responses use the same wall face and cumulative displacement reference, without resetting displacements. The 83.86 mm non-converged benchmark output in Table 4 is not used in these increment or β calculations.

All interaction-parameter analyses followed the same construction sequence to 29.5 m excavation depth, with all nine support levels installed. Each degradation scenario started independently from the same equilibrated baseline and retained the domain and boundary conditions in Section 3.1. Responses were extracted after equilibrium in the degradation phase.

The response U is defined as the maximum absolute cumulative horizontal displacement in the transverse X direction over all nodes on the selected retaining-wall face, covering its full height and length. This spatial extraction definition also applies to indicator O_1 . For the equilibrated 29.5 m reference state, $U_0 = 32.75$ mm. The 83.86 mm value in Table 4 is instead the displacement at a wall observation depth of 29.5 m in the last output of the non-converged 33.5 m benchmark excavation phase. It is a terminal instability response, whereas $U_0 = 32.75$ mm is an equilibrated reference response. The two values therefore belong to different calculation states and displacement fields; the terminal benchmark value is not used to calculate β . The same wall face, displacement component, and accumulation reference are used for all baseline, single-operation, and paired-operation analyses. Displacements are not reset at the start of the additional-operation phase. Responses are extracted after equilibrium is reached in each scenario.

The baseline, two single-operation, and paired-operation responses were denoted as U_0 , U_i , U_j , and $U_{i,j}$, respectively. Their increments were calculated as $\Delta U_i = U_i - U_0$, $\Delta U_j = U_j - U_0$, and $\Delta U_{i,j} = U_{i,j} - U_0$. Peak locations may differ between analyses; these increments therefore represent differences between wall-domain peaks, rather than displacement

changes at a common node or the maximum of a node-by-node displacement-difference field.

For a positive sum of the single-operation response increments that is not close to zero, the interaction parameter was calculated as

$$\beta_{ij} = \frac{\Delta U_{ij} - (\Delta U_i + \Delta U_j)}{\Delta U_i + \Delta U_j} \quad (7)$$

Positive, zero, and negative β indicate paired increments above, equal to, and below the additive reference, respectively. The resulting β is used as the multiplier in Equation (2) under the adopted indicator mapping assumption.

Table 10 reports the response increments and β values. For example, $\beta \approx 1.64$ means that the paired increment is approximately 2.64 times the sum of the single-operation increments. Negative coefficients are reported but contribute zero in Equation (2), as specified in Section 2.3.

3.4. Distance–Decay Formulation of the Neighboring-Excavation Contribution

The illustrative implementation considers excavations with identical geometry, support systems, and soil stratigraphy. It evaluates the one-way contribution from a designated source to a target, with normalized spacing as the explicit variable.

The exponential distance weight is defined by Equation (8):

$$\Phi_{(k' \rightarrow k)} = \exp(-\lambda \cdot \frac{d_{k'k}}{H_{avg}}) \quad (8)$$

Here, λ is the dimensionless attenuation coefficient describing the assumed distance-dependent decay in the neighboring-excavation contribution. The parameter $d_{k'k}$ is the spacing between the source and the target excavations, and H_{avg} is the average excavation depth of the two excavations.

The reference distance is $d_{ref} = \alpha H_{avg}$. Table 11 summarizes the reported normalized ground-movement influence distances L_{max}/H used to inform the illustrative choice $\alpha = 3.13$, where H is the depth of each reported excavation.

Table 11. Representative case-history data for excavation-induced ground-movement influence ranges.

Author and Publication Year	Excavation Depth	Maximum Influence Distance	Relative Influence Radius α
	H (m)	L_{max} (m)	
Wang et al. [34]	H	1.5H–3.5H	1.5–3.5
Ou et al. [35]	19.7	60.0	3.05
Li et al. [36]	19.7	61.4	3.10
Wang et al. [37]	11.0	≥ 33.0	≥ 3.0
Luca & Russo [38]	14.0	44.1	3.15
Ying et al. [39]	H	4.0H	4.00
Adopted reference scale α	-	-	3.13

Representing the interval 1.5–3.5 by its midpoint 2.5 and the lower bound ≥ 3.0 by 3.0 gives $[2.5 + 60/19.7 + 61.4/19.7 + 3.0 + 44.1/14 + 4.0]/6 \approx 3.13$. The baseline $\alpha = 3.13$ is retained as an adopted approximate reference scale.

The residual distance weight ε satisfies $\exp(-\lambda\alpha) = \varepsilon$, giving $\lambda = -\ln(\varepsilon)/\alpha$. With $\alpha = 3.13$ and $\varepsilon = 0.01$, $\lambda \approx 1.47$. Equation (8) can therefore be written as Equation (9). No hard cutoff is imposed: the weight continues to decrease smoothly beyond d_{ref} . Section 4.4.3 also examines $\varepsilon = 0.025$ and 0.05 at fixed α .

$$\Phi_{(k' \rightarrow k)} = \varepsilon^{d_{k'k}/(\alpha H_{avg})}, d_{k'k} \geq 0 \quad (9)$$

The distance weight in Equation (9) is substituted into Equation (5), together with the isolated-reference source and target indices, to calculate the final group-influenced safety-condition index.

4. Application to Twin Excavations and Result Analysis

This section applies the framework to a twin-excavation configuration, compares baseline and interaction-adjusted indices, and evaluates neighboring contributions and parameter sensitivity.

4.1. Twin-Excavation Model and Configuration

To demonstrate the proposed framework under adjacent excavation conditions, as shown in Figure 4, two adjacent excavations, denoted as Excavation A and B, were considered in the illustrative analysis. Both excavations adopt the geometry and support system described in Section 3.1. Each excavation has a width of 20 m and a design depth of 33.5 m, and is supported by diaphragm walls and nine levels of internal steel struts. Jet-grouted reinforcement layers were arranged in the lower part of the excavation to improve basal stability. The indicator inputs for the inherent indices of A and B were obtained from their respective isolated-excavation reference analyses, with neighboring-excavation influence excluded. The twin-excavation configuration defines the assessment scenario; its coupled responses are not used as the inherent-index inputs to the additive transfer formulation. Figure 4 defines the spatial configuration considered in the assessment; neighboring influence is introduced at the assessment stage through the transfer operator in Section 3.4.

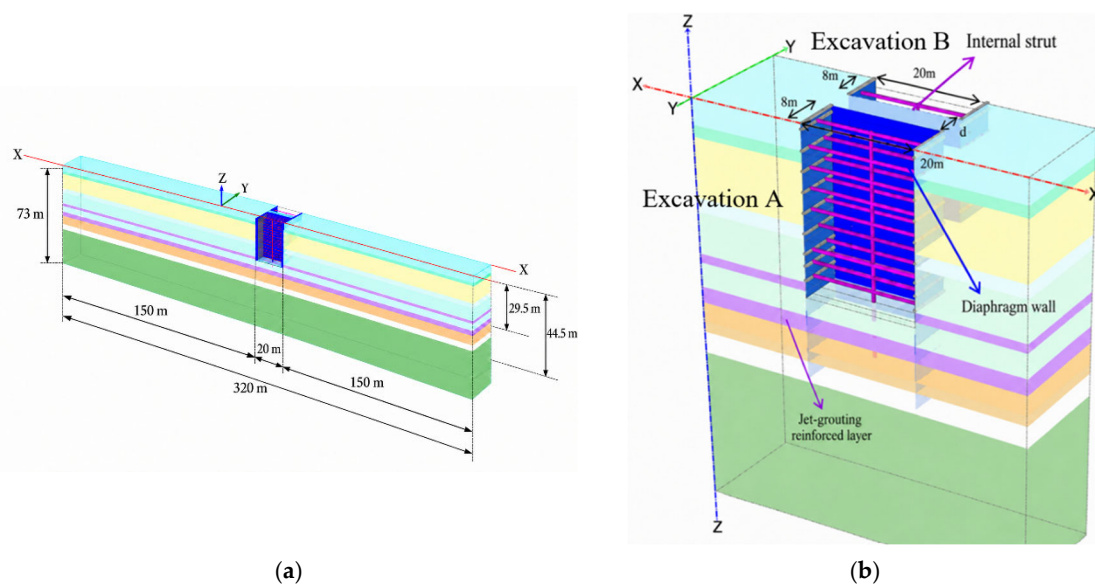


Figure 4. (a) Computational domain and principal dimensions of the twin-excavation configuration; (b) spatial arrangement and support system of the twin excavations.

The assessment combines indicator scores from the isolated-excavation reference states with the interaction coefficients estimated from the prescribed numerical operations in Section 3.3. Support-level scores are calculated to identify the governing assessment unit under the adopted rules.

The two excavations were assumed to be excavated synchronously as the selected illustrative construction condition. Under this assumption, excavation-induced deformation fields develop concurrently; the configuration is not established as a mechanical upper bound.

4.2. Calculation of Individual Excavations Condition Indices

The purpose of this subsection is to demonstrate how normalized indicator scores, indicator weights, and nonlinear interaction terms are integrated into a single-excavation condition index. In the illustrative analysis, three active indicators were selected: the strut–force imbalance indicator P_3 , the maximum horizontal displacement of the retaining wall O_1 , and the relative vertical displacement of the retaining wall O_2 .

For the illustrative assessment, the maximum horizontal displacement over the selected wall face (O_1) and the wall-top vertical displacement (O_2) are taken as 29.25 and 9.95 mm, respectively. These values are obtained from the isolated-reference analysis of Excavation A, with neighboring-excavation influence excluded. This analysis is separate from the single-excavation prototype analysis used to estimate β in Section 3.3, whose reference response is $U_0 = 32.75$ mm. O_1 and U share the same spatial extraction definition, but the two reported values belong to separate reference analyses. The 29.25 mm input is therefore retained for indicator scoring, whereas 32.75 mm is retained as the baseline for the response-increment calculations. In particular, the values of 34.42 and 40.02 mm in Table 10 are horizontal-displacement responses under the column-deactivation and wall-stiffness-reduction scenarios, respectively; they are not the O_1 and O_2 scoring inputs.

For indicator scoring, the maximum horizontal displacement over the selected wall face O_1 and wall-top vertical displacement O_2 were taken as 29.25 mm and 9.95 mm, respectively, yielding scores of 56.67 and 59.33 according to Table 8 and Equation (6). These scoring inputs differ from the responses of 34.42 mm and 40.02 mm reported in Table 10 for the single-factor degradation scenarios associated with O_1 and O_2 . Both values represent the maximum horizontal wall displacement U used to estimate β in Section 3.3, which consistently represents the maximum horizontal wall displacement under each degradation scenario, including the scenario associated with O_2 .

Table 12 reports the strut axial forces in the reference assessment state of Excavation A, before additional strut deactivation, column deactivation, or wall stiffness reduction. Both struts in each assessed pair remain active. These forces are used to calculate $P_3(m)$ and its condition score. The O_1 and O_2 scoring inputs are obtained from the same reference assessment state.

Table 12. Strut axial forces and P_3 scores in the reference assessment state of Excavation A.

Level (m)	F_1 (kN)	F_2 (kN)	F_{c1} (kN)	F_{c2} (kN)	N_i	N_j	P_3 (m)	x_{p3} (m)
2	846	1193	7120	7120	0.1188	0.1676	0.3404	48.07
3	1957	2774	9120	9120	0.2146	0.3042	0.3454	49.08
4	2468	3503	9120	9120	0.2706	0.3841	0.3467	49.34
5	2387	3313	9120	9120	0.2617	0.3633	0.3249	44.98
7	2395	1708	7120	7120	0.3364	0.2399	0.3349	46.98
8	1207	834	7120	7120	0.1695	0.1171	0.3655	53.10
9	1306	1794	7120	7120	0.1834	0.2520	0.3148	42.97

The independent single-operation and paired-operation responses in Table 10 are used only to estimate β . These coefficients are subsequently applied to the reference-state indicator scores through the adopted interaction formulation. Accordingly, the resulting composite scores assess the reference state under this parameterization and do not represent indicator scores extracted after individual-strut failure.

As described in Section 3.2, the global weights of the active indicators are renormalized within the set $\{O_1, O_2, P_3\}$. The normalized weights are 0.58, 0.22, and 0.20 for O_1 , O_2 , and P_3 , respectively. The same weights are used for both the baseline and interaction terms in Equations (1)–(3).

The nonlinear interaction terms were then introduced for the active interaction pairs P_3 – O_1 , P_3 – O_2 , and O_1 – O_2 . Accordingly, the integrated source-excavation evaluation index is calculated by Equations (1)–(3) and the resulting condition scores are listed in Table 13.

Table 13. Support-level composite safety-condition scores considering P_3 , O_1 , and O_2 of excavation A.

Scenario	Level	x_{p3}	x_{o1}	x_{o2}	$E_{k,m}^0$	$\Delta E_{p3,o1}^{k,m}$	$\Delta E_{p3,o2}^{k,m}$	$\Delta E_{o1,o2}^{k,m}$	$E_{k,m}$	Score Class
P-201/202	2	48.07	56.67	59.33	55.54	0.00	0.00	0.00	55.54	III
P-301/302	3	49.08	56.67	59.33	55.74	0.00	0.00	0.00	55.74	III
P-401/402	4	49.34	56.67	59.33	55.79	0.18	0.25	0.00	56.22	III
P-501/502	5	44.98	56.67	59.33	54.92	14.20	0.52	0.00	69.64	IV
P-701/702	7	46.98	56.67	59.33	55.32	10.39	2.48	0.00	68.18	IV
P-801/802	8	53.10	56.67	59.33	56.54	10.03	0.63	0.00	67.20	IV
P-901/902	9	42.97	56.67	59.33	54.51	0.00	0.00	0.00	54.51	III

Table 13 presents support-level composite scores for the reference assessment state. Differences among levels reflect the level-specific P_3 scores and interaction coefficients, while O_1 and O_2 are shared wall-level inputs. The scores at levels 5, 7, and 8 are 69.64, 68.18, and 67.20, respectively, with level 5 governing the excavation-level index under Equation (4). These increases relative to linear aggregation quantify the contributions of the prescribed interaction terms; they do not represent observed changes in the index following strut failure. Table 14 summarizes the corresponding excavation-level indices.

Table 14. Summary of isolated-reference condition indices for the twin excavations.

Excavation	Indicator Scores	Baseline Weighted Condition Index (E_0)	Nonlinear Interaction Contribution (ΔE_β)	Isolated-Reference Condition Index (E_k)
Excavation A	$P_3 = 44.98$ $O_1 = 56.67$ $O_2 = 59.33$	54.92	14.72	69.64
Excavation B	$P_3 = 39.60$ $O_1 = 46.60$ $O_2 = 52.48$	46.49	10.68	57.18

4.3. Target Excavation Condition Index Considering Neighboring Influence

After determining the source-excavation condition index, the neighboring influence was evaluated through the neighboring-influence functions Equations (8) and (9) in Section 3.4. The neighboring-excavation contribution was calculated using the source-index magnitude and the normalized excavation spacing. Table 15 summarizes the neighboring-excavation contribution under different source excavation scenarios and spacing conditions. The contribution calculated from the isolated-reference source index was added to the isolated-reference target index, so that neighboring influence enters the assessment only at this stage.

Table 15. Distance-weighted contributions from the support-level scores of excavation A at different normalized spacings.

Scenario	Source Score E_k	Spacing			
		0.50H	0.75H	1.00H	2.00H
P-201/202	55.54	26.61	18.42	12.75	2.93
P-301/302	55.74	26.71	18.49	12.80	2.94
P-401/402	56.22	26.94	18.65	12.91	2.96
P-501/502	69.64	33.37	23.10	15.99	3.67
P-701/702	68.18	32.67	22.62	15.66	3.60
P-801/802	67.20	32.20	22.29	15.43	3.54
P-901/902	54.51	26.12	18.08	12.52	2.87

The results show that the neighboring-excavation contribution is governed by both the source excavation condition and the spatial separation between excavations. Excavation scenarios with higher source indices generated consistently larger neighboring contribution. For example, in excavation A, the P-501/502 scenario with a source-index of 69.64 produced neighboring-excavation contributions of 33.37 and 3.67 at spacing ratios of 0.50H and 2.00H, respectively.

However, the neighboring-excavation contribution alone does not represent the overall assessed condition of the target excavation, because the target excavation may possess its own isolated-reference condition index before considering neighboring influence. Therefore, the composite condition index was calculated by combining the target excavation isolated-reference condition index with the transferred contribution.

In this illustrative case, Excavation A exhibited a higher isolated-reference condition index and was therefore selected as the representative source excavation. The transferred contribution from A to B was evaluated to demonstrate the composite condition index evolution of the neighboring excavation. As shown in Table 16, the neighboring excavation significantly modifies the target excavation condition index under close spacing conditions. When the spacing decreased to 0.75H, the neighboring-excavation contribution increased the target excavation condition index from 57.18 to 80.28. With increasing spacing, the additional influence gradually diminished, and the composite condition index approached the isolated-reference condition index of the target excavation.

Table 16. Composite condition index of target Excavation B at different excavation spacings.

Source Excavation	Target Isolated-Reference Condition Index	Spacing	Neighboring Contribution	composite Condition Index
P-501/502	57.18	0.50H	33.37	90.55
		0.75H	23.10	80.28
		1.00H	15.99	73.17
		2.00H	3.67	60.85

4.4. Sensitivity Analysis

A sensitivity analysis was conducted for the three updateable parameter groups, namely the indicator weights ω , the internal interaction coefficient β , and the spatial attenuation coefficient λ . Because these parameters act at different levels of the framework, their sensitivities were evaluated using different model outputs. Perturbations in ω and β were evaluated using the source-excavation composite condition index E_k , whereas perturbations in λ were evaluated using the neighboring-excavation

contribution at different normalized excavation spacings. Each parameter was varied by -20% , -10% , 0 , $+10\%$, and $+20\%$, while the remaining parameters were kept unchanged.

4.4.1. Sensitivity Analysis of the Indicator Weight ω

A sensitivity analysis was conducted for the weights of P_3 , O_1 , and O_2 . For each indicator, its baseline weight was varied from -20% to $+20\%$ at intervals of 10% , while the other two weights were kept unchanged before renormalization. The indicator values and nonlinear interaction coefficients were fixed throughout the analysis.

Table 17 shows that the composite condition index of excavation A remained within a narrow range under one-at-a-time perturbations in the indicator weights. Compared with the baseline weighted condition index of 69.64, the maximum variation in E_k was limited to 0.85% , indicating that the proposed assessment framework is relatively robust to moderate uncertainty in expert-derived weights.

Table 17. Sensitivity analysis of indicator weights ω .

Scenario	ω_{P3}	ω_{O1}	ω_{O2}	E_0	ΔE	E_k	Relative Change in E_k
(1.0 P_3 , 1.0 O_1 , 1.0 O_2)	0.20	0.58	0.22	54.92	14.72	69.64	–
(0.8 P_3 , 1.0 O_1 , 1.0 O_2)	0.17	0.60	0.23	55.33	13.71	69.04	-0.85%
(0.9 P_3 , 1.0 O_1 , 1.0 O_2)	0.18	0.59	0.22	55.12	14.25	69.37	-0.38%
(1.1 P_3 , 1.0 O_1 , 1.0 O_2)	0.22	0.57	0.22	54.72	15.13	69.86	0.32%
(1.2 P_3 , 1.0 O_1 , 1.0 O_2)	0.23	0.56	0.21	54.54	15.50	70.04	0.58%
(1.0 P_3 , 0.8 O_1 , 1.0 O_2)	0.23	0.52	0.25	54.69	14.95	69.64	0.01%
(1.0 P_3 , 0.9 O_1 , 1.0 O_2)	0.21	0.55	0.23	54.81	14.85	69.66	0.04%
(1.0 P_3 , 1.1 O_1 , 1.0 O_2)	0.19	0.60	0.21	55.01	14.57	69.58	-0.08%
(1.0 P_3 , 1.2 O_1 , 1.0 O_2)	0.18	0.62	0.20	55.10	14.40	69.50	-0.19%
(1.0 P_3 , 1.0 O_1 , 0.8 O_2)	0.21	0.61	0.18	54.71	15.34	70.05	0.60%
(1.0 P_3 , 1.0 O_1 , 0.9 O_2)	0.20	0.59	0.20	54.82	15.02	69.84	0.29%
(1.0 P_3 , 1.0 O_1 , 1.1 O_2)	0.20	0.57	0.24	55.01	14.43	69.44	-0.28%
(1.0 P_3 , 1.0 O_1 , 1.2 O_2)	0.19	0.56	0.25	55.10	14.15	69.25	-0.56%

Increasing the P_3 weight decreases the direct weighted component but increases the interaction contribution; the latter effect dominates, producing a modest increase in the composite score.

In contrast, variations in the O_1 weight produced negligible changes because the increase or decrease in the baseline component was largely offset by the corresponding variation in the interaction contribution. The influence of O_2 showed an opposite trend: reducing its weight slightly increased the final condition index. This behavior does not imply that a lower O_2 weight universally leads to a higher condition index; rather, it results from the weight normalization process, where the reduced O_2 contribution is redistributed to other indicators and strengthens the dominant P_3 – O_1 interaction term.

Overall, the sensitivity results demonstrate that the proposed condition-index calculation is not dominated by a single expert-defined weight, while preserving the influence of mechanically meaningful indicator interactions.

4.4.2. Sensitivity Analysis of the Internal Coupling Coefficient β

Table 18 presents the sensitivity of the source-excavation composite condition index to variations in the internal coupling coefficient β . The perturbation of β only modifies the nonlinear interaction contribution, while the baseline weighted condition index E_0 remains unchanged at 54.92 for all scenarios. As β increases from 0.8β to 1.2β , the nonlinear contribution ΔE_β increases from 11.77 to 17.66, resulting in an increase in the composite condition index E_k from 66.69 to 72.58.

Table 18. Sensitivity analysis of the internal coupling coefficient β .

Scenario	E_0	ΔE_{p3-O1}	ΔE_{p3-O2}	ΔE_β	E_k	Relative Change in E_k
0.8 β	54.92	11.36	0.42	11.77	66.69	−4.23%
0.9 β	54.92	12.78	0.47	13.25	68.16	−2.11%
β	54.92	14.20	0.52	14.72	69.64	–
1.1 β	54.92	15.62	0.58	16.19	71.11	2.11%
1.2 β	54.92	17.04	0.63	17.66	72.58	4.23%

The variation in ΔE_β is mainly governed by the P_3 - O_1 interaction term, which increases from 11.36 to 17.04, whereas the P_3 - O_2 contribution remains relatively small, increasing only from 0.42 to 0.63. This indicates that the coupling between support–force imbalance and retaining-wall horizontal displacement dominates the nonlinear amplification under the investigated condition.

Although β was increased by 20%, the composite condition index increased by only 4.23% compared with the baseline value. This difference results from the structure of the proposed condition-index formulation, where β affects only the nonlinear interaction component, while the baseline contribution E_0 remains unchanged. Therefore, the sensitivity of the total score is moderated by the dominant baseline component. The results indicate that uncertainty in the internal coupling coefficient has a direct influence on the interaction’s contribution to the condition index, but its effect on the final condition index remains bounded within the investigated range.

4.4.3. Sensitivity to the Spatial Attenuation Coefficient λ

Table 19 presents the sensitivity of the neighboring-excavation contribution to variations in the spatial attenuation coefficient λ . Unlike ω and β , which affect the source-excavation condition index, λ directly controls the attenuation process between adjacent excavations. The source-excavation condition index was maintained at 69.64, while λ was varied from $0.8\lambda_0$ to $1.2\lambda_0$.

Table 19. Sensitivity analysis of the spatial attenuation coefficient λ .

Scenario	0.10H	0.25H	0.50H	0.75H	1.00H	2.00H
0.8 λ_0	61.90	51.88	38.66	28.80	21.46	6.61
0.9 λ_0	61.00	50.01	35.92	25.69	18.52	4.93
λ_0	60.11	48.20	33.37	23.10	15.99	3.67
1.1 λ_0	59.23	46.46	31.00	20.79	13.80	2.74
1.2 λ_0	58.37	44.79	28.80	18.52	11.91	2.04

The results show that the neighboring-excavation contribution decreases consistently with increasing λ . For example, at a normalized spacing of 0.50H, the neighboring contribution decreases from 38.66 to 28.80 when λ increases from $0.8\lambda_0$ to $1.2\lambda_0$. This confirms that a larger attenuation coefficient represents a faster reduction in adjacent excavation influence.

The sensitivity of the neighboring-excavation contribution becomes more pronounced at larger normalized spacing when evaluated using relative changes. At 2.00H, reducing λ by 20% increases the neighboring contribution from 3.67 to 6.61, corresponding to a relative increase of approximately 80.1%. However, this large percentage change is associated with a relatively small baseline weighted condition index value. In terms of absolute change in the neighboring contribution, intermediate spacings exhibit more substantial changes. Therefore, both relative and absolute sensitivity should be considered when interpreting the influence of λ . Among all investigated perturbation scenarios, the largest relative deviation from the baseline occurred at $d/H = 2.00$ when λ

was reduced by 20%, whereas the largest absolute change occurred at $d/H = 0.75$ under the same λ perturbation.

To examine the prescribed residual-weight assumption, ε was set to 1%, 2.5%, and 5%, while $\alpha = 3.13$, the source score of 69.64, the target pre-transfer score of 57.18, and all other inputs were held constant. The corresponding λ values were calculated as $-\ln(\varepsilon)/\alpha$. Table 20 compares neighboring contributions, target composite scores, and assessment levels. For fixed α , ε and λ are deterministically related and were not varied independently.

Table 20. Sensitivity of neighboring-excavation contributions and target composite scores to the reference residual weight.

Normalized Spacing d/H_{avg}	Residual Weight ε (%)	Attenuation Coefficient λ	Neighboring Contribution	Target Composite Score	Assessment Level
0.50	1.0	1.4713	33.37	90.55	V
0.50	2.5	1.1786	38.63	95.81	V
0.50	5.0	0.9571	43.15	100.00	V
0.75	1.0	1.4713	23.10	80.28	V
0.75	2.5	1.1786	28.77	85.95	V
0.75	5.0	0.9571	33.97	91.15	V
1.00	1.0	1.4713	15.99	73.17	IV
1.00	2.5	1.1786	21.43	78.61	IV
1.00	5.0	0.9571	26.74	83.92	V
2.00	1.0	1.4713	3.67	60.85	IV
2.00	2.5	1.1786	6.59	63.77	IV
2.00	5.0	0.9571	10.27	67.45	IV

Table 20 shows that increasing the reference residual weight from 1% to 5% increases the calculated neighboring contribution and target composite score at all investigated spacings. When the reference residual weight increases from 1% to 5%, the target composite score increases by 6.60–10.87 points across the investigated spacings after application of the 100-point cap. At a normalized spacing of 1.00, the assessment changes from Level IV to Level V, whereas the levels remain unchanged at the other investigated spacings. At a normalized spacing of 0.50, the uncapped aggregate score reaches 100.33 under the 5% setting. These results indicate that the absolute scores and, in some scenarios, assessment levels depend on the selected residual weight. The analysis quantifies this parameter dependence but does not establish the physical validity of the transfer operator.

5. Discussion

5.1. Internal Nonlinear Coupling and Comparison with Linear Aggregation

Support–force imbalance and retaining-wall deformation are mechanically related in braced excavations, motivating the inclusion of selected interaction terms. At the governing support level, the baseline and interaction-adjusted condition indices were 54.92 and 69.64, respectively. The nonlinear contribution was 14.72 points, accounting for approximately 21.14% of the final index. These values quantify the contribution of the prescribed interaction terms within the assessment model. The maximum scenario-based interaction coefficient β was approximately 1.64.

Renormalizing the weights of O_1 , O_2 , and P_3 preserves their relative proportions while expressing importance within the active indicator set. Under the adopted interaction formulation, renormalization scales the direct and interaction contributions

equally before capping, preserving their relative balance. The fixed indicator set, normalized weights, and scoring functions provide a common basis for comparison among the investigated scenarios.

Equation (4) selects the maximum support-level score as the excavation-level index. This aggregation retains the controlling condition without diluting it through averaging and provides the source-excavation input for the neighboring-influence calculation. The support-level results complement this index by showing the distribution of adverse conditions.

5.2. Modeled Neighboring-Excavation Influence

The neighboring-excavation contribution combines the isolated-reference source index of 69.64 with an assumed exponential distance weight. The adopted reference scale $\alpha = 3.13$ and residual weight $\varepsilon = 0.01$ give $\lambda \approx 1.47$. For fixed attenuation parameters, the source index and normalized spacing determine the contribution in index points. Its decrease with spacing follows from the prescribed exponential function.

Equation (5) adds this contribution to the target excavation's isolated-reference index. Using isolated-reference inputs ensures that neighboring influence enters the calculation only through the added term, avoiding double counting. This separation makes the target's own condition and the modeled neighboring contribution explicit.

5.3. Numerical Demonstration and Parameter Sensitivity

The numerical example demonstrates the calculation procedure, comparison of the investigated scenarios, and identification of the governing support level. The benchmark comparison with published numerical mechanical responses concerns the numerical model, while the sensitivity analyses examine how the assessment outputs respond to changes in the three updateable parameter groups.

Within the investigated ranges, indicator-weight variations produced limited changes in the composite index, β primarily affected the nonlinear contribution, and λ governed the distance dependence of neighboring influence. Increasing ε from 1% to 5% changed the target score by 6.60–10.87 points and changed the assessment level at $d/H_{\text{avg}} = 1.00$. This result highlights the influence of the residual-weight assumption on both the score and its classification.

6. Conclusions

This study proposes a framework for adjacent deep excavations that integrates response-based indicator scoring, expert-derived weights, selected nonlinear interactions, support-level aggregation, and modeled neighboring influence. The numerical example demonstrates its implementation and identifies a governing source-excavation index of 69.64. The interaction terms increase the index relative to linear aggregation, while the assumed distance weight controls the neighboring contribution. Together, the example and sensitivity analyses illustrate the calculation procedure and its dependence on the adopted parameters.

The framework has several limitations. Its output is a composite safety-condition index on a conventional 0–100 scale, with higher values indicating more adverse conditions under the selected assessment rules; it is not a failure probability or expected loss. The index has not been calibrated against observed damage or consequences or independently validated using field monitoring and documented engineering conditions. Neither the numerical benchmark nor local parameter sensitivity establishes predictive accuracy or applicability across projects.

The interaction coefficients are derived from a limited set of degradation scenarios. Mapping displacement-based interactions to the bilinear score term remains an

approximation; consistency under a common linear scoring transformation does not validate the product form, its scaling, or its applicability across the full score range. Accordingly, the increase over linear aggregation does not establish an underestimate of actual engineering risk. The spatial operator and λ are likewise based on a literature-informed reference scale and a prescribed residual weight, rather than calibration against coupled twin-excavation responses. Avoiding double counting does not independently validate this transfer approximation.

The three-indicator assessment excludes other responses without assigning them zero adverse contribution, and scores from different indicator subsets are not directly comparable. Maximum aggregation describes the controlling score rather than the average condition or the location of first failure, and cannot distinguish an isolated high value from several similarly high values. The twin-excavation example uses identical geometry and support conditions and explicitly parameterizes only normalized spacing. The sampled spacings are not universal critical thresholds, and the target's pre-transfer score does not represent its susceptibility to external disturbance. Soil variability, hydraulic connectivity, construction timing, asymmetric geometry, and surcharge are not explicitly represented.

Future work should combine field monitoring with independently documented engineering conditions, using separate data for calibration and validation. Priorities are to test the mechanical-to-score mapping across degradation severities and to evaluate the spatial operator against coupled numerical and field responses across spacings and construction stages, including target responses and corresponding score increments. Subsequent extensions should incorporate additional structural, deformation, and hydrological indicators; geological, hydraulic, temporal, and target-susceptibility factors; asymmetric layouts and multiple neighbors; and monitoring-based parameter updates. These steps would establish the framework's engineering applicability and support its development beyond the present illustrative case.

Supplementary Materials: The following supporting information can be downloaded at: <https://www.mdpi.com/article/10.3390/app162010334/s1>. The supplementary material includes additional analyses of indicator responses and fixed-coefficient contributions at different degradation severities (Section S1, Tables S1 and S2), neighboring-excavation contributions across three construction stages (Section S2, Table S3), and numerical sensitivity and baseline displacement definitions (Section S3, Table S4). Reference [31] is cited in the Supplementary Materials.

Author Contributions: Conceptualization, C.H. and Y.L.; methodology, C.H. and Z.Z.; software, Z.Z. and W.L.; validation, Z.Z., Y.L. and W.L.; formal analysis, C.H. and Z.Z.; investigation, Z.Z. and W.L.; resources, C.H. and Y.L.; data curation, Z.Z. and W.L.; writing—original draft preparation, C.H. and Z.Z.; writing—review and editing, C.H. and Y.L.; visualization, Z.Z. and W.L.; supervision, C.H. and Y.L.; project administration, C.H.; funding acquisition, C.H. and Y.L. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Key R&D Program of China, grant numbers 2023YFC3009300 and 2023YFC3009302.

Data Availability Statement: The additional numerical analyses, supporting calculation workbook, and anonymized expert assessment data are provided in the Supplementary Materials. Additional model information is available from the corresponding author upon reasonable request.

Conflicts of Interest: The authors declare no conflict of interest.

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