

Article

Interpreting the Mechanical Behaviour of Carbonate Sand-Fine Mixtures Using the Modified Interfine Void Ratio

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Abstract: Understanding the mechanical behaviour of carbonate sand-fine mixtures is crucial for modelling geotechnical-related gas exploitation problems in potential gas hydrate reservoirs. This paper presents a laboratory investigation of the mechanical behaviour of carbonate sand-fine mixtures with a carbonate sand content of no more than 40%. By means of computer tomography, the internal porosity of carbonate sand grains was measured, and the interfine void ratio was modified. For carbonate sand-fine mixtures with a similar void ratio, all mixtures exhibited strain-softening behaviour, and the peak strength decreased with increasing carbonate sand content. Compared to the mixture whose density was controlled by the void ratio, the change rule of mechanical behaviour influenced by the carbonate sand content was precisely the opposite for that controlled by the interfine void ratio. The mixture with the smallest modified interfine void ratio exhibited the highest peak strength and the strongest dilative volumetric behaviour. By adopting the modified interfine void ratio as a state variable, a unique critical state line can be identified with a carbonate sand content of no more than 40%, leading to a framework that coherently characterises the mechanical behaviour of these gap-graded soil mixtures.



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Keywords: soil mixture; mechanical behaviour; carbonate sand; modified interfine void ratio

1. Introduction

The simple classification of soil into coarse- and fine-grained is a conventional approach in soil mechanics. However, most natural soils and manmade fills are mixtures of coarse- and fine-grained soils [1,2]. Recent studies on sand-fine mixtures have shown that the void ratio may not be an appropriate state variable to describe the mechanical behaviour of these soil mixtures [3]. For sand-fine mixtures where sand is the major component and plays a dominant role in the soil skeleton, experimental studies have shown that an addition of a small amount of fines in sand decreases its void ratio, but the stress–strain behaviour essentially remains the same, implying that the void ratio is no longer a suitable state variable to represent the stress–strain response of sand-fine mixtures [4]. The main reason for such behaviour is the inactive contribution of fines to the force chains of the sand matrix. To properly describe the mechanical behaviour of sand-fine mixtures, a new density parameter, the intergranular void ratio e_g [5,6], has been proposed by considering fines as voids:

$$e_g = \frac{e + f_c}{1 - f_c} \quad (1)$$

where e is the void ratio, and f_c is the fines content as a percentage, presenting the volume of fines when the volume of soil mixture is equal to 1.

Similarly, for sand-fine mixtures where fines are the major components and play a dominant role in the soil skeleton, sand grains may not take part in the force chains of the fine matrix. In this case, the volume of sand can be ignored. Based on this assumption, Thevanayagam et al. [7,8] further proposed an interfine void ratio e_f defined by neglecting the volume of sand grains to describe the mechanical behaviour of sand-fine mixtures:

$$e_f = \frac{e}{f_c} \quad (2)$$

where the fines content f_c (the volume of fines) is equal to the volume of soil mixture when the volume of sand grains is ignored.

Past studies on sand-fine mixtures mostly focused on quartz sands [9–15]. However, in the submarine sediment, carbonate sand (CS) is widely distributed along the continental shelves and the coastlines. The engineering properties of carbonate sand have received widespread attention and have been intensively studied [16–22]. Recent explorations have reported that a large amount of natural methane hydrate was found in the submarine fine-grained sediments containing carbonate materials in the northern South China Sea [23–27]. Methane hydrate is formed and preserved in locations where there is an ample supply of methane, as well as where the temperature and pressure requirements have met the stability conditions [28,29]. The occurrence of natural methane hydrate has been verified through core drilling at locations all over the world, and the large number of methane hydrate reserves highlights it as a potential future energy source [30–36]. As an unconventional energy source, the exploitation of methane hydrate has attracted attention worldwide [37,38]. By increasing temperature and reducing the pressure, methane hydrate can decompose into methane gas and water. Multiple approaches have been proposed and employed to extract methane from hydrate-bearing sediments, such as depressurisation, thermal stimulation, gas exchange, and other innovative methods [39–42]. However, geological disasters and engineering accidents could occur from the production of methane hydrate, such as subsea landslides, seabed settlements, mining holes, and production platform collapse [43–45]. Thus, understanding the mechanical behaviour of methane hydrate-bearing sediments is essential for their safe extraction.

Further investigation into the sediment drilled from the northern South China Sea showed that the methane hydrate content was significantly influenced by the amount of carbonate material found in the fine-grained sediment [46–49]. Hydrate formation in laboratory specimens has shown that the internal pores of carbonate sand provide ideal room for the growth and accumulation of hydrate [50]. Therefore, the mechanical behaviour of carbonate sand-fine mixtures is crucial for modelling the geotechnical-related gas exploitation problems in potential gas hydrate reservoirs. Ma et al. [51] investigated the breakage mechanism of particles for carbonate sand-fine mixtures. It was found that with a fine content of more than 40%, the breakage of the carbonate sand grains is insignificant during compression because fines are the dominant component in the soil matrix, and carbonate sand grains do not actively participate in the load bearing. Studies on the mechanical properties of carbonate sand-fine mixtures are rarely reported.

In this paper, a laboratory study on the effects of carbonate sand content on the mechanical behaviour of carbonate sand-fine mixtures is first presented. The internal porosity of carbonate sand is innovatively quantified by using computer tomography. Then, a modified interfine void ratio is derived considering the influence of internal pores of carbonate sand and the difference in specific gravity between carbonate sand and fines.

Finally, the experimental data are interpreted within a framework based on the modified interfine void ratio.

2. Materials and Methods

2.1. Materials

The tested soils were carbonate sand and quartz fines. The specific gravity of the tested carbonate sand and fines was 2.77 and 2.63, respectively. The average grain diameter (d_{50}) of carbonate sand was 0.75 mm. The maximum grain size of the fines was smaller than 75 μm , which are classified as fine particles according to ASTM Standard Practice for Classification of Soils for Engineering Purposes (D 2487). Two carbonate sand-fine mixtures were formed by mixing 20 and 40 wt.% carbonate sand with fines, which is termed as the carbonate sand content, representing the mass proportion of carbonate sand in the mixtures. In this study, the fine content remained greater than the carbonate sand content for the tested carbonate sand-fine mixtures. The grain size distributions of the soil mixtures are shown in Figure 1. It was found that the soil mixtures comprised gap-graded soils for carbonate sand contents between 20 and 40%.

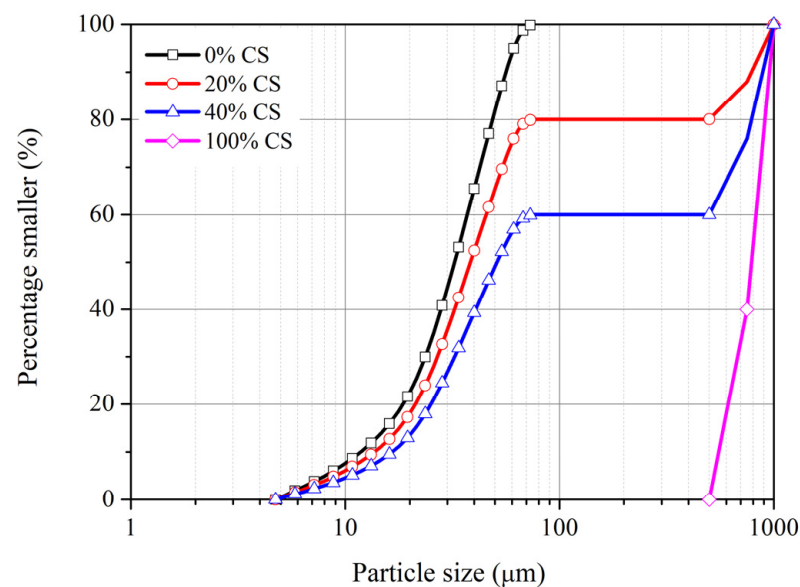


Figure 1. Grain size distributions of carbonate sand-fine mixtures.

The maximum and minimum void ratios of carbonate sand-fine mixtures with different carbonate sand contents were obtained using the ASTM procedures specified in ASTM Test Methods for Maximum Index Density and Unit Weight of Soils Using a Vibratory Table (D 4253) and Test Method for Minimum Index Density and Unit Weight of Soils and Calculation of Relative Density (D 4254). The results are illustrated in Figure 2. It is observed that both the maximum and minimum void ratios tend to decrease first and then increase with the increase in the carbonate sand content. The threshold value of the carbonate sand content denoting a change in soil fabric from “sand-in-fines” to “fines-in-sand” is approximately 60%. Therefore, for the carbonate sand-fine mixtures studied in this research, it was assumed that fines played a dominant role in the mixture skeleton since the maximum carbonate sand content was much less than 60%.

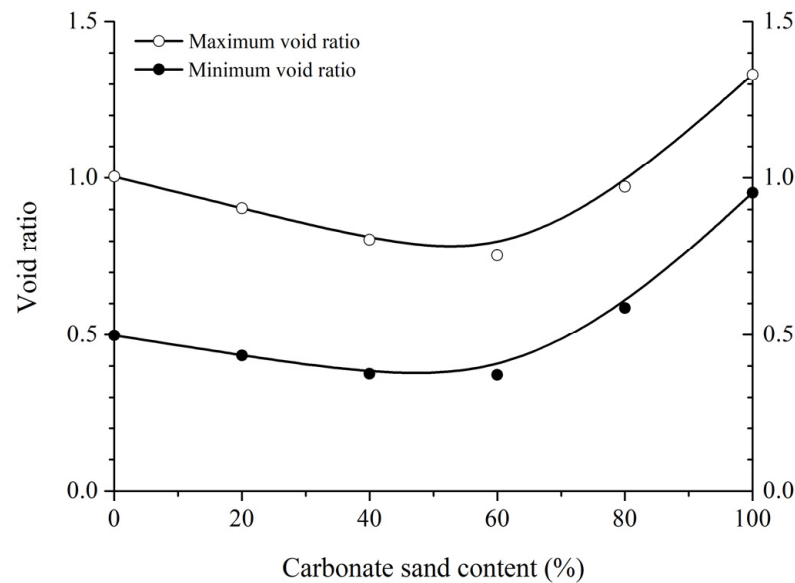


Figure 2. Maximum and minimum void ratios of carbonate sand-fine mixtures.

2.2. Triaxial Tests

Triaxial tests were carried out on specimens 50 mm in diameter and 100 mm in height. The specimens were prepared using the moist-tamping method [52]. The back pressure method was adopted to saturate the specimens. A maximum back pressure of 8 MPa was used to ensure that the pore pressure parameter B could reach a minimum value of 0.95. It should be noted that a high pore water pressure is required to reach the stability conditions of gas hydrate. Hence, the test results of this study can be used to compare with those of hydrate-bearing soil mixtures that are not reported in this article. Thereafter, the specimens were isotropically compressed to an effective confining pressure ranging from 0.5 to 3 MPa. After the completion of consolidation, drained shear tests were conducted at an axial strain rate of 0.1%/min.

According to past studies on sand-fine mixtures, the interfine void ratio is a potential density parameter for carbonate sand-fine mixtures in addition to the conventional void ratio [7,8]. Therefore, two series of triaxial tests were conducted by controlling the void ratio and interfine void ratio. A total of fifteen tests were carried out, and the test conditions are summarised in Table 1. In the first series of triaxial tests, nine specimens were controlled with a similar initial void ratio of around 0.580. The remaining six specimens were controlled with a similar initial interfine void ratio of around 0.720 in the other series of tests. For the specimens with a similar initial interfine void ratio, a target value (0.720) of the interfine void ratio was set, and the initial void ratio was back-calculated based on Equation (2). According to the calculated initial void ratio, the specimen was prepared and the real initial void ratio was measured after the specimen production was completed. Then, the real initial interfine void ratio was obtained by recalculation. The reference specimens of the mixtures are C5_V20, C10_V20 and C30_V20 under the effective confining pressures of 0.5, 1 and 3 MPa, respectively. The void ratio or interfine void ratio of other specimens are all referred to these three specimens under the condition of similar void ratios or interfine void ratios.

Table 1. Testing conditions of drained triaxial tests for carbonate sand-fine mixtures.

Specimen Category	Specimen Identity	Initial Void Ratio	Initial Interfine Void Ratio	Effective Confining Pressure (MPa)	Carbonate Sand Content (%)
Similar initial void ratio	C5_V0	0.583	0.583	0.5	0
	C5_V20	0.576	0.720	0.5	20
	C5_V40	0.573	0.955	0.5	40
	C10_V0	0.589	0.589	1	0
	C10_V20	0.580	0.725	1	20
	C10_V40	0.575	0.958	1	40
	C30_V0	0.590	0.590	3	0
	C30_V20	0.572	0.715	3	20
	C30_V40	0.570	0.950	3	40
Similar initial interfine void ratio	C5_I0	0.698	0.698	0.5	0
	C5_I40	0.442	0.736	0.5	40
	C10_I0	0.695	0.695	1	0
	C10_I40	0.443	0.738	1	40
	C30_I0	0.704	0.704	3	0
	C30_I40	0.444	0.740	3	40

2.3. Microscopic Investigation

For a carbonate sand-fine mixture, there are two types of pores: internal pores of carbonate sand grains and intergranular pores between soil particles. In this study, internal pores refer to all the pores within the solid boundary range of the carbonate sand grains, and the intergranular pores refer to the pore spaces existing between sand–sand particles, sand-fine particles, and fine–fine particles. If carbonate sand grains do not contribute to the force chains of the fine matrix, the internal pores of carbonate sand grains will be inactive. Hence, the volume change in the soil mixture is only the result of the volume change in intergranular pores between soil particles, and the volume of internal pores of carbonate sand grains should be neglected in evaluating the “actual” void ratio or porosity of the carbonate sand-fine mixture. In this study, computer tomography (CT) was adopted to measure the internal porosity of carbonate sand grains and investigate the microscopic structure of carbonate sand-fine mixtures.

A total of one hundred grains were randomly selected from the carbonate sand of all tested specimens to measure their internal porosities. A laboratory μ CT system, equipped with a high-resolution X-ray tube (XTH225/320 LC μ CT scan from Nikon, Tokyo, Japan), was used to scan the high-resolution images of the specimen. In order to conduct μ CT scanning, the sand grains were loosely deposited layer by layer into a cylindrical container 5 mm in diameter and 10 mm in height. A plastic sheet was used to cover the top of a layer of sand grains before depositing a subsequent layer of sand grains. This specimen preparation procedure enables the separation of sand grains, thus facilitating the identification of the internal pores of each sand grain from the CT image. Through image processing, a resolution of approximately 5 μ m could be achieved for each CT image produced in this study. This is favourably smaller than the diameter of the tested carbonate sand grains which ranges between 500 and 1000 μ m, ensuring the accuracy of measurement results. In addition, two carbonate sand-fine mixture specimens with 20 and 40% carbonate sand contents were also prepared using the same sample preparation method with triaxial tests to ensure the consistency of the specimens. Then, the specimens were scanned to investigate the microscopic structure.

3. Results and Discussion

3.1. Under the Condition of Similar Void Ratios

The effects of the carbonate sand content on the stress–strain behaviour of carbonate sand–fine mixtures were very similar for the range of confining pressures studied. Thus, only the triaxial drained test results obtained under an effective confining pressure of 0.5 MPa are presented, as shown in Figure 3. The specimen identities of the three specimens tested are C5_V0, C5_V20 and C5_V40. In the figure, deviator stress (q), which is defined as the difference between the major and minor principal stresses, represents the strength of specimens. Specimens were prepared at a similar initial void ratio ranging between 0.573 and 0.583. As observed in Figure 3, the peak strength of the specimen decreased with increasing carbonate sand content, and both deviator stress and volumetric strain maintain a steady value around an axial strain of 30%. In addition, the specimen changed from dilative to contractive behaviour with an increase in carbonate sand content. As the carbonate sand content increased, the specimen seemed to become “looser” despite being under the condition of similar initial void ratios. The three specimens all exhibited strain-softening behaviour. However, the shear strength of the three specimens approached almost the same value at large deformation under the same effective confining pressure. Considering the significant convergence of the stress–strain and volumetric deformation curves, it is assumed that the critical state has been achieved at the end of shearing for all specimens. [53]. The critical state data of triaxial tests under the condition of similar void ratios are summarised in Table 2. The mean effective stress (p') is defined as $p' = (\sigma_1' + 2\sigma_3')/3$, where σ_1' and σ_3' are the major and minor principal stresses, respectively.

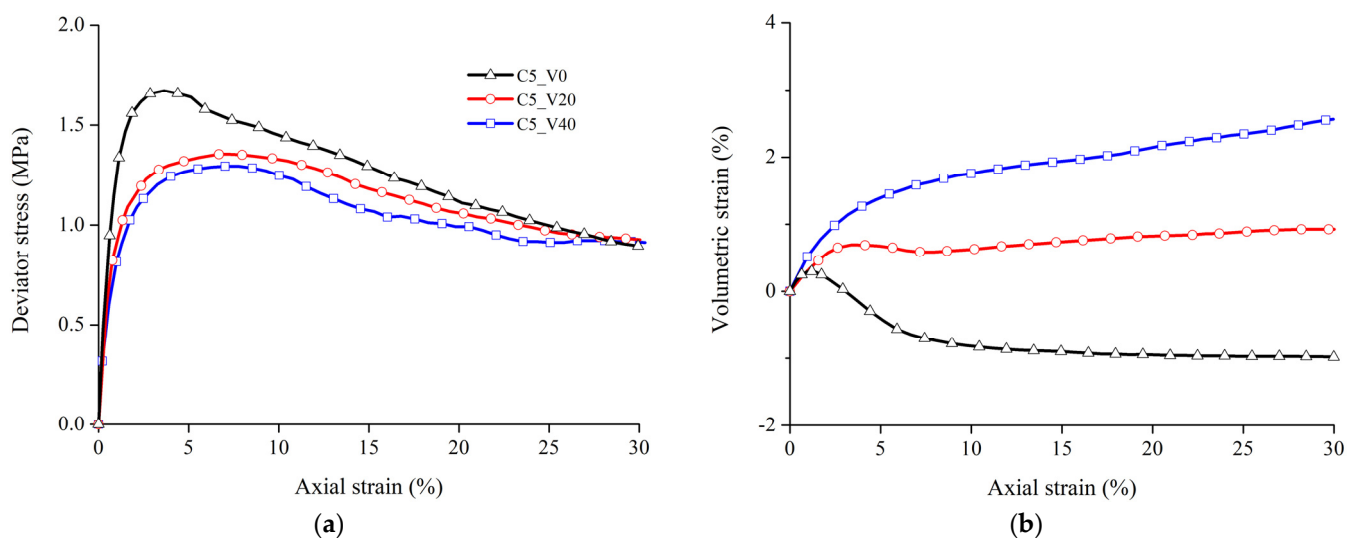


Figure 3. Triaxial test results of carbonate sand–fine mixtures with similar void ratios under a confining pressure of 0.5 MPa: (a) stress–strain curves; (b) volumetric deformation curves.

Figure 4a,b show the critical state data of the triaxial drained tests in the p' – q and e – $\ln p'$ planes for specimens with similar initial void ratios, respectively. The critical state in the p' – q plane is represented by a linear best-fit line passing through the origin. Clearly, the carbonate sand content had no significant effect on the critical state stress ratio for carbonate sand–fine mixtures. For the range of the carbonate sand contents tested, this finding is consistent with the aforementioned assumption that for sand–fine mixtures where fines played a dominant role in the soil skeleton, carbonate sand did not contribute to the force chains of the fine matrix, leading to a similar stress ratio at the critical state for the three tested carbonate sand–fine mixtures. Regarding the critical state data in the e – $\ln p'$ plane, a linear relationship can also be used to represent the critical state for each carbonate

sand-fine mixture. The critical state lines (CSLs) of carbonate sand-fine mixtures with 20 and 40% carbonate sand content were parallel to that of fines only (i.e., 0% carbonate sand content). In addition, the CSL shifted downwards with an increase in carbonate sand content. In other words, the carbonate sand-fine mixture with a higher carbonate sand content exhibited a lower void ratio at the critical state for a given mean effective stress. In the e – $\ln p'$ plane, the CSL of carbonate sand-fine mixtures was clearly influenced by the carbonate sand content, and no single CSL was observed with different carbonate sand content.

Table 2. Critical state data of triaxial tests under the condition of similar void ratios.

Specimen Identity	Initial State		Critical State		
	Void Ratio	Void Ratio After Consolidation	Mean Effective Stress (MPa)	Deviator Stress (MPa)	Void Ratio
C5_V0	0.583	0.581	0.789	0.895	0.597
C5_V20	0.576	0.560	0.809	0.926	0.545
C5_V40	0.573	0.531	0.804	0.913	0.492
C10_V0	0.589	0.587	1.564	1.693	0.577
C10_V20	0.580	0.556	1.657	1.972	0.521
C10_V40	0.575	0.513	1.624	1.873	0.460
C30_V0	0.590	0.587	4.609	4.828	0.509
C30_V20	0.572	0.541	4.741	5.223	0.447
C30_V40	0.570	0.510	4.682	5.046	0.412

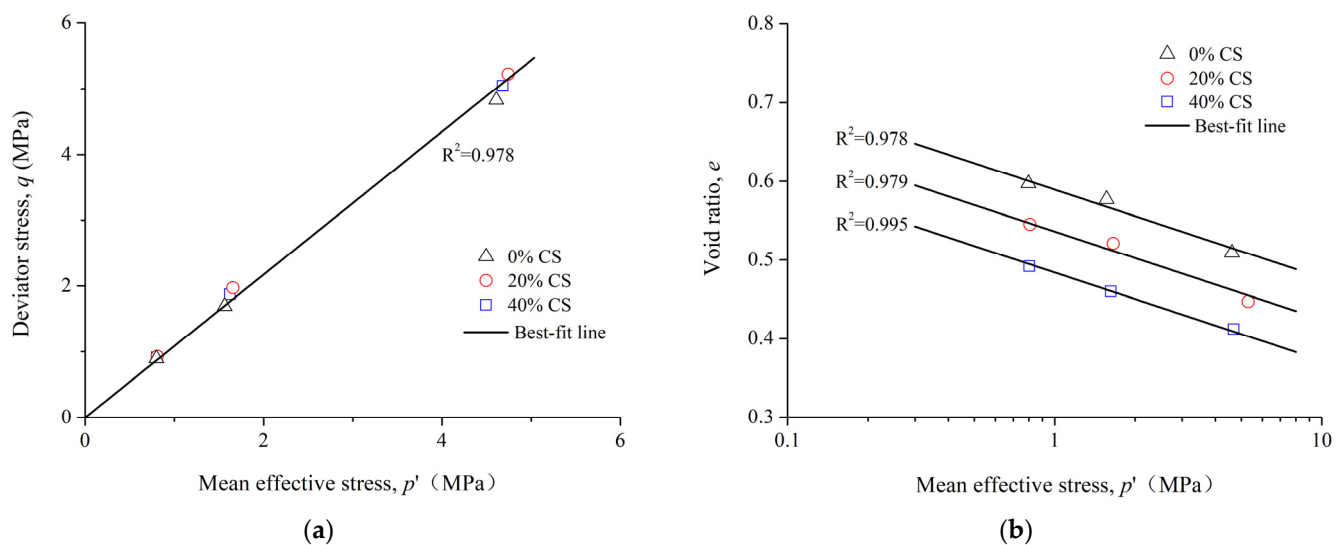


Figure 4. Critical state lines of carbonate sand-fine mixtures with similar void ratios in (a) p' – q and (b) e – $\ln p'$ planes.

3.2. Under the Condition of Similar Interfine Void Ratios

The specimens were prepared at similar initial interfine void ratios ranging between 0.698 and 0.736. Likewise, only the triaxial drained test results obtained under an effective confining pressure of 0.5 MPa are presented, as shown in Figure 5. The peak strength of the specimen increased with an increase in carbonate sand content, and both deviator stress and volumetric strain maintained a steady value around an axial strain of 30%. In addition, the specimen changed from dilative to contractive behaviour with a decrease in carbonate sand content. As the carbonate sand content increased, the specimen became “denser”

despite being under the condition of similar initial interfine void ratios. Compared to the specimens whose density was controlled by the void ratio, the carbonate sand content had the opposite effect on mechanical properties in the specimens controlled by the interfine void ratio. Similarly, the three specimens all exhibited strain-softening behaviour, and the shear strength of the three specimens approached a similar value at large deformation under the same effective confining pressure. Thus, under the condition of similar initial interfine void ratios, all tested specimens presumably reached the critical state at the end of shearing. The critical state data of triaxial tests under the condition of similar interfine void ratios are summarised in Table 3.

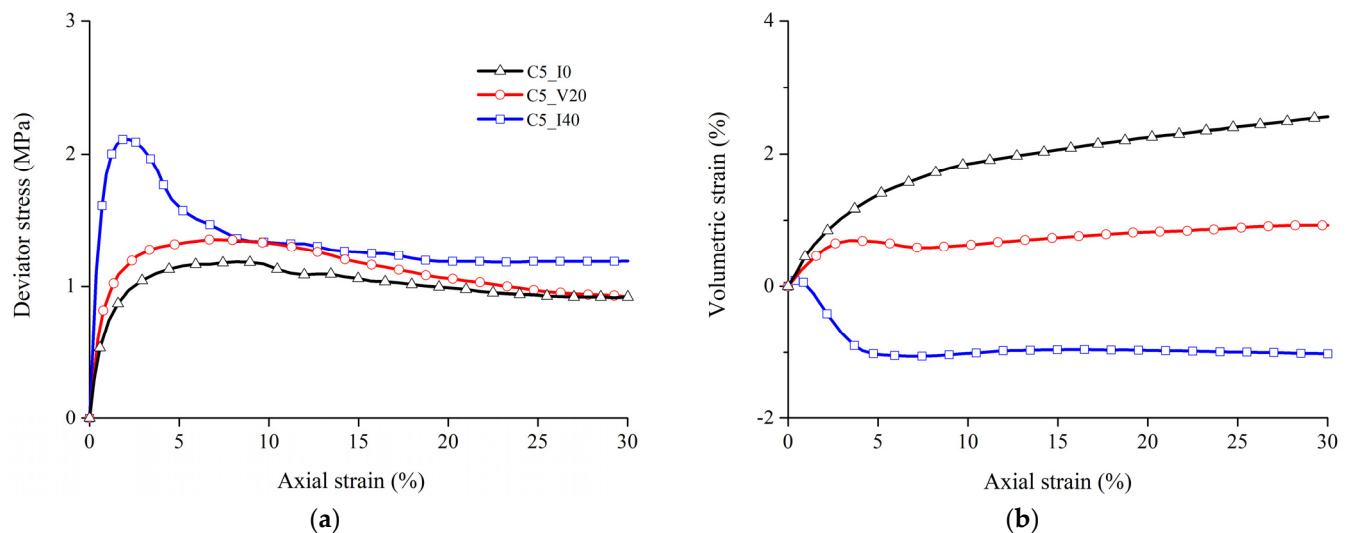


Figure 5. Triaxial test results of carbonate sand-fine mixtures with similar interfine void ratios under a confining pressure of 0.5 MPa: (a) stress–strain curves; (b) volumetric deformation curves.

Table 3. Critical state data of triaxial tests under the condition of similar interfine void ratios.

Specimen Identity	Initial State		Critical State		
	Interfine Void Ratio	Interfine Void Ratio After Consolidation	Mean Effective Stress (MPa)	Deviator Stress (MPa)	Interfine Void Ratio
C5_I0	0.698	0.663	0.807	0.922	0.620
C5_V20	0.720	0.701	0.809	0.926	0.681
C5_I40	0.736	0.733	0.897	1.190	0.773
C10_I0	0.695	0.670	1.611	1.833	0.589
C10_V20	0.725	0.695	1.657	1.972	0.651
C10_I40	0.738	0.735	1.682	2.046	0.742
C30_I0	0.704	0.655	4.829	5.487	0.529
C30_V20	0.715	0.676	4.741	5.223	0.559
C30_I40	0.740	0.736	4.837	5.511	0.684

Figure 6a,b show the critical state data of the triaxial drained tests in the $p'-q$ and $e_f-\ln p'$ planes for specimens with similar initial interfine void ratios, respectively. Similar to the previous scenario, the critical state in the $p'-q$ plane is represented by a linear best-fit line passing through the origin. The CSL of carbonate sand-fine mixtures in the $p'-q$ plane is unique despite different carbonate sand contents; the potential reason for this was explained previously. In the $e_f-\ln p'$ plane, the CSLs of carbonate sand-fine mixtures with 20 and 40% carbonate sand content were parallel to that of fines only (i.e., 0% carbonate sand

content). However, the CSL shifted upwards with an increase in carbonate sand content. This means that the carbonate sand-fine mixture with a higher carbonate sand content presented a higher interfine void ratio at the critical state for a given mean effective stress. Compared to carbonate sand-fine mixtures in which the density is controlled by the void ratio, the change rule of CSL influenced by the carbonate sand content was precisely the opposite in specimens controlled by the interfine void ratio.

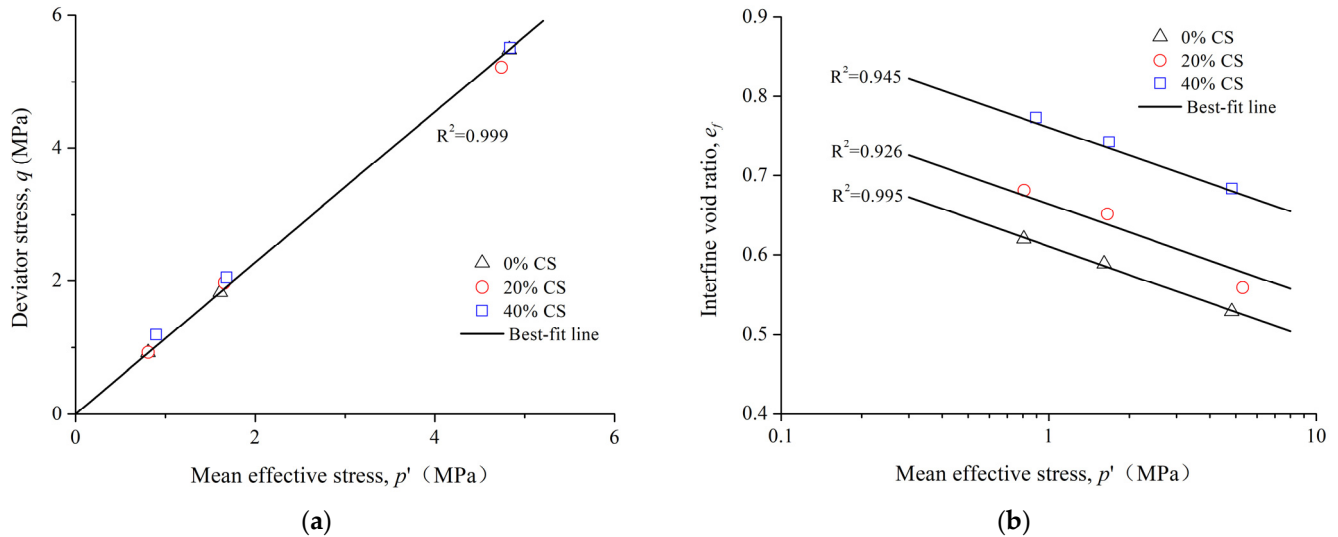


Figure 6. Critical state lines of carbonate sand-fine mixtures with similar interfine void ratio in (a) p' – q and (b) e_f – $\ln p'$ planes.

3.3. Internal Porosity of Carbonate Sand

Typical CT images presented in terms of three-dimensional and cross-sectional views for carbonate sand grains are shown in Figure 7. The image in the lower right corner was the three-dimensional picture of carbonate sand specimen, and that in the upper left corner was the cut image of the cross section (x-y section). In the upper right and lower left corners, they were the cut images of the vertical sections (x-z and y-z sections).

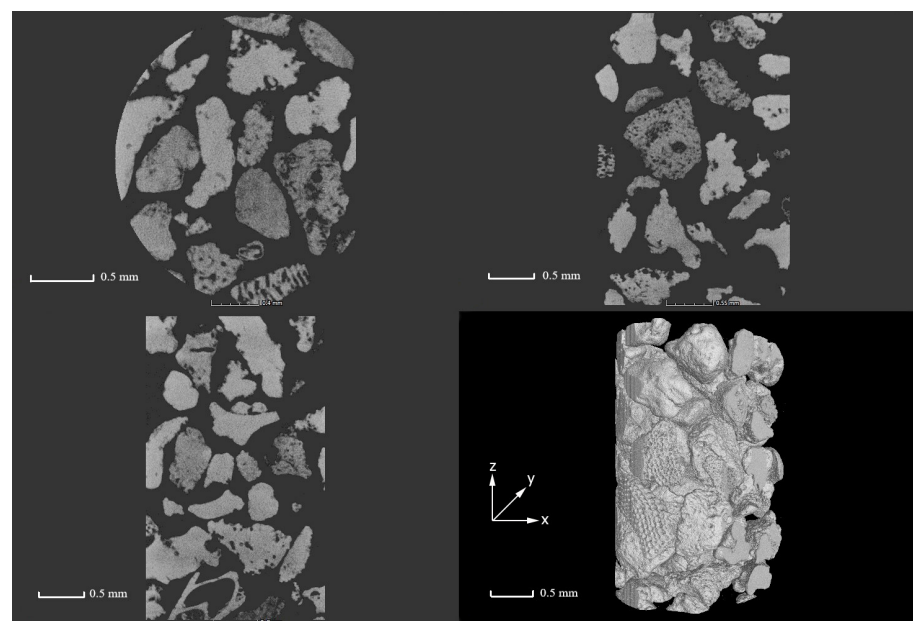


Figure 7. Three-dimensional and cross-sectional views of carbonate sand grains.

A two-dimensional cross-sectional CT image was scanned for each 5 μm interval along the height of the container, as shown in Figure 8a. The figure clearly reveals that most carbonate sand grains contain a substantial number of internal pores. ImageJ was used to analyse the CT images. First, the original greyscale image was converted to a binary image as depicted in Figure 8b. In the binary image, the pore and solid are represented by black and white colour, respectively. A three-dimensional surface boundary of a single sand grain was identified based on a series of sequential two-dimensional cross-sectional images. The internal porosity n_{in} of each sand grain was estimated using the following equation:

$$n_{in} = \frac{V_{in}}{V} = \frac{\int S_{in} dh}{\int S dh} \approx \frac{\sum S_{in} \Delta h}{\sum S \Delta h} = \frac{\sum S_{in}}{\sum S} \quad (3)$$

where V_{in} and V are volumes of the internal pores and sand grain, respectively; S_{in} and S are the areas of the internal pores and sand grain in one cross section, respectively; and dh and Δh are the spacing between neighbouring cross sections, which was 5 μm in this study. S_{in} and S were estimated based on the amount of pixel points of the binary image, as shown in Figure 8b.

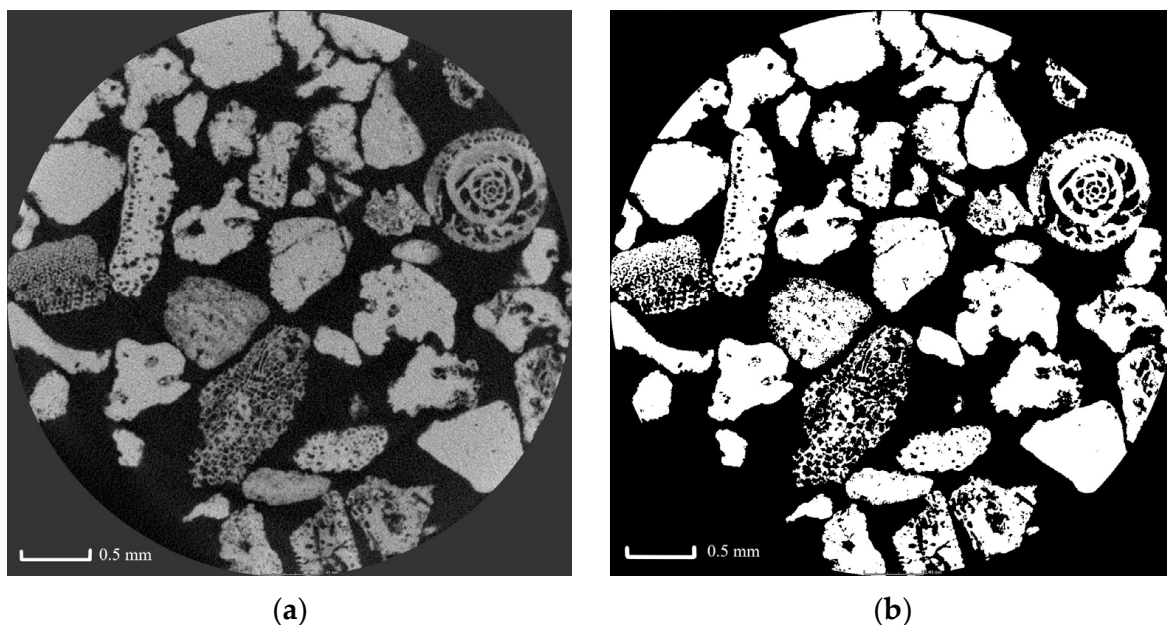


Figure 8. Typical CT image of a two-dimensional cross section of a carbonate sand specimen: (a) greyscale image; (b) binary image (black indicates void areas, and white represents solid areas).

In this study, a total of one hundred carbonate sand grains were analysed. The statistical results of the internal porosity are shown in Figure 9. It was found that the value of internal porosity ranged between 0 and 0.5. The statistical results conformed to the normal distribution with a mean value of 0.238. As a first approximation, 0.238 was used as the mean value of internal porosity for all tested carbonate sand in the subsequent analysis. The one hundred carbonate sand grains were randomly selected from the specimens of triaxial tests, having a certain representativeness in terms of selection method. However, they were not selected proportionally according to the composition of carbonate sand. If the proportion of a certain type of carbonate sand is too high in the one hundred carbonate sand grains, it may affect the mean value of internal porosity.

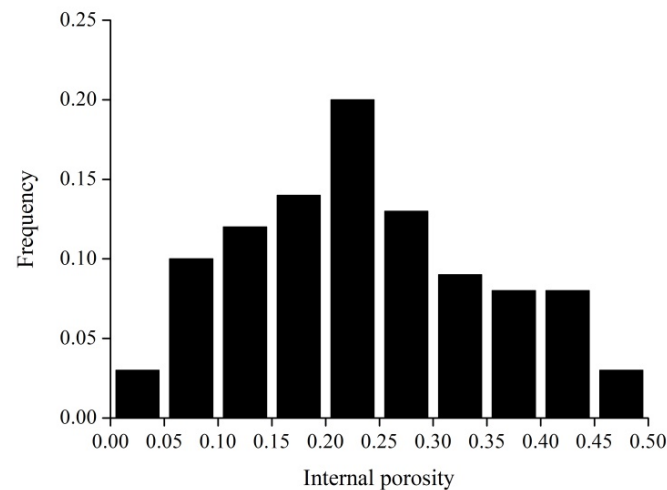
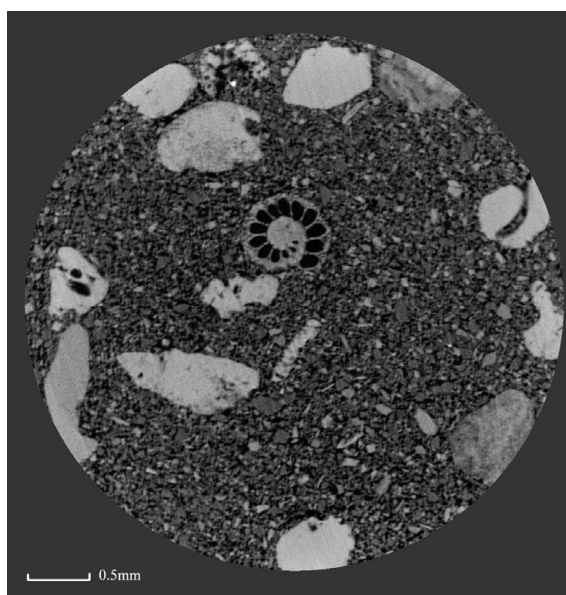


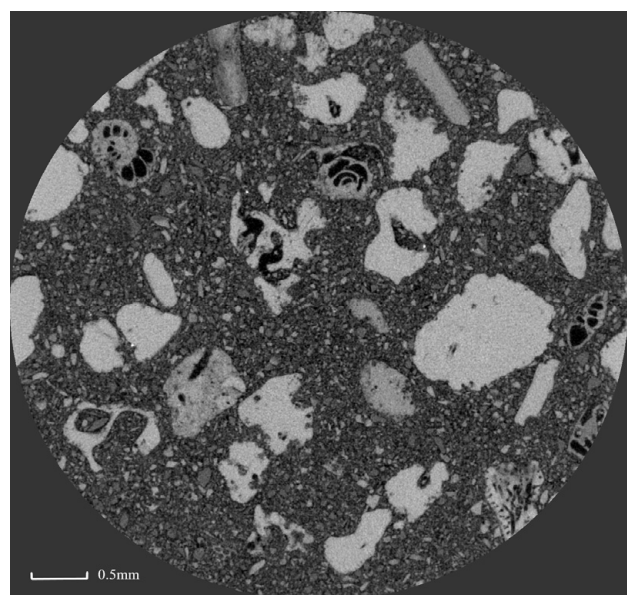
Figure 9. Statistical distribution of internal porosity of carbonate sand grains.

3.4. Microscopic Structure of Carbonate Sand-Fine Mixtures

Figure 10a shows a two-dimensional cross-sectional CT image of the carbonate sand-fine mixture with 20% carbonate sand content. Based on the microscopic image, it was found that carbonate sand grains were dispersed among fines and completely wrapped by fine particles. There was hardly any contact between two neighbouring carbonate sand grains. In this case, fines were the major components and clearly played a dominant role in the mixture skeleton. Figure 10b shows a CT image of the carbonate sand-fine mixture with 40% carbonate sand content. Compared to the mixture with 20% carbonate sand content, the number of carbonate sand grains significantly increased and were evenly distributed among the fine particles. Most of the carbonate sand grains were still not in direct contact with each other. Even if partial contact was observed between a few carbonate sand grains, the space around the contact angle was tightly filled with fine particles. Carbonate sand grains likely did not contribute to the force chains of the fine matrix when the carbonate sand content was not more than 40%.



(a)



(b)

Figure 10. Microscopic images of carbonate sand-fine mixtures with (a) 20 and (b) 40% carbonate sand content.

4. Modified Interfine Void Ratio

Compared to the mixture in which the density was controlled by the void ratio, the change rule of mechanical behaviour influenced by the carbonate sand content was precisely the opposite in specimens controlled by the interfine void ratio. Therefore, by adopting different density parameters, the effect of carbonate sand content on the mechanical properties of the mixtures was also very different. Past studies have proved the rationality of the interfine void ratio as the density parameter for sand-fine mixtures. Therefore, an appropriate density parameter should be determined that can be used independently to describe the state (including initial and critical states) and characterise the mechanical properties of carbonate sand-fine mixtures.

For carbonate sand-fine mixtures, the internal pores are inactive. Based on the concept of the interfine void ratio defined by Thevanayagam et al. [7,8], their volume should be neglected when the interfine void ratio is evaluated. Thus, for carbonate sand-fine mixtures, the interfine void ratio is expressed as follows:

$$e_f = \frac{V_{int}}{V_f} \quad (4)$$

where V_{int} is the volume of intergranular pores, and V_f is the volume of fines.

The void ratio of carbonate sand-fine mixtures is calculated as follows:

$$e = \frac{V_{int} + V_{in}}{V_f + V_c} \quad (5)$$

where V_{in} and V_c are volumes of the internal pores and solid parts of carbonate sand grains, respectively. The relationship between V_{in} and V_c can be expressed using the following equation:

$$V_{in} = \frac{n_{in}}{1 - n_{in}} V_c \quad (6)$$

Substituting Equations (5) and (6) into Equation (4) yields the following:

$$e_f = \left(1 + \frac{V_c}{V_f}\right) e - \frac{n_{in}}{1 - n_{in}} \frac{V_c}{V_f} \quad (7)$$

If specific gravities of carbonate sand and fines are the same, the ratio of the volume of carbonate sand and fines (V_c/V_f) is equal to the ratio of their mass ($CS/(1 - CS)$). As mentioned previously, a certain deviation was observed in the specific gravity of carbonate sand and fines tested in this study. Accordingly, considering the influence of the specific gravity, the interfine void ratio for carbonate sand-fine mixtures was modified as follows:

$$e_{fc} = \left(\frac{G_f CS}{G_s(1 - CS)} + 1\right) e - \frac{n_{in}}{1 - n_{in}} \frac{G_f CS}{G_s(1 - CS)} \quad (8)$$

where e_{fc} is the modified interfine void ratio for carbonate sand-fine mixtures. G_s and G_f are specific gravities of carbonate sand and fines, respectively. When the carbonate sand content is zero, the modified interfine void ratio is still equal to the conventional void ratio.

Based on Equation (8), the modified interfine void ratios of all fifteen tested specimens at the initial and critical states were calculated, and the results are summarised in Table 4. The drained triaxial test results of five carbonate sand-fine mixtures with different carbonate sand content shown in Figures 2 and 3 were analysed using the framework of the modified interfine void ratio.

Table 4. Critical state data of triaxial tests in terms of the modified interfine void ratio.

Specimen Identity	Modified Interfine Void Ratio After Consolidation	Critical State		
		Mean Effective Stress (MPa)	Deviator Stress (MPa)	Modified Interfine Void Ratio
C5_V0	0.581	0.789	0.895	0.597
C5_V20	0.625	0.809	0.926	0.606
C5_V40	0.685	0.804	0.913	0.618
C10_V0	0.587	1.564	1.693	0.577
C10_V20	0.620	1.657	1.972	0.576
C10_V40	0.654	1.624	1.873	0.564
C30_V0	0.587	4.609	4.828	0.509
C30_V20	0.601	4.741	5.223	0.482
C30_V40	0.649	4.682	5.046	0.482
C5_I0	0.663	0.807	0.922	0.620
C5_I40	0.530	0.897	1.190	0.570
C10_I0	0.670	1.611	1.833	0.589
C10_I40	0.532	1.682	2.046	0.551
C30_I0	0.655	4.829	5.487	0.529
C30_I40	0.533	4.837	5.511	0.479

Among the tested specimens with similar void ratios (Figure 2), specimen C5_V0 had the smallest modified interfine void ratio e_{fc} after consolidation (the densest specimen), resulting in the highest peak strength and strongest dilative volumetric behaviour. In addition, all three specimens had similar e_{fc} (0.597~0.618) at the critical state, leading to a similar critical state shear strength (0.895~0.913 MPa). The same analysis results were obtained for those specimens with similar interfine void ratios (Figure 3). Thus, the modified interfine void ratio e_{fc} can be used as a state variable to coherently characterise the mechanical behaviour of carbonate sand-fine mixtures with carbonate sand contents no more than 40%.

The critical state data shown in Figures 3 and 4 are presented in Figure 11 in terms of the modified interfine void ratio. As shown in Figure 11a, for all tested specimens with different carbonate sand contents, there is a clear linear relationship between the deviator stress and the mean effective stress at the critical state. The CSL in the p' - q plane is a straight line, and all the critical state data are precisely fitted by only one straight line passing through the zero point, indicating the uniqueness of the CSL for carbonate sand-fine mixtures in the p' - q plane. According to Equation (8), there is a linear relationship between the modified interfine void ratio e_{fc} and void ratio e . Given the established linear relationship of the critical state data in the e - $\ln p'$ plane, it follows that the relationship of the critical state data in the e_{fc} - $\ln p'$ plane should also exhibit linearity. Figure 11b shows that although the data are somewhat discrete, for all tested specimens with different carbonate sand content, a clear linear relationship is observed between the modified interfine void ratio and the logarithm of the mean effective stress at the critical state. The CSL in the e_{fc} - $\ln p'$ plane is depicted using only one straight line, implying that the CSL of carbonate sand-fine mixtures in the e_{fc} - $\ln p'$ plane is also unique. The above fitting and analysis results show that for carbonate sand-fine mixtures with a carbonate sand content of no more than 40%, the critical state line in the variable space represented by (p', q, e_{fc}) is unique and not affected by the carbonate sand content. This further proves the rationality of the modified interfine void ratio as a state variable to describe the density of carbonate sand-fine mixtures.

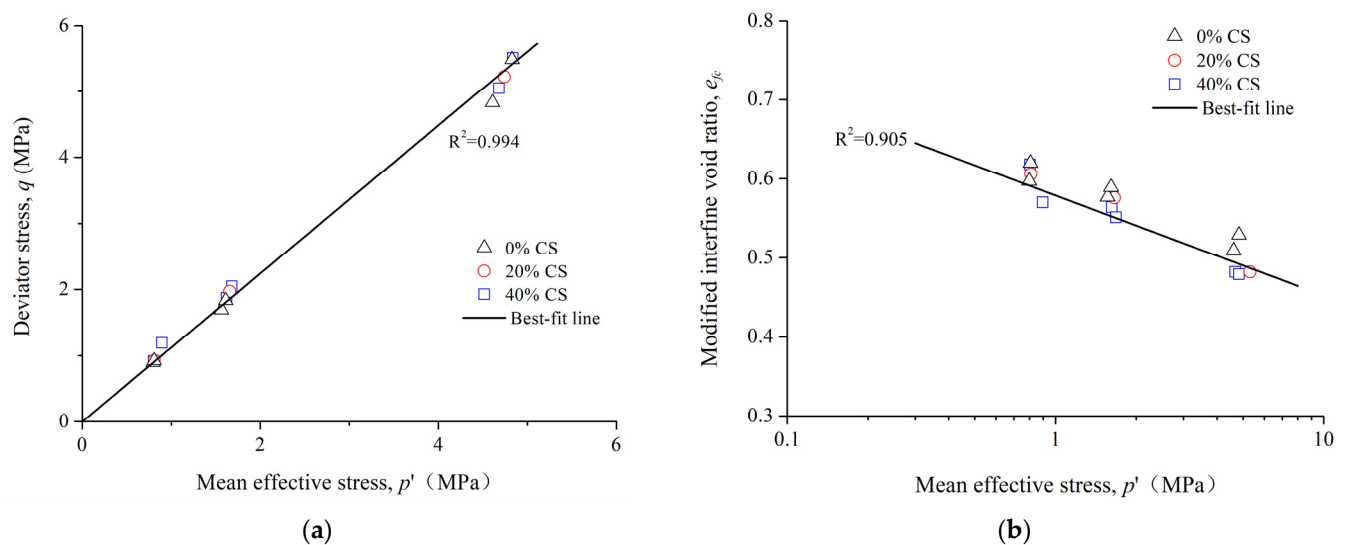


Figure 11. Critical state lines of carbonate sand-fine mixtures in (a) p' – q and (b) e_{fc} – $\ln p'$ planes.

5. Conclusions

In this study, drained triaxial tests were first carried out on carbonate sand-fine mixtures with carbonate sand contents ranging from 0 to 40%. As a potential density parameter, the interline void ratio was selected as a control variable in addition to the traditional void ratio. Therefore, two series of triaxial tests were conducted by controlling similar void ratios and interline void ratios. Based on the test results, the effect of carbonate sand content on the mechanical behaviour of carbonate sand-fine mixtures was investigated. Using computer tomography, the internal porosity of carbonate sand grains was measured, and the microscopic structure of carbonate sand-fine mixtures was studied. Furthermore, the equation of the interline void ratio was modified and proved effective in determining a state variable to describe the density of carbonate sand-fine mixtures. The conclusions are as follows:

- (1) For carbonate sand-fine mixtures with a similar initial void ratio, all mixtures exhibited strain-softening behaviour, and the peak strength decreased with an increase in carbonate sand content. The shear strength value was almost the same at large deformation under the same effective confining pressure. In addition, the volume changed from dilative to contractive behaviour with an increase in carbonate sand content. As the carbonate sand content increased, the mixture seemed to become “looser”. In the p' – q plane, a unique CSL was observed despite different carbonate sand contents, while the CSL shifted downwards with an increase in carbonate sand content in the e – $\ln p'$ plane.
- (2) For carbonate sand-fine mixtures with similar initial interline void ratios, the peak strength increased with an increase in carbonate sand content, and the volume changed from dilative to contractive behaviour with a decrease in carbonate sand content. The mixture became “denser” with an increase in carbonate sand content. In the e_{fc} – $\ln p'$ plane, the CSL shifted upwards with an increase in carbonate sand content. Compared to the mixture in which the density was controlled by the void ratio, the change rule of mechanical behaviour influenced by the carbonate sand content was precisely the opposite in specimens controlled by the interline void ratio.
- (3) Based on the two-dimensional cross-sectional CT images, it was found that most of the tested carbonate sand grains contained a substantial number of internal pores. For the tested carbonate sand, the internal porosities of one hundred carbonate sand grains were computed using an innovative integral calculation method. A normal

distribution of internal porosity was observed, with a mean value of 0.238. In addition, most of the carbonate sand grains were not in direct contact with each other, and the space around the contact angles between carbonate sand grains was tightly filled with fine particles. Carbonate sand grains likely did not contribute to the force chains of the fine matrix when the carbonate sand content was less than 40%.

- (4) According to the original definition of the interfine void ratio, except for the volume of carbonate sand grains, their internal pores should also be neglected. Considering the influence of internal pores, the equation of the modified interfine void ratio for carbonate sand-fine mixtures was established for the first time. The mixture with the smallest modified interfine void ratio exhibited the highest peak strength and strongest dilative volumetric behaviour. Using the modified interfine void ratio, unique CSLs were achieved in both the $p'-q$ and $e_{fc}-\ln p'$ planes, representing the critical state. As a result, the modified interfine void ratio can be used as a state variable to coherently characterise the mechanical behaviour of carbonate sand-fine mixtures with a carbonate sand content of no more than 40%.
- (5) The one hundred carbonate sand grains were randomly selected from the specimens of triaxial tests, having a certain representativeness in terms of selection method. However, they were not selected proportionally according to the composition of carbonate sand. If the proportion of a certain type of carbonate sand is too high in the one hundred carbonate sand grains, it may affect the mean value of internal porosity. By use of CT images, a resolution of approximately 5 μm could be achieved, which is favourably smaller than the diameter of the tested carbonate sand grains, ensuring the accuracy of measurement results. Particle breakage was not considered in this study, while it is an issue that cannot be ignored for carbonate sand especially under high confining pressures. Therefore, further research about the influences of particle breakage, as well as higher carbonate sand content, on the mechanical behaviour of the carbonate sand-fine mixtures are warranted.

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