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Review

Advances in Nonlinear Dynamics and Vibration Control of Clearance-Containing Flexible Oscillating Mechanisms

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Abstract

Clearance-containing flexible oscillating mechanisms are widely used in aerospace systems, robotic manipulators, precision manufacturing systems, and intelligent machinery, where their nonlinear dynamic behavior directly affects motion accuracy, stability, and operational reliability. The combined effects of joint clearance, structural flexibility, contact–impact, and constraint nonlinearities can induce rigid–flexible coupled vibration, bifurcation, chaos, and nonsmooth responses. This review systematically summarizes recent advances in dynamic modeling, nonlinear dynamic analysis, vibration and chaos control, experimental validation, and engineering applications of such mechanisms. Particular attention is given to contact–impact models, rigid–flexible and clearance–flexibility coupled formulations, bifurcation and chaos analysis, active, passive, and semi-active control, Nonlinear Energy Sinks (NESs), hybrid control, and Digital Twin-enabled validation. Existing studies indicate that continuous contact force models and flexible multibody formulations remain widely adopted, while the bidirectional coupling between joint clearance and structural deformation is a primary source of complex nonlinear behavior. Hybrid active–passive control is receiving increasing attention, and NESs show strong potential for broadband vibration suppression. However, current research is still limited by insufficient multiphysics coupling, incomplete understanding of high-dimensional nonsmooth dynamics, underdeveloped motion-accuracy-oriented control frameworks, and limited long-term experimental validation. Future research should emphasize high-fidelity multiphysics modeling, data-driven dynamic prediction, intelligent vibration control, NES optimization, and Digital Twin-based online validation.

Keywords: joint clearance; flexible oscillating mechanism; rigid–flexible coupling; nonlinear dynamics; vibration control; nonlinear energy sink

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1. Introduction

Modern aerospace systems, industrial robots, precision transmission devices, and advanced manufacturing equipment are increasingly required to operate at higher speeds with lighter structures, greater positioning accuracy, and improved reliability. Oscillating mechanisms are widely employed for motion transformation, attitude adjustment, and power transmission, and their dynamic performance therefore directly affects vibration levels, motion accuracy, operational reliability, and service life [1–6].

In practical mechanisms, manufacturing tolerances, assembly errors, long-term wear, thermal deformation, and variations in lubrication inevitably introduce clearances into kinematic joints. These clearances transform ideal continuous constraints into repeated contact–impact–separation processes accompanied by frictional wear, motion errors, and energy dissipation. When flexible components are involved, clearance effects interact with structural deformation, inertial coupling, and external excitation, producing rigid–flexible coupled vibration, bifurcation, chaos, and nonsmooth dynamic behavior and substantially increasing the difficulty of dynamic modeling and vibration control [7–12].

Progress in Flexible Multibody Dynamics (FMBD), the Absolute Nodal Coordinate Formulation (ANCF), contact–impact dynamics, and multiphysics modeling has improved the capability to describe these coupled mechanisms [13–18]. Meanwhile, Physics-Informed Neural Networks (PINNs), Scientific Machine Learning (SciML), Digital Twins, and reinforcement learning are increasingly being explored for parameter identification, dynamic prediction, model updating, and vibration control [19–24]. These developments further highlight the importance of clearance-related dynamics for the reliability, precision performance, and intelligent operation and maintenance of advanced mechanical systems [1,7,19].

1.1. State of the Art

Existing research on clearance-containing mechanical systems has established a broad knowledge framework encompassing five major directions: dynamic modeling, nonlinear dynamic analysis, vibration and chaos control, experimental validation and engineering applications, and intelligent modeling based on digital technologies [7,13,19,20].

(1) Dynamic modeling

Dynamic modeling constitutes the theoretical foundation for analyzing dynamic behavior and developing vibration control strategies for clearance-containing mechanisms. Early investigations primarily relied on rigid multibody dynamics, in which joint-impact processes were described using Hertzian contact theory, the Lankarani–Nikravesh contact force model, and modified friction formulations. Subsequently, flexible multibody dynamics (FMBD), the Absolute Nodal Coordinate Formulation (ANCF), the finite element method (FEM), and component mode synthesis (CMS) were progressively incorporated into the modeling of clearance-containing mechanisms, enabling the unified representation of structural deformation, elastic vibration, and contact–impact phenomena. More recently, research has evolved toward unified modeling frameworks capable of simultaneously considering multiple joint clearances, multiple flexible bodies, multiphysics coupling, and data-driven dynamic modeling [11,14,15,17,18,25–30].

(2) Nonlinear dynamic behavior

Because intermittent contact, impact, and friction induced by joint clearances inherently exhibit nonsmooth characteristics, clearance-containing mechanisms frequently display highly complex nonlinear dynamic responses. Considerable attention has therefore been devoted to the investigation of periodic motion, period-doubling bifurcation, Hopf bifurcation, grazing bifurcation, chaotic motion, Lyapunov exponents, Poincaré maps, and recurrence analysis. In parallel, uncertainty quantification, reliability analysis, and sensitivity analysis have been increasingly incorporated into nonlinear dynamic investigations to elucidate the influence of parameter variations on system responses and stability [31–40].

(3) Vibration and chaos control

To suppress vibration, mitigate impact loads, and improve motion accuracy, researchers have developed active, passive, semi-active, nonlinear energy sink (NES)-based, hybrid, and intelligent control strategies. Representative techniques include tuned mass dampers (TMDs), adjustable damping or stiffness devices, sliding mode control (SMC), adaptive control, model predictive control (MPC), and intelligent optimization. Semi-active approaches provide an intermediate solution between passive and fully active control by adjusting parameters such as stiffness, damping, or clearance with substantially lower control-energy requirements than direct active-force actuation. These control strategies have been investigated in flexible structures, robotic systems, aerospace mechanisms, and clearance-related mechanical systems [41–56].

(4) Experimental validation and engineering applications

With continuous advances in sensing and measurement technologies, increasing attention has been devoted to experimental verification of theoretical models and their practical implementation in engineering systems, including space manipulators, deployable solar arrays, aircraft landing gear, high-speed robotic systems, and precision transmission mechanisms. Meanwhile, advanced experimental techniques such as digital image correlation (DIC), laser vibrometry, high-speed imaging, and multisensor data fusion have substantially enhanced the capability to characterize complex dynamic behaviors and validate nonlinear dynamic models [57–60].

(5) Intelligent modeling and Digital Twins

Driven by rapid advances in intelligent computing, Physics-Informed Neural Networks (PINNs), DeepONet, neural operators, Scientific Machine Learning (SciML), and Digital Twin technologies have recently emerged as promising tools for dynamic modeling, model-order reduction, parameter identification, and structural health monitoring. The research emphasis has gradually shifted from conventional offline dynamic analysis toward model–data fusion, real-time prediction, and intelligent control, thereby providing new opportunities for investigating the dynamic behavior of complex clearance-containing mechanical systems [19,20,61–65].

Despite these advances, several gaps remain. Unified modeling frameworks simultaneously addressing multiple clearances, multiple flexible bodies, and multiphysics interactions are still limited, while dynamic modeling, nonlinear analysis, control design, and experimental validation are often studied separately. In addition, Digital Twin and Scientific Machine Learning approaches for real-time prediction and online control remain at an early stage, and systematic comparisons among existing methodologies are insufficient. These limitations motivate an integrated review that clarifies current progress, compares existing approaches, and identifies priorities for future research.

1.2. Objectives, Contributions, and the CFCC Framework

Motivated by the research gaps identified above, this review aims to provide an integrated synthesis of recent advances in the dynamics of clearance-containing flexible oscillating mechanisms, covering dynamic modeling, nonlinear dynamic analysis, vibration and chaos control, experimental validation, engineering applications, and emerging intelligent technologies. In addition to summarizing representative studies, the review compares the physical assumptions, analytical capabilities, computational characteristics, control mechanisms, engineering applicability, and major limitations of existing approaches. Particular emphasis is placed on representative studies published during 2021–2026, especially those reported between 2024 and 2026, while earlier landmark studies are retained where necessary to establish the theoretical foundations of contact mechanics, multibody dynamics, nonlinear dynamics, and vibration control. The overall organization and technical relationships among these research themes are summarized in Figure 1.

Existing review articles have primarily focused on individual topics, including joint-clearance dynamics, flexible multibody dynamics, contact–impact modeling, and specific vibration-control methods. Although these studies have established important theoretical and methodological foundations, the interactions among clearance-induced contact nonlinearities, structural flexibility, vibration control, experimental validation, and emerging cyber-physical technologies have received comparatively less integrated treatment. In particular, the increasing use of Digital Twins, Physics-Informed Neural Networks

(PINNs), and Scientific Machine Learning (SciML) has created a need to connect conventional physics-based dynamics with data-assisted prediction, online model updating, and intelligent decision-making within a common review framework [2,7–9,13,19,20].

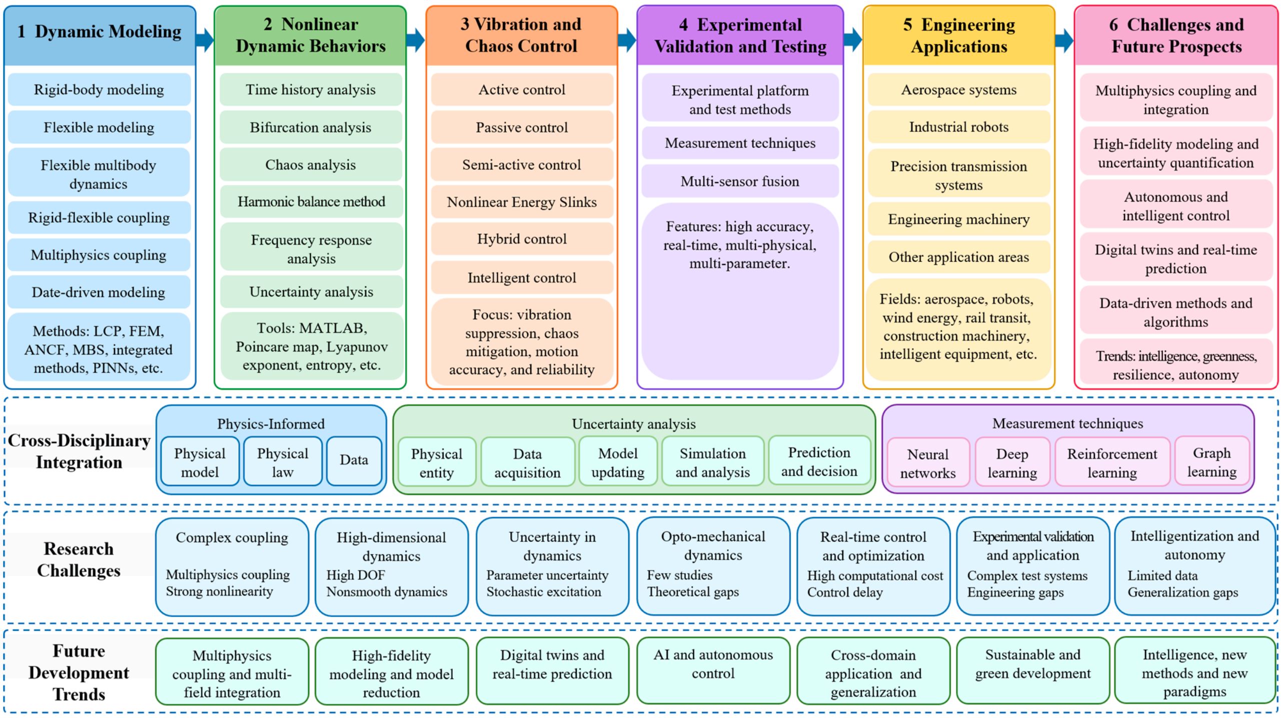


Figure 1. Comprehensive research framework for the dynamics of clearance-containing flexible oscillating mechanisms.

To organize these interconnected research directions, this review adopts the Complex Clearance–Flexible Coupling–Control Intelligence–Cyber Twin (CFCC) framework. The four components are derived from recurrent research streams identified in the reviewed literature. First, Complex Clearance (C) represents the nonlinear effects associated with joint-clearance geometry, intermittent contact–impact, friction, wear, lubrication, and related parameter uncertainties. Second, Flexible Coupling (F) describes the bidirectional interaction between joint clearance and structural deformation, including rigid–flexible coupling, flexible multibody dynamics, and multiphysics interactions. Third, Control Intelligence (C) covers passive, active, semi-active, Nonlinear Energy Sink (NES)-based, hybrid, and intelligent or data-driven control strategies for vibration suppression, chaos mitigation, motion-accuracy improvement, and reliability enhancement. Fourth, Cyber Twin (C) represents the integration of physical experiments, multisource sensing, model–data fusion, Digital Twins, PINNs, and SciML for online model updating, state prediction, parameter identification, and lifecycle-oriented decision support.

The CFCC framework is intended as a synthesis and organizational framework rather than a new mathematical formulation. Complex Clearance and Flexible Coupling provide the physical basis for the modeling and nonlinear analyses in Sections 2 and 3; Control Intelligence organizes the control strategies in Section 4; and Cyber Twin connects experimental validation, online sensing, model updating, and intelligent engineering applications in Sections 5 and 6. These dimensions are mutually coupled because clearance evolution influences structural response, structural response determines control requirements, and experimental or operational data can update both dynamic models and control strategies. This integrated perspective also helps distinguish established methods from emerging intelligent technologies.

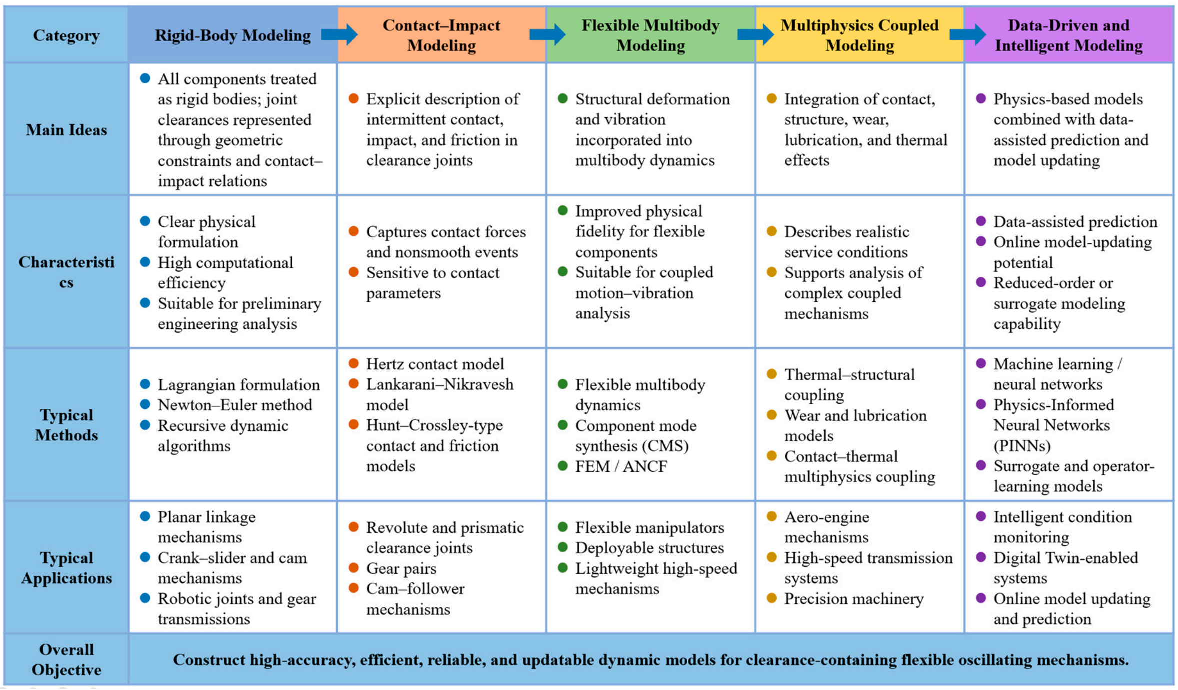
1.3. Literature Search and Selection Strategy

To improve transparency and reproducibility, the literature was searched using the Web of Science and CNKI databases, and the search was last updated on 15 July 2026. Search terms combined "joint clearance," "clearance joint," and "flexible oscillating mechanism" with keywords related to contact–impact modeling, flexible multibody dynamics, rigid–flexible coupling, bifurcation, chaos, nonsmooth dynamics, uncertainty, reliability, vibration control, Nonlinear Energy Sinks, Digital Twins, Physics-Informed Neural Networks, and Scientific Machine Learning. The primary publication window was 2021–2026, with particular attention to 2024–2026, while earlier seminal studies were retained when required to establish theoretical foundations. Studies directly relevant to clearance-related dynamics, control, validation, and intelligent modeling were included, whereas only peripherally related studies were excluded.

2. Advances in Dynamic Modeling of Clearance-Containing Oscillating Mechanisms

Dynamic modeling provides the basis for nonlinear analysis and vibration-control design of clearance-containing oscillating mechanisms. Joint clearances introduce contact–impact, friction, and energy dissipation, while structural flexibility, external loading, lubrication, wear, and environmental effects further increase model complexity. Consequently, modeling strategies must balance physical fidelity, computational efficiency, and engineering applicability [7,8,58,66–68].

Existing approaches can be broadly classified into five categories: rigid-body dynamic modeling, contact–impact dynamic modeling, flexible multibody dynamic modeling, rigid–flexible coupled and multiphysics dynamic modeling, and data-driven dynamic modeling. Their development has progressed from classical rigid multibody and contact formulations toward flexible and multiphysics frameworks and, more recently, physics-informed and data-assisted approaches [13–22]. Figure 2 summarizes the development of these techniques, while Table 1 compares their principal advantages, limitations, and representative applications.



Category	Rigid-Body Modeling	Contact–Impact Modeling	Flexible Multibody Modeling	Multiphysics Coupled Modeling	Data-Driven and Intelligent Modeling
Main Ideas	• All components treated as rigid bodies; joint clearances represented through geometric constraints and contact–impact relations	• Explicit description of intermittent contact, impact, and friction in clearance joints	• Structural deformation and vibration incorporated into multibody dynamics	• Integration of contact, structure, wear, lubrication, and thermal effects	• Physics-based models combined with data-assisted prediction and model updating
Characteristics	• Clear physical formulation • High computational efficiency • Suitable for preliminary engineering analysis	• Captures contact forces and nonsmooth events • Sensitive to contact parameters	• Improved physical fidelity for flexible components • Suitable for coupled motion–vibration analysis	• Describes realistic service conditions • Supports analysis of complex coupled mechanisms	• Data-assisted prediction • Online model-updating potential • Reduced-order or surrogate modeling capability
Typical Methods	• Lagrangian formulation • Newton–Euler method • Recursive dynamic algorithms	• Hertz contact model • Lankarani–Nikravesh model • Hunt–Crossley-type contact and friction models	• Flexible multibody dynamics • Component mode synthesis (CMS) • FEM / ANCF	• Thermal–structural coupling • Wear and lubrication models • Contact–thermal multiphysics coupling	• Machine learning / neural networks • Physics-Informed Neural Networks (PINNs) • Surrogate and operator-learning models
Typical Applications	• Planar linkage mechanisms • Crank–slider and cam mechanisms • Robotic joints and gear transmissions	• Revolute and prismatic clearance joints • Gear pairs • Cam–follower mechanisms	• Flexible manipulators • Deployable structures • Lightweight high-speed mechanisms	• Aero-engine mechanisms • High-speed transmission systems • Precision machinery	• Intelligent condition monitoring • Digital Twin-enabled systems • Online model updating and prediction
Overall Objective	Construct high-accuracy, efficient, reliable, and updatable dynamic models for clearance-containing flexible oscillating mechanisms.				

Figure 2. Evolution of dynamic modeling techniques for clearance-containing flexible oscillating mechanisms.

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Table 1. Comparison of Dynamic Modeling Methods for Clearance-Containing Flexible Oscillating Mechanisms.

Modeling Method	Major Advantages	Major Limitations	Typical Applications
Rigid-body Dynamics Modeling	Simple formulation, high computational efficiency, suitable for mechanism design and dynamic analysis	Neglects structural flexibility and cannot accurately capture high-speed nonlinear responses	Four-bar mechanisms, cam–follower mechanisms, robotic joints
Contact–Impact Dynamics Modeling	Accurately describes contact force, impact, and friction behaviors	Highly sensitive to model parameters and contact model selection	Rotating joints, gear pairs, cam mechanisms
Flexible Multibody Dynamics Modeling	Accurately captures structural deformation and dynamic responses with high prediction accuracy	Complex modeling procedure and high computational cost	Space deployable structures, robotic manipulators, aerospace structures
Rigid–Flexible Coupled Dynamics Modeling	Simultaneously considers rigid-body motion and structural vibration, offering strong engineering applicability	Large number of parameters and complex coupling relationships	Aerospace mechanisms, hybrid robotic systems, reconfigurable structures
Multiphysics Coupled Dynamics Modeling	Better represents real operating conditions and improves prediction accuracy	Difficult parameter identification and high computational complexity	Aero-engines, aircraft, high-speed transmission systems
Data-Driven Dynamics Modeling	High computational efficiency, supports online identification and model updating	Limited generalization capability and insufficient physical interpretability	Industrial robots, Digital Twin systems, intelligent manufacturing systems

2.1. Rigid-Body Dynamic Modeling

Rigid-body dynamic modeling represents the earliest and most widely adopted approach for investigating the dynamic behavior of clearance-containing oscillating mechanisms. Owing to its clear physical formulation, high computational efficiency, and compatibility with commercial multibody simulation software, it remains the theoretical foundation for engineering dynamic analysis. This approach generally assumes that all components are rigid bodies while accounting for geometric constraint variations, contact–impact interactions, and friction induced by joint clearances. The motion of the mechanism is subsequently governed by the multibody dynamic equations. Consequently, rigid-body models have been extensively employed in the dynamic analysis of planar linkage mechanisms, crank-slider systems, cam mechanisms, robotic joints, and gear transmission systems [9,11,14,69,70].

(1) Classical rigid-body dynamic models

Early studies primarily established multibody dynamic models using the Lagrangian formulation, the Newton–Euler method, and recursive dynamic algorithms, while representing clearance effects through geometric constraint relationships. Flores et al. first established a systematic dynamic framework for revolute joints with clearances by classifying joint motion into free flight, contact–impact, and continuous contact stages. Combined with contact force models, this framework enabled reliable prediction of the dynamic response of clearance-containing mechanisms and laid the foundation for subsequent investigations. Later, Lankarani, Machado, Pereira, and their co-workers further refined clearance modeling techniques, allowing rigid-body dynamic models to describe multiple joint clearances and complex contact processes more accurately [9,11,25,71,72].

With the rapid development of computational techniques, recursive dynamic algorithms have gradually become indispensable for engineering applications because of their high computational efficiency in large-scale multibody systems. More recently, Miao et al. proposed a recursive formulation for planar multibody systems with revolute joint clearances that significantly reduced computational complexity while maintaining modeling accuracy. Chen et al. subsequently developed a multiclearance dynamic model for planar mechanisms and systematically investigated the influence of clearance size on vibration characteristics and motion accuracy. These studies collectively demonstrate that rigid-body dynamic modeling continues to provide the fundamental basis for analyzing the dynamic behavior of complex mechanical systems [73,74].

(2) Dynamic modeling of multiple joint clearances

Multiple joint clearances can coexist within practical mechanisms, and their dynamic interactions introduce stronger nonlinearity and uncertainty than those in single-clearance systems [9,75–78].

Studies of multiclearance mechanisms show that interactions among clearance joints can induce secondary impacts and amplify vibration, while clearance distribution, thermal deformation, and multiple contact interfaces further modify contact states and nonlinear responses. These effects have been demonstrated in flexible planar mechanisms, spatial revolute-joint systems, and multiclearance gear transmissions. More recently, interval uncertainty analysis and tolerance-based clearance optimization have extended multiclearance modeling toward reliability-oriented design [9,68,76,78–80].

(3) Research trends

Rigid-body dynamic modeling remains valuable because of its clear formulation, computational efficiency, and engineering applicability, but its accuracy decreases when structural flexibility, thermal deformation, wear, and lubrication become significant [7,13,66–68]. Recent studies therefore increasingly address multiple clearances and stochastic parameters while improving computational efficiency through recursive algorithms. Machine learning surrogate models are also being explored for rapid prediction and online model updating, providing a basis for Digital Twin-enabled analysis [74,81–83].

2.2. Contact–Impact Dynamic Modeling

Contact–impact modeling constitutes one of the core components of dynamic modeling for clearance-containing oscillating mechanisms. Its primary objective is to accurately characterize the normal contact force, tangential friction force, and energy dissipation during joint contact. Because clearance joints repeatedly undergo transitions among free motion, contact–impact, continuous contact, and separation, the contact model directly determines the prediction accuracy of impact responses, vibration characteristics, nonlinear dynamic behavior, and subsequent control design. Consequently, developing contact–impact models that simultaneously achieve physical fidelity, numerical stability, and computational efficiency has remained a major research focus in the field of clearance dynamics [12,25,28,29,71,72,84,85].

(1) Classical contact–impact models

Early studies mainly established normal contact models based on Hertzian contact theory, in which the normal contact force is expressed as a nonlinear power-law function of the local contact deformation between two elastic bodies. Owing to its ability to accurately describe elastic contact behavior, Hertzian theory has become the foundation of most contact models for clearance-containing mechanisms. However, because the original Hertz model neglects impact-induced energy dissipation, its applicability is generally limited to ideal elastic collisions [26–28].

To overcome this limitation, Lankarani and Nikravesh introduced a hysteresis damping term into Hertzian theory and proposed the well-known Lankarani–Nikravesh (L–N) contact model, in which the coefficient of restitution is directly associated with energy dissipation during impact. This model significantly improved the accuracy of dynamic simulations and has since become one of the most widely adopted contact formulations in multibody dynamics. Flores et al. subsequently extended the L–N model to clearance-containing multibody systems and established a unified framework applicable to revolute joints, prismatic joints, and mechanisms containing multiple clearance joints. Owing to its simple mathematical formulation and clearly defined parameters, the L–N model remains the standard contact model implemented in most commercial multibody dynamic software packages [8,9,11,25].

In addition to the L–N model, the Hunt–Crossley model, the Gonthier model, and Johnson's contact theory have also been widely employed for various contact analyses. Johnson systematically established the theoretical foundation of elastic contact mechanics, while Machado et al. comparatively evaluated different contact force models in multibody dynamics and demonstrated that their numerical performance varies considerably under high-speed impacts and low-speed continuous-contact conditions [27,71,72].

(2) Friction modeling and improvements to contact models

Besides normal contact forces, tangential friction modeling also plays a crucial role in determining the dynamic response of clearance-containing mechanisms. Conventional analyses generally employ the Coulomb friction model to describe sliding friction between contacting bodies; however, the discontinuity of the Coulomb model near zero relative velocity often causes numerical oscillations. To improve computational robustness, Marques et al. systematically compared several continuous friction formulations and demonstrated that the LuGre, Stribeck, and modified Coulomb models more accurately reproduce practical friction characteristics. Pennestrì et al. further evaluated the applicability of different dry-friction models in multibody dynamics, providing useful guidelines for engineering applications [12,30,86].

With the increasing demand for high-speed and high-precision mechanical systems, recent studies have shifted from purely contact modeling toward coupled contact–friction–lubrication formulations. Peng et al. established a lubricated revolute-joint contact model to investigate the influence of lubricant film thickness, contact stiffness, and damping on system dynamics. Wang et al. proposed a nonlinear friction-coefficient model incorporating sliding velocity, normal load, and lubrication conditions, thereby improving prediction accuracy under complex operating conditions. Furthermore, Han et al. incorporated thermal deformation into dynamic modeling, enabling real-time updating of contact parameters under thermo-mechanical coupling conditions [67,68,87].

(3) Recent advances

Recent contact–impact research increasingly emphasizes physically identifiable contact parameters obtained through inverse analysis, experimental identification, and multiscale simulation. At the same time, contact models are being extended to thermal, lubrication, and wear effects because these factors modify contact stiffness and friction under realistic operating conditions. PINNs and surrogate models are also being explored for contact-parameter identification and online model updating, linking contact modeling more closely with experimental validation and Digital Twin-enabled analysis [12,21,22,84,85].

2.3. Flexible Multibody Dynamic Modeling

With the continuous trend toward higher operating speeds, lightweight structures, and large-scale mechanical systems, conventional rigid-body dynamic models have become increasingly inadequate for accurately describing the elastic deformation and vibration of flexible components during operation. This limitation is particularly evident in deployable space structures, robotic joints, aerospace actuation systems, and flexible oscillating mechanisms, where structural flexibility and joint clearances are strongly coupled, giving rise to complex nonlinear dynamic responses. Consequently, Flexible Multibody Dynamics (FMBD) has become one of the principal approaches for dynamic modeling of clearance-containing oscillating mechanisms and remains one of the most active research areas in recent years [2,13,15,16].

(1) Development of Flexible Multibody Dynamics

The fundamental objective of FMBD is to simultaneously describe the overall rigid-body motion and elastic deformation of structural components within a unified dynamic framework, thereby enabling integrated analysis of system motion and structural vibration. Early investigations primarily employed the Finite Element Method (FEM) to model flexible components, while Component Mode Synthesis (CMS) was introduced to reduce computational complexity. Shabana established the theoretical foundation of flexible multibody dynamics, providing a systematic framework for the dynamic analysis of complex mechanical systems. Subsequently, Bauchau further extended FMBD to beams, plates, shells, and spatial mechanisms, significantly promoting its application in aerospace engineering and robotic systems [14–16,70].

With the continuous improvement of computational capability, FMBD has gradually evolved from small-deformation analysis to formulations capable of accurately describing large displacements, large rotations, and large deformations. Gufler et al. reviewed recent developments in the application of flexible multibody dynamics to structural optimization and highlighted its increasing importance in the design of modern mechanical systems [13,17,18,88–90].

(2) Absolute Nodal Coordinate Formulation and High-Fidelity Flexible Modeling

In recent years, the Absolute Nodal Coordinate Formulation (ANCF) has emerged as one of the most effective approaches for modeling clearance-containing flexible mechanisms because of its capability to accurately represent large displacements, large rotations, and large deformations. By employing absolute nodal coordinates as generalized coordinates, ANCF eliminates the need for local coordinate transformations, thereby providing a unified formulation with high modeling accuracy for complex flexible structures [17,18,88,89].

Hu et al. developed an ANCF-based dynamic model for spatial flexible revolute joints with clearances, enabling unified modeling of structural deformation, joint contact, and system vibration. Jia et al. further investigated the coupled effects of structural flexibility and joint clearances in hybrid robotic systems and improved computational efficiency through model-order reduction techniques. Zhang et al. established a rigid–flexible coupled dynamic model for morphing wings and demonstrated the influence of joint clearances on structural vibration and motion accuracy. These studies collectively indicate that ANCF has become one of the major technological developments in high-fidelity dynamic modeling of flexible multibody systems [6,91,92].

(3) Recent advances

Recent FMBD research increasingly addresses systems containing multiple flexible bodies and joints while improving computational efficiency through model-order reduction, GPU-accelerated computing, and recursive algorithms. Thermal deformation, wear, lubrication, and stochastic parameters are also being incorporated to better represent practical service conditions. In parallel, PINNs, Neural-DAE methods, and surrogate models are being explored for rapid prediction and parameter identification, extending FMBD toward model–data-integrated and Digital Twin-enabled applications [21,22,68,81,91,92].

2.4. Rigid–Flexible Coupled and Multiphysics Dynamic Modeling

As modern mechanical systems continue to evolve toward higher operating speeds, lightweight designs, and enhanced reliability, their dynamic behavior is increasingly governed by the coupled effects of structural flexibility, joint clearances, contact–impact interactions, friction, wear, thermal deformation, lubrication, and manufacturing tolerances, rather than by any single factor. Consequently, conventional rigid-body or flexible multibody models alone are no longer sufficient to accurately represent the dynamic responses of practical engineering systems. Rigid–flexible coupled and multiphysics dynamic modeling has therefore become a major research direction for clearance-containing oscillating mechanisms [5,6,66–68,93].

(1) Development of rigid–flexible coupled dynamic modeling

The fundamental objective of rigid–flexible coupled dynamic modeling is to simultaneously describe the overall rigid-body motion and local elastic deformation within a unified dynamic framework, thereby capturing both the kinematic constraints of the mechanism and the interaction between structural vibration and joint impacts. Benefiting from advances in finite element analysis and Flexible Multibody Dynamics (FMBD), an increasing number of studies have incorporated flexible beams, plates, and complex spatial structures into multibody dynamic models [5,6,93].

Representative studies have applied rigid–flexible coupled formulations to deployable satellite structures, morphing wings, and aircraft control actuation systems, revealing strong interactions among structural deformation, clearance impacts, aerodynamic loading, and lubrication. These formulations provide a more realistic description of high-speed aerospace, robotic, and deployable mechanisms than approaches that treat structural flexibility and clearance effects independently [5,6,93].

(2) Multiphysics dynamic modeling

In recent years, multiphysics coupling has become an important trend in dynamic modeling. It is increasingly being recognized that joint clearances are influenced not only by structural parameters but also by thermal effects, lubrication conditions, wear evolution, and environmental loading, all of which continuously modify the dynamic behavior of mechanical systems [58,66–68,94].

Multiphysics models increasingly incorporate thermal deformation, lubrication, wear evolution, and uncertainty because these factors continuously modify joint clearance, contact stiffness, friction, and long-term system behavior. Thermal-clearance coupling, lubricated contact, wear–clearance co-evolution, interval uncertainty analysis, and tolerance-based clearance optimization have therefore been introduced to support more realistic and reliability-oriented dynamic prediction [66–68,79,80].

(3) Recent advances

Recent rigid–flexible and multiphysics models increasingly integrate structural flexibility, clearance, thermal effects, lubrication, wear, manufacturing uncertainty, and environmental loading. Model-order reduction, GPU-accelerated computation, and recursive algorithms have simultaneously improved the efficiency of large-scale coupled simulations. Digital Twin and surrogate-model approaches are also being explored for online model updating and rapid prediction, although unified high-fidelity models with reliable real-time updating remain an important research challenge [5,6,66–68,79–81,93].

2.5. Data-Driven Dynamic Modeling and Digital Twins

Recent advances in machine learning, big data analytics, and Digital Twin technologies have promoted the integration of physics-based and data-driven approaches, giving rise to a new paradigm for dynamic modeling. Compared with conventional physics-based models, data-driven approaches exploit experimental measurements, operational data, and historical system information to perform parameter identification, rapid prediction, and online model updating. These capabilities have demonstrated considerable potential for real-time dynamic analysis and Digital Twin applications in complex mechanical systems [19,20,63,95].

(1) Physics-Informed Neural Networks

Physics-Informed Neural Networks (PINNs) integrate governing equations, boundary conditions, and initial conditions into the neural network training process, thereby combining physical constraints with data-driven learning. As one of the most representative developments in Scientific Machine Learning (SciML), PINNs provide a new framework for dynamic modeling. Karniadakis et al. established the theoretical foundation of physics-informed machine learning, while Raissi et al. introduced PINNs for solving nonlinear partial differential equations. Subsequently, Lu et al. developed the DeepXDE platform, facilitating the application of PINNs to complex dynamic systems [20,61,65].

More recently, PINNs have been increasingly applied to robot dynamics, contact parameter identification, and flexible structural dynamics. Hu et al. proposed a PINN-based dynamic model incorporating friction effects for rapid prediction of robotic dynamics, whereas Sun et al. developed a Neural-DAE framework for online identification of complex dynamic systems. These studies demonstrate the potential of physics-informed and Neural-DAE approaches for efficient dynamic prediction and parameter identification [21,22].

(2) Digital Twin-based dynamic modeling

Digital Twin technology establishes a real-time connection between physical systems and virtual models, enabling online updating, state prediction, and optimization. Applications to industrial robots, aerospace systems, hydraulic equipment, and rotorcraft demonstrate its potential for frequency-response prediction, dynamic-state updating, path optimization, and reliability assessment. For clearance-containing oscillating mechanisms, the principal value of Digital Twins lies in using operational data to update dynamic models and support online prediction and health management [19,24,83,96–102].

(3) Scientific Machine Learning and surrogate modeling

DeepONet, neural operators, Gaussian Process models, and related surrogate techniques are increasingly being used for operator learning, high-dimensional reliability analysis, and multibody dynamic prediction. By combining physics-based information with efficient data-driven approximation, these approaches can substantially reduce computational cost while maintaining useful prediction accuracy, making them attractive for real-time analysis of complex mechanical systems [63,64,81,103–105].

(4) Recent advances

Recent developments increasingly emphasize physics-informed rather than purely data-driven learning, Digital Twin-based online model updating, and operator or surrogate models for efficient prediction. Data-driven modeling is also becoming more closely coupled with Bayesian optimization, reinforcement learning, and model predictive control. These approaches provide an important complement to conventional physics-based modeling, but their physical interpretability, generalization capability, and engineering validation remain important considerations [20–23,62,64,81,104,106–108].

2.6. Comparison of Dynamic Modeling Approaches and Future Perspectives

Table 1 compares the principal dynamic modeling approaches in terms of their major advantages, limitations, and representative applications. The comparison highlights the trade-off between computational efficiency, physical fidelity, parameter requirements, and engineering applicability.

Table 1 highlights the complementary characteristics of the principal modeling approaches. Rigid-body models retain advantages in computational efficiency and preliminary mechanism design, whereas contact–impact models provide more detailed descriptions of contact force, friction, and energy dissipation. FMBD and rigid–flexible coupled models improve physical fidelity when structural deformation becomes important, while multiphysics models further incorporate thermal, lubrication, wear, and uncertainty effects [7,13,15,16,71].

Digital Twins, PINNs, and SciML are extending these physics-based approaches toward online model updating, rapid prediction, and intelligent decision-making. However, systems involving multiple clearances, multiple flexible bodies, and strong multiphysics coupling still face high model complexity, difficult parameter identification, and trade-offs between computational efficiency and prediction accuracy [19–22,63,64]. These modeling limitations provide the basis for the nonlinear dynamic analyses reviewed in the following section, while the broader future research priorities are discussed in Section 6.

3. Advances in Nonlinear Dynamics of Clearance-Containing Oscillating Mechanisms

Nonlinear dynamic analysis reveals how joint clearance, contact–impact, friction, structural flexibility, and external excitation interact to determine the response of clearance-containing oscillating mechanisms. Repeated transitions among contact, separation, and re-contact introduce strong nonlinearity, nonsmooth behavior, and parameter sensitivity, which may lead to broadband vibration, bifurcation, quasiperiodicity, chaos, and dynamic instability and consequently degrade motion accuracy and reliability [73,75,76,109–112].

These behaviors are investigated using phase portraits, Poincaré maps, bifurcation diagrams, Lyapunov exponents, spectral analysis, recurrence analysis, entropy-based methods, and uncertainty analysis [36–40,113,114]. As summarized in Figure 3, the literature can be organized into five closely connected topics: vibration response characteristics, bifurcation behavior, chaotic motion, nonsmooth dynamics, and uncertainty and reliability analysis.

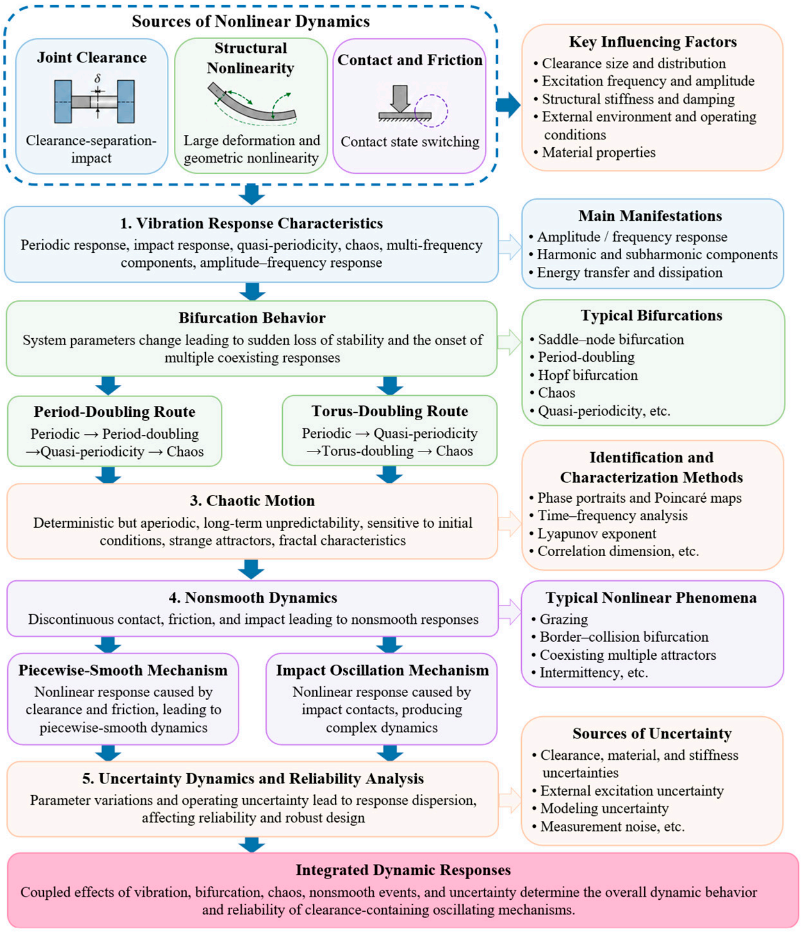

Figure 3. Framework of nonlinear dynamic analysis for clearance-containing flexible oscillating mechanisms.

3.1. Vibration Response Characteristics

Vibration response represents the most direct and fundamental manifestation of the dynamic behavior of clearance-containing oscillating mechanisms and provides the basis for subsequent investigations of bifurcation, chaos, and dynamic instability. Joint clearances alter the original kinematic constraints of the mechanism by introducing repeated contact, separation, and frictional sliding between moving components, thereby generating impact-induced vibration, high-frequency harmonics, and accumulated motion errors. As the clearance size, external loading, and operating speed vary, the vibration response generally exhibits pronounced nonlinear characteristics accompanied by frequency shifts, abrupt amplitude variations, and energy redistribution [69,73,110,115].

(1) Vibration response of single-clearance mechanisms

Early studies primarily focused on mechanisms containing a single revolute or prismatic joint clearance. By combining time-domain analysis, phase-plane trajectories, and frequency-spectrum analysis, researchers investigated the influence of clearance parameters on vibration characteristics. Flores et al. demonstrated that increasing joint clearance significantly increases both the contact frequency and impact amplitude, causing the system response to evolve from stable periodic motion to complex aperiodic behavior. Subsequent studies further examined the effects of contact stiffness, restitution coefficients, and friction parameters, showing that joint clearances not only modify the natural frequencies of the system but also excite higher-order frequency components, resulting in broadband vibration responses [9,11,69,71].

More recently, Chen et al. established a dynamic model for high-precision mechanisms with joint clearances and systematically investigated the influence of clearance size, operating speed, and contact parameters on vibration responses and spectral characteristics. Their results demonstrated that increasing joint clearance enhances high-frequency impact components and gradually transforms the frequency spectrum from a single dominant peak into a broadband, multi-peak distribution. These findings suggest that vibration responses not only characterize clearance-induced dynamic behavior but also provide an effective basis for condition assessment and health monitoring of mechanical systems [110,115].

(2) Vibration response of multiclearance and rigid–flexible coupled systems

Multiple joint clearances and rigid–flexible coupling introduce additional pathways for impact interaction and energy redistribution, producing stronger nonlinearity and variability in vibration responses [75,78,92,109]. Studies of spatial mechanisms, gear transmissions, and hybrid robotic systems show that multiclearance coupling can redistribute vibration energy, create additional resonance regions and combination frequencies, and amplify local impacts through elastic deformation [75,78,92].

Similar interactions are important in deployable aerospace structures, where clearance impacts can affect overall vibration, deployment synchronization, and structural stability. Experimental and digital-model studies of flexible solar arrays and related space mechanisms have confirmed the strong influence of rigid–flexible coupling on these responses [3,5,60,116].

(3) Recent advances

Recent vibration-response studies increasingly emphasize the quantitative characterization of impact-induced broadband responses and their implications for motion accuracy and reliability. Time–frequency, recurrence, and energy-based analyses provide complementary information beyond conventional time- and frequency-domain methods, thereby improving the identification of response transitions and energy redistribution. These developments strengthen vibration-response analysis as a quantitative basis for subsequent investigations of bifurcation, chaos, and reliability [24,75,92,94,110,117,118].

3.2. Bifurcation Behavior

Bifurcation is a fundamental phenomenon in nonlinear dynamical systems, referring to abrupt transitions in system motion caused by continuous variations in control parameters. For clearance-containing oscillating mechanisms, bifurcation represents a critical stage in the evolution from stable operation to complex nonlinear behavior. Because joint clearances continuously modify system stiffness and kinematic constraints through intermittent contact, mechanisms may evolve from periodic motion to period-doubling oscillations, quasiperiodic responses, and eventually chaotic motion. Consequently, bifurcation analysis is essential for understanding nonlinear dynamic mechanisms and provides important guidance for parameter optimization, stability assessment, and vibration control [31,32,38–40].

(1) Typical bifurcation phenomena

Existing studies have shown that the most common bifurcation phenomena in clearance-containing oscillating mechanisms include period-doubling bifurcation, Hopf bifurcation, grazing bifurcation, and border-collision bifurcation. Period-doubling bifurcation generally occurs as contact stiffness or excitation frequency varies and is widely recognized as one of the primary routes to chaos. Hopf bifurcation describes the transition from a stable equilibrium state to limit-cycle oscillation, whereas grazing and border-collision bifurcations arise from nonsmooth contact dynamics and constitute characteristic features that distinguish clearance-containing systems from conventional smooth dynamical systems [31,32,35,38,39].

Ren et al. investigated the dynamics of planar four-bar mechanisms operating under microgravity conditions and demonstrated that increasing joint clearance drives the system through successive stages of stable periodic motion, period-doubling bifurcation, and chaotic behavior. They further established comprehensive bifurcation diagrams to illustrate the dynamic evolution process. Xin et al. extended this work to three-dimensional hybrid mechanisms and showed that spatial revolute joint clearances not only alter the critical bifurcation conditions but also introduce multiple coexisting bifurcation paths, resulting in significantly more complex nonlinear responses [76,119].

(2) Parameter sensitivity and bifurcation mechanisms

Recent investigations have increasingly considered joint clearance size, contact stiffness, friction coefficient, damping, and operating speed as bifurcation control parameters to evaluate their influence on system stability. The results consistently indicate that increasing joint clearance reduces the stable operating region while significantly increasing the likelihood of period-doubling and grazing bifurcations. In contrast, appropriate increases in damping or optimization of contact parameters can effectively delay the onset of bifurcation and improve dynamic stability [76,112,119,120].

The presence of multiple joint clearances and structural flexibility further increases the complexity of bifurcation behavior. Studies on spatial multibody mechanisms have shown that interactions among multiple local bifurcations may occur simultaneously, making conventional single-parameter analyses insufficient for accurately describing the dynamic evolution of complex systems. Consequently, multiparameter bifurcation analysis has gradually become an important direction in recent nonlinear dynamic research [75,76,112].

(3) Recent advances (2024–2026)

Recent bifurcation studies increasingly employ multiparameter analyses to determine stability boundaries under the coupled effects of joint clearance, structural flexibility, thermal variation, and parameter uncertainty. The combined use of bifurcation diagrams, Lyapunov exponents, Poincaré maps, and spectral analysis improves the identification of critical transitions in high-dimensional parameter spaces. These results also provide a more direct basis for reliability-oriented parameter optimization and vibration-control design [68,76,80,112,119,120].

3.3. Chaotic Motion

Chaotic motion is one of the most representative nonlinear dynamic phenomena in clearance-containing oscillating mechanisms. It is characterized by extreme sensitivity to initial conditions, long-term unpredictability of system trajectories, and broadband random-like vibration. Owing to the combined effects of periodic contact–impact, frictional sliding, and structural flexibility, these mechanisms typically evolve through a sequence of periodic motion, period-doubling bifurcation, quasiperiodic motion, and chaotic motion. Consequently, chaos analysis has become an essential approach for understanding the underlying mechanisms of complex nonlinear dynamics [38,109,111].

(1) Mechanisms of chaotic motion

Previous studies have demonstrated that chaotic behavior in clearance-containing oscillating mechanisms primarily originates from discontinuous constraint switching caused by repeated contact–impact events. During contact, instantaneous impacts between mating joints lead to rapid redistribution of system energy, whereas separation returns the mechanism to a free-motion state. The repeated alternation between these motion regimes introduces pronounced nonsmooth characteristics into the system dynamics and ultimately gives rise to complex chaotic attractors [31–34].

Flores et al. reported that increasing joint clearance gradually transforms the system trajectory from a single limit cycle into a complex chaotic attractor, accompanied by increasingly pronounced chaotic characteristics. Subsequent investigations further demonstrated that contact stiffness, friction parameters, and excitation frequency strongly influence the onset and evolution of chaotic motion, with both impact frequency and contact energy dissipation governing the rate of chaotic evolution. More recently, Zhang et al. established a dynamic model for spatial multibody mechanisms and revealed strong chaotic coupling among multiple clearance joints, showing that impacts occurring at one joint may trigger chaotic responses throughout the entire system. Xin et al. further demonstrated that spatial revolute joint clearances introduce additional degrees of freedom, resulting in more complex topological structures of chaotic attractors [69,75,76,109].

(2) Chaos identification methods

With the continuous development of nonlinear dynamics, numerous techniques have been proposed for identifying chaotic motion. Commonly used methods include Lyapunov exponents, Poincaré maps, phase portraits, spectral analysis, recurrence analysis, and entropy-based measures [36,37,113,114].

Among these approaches, the Lyapunov exponent quantitatively characterizes the sensitivity of system trajectories to initial conditions and is widely regarded as one of the most reliable indicators of chaos. Poincaré maps reveal the transition from periodic to chaotic motion through changes in intersection patterns, whereas phase portraits provide intuitive visualization of the evolution of chaotic attractors. More recently, recurrence analysis and sample entropy have been increasingly employed to identify hidden dynamic characteristics from complex nonlinear responses. Furthermore, combining multiple analytical techniques has become the prevailing practice. In particular, Lyapunov exponents are frequently used together with spectral analysis, Poincaré maps, and phase portraits to improve the reliability of chaos identification [36,37,113,114].

(3) Recent advances

Recent chaos studies increasingly emphasize quantitative state identification and prediction using recurrence analysis and entropy-based indicators, while machine learning-assisted methods are also being explored in related nonlinear state-identification and prediction problems [104,109,121–124]. Nevertheless, different identification tools remain complementary: Lyapunov exponents provide rigorous quantitative criteria, Poincaré maps and phase portraits reveal dynamic transitions visually, while recurrence and entropy methods are particularly useful for complex experimental signals. Combined use of these methods therefore provides more reliable characterization of chaotic behavior [36,37].

3.4. Nonsmooth Dynamics

Unlike conventional continuous dynamical systems, clearance-containing oscillating mechanisms undergo repeated transitions among contact, impact, and separation, resulting in piecewise continuous governing equations. Such systems are therefore classified as nonsmooth dynamical systems. Nonsmooth dynamics not only governs impact responses and energy dissipation during contact events but also plays a decisive role in bifurcation evolution, the onset of chaotic motion, and overall system stability. Consequently, it has become an important topic in the study of nonlinear dynamics of clearance-containing mechanisms [31–34].

(1) Characteristics of nonsmooth dynamics

In clearance-containing oscillating mechanisms, the kinematic constraints continuously change with the contact state, causing the governing equations to switch between different motion regimes. During impact, abrupt changes in velocity and acceleration introduce pronounced discontinuities into the system response, distinguishing clearance-containing mechanisms from conventional smooth dynamical systems.

Leine and Nijmeijer established a systematic theoretical framework for nonsmooth dynamics, providing a unified basis for impact dynamic analysis. Brogliato subsequently developed nonsmooth contact dynamic models that comprehensively describe energy transfer and constraint switching during impact processes. These theoretical developments have become fundamental to contemporary research on the dynamics of clearance-containing mechanisms [31,33].

(2) Grazing bifurcation and border-collision bifurcation

Among the most representative phenomena in nonsmooth dynamical systems are grazing bifurcation and border-collision bifurcation. Nordmark first established the theoretical framework for grazing bifurcation, demonstrating that abrupt transitions in system dynamics may occur when system trajectories become tangent to constraint boundaries, leading to the sudden destabilization of periodic orbits. Subsequent studies extended grazing bifurcation theory to clearance-containing mechanisms and showed that both joint clearance size and impact stiffness significantly affect the occurrence and critical conditions of grazing bifurcation, as well as overall system stability [35].

More recently, Dang et al. investigated complex orbital systems and analyzed the influence of impact events and nonsmooth constraints on dynamic stability under high-speed operating conditions. Krysko et al. studied contact dynamics in porous beam structures and revealed the evolution of system stability under complex contact conditions. These studies have further broadened the application of nonsmooth dynamic theory to complex engineering systems [111,125].

(3) Recent advances (2024–2026)

Recent nonsmooth-dynamics research increasingly combines analytical and numerical methods with uncertainty assessment and experimental validation. High-speed imaging, DIC, and laser vibrometry provide direct measurements of contact-state transitions and impact responses, while data-driven methods are being explored for impact-state recognition and online parameter identification [21,22,58,59]. Analytical methods offer physical insight, numerical simulation accommodates complex multibody systems, and experiments provide direct validation; their integration is therefore essential for improving the accuracy and engineering applicability of nonsmooth dynamic analysis [7,58,59].

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3.5. Uncertainty Dynamics and Reliability Analysis

In practical engineering applications, clearance-containing oscillating mechanisms are influenced not only by joint clearances but also by multiple sources of uncertainty, including manufacturing tolerances, assembly errors, material variability, wear evolution, lubrication conditions, and fluctuating environmental loads. These uncertainties introduce significant variability into the system parameters, resulting in stochastic dynamic responses, shifts in stability boundaries, and degradation of system reliability. Consequently, uncertainty dynamics and reliability analysis have become an important research direction in the study of complex mechanical systems and a crucial step toward bridging dynamic analysis with engineering applications [58,79,80,82,94,116,118,126–129].

(1) Uncertainty Modeling Methods

Early studies primarily employed parametric sensitivity analysis to investigate the effects of variations in clearance size, contact stiffness, and damping on dynamic responses. Although effective for identifying local parameter influences, this approach is insufficient to characterize the inherent randomness of complex engineering systems.

More recently, probabilistic methods, interval analysis, and stochastic finite element methods have been increasingly adopted for the dynamic analysis of clearance mechanisms. Interval-based approaches have been introduced to quantify the propagation of dynamic responses under parameter uncertainty, while dynamic models incorporating multiple uncertainty sources have enabled systematic investigations of the combined effects of joint clearances, structural flexibility, and external loading on system reliability. In addition, uncertainty analysis integrated with tolerance design has provided theoretical support for clearance optimization in high-reliability mechanical systems [79,80,116,126].

With the continued development of reliability theory, research has gradually evolved from single-parameter sensitivity analysis toward global sensitivity analysis, multi-parameter coupled uncertainty analysis, and reliability-based design optimization, thereby strengthening the integration of dynamic analysis with engineering design [94,117,118].

(2) Reliability Evaluation and Parameter Optimization

In recent years, Liang et al. developed a reliability analysis model for complex multibody mechanisms considering wear evolution and joint-clearance development, demonstrating that wear not only alters the dynamic response but also continuously degrades system reliability throughout the service life. Zuo et al. further investigated the influence of prismatic joint clearances on motion-accuracy reliability and proposed a reliability-based optimization method for mechanism design. In addition, Jia et al. integrated sensitivity analysis with dynamic modeling to perform parameter screening and model-order reduction for complex robotic systems, thereby improving the computational efficiency of reliability analysis [92,94,117].

These developments have shifted reliability analysis from post-evaluation toward predictive assessment and reliability-oriented design [79,94,117]. Recent studies increasingly combine probabilistic and interval approaches with surrogate and data-driven models to improve efficiency in multiparameter problems. Digital Twin and physics-informed methods are also being explored for online reliability updating and remaining-useful-life prediction, although their maturity for clearance-containing flexible mechanisms remains limited [80,82,96,99,104,116,122,126,129].

(3) Comparative Analysis

Probabilistic, interval, and surrogate-based approaches remain complementary. Probabilistic methods provide detailed descriptions of stochastic variation but may require high computational cost; interval methods are useful when probability distributions are incomplete; and surrogate models improve computational efficiency but depend on training-data quality. Hybrid use of these methods can therefore provide a better balance between prediction accuracy and computational efficiency [80,82,99,104,126].

3.6. Comparative Analysis and Research Trends

Research on the nonlinear dynamics of clearance-containing oscillating mechanisms has established a framework encompassing vibration response, bifurcation behavior, chaotic motion, nonsmooth dynamics, and uncertainty analysis. Studies increasingly consider the coupled effects of multiple clearances, structural flexibility, friction, lubrication, thermal effects, wear, and parameter uncertainty, while analytical tools have expanded from conventional time- and frequency-domain analysis to Poincaré maps, Lyapunov exponents, recurrence analysis, entropy-based indicators, and reliability-oriented methods [7,58,73,75,76,94,109,111].

As summarized in Figures 3 and 4 and Table 2, three developments are particularly important. First, nonlinear analysis increasingly addresses realistic operating conditions involving several interacting physical factors rather than isolated deterministic effects. Second, complementary analytical, numerical, and statistical methods are increasingly combined to identify critical transitions, stability boundaries, and nonlinear states in high-dimensional systems. Third, Digital Twins, PINNs, and SciML are being explored for state identification, parameter estimation, and online prediction, extending conventional analysis toward model–data-integrated frameworks [19–22,58,66–68].

Important limitations nevertheless remain. Strong coupling among clearance, flexibility, friction, lubrication, thermal effects, and uncertainty is not yet fully represented within unified analytical frameworks, while emerging intelligent methods still require improvements in physical consistency and engineering validation [7,13,19–21]. These cross-cutting issues are discussed further in Section 6. The following section focuses on control strategies that translate nonlinear dynamic understanding into vibration suppression, stability enhancement, and motion-accuracy improvement [19–21,104].

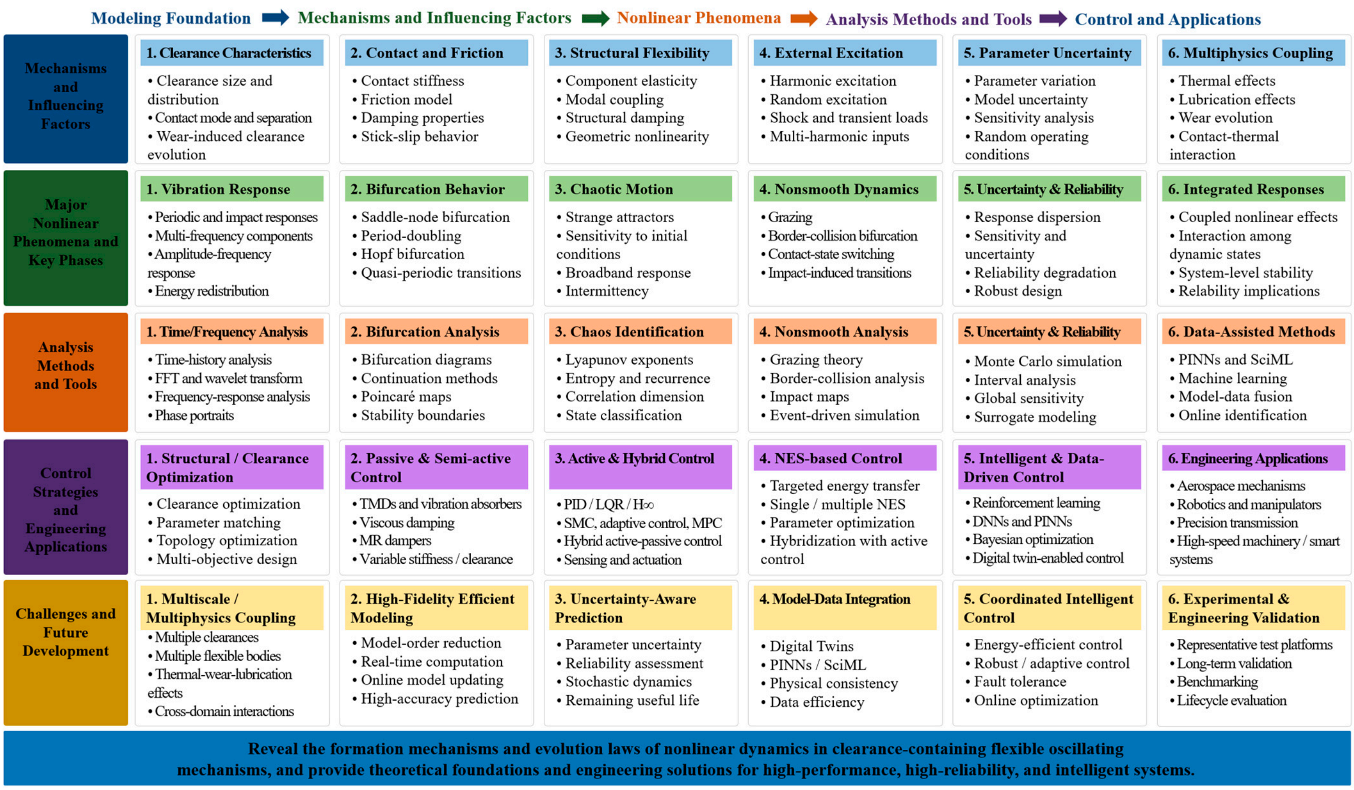

Figure 4. Comprehensive framework of nonlinear dynamic analysis, control strategies, and engineering applications for clearance-containing flexible oscillating mechanisms.

Table 2. Comparison of Nonlinear Dynamic Analysis Methods for Clearance-Containing Flexible Oscillating Mechanisms.

Dynamic Phenomenon	Primary Influencing Factors	Representative Analysis Methods	Typical Applications	Research Trends
Vibration response	Clearance, structural stiffness, excitation	Time-history analysis, FFT, wavelet transform	Dynamic response evaluation and condition monitoring	Intelligent monitoring and online state prediction
Bifurcation	Clearance, damping, rotational speed	Bifurcation diagrams, Poincaré maps	Stability analysis and parameter optimization	Multi-parameter bifurcation analysis
Chaotic motion	Impact, friction, structural flexibility	Lyapunov exponent, entropy analysis	Fault diagnosis and nonlinear state identification	AI-assisted nonlinear dynamics analysis
Nonsmooth dynamics	Contact state switching	Grazing bifurcation theory, border-collision analysis	Contact behavior analysis and dynamic optimization	Hybrid physics–data-driven modeling
Uncertainty analysis	Wear, manufacturing tolerances, random excitation	Monte Carlo simulation, interval analysis	Reliability assessment and robust design	PINNs-based uncertainty quantification

4. Advances in Vibration and Chaos Control of Clearance-Containing Oscillating Mechanisms

Nonlinear dynamic analysis identifies the mechanisms underlying vibration, bifurcation, chaos, and nonsmooth behavior, but practical systems additionally require effective suppression of excessive vibration and repeated impacts to maintain motion accuracy, reliability, and service life [3,42–46,120].

The control strategies reviewed here are organized into six complementary categories: active, passive, semi-active, Nonlinear Energy Sink (NES)-based, hybrid active–passive, and intelligent or data-driven control. Active methods provide high control authority; passive methods dissipate vibration energy without continuous external power; semi-active methods adjust damping, stiffness, or clearance with lower energy demand than fully active actuation [54–56]; and NES-based methods exploit targeted energy transfer. Hybrid approaches combine passive robustness and active adaptability, while PINNs, SciML, and Digital Twin technologies are being explored for state prediction, parameter identification, and adaptive optimization [19,20,41,48–50]. Because these intelligent approaches remain less mature than established passive and active methods, they are treated as emerging rather than replacement technologies.

4.1. Active Vibration Control

Active vibration control (AVC) suppresses structural vibrations, improves motion accuracy, and enhances dynamic stability by acquiring the real-time operating state of the system through sensors, computing the control input using a controller, and applying active control forces or torques via actuators. Compared with passive control, AVC continuously adjusts control parameters according to the system state, providing superior vibration attenuation under complex operating conditions and broadband excitations. Consequently, it has been widely applied in aerospace systems, robotic manipulators, flexible mechanisms, and precision systems [44,51–53].

(1) Classical active control methods

Early studies primarily focused on classical control strategies, including proportional–integral–derivative (PID) control, state-feedback control and optimal control. Owing to its simplicity and ease of implementation, PID control has been extensively employed in robotic joints, servo drives, and precision positioning systems. However, because clearance-containing oscillating mechanisms exhibit pronounced nonlinearities and time-varying parameters, conventional PID controllers often fail to maintain satisfactory performance under complex operating conditions. To improve control accuracy, modern control approaches, including state-feedback control, linear quadratic regulator (LQR) control and H_∞ control, have been introduced to optimize dynamic performance [51–53].

With the development of nonlinear control theory, sliding mode control (SMC) has attracted considerable attention because of its strong robustness. Previous studies have shown that SMC effectively suppresses the influence of parameter perturbations and external disturbances on the dynamic response while maintaining high control accuracy in the presence of joint clearances and friction nonlinearities. Adaptive control further improves system performance by estimating system parameters online and updating the control law in real time, thereby compensating for clearance variations and model uncertainties in complex mechanical systems [43,44,50].

(2) Active control of complex mechanisms

Active control of flexible multibody and space mechanisms must simultaneously suppress structural vibration and accommodate coupling between flexible deformation and joint clearances, which substantially increases the control difficulty [3,44,51–53]. Model Predictive Control (MPC) is particularly useful because system constraints and future-state predictions can be incorporated directly into the control process. In flexible manipulators, deployable structures, and precision servo systems, MPC has been used to coordinate vibration suppression and motion-error reduction, while state observers and parameter-identification methods further compensate for clearance variation and external disturbances [23,44,52].

(3) Recent advances (2024–2026)

Recent active-control research increasingly combines state estimation, parameter identification, predictive control, and data-driven optimization to compensate for clearance variation and model uncertainty. Digital Twin and physics-informed methods are also being explored for online controller adaptation and multi-objective optimization involving vibration, motion accuracy, energy consumption, reliability, and service life [21,23,24,106].

(4) Comparative analysis

Active control provides high control authority, rapid response, and real-time adaptability, but its engineering implementation depends on reliable and continuous external power, sensing, actuation, and control hardware. Power interruption, actuator failure or saturation, sensing errors, and model uncertainty may degrade performance or compromise stability; robust, adaptive, fault-aware, and state-estimation methods are therefore increasingly important [43,44,52,53].

4.2. Passive Vibration Control

Passive vibration control suppresses dynamic responses by increasing damping, modifying structural stiffness, or optimizing structural parameters to dissipate vibration energy. Unlike active control, passive control does not require external power and is characterized by simple structures, high reliability, and low maintenance costs. Consequently, it has been widely applied in aerospace, vehicle engineering, mechanical transmission systems, and robotic systems [45,46,129–131].

Common passive techniques include tuned mass dampers (TMDs), dynamic vibration absorbers (DVAs), viscous dampers, isolation devices, and damping materials. TMDs are mature and effective near their tuning frequency, whereas DVAs are useful under periodic excitation but become less effective under broadband excitation and significant parameter variation [129–131]. Recent developments in damping materials, multi-stage absorbers, flexible supports, and hybrid isolation structures have broadened the applicable frequency range [129–131].

Passive control therefore offers simple implementation, high reliability, low maintenance, and no requirement for continuous external power, but its fixed parameters limit adaptability to changing operating conditions. For this reason, passive devices are increasingly being combined with active control or NES-based methods to provide baseline isolation together with broader vibration suppression capability [41,45,46,131].

4.3. Semi-Active Vibration Control

Semi-active vibration control combines the reliability and relatively low energy demand of passive devices with the adaptability of active regulation. Instead of directly applying large external control forces, semi-active systems modify selected mechanical properties, such as damping, stiffness, or clearance, according to measured operating states. Representative implementations include magnetorheological dampers, variable-stiffness joints, and adjustable-clearance devices. This characteristic makes semi-active control particularly attractive when continuous high-power actuation is undesirable, but fixed passive parameters cannot provide adequate performance under varying operating conditions [54–56].

Variable-stiffness regulation has been investigated for flexible jointed structures, where modification of joint torsional stiffness changes the system natural frequencies and can therefore move the structure away from resonant operating regions. Semi-active clearance adjustment provides another relevant mechanism. Adjustable bearing concepts have demonstrated that changes in radial clearance modify the stiffness and damping characteristics of the supporting system and can reduce vibration responses under different operating conditions. More recently, magnetorheological damping structures have been investigated for flexible manipulators, where the damping force can be regulated through low-power electrical input to suppress residual vibration [54–56].

Compared with fully active control, semi-active strategies generally require less external energy and do not rely on continuous generation of large control forces. They can therefore provide a useful compromise among vibration attenuation, adaptability, power demand, and engineering reliability. Their control authority is nevertheless constrained by the achievable range of damping, stiffness, or clearance adjustment, and their performance depends on sensing accuracy, device dynamics, and the effectiveness of the parameter-adjustment law. Moreover, direct experimental studies on semi-active control of clearance-containing flexible mechanisms remain limited, particularly under impact, wear, uncertainty, and time-varying operating conditions [54–56].

4.4. Nonlinear Energy Sink (NES) Control

A Nonlinear Energy Sink (NES) is an advanced vibration control technique that exploits nonlinear stiffness to achieve Targeted Energy Transfer (TET) from the primary system to an attached absorber. Unlike conventional tuned mass dampers, an NES does not rely on the natural frequency of the primary structure. Instead, vibration energy is continuously transferred and dissipated through nonlinear coupling, providing broadband vibration attenuation, strong robustness, and superior adaptability to complex nonlinear systems. Consequently, NES technology has become one of the most active research areas in the vibration control of clearance-containing oscillating mechanisms [41,47,132–136].

The theory of targeted energy transfer established by Vakakis and subsequent studies by Kerschen, Gendelman, and others provides the principal theoretical basis for NES-based vibration suppression. These studies demonstrate that nonlinear coupling can transfer energy from the primary structure to the NES and effectively attenuate transient, impact-induced, and nonlinear resonant responses without requiring precise linear frequency tuning [41,47,132–136].

For clearance-containing oscillating mechanisms, several enhanced NES configurations have been proposed, including rotational NESs, distributed NESs, inertial NESs, and impact NESs. The fundamental principle of targeted energy transfer and energy dissipation in NES systems is schematically illustrated in Figure 5. These designs have been integrated with rigid–flexible coupled dynamic models to investigate the influence of NES parameters on vibration responses, bifurcation behavior, and chaotic motion. The results consistently indicate that appropriate selection of the mass ratio, nonlinear stiffness, and damping parameters significantly reduces contact–impact responses and chaotic vibrations while improving the dynamic stability of the mechanism. Furthermore, NES technology has increasingly been applied to flexible structures, aircraft landing gear, robotic joints, and high-speed rotating machinery, demonstrating its effectiveness for broadband vibration suppression and impact mitigation in complex engineering systems [42,45,46,131,134].

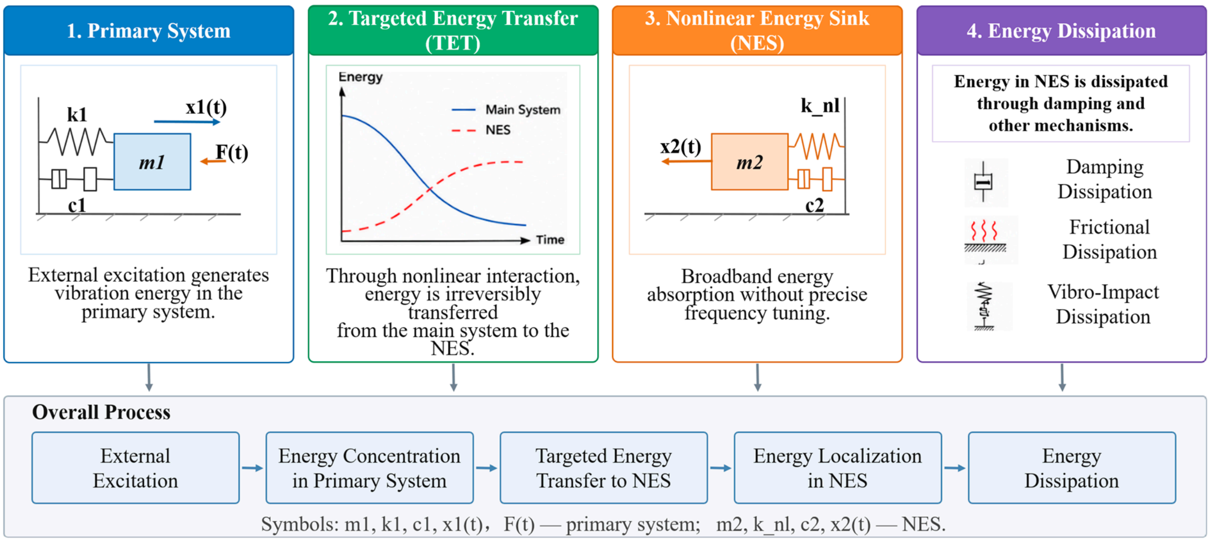

Figure 5. Schematic illustration of targeted energy transfer and energy dissipation in nonlinear energy sink (NES) systems.

Current NES research focuses on multi-NES configurations, parameter optimization, and hybridization with active control to improve broadband vibration attenuation and robustness under varying operating conditions. Digital Twin-assisted parameter tuning may provide an emerging route for adapting NES performance to changes in system dynamics, although direct engineering validation remains limited [23,41,45,46].

4.5. Hybrid Active–Passive Control

Hybrid active–passive control combines passive vibration isolation or energy dissipation with active compensation for parameter variations, thereby providing baseline robustness together with real-time adaptability [41,51–53,131]. Representative configurations include TMD-assisted active control, NES-assisted active control, and damper-based feedback control. For systems involving joint clearances, structural flexibility, and uncertainty, these combinations can improve vibration suppression, motion accuracy, stability, and energy efficiency compared with either strategy used independently [41,44,129,131].

MPC, adaptive control, and Digital Twin-based parameter identification are increasingly being incorporated into hybrid systems to update control strategies online and coordinate active inputs with passive devices. Reinforcement learning and Bayesian optimization are also being explored for controller tuning under changing operating conditions [23,44,106].

4.6. Intelligent Control and Data-Driven Control

Driven by rapid advances in artificial intelligence (AI), Digital Twin technology, and Scientific Machine Learning (SciML), vibration control of clearance-containing oscillating mechanisms has gradually evolved from conventional model-based approaches toward data-driven control, adaptive optimization, and intelligent decision-making. By integrating dynamic models, online monitoring data, and intelligent algorithms, intelligent control enables real-time updating of control parameters and prediction of system states, thereby providing new solutions for vibration suppression and stability enhancement under complex operating conditions [19–23].

In recent years, reinforcement learning (RL), deep neural networks (DNNs), Bayesian optimization, and Physics-Informed Neural Networks (PINNs) have been increasingly applied to vibration control of mechanical systems. Reinforcement learning optimizes control policies through interaction with the operating environment and has shown potential for improving controller adaptability in complex nonlinear systems. PINNs integrate governing equations into the neural network training process, enabling the fusion of physics-based and data-driven models and demonstrating considerable potential for control parameter identification, state prediction, and model updating. In parallel, Digital Twin technology establishes a real-time mapping between physical systems and virtual models, enabling online correction of dynamic models and adaptive optimization of control strategies according to operating conditions, thereby improving system stability and operational reliability [20,21,23,65,106].

Recent intelligent-control research increasingly integrates physics-based and data-driven approaches to improve physical consistency and adaptability. Digital Twins are also being combined with online monitoring and fault diagnosis for state perception and predictive control, while distributed and cooperative computing provides additional possibilities for lifecycle-oriented control of robotic, aerospace, and intelligent manufacturing systems [23,24,83,96,97,100].

4.7. Comparative Analysis and Future Trends

The reviewed strategies provide complementary rather than interchangeable capabilities. Passive control offers high reliability and low energy demand but limited adaptability; active control provides high control authority but relies on continuous power, sensing, and actuation; semi-active control offers lower-energy parameter adjustment; NES-based control provides broadband targeted energy transfer; hybrid systems combine passive robustness with active adaptability; and intelligent approaches enable online prediction and optimization but remain less mature in engineering validation [23,41,43–46,54–56].

As summarized in Figure 6 and Table 3, control selection should account for vibration bandwidth, impact severity, motion-accuracy requirements, available energy, model uncertainty, and implementation constraints. For strongly coupled clearance–flexibility systems, control design should simultaneously consider vibration attenuation, contact–impact mitigation, motion accuracy, reliability, and energy consumption. State estimation, parameter identification, and model updating may further improve robustness under time-varying clearance, wear, structural properties, and operating conditions [3,23,24,92,106].

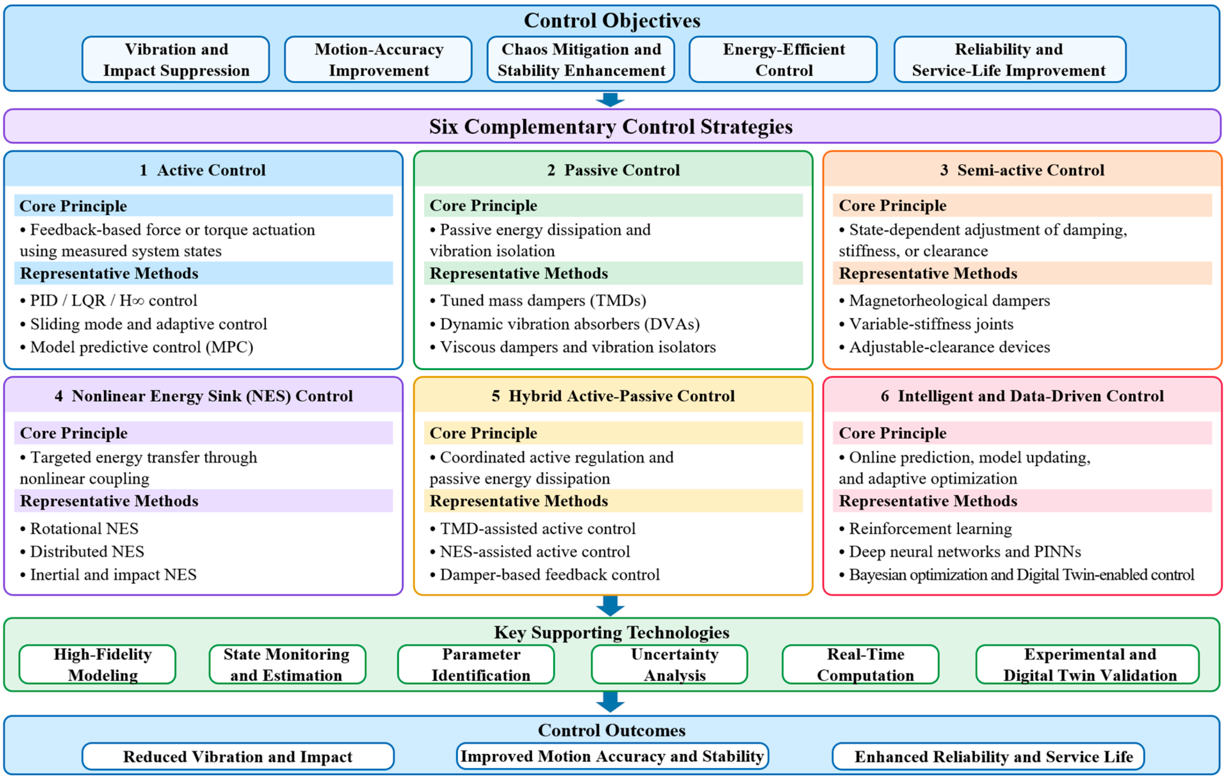

Figure 6. Comprehensive framework for vibration, impact, and chaos control of clearance-containing flexible oscillating mechanisms.

Table 3. Comparison of Vibration and Chaos Control Strategies for Clearance-Containing Flexible Oscillating Mechanisms.

Control Strategy	Principle	Target Systems	Major Advantages	Major Limitations	Representative Applications
Active Control	Sensors, controllers, and actuators generate real-time control forces or torques	High-speed, high-precision, and flexible mechanisms	High control authority, rapid response, and real-time adaptability	Dependence on reliable and continuous external power supply; sensitivity to actuator/sensor failures and model uncertainty; relatively high implementation cost	Aerospace mechanisms, robotic manipulators, precision servo systems
Passive Control	Dampers, isolators, or auxiliary masses dissipate or redistribute vibration energy	Systems with relatively stable operating conditions	Simple structure, high reliability, low maintenance, and no continuous external power requirement	Fixed parameters and limited adaptability to varying operating conditions	Tuned mass dampers, dynamic vibration absorbers, vibration isolators
Semi-active Control	Adjusts damping, stiffness, or clearance according to the measured system state	Flexible or clearance-related systems with varying operating conditions	Lower power demand than fully active control; adjustable mechanical properties; improved adaptability compared with fixed passive devices	Limited control authority; dependence on sensing and adjustment devices; device nonlinearities and limited validation in clearance-containing flexible mechanisms	Magnetorheological dampers, variable-stiffness joints, adjustable-clearance bearing systems
Nonlinear Energy Sink (NES) Control	Transfers vibration energy irreversibly through targeted energy transfer	Impact vibration, broadband vibration, and nonlinear systems	Broadband vibration suppression without precise frequency tuning; strong robustness	Complex parameter design and incomplete theoretical framework for strongly coupled engineering systems	Aerospace structures, flexible structures, rotating machinery
Hybrid Active–Passive Control	Combines active regulation with passive vibration isolation or energy dissipation	Multi-degree-of-freedom and complex coupled systems	Improved stability, vibration suppression, robustness, and control accuracy	Complex control architecture, parameter coordination, and additional energy requirements	Flexible robots, space structures, complex mechanical systems

Intelligent Control and Digital Twins	Integrates learning algorithms, physics-informed models, and Digital Twins for prediction and adaptive optimization	Systems with time-varying parameters, uncertainty, and multiphysics coupling	Online model updating, adaptive optimization, and state prediction	Limited engineering validation, computational cost, data dependence, and unresolved robustness issues	Intelligent robots, Digital Twin-enabled systems, smart manufacturing

Important implementation challenges remain, including reliable energy delivery and fault-tolerant hardware for active and hybrid systems, systematic parameter design for semi-active and NES-based methods, and physical consistency, real-time computation, robustness, and long-term validation for intelligent or Digital Twin-enabled control [19,20,23]. These broader research priorities are discussed further in Section 6. Experimental validation and engineering applications are considered next.

5. Advances in Experimental Validation and Engineering Applications

Experimental validation is essential for assessing dynamic-model accuracy and control effectiveness under manufacturing errors, material variability, contact uncertainty, and environmental disturbances [7,57–59]. Advances in high-precision testing, high-speed data acquisition, full-field measurement, and multisource sensing now enable quantitative characterization of contact–impact, rigid–flexible coupling, vibration, and motion accuracy under representative operating conditions. Engineering validation has consequently expanded across aerospace, robotics, precision manufacturing, and intelligent mechanical systems, as summarized in Figure 7 [57–59].

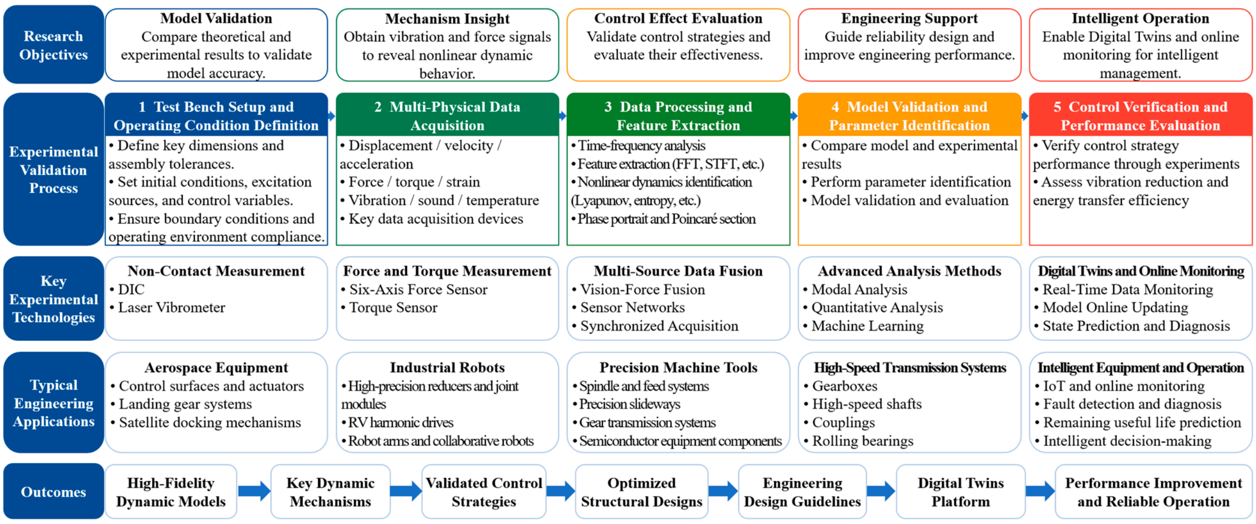

Figure 7. Comprehensive framework for experimental validation and engineering applications of clearance-containing flexible oscillating mechanisms.

5.1. Advances in Experimental Validation Methods

Experimental validation is essential for evaluating the accuracy of dynamic models and the effectiveness of control strategies and provides a critical basis for translating theoretical research into engineering applications. To investigate the complex contact–impact and nonlinear dynamic behavior of clearance-containing oscillating mechanisms, researchers have developed various experimental methods for the quantitative measurement of motion trajectories, impact forces, vibration responses, and energy transfer processes [57–59].

Early experimental studies primarily employed displacement sensors, accelerometers, and strain gauges to acquire the operating states of mechanisms, together with high-speed imaging systems to analyze impact processes within joints. With advances in measurement technology, Digital Image Correlation (DIC) has gradually been introduced into dynamic experiments on flexible mechanisms, enabling noncontact full-field measurement of structural displacement and strain and improving the measurement accuracy of flexible deformation and impact responses. Meanwhile, Laser Doppler Vibrometers (LDVs) and high-speed vision measurement systems have been widely used to characterize complex vibration responses, allowing high-accuracy identification of frequency characteristics, impact responses, and modal parameters [58,59].

In recent years, multisource sensor fusion has become an important direction in experimental validation. Force sensors, Inertial Measurement Units (IMUs), visual measurement systems, and acoustic emission sensors have been integrated to synchronously acquire information on impact processes, friction states, and dynamic parameters, thereby improving the identification of complex dynamic behavior. With the development of Digital Twin technology, experimental data are used not only for model validation but also for real-time correction of dynamic models, enabling online parameter updating and prediction of dynamic states [19,24,96,100].

Together, these developments have shifted experimental validation toward multisource, full-field, and online measurement frameworks for evaluating complex dynamic models and control strategies [7,58,59,100].

5.2. Advances in Experimental Platforms and Measurement Technologies

Experimental platforms for clearance-containing oscillating mechanisms mainly include joint-clearance dynamic test rigs, integrated flexible-mechanism platforms, robotic-joint test systems, and ground simulation systems for deployable space mechanisms. These platforms enable investigations at different scales and under adjustable clearance, speed, loading, and structural conditions [3,58–60].

Typical joint-clearance dynamic test rigs are equipped with high-precision servo drive systems, adjustable-clearance mechanisms, six-axis force sensors, and high-speed data-acquisition systems, enabling dynamic response measurements under different clearance sizes, operating speeds, and loads. For flexible oscillating mechanisms, laser vibrometry, Digital Image Correlation (DIC), and modal testing are commonly combined to characterize flexible deformation, impact responses, and energy dissipation. In recent years, visual measurement and online parameter identification have also been introduced into robotic dynamic test platforms, allowing the simultaneous evaluation of motion accuracy, joint clearance, and control performance [58,59,92].

Meanwhile, Digital Twin-based validation platforms have become an emerging research focus. By establishing real-time mappings between virtual models and physical systems, experimental data can be used to continuously update dynamic models and integrate experimental validation, parameter identification, and control optimization. Digital Twin platforms have been applied to robots, aircraft actuators, and intelligent manufacturing systems for dynamic-state prediction, fault diagnosis, and online optimization of control strategies, providing a new technical approach for experimental investigations of complex systems [19,24,83,96,100].

Experimental platforms are therefore becoming important interfaces among dynamic modeling, model updating, control optimization, and engineering validation [19,24,58].

5.3. Advances in Engineering Applications

Clearance-related dynamic modeling and control methods have been applied to aerospace systems, industrial robots, precision manufacturing, high-speed transmission systems, and intelligent equipment. In these applications, manufacturing tolerances, assembly errors, and wear make joint clearance an important factor affecting motion accuracy and operational reliability [1,3,4,57,59,78,93,137–140].

Aerospace engineering represents one of the most typical application areas of clearance dynamics. In satellite solar-array deployment mechanisms, aircraft actuators, landing gear, and space manipulators, joint clearance not only affects deployment accuracy but may also increase impact loads and attitude-control errors. In recent years, high-accuracy dynamic models combined with experimental testing have been used to systematically investigate deployable space mechanisms, flexible solar arrays, and aircraft servo mechanisms, enabling clearance-parameter optimization and lightweight structural design [3,5,45,46,59,60,93,116,139,140].

Industrial robots constitute another important application area. As robotic systems continue to develop toward higher speeds and greater precision, the effects of joint clearance and reducer backlash on trajectory accuracy and end-effector positioning errors have become increasingly prominent. Dynamic modeling, online parameter identification, and control compensation have been combined to improve joint-clearance identification and robotic motion accuracy, thereby supporting performance optimization of intelligent manufacturing systems [1,21,22,44,52,53,92,118,141].

Dynamic analysis and control methods have also been widely applied to high-speed gear transmissions, computer numerical control machine tools, wind-energy systems, and medical devices. Overall, engineering application research has evolved from isolated dynamic analysis toward an integrated technical framework combining design optimization, condition monitoring, service-life prediction, and intelligent maintenance [24,57,78,98].

5.4. Advances in Digital Twin-Based and Intelligent Validation

Digital Twin technology establishes a real-time connection between physical systems and virtual models, enabling online model updating, state prediction, and control optimization beyond conventional offline validation [19,24,83,96–102]. Applications in robotics, aerospace systems, intelligent manufacturing, and transmission systems increasingly combine operational data with dynamic models to assess vibration, wear, health state, and remaining useful life [24,83,96–100].

Integration with PINNs, reinforcement learning, and SciML further supports parameter learning and adaptive optimization and may reduce the dependence on repeated physical testing. However, the resulting models must still demonstrate physical consistency, computational efficiency, robustness, and reliable performance under complex operating conditions [20,21,23,104,122].

Digital Twin-based validation therefore provides an important bridge among theoretical modeling, experimental data, and engineering applications but should be regarded as an emerging complement to rather than a replacement for physical experiments [19,24,83,100].

As summarized in Table 4, engineering applications increasingly integrate dynamic analysis, experimental validation, Digital Twin-based model updating, and intelligent monitoring across aerospace, robotic, precision-manufacturing, high-speed transmission, and intelligent systems [3,24,58,59,92,96,100].

Table 4. Comparison of Experimental Validation and Engineering Applications of Clearance-Containing Oscillating Mechanisms.

Application	Typical Mechanisms	Dynamic Issues	Validation Methods
Aerospace systems	Space manipulators, solar-array deployment mechanisms, actuators, and landing gear	Joint clearance, rigid–flexible coupling, impact vibration, and deployment stability	High-speed photography, laser vibrometry, DIC, and modal testing

Industrial robots	RV reducer joints, collaborative robots, and serial robots	Backlash, trajectory errors, and joint impacts	Six-axis force sensing, visual measurement, and encoders
Precision manufacturing systems	CNC machine tools, precision positioning platforms, and automated assembly systems	Micro-clearance, micro-vibration, and positioning errors	Laser interferometry, DIC, and vibration testing
High-speed transmission systems	Gear transmissions, bearing systems, and couplings	Contact–impact, meshing vibration, and wear evolution	Vibration testing, acoustic emission, and spectral analysis
Intelligent systems and operation and maintenance	Intelligent production lines, Digital Twin-enabled systems, and intelligent inspection platforms	Parameter drift, multisource information fusion, and state prediction	Multisensor fusion, online monitoring, and Digital Twins

5.5. Comparative Analysis and Development Trends in Engineering Applications

Experimental validation and engineering applications provide an essential link among dynamic theory, vibration control, and practical implementation. Advances in high-speed measurement, DIC, laser vibrometry, multisource sensing, and Digital Twin technologies have extended validation from conventional parameter measurements toward full-field measurement, online monitoring, and integrated analysis [7,19,58,59].

These developments support model validation, parameter identification, vibration-control assessment, and reliability analysis in aerospace systems, industrial robots, precision manufacturing, high-speed transmission systems, and intelligent equipment. Digital Twins and intelligent technologies further enable online model updating, state prediction, and lifecycle-oriented monitoring, although their long-term engineering validation remains limited [1,3,24,57,59,78,92,93,96,137].

Major challenges therefore include reproducing complex service conditions, integrating multisource data, standardizing experimental platforms, and establishing long-term validation procedures. These issues motivate closer coordination among dynamic modeling, experiments, Digital Twins, and intelligent control; they are discussed further in Section 6.

6. Challenges and Future Research Directions

The preceding review identifies several cross-cutting challenges in the modeling, nonlinear analysis, control, and experimental validation of clearance-containing flexible oscillating mechanisms. Multiple clearances, multiple flexible bodies, multiphysics interactions, and time-varying operating conditions substantially increase modeling and control complexity, while Digital Twins, SciML, PINNs, and intelligent optimization introduce new opportunities together with requirements for physical consistency, real-time computation, and engineering validation. The following subsections therefore focus on four priorities: high-accuracy multiphysics modeling, uncertainty-aware nonlinear prediction, coordinated vibration control, and long-term experimental and engineering validation.

6.1. High-Accuracy Dynamic Modeling and Multiphysics Coupling

High-fidelity dynamic modeling remains fundamental to nonlinear analysis, vibration control, and engineering application. Although rigid-body, FMBD, rigid–flexible coupled, and data-driven approaches have advanced considerably, unified representation of clearance, structural flexibility, friction, lubrication, thermal deformation, wear, and uncertainty under realistic service conditions remains incomplete.

Future developments in dynamic modeling should focus on several aspects. First, unified multiphysics dynamic models should be established by incorporating contact–impact, heat transfer, lubrication characteristics, wear evolution, and material nonlinearities into a common analytical framework, enabling high-accuracy characterization of dynamic behavior under complex service conditions. Second, multiscale modeling theories should be further developed to establish relationships between microscopic contact behavior and macroscopic dynamic responses, thereby improving the predictive capability of models for complex mechanisms. In addition, advances in high-performance computing should promote the application of model-order reduction, parallel computing, and adaptive solution algorithms, improving computational efficiency while maintaining accuracy and supporting real-time dynamic analysis of large-scale complex systems.

The development of Digital Twins, PINNs, and Scientific Machine Learning also provides new opportunities for dynamic modeling. Future research should combine the physical interpretability of physics-based models with the rapid learning capability of data-driven models, promoting deeper integration among physics-based models, experimental data, and data-driven learning models. This integration will facilitate the development of next-generation dynamic models with online updating, adaptive correction, and autonomous learning capabilities, enabling the transition of complex mechanical systems from offline modeling to real-time prediction.

6.2. Nonlinear Dynamic Analysis and Intelligent Prediction

Nonlinear dynamic analysis must increasingly address uncertainty and time-varying operating conditions rather than deterministic responses alone. Although vibration, bifurcation, chaos, and nonsmooth dynamics have been extensively studied, the evolution of nonlinear states under random clearances, multisource disturbances, wear, and long-term service conditions remains insufficiently understood. Jump resonance is another important nonlinear phenomenon that deserves attention in forced nonlinear systems. Recent studies have shown that nonlinear systems may exhibit multiple hysteretic jump resonances, with abrupt transitions among coexisting response branches as excitation conditions vary [142,143]. These results provide a useful perspective for interpreting abrupt amplitude changes, multistability, and stability transitions in strongly nonlinear systems, although their direct extension to clearance-containing flexible mechanisms requires further investigation.

Future research should establish nonlinear dynamic analysis frameworks for complex operating conditions by integrating stochastic dynamics, reliability analysis, and data-driven methods to provide a unified description of complex dynamic behavior. Recurrence analysis, complex network analysis, information entropy, and time–frequency analysis should also be further integrated to improve nonlinear-state identification and characterization of dynamic evolution. Moreover, by combining Digital Twins with online monitoring data, real-time identification and prediction models should be developed to shift dynamic analysis from post-event assessment toward predictive evaluation, thereby providing theoretical support for fault warning and health management.

6.3. Vibration Control and Autonomous Optimization

Current vibration-control approaches still face limited adaptability to parameter variation and complex operating conditions, and active, passive, semi-active, and NES-based methods are often designed independently. Coordinated control frameworks that jointly consider dynamic models, energy availability, sensing reliability, clearance evolution, and structural uncertainty therefore remain necessary.

For control design, a more immediate priority is to coordinate vibration suppression with energy availability, sensing reliability, and clearance evolution. This is particularly important for semi-active and hybrid systems operating under time-varying conditions. In particular, advances in Digital Twins, reinforcement learning, and Model Predictive Control (MPC) provide new technical pathways for real-time control of complex mechanical systems. Integrating multisource sensor data with intelligent algorithms can enable dynamic adjustment of control parameters and improve system stability and robustness under complex operating conditions. Further attention should also be devoted to broadband vibration suppression, multi-objective optimization, and low-energy control to simultaneously improve vibration attenuation, motion accuracy, and energy efficiency.

6.4. Experimental Validation, Digital Twins, and Engineering Applications

Experimental validation remains an essential foundation for the development of dynamic theory and a critical step in transferring research outcomes to engineering applications. Recent advances in high-speed imaging, Digital Image Correlation (DIC), laser vibrometry, multisource sensing, and Digital Twin technology have improved the validation of dynamic models and the monitoring of operating states. However, current studies remain limited in their ability to reproduce complex operating conditions, integrate multisource data, and conduct long-term service validation, leaving a gap between theoretical models and practical engineering systems.

Long-duration experiments are particularly needed to determine how clearance growth, wear, temperature, and repeated impacts affect model accuracy and control performance over service time. Such data would also provide a more credible basis for Digital Twin model updating. Experimental validation, Digital Twins, and online monitoring should be deeply integrated to enable continuous updating of dynamic models and real-time assessment of operating states. Long-term validation should also be strengthened in robotics, aerospace engineering, precision manufacturing, and intelligent systems to improve the engineering credibility and transferability of theoretical and experimental research.

6.5. Summary

The principal future priorities are high-fidelity multiphysics modeling, reliable prediction under uncertainty, coordinated vibration control, and long-term experimental and engineering validation. Progress will require closer integration of modeling, condition monitoring, prediction, control, and experiments. Digital Twins, PINNs, SciML, and intelligent optimization may support this integration, but their effectiveness must ultimately be demonstrated through physically consistent models and representative long-term validation.

7. Conclusions

This review synthesizes the principal advances in the nonlinear dynamics and vibration control of clearance-containing flexible oscillating mechanisms from the perspectives of dynamic modeling, nonlinear response analysis, vibration control, experimental validation, and engineering applications. The reviewed studies demonstrate that joint clearance cannot be treated independently of structural flexibility, contact–impact, friction, wear, lubrication, and operating uncertainty, because their interactions determine vibration responses, stability boundaries, motion accuracy, and long-term reliability.

In dynamic modeling and nonlinear analysis, rigid-body and contact–impact formulations remain important for efficient mechanism-level analysis, whereas flexible multibody and rigid–flexible coupled approaches provide higher fidelity when structural deformation becomes significant. Multiphysics modeling further improves the representation of thermal, lubrication, wear, and uncertainty effects. Bifurcation, chaos, nonsmooth dynamics, and reliability analyses complement conventional vibration-response analysis by identifying critical transitions, stability boundaries, and parameter sensitivities that cannot be captured by time-domain responses alone.

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For vibration suppression, passive, active, semi-active, NES-based, hybrid, and intelligent strategies provide complementary capabilities with different requirements for energy supply, model accuracy, robustness, bandwidth, and implementation complexity. NESs are particularly attractive for broadband energy transfer, whereas active and hybrid strategies offer greater adaptability when reliable sensing, actuation, and external power are available. Experimental techniques, including high-speed imaging, Digital Image Correlation, laser vibrometry, and multisensor measurements, remain essential for evaluating model credibility and control effectiveness under realistic operating conditions.

A clear difference in technology readiness should nevertheless be recognized. Contact–impact modeling, flexible multibody dynamics, and conventional passive and active control have relatively mature theoretical and experimental foundations. In contrast, PINNs, reinforcement learning, Scientific Machine Learning, and Digital Twin-enabled prediction and control continue to emerge for clearance-containing flexible mechanisms, particularly with respect to physical consistency, real-time computation, robustness, and long-term experimental validation. Future progress therefore depends not simply on introducing more intelligent algorithms but on establishing experimentally validated closed-loop frameworks that integrate high-fidelity modeling, state perception, uncertainty-aware prediction, energy-efficient control, and lifecycle engineering validation.

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