

Article

Method of Embankment Stability Analysis Taking into Account the Change in Undrained Shear Strength Caused by the Principal Stress Rotation in the Subsoil

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Abstract

This study investigates the selection of shear strength parameters for cohesive soil subsoils in the stability analysis of embankments in undrained conditions. It specifically examines how rotation of the principal stresses, induced by embankment construction, affects the undrained shear strength of the subsoil. Laboratory tests were conducted using a hollow cylinder apparatus on two types of cohesive soils to determine undrained shear strength at principal stress rotation angles of 0° , 15° , 30° , 45° , 60° , 75° , and 90° . Analysis of the laboratory results, combined with numerical modelling, enabled the development of a new stability analysis method that incorporates the influence of principal stress rotation on undrained shear strength of subsoil.

Keywords: embankment stability analysis; safety factor; undrained shear strength; principal stress rotation; subsoil

1. Introduction

The analysis of embankment stability is a broad field within civil engineering and an important topic of research. There are many factors that have not yet been fully elucidated and that influence the stability of earthworks to varying degrees, yet are overlooked or simplified in existing calculation methods. Consequently, ongoing research into embankment stability analysis aims to ensure that stability calculations reflect actual conditions as accurately as possible, allow for the optimal design of the planned earthwork structure, whilst maintaining an adequate safety margin.

Currently, there are many methods for analysing embankment stability, which determine the stability factor with varying degrees of accuracy [1,2]. Methods based on limit equilibrium analysis [3] and numerical methods based on the finite element method [4] are commonly used. The former methods were based on simple analyses due to the lack of advanced computational tools. The development of computers has led to the emergence of stability analysis methods that allow consideration of a greater number of factors affecting stability, thereby achieving more accurate calculation results.

In the preliminary design stage of an embankment, stability can be assessed using simple formulas or nomograms [5]. The safe height of the embankment can be estimated using Terzaghi's formula [6] or Taylor's stability nomogram [7], as well as the procedures proposed by Poulos and his team [8]. Limit equilibrium methods are commonly used for stability analysis [3,8–10]; however, these methods require separating the assessment of



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changes in soil properties during loading from that of equilibrium conditions. A comprehensive description of soil behaviour under loading is enabled by numerical methods based on the finite element method [4,11–14], which account for the non-linear stress–strain relationship in their calculations. These are becoming increasingly common engineering practices in the design of embankments and other civil engineering structures. Stability analysis can also be carried out using methods of plasticity theory, in which the boundary condition allows the slope contour in equilibrium to be determined ([15], based on Sokołowski’s 1960 publication), as well as using kinematic methods (KEMs) belonging to a group of methods derived from plasticity theory [16]. Probabilistic methods are also used (e.g., [17–21]), in which factors influencing slope stability are treated as random variables. Recently, artificial neural networks and genetic algorithms based on relationships between individual parameters have also been gaining popularity (e.g., [22–25]).

When assessing the reliability of the results, it is important to bear in mind that, in addition to the accuracy of the calculation method, a detailed investigation of the subsoil conditions and precise modelling of its behaviour in response to changes in stress caused by loading are paramount.

Cohesive soils are characterised by varying undrained shear strength and exhibit different deformations under applied load, depending on their consistency. Incorrect selection of the loading rate and construction rate increases pore water pressure by only a slight amount, resulting in a slight increase in effective stress, which may ultimately lead to a loss of stability in the embankment under construction.

The stability analysis is carried out according to characteristic load paths corresponding to the individual phases of the embankment’s construction and operation. From a practical point of view, the most critical conditions occur during the construction stage; however, a stability assessment is also required after construction, in accordance with diagrams representing live loads and additional loads anticipated during the operational period.

When analysing the stability of an embankment, attention should also be paid to the fact that the deformations causing damage are not uniform across the entire slip surface at any given moment in time. Deformations are progressive in nature and differ at the toe of the embankment slope and at the crest of the embankment at a given time (Figure 1) [26]. This highlights the need to account for the non-uniform distribution of failure-inducing deformations along the slip surface in embankment stability analysis.

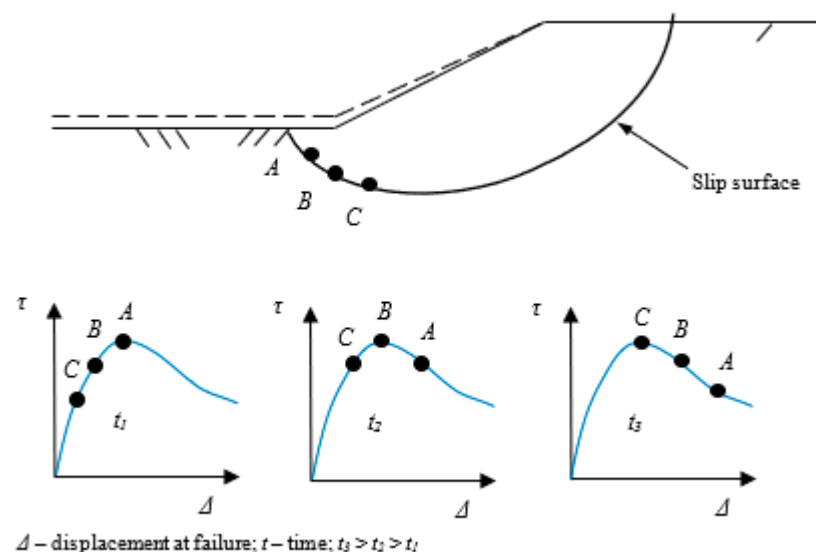


Figure 1. Distribution of failure-inducing strains along a potential slip surface [26].

Tschuchnigg [27] points out that in embankment stability analyses, the directions and magnitudes of displacements, as well as the velocity vectors of these displacements, must also be considered, as time-varying displacements affect the stability of the entire structure.

The shear strength of cohesive soil is one of the most important characteristics in embankment design. This parameter is determined on the basis of laboratory or field tests, the methods for which have undergone significant development over the years; as a result, the available measuring instruments meet the highest standards of accuracy. A change in the directions of the principal stresses in the subsoil occurs when the directions of the increments in principal stresses arising from a change in the stress state of the subsoil, caused by loading or unloading, do not coincide with the original directions of these stresses [28,29]. Consequently, the directions of all three principal stresses deviate from their original directions by angles denoted respectively as α , β and γ (Figure 2). This phenomenon is observed during the construction of almost all types of geotechnical structures [30]. Given the focus of this study, only the change in the directions of the principal stresses in the subsoil caused by embankment construction was subjected to further analysis. One method for determining the directions of the principal stresses in the subsoil is to perform a numerical analysis using the finite element method. Jardine and Menkiti [31] presented isolines of the rotation angles of the principal stress directions in the subsoil, resulting from the construction of an embankment on a subsoil composed of soft plastic clay (Figure 3). As shown by the calculations, changes in the stress state of the subsoil caused by the construction of a 3 m high embankment do not occur only immediately below the subsoil surface, but extend to depths of up to 10 m. Based on the contour lines presented, it is also possible to identify zones along the potential slip surface where soil failure occurs at a specific value of α .

The phenomenon of a change in the directions of the principal stresses in the subsoil following loading by an embankment leads to the formation of different soil shear patterns, which contribute to the development of anisotropy in undrained shear strength. In the literature, this phenomenon is most commonly referred to as induced anisotropy [32,33]. Of course, even under natural conditions, soil is usually an anisotropic material in terms of its mechanical and physical properties, which results, among other things, from the arrangement of soil particles, the nature of the contact between them, and the pore distribution. This phenomenon is known as structural anisotropy. Distinguishing the effects of both types of anisotropy is a difficult task; therefore, the focus of discussion is usually on so-called combined anisotropy [34]. However, if the soil in the subsoil is homogeneous in terms of its physical properties, a change in the directions of the principal stresses is the primary factor causing the anisotropy of undrained shear strength. In this case, determining the change in shear strength using appropriate testing equipment poses no problem. The rotation of principal stress directions in the soil is the rule rather than the exception. Therefore, the issue of changes in undrained shear strength should not be overlooked when modelling soil behaviour under load. Regardless of factors such as soil type, stress history, embankment geometry, or loading rate, the phenomenon of rotation of principal stress directions occurs in the subsoil and affects the undrained shear strength as well as the stability of the embankment. Existing methods of stability analysis, whether based on limit equilibrium methods or soil constitutive equations used in the finite element method, mostly do not account for changes in the undrained shear strength of the soil resulting from the rotation of principal stress directions and therefore require modification.

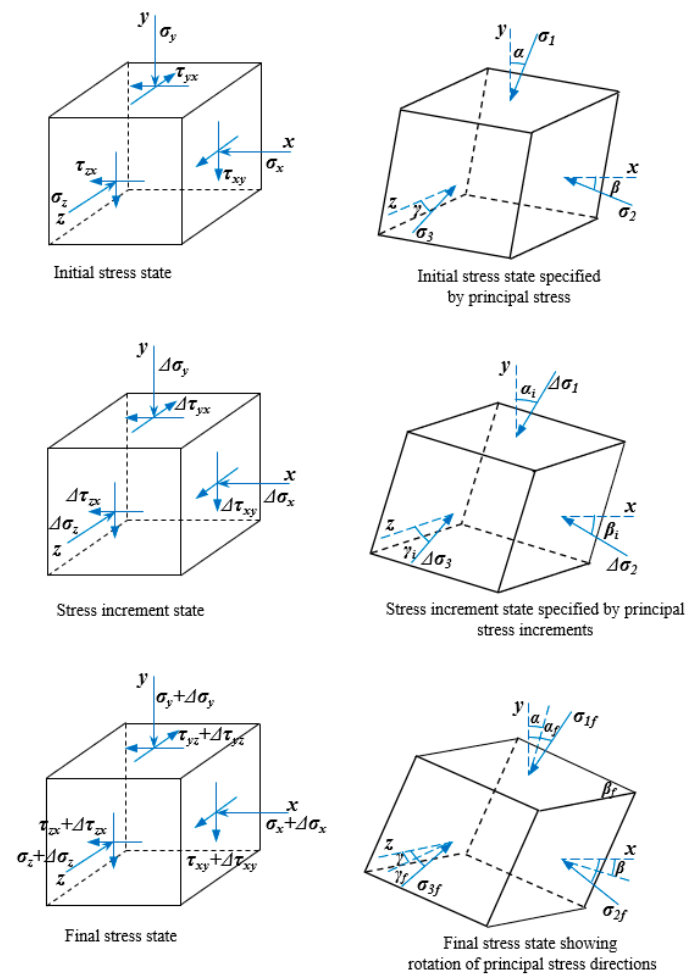


Figure 2. The phenomenon of changes in the direction of principal stresses at any point in the subsoil [28].

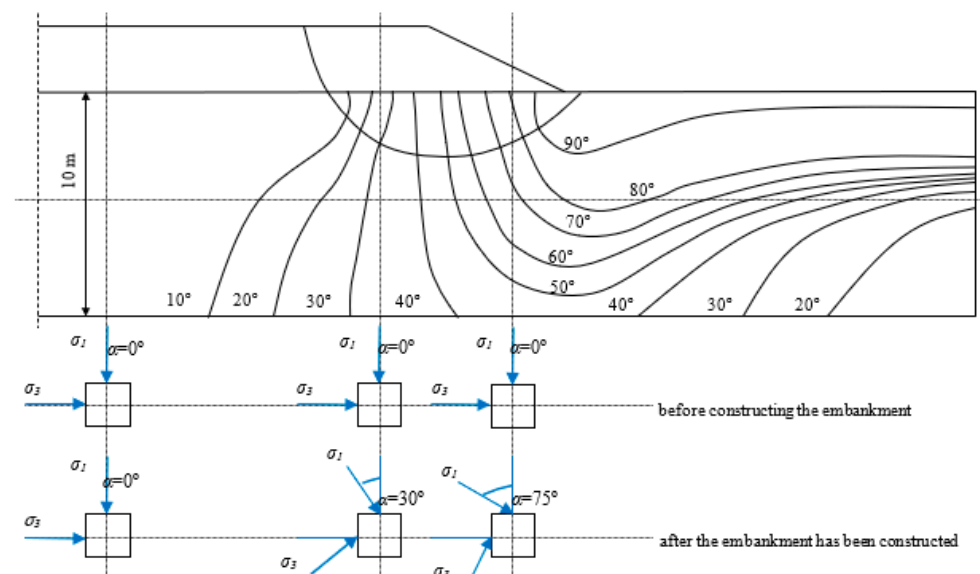


Figure 3. Isolines of the angles of the principal stress rotation in the embankment subsoil [31].

The subject of this paper is the influence of changes in undrained shear strength caused by the principal stress rotation in the subsoil on embankment stability. Consequently, the anisotropy of undrained shear strength arising from the loading of the subsoil by the embankment was subjected to further consideration. To this end, laboratory tests were

carried out in a hollow cylinder apparatus to determine the shear strength at various angles of rotation of the principal stress directions. Based on the tests conducted, a method was proposed to determine the embankment stability factor, accounting for variation in undrained shear strength in the subsoil. The results of the research presented in this paper indicate that a change in the directions of the principal stresses in the subsoil significantly affects the value of the undrained shear strength. The phenomenon of rotation of the principal stress directions in the subsoil loaded by an embankment is inevitable, and existing methods for analysing embankment stability do not provide a clear way to account for its effects in calculations. In light of the considerations regarding embankment stability analysis and undrained shear strength, a new method for analysing embankment stability is presented, which accounts for the influence of principal stress directions on the undrained shear strength of cohesive subsoil. The novelty of the proposed method lies in comparing the values of shear stress τ along the potential slip surface with the values of the limit shear stress τ_F (corresponding to τ_{fu}). The values of τ_F take into account changes resulting from the rotation of the principal stress directions. The approach presented in this article distinguishes the proposed method for analysing embankment stability from existing, widely known calculation methods. Conventional limit equilibrium methods and standard finite element analyses generally assume that the undrained shear strength remains constant within a given soil layer and do not explicitly account for strength degradation due to principal stress rotation. In contrast, the proposed approach incorporates experimentally determined reductions in undrained shear strength associated with different principal stress rotation angles and directly includes these changes in the calculation of the stability factor. As a result, the method enables a more realistic representation of the stress conditions developing in cohesive subsoil during embankment construction.

The following sections of this paper describe the types of soil for which physical property tests and undrained shear strength tests were conducted. The tests were conducted using a hollow cylinder apparatus on two selected types of cohesive soils to determine the undrained shear strength at principal stress rotation angles of 0° , 15° , 30° , 45° , 60° , 75° , and 90° . In the next section of the paper, an analysis of the laboratory results was conducted in conjunction with numerical modelling. Furthermore, a method for stability analysis was proposed, taking into account the influence of principal stress rotation on the undrained shear strength of the subsoil.

It is hypothesised that neglecting rotation of principal stress directions in embankment stability analysis leads to a systematic overestimation of the factor of safety due to an overprediction of undrained shear strength in cohesive subsoils. Furthermore, it is assumed that incorporating stress-rotation-induced strength anisotropy into stability calculations results in a more realistic prediction of embankment safety under complex loading paths.

2. Materials and Methods

The tests were conducted on two cohesive soils with undisturbed structures. The average values of the soil physical properties tested are summarised in Table 1.

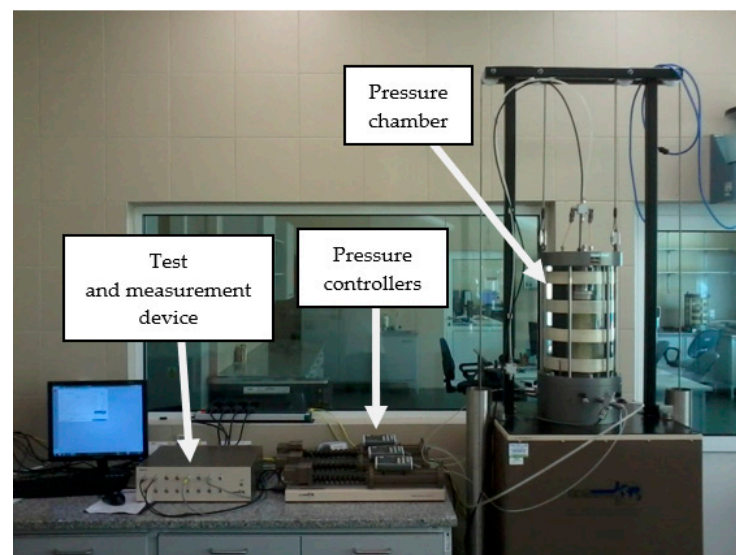
The results of the granulometric composition tests indicate that the soils analysed, in accordance with PN-EN ISO 14688-2:2006 [35] and PN-EN ISO 14688-2:2006/ Ap2:2012 [36], are sandy silty clays (sasiCl). In accordance with the PN-EN ISO 14688-2:2006 standard, sandy silty clays are characterised by a sand fraction content in the range of 20–22%, a silt fraction in the range of 49–53% and a clay fraction in the range of 26–30%.

Table 1. Summary of average physical properties of the cohesive soils tested.

Type of Soil	w_n [%]	w_L [%]	w_p [%]	I_p [%]	I_L [-]	I_C [-]	ρ [t/m ³]	ρ_d [t/m ³]	OCR [-]
sasiCl (1)	28.7	59.0	24.3	34.7	0.13	0.87	1.99	1.55	2.7
sasiCl (2)	29.6	44.7	17.3	27.4	0.45	0.55	2.03	1.57	2.2

where w_n —water content; w_L —liquid limit; w_p —plastic limit; I_p —index plasticity; I_L —plasticity index; I_C —consistency index; ρ —density; ρ_d —relative density; OCR—pre-consolidation ratio.

The tests carried out in the hollow cylinder apparatus (Figure 4) included HCACAU tests (with anisotropic consolidation and shear under undrained conditions), which were used to determine the undrained shear strength at selected rotation angles of the principal stress directions. These tests made it possible to model the failure mechanisms occurring in the subsoil as a result of loading it with an embankment. The tests were conducted for 7 different values of α : 0°, 15°, 30°, 45°, 60°, 75°, and 90°. These tests offer an advantage over other devices in which the undrained shear strength is determined only for a single selected failure mechanism.

**Figure 4.** Hollow cylinder apparatus-view of the test setup.

In the hollow cylinder apparatus, soil samples in the form of a hollow cylinder with a height of 200 mm, an external diameter of 100 mm and an internal bore diameter of 60 mm were tested. The samples were prepared in a machine tool using a set of cutting blades and a steel wire. The preparation of the hollow-cylinder-shaped soil sample began with a cylindrical soil sample with a diameter of 100 mm and a length of approximately 215 mm. The cylindrical sample was then placed in a two-part steel mould lined on the inside with a thin plastic sheet, inserted into the machine tool and secured by applying two-part clamps from below and above, tightened with screws. The hole was drilled along the axis of the cylinder using a drill bit fitted with an adjustable-diameter cutting blade. The hole was drilled in 25 consecutive stages of varying diameters. Drilling began at 12 mm and increased by 2 mm each time until 60 mm was reached. A steel wire was also used to drill the hole without significantly disturbing the soil structure. Drilling the hole took approximately 8–10 h of continuous work. After drilling a 60 mm diameter hole, the sample was removed from the machine and trimmed using a steel wire and a specialised cutter to a target length of 200 mm. The samples were cut in a constant-humidity chamber, which limited the effects of time and temperature on the samples.

Tests in the hollow cylinder apparatus consisted of the following successive stages: preliminary saturation, i.e., flushing; actual saturation; consolidation; changing the intermediate principal stress parameter b ; changing the angle of rotation of the principal stress directions α ; and shearing. The parameter b was varied from 0 to 0.5 or 0.3, depending on the test, so that the heterogeneity of the stress state in the specimen caused by the dimensions and curvature of the specimen under test would have as little effect as possible on the results obtained [28,37]. This stage lasted 24 h. The next stage of the tests also lasted 24 h and involved changing the rotation angle of the principal stress directions α . The tests were carried out at α values of 0° , 15° , 30° , 45° , 60° , 75° , and 90° . Once the pressures in the specimen had been equalised, shearing was commenced. The specimens were sheared under steady-state conditions with a stress path characterised by an increase in the stress deviator q and a constant mean total stress p , whilst the intermediate principal stress parameter b and the angle of rotation of the principal stress directions α remained constant. The constant angle α during shearing of the specimen enabled the implementation of stress paths in various planes of the q - p - α space. The selected sample before and after testing in the cylindrical torsion apparatus is shown in Figure 5.

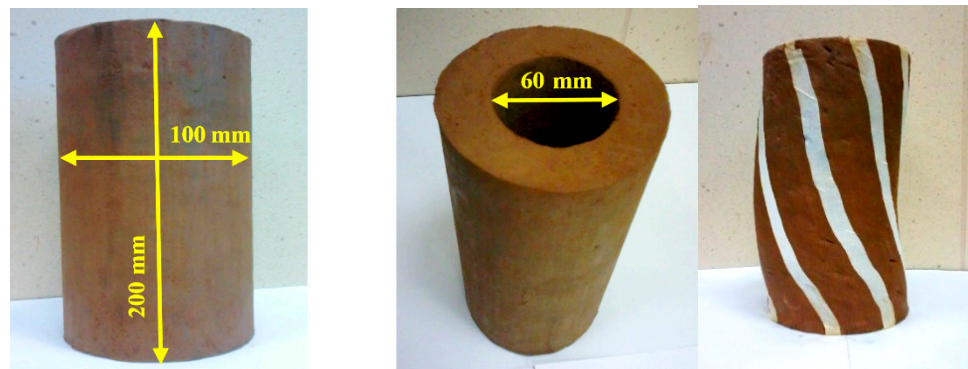


Figure 5. Selected sample before and after testing in a hollow cylinder apparatus.

The control and monitoring of the individual stages were carried out using a computer with GDSLab software, a digital measuring unit, and pressure transducers within the specimen, outside the specimen, and inside the borehole. For each angle of rotation of the principal stress directions, one baseline test and one verification test were performed. Similar results were obtained in all the repetitions, confirming the validity of the tests and the results obtained.

The selected total stress paths in the principal stress space σ_1 - σ_2 - σ_3 , total stress paths in the q - p - α space, and effective stress paths in the t - p' - α space are shown in Figures 6–8. Based on the obtained characteristics, individual undrained shear strength values were determined for the tested soils using the maximum stress deviator and the ratio of the largest to the smallest effective principal stress as failure criteria. The results of the undrained shear strength values obtained for both failure criteria analysed were similar. For further analysis, the maximum ratio of principal stresses was adopted as the failure criterion.

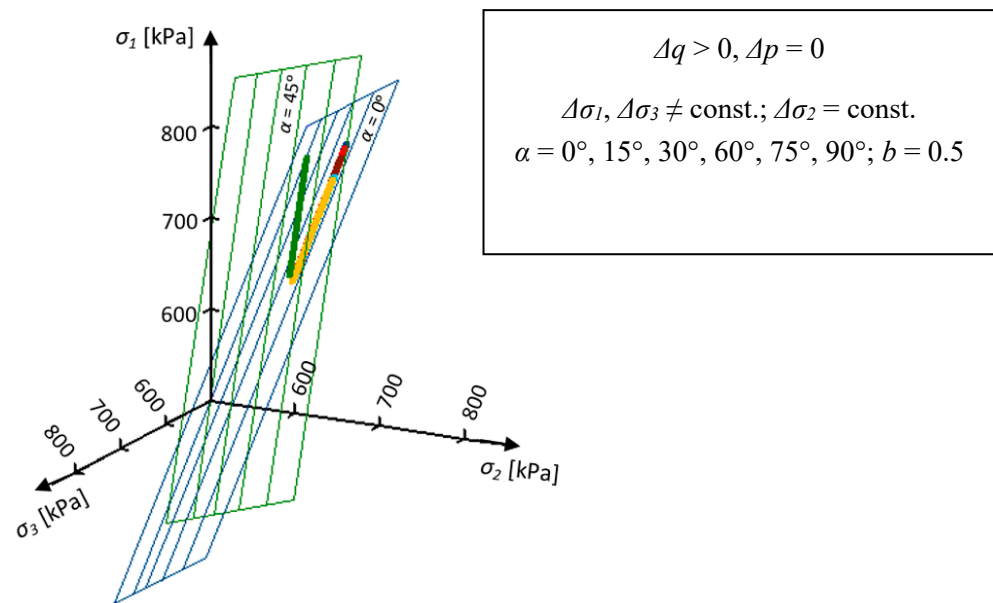


Figure 6. Total stress paths relative to the principal stress space from tests on sandy silty clay—sasiCl (1).

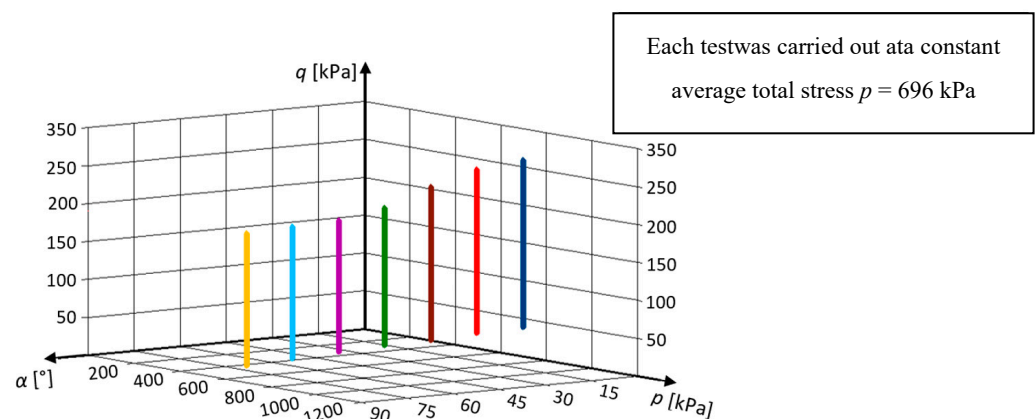


Figure 7. The total stress paths in q - p - α space from tests on sandy silty clay—sasiCl (1).

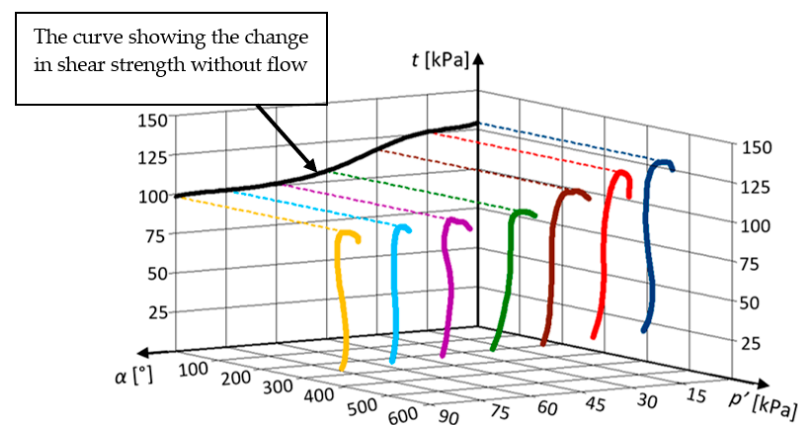


Figure 8. Effective stress paths in the t - p' - α space from tests on sandy silty clay—sasiCl (1).

3. Results

Tables 2 and 3 summarise the values of undrained shear strength τ_{fu} and normalised undrained shear strength τ_{fu}/σ'_{vo} obtained for individual soil types, together with the corresponding vertical deformations, assuming specific failure criteria. The results obtained

are also presented as a relationship between the normalised undrained shear strength and the angle of rotation of the principal stress directions (Figures 9 and 10).

Table 2. Results of tests on samples of sandy silty clay sasiCl (1) carried out in a hollow cylinder apparatus.

Rotation Angle of Principal Stress Directions α [°]	Failure Criterion	Undrained Shear Strength τ_{fu} [kPa]	Normalised Undrained Shear Strength τ_{fu}/σ'_{vo} [-] $\sigma'_{vo} = 220$ kPa	Vertical Strain ε_z [%]
0	Maks. σ'_1/σ'_3	128.0	0.582	7.1
15	Maks. σ'_1/σ'_3	123.1	0.560	6.0
30	Maks. σ'_1/σ'_3	115.6	0.525	7.9
45	Maks. σ'_1/σ'_3	104.3	0.474	8.6
60	Maks. σ'_1/σ'_3	95.8	0.435	9.1
75	Maks. σ'_1/σ'_3	95.3	0.433	8.9
90	Maks. σ'_1/σ'_3	93.3	0.425	7.5

Table 3. Results of tests on samples of sandy silty clay sasiCl (2) carried out in a hollow cylinder apparatus.

Rotation Angle of Principal Stress Directions α [°]	Failure Criterion	Undrained Shear Strength τ_{fu} [kPa]	Normalised Undrained Shear Strength τ_{fu}/σ'_{vo} [-] $\sigma'_{vo} = 220$ kPa	Vertical Strain ε_z [%]
0	Maks. σ'_1/σ'_3	129.3	0.588	9.4
15	Maks. σ'_1/σ'_3	125.8	0.572	10.3
30	Maks. σ'_1/σ'_3	117.7	0.535	11.2
45	Maks. σ'_1/σ'_3	106.6	0.485	14.4
60	Maks. σ'_1/σ'_3	101.4	0.461	14.9
75	Maks. σ'_1/σ'_3	99.7	0.454	13.4
90	Maks. σ'_1/σ'_3	98.4	0.447	12.3

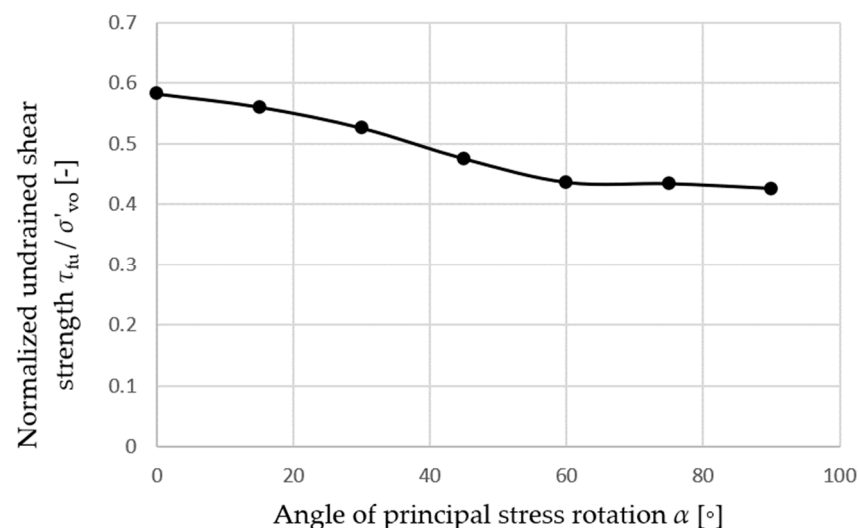


Figure 9. Variation in the standardised undrained shear strength as a function of the angle of principal stress rotation for sasiCl (1).

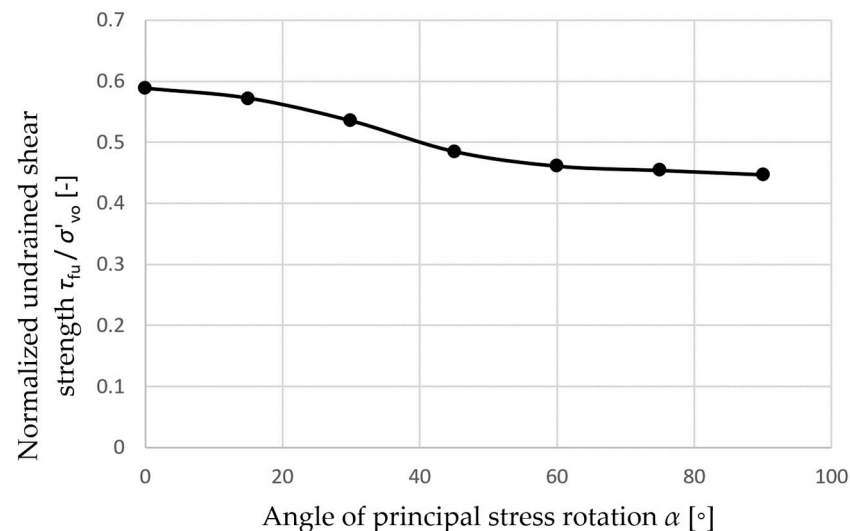


Figure 10. Variation in the standardised undrained shear strength as a function of the angle of principal stress rotation for sasiCl (2).

The results obtained in the present study confirm that principal stress rotation has a substantial influence on the undrained shear strength of the tested cohesive soils. For both analysed materials, the most significant reduction in normalised undrained shear strength occurred within the range of $\alpha = 0\text{--}45^\circ$, after which the rate of strength degradation gradually decreased, indicating a tendency towards stabilisation of the soil response at higher stress rotation angles.

For the sasiCl (1) soil, the undrained shear strength decreased by 19% at $\alpha = 45^\circ$ and by 27% at $\alpha = 90^\circ$ relative to the reference triaxial compression condition ($\alpha = 0^\circ$). Comparable reductions were observed for sasiCl (2), amounting to 18% and 24%, respectively. These results indicate that the assumption of constant principal stress orientation, commonly adopted in conventional triaxial testing, may lead to an overestimation of the in situ undrained shear strength of cohesive soils subjected to complex stress paths.

The observed strength degradation is consistent with the findings of Lade and Kirk-gard [38], who demonstrated that principal stress rotation can reduce the undrained shear strength of overconsolidated clays by up to approximately 30%, depending on the imposed stress path. Similarly, Nishimura et al. [39] reported significant reductions in clay stiffness and strength under rotational shear loading, attributing this effect to progressive destructuration and anisotropic rearrangement of the soil fabric. Furthermore, Jardine [37] highlighted that principal stress rotation is one of the governing mechanisms responsible for discrepancies between laboratory-derived and field-observed undrained shear strength parameters in foundation and offshore geotechnical engineering.

Mechanistically, the reduction in undrained shear strength observed in the present study may be attributed to the progressive breakdown of inherent soil fabric anisotropy and degradation of interparticle bonding caused by non-coaxial stress paths. As noted by Qian et al. [40], rotation of principal stresses induces additional plastic strains even at constant stress magnitude, leading to cumulative damage within the soil skeleton and reduced mobilisation of peak shear resistance. This phenomenon is particularly relevant for naturally structured cohesive soils, where bonding and particle orientation contribute significantly to peak undrained strength.

The slightly lower sensitivity of sasiCl (2) to principal stress rotation compared to sasiCl (1) may indicate differences in mineralogical composition, particle arrangement, or microstructural anisotropy. Similar material-dependent responses were reported by

Nakai and Mihara [41], who showed that the influence of stress rotation becomes more pronounced in soils exhibiting stronger inherent anisotropy and more structured fabric.

From an engineering perspective, the results presented demonstrate that neglecting principal stress rotation effects in geotechnical design may lead to an unconservative overestimation of undrained shear strength and bearing capacity, particularly in problems involving complex loading paths, such as foundations under combined loading, embankment toes, retaining structures, and offshore geotechnical structures. Therefore, incorporating stress-rotation-dependent strength degradation into constitutive modelling and geotechnical design methodologies should be considered in advanced engineering analyses.

4. Proposed Techniques of Determination of Embankment Stability

4.1. Proposed Method

The research shows that changes in undrained shear strength due to rotation of the principal stresses in the subsoil must be accounted for in embankment stability analyses. To account for the phenomenon under analysis, a method for determining the embankment stability factor has been proposed, which requires a numerical analysis using the finite element method. To this end, a computational model must be developed that accurately reflects field conditions. In the computational model, the geometry of the embankment and the subsoil layers must be defined, a constitutive model of the soil that optimally describes the behaviour of the embankment and the subsoil layers must be selected, geotechnical parameters for the embankment and the individual subsoil layers must be entered, determine the boundary and initial conditions, discretise the created computational model into finite elements, define the soil–water conditions, specify the initial stress state, and adopt a loading path consistent with the actual loading of the subsoil by the embankment.

Using the created calculation scheme, the increments in total displacement resulting from the embankment loading of the cohesive subsoil should be determined, based on which the path of the potential slip surface is defined. Next, along the determined slip surface, the values of shear stresses τ , the limit values of shear stresses at failure τ_F , and the principal stress rotation at failure α_F are determined. The values of the ultimate shear stresses and the principal stress rotation at failure are determined from a numerical simulation of the embankment loading the subsoil until loss of stability, by continuing the assumed loading path. The values of the limiting shear stresses at failure τ_F correspond to the value of the undrained shear strength τ_{fu} . Subsequently, changes in the limiting shear stresses due to rotation of the principal stress directions are accounted for using the equation proposed by Bishop [42]. Next, along the determined slip surface, zones of length l_i are defined, characterised by specific values of shear stress τ_i and values of the limit shear stress at failure τ_{Fi} . Based on the results obtained, local stability factors of the embankment are determined in the individual zones of the slip surface using the following equation:

$$F_l = \frac{\tau_{Fi} \cdot l_i}{\tau_i \cdot l_i} \quad (1)$$

The global stability factor (F) of the embankment, expressed as the ratio of the sum of the ultimate shear stresses at failure to the sum of the shear stresses in the individual zones of the slip surface, is determined using the following equation:

$$F = \frac{\sum_{i=1}^n \tau_{Fi} \cdot l_i}{\sum_{i=1}^n \tau_i \cdot l_i} \quad (2)$$

Figure 11 shows a flowchart illustrating the procedure for determining the embankment stability factor using the proposed method.

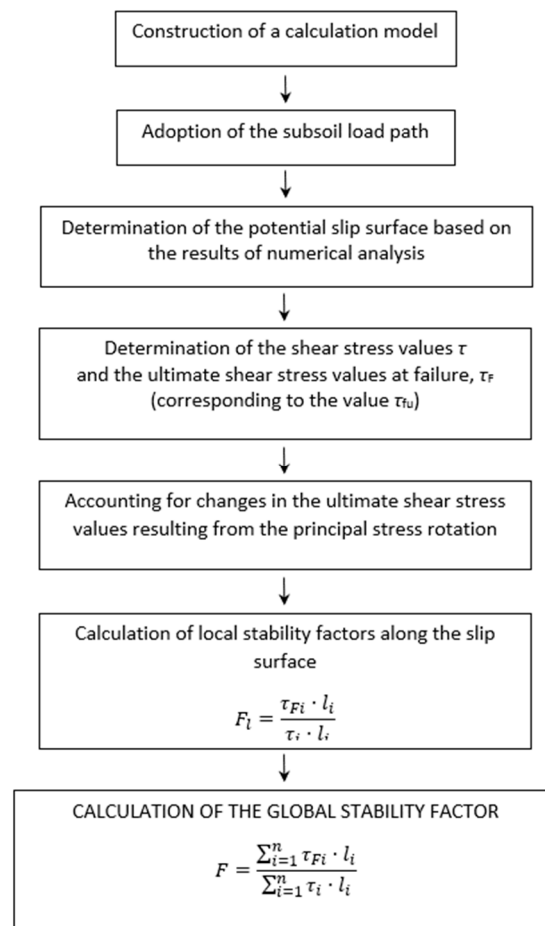


Figure 11. Flowchart of the procedure for determining the embankment stability factor using the proposed method.

The proposed stability analysis method applies to embankments on cohesive subsoil and allows any load path on the subsoil to be assumed during embankment construction.

4.2. Analysis of a Selected Example Using the Proposed Calculation Method

In accordance with the proposed method, a stability analysis of the embankment was carried out using calculation models created in Plaxis, applying an elastic–ideally plastic model with the Coulomb–Mohr plasticity criterion. For the cohesive soil layer described by an elastic–ideally plastic model with the Coulomb–Mohr plasticity criterion, parameter values obtained from tests conducted on sandy silty clay (sasiCl) were used. To reflect the actual conditions in the cohesive soil, it was assumed that the undrained shear strength τ_{fu} and the modulus of elasticity E vary with depth z . In the numerical analysis, it was assumed that the value of the undrained shear strength τ_{fu} and the modulus of elasticity E increase by 10% per 1 m of depth. The calculations were performed for an embankment founded on cohesive soil, assuming two loading paths. Load path no. 1 involved the immediate loading of the subsoil by the embankment, whilst load path no. 2 involved the gradual loading of the subsoil over time. Load path no. 1 reflects an immediate load on the subsoil, while load path no. 2 reflects the phased construction of the embankment, taking into account the strengthening of the subsoil resulting from the consolidation process. For load path no. 2, the construction of a single embankment layer with a thickness of 0.5 m every 1 day was assumed. The load paths applied to the subsoil by the embankment are shown in Figures 12 and 13. A numerical simulation of the subsoil loading by the embankment until loss of stability was performed by increasing the embankment height by

0.1 m. For load path no. 1, at an embankment height of 9.2 m, there was an increase in total displacements; consequently, it was concluded that stability was lost at this embankment height. In the case of load path no. 2, at an embankment height of 9.5 m, there was an increase in total displacements.

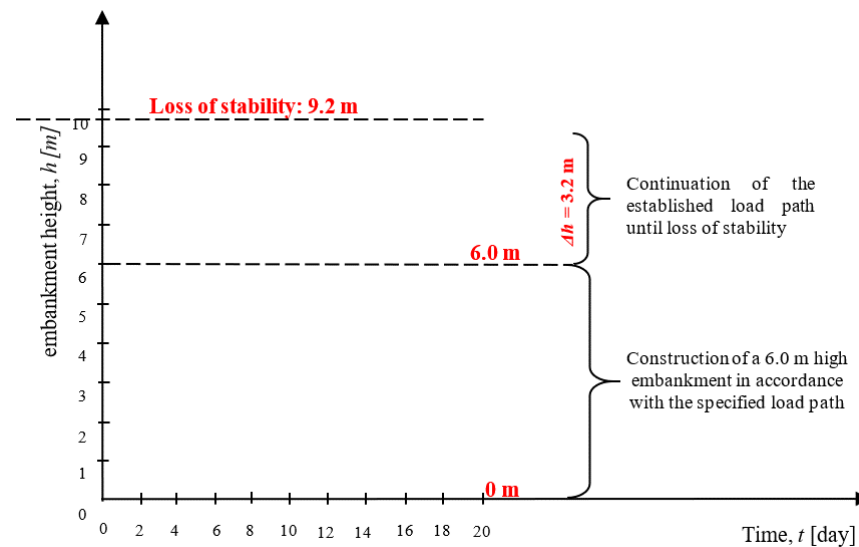


Figure 12. The course of the numerical simulation of the subsoil loading until loss of stability for the analysed embankment design (load path no. 1).

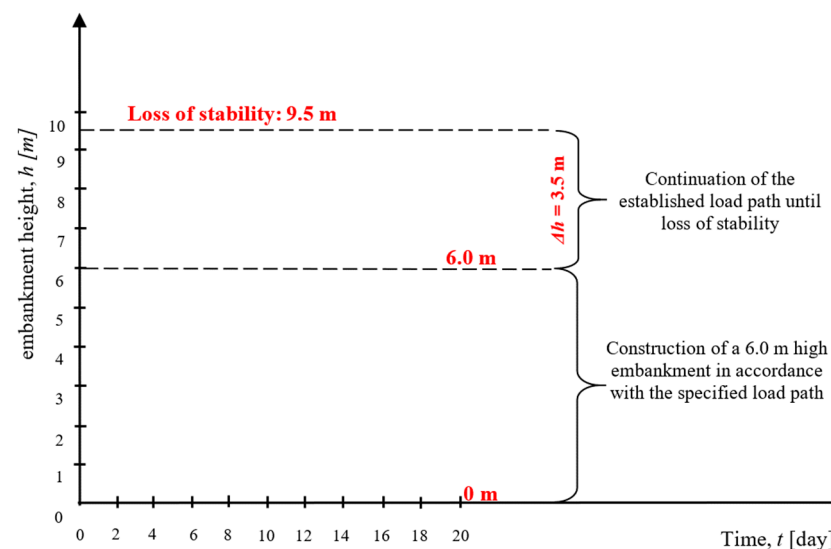


Figure 13. The course of the numerical simulation of the subsoil loading until loss of stability for the analysed embankment design (load path no. 2).

Potential slip surfaces were identified for the embankment configurations under consideration (Figure 14). Their paths were determined from analyses of total displacement increments.

The numerical analysis began with the creation of a computational model (Figure 15). The studied embankment was 6 m in height, 12 m in crest width and 1:2.5 on the side slopes. The geometry of the embankment corresponded to the analysed design case. The numerical domain was extended to four times the embankment width in order to minimise the influence of boundary conditions on the calculated stress and displacement fields. The embankment and the thickness of the cohesive soil layer were the ones considered in the

embankment construction case. As there is a layer in the ground that is not cohesive but has much higher stiffness, the layer is chosen as the lower boundary of the numerical model.

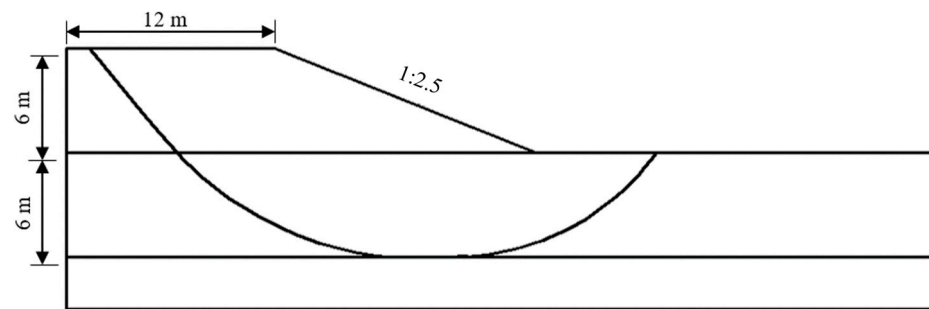


Figure 14. The path of the slip surface determined for embankment construction (load path no. 1).

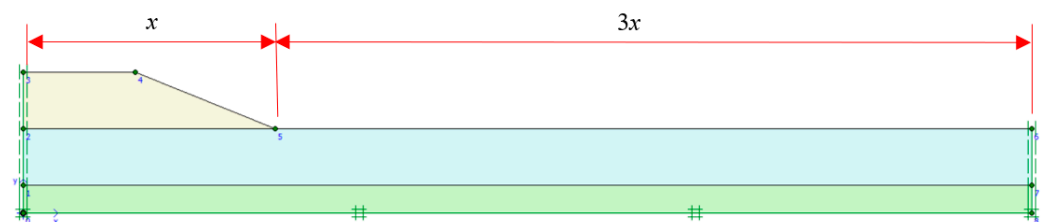


Figure 15. Calculation scheme used in the finite element analysis of embankment stability.

The boundary condition of the models is based on fixed-hinged supports at the bottom edge to prevent displacements in the vertical ($u_y = 0$) and horizontal ($u_x = 0$) directions, and sliding-hinged supports at the side edges to prevent displacements in the horizontal direction ($u_x = 0$) but allow free vertical displacements ($u_y \neq 0$).

The discretisation of the computational model to finite elements is shown in Figure 16. The finite element model has been discretised with 15-node triangular elements available in PLAXIS 2D. A uniformly refined finite element mesh was adopted throughout the analysed domain in order to ensure an accurate representation of stress and displacement fields. The material parameters used in the numerical analysis are summarised in Table 4.

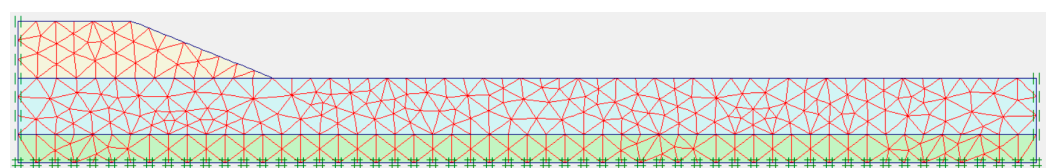


Figure 16. Division of the computational model into finite elements.

Table 4. Material parameters adopted in the numerical analyses.

Layer	Parameters					
	Unit Weight γ [kN/m ³]	Effective Cohesion c' [kPa]	Effective Angle of Internal Friction φ' [°]	Undrained Shear Strength τ_{fu} [kPa]	Young Modulus E [MPa]	Poisson Ratio ν [-]
Embankment fill	18.5	0	33	-	80	0.30
Cohesive soil (Surface)	19.9	-	-	$20 + 2 \cdot z$	$30 + 3 \cdot z$	0.35
Sand layer	22.0	0	35	-	150	0.30

Since the analysis assumes undrained conditions, the groundwater table has been set equal to ground level. For the embankment loading cases, numerical simulations provided stress and strain distributions in the subsoil. The results are shown graphically as well as numerically for the total displacement and the directions of the effective principal stresses as shown in Figures 17 and 18.

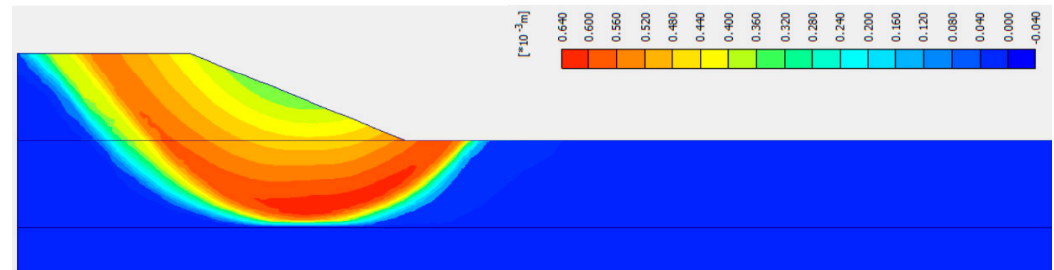


Figure 17. Increments in total displacement.

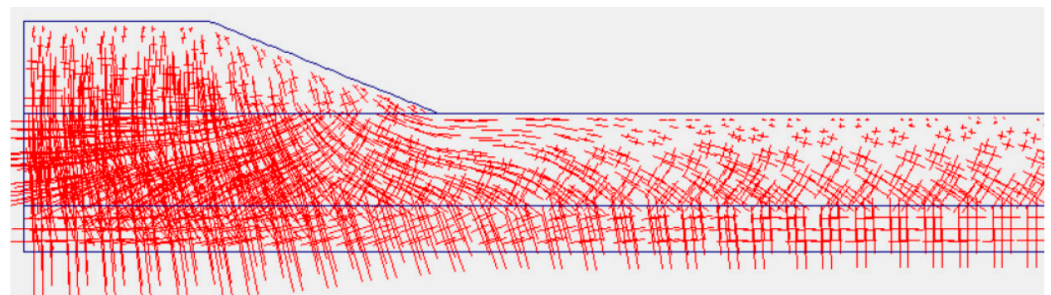


Figure 18. Directions of the effective principal stresses.

Subsequently, the values of shear stresses τ , the ultimate shear stresses at failure τ_F , and the principal stress rotation at failure α_F were determined. The values of the ultimate shear stresses and the principal stress rotation at failure were determined based on a numerical simulation of the embankment load on the subsoil until loss of stability, by continuing the assumed load path. The isolines of the distribution of shear stresses (Figure 19) and the limit shear stresses at failure (Figure 20) were determined based on an analysis carried out in the SURFER software (version 10) for the analysed embankment design. Subsequently, along the determined slip surface, zones of length l were defined, characterised by the rotation angles of the principal stress directions, for which tests were carried out in a hollow cylinder apparatus (Figure 21).

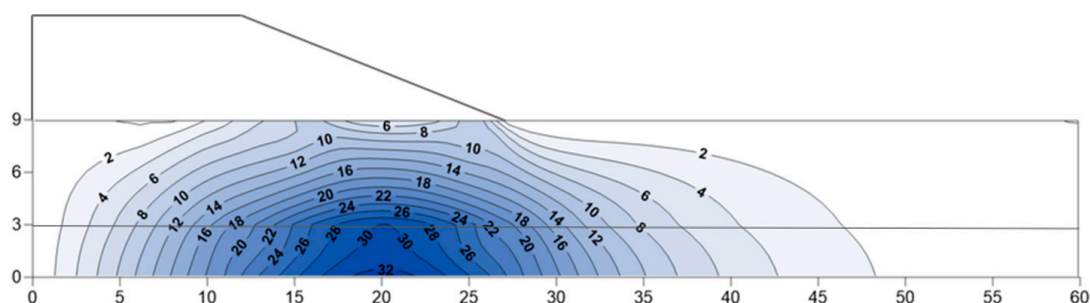


Figure 19. Isolines of shear stress τ for embankment design (load path no. 1).

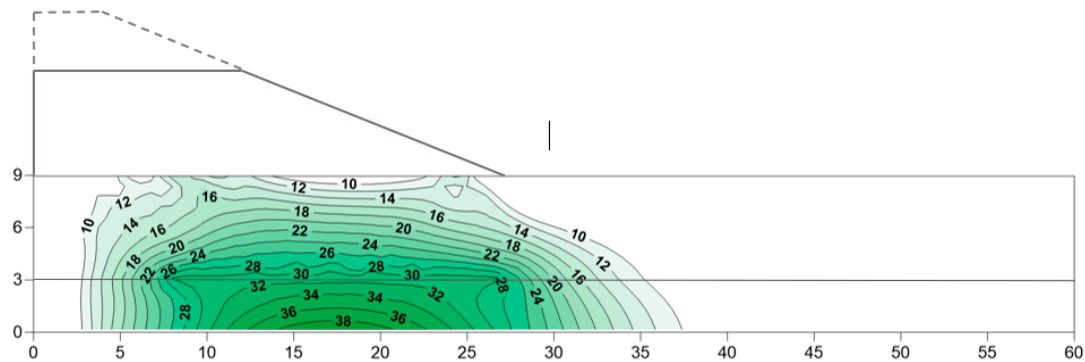


Figure 20. Isolines of the ultimate shear stress at failure τ_F , determined by continuing the load path for embankment design (load path no. 1).

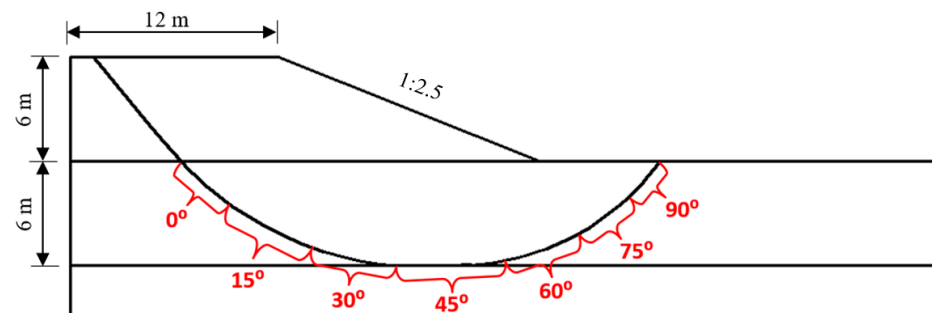


Figure 21. Division of the embankment slip surface into zones corresponding to the tests carried out in the hollow cylinder apparatus (load path no. 1).

Subsequently, the values of shear stresses τ_i and the limit values of shear stresses at failure τ_{Fi} at the predicted moment of loss of stability were assigned to the individual zones. The limit shear stress values were adjusted to account for variations resulting from the specific principal stress rotation at failure. Based on the analysis, local stability coefficients and global stability coefficients were determined using Equations (1) and (2). In order to verify the correctness of the determined slip surface in the foundation, isolines of the tangential stress mobilisation factor η_F (Figure 22) were determined, expressed as the ratio of the tangential stress τ to the ultimate tangential stress τ_F :

$$\eta_F = \frac{\tau}{\tau_F} \quad (3)$$

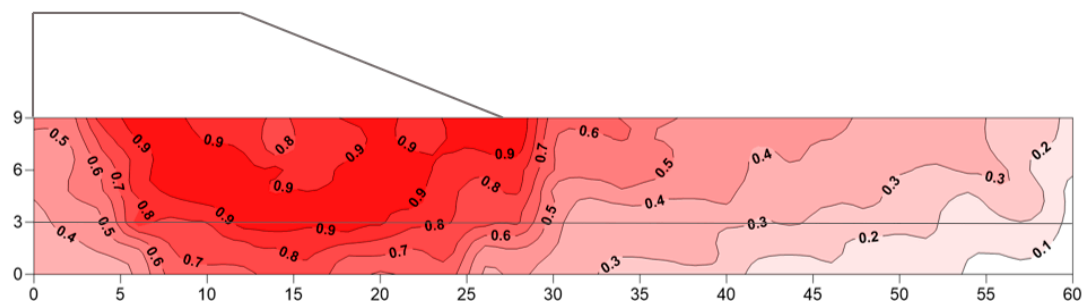


Figure 22. Isolines of the tangential stress mobilisation factor η_F for embankment (load path no. 1).

Based on the isoclines of the tangential stress mobilisation factor, it was found that the slip surfaces assumed for the embankment configurations under consideration pass through the zones with the highest values of the tangential stress mobilisation factor η_F .

Calculations of the stability factor F were carried out for the analysed embankment design, taking into account changes in the undrained shear strength of the subsoil resulting from principal stress rotation, in accordance with the described procedure.

To compare the impact of the analysed issue on the embankment stability factor, a stability assessment was also carried out, accounting for the principal stress rotation in the undrained shear strength. To this end, calculations were carried out according to the described procedure, but changes in the ultimate tangential stresses at failure due to the principal stress rotation were not accounted for. A summary of the values of the embankment stability factors determined using the proposed method is presented in Table 5. Table 4 also summarises the stability coefficients determined according to the described procedure, without accounting for changes resulting from the principal stress rotation along the potential slip surface, as well as those determined using the limit equilibrium method.

Table 5. Summary of the stability factors F determined for the analysed embankment designs.

Embankment	Method for Determining the Embankment Stability Factor	
	Proposed Method	Calculations Without Considering Principal Stress Rotation
Loading Path No. 1	1.30	1.41
Loading Path No. 2	1.37	1.49

The stability factors obtained using the proposed method—both with and without accounting for the principal stress rotation—indicate that, in the analysed cases, the embankment structure did not lose stability. Embankment construction according to load path no. 2 results in higher stability factor values compared to embankment construction according to load path no. 1. Higher values of the embankment stability factor for staged embankment loading result from the strengthening of the subsoil due to the consolidation process.

5. Conclusions

The proposed method for embankment stability analysis accounts for the influence of principal stress rotation on subsoil undrained shear strength, providing a significant advantage over conventional stability assessment approaches. The method enables calculations for a specified loading path, allows selection of constitutive models appropriate to individual soil layers, and permits determination of slip surfaces of arbitrary geometry. In the case of staged embankment construction, the method also enables stability assessment at any construction stage, accounting for subsoil strengthening from consolidation.

The determination of local stability factors enables the identification of zones of instability initiation along the analysed slip surface and monitoring their development during progressive embankment construction.

Based on the analyses, it was found that neglecting the effect of principal stress rotation can lead to a significant overestimation of embankment stability. For the analysed cases, the calculated factor of safety F decreased from 1.41 to 1.30 for loading path no. 1 and from 1.49 to 1.37 for loading path no. 2 when the influence of principal stress rotation was considered, corresponding to reductions of approximately 7.8% and 8.1%, respectively. Furthermore, laboratory hollow cylindrical shear tests demonstrated that the undrained shear strength of the tested cohesive soils decreased by up to 27% with increasing principal stress rotation, confirming the substantial influence of stress rotation on soil strength mobilisation.

The conducted analyses confirmed the research hypothesis that principal stress rotation reduces the undrained shear strength of cohesive soils, and that omitting this effect in

conventional stability calculations results in an unconservative overestimation of embankment stability. The results indicate that using traditional stability assessment methods for embankments founded on cohesive soils may lead to unconservative safety predictions, particularly when complex stress paths and toe wall structures are present. Although the proposed method has been developed for embankments founded on cohesive soils, it provides a basis for further research to extend the approach to other soil types and to refine the modelling of stress-rotation-induced strength degradation.

The proposed method provides an opportunity for future investigations to understand the effects of principal stress rotation in other soil types to determine the stability of embankments. Although the results for two cohesive soils and two loading paths showed that the factor of safety was reduced by principal stress rotation, the present work is an initial proof-of-concept.

The proposed method was developed and tested using laboratory data from hollow cylinder tests and numerical simulations of embankment loading. Therefore, further research is required to validate the proposed method across a wider range of geotechnical settings. Future work should focus on validating the proposed method through field observations, centrifuge model tests, and other laboratory data.

Moreover, sensitivity and uncertainty analyses should have been carried out to assess the influence of key parameters, such as undrained shear strength, stiffness, loading path, principal stress rotation angle, constitutive model selection, soil anisotropy, and overconsolidation ratio, on the predicted stability response. This will allow for practical recommendations for engineering applications and should contribute to further refinement of our stability assessment model.

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