

## Article

# Research on Dynamic Characteristics of High-Speed Helical Gears with Crack Faults in Electric Vehicle Deceleration Systems

Hongyuan Zhang , Dongsheng Li, He Wang and Hongyun Sun

School of Automotive and Transportation, Shenyang Ligong University, Shenyang 110159, China; 2302620755@stu.sylu.edu.cn (D.L.); 2302610196@stu.sylu.edu.cn (H.W.); shy5006@sylu.edu.cn (H.S.)

\* Correspondence: zhy\_sylu@163.com; Tel.: +86-13940049202

## Abstract

As a key component of pure electric vehicles, the reducer plays a vital role in power transmission and overall drive system performance. This study investigates the nonlinear dynamic characteristics of helical gears with tooth root crack faults in high-speed reducers. A coupled bending–torsional–shaft dynamic model is developed, in which the time-varying mesh stiffness of cracked helical gears is calculated using an improved potential energy method. The system’s nonlinear dynamic responses under varying mesh error excitation, gear backlash, and damping ratio are numerically obtained via the variable-step Runge–Kutta method. The results reveal that under high input speed conditions, the motion of the faulted system evolves from single-period to quasi-periodic motion as bifurcation parameters change. In the stable state, fault characteristic signals are apparent, whereas under strong nonlinear vibrations and chaotic motion, they become difficult to distinguish in traditional time- and frequency-domain analyses. To address this limitation, the DBSCAN clustering algorithm is introduced, which applies machine learning to cluster the Poincaré cross-sections of the system under different motion states. This approach enables the effective classification and identification of crack-induced and fault-related noise, thereby improving the accuracy of fault detection in nonlinear dynamic gear systems.

**Keywords:** time-varying meshing stiffness; helical gear system; bifurcation and chaos; crack; dynamic response



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## 1. Introduction

Root crack failure is one of the common fault forms in helical gear transmission systems. In electric vehicles, frequent rapid acceleration, rapid deceleration or prolonged climbing can cause the gears to be subjected to overload impacts, exceeding the design load limit, which in turn leads to cracks at the tooth root. Poor lubrication can accelerate the formation of fatigue cracks [1]. For instance, insufficient or aged lubricating oil will increase the coefficient of friction, and under the action of alternating stress, it will accelerate the propagation of microcracks. In addition, when vehicles operate for a long time in harsh environments such as damp and acidic conditions, the corrosion of tooth surfaces will reduce the strength of gears. Corrosion pits may also develop into crack sources, especially in high-humidity or chemically polluted environments, where corrosion products will weaken the toughness of materials. Therefore, it is essential to investigate the fault dynamics of helical gears from both design and diagnostic perspectives. From the design aspect, optimizing geometric parameters—such as improving tooth profiles through finite element analysis—can effectively enhance the fatigue resistance of gears. From the fault-diagnosis

aspect, early crack initiation can be identified by analyzing characteristic features in the vibration signals. These efforts are of great significance for ensuring the operational safety and reliability of electric vehicle transmission systems, preventing unexpected shutdowns or failures of the drivetrain [2,3].

In the context of studying the time-varying stiffness of a helical gear, Wan et al. developed a model to quantify the time-varying stiffness. This model was based on the principle of cumulative integral functions. It evaluated key geometric parameters, such as the number of teeth and the tooth width, as well as their interactions [4,5]. In the context of idealized conditions involving negligible surface friction, Liu et al. have refined the traditional method of computing the forces exerted by a gear tooth on a gear tooth [6]. Zhu et al. employed a computational approach to analyze the surface contact variations in a clamped-pitch gear, considering the effects of surface roughness on the contact stiffness [7]. He et al. calculated the time-varying meshing stiffness of gears considering eccentric defects based on the improved potential energy method, and also studied the influence of gear eccentricity on the lateral and torsional responses and dynamic transmission errors [8]. In this study, Hu et al. employed the Electric Constant Velocity Transmission (ECVT) as a model to calculate the time-varying stiffness of the gear mesh under various torque conditions. They then proceeded to analyze the impact of torque and time-varying stiffness on the nonlinear characteristics of the gear mesh [9]. In considering the condition that the core and the teeth are not coplanar, Yang et al. propose a method for calculating the instantaneous inter-engagement stiffness of a helical gear system. This method is more analogous to finite element methods [10]. Zhang et al. have proposed an enhanced finite element method that considers the non-overlapping phenomenon of the root and base circles, as well as the root transition circle angle, significantly improving calculation accuracy. Combined with simulations performed using KISSsoft v2022, a professional gear-analysis software widely used for gear design, strength evaluation, and meshing performance prediction, the influence of crack depth and angle on local strain degradation has been elucidated [11].

In the research on the dynamic characteristics of gears with faults, Mo et al. developed a dynamic model that considers the effects of friction and crack formation on the teeth. This model was used to calculate the time-varying coupling stiffness, the overall error, the load distribution ratio, and the displacement trend under conditions of crack formation and friction [12]. Ning et al. took into consideration the influence of the adjacent teeth's flexibility on their interaction, as well as the effect of the presence of cracks on the teeth's ability to engage with the gears. They also calculated the shear strength of the gears with cracks, thereby significantly improving the accuracy of the calculations [13]. In consideration of gearbox flexibility, Liu et al. analyzed the dynamic characteristics of a straight cylindrical gear with a tooth-root crack [14]. Vikash Kumar et al. have proposed a model for the analysis of nonlinear systems, specifically a direct-axis rotor system. This model is based on the analysis of the dynamics of a system with different levels of crack propagation. The model utilizes the crack propagation depth at the rotor's root as a variable to study the nonlinear dynamics of the rotor system [15]. Jiang et al. studied the time-varying meshing stiffness of three different types of cracks and used statistical indicators extracted in the time domain and frequency domain to evaluate the influence of cracks on dynamic responses [16]. Zhang et al. employed the method of energy to calculate the time-varying tooth-tooth interaction stiffness under various conditions of crack, and established a model of the high-speed train's gear transmission system's torsional vibration. They then computed the system's response to vibrations due to cracks [17]. Among them, the energy-based method for calculating the time-varying meshing stiffness of helical gears applies a slicing and integration approach to distribute the meshing deformation

into corresponding energy components. Yang et al. studied the dynamic response of the gear transmission system with wear faults in high-speed trains and conducted experimental research [18]. Jiang et al. proposed a gear dynamics model for calculating spalling defects and took into account the influence of friction [19]. In view of the influence of elastohydrodynamic lubrication and friction theory on the root crack fault, Braumann, L. et al. comprehensively described the gear tribology technology under the condition of lubrication loss [20]. Based on the improved potential energy method, Zhao J established the dynamic model of spur gear with root crack fault, analyzed the influence of different crack degrees on the tribological characteristics of the system and predicted the influence of root crack on the friction dynamic behavior of spur gear pair [21]. Wang et al. calculated the meshing stiffness of cracked planetary gears under the friction force obtained under the condition of mixed elastohydrodynamic lubrication (EHL), and analyzed the influence of crack propagation conditions on the meshing stiffness of planetary gear systems [22]. Wu et al. proposed an EHL-based micro-contact thermodynamic model that accurately captures the effects of sliding speed, oil temperature, and surface pressure on interfacial thermodynamic behavior and clarifies the failure mechanism of wet friction discs [23].

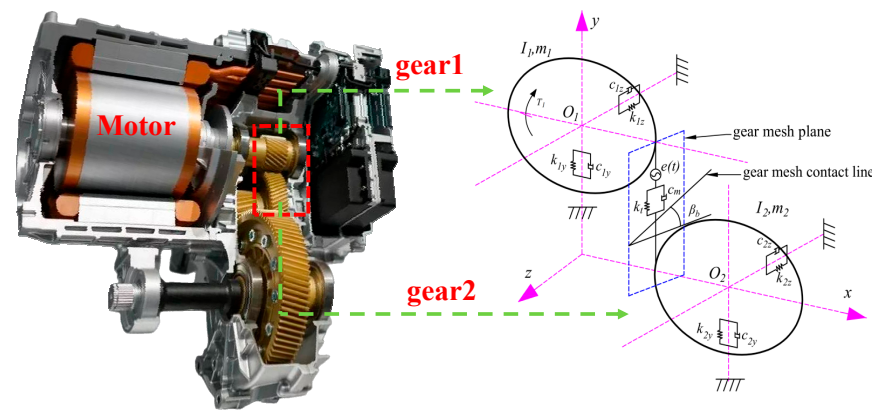
For the study of clustering algorithms, Ezugwu et al. presented a comprehensive and up-to-date review of traditional and state-of-the-art clustering techniques, emphasizing their practical relevance across diverse domains such as education, medicine, marketing, biology, and bioinformatics. Their survey highlights current challenges with conventional methods, discusses emerging applications in big data, AI, and robotics, and serves as a valuable reference for designing improved clustering algorithms [24]. Bhattacharjee et al., provided a comprehensive survey of density-based clustering algorithms, providing a detailed taxonomy, comparative analysis, application domains, and future research directions for DBCLAs developed over the past two decades [25]. Kulkarni et al. reviewed DBSCAN and its variants for big data clustering, summarizing their characteristics and providing a detailed comparative analysis of these density-based methods [26].

Based on an extensive review of the literature, it is evident that current research primarily focuses on improving algorithms for calculating the time-varying meshing stiffness of helical gears. Studies on fault dynamics often concentrate solely on linear signal characteristics in the time and frequency domains. While these methods can indirectly reflect the system's motion state, they fail to capture the complexity of nonlinear dynamics. In this study, a six-degree-of-freedom bending-torsion-axial coupling dynamic model of a high-speed helical gear pair in an electric vehicle reducer system is established using the lumped parameter method. The time-varying meshing stiffness of helical gears with root crack faults is calculated based on an improved potential energy method. The dynamic responses of the high-speed helical gear transmission system under varying parameters after introducing root crack faults are analyzed. Furthermore, the nonlinear characteristics of both healthy and faulty helical gear transmission systems are investigated.

The first chapter provides a comprehensive overview of recent research on time-varying meshing stiffness and dynamic analysis of healthy and faulty helical gears. The second chapter presents the dynamic modeling of the helical gear pair in the high-speed stage of an electric vehicle reducer. In the third chapter, the time-varying meshing stiffness for both healthy and root crack fault conditions is calculated. The fourth chapter conducts a nonlinear dynamic analysis of the faulty helical gear pair, incorporating the DBSCAN clustering algorithm to analyze points on the Poincaré section, enabling clear discrimination between chaotic and fault-induced signal characteristics. The final chapter summarizes the conclusions of the study.

## 2. Establishment of a Dynamic Model of a Helical Gear Transmission System

The present study focuses on the high-speed input gear of a certain electric vehicle's secondary helical gear. The establishment of a six-degree-of-freedom model of a helical gear transmission system utilizing the concentration parameter method is illustrated in Figure 1.



**Figure 1.** The dynamic model of the helical gear pair.

The following system of differential equations is derived from the model depicted in Figure 1:

$$\begin{cases} m_1 \ddot{y}_1 + c_{1y} \dot{y}_1 + k_{1y} y_1 = -F_y \\ m_1 \ddot{z}_1 + c_{1z} \dot{z}_1 + k_{1z} z_1 = -F_z \\ I_1 \ddot{\theta}_1 = T_1 - F_y r_{b1} \\ m_2 \ddot{y}_2 + c_{2y} \dot{y}_2 + k_{2y} y_2 = F_y \\ m_2 \ddot{z}_2 + c_{2z} \dot{z}_2 + k_{2z} z_2 = F_z \\ I_2 \ddot{\theta}_2 = -T_2 + F_y r_{b2} \end{cases} \quad (1)$$

where  $m_1$  and  $m_2$  are the main and driven gear mass, respectively.  $k_{1y}$  and  $k_{2y}$  are the Y-direction equivalent support stiffness of the main and driven gears, respectively.  $k_{1z}$  and  $k_{2z}$  are the Z-direction equivalent support stiffness of the main and driven gears, respectively.  $c_{1y}$  and  $c_{2y}$  are the Y-direction equivalent support damping of the main and driven gears, respectively.  $c_{1z}$  and  $c_{2z}$  are the Z-direction equivalent support damping of the main and driven gears, respectively.  $y_1$  and  $y_2$  are the main and auxiliary gears, respectively, in the Y direction.  $z_1$  and  $z_2$  are the main and auxiliary gears, respectively, on the Z-axis.  $r_{b1}$  and  $r_{b2}$  are the main components, and the diameter of the moving wheel is the same as its base.  $I_1$  and  $I_2$  are the inertia of the main and driven gears, respectively.  $\theta_1$  and  $\theta_2$  are the points where the center of the gear center rotates.

Due to the presence of vibration and error, the dynamic meshing deformation amount  $\delta$  between the two helical gears at the equivalent meshing point along the meshing line is expressed as follows:

$$\delta = (y_1 - y_2) - (z_1 - z_2) \sin \beta - (r_{b1} \theta_1 - r_{b2} \theta_2) \cos \beta - e(t) \quad (2)$$

The dynamic contact force of the spiral helical gear in the Y and Z directions is expressed as follows:

$$\begin{cases} F_m = c_m \dot{\delta} + k_t f(\delta) \\ F_y = F_m \cos \beta_b \\ F_z = F_m \sin \beta_b \end{cases} \quad (3)$$

where  $\beta_b$  is the helix Angle of the gear base circle and  $\beta_b = \arctan(\tan \beta \cos \alpha_n)$ .  $k_t$  is the variable stiffness of the meshing of the helical gear.  $f(\delta)$  is the nonlinear function of the gear backlash, and the definition of the backlash can be expressed as  $2b$ :

$$f(\delta) = \begin{cases} \delta - b, & \delta > b; \\ 0, & |\delta| \leq b; \\ \delta + b, & \delta < -b; \end{cases} \quad (4)$$

$c_m$  is the gear meshing damping and can be expressed as:

$$c_m = 2\zeta_m \sqrt{k_t \frac{r_{b1}^2 r_{b2}^2 I_1 I_2}{I_1 r_{b1}^2 + I_2 r_{b2}^2}} \quad (5)$$

Among them,  $\zeta_m$  is the contact damping ratio.

$e(t)$  is the dynamic gear meshing error, whose expression is:

$$e(t) = e_0 + \sum_{i=1}^n (er_{2i} \sin(i\omega t) + er_{2i} \cos(i\omega t)) = e_0 + \sum_{i=1}^n er_i \cos(i\omega_m t + \phi_i) \quad (6)$$

Among them,  $e_0$  is the mean value of the gear meshing error,  $er_i$  is the amplitude of the  $i$ -th harmonic of the meshing error,  $\omega_m$  is the meshing frequency, and  $\phi_i$  is the initial phase angle of the  $i$ -th harmonic and  $\phi_i = \arctan(-er_{2i-i}/er_{2i})$ , let  $\bar{er}_i = \sqrt{er_{2i-i}^2 + er_{2i}^2}$ . Since this paper takes into account various nonlinear factors, the transmission error in this paper is taken as the first harmonic term, that is,  $i = 1$ :

$$e(t) = e_0 + er_1 \cos(\omega t + \phi_1) \quad (7)$$

$er_1$  represents the amplitude of the first harmonic of the transmission error, and  $\phi_1$  represents the initial phase of the first harmonic.

To ensure engineering relevance, the fundamental parameters of this component are presented in Table 1. These parameters originate from the reducer gear pair of an existing commercial new-energy vehicle. As such, they are representative of typical high-speed helical gears used in modern electric vehicle drivetrains, and therefore provide a realistic basis for evaluating the applicability and generality of the dynamic characteristics discussed in this study.

**Table 1.** Basic parameters of helical gear pair.

| Properties                                  | Unit | Pinion            | Gear              |
|---------------------------------------------|------|-------------------|-------------------|
| Number of teeth $Z$                         | -    | 21                | 65                |
| Modulus $m_n$                               | mm   | 1.6               | 1.6               |
| Pressure angle $\alpha$                     | °    | 19                | 19                |
| Helix angle $\beta$                         | °    | 23.5              | 23.5              |
| Gear width $B$                              | mm   | 32                | 32                |
| Mass $m$                                    | kg   | 0.92              | 1.45              |
| Vertical bearing equivalent stiffness $k_y$ | N/m  | $6.0 \times 10^8$ | $1.5 \times 10^8$ |
| Axial bearing equivalent stiffness $k_z$    | N/m  | $6.0 \times 10^8$ | $1.5 \times 10^8$ |
| Rotation direction                          | -    | Right             | Left              |
| Poisson's ratio $\mu$                       | -    | 0.3               | 0.3               |
| Modulus of elasticity $E$                   | GPa  | 206               | 206               |

In constructing the dynamic model of the helical gear system, several assumptions were made to ensure mathematical tractability and computational feasibility. These as-

sumptions are commonly adopted in gear dynamics research; however, their potential influence on model accuracy deserves further clarification.

- (1) Rigid gear bodies. The gears were modeled as rigid disks with local tooth flexibility represented by meshing stiffness. This assumption is widely used in high-speed gear dynamics because the bending and torsional deformations of the gear bodies are significantly smaller than the local tooth compliance. Therefore, the rigid-body assumption has only a minor influence on the predicted meshing vibration.
- (2) Time-varying meshing stiffness based on cantilever-beam theory. The meshing stiffness was calculated using a discrete integration method combined with a cantilever-beam representation of the tooth. Although this introduces slight simplifications to the real tooth geometry, previous studies have confirmed that this method can accurately capture the primary stiffness fluctuation pattern, even in the presence of cracks [4].
- (3) Neglecting friction-induced excitation in the axial direction. Frictional forces were included only in the transverse plane. This simplification is reasonable because axial friction forces are typically much smaller compared with the normal and tangential contact forces. Consequently, their influence on the overall dynamic response is limited.
- (4) Crack modeled as a stiffness-reduction zone. The tooth crack was represented by a local reduction in the tooth-root stiffness. This approximation has been validated in earlier works and effectively describes the dominant influence of root cracks on the meshing characteristics. Nevertheless, secondary effects such as crack propagation direction and stress redistribution are not considered, which may cause slight deviations in extreme loading conditions.

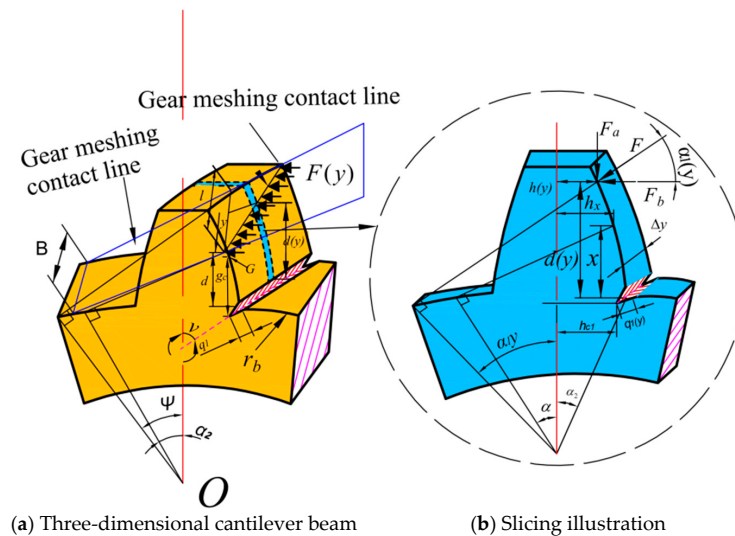
Overall, these assumptions simplify the model while preserving the essential nonlinear dynamic behaviors of interest. Although certain high-frequency or secondary effects may be neglected, the resulting model captures the key dynamic characteristics of cracked helical gears with sufficient accuracy for engineering applications.

### 3. Calculation of Time-Varying Meshing Stiffness of Helical Gear with Tooth Root Crack

The energy method for determining the time-varying meshing stiffness (TVMS) of helical gears is based on the principle of equating the total elastic deformation energy stored in the gear tooth during meshing to an equivalent spring system. Owing to the gradual, multi-tooth engagement introduced by the helix angle, the meshing region is discretized along the face width into a series of two-dimensional micro-slices, each treated as an equivalent spur gear element. For each slice, the strain energy contributions arising from bending, shear, axial compression, and Hertzian contact deformation are computed using established analytical formulations. By integrating the strain energy of all slices over the entire meshing width, the total elastic energy of the helical tooth is obtained. The procedure for calculating the meshing stiffness of a healthy gear using the energy method can be found in Ref. [5]. The instantaneous meshing stiffness is then derived from the relationship between this total strain energy and the corresponding meshing displacement, providing an accurate and physically grounded estimation of the TVMS that captures the three-dimensional deformation characteristics governing the dynamic response of helical gear systems.

The tooth root crack is defined by the crack angle  $\nu$  (constant) and the crack depth  $q$ . This paper assumes that the crack propagation shape is a parallelogram with a tangent length running through the entire tooth width, the crack propagation path is linear and the depth is equal along the direction of the dashed line in the figure, and the influence of

crack thickness (opening degree) on the overall position of the gear teeth is ignored. The three-dimensional slice diagram of helical gear with tooth root crack is shown in Figure 2.

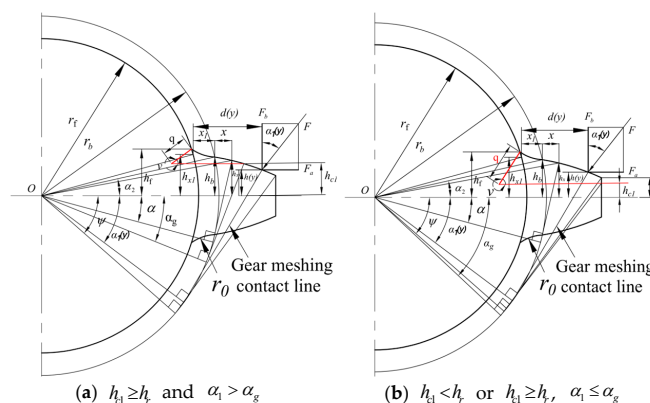


**Figure 2.** Slicing diagram of helical gear with crack fault.

In the event that the base circle and the root circle are not congruent, as illustrated in Figure 2, the following parameters can be calculated: bending stiffness  $k_b$ , shear stiffness  $k_s$ , axial compression stiffness  $k_a$ , hertz contact stiffness  $k_h$ , and elastic stiffness of the matrix  $k_f$ .

As illustrated in Figure 2, when a tooth root crack defect occurs in a gear, the length of the contact line on the tooth surface remains constant at the same meshing position. Consequently, such a root crack does not influence the Hertzian contact stiffness within the single-tooth meshing stiffness of the gear. Moreover, the presence of a crack does not affect the gear's capacity to withstand radial loads. Therefore, in the evaluation of load-bearing stiffness against axial loads, the calculation method applicable to healthy gears can still be employed. For gear pairs with tooth root cracks, after recalculating the bending and shear stiffness under the cracked condition, the overall meshing stiffness can be determined indirectly.

According to Reference [27], the helical gears with root cracks discussed in this study are analyzed under four different scenarios: When the radius of the base circle is greater than the radius of the tooth root circle, the cantilever beam models of Situation 1 and 2 are shown in Figure 3. The analytical expressions for the time-varying meshing stiffness are based on the improved potential energy method of Zhang et al. [11], which has been widely used in helical gear dynamics.



**Figure 3.**  $r_b > r_f$ , crack fault helical gear slice cantilever beam model.

When  $r_b > r_f$ ,  $h_{c1} \geq h_r$  and  $\alpha_1 > \alpha_g$ , as shown in Figure 3a, the inertia distance and cross-sectional area of the cracked gear tooth from the base circle  $x$  are recorded as  $I'_x$  and  $A'_x$ , respectively. Its expression is as follows:

$$\begin{cases} I'_x = \frac{1}{12}(h_{c1} + h_x)^3 B & x \leq g_c \\ I'_x = \frac{1}{12}(2h_x)^3 B & x > g_c \end{cases} \quad (8)$$

$$\begin{cases} A'_x = (h_{c1} + h_x) B & x \leq g_c \\ A'_x = 2h_x B & x > g_c \end{cases} \quad (9)$$

In the formula:  $h_{c1}$  is the vertical distance from the crack tip to the center line of the gear,  $h_{c1} = R_{b1} \sin \alpha_2 - q \sin v$ .

In this case, the bending stiffness  $k_b$  and shear stiffness  $k_s$  with tooth root cracks can be expressed as:

$$\begin{aligned} k_{bcrack} = \sum_{i=1}^N 1/ & \left\{ \frac{12}{E} \int_{-\alpha_g}^{\alpha_2} \frac{\{1 + \cos \alpha'_1 [(\alpha_2 - \alpha) \sin \alpha - \cos \alpha]\}^2 (\alpha_2 - \alpha) \cos \alpha}{\left[ \sin \alpha_2 - \left( \frac{q_1}{r_{b1}} \right) \sin v + \sin \alpha + (\alpha_2 - \alpha) \cos \alpha \right]^3 \Delta y} d\alpha \right\} \\ & + \sum_{i=1}^N 1/ \left\{ \frac{1.5}{E} \int_{-\alpha'_1}^{-\alpha_g} \frac{\{1 + \cos \alpha'_1 [(\alpha_2 - \alpha) \sin \alpha - \cos \alpha]\}^2 (\alpha_2 - \alpha) \cos \alpha}{[\sin \alpha + (\alpha_2 - \alpha) \cos \alpha]^3 \Delta y} d\alpha \right\} \\ & + \int_0^{r_b - r_f} \frac{1.5 \{ [d(y) + x_1] \cos \alpha'_1 - h(y) \sin \alpha'_1 \}^2}{2Eh_{x1}^3} dx_1 \end{aligned} \quad (10)$$

$$\begin{aligned} k_{scrack} = \sum_{i=1}^N 1/ & \left\{ \frac{1.2}{E} \int_{-\alpha_g}^{\alpha_2} \frac{(1 + v)(\alpha_2 - \alpha) \cos \alpha \cos \alpha'^2_1}{\left[ \sin \alpha_2 - \left( \frac{q_1}{r_{b1}} \right) \sin v + \sin \alpha + (\alpha_2 - \alpha) \cos \alpha \right] \Delta y} d\alpha \right\} \\ & + \sum_{i=1}^N 1/ \left\{ \frac{1.2}{E} \int_{-\alpha'_1}^{-\alpha_g} \frac{\{1 + \cos \alpha'_1 [(\alpha_2 - \alpha) \sin \alpha - \cos \alpha]\}^2 (\alpha_2 - \alpha) \cos \alpha}{\left[ \sin \alpha_2 + \left( \frac{q_1}{r_{b1}} \right) \sin v + \sin \alpha + (\alpha_2 - \alpha) \cos \alpha \right] \Delta y} d\alpha \right\} \\ & + \int_0^{r_b - r_f} \frac{1.2 \cos \alpha'^2_1}{GA_{x1}} dx_1 \end{aligned} \quad (11)$$

Here,  $N$  represents the number of parts that the helical gear is cut into;  $\Delta y = l/N$ ;  $l$  is the component of the contact wire length in the direction of tooth width.

Situation 2: When  $r_b > r_f$ , satisfies  $h_{c1} < h_r$  or  $h_{c1} \geq h_r$ ,  $\alpha_1 \leq \alpha_g$ , as shown in Figure 3b, the inequality of  $x \leq g_c$  is constant, and  $I'_x$  and  $A'_x$  can be simplified to:

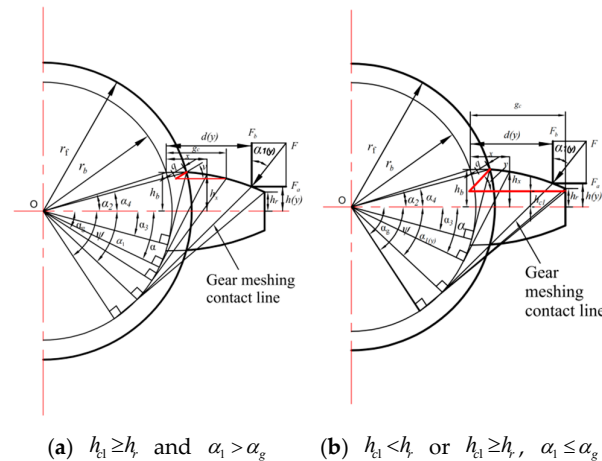
$$\begin{cases} I'_x = \frac{1}{12}(h_{c1} + h_x)^3 B \\ A'_x = (h_{c1} + h_x) B \end{cases} \quad (12)$$

In this case,  $k_b$  and  $k_s$  can be expressed as:

$$\begin{aligned} k_{bcrack} = \sum_{i=1}^N 1/ & \left\{ \frac{12}{E} \int_{-\alpha'_1}^{\alpha_2} \frac{\{1 + \cos \alpha'_1 [(\alpha_2 - \alpha) \sin \alpha - \cos \alpha]\}^2 (\alpha_2 - \alpha) \cos \alpha}{\left[ \sin \alpha_2 - \left( \frac{q_1}{r_{b1}} \right) \sin v + \sin \alpha + (\alpha_2 - \alpha) \cos \alpha \right]^3 \Delta y} d\alpha \right\} \\ & + \int_0^{r_b - r_f} \frac{1.5 \{ [d(y) + x_1] \cos \alpha'_1 - h(y) \sin \alpha'_1 \}^2}{2Eh_{x1}^3} dx_1 \end{aligned} \quad (13)$$

$$k_{scrack} = \sum_{i=1}^N 1/ \left\{ \frac{1.2}{E} \int_{-\alpha'_1}^{\alpha_2} \frac{(1+v)(\alpha_2 - \alpha) \cos \alpha \cos \alpha'^2_1}{\left[ \sin \alpha_2 - \left( \frac{q_1}{r_{b1}} \right) \sin v + \sin \alpha + (\alpha_2 - \alpha) \cos \alpha \right] \Delta y} d\alpha \right\} + \int_0^{r_b - r_f} \frac{1.2 \cos \alpha'^2_1}{GA_{x1}} dx_1 \quad (14)$$

When the radius of the base circle of the gear is less than the radius of the tooth root circle, the cantilever beam model of Situations 3 and 4 is shown in Figure 4.



**Figure 4.**  $r_b < r_f$ , crack fault helical gear slice cantilever beam model.

Situation 3: When  $r_b < r_f$ ,  $h_{c1} \geq h_r$  and  $\alpha_1 > \alpha_g$  are satisfied, as shown in Figure 4a. In this case,  $k_b$  and  $k_s$  can be expressed as:

$$k_{bcrack} = \sum_{i=1}^N 1/ \left\{ \frac{12}{E} \int_{-\alpha_g}^{f(\alpha)} \frac{\{1 + \cos \alpha'_1 [(\alpha_2 - \alpha) \sin \alpha - \cos \alpha]\}^2 (\alpha_2 - \alpha) \cos \alpha}{\left[ \sin \alpha_2 - \left( \frac{q_1}{r_{b1}} \right) \sin v + \sin \alpha + (\alpha_2 - \alpha) \cos \alpha \right]^3 \Delta y} d\alpha \right\} + \sum_{i=1}^N 1/ \left\{ \frac{1.5}{E} \int_{-\alpha'_1}^{-\alpha_g} \frac{\{1 + \cos \alpha'_1 [(\alpha_2 - \alpha) \sin \alpha - \cos \alpha]\}^2 (\alpha_2 - \alpha) \cos \alpha}{[\sin \alpha + (\alpha_2 - \alpha) \cos \alpha]^3 \Delta y} d\alpha \right\} \quad (15)$$

$$k_{scrack} = \sum_{i=1}^N 1/ \left\{ \frac{1.2}{E} \int_{-\alpha_g}^{f(\alpha)} \frac{(1+v)(\alpha_2 - \alpha) \cos \alpha \cos \alpha'^2_1}{\left[ \sin \alpha_2 - \left( \frac{q_1}{r_{b1}} \right) \sin v + \sin \alpha + (\alpha_2 - \alpha) \cos \alpha \right] \Delta y} d\alpha \right\} + \sum_{i=1}^N 1/ \left\{ \frac{1.2}{E} \int_{-\alpha'_1}^{-\alpha_g} \frac{\{1 + \cos \alpha'_1 [(\alpha_2 - \alpha) \sin \alpha - \cos \alpha]\}^2 (\alpha_2 - \alpha) \cos \alpha}{\left[ \sin \alpha_2 + \left( \frac{q_1}{r_{b1}} \right) \sin v + \sin \alpha + (\alpha_2 - \alpha) \cos \alpha \right] \Delta y} d\alpha \right\} \quad (16)$$

Situation 4: When  $r_b < r_f$ ,  $h_{c1} < h_r$ , or  $h_{c1} \geq h_r$  and  $\alpha_1 \leq \alpha_g$ , as shown in Figure 4b. In this case,  $k_b$  and  $k_s$  can be expressed as follows:

$$k_{bcrack} = \sum_{i=1}^N 1/ \left\{ \frac{12}{E} \int_{-\alpha'_1}^{f(\alpha)} \frac{\{1 + \cos \alpha'_1 [(\alpha_2 - \alpha) \sin \alpha - \cos \alpha]\}^2 (\alpha_2 - \alpha) \cos \alpha}{\left[ \sin \alpha_2 - \left( \frac{q_1}{r_{b1}} \right) \sin v + \sin \alpha + (\alpha_2 - \alpha) \cos \alpha \right]^3 \Delta y} d\alpha \right\} \quad (17)$$

$$k_{scrack} = \sum_{i=1}^N 1 / \left\{ \frac{1.2}{E} \int_{-\alpha'_1}^{f(\alpha)} \frac{(1+v)(\alpha_2 - \alpha) \cos \alpha \cos \alpha_1'^2}{\left[ \sin \alpha_2 - \left( \frac{q_1}{r_{b1}} \right) \sin v + \sin \alpha + (\alpha_2 - \alpha) \cos \alpha \right] \Delta y} d\alpha \right\} \quad (18)$$

The existence of root cracks will affect the time-varying meshing stiffness of helical gears. Based on the previous text, this paper, respectively, studies the influence of the crack depth and crack angle at the tooth root on the meshing stiffness of helical gears. The selected parameter data are shown in Tables 2 and 3:

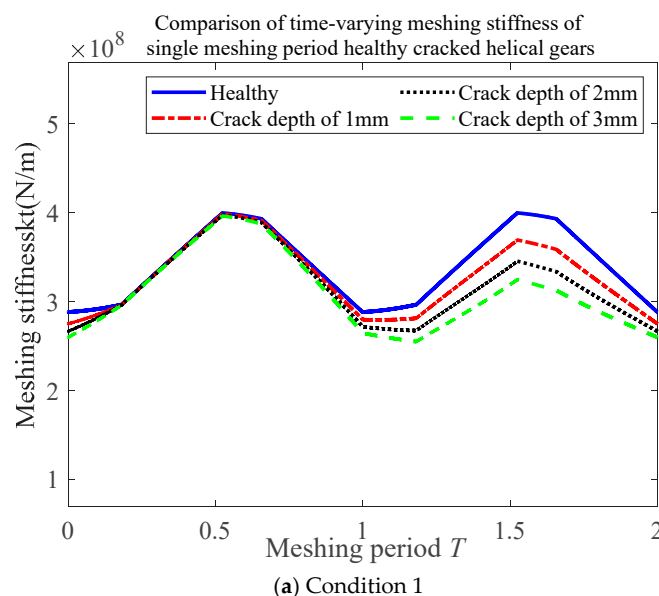
**Table 2.** Crack Condition 1.

| Parameter/Sort | 1    | 2    | 3    | 4    |
|----------------|------|------|------|------|
| q              | 0 mm | 1 mm | 2 mm | 3 mm |
| v              | -    | 20°  | 20°  | 20°  |

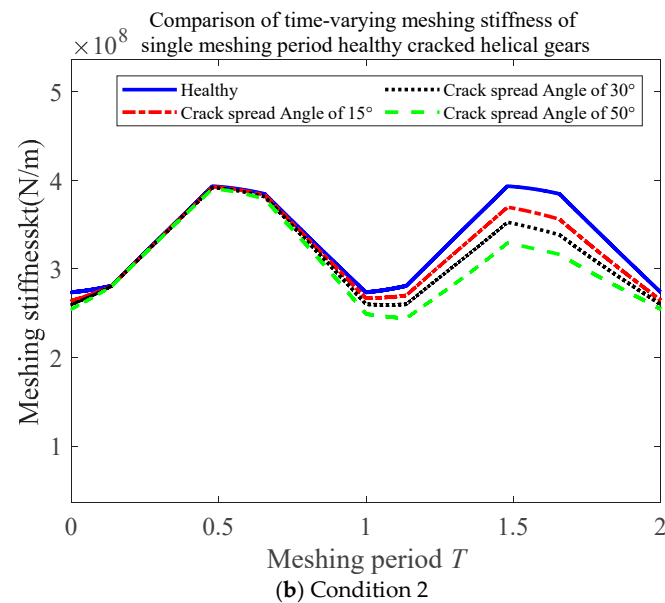
**Table 3.** Crack Condition 2.

| Parameter/Sort | 1    | 2    | 3    | 4    |
|----------------|------|------|------|------|
| q              | 0 mm | 1 mm | 1 mm | 1 mm |
| v              | -    | 15°  | 30°  | 50°  |

It can be seen from Figure ?? that as the crack depth increases from 0 mm to 3 mm, the meshing stiffness of helical gears continuously decreases, and the attenuation rate gradually accelerates. The mechanism lies in that the increase in crack depth will intensify the stress concentration at the tooth root and weaken the local load-bearing capacity. As the crack extends towards the centerline of the gear, the risk of tooth breakage increases and the decrease in stiffness becomes more significant. As the crack propagation angle increases, the stiffness also decreases, but the rate of reduction slows down. In comparison, the weakening effect of crack depth on stiffness is more significant; especially when the depth exceeds 2 mm, the rate of stiffness decline accelerates sharply, indicating that the time-varying meshing stiffness is more sensitive to crack depth.



**Figure 5.** Cont.



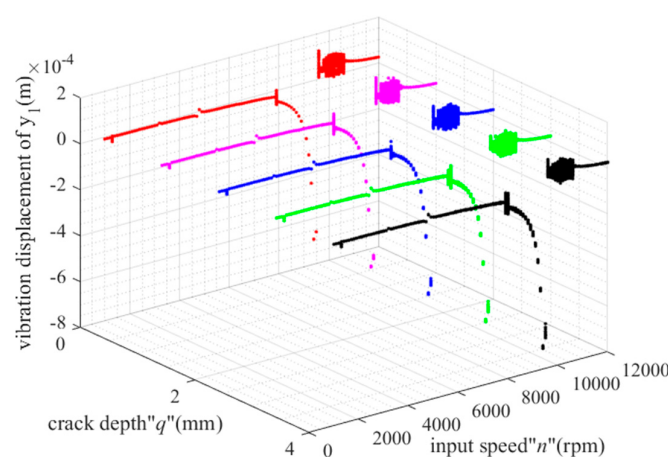
**Figure 5.** Time-varying meshing stiffness of healthy and faulty helical gears.

#### 4. Analysis of Dynamic Characteristics of Helical Gear Transmission System with Root Crack Fault

This section presents a comprehensive nonlinear dynamic analysis of the helical gear transmission system under root crack fault conditions. Various key parameters—including input speed, backlash, error excitation amplitude, and meshing damping ratio—are investigated to evaluate their influence on system responses. The following subsections detail the effects of these parameters individually.

##### 4.1. The Influence of Input Speed on the System Under the Fault of Tooth Root Crack

Taking the input rotational speed as the bifurcation parameter, the crack depth  $q$  at the tooth root was set to 0 mm, 1 mm, 2 mm, 3 mm, and 4 mm, respectively. The gear back lash, error excitation amplitude, and damping ratio were assigned specific values. The resulting bifurcation diagram of the driving gear in the  $y$ -direction is presented in Figure 6.

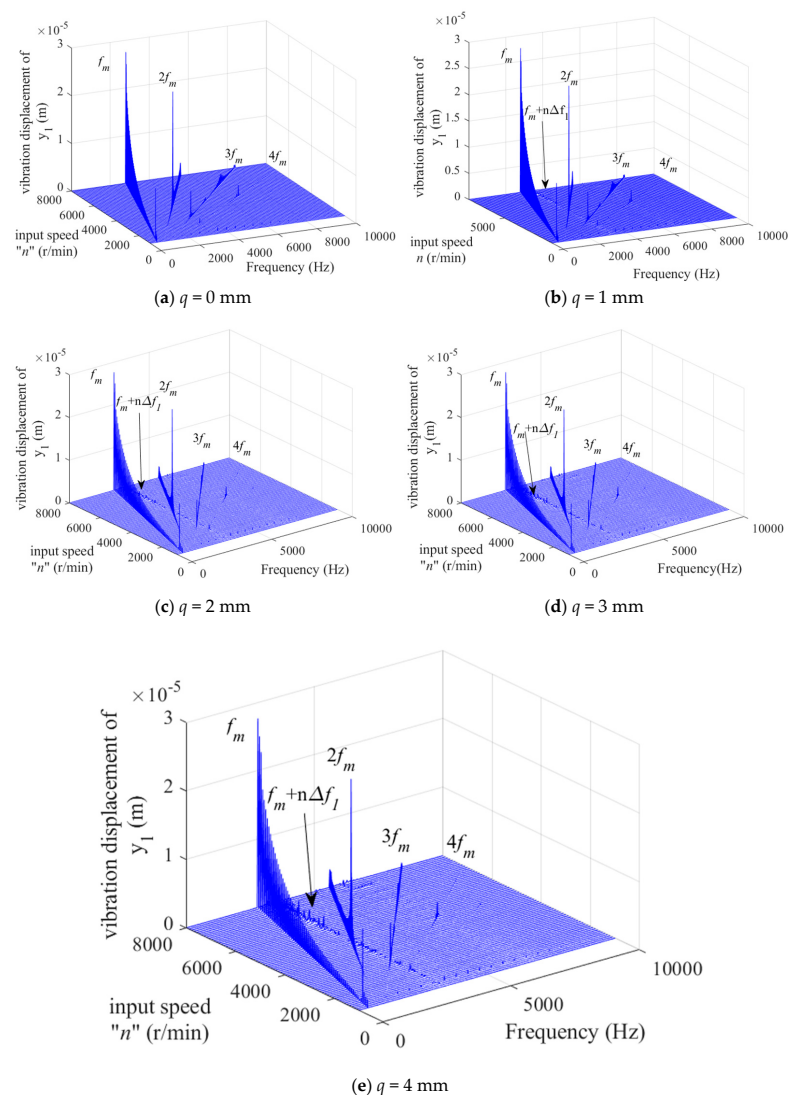


**Figure 6.** The influence of crack depth on the rotational speed bifurcation diagram.

As illustrated in Figure 6, within the low-speed operating range, the influence of crack faults on the system's motion state is relatively minimal, and the dynamic response remains largely consistent with that of an intact system. Under high-speed operating conditions, crack faults can lead to a transition of the system behavior from single-periodic

motion to quasi-periodic motion exhibiting pronounced nonlinear characteristics. Moreover, the extent of this transformation demonstrates a clear intensification trend as the crack depth  $q$  increases.

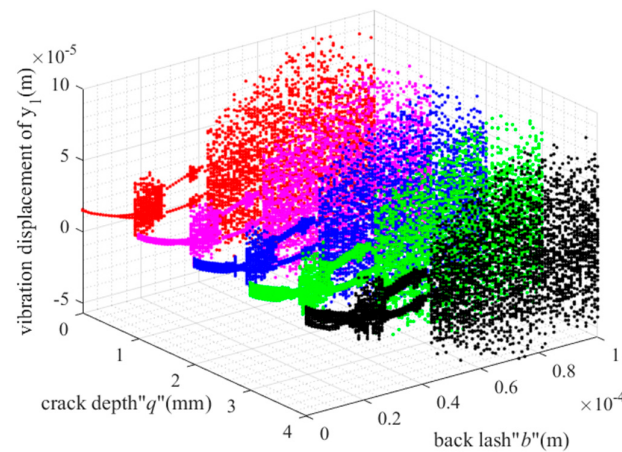
Based on a tooth root crack depth of  $q = 3$  mm, the influence of crack faults on the dynamic characteristics of the system during the speed-up process is further investigated. As shown in Figure 7, within the subcritical speed range, the effect of the tooth root crack on the radial vibration of the helical gear system is relatively insignificant. However, as the rotational speed approaches the critical value, the critical speed of the cracked system becomes slightly higher than that of the healthy system. This phenomenon can be attributed to the local reduction in stiffness caused by the tooth root crack, which alters the overall stiffness distribution of the system, thereby modifying its natural frequency and critical speed. At the critical speed, the displacement amplitude under the cracked condition is markedly higher than that under the healthy condition, indicating that the presence of the crack amplifies the resonance response of the system. This occurs because the crack weakens the gear meshing stiffness, reducing the system's damping capacity at resonance. Consequently, vibration energy tends to accumulate and be released more intensely, resulting in a larger displacement amplitude. Even beyond the critical speed, the influence of the crack on the system's dynamic characteristics remains evident.



**Figure 7.** Three-dimensional waterfall diagram in the y direction of the driving wheel.

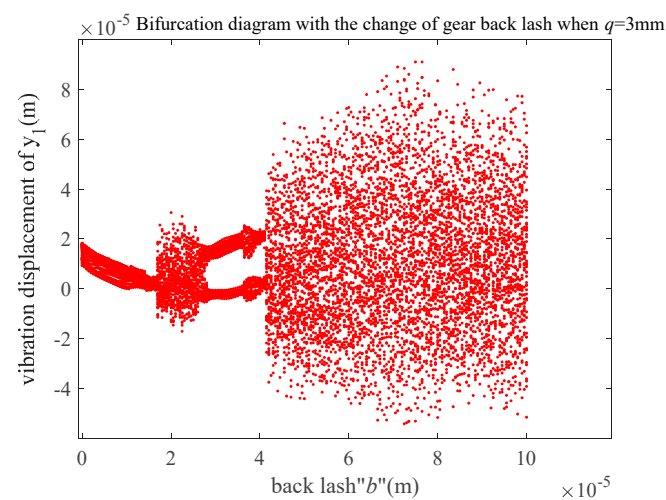
#### 4.2. The Influence of Backlash on the System Under Tooth Root Crack Fault

Taking the input speed as 10,000 r/min and using the gear backlash as the bifurcation parameter, the crack depth  $q$  at the tooth root is set to 0 mm, 1 mm, 2 mm, 3 mm, and 4 mm, respectively. With fixed values for the meshing damping ratio and error excitation amplitude, the bifurcation diagram in the y-direction of the driving gear is obtained, as illustrated in Figure 8.



**Figure 8.** The influence of crack depth on the gear backlash bifurcation diagram.

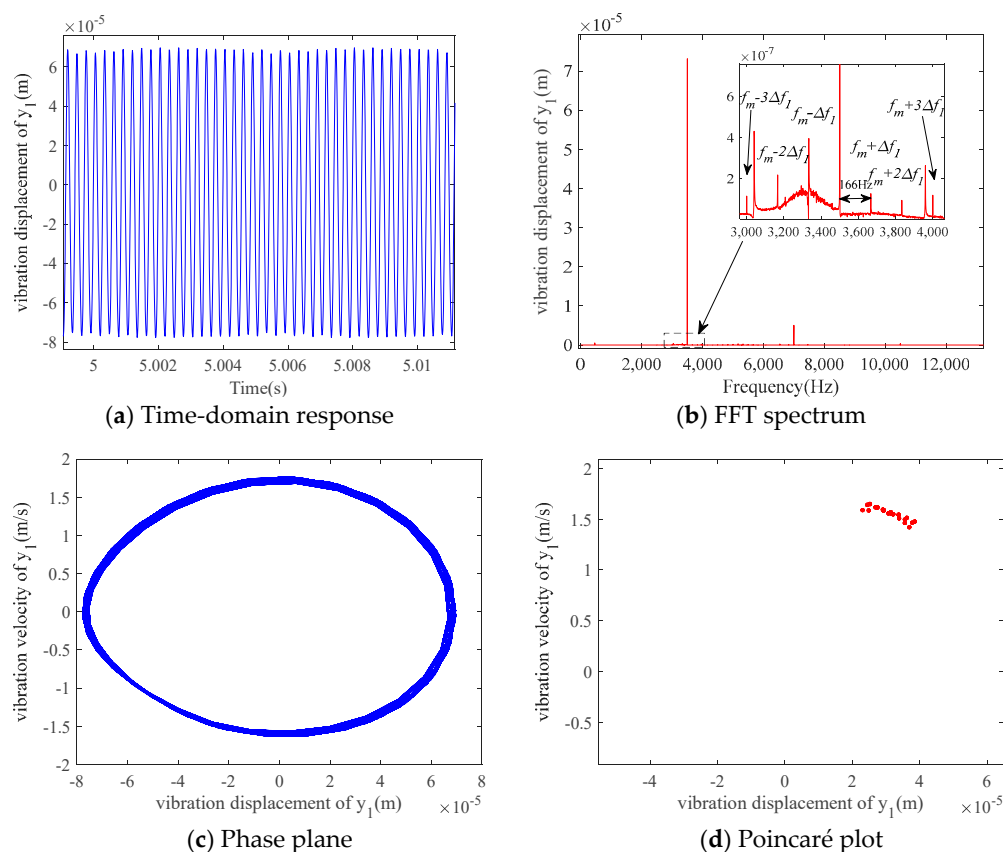
As illustrated in Figure 8, with an increase in the crack depth  $q$  at the tooth root, within the backlash intervals of  $b \in [0, 1.2 \times 10^{-5}]$  m and  $b \in [2.9 \times 10^{-5}, 3.6 \times 10^{-5}]$  m, the system's motion state transitions from periodic to quasi-periodic motion, accompanied by significantly enhanced nonlinear characteristics. This transition results in the weakening and eventual disappearance of the first bifurcation phenomenon observed in the healthy system. However, in the chaotic intervals where  $b \in [1.75 \times 10^{-5}, 2.80 \times 10^{-5}]$  m and  $b > 4.25 \times 10^{-5}$  m, no significant changes are observed with variations in crack depth. Taking a crack depth  $q$  of 3 mm at the tooth root as an example, as shown in Figure 9, the backlash values at key nodes are selected as  $1 \times 10^{-5}$  m and  $4.5 \times 10^{-5}$  m for detailed analysis.



**Figure 9.** The gear backlash bifurcation diagram under the tooth root crack depth of 3 mm.

According to the analysis, as shown in Figure 10, in the case of a tooth root crack fault, when the gear backlash is  $b = 1 \times 10^{-5}$  m, periodic signals appear in the vibration displacement time-domain diagram of the system. The peak interval between two ad-

jacent impacts corresponds to one rotational period of the faulty gear, with a period of  $T = 60/n = 0.006$  s. As shown in the local magnification of Figure 10b, the spectrum diagram of the driving gear exhibits modulated sideband frequencies centered at the gear meshing frequencies  $f_m$  and  $n f_m$ . The sideband frequency spacing equals the rotational frequency of the faulty gear, which is  $\Delta f_1 = n/60 = 10000/60 = 166.67$  Hz. The main frequency components of the system under the driving gear crack fault are  $f_m$  and  $n f_m \pm m \Delta f_1$ . At this stage, the phase diagram consists of a closed loop formed by multiple densely packed curves, while the Poincaré section displays a relatively scattered set of points, indicating that the system is in a quasi-periodic motion state.



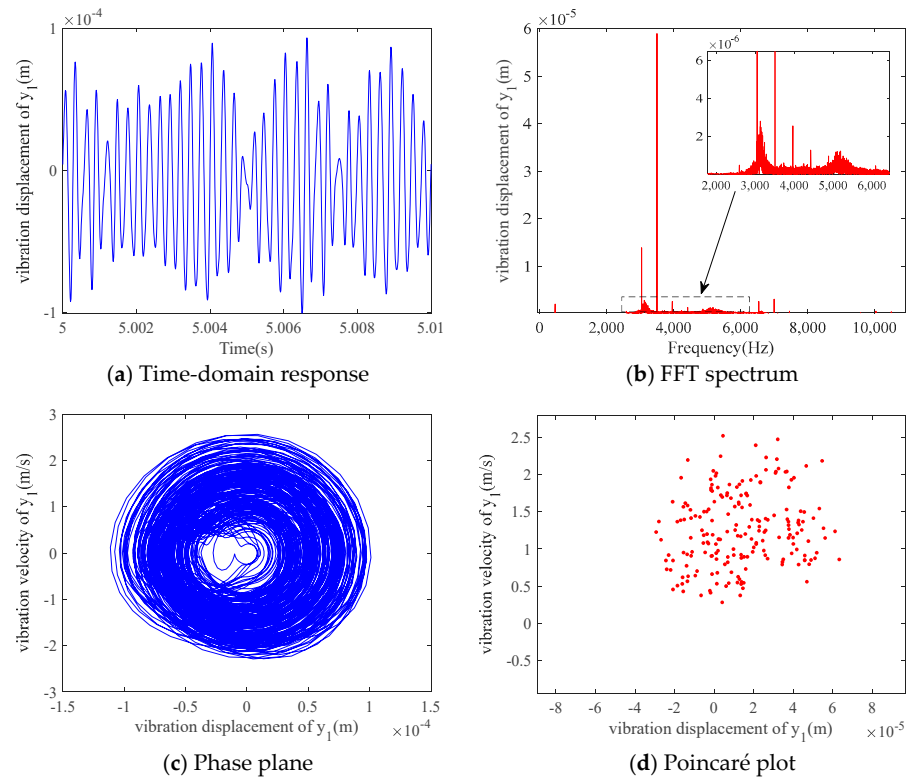
**Figure 10.** The response of the system when the backlash  $b = 1 \times 10^{-5}$  m under the tooth root crack depth of 3 mm.

When the gear backlash is  $b = 4.5 \times 10^{-5}$  m, as shown in Figure 11, the time-domain response of the system's vibration displacement exhibits non-periodic and irregular oscillations. The power spectral density displays a multi-peak continuous structure over a wideband background. Meanwhile, the phase trajectory diffuses into a complex and non-repetitive pattern, and the Poincaré section reveals an irregularly scattered point cloud. These dynamic behaviors show no significant differences compared to the response characteristics of a healthy helical gear system. This observation fully indicates that the wideband random features of chaotic motion completely mask the periodic modulation signals induced by the tooth root crack fault, rendering the fault characteristics undetectable in both the time-frequency domain and the phase space.

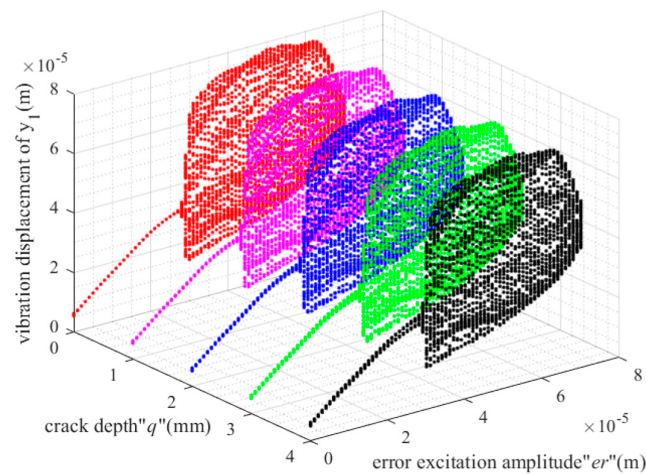
#### 4.3. The Influence of Error Excitation Amplitude on the System Under Tooth Root Crack Fault

Taking the input speed as 10,000 r/min and the error excitation amplitude as the bifurcation parameter, the crack depth  $q$  at the tooth root was set to 0 mm, 1 mm, 2 mm, 3 mm, and 4 mm, respectively, and the gear backlash and meshing damping ratio were

set to certain values. The bifurcation diagram in the  $y$  direction of the driving wheel was drawn as shown in Figure 12.

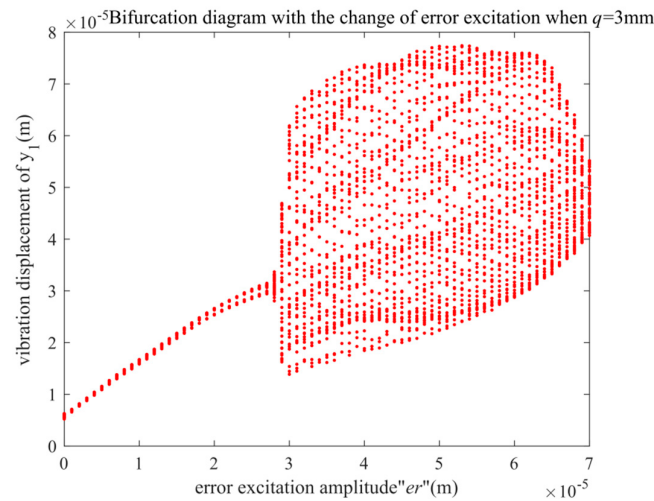


**Figure 11.** The response of the system when the backlash  $b = 4.5 \times 10^{-5}$  m under the tooth root crack depth of 3 mm.



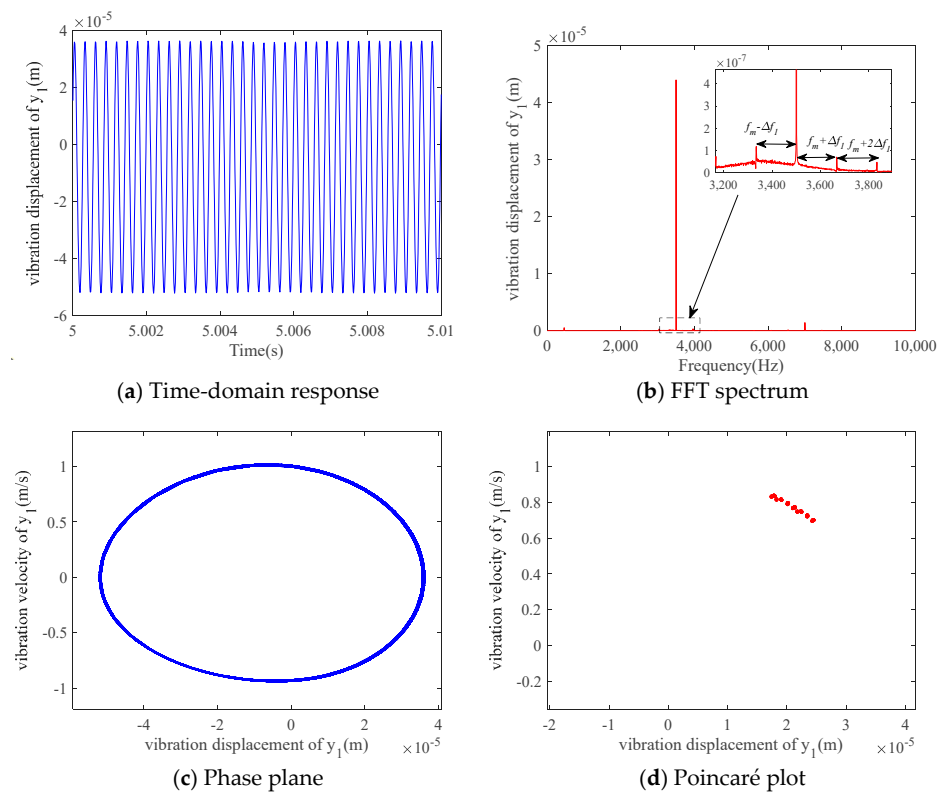
**Figure 12.** The influence of crack depth on the error excitation bifurcation diagram.

As illustrated in Figure 12, when the error excitation amplitude  $er < 2.7 \times 10^{-5}$  m, the system's motion transitions from single-periodic to quasi-periodic. Furthermore, as the crack depth at the tooth root increases, the nonlinear characteristics of the quasi-periodic motion become more pronounced. When the error excitation amplitude  $er > 2.7 \times 10^{-5}$  m, the chaotic behavior of the cracked system remains largely unchanged compared to that of the healthy system. To further analyze this behavior, Figure 13 presents the bifurcation diagram for a root crack depth  $q$  of 3 mm, with selected error excitation amplitudes of  $1 \times 10^{-5}$  m and  $4 \times 10^{-5}$  m corresponding to key operational nodes.



**Figure 13.** The error excitation amplitude bifurcation diagram under the tooth root crack depth of 3 mm.

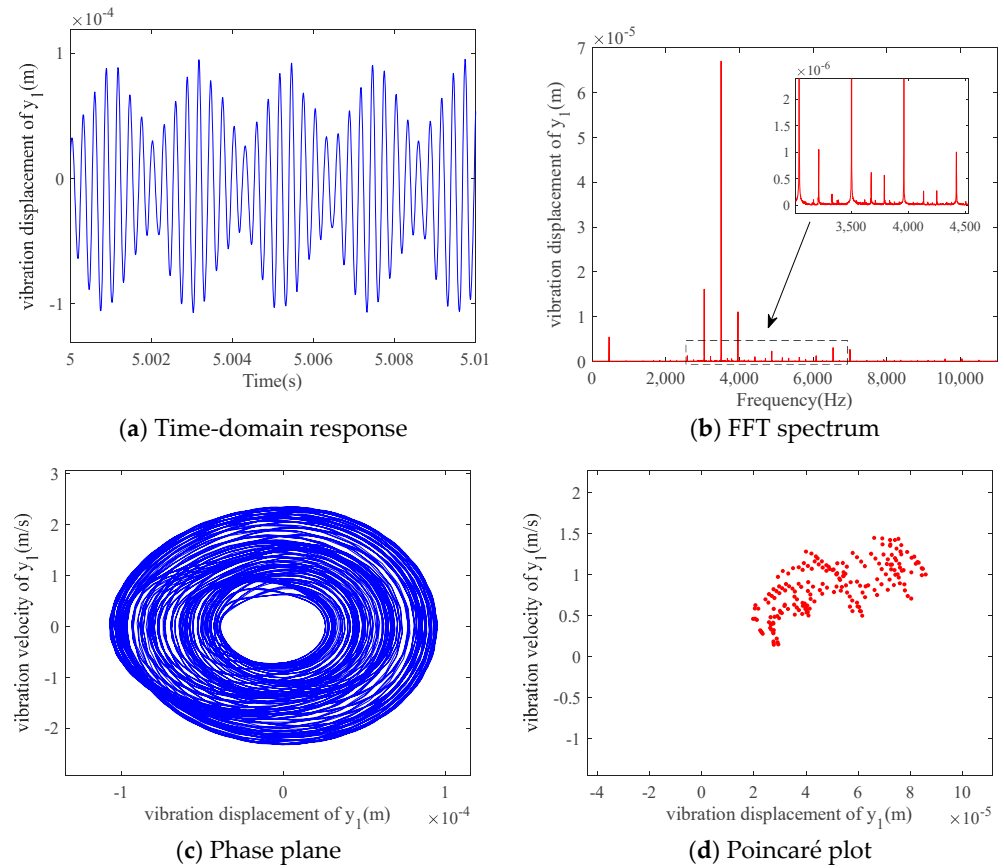
The introduction of crack faults significantly alters the system's motion characteristics. When the error excitation amplitude is  $er = 1 \times 10^{-5}$  m, the system transitions from single-period motion in the healthy state to quasi-periodic motion. This transition is relatively subtle, and the amplitude of the fault-induced modulated sidebands in the spectrum remains low, as illustrated in Figure 14b.



**Figure 14.** The response of the system when the error excitation amplitude  $er = 1 \times 10^{-5}$  m under the tooth root crack depth of 3 mm.

Under a higher error excitation amplitude ( $er = 1 \times 10^{-5}$  m) as shown in Figure 15, where chaotic motion occurs, both the faulty and healthy systems exhibit highly similar time-domain irregular oscillations and wideband continuous spectral features. This sug-

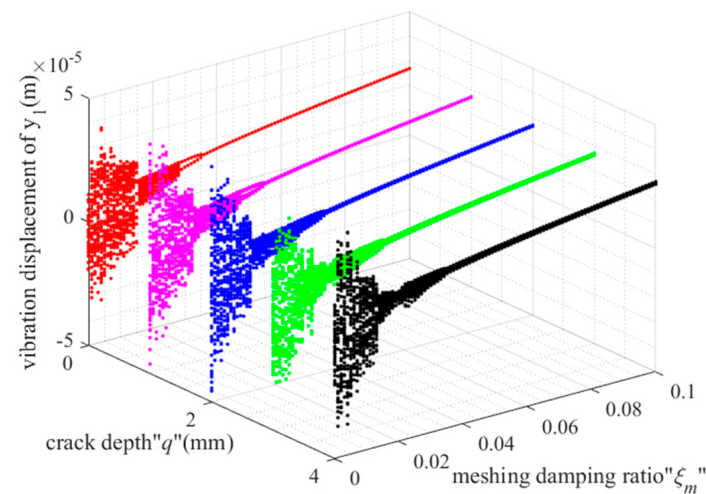
gests that the strong noise background associated with chaotic motion masks the fault characteristic signals.



**Figure 15.** The response of the system when the error excitation amplitude  $e_r = 4 \times 10^{-5}$  m under the tooth root crack depth of 3 mm.

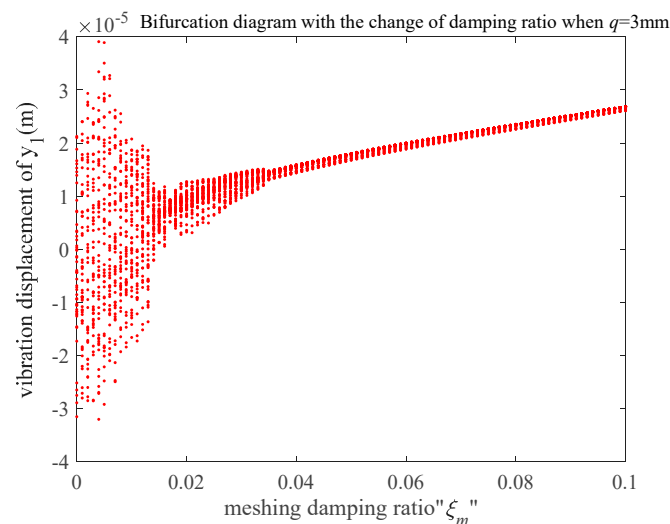
#### 4.4. The Influence of Meshing Damping Ratio on the System Under Tooth Root Crack Fault

Taking the input speed as 10,000 r/min, with the meshing damping ratio  $\xi_m$  as the bifurcation parameter, and the crack depth  $q$  at the tooth root as 0 mm, 1 mm, 2 mm, 3 mm, and 4 mm, respectively, and taking certain values for the gear back lash and error excitation amplitude, the bifurcation diagram in the y direction of the driving wheel is drawn as shown in Figure 16.



**Figure 16.** The influence of crack depth on damping ratio bifurcation diagram.

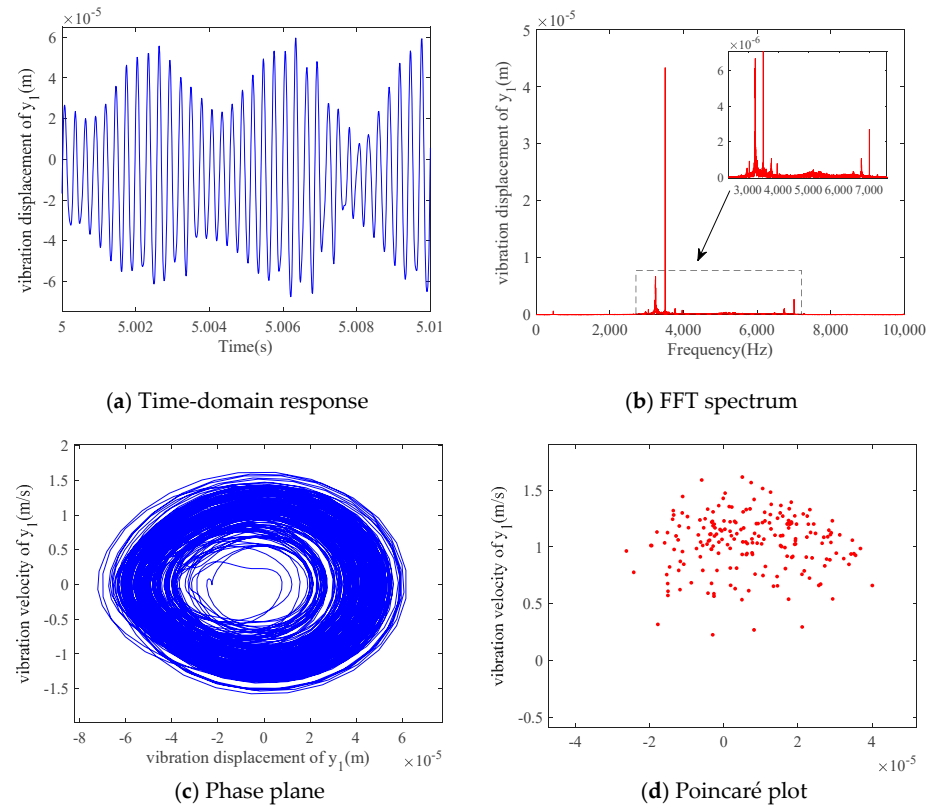
It can be observed from Figure 16 that as the crack depth at the tooth root increases, the system's motion state transitions from stable periodic motion to quasi-periodic motion when the damping ratio  $\zeta_m > 0.036$ , accompanied by a significant enhancement of nonlinear characteristics. Notably, in the healthy system, a typical period-doubling motion is present within the damping ratio range of 0.029 to 0.036. However, as the crack depth increases, the period-doubling characteristics within this range are markedly diminished. Taking the case with a tooth root crack depth of 3 mm as an example, as illustrated in Figure 17, a detailed analysis was conducted by selecting meshing damping ratios  $\zeta_m$  of key nodes at three representative values: 0.005, 0.03, and 0.05.



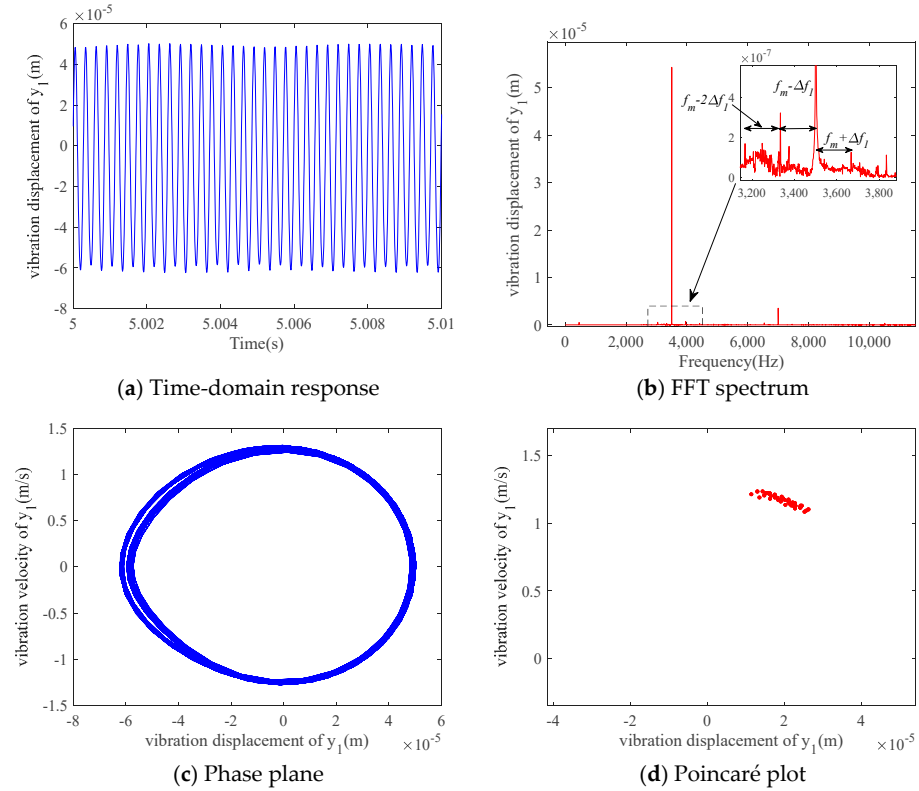
**Figure 17.** The meshing damping ratio bifurcation diagram under the tooth root crack depth of 3 mm.

From the analysis, it can be known that when the system damping is relatively small ( $\zeta_m < 0.03$ ), its motion state is more complex and will go through chaotic motion and quasi-periodic motion, as shown in Figure 18. Within this damping ratio range, the system exhibits distinct wideband chaotic response characteristics. This intense chaotic background signal is widely distributed, completely masking the periodic modulated signal generated by the tooth root crack fault. At this point, even if there is a crack fault, its unique periodic characteristics are difficult to be manifested in the response signal, making it extremely difficult to identify the fault through conventional analysis methods. The fault features are submerged by the complex dynamic behavior of chaotic motion.

When the damping ratio of the system reaches  $\zeta_m = 0.03$ , as shown in Figure 19, compared with the motion state of a healthy system, the motion form of the system undergoes a significant transformation, evolving from the previous chaotic and quasi-periodic alternating state to a relatively stable quasi-periodic motion. With the weakening of chaotic characteristics, the influence of wideband background noise is reduced. In the corresponding spectrum diagram, the characteristic frequency components induced by crack faults can begin to be observed. Although these characteristic frequency components have just emerged with relatively small amplitudes and have not yet formed obvious prominent peaks, they have already broken the signal ambiguity state under the chaotic background, providing initial identifiable clues for subsequent fault identification.

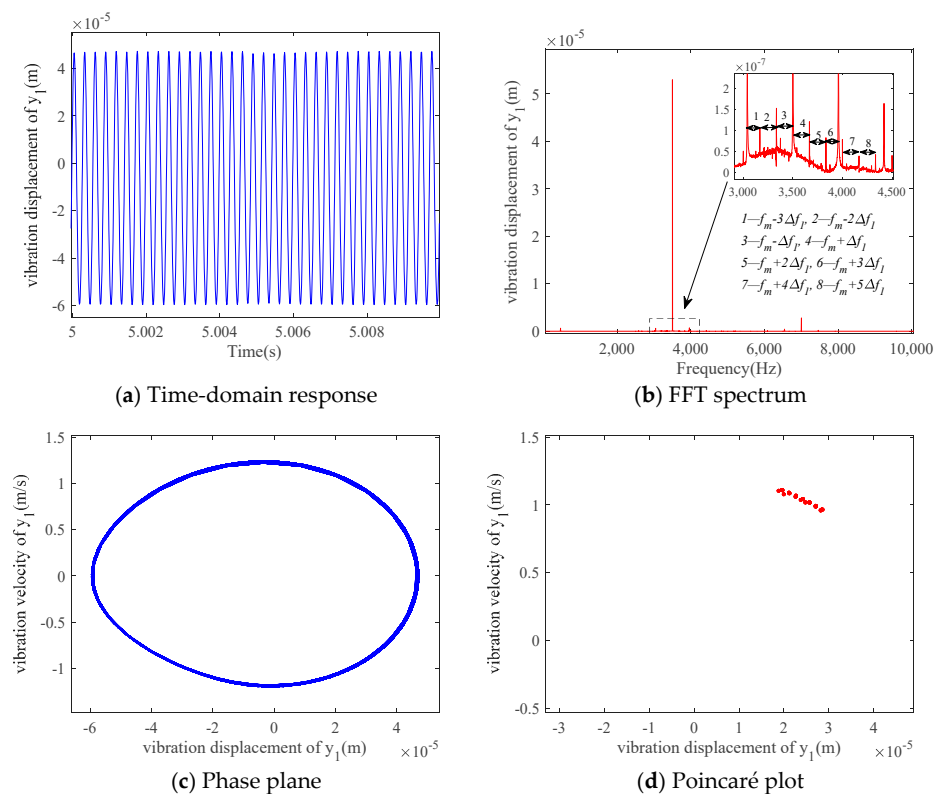


**Figure 18.** The response of the system when the meshing damping ratio  $\xi_m = 0.005$  under the tooth root crack depth of 3 mm.



**Figure 19.** The response of the system when the meshing damping ratio  $\xi_m = 0.03$  under the tooth root crack depth of 3 mm.

When the damping ratio of the system further increases to  $\xi_m = 0.05$ , as shown in Figure 20, the motion state of the system tends to stabilize, and the regularity of the dynamic response is significantly enhanced. At this point, the chaotic background noise has been effectively suppressed. The amplitude of the fault characteristic frequency in the spectrum diagram shows a significant increasing trend, and at the same time, the signal-to-noise ratio of the signal has also greatly improved. Compared with the analysis results under other damping ratio parameters, the crack fault signal under this damping ratio shows extremely prominent recognizability: The fault characteristic frequency forms clear and sharp peaks in the spectrum diagram. Its energy is concentrated and clearly distinguishable from the surrounding background noise. Not only is the position of the characteristic frequency accurately identifiable, but also the amplitude advantage is significant, enabling the existence of the tooth root crack fault to be intuitively and clearly identified, providing a strong basis for the precise diagnosis of the fault.

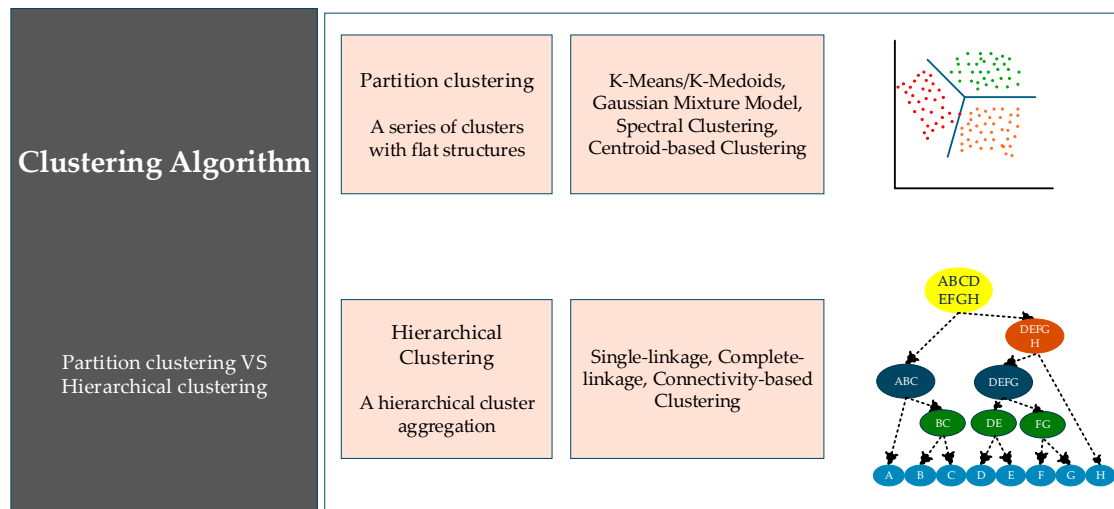


**Figure 20.** The response of the system when the meshing damping ratio  $\xi_m = 0.05$  under the tooth root crack depth of 3 mm.

#### 4.5. Recognition Verification Based on the DBSCAN Clustering Algorithm

In the fields of data mining and machine learning, clustering is an indispensable part. It can divide data objects into different groups, enabling people to better understand the internal structure of the data. Clustering is different from classification, the principle is shown in Figure 21. It is an unsupervised learning method. The mainstream clustering is divided into two types: Partition clustering and Hierarchical clustering. Partition clustering algorithms provide a series of flat-structured clusters, among which there is no explicit structure to indicate their interrelationships. Common algorithms include K-Means/K-Medoids, Gaussian Mixture Model, Spectral Clustering, Centroid-based Clustering, etc. Hierarchical clustering outputs a cluster set with a hierarchical structure, and thus can provide more abundant information than the unstructured cluster set output by partition

clustering. Hierarchical clustering can be regarded as nested partition clustering. Common algorithms include Single-linkage, Complete-linkage, Connectivity-based Clustering, etc.



**Figure 21.** Classification by clustering algorithm.

In this study, the DBSCAN algorithm was used to classify the Poincaré sections under different motion states. DBSCAN is the acronym for Density-Based Spatial Clustering of Applications with Noise, a density-based unsupervised clustering method proposed by Ester et al. [28]. It groups data points into clusters based on the notion of density connectivity, while automatically identifying outliers as noise. Unlike K-means or other centroid-based methods, DBSCAN does not require predefining the number of clusters and can effectively detect clusters with arbitrary shapes, which makes it particularly suitable for analyzing nonlinear and chaotic responses in gear dynamics.

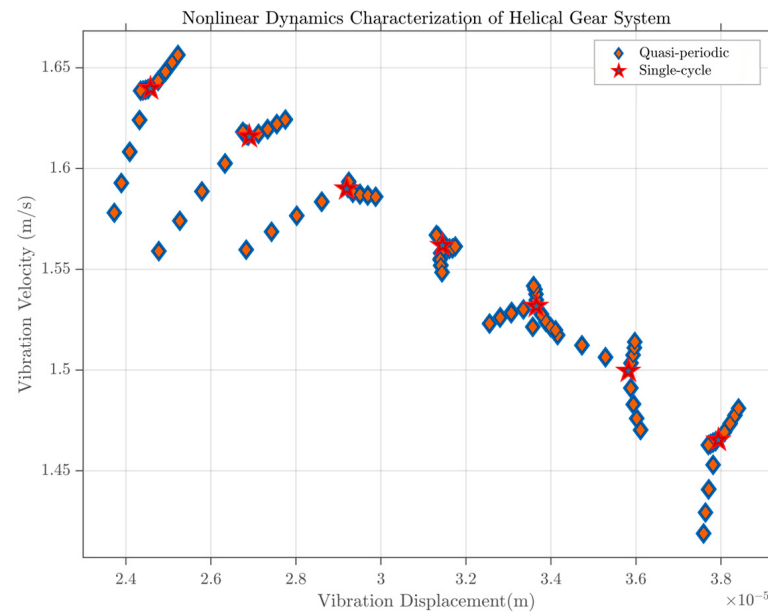
Traditional clustering algorithms, such as K-Means and its improved variant K-Medoids, are efficient and straightforward but require the number of clusters to be predefined and struggle to identify non-spherical cluster structures. Moreover, they are relatively sensitive to noise and outliers. Hierarchical clustering, while capable of revealing the hierarchical relationships among data, suffers from high computational complexity. In contrast, as a density-based algorithm, DBSCAN possesses the distinct advantage of automatically detecting clusters of arbitrary shapes without the need to preset the number of clusters. It can also effectively identify and separate noise points, thereby exhibiting superior robustness and flexibility when handling complex real-world datasets.

$$N_{Eps}(p) = \{q \in D \mid dist(p, q) \leq Eps\} \quad (19)$$

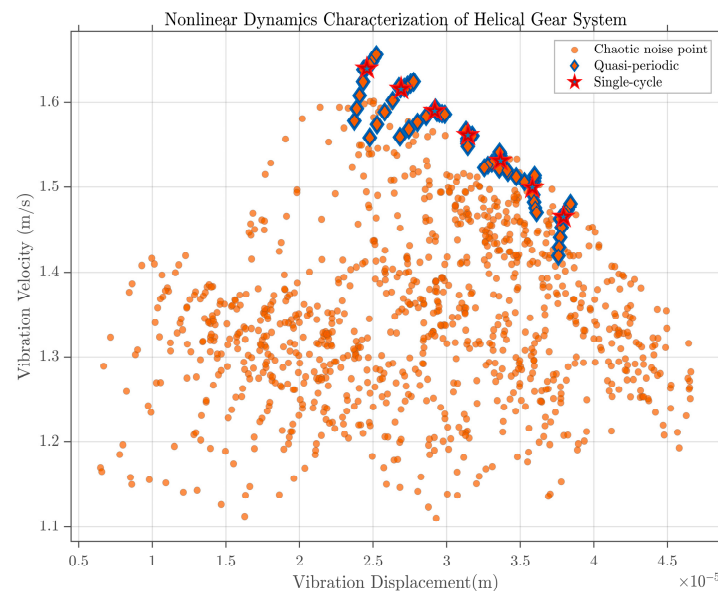
In the formula:  $p$  and  $q$  are points in the Poincaré cross-section,  $Eps$  is the neighborhood radius of the point set. In this article, the neighborhood radius and the minimum number of points are derived based on the motion state of healthy gears. In the DBSCAN algorithm, input the neighborhood radius range and find the optimal solution of  $Eps$ . In the health system, the minimum number of points (Minpts) is 5.

As discussed earlier, the motion state of the system is highly sensitive to variations in tooth-side clearance. Therefore, in this section, the tooth-side clearance is treated as a self-varying parameter, and the system behavior is analyzed using a clustering algorithm, as illustrated in Figures 22 and 23. Figure 22 shows that, after the introduction of crack faults, the points on the Poincaré cross-section can be approximately clustered into two categories: single-period point sets and quasi-periodic noise point sets. As the crack depth increases, the distance between the outermost noise points and the center

of the single-period point set gradually increases. In Figure 23, as the system transitions into a chaotic state, the Poincaré plots become increasingly scattered. Previous analyses indicate that once the system exhibits chaotic motion, the resulting chaotic noise tends to obscure fault-induced noise, making it difficult to identify fault signals using conventional time-domain or frequency-domain methods. In contrast, the DBSCAN clustering algorithm can effectively distinguish single-period clusters, fault-related noise points, and chaotic noise points, demonstrating strong potential for fault identification in nonlinear dynamic systems.



**Figure 22.** Visualization of cluster analysis of fault-containing systems with cracks when “ $b = 1 \times 10^{-5} \text{ m}$ ”.



**Figure 23.** Visualization of cluster analysis of fault-containing systems with cracks when “ $b = 2 \times 10^{-5} \text{ m}$ ”.

## 5. Conclusions

This study focuses on the helical gear pair located in the high-speed stage of a specific pure electric vehicle reducer. A torsional-bending coupled dynamic model of the helical gear pair is established using the lumped parameter method. Based on an improved potential energy approach, the time-varying meshing stiffness of the helical gear pair under

both healthy and root crack fault conditions is calculated. The dynamic model is then solved using the Runge–Kutta method. Bifurcation diagrams of the system are generated with respect to various parameters, enabling a quantitative analysis of the system's dynamic response under different operating conditions. The following conclusions are drawn:

- (1) For the helical gear transmission system used in electric vehicle reducers, the system demonstrates a highly stable single-period motion state within the low-speed range. As the input speed increases, the system experiences abrupt transitions and exhibits pronounced nonlinear behavior along with complex motion state transformations. Therefore, investigating the dynamic characteristics of helical gear systems under high-speed input conditions holds considerable theoretical and practical significance. Such research is of great engineering value in ensuring the operational reliability and performance of high-speed electric drive systems.
- (2) For helical gear transmission systems with crack faults, the introduction of fault characteristics significantly alters the dynamic response characteristics of the system. Within the parameter range where the gear backlash and error excitation amplitude are relatively small and the meshing damping is relatively high, the original single-period or double-period motion has evolved into quasi-period motion. The quasi-periodic behavior caused by the fault masked the first bifurcation phenomenon that occurred in the healthy system under small gaps. When the system operates in a stable motion state, the fault characteristics have high identifiability. Periodic impact components caused by crack propagation appear in the time-domain response, while obvious fault modulation sideband and its multiple frequency components appear on both sides of the meshing frequency in the spectrum. However, when the system enters the chaotic motion state, the intense nonlinear background noise drowns out the fault characteristic signals, making them undetectable in both the time domain and the frequency domain. By using the DBSCAN clustering algorithm to conduct cluster analysis on the Poincare section points, single-period classes, fault noise points, and chaotic noise points can be easily identified.

**Author Contributions:** Conceptualization, H.Z. and D.L.; methodology, D.L.; software, D.L.; validation, H.Z., D.L. and H.W.; formal analysis, H.S.; investigation, D.L. and H.S.; resources, H.Z.; data curation, D.L.; writing—original draft preparation, D.L.; writing—review and editing, H.Z. and D.L.; visualization, H.S.; project administration, H.Z. All authors have read and agreed to the published version of the manuscript.

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