

Article

Kinematic Upper-Bound Analysis of Safety Performance for Precast 3D Composite Concrete Structure with Extended Mohr–Coulomb Criterion

Taoxiang Feng ^{1,2}, De Zhou ^{2,*} and Qiang Chen ¹

¹ Sinohydro Engineering Bureau 8 Co., Ltd., Changsha 410083, China; 123274@powerchina.cn (T.F.); 123215@powerchina.cn (Q.C.)

² School of Civil Engineering, Central South University, Changsha 410075, China

* Correspondence: 210026@csu.edu.cn

Abstract

This study develops a systematic kinematic upper-bound framework to evaluate the ultimate bearing capacity and failure mechanisms of prefabricated cast-in-place slab–wall joints in overlapped metro stations. Recognizing the complex shear–compression interaction in these critical structural nodes, a novel three-dimensional short-block shear failure model is established based on the principle of energy balance. The analysis employs a modified Mohr–Coulomb strength criterion incorporating a finite tensile strength cut-off, enabling more accurate representation of cracking and tensile resistance effects. Analytical solutions are derived to predict the ultimate capacity and critical failure angle, followed by a comprehensive parametric analysis. The results reveal that cross-sectional dimensions dominate the bearing capacity, while the internal friction angle and tensile-to-compressive strength ratio significantly influence both the magnitude and mode of failure. A narrower load distribution width enhances capacity and reduces the optimal failure angle. Overall, the proposed 3D model provides a rigorous and efficient theoretical tool for the design optimization and safety assessment of prefabricated underground structures.

Keywords: kinematic upper-bound method; 3D failure mechanism; joint bearing capacity; prefabricated assembled



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1. Introduction

In the context of rapid urbanization and unprecedented growth in metropolitan populations, metro stations have emerged as a cornerstone of urban underground transportation infrastructure, playing an indispensable role in facilitating mass transit, reducing surface traffic congestion, and supporting sustainable urban development [1,2]. The increasing demand for high-capacity, safe, and reliable transit systems has placed significant emphasis on the structural performance and durability of metro stations. In addition to carrying enormous live loads from passengers and trains, underground metro structures must withstand a wide range of environmental and operational influences, including soil–structure interaction, groundwater pressures, seismic activity, and temperature fluctuations. Moreover, urban construction sites are often located in densely populated areas, where space constraints, existing infrastructure, and environmental considerations pose additional challenges for conventional construction methods. Under such circumstances, construction techniques that can simultaneously ensure structural safety, reduce environmental impact, and enhance construction efficiency are highly desirable.

Traditionally, cut-and-cover construction has been the most prevalent method for metro station development [3]. This method, often combined with conventional support systems such as cast-in-place reinforced concrete or bored piles, offers reliability but suffers from several limitations. Drilling and casting processes are slow, prone to wall collapse, generate significant amounts of slurry requiring complex disposal, and result in high levels of noise and dust, adversely affecting nearby communities. From an environmental perspective, these issues not only increase the carbon footprint and on-site pollution but also conflict with modern sustainability goals that prioritize energy efficiency and emission reduction during urban infrastructure construction. In contrast, prefabricated construction allows for the mass production of structural components under controlled factory conditions, enhancing quality control, improving dimensional accuracy, and reducing both construction duration and site disturbances. Furthermore, by minimizing on-site concrete pouring and waste generation, prefabricated systems contribute to cleaner construction environments and better compliance with “green building” principles increasingly promoted in metropolitan development. As urban projects increasingly adopt sustainable and “green” design principles, hybrid construction systems that integrate temporary support and permanent structural components have become more attractive. By combining prefabricated elements with cast-in-place concrete connections, these systems provide improved construction efficiency, better environmental performance, and enhanced structural integrity, while also offering cost-effective solutions for complex underground projects. Such approaches have facilitated the industrialization of underground construction, enabling precise scheduling, reduced labor intensity, and more predictable structural behavior, which is critical for high-traffic metro stations and long-term urban resilience.

Hybrid metro station systems, often referred to as “prefabricated cast-in-place” or composite construction, involve the off-site fabrication of key load-bearing elements, including walls, slabs, beams, and columns, which are subsequently transported and assembled on-site. Following assembly, critical joints and interface regions are cast in place to achieve structural continuity, load transfer, and overall integrity [4]. This “prefabrication plus cast-in-place” approach offers the dual advantage of reduced construction duration and minimal interference with existing urban activities while maintaining high structural performance, including enhanced water tightness, seismic resistance, and durability. Within such systems, slab–wall joints serve as essential structural nodes responsible for transferring loads between horizontal and vertical elements. The performance of these joints is directly linked to the station’s stability and long-term safety. Under operational conditions, these joints are subjected to complex three-dimensional (3D) stress states, including in-plane bending and shear, out-of-plane bending, torsion, and multi-directional shear forces. Accurate assessment of their mechanical behavior and ultimate load-bearing capacity is therefore critical for safe and efficient structural design.

Substantial research has been conducted both domestically and internationally on slab–wall joints, predominantly focusing on cast-in-place reinforced concrete shear wall systems [5–7]. Previous studies have examined bearing capacity and deformation behavior under in-plane bending and shear [8–10], lateral shear response [11–13], and energy dissipation and ductility degradation under cyclic or repeated loading conditions [14]. Other investigations have compared the mechanical performance of prefabricated and cast-in-place joints, confirming the feasibility of prefabricated systems for load-bearing applications and highlighting the benefits of factory-controlled component quality [15]. However, most existing research remains limited to 2D analyses or relies heavily on experimental testing and numerical simulations. Consequently, there is a lack of systematic 3D analytical frameworks capable of capturing the complex failure mechanisms, multi-axial stress states, and spatial deformation behavior of slab–wall joints under realistic operational conditions.

Limit analysis, grounded in plasticity theory and the principle of energy balance, provides a rigorous theoretical basis for evaluating ultimate loads, failure mechanisms, and collapse behavior in underground and other structural systems [16–24]. Applications have included tunnels, deep excavations, slopes, and retaining structures. Among its variants, the kinematic upper-bound method is widely employed due to its balance of computational efficiency and theoretical rigor. The approach involves the potential failure mechanisms, constructing kinematically admissible 3D velocity fields, and computing the balance between external work and internal plastic dissipation. When the external power equals or exceeds internal dissipation, the proposed mechanism is considered admissible. Optimization of the mechanism yields the minimum upper-bound estimate of the structure's collapse load. The optimization is based on (1) the material deforms according to plastic flow rules once yielding occurs, and (2) at the ultimate state, external work is fully balanced by internal plastic dissipation.

In recent years, significant progress has been made in extending and applying upper-bound limit analysis to masonry and concrete-like materials. For instance, Buzzetti et al. [25] proposed a finite element upper-bound framework capable of automatically identifying collapse mechanisms in masonry structures without the need to predefine failure patterns, enabling objective assessment of structural safety under seismic excitation. Similarly, Di Gennaro et al. [26] examined the effects of localized damage on the stability of masonry arches under horizontal loads, highlighting the sensitivity of collapse multipliers to geometric configuration and degradation. Zona et al. [27] introduced a semi-analytical limit analysis for conical and parabolic domes, deriving admissible stress fields and collapse loads based on Melan's theorem. Furthermore, Grillanda and Mallardo [28] formulated a compatible strain-based upper-bound model for in-plane loaded masonry walls, which homogenizes plastic strain rates to ensure velocity compatibility within the continuum. To facilitate practical application, Hua et al. [29] developed a computer-based limit analysis tool (LACT3) that performs kinematic collapse evaluations of two-dimensional masonry walls using the Mohr–Coulomb yield criterion within a graphical interface.

These studies collectively demonstrate the versatility of limit analysis in identifying failure mechanisms, optimizing structural performance, and bridging the gap between theoretical plasticity models and engineering implementation. Moreover, their findings underscore the method's potential for extending from two-dimensional masonry systems to more complex 3D concrete structures, such as hybrid metro station joints.

Nevertheless, within concrete structural applications, most limit analysis studies have been restricted to 2D problems. For instance, Mousavian and Casapulla [30] proposed a novel optimization-based approach for 2D arched structures, employing classic limit analysis and discontinuity layout optimization to determine optimal joint configurations that maximize load-bearing capacity. Larsen [31] developed lower-bound formulas for concrete bearing capacity. Mazars [32] developed a 2D finite element damage model for reinforced concrete, incorporating monotonic and cyclic loading effects, to realistically capture nonlinear behaviors such as unilateral effects, permanent strains, and steel–concrete debonding. Salim and Sebastian [33] applied upper-bound plasticity theory to predict shear capacity of unreinforced or unconstrained concrete slabs. Pisano et al. [34,35] developed numerical limit analysis tools for predicting peak load capacities and applied them to full-scale concrete walls and slabs. While these studies provide valuable insights, they largely neglect 3D effects that are particularly critical in slab–wall joints, where out-of-plane moments, torsion, and multi-directional shear forces interact in a complex manner. As such, the failure mechanisms, ultimate load capacity, and design methodologies for 3D joints remain underexplored.

To ensure analytical tractability, the proposed method adopts a rigid–plastic assumption and neglects reinforcement, interface friction, and post-peak softening, which makes it particularly suitable for short block-type joints or compression-dominated regions where bending effects are limited. Addressing these research gaps, the present study applies limit analysis [36,37] to investigate the 3D load-bearing performance of slab–wall joints in hybrid metro stations. A 3D shear model is proposed, admissible 3D velocity fields are constructed, and a modified Mohr–Coulomb (MC) criterion is incorporated to derive generalized analytical solutions for the safety calculation of joint capacity. By balancing external power and internal plastic dissipation, the method can provide a rapid and theoretically rigorous means of predicting joint behavior under realistic 3D loading conditions. The findings not only contribute to a more accurate and comprehensive understanding of slab–wall joint mechanics but also offer practical guidance for the design, optimization, and safety assessment of prefabricated cast-in-place joints in underground metro structures, ultimately supporting the advancement of sustainable urban transit infrastructure.

2. Concrete Material Model

Concrete, as a quasi-brittle material, exhibits mechanical characteristics that are intermediate between those of ductile metals and purely brittle rocks. Similarly to natural rock, concrete displays a pronounced brittleness under tensile loading, where crack initiation and propagation occur rapidly once the ultimate tensile strength is exceeded. In contrast, when subjected to compressive loading, the material retains a certain degree of deformability and load-bearing capacity, even beyond the peak stress, thereby showing a relatively more ductile behavior. This curve highlights that the compressive strength of concrete f'_c is generally 8–12 times higher than its tensile counterpart f'_t , while the corresponding compressive strain is approximately proportional to the tensile strain at failure. Failure under compression is usually initiated in the unstable descending branch of the stress–strain curve, where a rapid reduction in strength accompanies the development of microcracking and localized damage.

However, when concrete is subjected to sufficiently high confining pressure, its mechanical behavior undergoes a significant transformation. The application of confinement not only enhances the peak compressive strength but also substantially increases the corresponding strain capacity. This effect delays the onset of instability in the post-peak branch and promotes a transition from brittle softening to ductile plastic flow. Two important consequences of confinement can thus be summarized as follows (1) the peak compressive strength and its associated strain are markedly improved, and (2) the post-peak mechanical response exhibits enhanced stability, eventually suppressing the otherwise unstable softening behavior that characterizes unconfined concrete.

In the framework of limit analysis commonly employed in engineering mechanics, the tensile resistance of concrete is often simplified as negligible. This discussion, though idealized, is considered reasonable because the tensile strength of concrete is very small compared to its compressive strength. By adopting this simplification, the rigid–plastic limit theorems remain strictly applicable, and the material model asymptotically approximates the actual behavior of concrete under loading. Traditional approaches sometimes consider that the tensile strength is a fixed fraction of the compressive strength; however, neglecting it entirely not only simplifies the formulation but also ensures a conservative design outcome. Nevertheless, extensive experimental investigations have demonstrated that compressive failure in concrete is not purely ductile. Instead, it is typically accompanied by brittle fracture, progressive cracking, and strain-softening behavior after the peak stress, which deviates from the perfectly plastic models.

For monotonic loading cases, particularly when compressive strains remain relatively small, concrete exhibits noticeable deformability prior to substantial strength degradation. This characteristic provides justification for approximating concrete as a perfectly plastic material in limit analysis, with its compressive yield stress taken as the ultimate compressive strength determined in standard tests. Although such an approximation neglects tensile capacity and post-peak softening, it provides a practical and computationally efficient means of estimating structural bearing capacity.

From the standpoint of conservative structural design, if concrete is considered incapable of sustaining tensile stress, the allowable bearing capacity of the compressive zone should not exceed the reference compressive strength obtained from unconfined prism specimens. This forms the lower bound of capacity estimates. Nevertheless, a wealth of experimental data indicates that the actual bearing capacity of concrete is not a fixed value but instead increases significantly with the loaded area ratio. Beyond a certain threshold, this increase stabilizes and approaches a limiting value. Such findings highlight the scale effect in concrete strength, where larger loaded areas promote a redistribution of stresses and delay failure localization.

Therefore, incorporating the limited but non-negligible tensile strength of concrete into theoretical formulations can improve the predictive accuracy of analytical models. By doing so, the agreement between theoretical predictions and experimental measurements can be substantially enhanced. This refinement leads to more reliable engineering evaluations, especially in the context of safety assessment and optimal material utilization. Ultimately, the accurate representation of concrete's dual behavior—brittle under tension and relatively ductile under compression—plays a critical role in developing robust material models that balance both safety and efficiency in structural design.

3. Modified MC Strength Criterion

3.1. MC Yield Criterion Modified with Zero-Tensile Cutoff

In the classical MC strength criterion, concrete is typically idealized as a material that can resist compressive stresses but exhibits negligible tensile capacity, reflecting its inherent brittleness under tension [36,37]. The stress–strain behavior under this “zero-tensile” assumption is schematically illustrated in Figure 1a, highlighting the significant disparity between compressive and tensile strengths. To represent the tensile cutoff on the principal stress plane, a tensile failure circle tangent to the vertical axis (σ_1 -axis) is introduced. The envelope of this circle forms a specific angle with the horizontal axis (σ_3 -axis), which corresponds to the material's internal friction angle, ϕ , while the intersection with the vertical axis occurs at a distance equal to the cohesion parameter, c , characterizing the adhesive strength inherent to the concrete matrix.

Within this idealization, the analysis focuses exclusively on rigid-body velocity discontinuities along potential failure surfaces, which allows for the rigorous application of upper-bound approach. On each discontinuity, the energy dissipation rate per unit area can be expressed as a function of the local stress state and the velocity jump across the interface, effectively capturing the contribution of frictional sliding while neglecting tensile effects. This formulation is particularly advantageous for evaluating failure mechanisms dominated by compressive stresses, such as crushing or sliding along shear planes, where tensile stresses play a minor role. Furthermore, by neglecting tensile resistance, the model simplifies the constitutive description, allowing for tractable analytical derivations of ultimate load-bearing capacity, while still preserving the essential physical characteristics of concrete under practical engineering conditions.

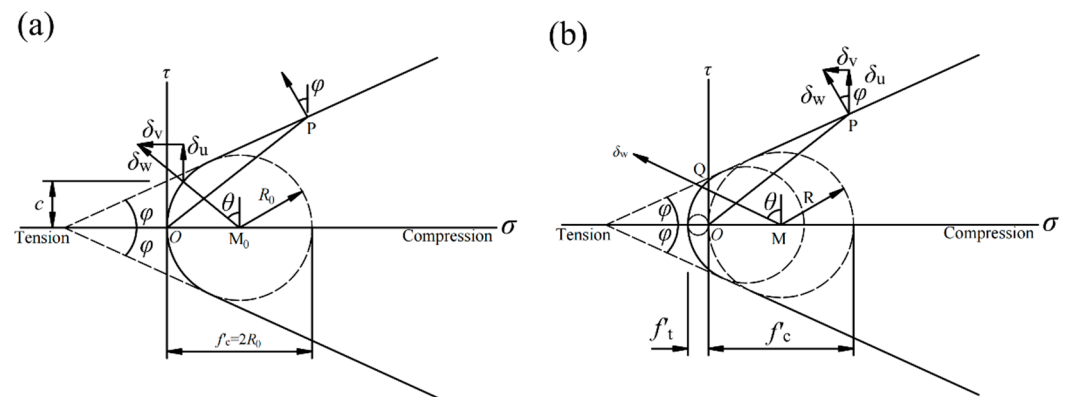


Figure 1. Modified Mohr–Coulomb criterion stress–strain envelopes: (a) zero tensile cutoff; (b) low tensile cutoff.

Overall, this approach provides a robust theoretical framework for predicting the ultimate structural performance of concrete elements in situations where compressive stresses dominate, offering insights that are directly applicable to both design optimization and safety assessment of reinforced and unreinforced concrete structures. Additionally, it serves as a foundation for extending the analysis to more complex conditions, including confined concrete, multi-axial loading, and the interaction with reinforcement or other boundary constraints, which are critical for realistic 3D structural evaluations.

Under the above case, the energy dissipation rate per unit area can be expressed as follows:

$$D_A = T\delta_u - \sigma\delta_v = \delta_u(T - \sigma \tan \theta) \quad (1)$$

where δ_u denotes the tangential velocity along the discontinuity surface; δ_v represents the normal separation velocity; δ_w corresponds to the relative velocity vector between the two sides of the discontinuity, and θ defines the angle between δ_w and δ_u .

This strength criterion is required to satisfy the following constraints:

$$\begin{cases} \tau = R_0 \cos \theta \\ \sigma = R_0 - R_0 \sin \theta \\ \delta_v = \delta_u \sigma \tan \theta \\ \theta \geq \varphi \end{cases} \quad (2)$$

By solving Equations (1) and (2) simultaneously, the following result can be obtained:

$$D_A = c\delta_u \frac{\tan(45 + \varphi/2)}{\tan(45 + \theta/2)} \quad (3)$$

3.2. MC Yield Criterion Modified with Small-Tensile Cutoff

Extensive experimental investigations on concrete have shown that the classical Coulomb criterion is generally effective in describing the failure behavior of concrete under various stress conditions. Nonetheless, the traditional formulation exhibits notable limitations when predicting the increase in maximum principal stress at failure under elevated confining pressures. Specifically, it tends to overestimate the strengthening effect associated with confinement, which contradicts experimental evidence indicating that the tensile strength of concrete is largely insensitive to variations in the intermediate principal stress perpendicular to the direction of tension. Such discrepancies highlight the need for an enhanced constitutive criterion that more accurately reflects concrete's mechanical response across different loading scenarios.

To address this limitation, the present study adopts a modified criterion that extends the classical MC model by introducing two key features: a finite tensile strength and a controlled local deformability. This enhanced criterion is capable of capturing both the compressive-dominated failure mechanisms and the subtle effects of limited tensile capacity, providing a more realistic representation of concrete behavior under complex stress states in Figure 1b. Here, the Mohr circles for unconfined compression and uniaxial tension are depicted, and the intersections with the horizontal axis define the characteristic stresses f'_c and f'_t , which quantify the material's failure response under different combinations of normal and shear stresses.

By incorporating a finite tensile strength, the modified criterion allows for a moderate contribution of tensile resistance to the overall load-bearing capacity, which becomes particularly relevant in partially confined or lightly loaded regions. Meanwhile, the inclusion of local deformability accounts for the post-peak softening observed in experiments, improving the correspondence between theoretical predictions and empirical measurements. Collectively, these enhancements provide a more robust framework for modeling concrete failure, enabling more accurate estimations of energy dissipation, stress redistribution, and ultimate capacity in structural elements. Consequently, the proposed criterion is well-suited for application in a rigid plastic upper-bound method, where it facilitates the derivation of reliable analytical solutions for the ultimate load-bearing behavior of concrete under both monotonic and complex multi-axial loading conditions.

Under the idealized rigid-plastic assumption, the expression for the energy dissipation rate per unit area can be derived. When a velocity coordinate system is superimposed on the stress space, as illustrated in Figure 1b, the tangential slip velocity and normal separation velocity components on the failure plane are orthogonal to the yield envelope. Accordingly, the energy dissipation rate per unit area, D_A , can be expressed as the dot product of the stress vector (σ, τ) and the relative velocity vector $(\delta v, \delta u)$. The stress vector is represented by either OP or OQ , and the relative velocity vector by δw . Notably, the stress vector OQ can be decomposed into the sum of vectors OM and MQ , leading to the following relationship:

$$D_A = \begin{cases} R\delta w - (R - f'_t)\delta v \\ \delta w(f'_c \frac{1-\sin\theta}{2} + f'_t \frac{\sin\theta - \sin\varphi}{1-\sin\varphi}) \end{cases} \quad (4)$$

The adopted strength criterion must satisfy the following constraint conditions:

$$\begin{cases} \sigma = R \sin \theta - (R - f'_t) \\ \tau = R \cos \theta \\ \tan \theta = \frac{\delta v}{\delta u} \geq \tan \varphi \\ R = \frac{1}{2}f'_c - f'_t \frac{\sin \varphi}{1 - \sin \varphi} \end{cases} \quad (5)$$

4. Shear Failure of Concrete Slab–Wall Assemblies

4.1. Engineering Background

The Shenzhen Metro Line 15, as the first line to commence under the Fifth-Phase Rail Transit Construction Plan (2023–2028), is planned as an urban-speed loop line traversing major development zones such as Qianhai, Nanshan, and Bao'an Central District. Upon completion, the line will significantly enhance the transfer efficiency of the rail transit network, while connecting densely populated and employment-concentrated clusters including Mawan, Shekou, Houhai, the Hi-Tech Park, Liuxiandong, Xin'an, Xixiang, and Dachan Bay. This development is of great strategic importance in optimizing Shenzhen's

urban spatial structure, alleviating transportation pressure, and advancing the high-quality integration of the Guangdong–Hong Kong–Macao Greater Bay Area.

Unlike traditional cast-in-place construction, Line 15 adopts a large proportion of prefabricated and assembled components for its station structures. This approach not only shortens construction periods and reduces on-site labor intensity but also promotes structural standardization and eco-friendliness while maintaining safety and durability. The extensive application of prefabrication in metro construction reflects an inevitable trend toward industrialized and sustainable underground engineering practices.

Among the various prefabricated elements, the connection between the base slab and the prefabricated sidewalls represents a critical structural interface. This joint directly governs the global load transfer mechanism, shear resistance, and long-term durability of the station. Improper detailing or execution at this interface may lead to uneven stress distribution, durability reduction, or even leakage problems. Hence, ensuring the reliable and stable performance of slab-to-wall connections has become both a technical challenge and a decisive factor in the large-scale adoption of prefabricated metro stations.

To better illustrate the construction details and structural features, representative components from the Line 15 project are presented. Figure 2 shows a field photograph of the slot openings designed for the slab-to-wall connection, while Figure 3a,b presents the prefabricated sidewall. These images provide direct engineering evidence to support the subsequent mechanical analysis and structural investigation.

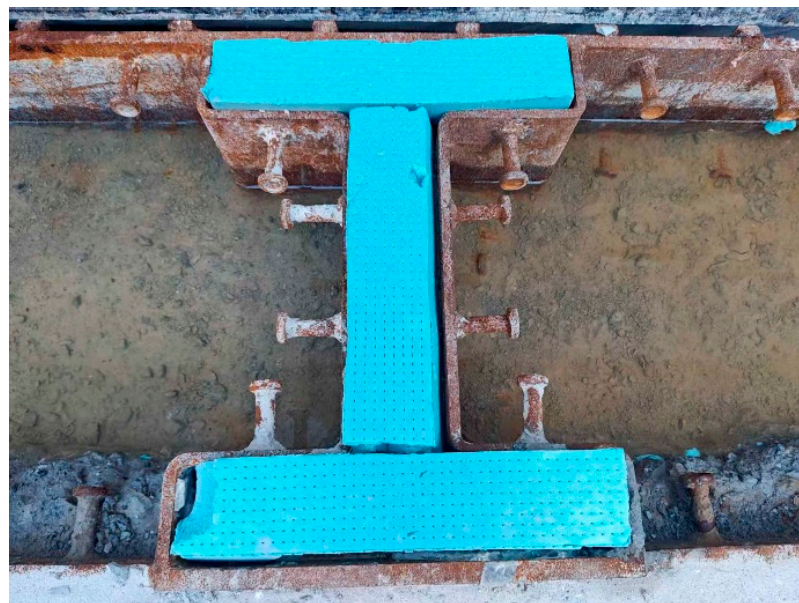


Figure 2. Prefabricated slot opening of the base slab at Chanwan Station of Shenzhen Metro Line 15.

4.2. Three-Dimensional Failure Calculation Model

In prefabricated structural systems, the connection between the sidewall and the base slab represents a critical component that governs the overall mechanical performance and structural safety, as this joint not only ensures the efficient transfer of high-magnitude vertical loads but must also reliably withstand complex foundation reactions, dynamic lateral earth pressures, and the combined effects of multiple operational load cases. Therefore, an accurate and theoretically sound understanding of the mechanical behavior of the slab-to-wall connection is essential for evaluating both the structure's load-bearing capacity and its long-term durability, yet the inherent structural discontinuities—including reserved slots, heterogeneous grouting materials, and interfacial transition zones—introduce significant material heterogeneity and highly localized stress concentrations, posing substantial

challenges to conventional, homogeneous analytical approaches. To facilitate the investigation of the instability mechanism and the ultimate load capacity of this complex joint, the connection is idealized in this study as a 3D block model subjected to line loading, as conceptually illustrated in Figure 4. This idealization is justified because it captures the essential geometric constraint and load-transfer characteristics of the joint—where failure is dominated by shear and compression/tension failure along a potential plane—thereby providing a robust theoretical foundation for developing calculation methods tailored specifically to prefabricated metro station structures. The selection of this simplified 3D block model, however, necessitates a detailed and rigorous discussion regarding the underlying parameters and constitutive assumptions. For the cementitious materials typical of this connection, the strength criteria are intrinsically defined by key parameters: the uniaxial compressive strength (f'_c), the internal friction angle (ϕ), and the tensile strength (f'_t). In the development of the analytical formulation, a critical modeling decision involves the treatment of tensile strength. While a common and often accepted engineering practice in the limit analysis of brittle geomaterials is the conservative assumption of zero or negligible tensile strength ($f'_t \approx 0$), primarily justified by the typically large f'_c/f'_t ratio (often exceeding 10:1), this study adopts a more comprehensive approach by utilizing a general strength criterion that incorporates a non-zero tensile-to-compressive strength ratio (f'_c/f'_t). This methodological choice is paramount: ignoring tensile strength altogether constitutes a critical oversimplification that could lead to a non-conservative prediction of the overall load capacity and, more importantly, an inaccurate determination of the critical failure angle (α). This is particularly true in the near-surface zones of the joint where confinement is limited and the stress field may exhibit significant tensile stress components that initiate splitting failure, a mode which is directly controlled by f'_t . By retaining f'_c/f'_t as a primary governing parameter, the model allows for a rigorous sensitivity analysis, enabling a precise, quantitative understanding of the coupled influence of tensile resistance on the ultimate bearing capacity and the precise evolution of the failure mechanism. The detailed investigation of f'_c/f'_t effects, specifically its impact on both the magnitude of capacity and the shift in the optimal failure angle, is presented in the following analysis section, underscoring the necessity of properly accounting for this parameter for a robust and accurate evaluation of the prefabricated joint's stability.

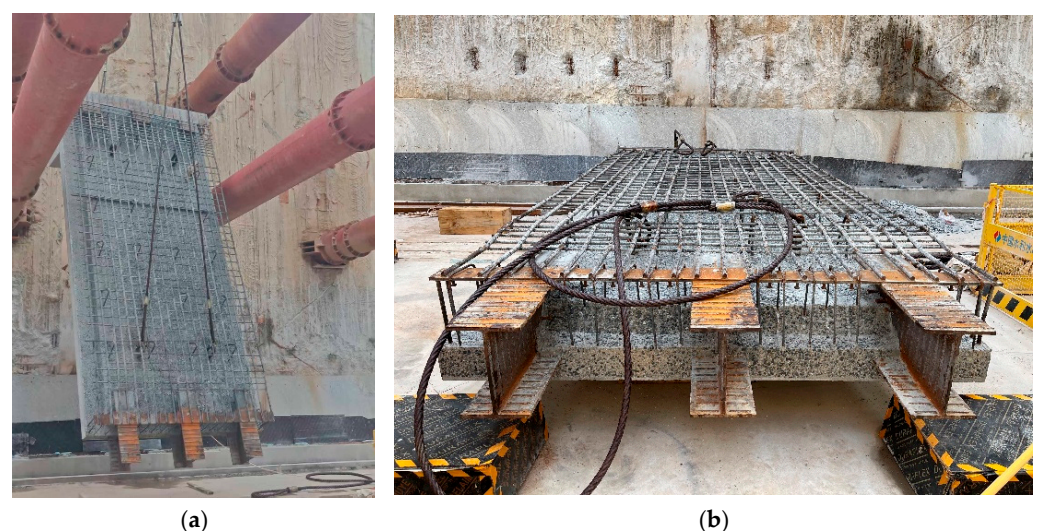


Figure 3. Prefabricated sidewall at Chanwan Station (a) Sectional view; (b) Planar view.

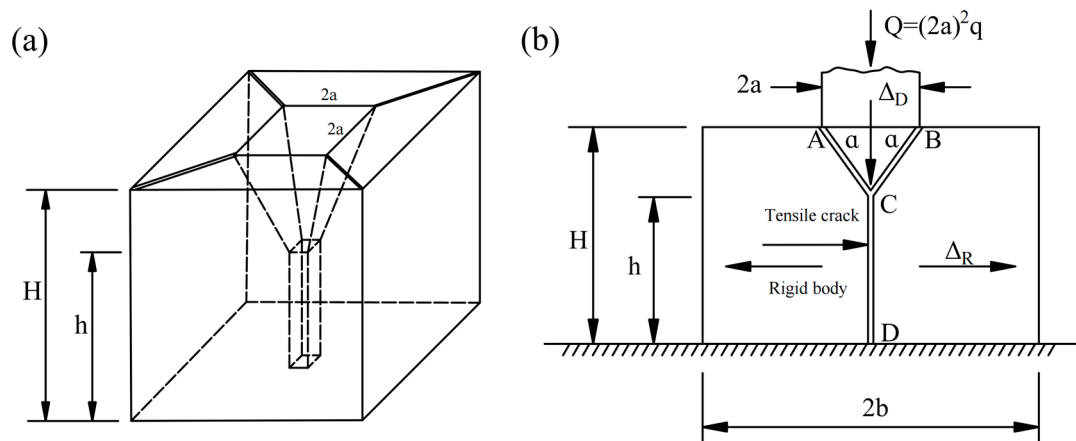


Figure 4. Three-Dimensional shear failure model: (a) schematic of 3D failure mechanism; (b) sectional view of failure mechanism.

Figure 4 illustrates the simplified loading model, in which a prismatic block of height H and plan dimensions of $2b \times 2b$ is subjected to a uniformly distributed surface load Q uniformly distributed along its width. To explore the 3D failure mechanism of this block under loading, a corresponding kinematically admissible velocity field is constructed.

The proposed mechanism consists of two components: (i) a rigid wedge-shaped failure region ABC with an apex angle 2α , and (ii) a simple tensile crack CD that is oriented vertically and perpendicular to the smooth, frictionless foundation. Under the action of the applied load, the wedge-shaped block undergoes a rigid-body downward displacement, accompanied by lateral movements in the side regions, thereby capturing the localized deformation characteristics of the material in 3D space.

Furthermore, at every point along the discontinuity surfaces AC and BC , the relative velocity vector forms an angle φ with the discontinuity plane, ensuring the compatibility of the velocity field. This construction allows the model to realistically represent the velocity distribution and energy dissipation during the failure process. The overall configuration of this velocity field is compatible, which provides a clear illustration of the geometric features and motion pattern of the proposed 3D block failure mechanism.

4.3. Upper-Bound Solution

In the context of limit analysis, the upper-bound method serves as an efficient approach to estimate the collapse load of structural systems, and it is particularly suitable for 3D short-block models. The essence of this method lies in constructing a kinematically compatible velocity field that satisfies all imposed boundary conditions while remaining consistent with the potential failure mechanisms. By enforcing compatibility along all discontinuities and equating the external work rate with the internal plastic dissipation rate, an upper-bound estimate of the ultimate load can be obtained.

For the short-block problem considered here, the velocity boundary conditions require the loaded area to remain constant, while the base must stay planar, as both the mold and foundation are rigid. Under this velocity field, balancing external work and internal dissipation allows the determination of the minimum average pressure on the block, providing an upper-bound solution for the collapse load.

Figure 4 depicts a block with thickness H and length and width of $2b$, subjected to a uniformly distributed surface load Q uniformly distributed along the width. The compatible velocity field is parameterized by α and θ , from which the dissipation rate along discontinuities can be systematically derived. Ultimately, the resulting upper-bound

solution q_u can be expressed as a function of tensile strength f'_t and compressive strength f'_c , effectively capturing the 3D failure behavior of the short block:

$$q_u = \frac{f'_c \left(\frac{1-\sin\theta}{2} \right)}{\sin\alpha \cos(\alpha+\theta)} + \frac{f'_t \left[\left(\frac{\sin\theta - \sin\varphi}{1-\sin\varphi} \right) + \sin(\alpha+\theta) \left(\frac{2Hb}{a^2} \sin\alpha - \cos\alpha \right) \right]}{\sin\alpha \cos(\alpha+\theta)} \quad (6)$$

where Q_u represents the ultimate surface load.

The minimum value of Equation (10) can be determined by performing a differential analysis, which allows the identification of the critical point corresponding to the lowest possible value under the given constraints. The differential equation governing this analysis can be expressed in the following form:

$$\frac{\partial}{\partial\alpha} \left(\frac{Q}{2af'_t} \right) = \frac{\partial}{\partial\alpha} \left[\frac{g_1(\alpha)}{g_2(\alpha)} \right] = \frac{g_2(\alpha) \frac{\partial g_1(\alpha)}{\partial\alpha} - g_1(\alpha) \frac{\partial g_2(\alpha)}{\partial\alpha}}{[g_2(\alpha)]^2} \quad (7)$$

where $g_1(\alpha)$ and $g_2(\alpha)$ represent the two characteristic functions associated with Equation (6), respectively. To proceed with the analysis, the differential equations listed above are next reformulated by setting their right-hand sides equal to zero. This transformation allows the system to be expressed in a homogeneous form, which is essential for deriving analytical solutions and further evaluating the upper-bound collapse load:

$$\frac{\partial g_2(\alpha)}{\partial\alpha} g_1(\alpha) = \frac{\partial g_1(\alpha)}{\partial\alpha} g_2(\alpha) \quad (8)$$

When Equation (8) holds true, the upper-bound solution reaches its lowest value. Inserting this result into Equation (7) provides the critical value associated with the considered failure mechanism:

$$q_u = \frac{Q_u}{2a} = f'_t \frac{\frac{\partial g_1(\alpha)}{\partial\alpha}}{\frac{\partial g_2(\alpha)}{\partial\alpha}} = f'_t \left[\frac{2bH}{a^2} \tan(2\alpha_c + \varphi) - 1 \right] \quad (9)$$

This solution is valid if and only if the parameter α satisfies the following condition:

$$\cot\alpha = \cot\alpha_c = \tan\varphi + \sec\varphi \left[1 + \frac{\frac{2bH}{a^2} \cos\varphi}{\left(\frac{f'_c}{f'_t} \right) \left(\frac{1-\sin\varphi}{2} \right) - \sin\varphi} \right]^{\frac{1}{2}} \quad (10)$$

The critical value in the equation is only applicable for the 3D mechanism and can be determined from the model's geometric constraints:

$$\frac{1}{2}\pi > \alpha_c > \tan^{-1} \frac{a^2}{2bH} = \alpha_{\min} \quad (11)$$

5. Results and Discussion

To provide a more intuitive and systematic understanding of the influence of different governing variables on the ultimate bearing capacity of 3D short blocks, this study employs the analytical formulation derived in the previous section to conduct a comprehensive series of calculations under multiple scenarios. The results encompass variations with respect to internal friction angle, cross-sectional dimensions, and material mechanical properties, and are presented in graphical form to facilitate comparative analysis. This approach enables a clearer identification of the coupled effects among these factors and their impact on the evolution of the ultimate bearing capacity. Figure 5 illustrates the variation in the ultimate bearing capacity of short blocks with respect to angle α under different internal friction angles (φ) for three representative cross-sectional sizes. It is evident that each curve exhibits

a distinct minimum value; however, the corresponding angle α at which this minimum occurs is not identical across cases. Specifically, under the same φ , a smaller cross-sectional size results in a generally lower ultimate bearing capacity, while the associated minimum tends to occur at a larger angle α . This implies that as the cross-section becomes smaller, the block is more prone to reach its ultimate state at a steeper failure inclination. Conversely, when the cross-sectional size remains constant, an increase in φ leads to a notable rise in the ultimate bearing capacity, whereas the minimum shifts toward a smaller angle α . This indicates that a larger internal friction angle enhances the shear resistance of the block, enabling failure at less inclined critical angles. These observations not only highlight the coupled effect of cross-sectional geometry and frictional strength on both the magnitude of ultimate capacity and the optimal failure angle, but also provide valuable physical insight: cross-sectional size predominantly governs the scale of bearing capacity, while the internal friction angle exerts a decisive influence on the location of the critical failure angle.

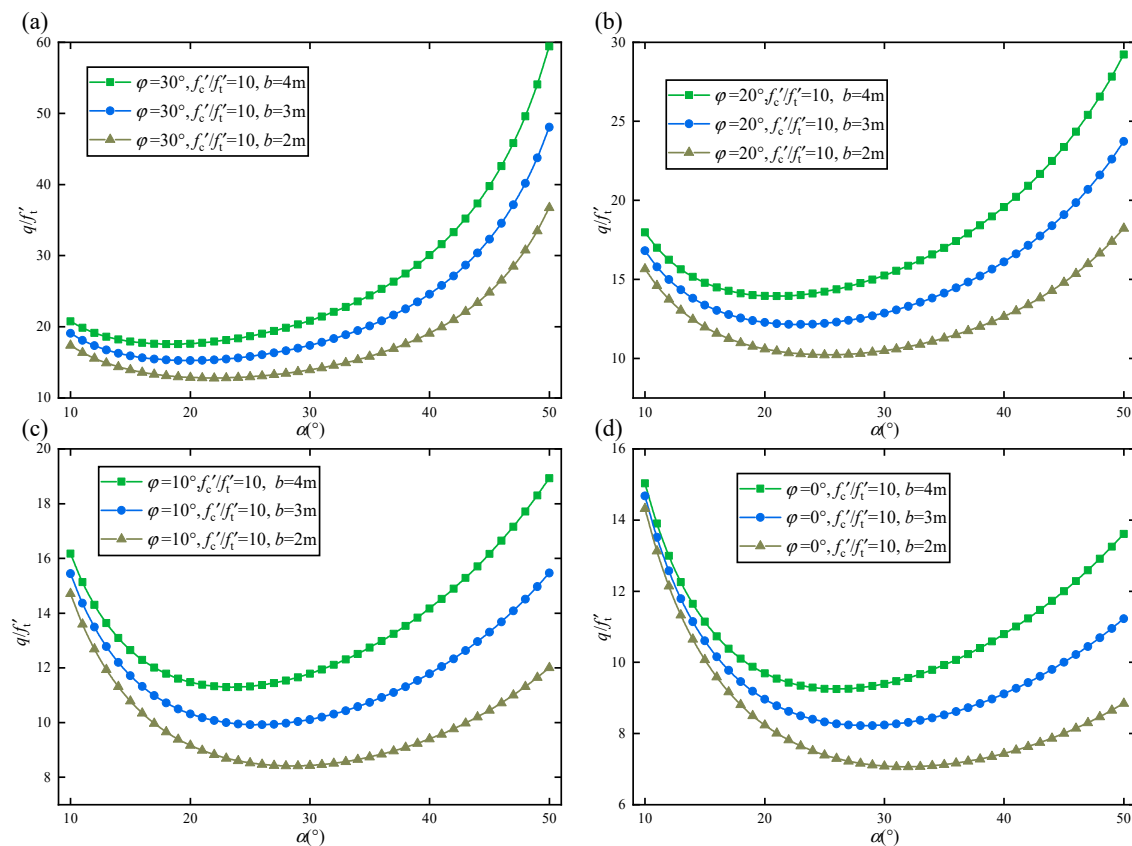


Figure 5. Variation in the upper-bound solution with respect to α for different cross-sectional dimensions b ($f_c'/f_t' = 10$): (a) $\varphi = 30^\circ$; (b) $\varphi = 20^\circ$ (c) $\varphi = 10^\circ$ (d) $\varphi = 0^\circ$.

Figure 6 illustrates the variation in the ultimate bearing capacity of the short block with respect to angle α under different load distribution widths. It is evident that this trend differs significantly from the influence of cross-sectional dimensions: as the load distribution width decreases, the ultimate bearing capacity of the short block increases, while the angle α corresponding to the minimum bearing capacity shifts to smaller values. A closer examination of the curve shapes reveals that when the internal friction angle φ is relatively large, the descending portion of the curve is less pronounced, with the initial value considerably lower than the final value. Conversely, as φ decreases, the descending trend becomes more distinct, and the difference between the initial and final values diminishes progressively. These findings highlight a clear coupling effect between the load distribution width and the internal friction angle: the former primarily governs

the magnitude of bearing capacity enhancement and the location of the optimal failure angle, while the latter dictates the slope of the curve and the extent of bearing capacity variation. Together, they jointly control the stability characteristics of the 3D short block.

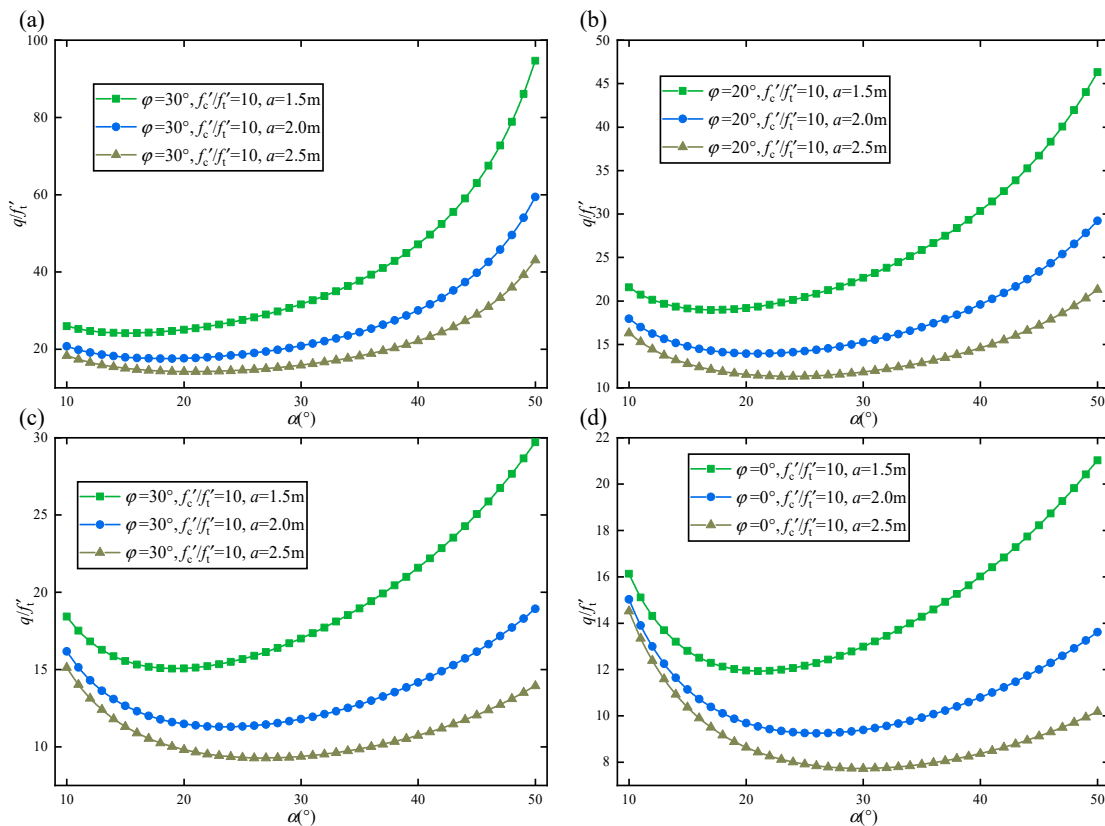


Figure 6. Variation in the upper-bound solution with respect to α for different load distribution width a ($f_c'/f_t' = 10$): (a) $\phi = 30^\circ$; (b) $\phi = 20^\circ$ (c) $\phi = 10^\circ$ (d) $\phi = 0^\circ$.

To further investigate the influence of material mechanical properties on the ultimate bearing capacity of short blocks, special attention is given to the effect of the tensile-to-compressive strength ratio (f_c'/f_t'), thereby allowing for a detailed discussion on model parameter selection and the underlying assumptions. While the tensile strength (f_t') is often neglected or conservatively simplified in many analytical models for brittle materials due to its characteristically low magnitude, such a simplification may compromise the accurate assessment of the failure mechanism. Hence, this analytical formulation retains the parameter to conduct a sensitivity analysis and quantify its impact. Figure 7 is presented under the same parameter settings as Figure 5, except that the tensile-to-compressive strength ratio of the material is reduced from 10 to 5. A comparison between Figures 5 and 7 clearly indicates that a decrease in the f_c'/f_t' ratio leads to a reduction in the ultimate bearing capacity. Moreover, under otherwise identical conditions, the critical angle α corresponding to the minimum capacity also shifts to a smaller value. This observation demonstrates that the tensile-to-compressive ratio not only governs the overall magnitude of the ultimate capacity but also fundamentally affects the optimal failure angle of the short block. When the tensile resistance of the material is relatively weakened, the block becomes more susceptible to tensile failure or crack extension within the stress field, thereby lowering its bearing capacity and shifting the critical failure mechanism to a less inclined angle (α) which involves a greater component of tensile stress. These findings highlight the significant role of the f_c'/f_t' ratio in determining the load-bearing performance of

short blocks and underscore the necessity of appropriately considering material strength parameters, rather than simply neglecting tensile strength, in engineering design.

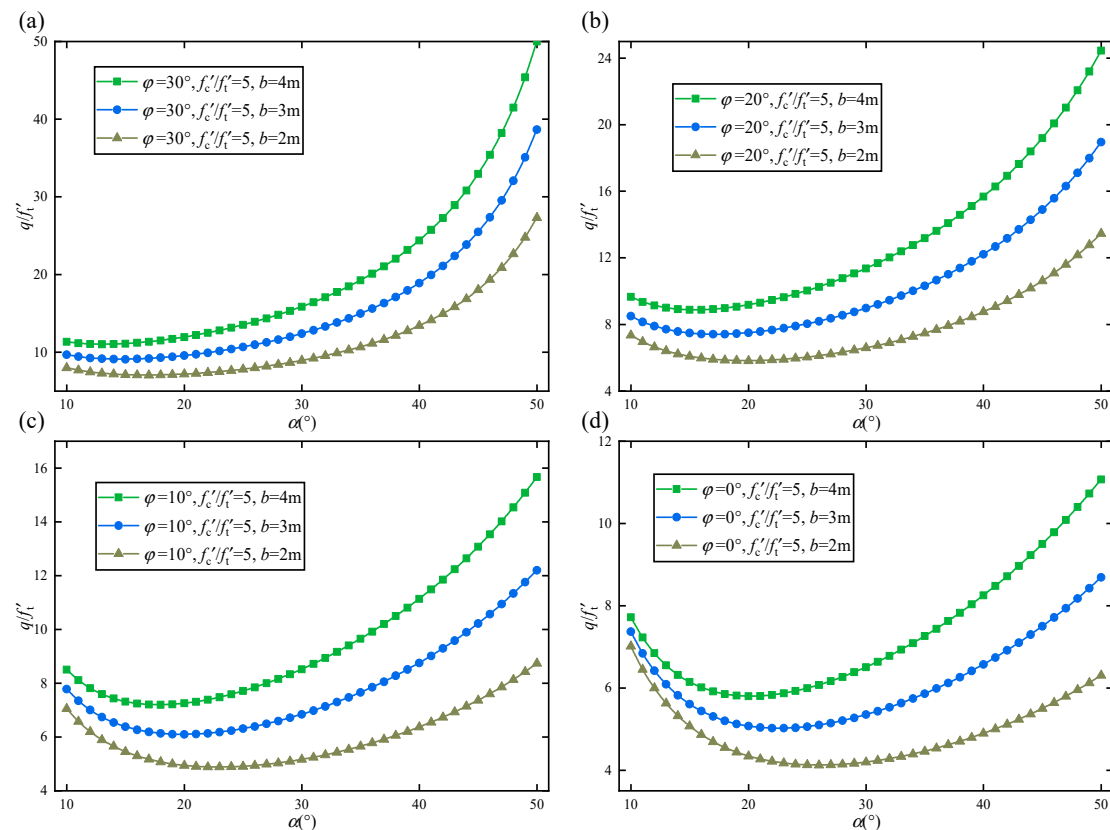


Figure 7. Variation in the upper bound solution with respect to α for different cross-sectional dimensions b ($f_c'/f_t' = 5$): (a) $\phi = 30^\circ$; (b) $\phi = 20^\circ$ (c) $\phi = 10^\circ$ (d) $\phi = 0^\circ$.

6. Conclusions

In this study, the complex shear failure of prefabricated cast-in-place slab–wall joints was rigorously investigated through the establishment of a novel three-dimensional (3D) short-block shear failure model. By employing the kinematic upper-bound approach and incorporating a modified Mohr–Coulomb criterion that accounts for the finite tensile strength cut-off, generalized analytical solutions were successfully derived to systematically predict the ultimate bearing capacity and the corresponding failure mechanisms of the short blocks. The comprehensive parametric analysis yielded several critical and quantitative conclusions with clear engineering implications.

Specifically, the study confirmed that cross-sectional dimensions have a dominant influence on the overall magnitude of the bearing capacity, where smaller cross-sections result in a notable reduction in ultimate capacity, concurrently leading to an increase in the critical failure angle, underscoring the sensitivity of the joint's stability to size effects in design. Conversely, the internal friction angle exerts a critical control over the failure mode: its increase not only substantially enhances the capacity but also shifts the optimal failure angle downward to a smaller value, demonstrating the primacy of frictional resistance in controlling joint stability. Furthermore, the analysis rigorously proved that reducing the load distribution width significantly promotes a higher ultimate capacity while concurrently decreasing the critical failure angle, providing a clear design directive for optimizing the load transfer zone. Most significantly, the research provided direct, rigorous evidence that a reduction in the material tensile-to-compressive strength ratio decreases the ultimate capacity and shifts the optimal failure angle downward. This finding

compellingly demonstrates that tensile capacity is an indispensable factor influencing the failure mode, theoretically validating the necessity of incorporating finite tensile strength into the analytical framework, which is expected to enhance the consistency between theoretical prediction and experimental observation.

While the proposed 3D shear model offers a reliable and theoretically sound framework for rapid analytical calculation, certain limitations exist, primarily stemming from the idealized assumptions inherent to the rigid-plastic limit analysis framework, such as neglecting the effects of reinforcement and long-term strain softening. Moving forward, future research should focus on three main directions: first, integrating the influence of reinforcement effectively into the velocity field construction to capture the interaction between concrete and steel; second, extending the model to incorporate cyclic loading effects relevant to seismic activity and long-term durability; and third, conducting extensive physical experiments or advanced numerical simulations to rigorously verify the accuracy of the predicted critical failure angles, especially under conditions where the tensile strength significantly influences the failure path. Overall, the present findings offer valuable theoretical guidance and practical reference for the design optimization and safety assessment of prefabricated cast-in-place connections in underground structures.

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