

Article

Interpolation-Based Evaluation and Prediction of Vortex Efficiency in Torque-Flow Pumps

Vladyslav Kondus ¹, Ivan Pavlenko ², Marek Ochowiak ^{3,*}, Andżelika Krupińska ³, Magdalena Matuszak ³ and Sylwia Włodarczak ^{3,*}

¹ Department of Applied Hydroaeromechanics, Sumy State University, 116, Kharkivska St., 40007 Sumy, Ukraine; v.kondus@pgm.sumdu.edu.ua

² Department of Computational Mechanics Named After Volodymyr Martynovskyy, Sumy State University, 116, Kharkivska St., 40007 Sumy, Ukraine; i.pavlenko@cm.sumdu.edu.ua

³ Department of Chemical Engineering and Equipment, Poznan University of Technology, 4, Berdychowo St., 60-965 Poznan, Poland; andzelika.krupinska@put.poznan.pl (A.K.); magdalena.matuszak@put.poznan.pl (M.M.)

* Correspondence: marek.ochowiak@put.poznan.pl (M.O.); sylwia.wlodarczak@put.poznan.pl (S.W.)

Abstract

The article presents the results of a study on the energy efficiency of torque-flow pumps with a particular focus on isolating the efficiency of the vortex operating process. The relevance of the topic is determined by the complexity of describing the combined operating process, which includes both blade and vortex components, the latter of which has remained insufficiently studied until now. The research aimed to improve the accuracy of predicting the efficiency of torque-flow pumps by analytically determining the effectiveness of the vortex operating process over the full range of specific speed coefficients. The study employed empirical data from bench tests of prototype pumps, supplemented by well-known empirical methodologies. To construct analytical dependencies, a Lagrange interpolation polynomial with equally spaced nodes in the range $n_s = 10\text{--}220$ was applied. This made it possible to obtain generalized functions for the efficiency of torque-flow pumps, centrifugal (blade) pumps, and, for the first time, to determine the complete characteristic of the vortex operating process efficiency. The proposed interpolation method reproduces the empirical efficiency characteristics with an accuracy better than 1.1–1.2%, enabling reliable prediction of the intrinsic vortex process efficiency across the entire range of specific speeds. It was established that the maximum value of the vortex operating process efficiency is 66.6% at n_s , while the optimal operating range of the pumps corresponds to $n_s = 70\text{--}140$. The practical significance of the obtained results lies in their applicability for pump design and optimization for the transportation of liquids containing inclusions, as well as in ensuring higher accuracy of engineering calculations without the need for extensive experimental testing. The importance of the study is further emphasized by its contribution to achieving the United Nations Sustainable Development Goals, particularly in the areas of energy efficiency, industrial innovation, and resilient infrastructure.

Keywords: torque-flow pumps; energy efficiency; vortex operating process; efficiency; Lagrange polynomial interpolation; pump performance prediction; Sustainable Development Goals (SDGs)



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1. Introduction

Improving the energy efficiency of hydraulic machines is one of the leading directions in modern mechanical engineering [1]. Among them, torque-flow (Figure 1) pumps occupy a special place, as they combine the properties of centrifugal and vortex machines and are capable of handling liquids containing a significant amount of solid inclusions [2]. This makes them promising for applications in the chemical, oil, food, and municipal industries [3].

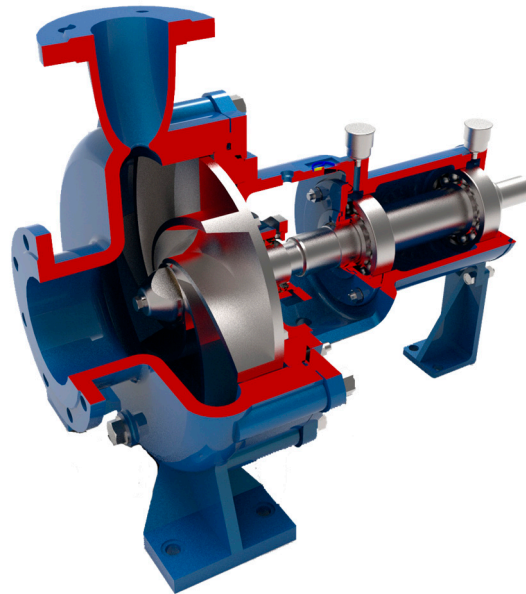


Figure 1. Torque-flow pump (TFP) of TURO type.

At the same time, the operating process in torque-flow pumps has a complex nature, consisting of blade and vortex components that differ significantly in their mechanisms of energy transfer [4]. The vortex component remains the least studied, despite its substantial influence on the overall efficiency of pumps of this type [5]. Traditionally, the assessment of energy efficiency is based on empirical graphical dependencies, which are characterized by increased error in the extreme ranges of specific speed [6]. This limits the possibilities for prediction and complicates the design of new pumps [7].

Therefore, the development of analytical methods capable of separating the vortex contribution to the pump efficiency is of key importance [8]. Interpolation techniques, particularly the Lagrange polynomial, provide a suitable mathematical tool for constructing generalized efficiency dependencies over a wide range of specific speeds [9].

Torque-flow pumps are widely used in conditions where conventional centrifugal pumps cannot maintain sufficient reliability due to contamination of the pumped liquid [10]. Therefore, accurately predicting their efficiency at the design stage is essential, as it reduces the need for extensive experimental testing and accelerates the development of new pump prototypes [11].

The quantitative determination of the vortex operating-process efficiency remains an unresolved problem, as previous studies provided only approximate values near the optimal specific-speed region [12,13]. Establishing the complete efficiency characteristic $\eta_{OP}(n_s)$ over the full specific-speed range addresses this gap and enables a comprehensive assessment of the vortex contribution to the overall pump performance. This result provides an analytical foundation for further optimizing the torque-flow pump flow passages.

The relevance of improving pump efficiency also extends to broader sustainability goals. This research aligns with several United Nations Sustainable Development Goals,

including SDG 7 (“Affordable and Clean Energy”), SDG 9 (“Industry, Innovation and Infrastructure”), and SDG 12 (“Responsible Consumption and Production”), as more accurate efficiency prediction supports the development of energy-efficient, reliable and resource-efficient pumping systems [14].

2. Literature Review

Studies of pump energy efficiency traditionally distinguish between hydraulic, volumetric, and mechanical components [15]. For centrifugal and axial pumps, extensive experimental and theoretical data have established well-known empirical efficiency correlations as functions of specific speed [16,17]. Torque-flow pumps differ fundamentally from these machines, as their operating process combines a blade component with a vortex component generated in a recessed flow passage [18–21]. While the blade mechanism is similar to that of centrifugal pumps, the vortex mechanism is governed by viscous energy transfer within a toroidal vortex, making its analytical description considerably more complex and limiting existing studies mainly to empirical determination of total pump efficiency [22].

One of the most commonly used approaches to evaluating torque-flow pump efficiency relies on graphical correlations obtained from large empirical datasets [23]. However, this method exhibits significant limitations: the error in the extreme specific-speed ranges ($n_s < 70$ and $n_s > 140$) can exceed 5%, leading to noticeable discrepancies between predicted and experimentally measured efficiencies [18,24,25]. As a result, accurate design based on such correlations is feasible only within a narrow range of specific speeds, typically $n_s = 70$ –120.

Previous studies [26,27] indicate that the vortex component plays a decisive role in shaping the overall efficiency characteristic of torque-flow pumps. However, its separate quantitative description is still lacking. To address this gap, modern prediction approaches increasingly rely on interpolation and approximation techniques [28], which enable the generalization of empirical data into analytical dependencies suitable for mathematical analysis, engineering calculations, and CFD-based design frameworks [29,30].

Thus, despite considerable progress in studying the overall efficiency of torque-flow pumps, the separate quantitative identification of the vortex operating-process efficiency remains unresolved. This gap defines the scientific novelty of the present work, which aims to establish the analytical dependence $\eta_{OP}(n_s)$ across the full range of specific-speed values.

The study aims to improve the accuracy of predicting the efficiency of torque-flow pumps by analytically determining the effectiveness of the vortex working process over the full range of specific speed coefficients. For this purpose, an interpolation approach was applied, which enables the transition from empirical graphical dependencies to mathematically substantiated functions.

To achieve this aim, the following tasks were set:

1. To analyze the existing methodologies for determining the efficiency of torque-flow pumps and to justify the feasibility of applying interpolation methods.
2. To construct analytical interpolation dependencies of the efficiency of torque-flow pumps $\eta_{TFP}(n_s)$ and centrifugal pumps $\eta_{BP}(n_s)$ based on specific speed n_s using initial computational and experimental data.
3. To develop a methodology for isolating the efficiency of the vortex operating process of torque-flow pumps $\eta_{OP}(n_s)$ by comparing the efficiency dependencies of torque-flow $\eta_{TFP}(n_s)$ and centrifugal (blade) pumps $\eta_{BP}(n_s)$ to specific speed n_s .
4. To determine the complete characteristic of the efficiency of the vortex operating process of torque-flow pumps $\eta_{OP}(n_s)$ within a wide range of specific speeds $n_s = 10$ –220.
5. To establish the optimal range of specific speed in which the maximum values of vortex process efficiency are achieved.

6. To evaluate the practical significance of the obtained dependencies for the design, optimization of geometry, and determination of operating modes of torque-flow pumps operating under complex fluid transportation conditions.

3. Materials and Methods

To improve the accuracy of predicting the energy efficiency of torque-flow pumps, an interpolation approach was applied, which enables the transition from empirical graphical dependencies to an analytical form [31].

The empirical efficiency data used in this study were not obtained from new experiments but were taken from previously published works by the authors [2,5,12,13]. These studies provide detailed descriptions of the pump prototypes, the hydraulic test bench, and the measurement accuracy in accordance with ISO 9906:2012 [32].

The original experiments were performed on a closed-loop laboratory test bench equipped with a variable-speed electric drive ($\pm 0.2\%$ accuracy), an electromagnetic flowmeter ($\pm 0.5\%$), calibrated pressure transducers ($\pm 0.3\%$), and a digital power meter ($\pm 1\%$). Each operating point was averaged over multiple repeated measurements to ensure statistical reliability.

In the present work, these empirical datasets are used solely as validated reference points for constructing interpolation curves. Since the focus of this study is the analytical reconstruction of efficiency characteristics, a detailed experimental campaign is not repeated here; instead, the previously validated empirical data serve as the foundation for interpolation-based modeling.

The initial data on the efficiency of torque-flow pumps, $\eta_{TFP}(n_s)$, were obtained using the well-known TURO design methodology for torque-flow pumps [33], as well as based on empirical data from bench tests of prototype torque-flow pumps conducted under laboratory conditions. This made it possible to form a computational basis for interpolation approximation and ensured higher reliability of the model.

The use of graphical efficiency charts may introduce an error exceeding 5%, due to the subjective nature of visual reading and selection of interpolation points [28,31]. This effect is particularly pronounced in the operating regions where the pump efficiency changes sharply with specific speed changing ($n_s < 80$ and $n_s > 140$). Considering this, the Lagrange interpolation method was applied as a more accurate analytical approach.

The general form of the interpolation function is expressed as:

$$L(n_s) = \sum_{i=0}^n \eta_i l_i(n_s), \quad (1)$$

where η_i —is the value of the function at point x_i , $l_i(x)$ is the fundamental Lagrange polynomial (basis polynomial), n_s —pump specific speed coefficient.

The fundamental Lagrange polynomial is defined as:

$$l_i(n_s) = \prod_{j=0, j \neq i}^n \frac{n_s - n_{sj}}{n_{si} - n_{sj}}, \quad (2)$$

with the following property:

$$l_i(n_{sj}) = \begin{cases} 0, & \text{if } j \neq i; \\ 1, & \text{if } j = i. \end{cases} \quad (3)$$

In this study, the pump specific speed n_s (dimensionless) is defined at the best efficiency point (BEP) as:

$$n_s = \frac{3.65 n \sqrt{Q}}{H^{\frac{3}{4}}}, \quad (4)$$

where H —the pump head, m; Q —the pump flow rate m^3/h ; n —the pump shaft rotational speed, rpm.

To construct the interpolation function, equally spaced interpolation nodes were chosen: $n_{s0} = 10$, $n_{s1} = 80$, $n_s = 150$, and $n_s = 220$. In this way, an analytical dependence of the efficiency of torque-flow pumps, $\eta_{TFP}(n_s)$, was obtained for the full range of specific speeds.

In parallel, using the same scheme, an interpolation function was constructed for the efficiency of centrifugal (blade) pumps $\eta_{BP}(n_s)$, with the initial values based on the classical decomposition of efficiency (hydraulic, volumetric, and mechanical).

Since the direct experimental determination of the efficiency of the vortex operating process (η_{OP}) is impossible, it was evaluated indirectly. In the first approximation, without considering volumetric losses, it was defined as:

$$\eta_{OP} = \frac{\eta_{TFP}}{\eta_{BP}}, \quad (5)$$

and with the inclusion of volumetric losses, which are characteristic of centrifugal pumps but absent in torque-flow pumps:

$$\eta_{OP} = \frac{\eta_{TFP}}{\eta_{BP} \cdot \eta_{VBP}}, \quad (6)$$

where η_{VBP} represents volumetric losses that occur due to leakage of the flow through the front shroud of the impeller from the outlet region to the inlet region.

Refined calculations were carried out by constructing a dedicated interpolation function for $\eta_{OP}(n_s)$ using the same nodes. The general form of the refined function is:

$$\eta_{OP}(n_s) = \sum_{i=0}^3 \eta_{OP}(n_{si}) l_i(n_s), \quad (7)$$

where $n_{si} \in \{10, 80, 150, 220\}$.

The obtained dependencies enable the determination of the efficiency of torque-flow pumps and the isolation of the contribution of the vortex working process across a wide range of specific speeds ($n_s = 10\text{--}220$) with accuracy that meets the requirements for engineering calculations.

The step-by-step procedure of interpolation-based prediction of efficiency is shown in Figure 2.

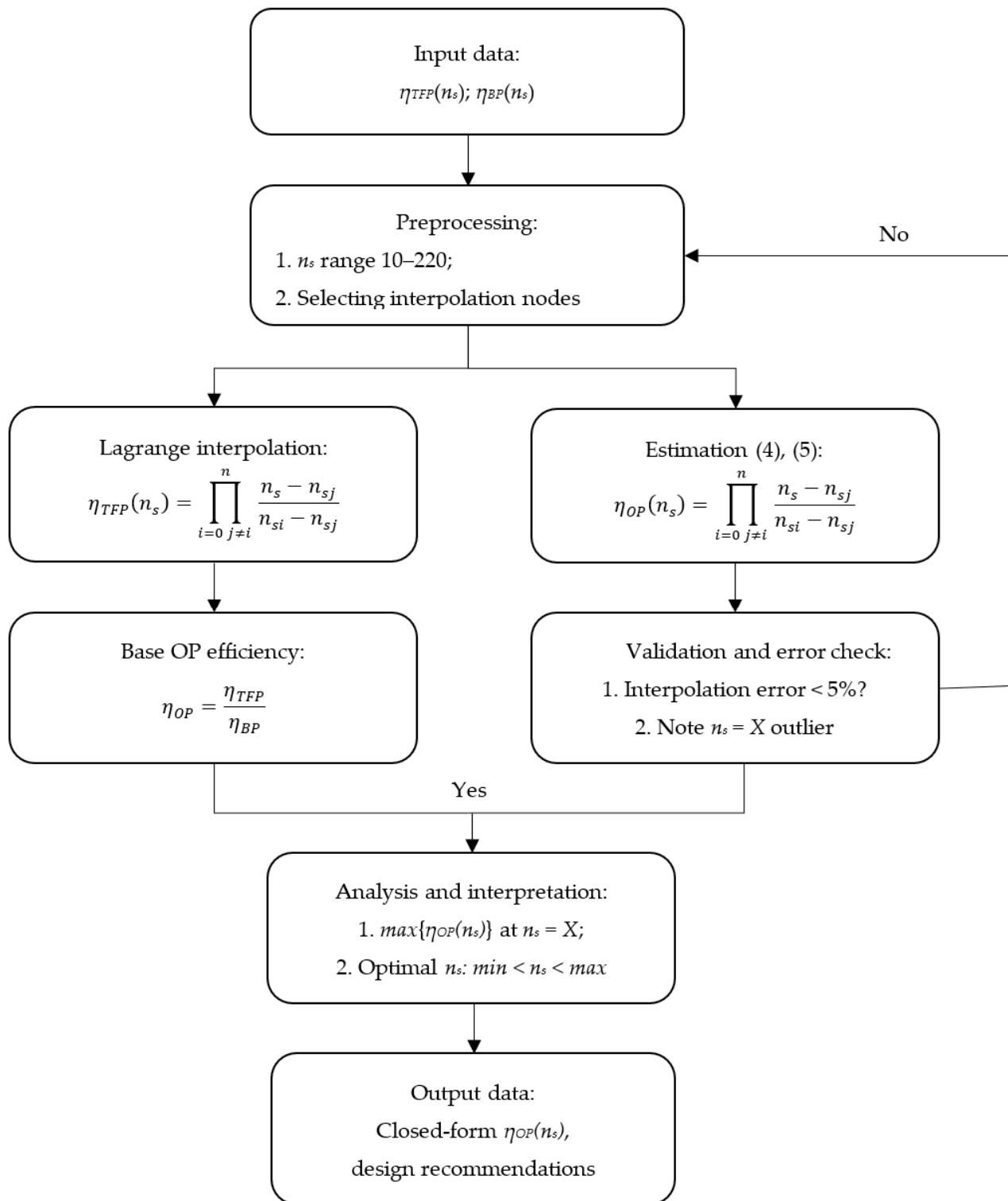


Figure 2. Flowchart of the interpolation-based procedure for predicting the efficiency of torque-flow pumps with iterative accuracy validation.

4. Results

4.1. Determination of Analytical Dependence for Torque-Flow Pumps

As a result of the calculations, interpolation dependencies of the efficiency (η) were constructed for torque-flow pumps $\eta_{TFP}(n_s)$, centrifugal pumps $\eta_{BP}(n_s)$, and a separate indicator of the energy efficiency of the vortex operating process $\eta_{OP}(n_s)$.

Based on the initial empirical data and the Lagrange interpolation polynomial, the function $\eta_{TFP}(n_s)$, was obtained, which reproduces the known graphical dependence (Figure 3) with high accuracy:

$$\eta_{TFP}(n_s) = 9.281 \cdot 10^{-8} n_s^3 + 5.824 \cdot 10^{-5} n_s^2 + 9.543 \cdot 10^{-3} n_s + 0.0593 \quad (8)$$

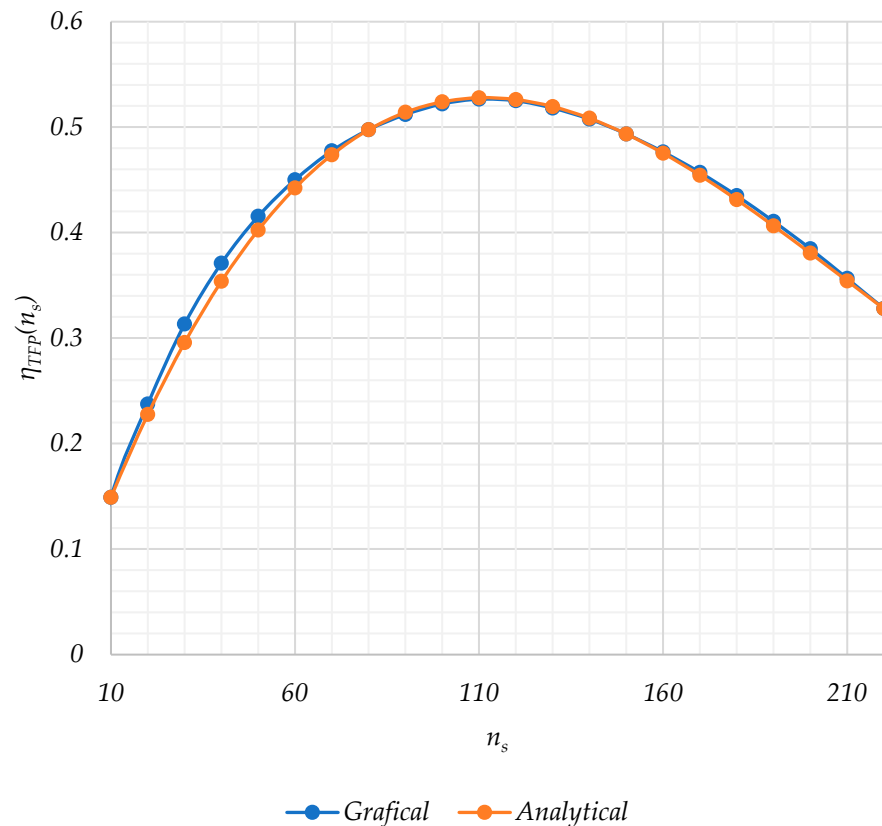


Figure 3. Comparison of the results of the graphical method (blue line) and the interpolation calculation (red line) for the efficiency of a torque-flow pump $\eta_{TFP}(n_s)$.

Notably, the efficiency curves used in this study (Figure 3) do not correspond to a single pump but represent a generalized dependence $\eta(n_s)$, obtained empirically in previous works for a wide range of torque-flow pump designs. Therefore, the present interpolation does not rely on specific dimensional parameters (Q , H , and n), but on the dimensionless specific speed n_s , which allows determining the theoretically achievable vortex operating process efficiency in a generalized form.

The comparison of values (Table 1) showed that the discrepancy between the graphical and analytical determinations does not exceed 1.1% in most cases, except in the zone of $n_s = 30$, where an increased deviation ($\approx 5.6\%$) was observed. This deviation may be explained by the sharp increase in hydraulic and dissipative losses at such low specific speeds, particularly when the pump operates at flow rates significantly below its best efficiency point ($Q < Q_{BEP}$), as reported in recent experimental and numerical studies [34,35].

Thus, the interpolation dependence (7) can be used as an analytical analog of the graphical method, suitable for calculations in the range $n_s = 10$ –220.

4.2. Determination of Analytical Dependence for Centrifugal Pumps

A similar approach was applied to single-stage centrifugal (blade) pumps, whose initial values were formed according to the classical dependencies of hydraulic, volumetric,

and mechanical efficiency. The dependence of the efficiency of centrifugal (blade) pumps $\eta_{BP}(n_s)$ is represented by the following equation:

$$\eta_{BP}(n_s) = 8.166 \cdot 10^{-8} n_s^3 + 3.835 \cdot 10^{-5} n_s^2 + 5.91 \cdot 10^{-3} n_s + 0.522.$$

Table 1. Dependence of the efficiency of torque-flow pumps $\eta_{TFP}(n_s)$ on the specific speed of the pump in the range $n_s = 10$ –220.

Specific Speed, n_s	Efficiency of the Torque-Flow Pump, η		Difference (Dimensionless)	Relative Difference, %
	Graphical Method	Lagrange Interpolation (7)		
0	0	0.0593	0.0593	-
10	0.149	0.1490	0	0
20	0.2375	0.2276	−0.0099	−4.2
30	0.3133	0.2957	−0.0176	−5.6
40	0.3710	0.3538	−0.0172	−4.6
50	0.4155	0.4024	−0.0131	−3.1
60	0.4500	0.4422	−0.0078	−1.7
70	0.4775	0.4737	−0.0038	−0.8
80	0.4975	0.4975	0	0
90	0.5120	0.5140	0.0020	0.4
100	0.5220	0.5240	0.0020	0.4
110	0.5265	0.5278	0.0013	0.2
120	0.5250	0.5261	0.0011	0.2
130	0.5180	0.5195	0.0015	0.3
140	0.5075	0.5084	0.0009	0.2
150	0.4935	0.4935	0	0
160	0.4765	0.4753	−0.0012	−0.3
170	0.4570	0.4543	−0.0027	−0.6
180	0.4350	0.4312	−0.0038	−0.9
190	0.4106	0.4064	−0.0042	−1.0
200	0.3847	0.3806	−0.0041	−1.1
210	0.3565	0.3542	−0.0023	−0.6
220	0.32	0.3279	0	0

Note: green color–grid nodes by Lagrange interpolation polynomial.

The reference centrifugal-pump efficiency $\eta_{BP}(n_s)$ corresponds to the generalized characteristic of classical single-stage pumps equipped with a closed impeller, as commonly presented in pump-design literature. In contrast, torque-flow pumps employ a semi-open impeller operating in a recessed free-vortex chamber, which forms the toroidal vortex responsible for the vortex operating process analyzed in this study.

The analytical description reliably reproduces the behavior of real pumps; the difference between the curves does not exceed 1.2% in the range $n_s = 70$ –220 (Figure 4).

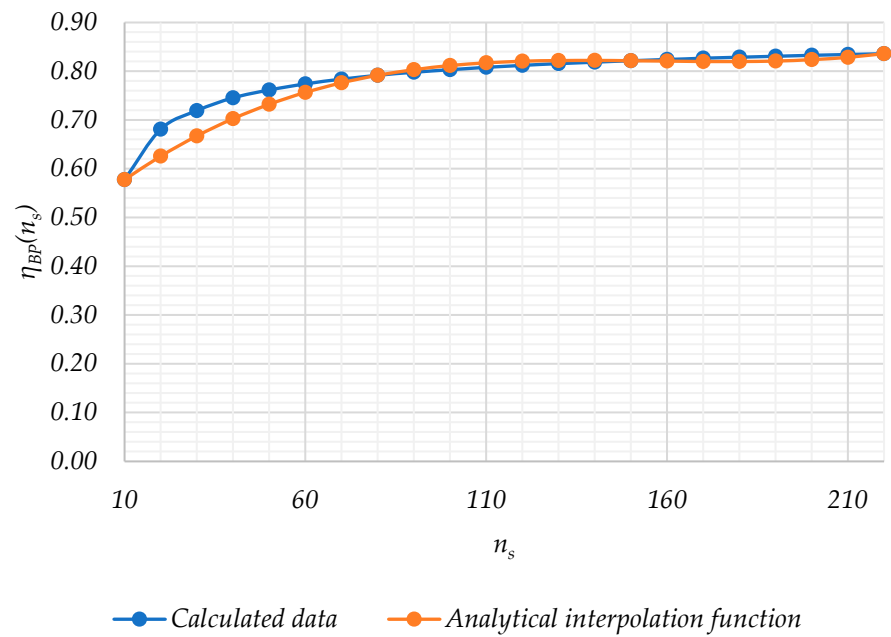


Figure 4. Comparison of the efficiency of centrifugal pumps: calculated data (blue line) and analytical interpolation function (red line).

The calculation results (Table 2) show that the analytical function $\eta_{BP}(n_s)$ accurately approximates the initial data. The discrepancy in most points does not exceed 1.2%. This confirms the correctness of using the interpolation approach for blade pumps as well.

The polynomial representation of $\eta_{BP}(n_s)$ was selected to provide a smooth analytical reference for the hydraulic (blade) component of efficiency. This form allows the determination of the intrinsic vortex-process efficiency $\eta_{OP}(n_s)$ by dividing the generalized torque-flow pump efficiency $\eta_{TFP}(n_s)$ by the baseline hydraulic efficiency of a classical centrifugal pump at the same specific speed $\eta_{BP}(n_s)$.

Table 2. Dependence of the efficiency of centrifugal (blade) pumps $\eta_{BP}(n_s)$ on the specific speed of the pump $n_s = 10$ –220.

Specific Speed, n_s	Efficiency of the Torque-Flow Pump, η		Difference (Dimensionless)	Relative Difference, %
	Graphical Method	Lagrange Interpolation (8)		
10	0.578	0.578	0	0
20	0.681	0.626	−0.055	−8.1
30	0.719	0.667	−0.052	−7.2
40	0.745	0.703	−0.043	−5.7
50	0.762	0.732	−0.029	−3.9
60	0.774	0.757	−0.017	−2.3
70	0.784	0.776	−0.008	−1.0
80	0.792	0.792	0	0
90	0.798	0.803	0.005	0.6
100	0.803	0.812	0.008	1.0
110	0.808	0.817	0.009	1.1
120	0.812	0.820	0.008	1.0
130	0.816	0.822	0.006	0.8

Table 2. Cont.

Specific Speed, n_s	Efficiency of the Torque-Flow Pump, η		Difference (Dimensionless)	Relative Difference, %
	Graphical Method	Lagrange Interpolation (8)		
140	0.819	0.822	0.004	0.4
150	0.822	0.822	0	0
160	0.824	0.821	−0.003	−0.4
170	0.827	0.820	−0.007	−0.8
180	0.829	0.820	−0.009	−1.1
190	0.831	0.821	−0.010	−1.2
200	0.833	0.824	−0.009	−1.1
210	0.834	0.829	−0.006	−0.7
220	0.836	0.836	0	0

Note: green color–grid nodes by Lagrange interpolation polynomial.

4.3. Isolation of the Efficiency of the Vortex Operating Process

The working process of vortex pumps in torque-flow pumps has a complex nature, and their efficiency cannot be determined directly by experimental methods. Therefore, it was evaluated by comparing the efficiency of torque-flow pumps $\eta_{TFP}(n_s)$ and centrifugal (blade) pumps $\eta_{BP}(n_s)$ under similar operating conditions (equal specific speed).

Calculations based on dependencies (4) and (5) made it possible to construct the dependence of the efficiency of the vortex operating process of torque-flow pumps, $\eta_{OP}(n_s)$, on the specific speed (Table 3). The analysis of these data allows several important conclusions to be drawn.

The dependence of the efficiency $\eta_{TFP}(n_s)$ of torque-flow pumps and the efficiency of the vortex operating process $\eta_{OP}(n_s)$ on the specific speed n_s (Figure 5) demonstrates that it is precisely the vortex process that determines the shape of the overall pump efficiency curve. The maximum $\eta_{OP}(n_s)$ is achieved at $n_s \approx 100$, while the optimal operating range of pumps is $n_s = 70\text{--}140$.

The maximum efficiency of the vortex working process of free-vortex pumps $\eta_{OP}(n_s)$ is 0.666 (64.6%) at $n_s = 100$. It is in this mode that the energy of the vortex process is transferred to the flow with the least losses.

The optimal specific speed range for the effective operation of the vortex operating process is within $n_s = 70\text{--}140$. During this interval, the value of the efficiency of the vortex operating process of the torque-flow pump, $\eta_{OP}(n_s)$, confirms the previously known value of 0.63, which can be considered one of the key elements of the scientific novelty of this study.

When the specific speed, n_s , is reduced to values less than 70 or increased to values greater than 140, a significant decrease in efficiency is observed. Thus, at the specific speed $n_s = 220$, the efficiency of the vortex operating process of the torque-flow pump $\eta_{OP}(n_s)$ is only 0.40, i.e., more than 25% is lost relative to the maximum. This results in a decrease of 5% or greater in the overall efficiency of the pump.

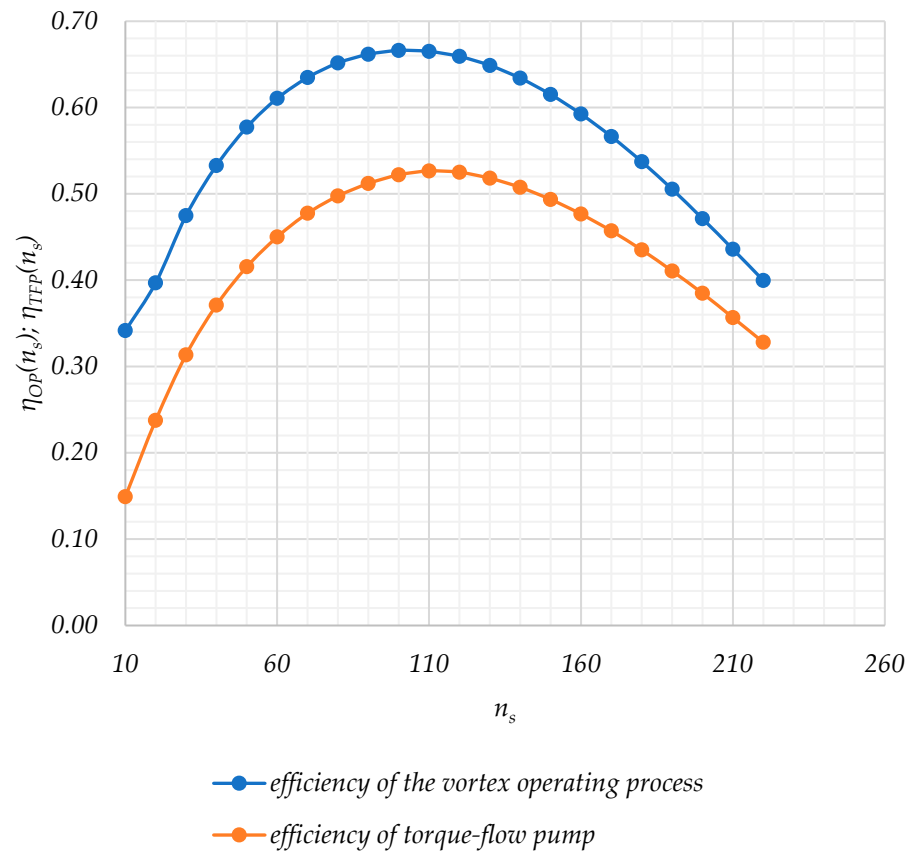


Figure 5. Dependence of the efficiency of torque-flow pumps $\eta_{TFP}(n_s)$ (red line) and the efficiency of the vortex working process $\eta_{OP}(n_s)$ (blue line) on the specific speed n_s .

Table 3. Dependence of the efficiency η of the vortex operating process of torque-flow pumps $\eta_{OP}(n_s)$ on the specific speed of the pump $n_s = 10$ –220.

Specific Speed, n_s	Efficiency of the Vortex Operating Process, η_{OP}	
	Not Including η_{VBP} (4)	Including η_{VBP} (5)
0	0.297	0.341
10	0.364	0.397
20	0.443	0.475
30	0.504	0.533
40	0.550	0.577
50	0.585	0.611
60	0.610	0.635
70	0.629	0.652
80	0.640	0.662
90	0.646	0.666
100	0.646	0.665
110	0.641	0.659
120	0.632	0.649
130	0.618	0.634
140	0.601	0.615
150	0.579	0.592

Table 3. Cont.

Specific Speed, n_s	Efficiency of the Vortex Operating Process, η_{OP}	
	Not Including η_{VBP} (4)	Including η_{VBP} (5)
160	0.554	0.566
170	0.526	0.537
180	0.495	0.505
190	0.462	0.471
200	0.428	0.436
210	0.297	0.341
220	0.364	0.397

Note: green color-grid nodes by Lagrange interpolation polynomial.

5. Discussion

The obtained results confirmed the key role of the vortex operating process in shaping the energy efficiency of torque-flow pumps. The interpolation approach made it possible not only to reproduce the known graphical dependencies but also to isolate the contribution of the vortex component, which was previously available only in qualitative assessments.

The obtained vortex-efficiency curve $\eta_{OP}(n_s)$ demonstrates a characteristic structure that reflects the internal hydrodynamic mechanisms governing the recessed-vortex flow in torque-flow pumps. At low specific-speed values ($n_s < 70$), the efficiency increases with n_s because the toroidal vortex inside the recessed chamber is only beginning to form. In this regime, the swirl induced by the rotating blade system is insufficient to generate a fully coherent recirculating structure. The flow is characterized by weak, irregular vortex patterns and local secondary motions, resulting in relatively high viscous losses. As the specific speed increases, the shear interaction between the blade wake and the recirculating zone intensifies, leading to a more stable and centered vortex core. This flow reorganization reduces dissipation and enhances the viscous entrainment mechanism responsible for energy transfer within the vortex chamber, which explains the monotonic rise of $\eta_{OP}(n_s)$ in the low-speed region.

The maximum efficiency, observed at approximately $n_s \approx 100$, corresponds to the regime in which the toroidal vortex achieves its highest coherence and stability. In this region, the rotational momentum imparted by the impeller matches the natural precession and circulation dynamics of the vortex core. As a result, the vortex remains axisymmetric, the shear layers are well-defined, and the majority of energy transfer occurs through organized viscous entrainment rather than through turbulent dissipation. This balance between imposed swirl and natural vortex dynamics yields the fundamental optimum of the vortex operating process and explains the peak value of $\eta_{OP}(n_s)$.

Beyond $n_s > 140$, the efficiency begins to decline due to the onset of flow instabilities and enhanced turbulent dissipation. Excessive swirl leads to an over-energy input into the recirculating flow, which destabilizes the vortex core and may trigger phenomena such as precession, vortex breakdown, or interaction between the vortex and the chamber walls. The shear layers become thicker and more chaotic, and secondary vortical structures emerge, all of which contribute to increased energy losses. At high specific speeds, the proportion of energy converted into useful head decreases, while the proportion dissipated through turbulence and viscous friction increases, producing the descending branch of the $\eta_{OP}(n_s)$ curve.

A comparison with centrifugal-pump efficiency characteristics highlights the fundamental difference between the two machine types. In classical centrifugal pumps, the

efficiency curve is primarily governed by blade incidence, slip, and secondary losses, which typically produce a smooth increase, a plateau near the best-efficiency point, and a decline associated with flow separation or cavitation. In torque-flow pumps, the dominant mechanism is the behavior of the toroidal vortex, which introduces a stronger dependence on internal flow coherence and stability. The bell-shaped form of the $\eta_{OP}(n_s)$ curve is therefore a direct consequence of the physics of the recessed-vortex system: efficiency rises as the vortex forms, peaks when the vortex is most coherent, and falls as the vortex becomes unstable and dissipative under excessive swirl.

This physical interpretation demonstrates that the efficiency of torque-flow pumps is intrinsically linked to the stability of the internal vortex structure, rather than solely to blade loading, as in centrifugal pumps. Consequently, the analytically reconstructed vortex-efficiency curve obtained in this study not only represents a mathematical result but also provides an essential hydrodynamic insight into the operation of torque-flow pumps. Such understanding forms a foundation for optimizing the geometry of the recessed chamber, selecting appropriate specific-speed ranges, and improving the predictive accuracy of design methodologies.

To evaluate the accuracy of the proposed analytical model, a quantitative error analysis was performed by comparing the Lagrange-interpolated efficiency values with empirical reference data obtained from previous experimental studies [2,5,12,13]. For torque-flow pumps, the maximum deviation between the interpolated curve $\eta_{TFP}(n_s)$ and the empirical values does not exceed 1.1% within the range $n_s = 70$ –220, except for the low-speed region around $n_s = 10$ –60, where the discrepancy reaches approximately 5.6% (Table 1). This deviation is attributed to the steep local slope of the efficiency characteristic in this region, where even minor uncertainties in the original graphical data may lead to amplified interpolation errors.

For centrifugal pumps, the analytical function $\eta_{BP}(n_s)$ exhibits similarly high accuracy, with a deviation remaining below 1.2% for $n_s \geq 70$. However, slightly higher differences are observed at very low specific speeds (Table 2). These results indicate that the interpolation-based model provides a reliable analytical representation of both centrifugal and torque-flow pump efficiencies over a wide operating range.

The potential sources of error in the reconstructed efficiency curves include:

- (1) uncertainties in the empirical datasets obtained initially from laboratory measurements;
- (2) digitization-related variations inherent to graphical efficiency curves from the literature;
- (3) the sensitivity of Lagrange interpolation to points located in regions with steep efficiency gradients;
- (4) the assumptions associated with separating centrifugal and vortex contributions using ratio-based definitions.

Despite these factors, the observed interpolation errors remain small and within the acceptable range for engineering modeling, confirming the robustness of the analytical approach used in this study.

The peak efficiency of the vortex process ($\eta_{OP} \approx 0.666$ at $n_s = 100$) indicates an optimal balance between the blade and vortex mechanisms of energy transfer. In this regime, the toroidal vortex forms a “liquid blade” that minimizes losses due to shear deformation and viscous friction. The results are in good agreement with previously known data [13,23], which confirms the correctness of the applied interpolation approach.

The optimal operating zone ($70 \leq n_s \leq 140$) reflects the balance between two extreme regimes. At low specific speed values, the vortex is formed insufficiently intensively, reducing its contribution to the total efficiency. In the case of high n_s values (more than 140), on the contrary, dissipation intensifies, and secondary flows develop, leading to a rapid increase in hydraulic losses.

Although the intrinsic vortex-process efficiency $\eta_{OP}(n_s)$ was reconstructed using Lagrange interpolation, the resulting curve can also be approximated with a low-order polynomial for engineering use. Such a polynomial may serve as a convenient predictive tool during the preliminary design stage, where the target specific speed is known and the designer must determine feasible impeller dimensions. In this context, $\eta_{OP}(n_s)$ provides an upper limit for the theoretically achievable efficiency of a torque-flow pump and can be used as a constraint when choosing geometric parameters or comparing alternative design concepts.

Of particular interest is the comparison with centrifugal pumps [31]. Even though η_{BP} shows a smooth growth and reaches high values over a wide range of specific speeds, it is the vortex component in torque-flow pumps that determines the characteristic “bell-shaped” efficiency characteristics of $\eta_{TFP}(n_s)$. This confirms the hypothesis that the vortex mechanism dominates in shaping the energy efficiency of this type of hydraulic machine.

From a practical point of view, the obtained dependencies enable the determination of a reasonable operating range for designing torque-flow pumps. Using machines with $n_s = 70$ –140 ensures stable efficiency when transporting liquids with solid inclusions, whereas operation outside this interval results in a decrease in overall efficiency by 5% or more. This makes the interpolation methodology a valuable tool for practical pump engineering, where accurate analytical approaches have been lacking to date.

The scientific novelty of this study lies in the fact that, for the first time, not only approximate values of the efficiency of the vortex operating process at the best efficiency point ($n_s = 100$) were obtained, but also the complete characteristic $\eta_{OP}(n_s)$ over the entire range of specific speeds. This enables the quantification of the vortex mechanism’s contribution under any pump operating condition, not just at the maximum efficiency point.

The practical significance of this result lies in the creation of an analytical basis for designing free-vortex pumps [36] with predictable energy efficiency indicators. Previously, engineers relied only on experimental graphs, which had significant errors outside the optimal range. Now, it is possible to apply analytical functions in software tools to optimize the geometry of flow passages, select operating modes, and perform preliminary efficiency assessments without the need for numerous experimental tests [37].

In addition, the obtained dependencies are of great importance for applied tasks, such as transporting liquids with a high content of solid inclusions, operating under variable loads, and designing pumping systems for the chemical, oil, and gas industries [38]. The ability to account for the real contribution of the vortex operating process at any specific speed opens the way to increasing the reliability and durability of pumping equipment.

Thus, the results of the study confirm the need to focus on medium values of the specific speed when designing advanced torque-flow pumps [39], as well as open up opportunities for further optimization of flow passage geometry based on the refined dependencies $\eta_{OP}(n_s)$.

It should be emphasized that the idealized operating framework adopted in this study was chosen deliberately in order to isolate the intrinsic efficiency of the toroidal vortex operating process. In a real torque-flow pump, the measured efficiency always contains contributions from mechanical, volumetric, leakage, and, in many cases, particle-induced losses, which cannot be experimentally separated from the vortex component. By removing these effects, the present analysis provides a baseline torque-flow pump operating process characteristic $\eta_{OP}(n_s)$ that reflects the underlying hydrodynamic mechanism in its pure form. This intrinsic dependence may subsequently be combined with additional loss models to reconstruct the complete efficiency of a real torque-flow pump. Therefore, the analytical characteristics derived in this work should be interpreted as a foundational

layer for further studies, including the incorporation of off-design operation ($Q < Q_{BEP}$), multiphase and slurry effects, and mechanical dissipation.

6. Conclusions

In this study, an analytical approach is proposed for the first time to determine the efficiency of a torque-flow pump and its vortex operating process, $\eta_{OP}(n_s)$, based on the Lagrange interpolation polynomial. This made it possible to move from graphical estimates to an exact mathematical description.

The obtained interpolation dependencies $\eta_{TFP}(n_s)$ and $\eta_{BP}(n_s)$ reliably reproduce the known computational and experimental data. The maximum discrepancies between the analytical and initial values do not exceed 1.1–1.2%, which meets the required accuracy for engineering calculations.

For the first time, the complete characteristics of the efficiency of the vortex working process were obtained for the entire range of specific speeds, $n_s = 10$ –220. Previously, such data were available only for selected optimal operating ranges of pumps.

It was established that the maximum efficiency of the vortex working process $\eta_{OP}(n_s)$ of torque-flow pumps is 0.666 (66.6%) at a specific speed of $n_s = 100$. The optimal operating range of specific speed is $70 \leq n_s \leq 140$, where η_{OP} remains close to the maximum value of 66.6%.

It was shown that a decrease in energy efficiency accompanies pump operation outside this range: for values of specific speed $n_s < 70$ and $n_s > 140$, the drop in $\eta_{OP}(n_s)$ is 3.3% or more, resulting in a reduction in overall pump efficiency of at least 5%.

The practical significance of the obtained results lies in the possibility of using the analytical dependencies $\eta_{TFP}(n_s)$, $\eta_{BP}(n_s)$ and $\eta_{OP}(n_s)$ in the design of new pumps, optimization of their geometry and operating ranges, as well as in the modeling of pumping systems under demanding operating conditions (transport of liquids with solid inclusions, variable loads, etc.).

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