

Article

Optimal Dimensional Synthesis of Ackermann Steering Mechanisms for Three-Axle, Six-Wheeled Vehicles

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Abstract: This study employs four metaheuristic optimization methods to optimize the dimensional synthesis of Ackermann steering mechanisms for three-axle, six-wheeled vehicles with front-axle steering mode and reverse-phase steering mode. The employed optimization methods include Particle Swarm Optimization (PSO), Hybrid Particle Swarm Optimization (HPSO), Differential Evolution with golden ratio (DE-gr), and Linearly Ensemble of Parameters and Mutation Strategies in Differential Evolution (L-EPSDE). With a front-wheel steering angle range of 70 degrees, two hundred optimization experiments were conducted for each method, and statistical analyses revealed that DE-gr and L-EPSDE methods outperformed PSO and HPSO methods in terms of standard deviation, mean value, and minimum error. These two methods exhibited superior convergence stability, faster convergence, and higher accuracy compared to PSO and HPSO. Reverse-phase ($K = 1$) steering mode outperformed front-axle steering mode, delivering reduced steering errors and turning radii. Considering the transmission ratio of front to rear axle (K) as a design variable in reverse-phase steering mode increased design flexibility and significantly lowered steering errors for the front and rear axle steering mechanisms. However, this comes with a slight increase in the turning radius of the vehicle's front part compared to when $K = 1$. The optimized mechanism, designed using the DE-gr method, was validated through kinematic simulations and steering analyses using MSC-ADAMS v2015 software, further confirming the effectiveness and reliability of the proposed design.

Keywords: three-axle six-wheeled vehicle; Ackermann steering mechanism; PSO; HPSO; DE-gr; L-EPSDE



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1. Introduction

The steering mechanism is a crucial subsystem for controlling a vehicle's steering motion and significantly impacts its stability and maneuverability. Traditional steering systems in two-axle, four-wheel vehicles generally include front-wheel drive, rear-wheel drive, and four-wheel drive. Four-wheel drive systems are further classified into four-wheel independent steering (4WIS) and four-wheel coordinated steering. Conventional steering systems often struggle to maintain stability and maneuverability when a vehicle navigates narrow paths or requires large turning angles. In vehicles with two or more axles, having only one steerable axle can lead to reduced agility and significant tire wear due to larger contact areas. To improve maneuverability and stability on complex roads, multi-axle steering systems are often employed, offering enhanced agility and reduced

tire wear. For example, a four-axle, eight-wheeled vehicle may be unsuitable for narrow paths due to its extended length. Conversely, a three-axle, six-wheeled vehicle, with a more compact design, is better equipped to navigate rugged terrain and narrow spaces, providing superior mobility and off-road capability.

The problem of dimensional synthesis of linkages can be divided into three types: function generation, motion generation/rigid body guidance, and path generation [1]. In a vehicle, the steering system is one of the important subsystems, and the steering system has a great influence on the maneuverability and flexibility of the vehicle. The Ackermann trapezoidal steering mechanism for front-wheel drive in conventional four-wheeled vehicles essentially is a four-bar function generator designed to align the extensions of the front and rear wheels' axles at a common point during steering, minimizing tire wear and preventing sideslip instability. Generally, the steering mechanism of a vehicle is designed so that the axles of steerable wheels and rear wheels axle intercept at one point, which is called Ackermann's condition. The vehicle steering mechanism essentially is a function generator; its output and input steering angles should satisfy the Ackermann steering geometry to prevent tire abrasion and vehicle side-slip instability. This article investigates the optimal dimensional synthesis of the Ackermann steering mechanism for a three-axle, six-wheeled vehicle as a function generation problem.

In general, there is no steering mechanism that can fully satisfy the Ackermann principle at every steering angle in the whole turning process, so it can only be designed to optimize the steering mechanism so that the overall steering errors are as small as possible. Many researchers have paid attention to finding the optimum solution for minimizing the steering error between the ideal Ackermann condition and the practical generated angle of the front outer wheel within the steering motion range. Many types of vehicle or robot steering mechanisms were proposed in the literature, such as non-circular gear steering [2,3], differential gear steering [4], pure rolling steering [5], rack-and-pinion steering mechanisms [6–8], Ackermann trapezoidal steering mechanisms [9–17], Ackermann-type steering mechanisms [18] and multi-link steering mechanisms [19–28].

Wolfe [9] proposed an analytical method for determining the dimensions of a planar Ackermann four-bar steering mechanism. Emura and Arakawa [2] proposed a new steering mechanism using noncircular gears capable of turning a carrier with a small radius. To facilitate analysis, the concept of a virtual two-wheeled vehicle was introduced. It was shown that the noncircular gear in the mechanism is an elliptical gear with a pitch curve represented by a simple equation. This mechanism is applicable to wheeled carrier robots in industrial production lines. Carcaterra et al. [4] presented a differential steering mechanism, derived from a six-bar steering linkage, with the addition of a differential mechanism. The differential mechanism, coupled with a higher pair, converts the single input from the steering wheel into two outputs, ensuring the condition for correct steering is satisfied. Simionescu et al. [6] developed a kinematic model for synthesizing the rack-and-pinion steering linkage of a four-wheel-drive light tractor and applied the Nelder-Mead Simplex optimization method to optimize the objective functions, which included the Ackermann motion criteria and motion transmissibility. De-Juan et al. [7] employed a local optimization method based on exact gradient determination, which requires an initial guess for the design variables, to synthesize four types of steering mechanisms used in road vehicles: rack-and-pinion, four-bar, six-bar, and double four-bar mechanisms. Slesongsom and Bureerat [8] proposed a multi-objective optimization approach for a rack-and-pinion steering linkage, aiming to simultaneously minimize steering error and turning radius. The optimization problem was addressed using a hybrid method that combines multi-objective population-based incremental learning with differential evolution.

Yao and Angeles [10] proposed a computational kinematics approach based on elimination procedures to synthesize a steering four-bar mechanism. The goal was to minimize the root-mean-square of the structural error within the steering motion range. The minimization problem was reformulated into two polynomial equations with two variables, and by eliminating one variable, a univariate polynomial equation was derived. The roots of this equation provided all locally optimal linkages. However, the global optimum identified from the locally optimal solutions exhibited dimensional unbalance. They addressed this drawback by employing a kinematically equivalent focal six-bar mechanism. Simionescu and Beale [11] utilized Brent's optimization algorithm to synthesize the Ackermann steering mechanism and developed parametric design charts to assist in selecting the optimal initial angles for implementing an Ackermann steering mechanism in a new vehicle for practical applications. Shariati and Norouzi [12] employed a gradient-based sequential quadratic programming (SQP) algorithm, which requires an initial estimate of variables, to optimize the objective function of a four-bar function generator and achieve five precision points within the range of motion. In the study, the distribution of precision points served as the optimization parameter. An Ackermann steering mechanism was successfully optimized. Stoicescu [13] applied two optimization methods, Simulated Annealing, and Differential Evolution, to synthesize the trailing Ackermann steering linkage and leading Ackermann steering linkage, respectively. The study considered two optimal criteria, the maximum absolute steering error and the root mean square (RMS) of the steering error. Etefagh and Javash [14] carried out an optimal synthesis of an Ackermann four-bar steering mechanism using an artificial immune system (AIS) algorithm and a genetic algorithm (GA). In their approach, the link length was selected as the optimization parameter in the first method, whereas the precision point distribution was employed in the second method. The optimal results revealed that the AIS algorithm outperformed the GA in the first method, whereas the GA produced superior results compared to the AIS algorithm in the second method. Liu et al. [15] studied the optimization of a steering trapezoid mechanism by minimizing the sum of absolute differences between the actual and theoretical rotational angles of the anterolateral steering wheel. An improved particle swarm optimization (IPSO) algorithm is introduced with over-flow exception handling to address nonlinear constraints. Performance comparisons demonstrated that IPSO outperforms traditional PSO and simulated annealing-based PSO (SA-PSO) in convergence speed and computational efficiency, though it slightly lags behind SA-PSO in calculation accuracy.

Pramanik and Thipse [16] presented the kinematic synthesis of a crossed four-bar mechanism for four-wheel automotive steering using the Hooke and Jeeves optimization method to achieve five precision points. The mechanism, incorporating two equal spur gears to coordinate outer wheel motion, achieved a maximum steering error of 0.12 degrees, making it suitable for long-wheelbase vehicles. Li et al. [17] optimized the design of a steering trapezoid mechanism for an agricultural tractor using the Powell Hybrid Genetic Algorithm (HGA), which combines the global search capability of genetic algorithms with the efficient local search ability of the Powell method. This approach enhanced the accuracy and efficiency of the tractor's steering mechanism design. Experimental verification demonstrated the reliability and feasibility of the HGA method for optimizing tractor steering mechanisms. Zhao et al. [18] proposed the synthesis of an Ackermann-type steering mechanism incorporating a five-bar mechanism coupled with incomplete noncircular gears, designed to precisely satisfy the Ackermann steering geometry. The synthesis process involved designing the pitch curves, addendum curves, dedendum curves, tooth profiles, and transition curves of the noncircular gear pair. The optimized design enhances vehicle steering performance while maintaining a structure similar to existing four-bar steering linkages.

The literature on the dimensional synthesis of three-axis, six-wheeled, or multi-axis, multi-wheeled steering mechanisms is relatively scarce. Wang and Dai [29] proposed a dual-axle steering system for a three-axle, six-wheeled vehicle with a rear non-steerable axle. The steering system comprised two symmetrical planar six-bar mechanisms integrated with a planar four-bar mechanism serving as the double-shaft linkage. The Newton-Raphson iterative method was utilized to linearize and solve the design equations, resulting in an optimized design with a maximum structural error of 0.8 degrees.

Zhou et al. [30] conducted the design and error analysis of a steering device consisting of five axles of a Watt-II center-arm six-bar mechanism for a multi-wheel heavy vehicle. Broyden's iterative algorithm was employed, with an appropriate initial value selected, to solve the linear simultaneous equations through seven precision points, ensuring compliance with the Ackermann steering principle. Additionally, ADAMS v2015 software was utilized for the kinematic simulation of the designed steering mechanism. Erdemir et al. [31] explored the optimization of a five-axle steering mechanism for heavy vehicles using a hybrid approach that combines the global search capabilities of the Bees algorithm with the local refinement efficiency of the Hooke-Jeeves algorithm. This method improved steering accuracy and performance, validating its effectiveness for complex multi-axle systems in heavy vehicles. Lima et al. [32] conducted the dimensional synthesis of steering mechanisms for a three-axle semi-trailer truck with front and rear steerable axles. The steering system, consisting of four-bar linkages, was optimized using the interior-point algorithm, which required an initial guess solution, to minimize the root-mean-square (RMS) error between the steering angles calculated from the Ackermann condition and the Freudenstein equation. The research offers design insights and practical solutions to enhance the steering performance and maneuverability of three-axle semi-trailers. Buzzi et al. [33] examined the optimal dimensional synthesis of a four-bar steering mechanism for a three-axle, six-wheeled steering system with front and rear steerable axles. The system comprised two four-bar mechanisms, one for each steerable axle, coordinated by a constant-pitch gear system. The optimization objective function consisted of two sets of quadratic polynomials involving five design parameters of the front and rear steering mechanisms, along with a weighting factor for the wheelbase. However, the paper does not specify the optimization method employed. Additionally, by simplifying the nonlinear relationship between the output and input steering angles into a linear one. For the optimally synthesized steering mechanism, the maximum steering error of the front four-bar mechanism was 6.8 degrees, while that of the rear four-bar mechanism was 15 degrees. Bu et al. [34] developed an independent steering system for a three-axle, six-wheeled electric vehicle powered by six in-wheel motors, aiming to enhance the flexibility of the vehicle on complex off-road terrains. The system integrates both mechanical and electronic control steering systems, each offering multiple steering modes. The study introduces these modes, builds a detailed CAD model using SolidWorks, and establishes a kinematic model for analysis. A segmented steering trapezoid mechanism was implemented to enhance the performance of the reverse-phase steering mode. Results show the steering system significantly improves vehicle flexibility, enabling stable and efficient maneuvering on complex terrains.

Traditional optimization methods, such as the Newton-Raphson, SQP, and Hooke-Jeeves methods, etc., require gradient calculations and an initial estimated solution, complicating the design process for complex nonlinear objective functions. Modern, well-developed, and well-popular metaheuristic optimizations, such as swarm-based algorithms: PSO, HPSO, MKH, CS, APT-FPSO, BAS, etc. [35], and evolutionary-based algorithms: GA, GA-KK, GA-CSP, DE, GA-FL, GA-DE, MUMSA, NSGAI, IOA^{s-at}, ICA-SA, DE-SRT, TS-MBFOA, SAP-TLBO, ICA-GA, TLBO, CMDE, CPF-DE, HLIDE, ADELI, DEC, ImHS, GSA, ATLBO-DA, GSEF-IAA, ODSRA+CP, REA, HCDJ, etc. [35], overcome these challenges by

eliminating the need for deriving the gradient of the objective function or providing the initial guess solution while efficiently and accurately finding the optimal results.

Kennedy et al. [36] proposed the Particle Swarm Optimization (PSO) algorithm in 1995, inspired by the predatory behavior of bird flocks. In 1997, Storn and Price [37] proposed the Differential Evolution (DE) algorithm to address the Chebyshev polynomial fitting problem. Kang et al. [35] developed the Hybrid Particle Swarm Optimization (HPSO) algorithm to enhance the convergence and diversity of improved PSO. HPSO incorporates the constriction factor (C) and inertia weight (w) from improved PSO while selecting the top 30% of particles with the best fitness values for the next generation. The remaining 70% undergo crossover and selection by Genetic Algorithms (GA). Kang et al. [35] incorporated the golden ratio values (0.618:0.382) into the DE algorithm as crossover and mutation rates. Mutation is performed first with a rate of 0.382, followed by crossover with a rate of 0.618, leading to the Differential Evolution with golden ratio (DE-gr). This approach significantly improves the accuracy and convergence efficiency of traditional DE, making DE-gr suitable for low- and medium-dimension optimization problems [35,38]. EPSDE, proposed by Mallipeddi et al. [39] in 2011, is an efficient metaheuristic optimization method known for its adaptive parameter adjustment, minimizing the need for extensive manual tuning. However, when applied to high-dimension engineering optimization problems, it may experience incomplete or slow convergence, requiring an increased population size or iteration count to achieve satisfactory results. To improve this, Kang et al. [35] introduced the concept of linear population size reduction (LPSR) into the EPSDE method, resulting in the L-EPSDE method. It eliminates poor individuals during iterations, linearly reduces population size, and enhances search efficiency and convergence speed. In [35], the performance of five metaheuristic optimization methods (PSO, HPSO, DE-gr, EPSDE, and L-EPSDE) was compared in solving the optimal dimensional synthesis problems of five representative four-bar path generators. The synthesis problems of four-bar path generators essentially are medium- or high-dimension problems. Results demonstrated that four metaheuristic methods (HPSO, DE-gr, EPSDE, and L-EPSDE) efficiently converged to the global optimal solution with minimal iterations. L-EPSDE was verified as particularly effective for high-dimension optimization problems.

Due to the limited literature addressing the optimal dimensional synthesis of steering mechanisms for three-axle, six-wheeled vehicles, this study focuses on optimizing Ackermann steering mechanisms for two steering modes in a three-axle, six-wheeled vehicle: front-axle steering (front axle steerable) and reverse-phase steering (central axle non-steerable). A comparison of turning radii between vehicles with one and two steerable axles reveals that two steerable axles significantly reduce the turning radius and enhance maneuverability. Due to space limitations, the steering mode involving the first two steerable axles will be addressed in a separate study.

Four metaheuristic optimization methods known for their convergence efficiency and ease of implementation, involving two swarm-based algorithms: Particle Swarm Optimization (PSO) and Hybrid Particle Swarm Optimization (HPSO), along with two evolutionary-based algorithms: Differential Evolution with golden ratio (DE-gr) and Linearly Ensemble of Parameters and Mutation Strategies in Differential Evolution (L-EPSDE), are employed to compare their effectiveness and efficiency in solving the optimal dimensional synthesis of Ackermann trapezoidal steering mechanisms for such vehicles. The details of the algorithms and flowcharts of the employed four metaheuristic optimization methods can be referred to in the literature [35].

Two hundred optimization numerical experiments were conducted using the four metaheuristic optimization methods. Statistical analysis reveals that evolutionary-based algorithms (DE-gr and L-EPSDE) outperform swarm-based ones (PSO and HPSO) in accu-

racy and efficiency. Additionally, intermediate-axle placement was considered a weighted design parameter in the optimal synthesis of the reverse-phase steering mechanism, allowing for more flexible design outcomes. The results of this study validate that the evolutionary-based optimization methods are effective and efficient approaches for synthesizing Ackermann steering mechanisms in three-axle, six-wheeled vehicles.

2. Position Analysis of Ackermann Steering Mechanism

2.1. The Ackermann Principle of Vehicle Turning

When a conventional front-wheel drive, two-axle, four-wheeled vehicle turns, the extensions of the axles of the front and rear wheels must intersect at a single point, the instantaneous center of turning, so that the vehicle can turn smoothly with minimum sideslip and to reduce tire wear, which is known as the Ackermann principle of vehicle steering. The Ackermann steering mechanism is a symmetric four-bar linkage with input and output links of equal length, capable of accurately satisfying the Ackermann principle at some specific angular positions (up to five positions). Figures 1 and 2 illustrate the phases of the Ackermann steering mechanism when the vehicle is traveling straight and turning left, respectively. The steering angles of the front wheels must meet the following relationship for proper functioning.

$$\cot\delta_{out} - \cot\delta_{in} = \frac{W}{L} \quad (1)$$

where W is the distance between the two front wheels (called wheel track), L is the distance between the front and rear wheel axles (called wheelbase), δ_{in} and δ_{out} are the steering angles of the front inner and outer wheels, respectively. The point C in Figure 2 is the vehicle's gravity center, a is the distance between the rear wheel axle and the center of gravity of the vehicle, and R_o is the perpendicular distance from the turning center O to the vehicle's central longitudinal axis. And R_c is the instantaneous turning radius of the gravity center of the vehicle, which can be obtained by the following formula:

$$R_c = \sqrt{a^2 + L^2 \cot^2 \delta} \quad (2)$$

In which

$$\cot\delta = \frac{\cot\delta_{in} + \cot\delta_{out}}{2} \quad (3)$$

$$\cot\delta_{in} = \frac{R_o - \frac{W}{2}}{L}, \cot\delta_{out} = \frac{R_o + \frac{W}{2}}{L} \quad (4)$$

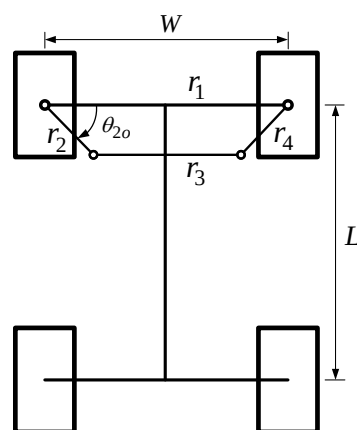


Figure 1. Trapezoidal steering mechanism when the vehicle is traveling straight.

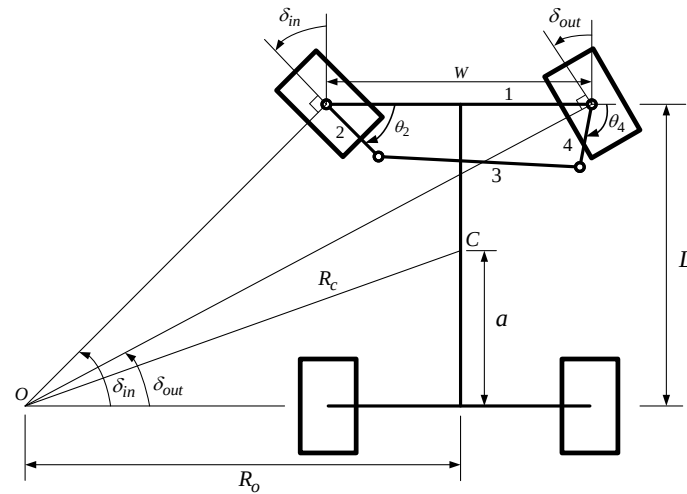


Figure 2. Trapezoidal steering mechanism when the vehicle is turning left.

2.2. Planar Four-Bar Steering Mechanism

The Ackermann steering mechanism or trapezoidal steering mechanism, as shown in Figure 3, is a symmetric four-bar linkage with equal lengths of the input link and the output link, $r_2 = r_4$, while the length of the connecting link r_3 is smaller or larger than the length of the fixed link r_1 . If r_3 smaller than r_1 , it is arranged as a trailing Ackermann steering mechanism, otherwise, if r_3 larger than r_1 , it is arranged as a leading Ackermann steering mechanism [7,13].

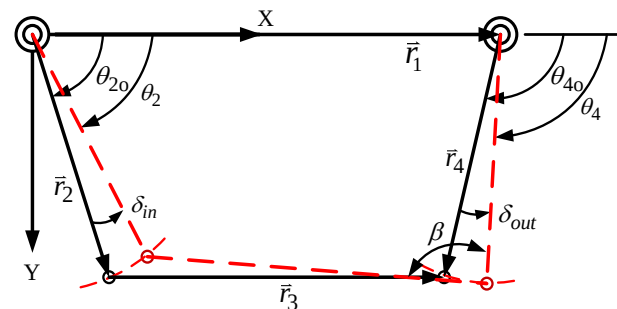


Figure 3. Ackerman trapezoidal steering mechanism.

In Figure 3, solid lines represent the trapezoidal steering mechanism during straight travel, while red dotted lines show it during a left turn. The vector loop equation for the four-bar mechanism is as follows:

$$\vec{r}_3 = \vec{r}_1 + \vec{r}_4 - \vec{r}_2 \quad (5)$$

Take the inner product itself on each side of Equation (5), as follows:

$$\vec{r}_3 \cdot \vec{r}_3 = (\vec{r}_1 + \vec{r}_4 - \vec{r}_2) \cdot (\vec{r}_1 + \vec{r}_4 - \vec{r}_2) \quad (6)$$

Expanding the inner product as follows:

$$(2r_1r_4\cos\theta_1 - 2r_2r_4\cos\theta_2)\cos\theta_4 + (2r_1r_4\sin\theta_1 - 2r_2r_4\sin\theta_2)\sin\theta_4 = r_3^2 - r_1^2 - r_2^2 - r_4^2 + 2r_1r_2\cos(\theta_2 - \theta_1) \quad (7)$$

Which can be briefly expressed as follows:

$$A\cos\theta_4 + B\sin\theta_4 = C \quad (8)$$

Since $\theta_1 = 0^\circ$, the coefficients can be expressed as follows:

$$\begin{aligned} A &= 2r_4(r_1 - r_2 \cos \theta_2) \\ B &= -2r_2 r_4 \sin \theta_2 \\ C &= r_3^2 - r_1^2 - r_2^2 - r_4^2 + 2r_1 r_2 \cos \theta_2 \end{aligned} \quad (9)$$

From the half-angle formula of the trigonometric function, the angular position of the output link of the trapezoidal steering mechanism can be expressed as follows:

$$\theta_4 = 2 \tan^{-1} \left(\frac{B \pm \sqrt{A^2 + B^2 - C^2}}{A + C} \right) \quad (10)$$

According to the assembly of the steering mechanism, the symbol before the radical sign in Equation (10) is taken positive sign. The angular position of the coupler link is as follows:

$$\theta_3 = \tan^{-1} \left(\frac{r_4 \sin \theta_4 - r_2 \sin \theta_2}{r_1 + r_4 \cos \theta_4 - r_2 \cos \theta_2} \right) \quad (11)$$

where

$$\theta_2 = \theta_{20} - \delta_{in} \quad (12)$$

Since $\theta_{40} = \pi - \theta_{20}$, the steering angle of the outer wheel δ_{out} can be expressed as follows:

$$\delta_{out} = \theta_{40} - \theta_4 = \pi - \theta_{20} - \theta_4 \quad (13)$$

In Equations (12) and (13), θ_2 , θ_4 are the angular position of the input and output link, respectively, and θ_{20} , θ_{40} are the initial input and output angle, respectively, when the vehicle travels straight (as shown in Figure 1).

As shown in Figure 3, the transmission angle, β , of the steering mechanism can be expressed as follows:

$$\beta = \cos^{-1} \left(\frac{r_3^2 + r_4^2 - r_1^2 - r_2^2 + 2r_1 r_2 \cos \theta_2}{2r_3 r_4} \right) \quad (14)$$

The transmission angle of a four-bar linkage is a transmission index for transmission efficiency. If the deviation of the transmission angle from 90 degrees, $|\beta - 90|$, is lower than 50 degrees then the transmission efficiency is good and satisfactory [40].

3. Steering Geometry of a Three-Axle, Six-Wheeled Vehicle

3.1. Steering of a Three-Axle, Six-Wheeled Vehicle

This paper aims to optimize the design of the Ackermann trapezoidal mechanism for a three-axle, six-wheeled vehicle, only considering two steering configurations: a front-axle (or called front two-wheel) steering mode (only front steerable axle) and a reverse-phase steering mode (front and rear steerable axles with middle non-steerable axle). The study compares these optimized steering mechanisms of different steering modes in terms of turning radius and steering error.

3.1.1. Front-Axle Steering Mode

In the six-wheeled front-axle steering system, only the front wheels need to be steering controlled, making the steering system relatively simple. As shown in Figure 4, the intermediate axle lies in the middle between axle 1 and axle 3. The instantaneous turning center (O) lies in the middle of axle 2 and axle 3. Here, L_o is the vertical distance from axle 1 to point O, and a_1 is the vertical distance from the vehicle's center of gravity C to point O.

This steering mode closely resembles the Ackermann steering system used in a front-wheel, two-axle, four-wheeled vehicle.

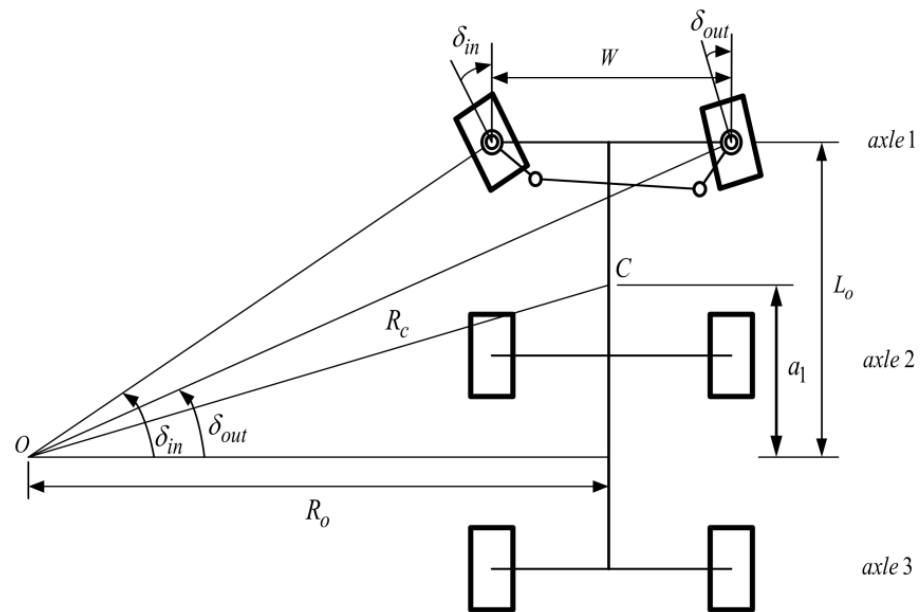


Figure 4. Front-axle steering system (left turn).

During the steering process, the turning radius of the vehicle at the center of gravity C is as follows:

$$R_c = \sqrt{a_1^2 + L_o^2 \cot^2 \delta} \quad (15)$$

where

$$\cot \delta = \frac{\cot \delta_{in} + \cot \delta_{out}}{2} \quad (16)$$

$$\cot \delta_{in} = \frac{R_o - \frac{W}{2}}{L_o}, \quad \cot \delta_{out} = \frac{R_o + \frac{W}{2}}{L_o} \quad (17)$$

3.1.2. Reverse-Phase Steering Mode

The six-wheeled reverse-phase steering system, as shown in Figures 5 and 6, has only the first (front) and third (rear) axles as steerable axles. The steering mechanisms on the front and rear axles steer the wheels, and the trapezoidal steering mechanism is arranged as shown and well-designed, which ensures the instantaneous turning center lies on the extension line of the middle axle. The two sets of steering mechanisms on the front and the rear shafts can be connected by gear or hydraulic system. Figure 5 illustrates the decomposition of the ideal Ackermann steering geometry for a left turn. In the figure, L is the distance between the front and rear axles, L_1 is the distance between the front and intermediate axles, L_3 is the distance between the intermediate and rear axles, W_f is the width between the front tires, and W_r is the width between the rear tires, δ_{in_f} and δ_{out_f} are the steering angles of the front inner and outer wheels. While δ_{in_r} and δ_{out_r} are the steering angles of the rear inner and outer wheels. Distance a_1 and a_3 are the perpendicular distance from the center of gravity of the front and rear parts of the vehicle, C_1 and C_3 , to the instantaneous turning center O . The steering angles of the outer wheel and inner wheel of the front and rear four-bar steering mechanisms should satisfy the Ackermann geometrical relations, respectively.

$$\cot \delta_{outA_f} - \cot \delta_{in_f} = \frac{W_f}{L_1} \quad (18)$$

$$\cot \delta_{outA_r} - \cot \delta_{in_r} = \frac{W_r}{L - L_1} \quad (19)$$

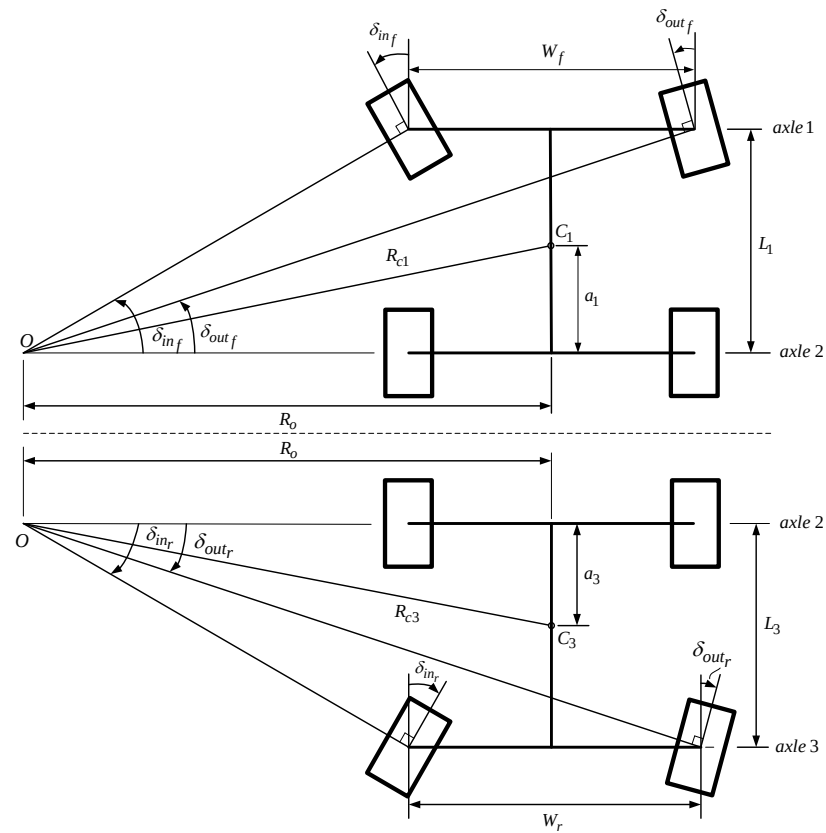


Figure 5. Ideal steering geometry of a three-axle vehicle (exploded view of left turning).

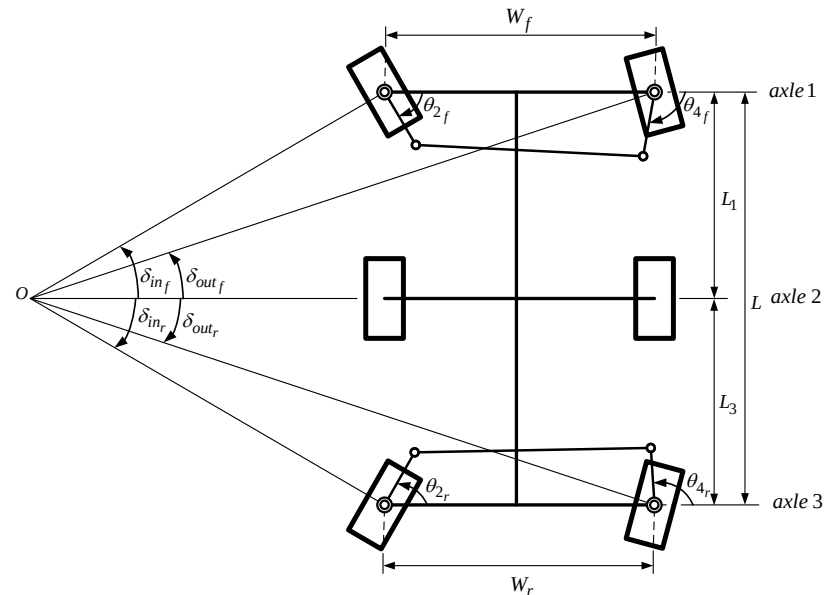


Figure 6. Reverse-phase steering system of three-axle, six-wheeled (left turning).

Here, $\delta_{outA_f}(i)$ and $\delta_{outA_r}(i)$ mean the ideal Ackermann steering angles of the outer wheels of the front and rear axles.

From the triangular relationship of Ackermann steering geometry in Figure 5, the steering angles of the inner wheels of rear and front axles, δ_{in_r} and δ_{in_f} , should further satisfy the following relationship.

$$\tan \delta_{in_r} = -K \tan \delta_{in_f} \quad (20)$$

where

$$K = \frac{L - L_1}{L_1} = \frac{L_3}{L_1} \quad (21)$$

Here, K is called the transmission ratio of front to rear axle. Equations (18) and (19) are the primary Ackermann geometry equations, and Equation (20) can be thought of as the dominant Ackermann geometry equation for a three-axle, six-wheeled vehicle with reverse-phase steering mode. Equation (20) can be substituted into Equation (19), and obtain the following equation:

$$\cot \delta_{outA\ r} + \frac{\cot \delta_{in_f}}{K} = \frac{W_r}{L_3} \quad (22)$$

It indicates that the ideal Ackermann steering angle of the outer wheel of the rear steering mechanism, $\delta_{outA\ r}$, is mainly affected by the parameter K .

From Figure 5. The instantaneous turning centers of the front and rear parts of the vehicle, R_{c1} and R_{c3} , can be expressed as follows:

$$R_{c1} = \sqrt{a_1^2 + L_1^2 \cot^2 \delta_f} \quad (23)$$

$$\cot \delta_f = \frac{\cot \delta_{in_f} + \cot \delta_{out_f}}{2} \quad (24)$$

$$\cot \delta_{in_f} = \frac{R_o - \frac{w_f}{2}}{L_1}, \quad \cot \delta_{out_f} = \frac{R_o + \frac{w_f}{2}}{L_1} \quad (25)$$

and

$$R_{c3} = \sqrt{a_3^2 + L_3^2 \cot^2 \delta_r} \quad (26)$$

$$\cot \delta_r = \frac{\cot \delta_{in_r} + \cot \delta_{out_r}}{2} \quad (27)$$

$$\cot \delta_{in_r} = \frac{R_o - \frac{w_r}{2}}{L - L_1}, \quad \cot \delta_{out_r} = \frac{R_o + \frac{w_r}{2}}{L - L_1} \quad (28)$$

When the intermediate axle lies in the middle position of the front and rear axles, then $L_1 = L_3$, $K = 1$. If $a_1 = a_3$ and $W_f = W_r$, then $R_{c1} = R_{c3}$.

If K is taken into account as a design parameter for optimization of the steering mechanism, then $K = 1$ can be considered as a special case.

As shown in Figure 6 of the three-axle, six-wheeled, reverse-phase steering system, the four-bar steering mechanisms on axle 1 (front axle) and axle 3 (rear axle) are both Ackermann steering mechanisms. When the vehicle turns left, the movement of the front and rear Ackermann steering mechanisms, is shown by red dash line in Figures 7 and 8, respectively. Among them, the input link lengths of the front and rear steering mechanisms are r_{2f} , r_{2r} , and the output link lengths are r_{4f} , r_{4r} , and $r_{4f} = r_{2f}$, $r_{4r} = r_{2r}$. And θ_{2of} , θ_{2or} are the initial angles of the input links of the front and rear steering mechanisms when the vehicle is traveling straight. The relationship of the lengths of coupler links r_{3f} , r_{3r} , the

angular position of the input links, and the steering angles of the front and rear steering mechanisms, θ_{2f} , θ_{2r} , are expressed as follows, respectively.

$$r_{3f} = W_f - 2r_2 f \cos \theta_{2of} \quad (29)$$

$$r_{3r} = W_r - 2r_{2r} \cos \theta_{2or} \quad (30)$$

$$\theta_{2f} = \theta_{2of} - \delta_{inf} \quad (31)$$

$$\theta_{2r} = \theta_{2or} + \delta_{inr} \quad (32)$$

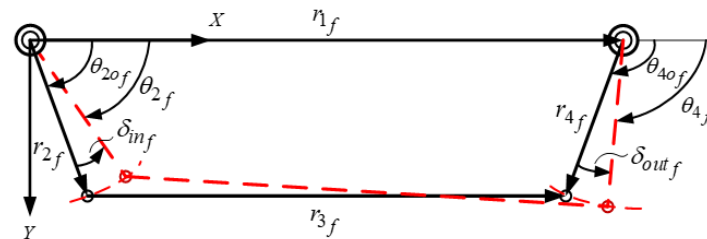


Figure 7. Ackermann steering mechanism on the front axle.

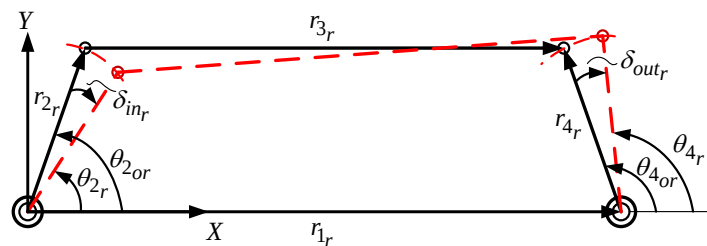


Figure 8. Ackermann steering mechanism on the rear axle.

The generated output steering angles of the synthesized front and rear trapezoidal steering mechanisms, δ_{outf} , δ_{outr} , are expressed as follows:

$$\delta_{outf} = \pi - \theta_{2of} - \theta_{4f} \quad (33)$$

$$\delta_{outr} = \theta_{4r} - \pi + \theta_{2or} \quad (34)$$

3.2. Turning Radii of the Three-Axle, Six-Wheeled Vehicle

Here, we take a comparison between the turning radii of the front-axle steering mode and the reverse-phase steering mode ($K = 1$) for a three-axle, six-wheeled vehicle. The data of the vehicle is as follows: the wheel tracks of the front and rear steering mechanisms are $W_f = 2180$ mm and $W_r = 2170$ mm, the wheelbase of the vehicle is $L = 4300$ mm, the steering range of the front inner wheel is $-30^\circ \leq \delta_{in} \leq 40^\circ$. Based on Equations (2), (23) and (26), the corresponding turning radii for the two steering modes are shown in the second to fourth columns of Table 1. Clearly, the reverse-phase steering mode ($K = 1$) achieves a smaller turning radius than the front-axle steering mode. A smaller turning radius improves maneuverability and efficiency, especially in tight spaces. With identical vehicle dimensions (same wheel track and wheelbase), the reverse-phase steering mode offers greater design flexibility, enabling a smaller turning radius and reduced steering error (for steering error of the front-axle steering mode and two cases of reverse-phase steering mode, $K = 1$ and $K \neq 1$, would be studied in Section 4.3), which are key advantages over the front-axle steering mode.

Table 1. Turning radii for two steering modes (unit: m).

Front Steering Angle (°)	Steering System				
	Front-Axle Steering Mode	Reverse-Phase ($K = 1$) Steering Mode		Reverse-Phase ($K \neq 1$) Steering Mode	
	R_c	R_{c1}	R_{c3}	R_{c1}	R_{c3}
40	5.046	3.568	1.896	5.199	2.947
30	6.892	4.575	3.365	6.730	4.802
20	9.955	6.939	5.564	9.841	8.114
10	19.209	12.873	11.830	19.149	17.554
−10	17.619	11.641	12.650	17.712	19.050
−20	8.246	5.335	6.434	8.139	9.638
−30	4.846	2.780	4.339	4.664	6.931

4. Optimization Synthesis of the Four-Bar Steering Mechanism of a Three-Axle, Six-Wheeled Vehicle

This study employs four metaheuristic optimizations, PSO, HPSO, DE-gr, and L-EPSDE, to solve the optimal dimensional synthesis problems of a three-axle, six-wheeled steering mechanism with a central non-steerable axle or only a front steerable axle.

4.1. The Objective Function and Constrictions

The optimization objective function for the design of the four-bar steering mechanism in the front-axle steering mode is defined as the mean of the squared steering errors at all selected positions during the steering motion, combined with a penalty function for constraint violations, as follows:

$$\text{Min } f(X) = \left\{ \sum_{i=1}^N \frac{[\delta_{out f(i)} - \delta_{outA f(i)}]^2}{N} \right\} + Mh_1 \quad (35)$$

In the reverse-phase steering mode, the optimization objective function is defined as the mean of the squared steering errors at all selected positions during the steering motion for four-bar steering mechanisms of the front and rear axles, combined with a penalty function for constraint violations, as follows:

$$\text{Min } f(X) = \sum_{i=1}^N \left(w_1 \times \frac{[\delta_{out f(i)} - \delta_{outA f(i)}]^2}{N} + w_3 \times \frac{[\delta_{out r(i)} - \delta_{outA r(i)}]^2}{N} \right) + Mh_1 + Mh_2 \quad (36)$$

In Equations (35) and (36), X is the vector of design parameters, N is the number of selected steering angular positions of the inner front wheel, w_1 and w_3 are the weighting factor on the placement of the intermediate axle, which can be expressed as a function of the transmission ratio of front to rear axle, K , as follows:

$$w_1 = \frac{L_1}{L} = \frac{1}{K+1}, w_3 = \frac{L_3}{L} = \frac{K}{K+1} \quad (37)$$

$\delta_{out f(i)}$ and $\delta_{out r(i)}$ are the practically generated steering angles of the outer wheels of the synthesized front and rear four-bar steering mechanisms, evaluated by Equations (33) and (34) and $\delta_{outA f(i)}$ and $\delta_{outA r(i)}$ are the ideal Ackermann steering angles of the outer wheels of the front and rear axles, evaluated by Equations (18)–(20). And $\delta_{out f(i)} - \delta_{outA f(i)}$, $\delta_{out r(i)} - \delta_{outA r(i)}$ are the steering errors of the outer wheels of the front and rear axles when the steering mechanisms move to the i -th steering angular

position. That is the structural error of the function generation mechanism. Mh_1 and Mh_2 are the penalty functions for violating the constraints, and the value of M is usually set to 10^6 , when the constraints are not met, h is set to 1; otherwise, it is set to 0.

The Ackermann four-bar steering mechanisms of the front and rear axles must simultaneously satisfy the constraint of a triple rocker mechanism of Grashof's criteria [41], the constraint inequalities of the link lengths are expressed as follows:

$$h_1 : 2 \left[\text{Max}(r_{1f}, r_{2f}, r_{3f}, r_{4f}) + \text{Min}(r_{1f}, r_{2f}, r_{3f}, r_{4f}) \right] > \text{Sum}(r_{1f}, r_{2f}, r_{3f}, r_{4f}) \quad (38)$$

$$h_2 : 2 \left[\text{Max}(r_{1r}, r_{2r}, r_{3r}, r_{4r}) + \text{Min}(r_{1r}, r_{2r}, r_{3r}, r_{4r}) \right] > \text{Sum}(r_{1r}, r_{2r}, r_{3r}, r_{4r}) \quad (39)$$

4.2. Design Variable

The design of the four-bar steering mechanism for the front-axle steering mode is relatively simple, with only two design variables, the design vector X is expressed as:

$$X = [x_1, x_2] = [r_{2f}, \theta_{2of}] \quad (40)$$

For the design of the four-bar steering mechanism in reverse-phase steering mode: There are two cases to be discussed.

- (i) If the intermediate axle lies in the middle of the front and rear axles, there are four design variables, the design vector X is represented as follows:

$$X = [x_1, x_2, x_3, x_4] = [r_{2f}, \theta_{2of}, r_{2r}, \theta_{2or}] \quad (41)$$

- (ii) If the transmission ratio of front to rear axle (K) is treated as a design variable, there are five design variables, the design vector X is expressed as follows:

$$X = [x_1, x_2, x_3, x_4, x_5] = [r_{2f}, \theta_{2of}, r_{2r}, \theta_{2or}, K] \quad (42)$$

4.3. Optimal Results of the Four-Bar Steering Mechanism

This paper develops MATLAB programs for four metaheuristic optimization methods—PSO, HPSO, DE-gr, and L-EPSDE—to optimize the dimensions and kinematic analysis of the four-bar steering mechanism. For the front-axle steering mode, the objective function of optimization is Equation (35), and Equation (38) is the constraint inequality. For the reverse-phase steering mode, the objective function of optimization is Equation (36), and Equations (38) and (39) are the constraint inequalities. The parameter settings for each optimization method are shown in Table 2, with a population size of 100 and a maximum iteration count of 1000.

Refer to Buzzi et al. [33] for the dimensions of a three-axle, six-wheeled, four-bar steering mechanism. The wheel tracks of front and rear steering mechanisms are $W_f = 2180$ mm and $W_r = 2170$ mm, the wheelbase between the front and rear axles is $L = 4300$ mm, the range of steering angle of a front inner wheel is $-30^\circ \leq \delta_{inf} \leq 40^\circ$. If $K = 1$, the corresponding range of steering angle of the rear inner wheel is $-40^\circ \leq \delta_{inr} \leq 30^\circ$. In this paper, the angular positions of optimal design are taken at every steering angle with equal spacing, so $N = 71$. The range of the design parameters of the steering mechanism is shown in Table 3. In the following, the optimal design of three steering mechanisms for two steering modes of a three-axle, six-wheeled, four-bar steering mechanism is discussed.

Table 2. The parameter setting of the four optimization methods.

Optimization Method	PSO	HPSO	DE-gr	L-EPSDE
Size of population (NP)	100	100	100	100
Minimum NP (NP_{min})	--	--	--	5
Crossover method	--	Single-point/ uniform	Multiple-point	Multiple-point
Crossover rate (CR)	--	--	0.618	0.1~0.9
Mutation method	--	--	DE/best/1	DE/best/2 DE/rand/1 DE/current-to-rand/1
Mutate rate	--	--	0.382	0.4~0.9
Selection method	--	--	Competition rule	Competition rule
Learning coefficients (C_1, C_2)	2.05	2.05	--	--
Inertia weight (ω)	0~1	0~1	--	--
Iteration number			1000	
Experimental times			200	

Table 3. Design range of four-bar steering mechanisms.

Parameters	r_{2f} (mm)	θ_{2of} (°)	r_{2r} (mm)	θ_{2or} (°)	K
Range of $K = 1$	200~300	60~120	200~300	60~120	1
Range of $K \neq 1$	200~300	60~90	200~300	60~150	0.3~0.9

4.3.1. Optimal Design of the Front-Axle, Four-Bar Steering Mechanism

In this steering mode, design parameters are r_{2f} , θ_{2of} , assume $L_1 = L_3$, $a_1 = a_3 = L_1/2$. Table 4 summarizes the statistical results of two hundred optimization experiments for the front-axle (or called front two-wheel) four-bar steering mechanism using four optimization methods. The red-marked values in each row represent the minimum values among the statistical results of the four methods. The DE-gr method achieves the smallest standard deviation (SD) (1.98×10^{-6}), mini-worst value (1.70×10^{-4}), and the second-best error value (1.51×10^{-4}). PSO method also achieves the smallest SD but slightly large mean value (1.58×10^{-4}) and max-best value (1.56×10^{-4}). The HPSO method has a slightly large SD (4.29×10^{-6}) and is superior to the PSO method in terms of mean value, and best value. While the L-EPSDE method delivers the smallest best value (1.47×10^{-4}) and mean value (1.49×10^{-4}) but a slightly large SD (3.61×10^{-6}).

Table 5 presents the optimal objective function values, dimensions, and maximum steering errors for the front-axle four-bar steering mechanisms synthesized from two hundred optimization experiments using four optimization methods. The L-EPSDE method achieves the smallest optimal objective function value (1.472×10^{-4}) and maximum steering error (8.9699°), and the HPSO method has the second smallest maximum steering error (8.9871°), while even the DE-gr method has the minimum SD (1.98×10^{-6}) and the second-

best optimal objective function value (1.511×10^{-4}), the maximum steering error is the largest (9.1156°).

Table 4. Statistical results of the four optimization methods for the front-axle mode (unit: rad^2).

Method	Best Error	Worst Error	Median	Mean	SD
PSO	1.56×10^{-4}	1.77×10^{-4}	1.58×10^{-4}	1.58×10^{-4}	1.98×10^{-6}
HPSO	1.52×10^{-4}	1.90×10^{-4}	1.52×10^{-4}	1.53×10^{-4}	4.29×10^{-6}
DE-gr	1.51×10^{-4}	1.70×10^{-4}	1.51×10^{-4}	1.51×10^{-4}	1.98×10^{-6}
L-EPSDE	1.47×10^{-4}	1.83×10^{-4}	1.49×10^{-4}	1.49×10^{-4}	3.61×10^{-6}

Table 5. Optimized front-axle four-bar steering mechanisms using four optimization methods.

Method	PSO	HPSO	DE-gr	L-EPSDE
Iteration number	1000	1000	1000	1000
Objective fun. (rad^2)	0.0001562	0.0001522	0.0001511	0.0001472
r_{2f} (mm)	284.8159	251.1985	238.8548	218.0000
θ_{2of} ($^\circ$)	66.8241	66.4574	66.4345	66.0907
Max. steering error ($^\circ$)	9.0083	8.9871	9.1156	8.9699

Figure 9 illustrates the convergence speed of the four optimization methods, with L-EPSDE showing the fastest convergence, followed by DE-gr. From Table 4 and Figure 9, overall, L-EPSDE and DE-gr methods outperform PSO and HPSO methods in accuracy, precision, and convergence speed for the front-axle steering mode. The steering angle relation, the turning radius, and the steering error of the optimized front-axle four-bar steering mechanism are shown in Figure 10, Figure 11, and Figure 12, respectively. For comparison, the part data of the turning radius (Figure 11) at the center of gravity of the vehicle are shown in the second column of Table 1. In Figures 10–12, the curves of the four methods almost overlap and are difficult to distinguish due to the similarity of their optimal results. The two enlarged figures on the right side of Figure 12 show the difference between the four methods at a small range around the maximum steering error and the end of the curves.

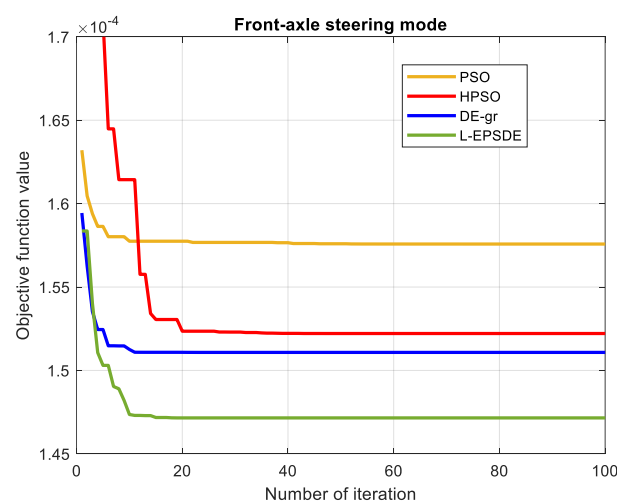


Figure 9. Convergence speed of the four optimization methods (Front-axle steering mode).

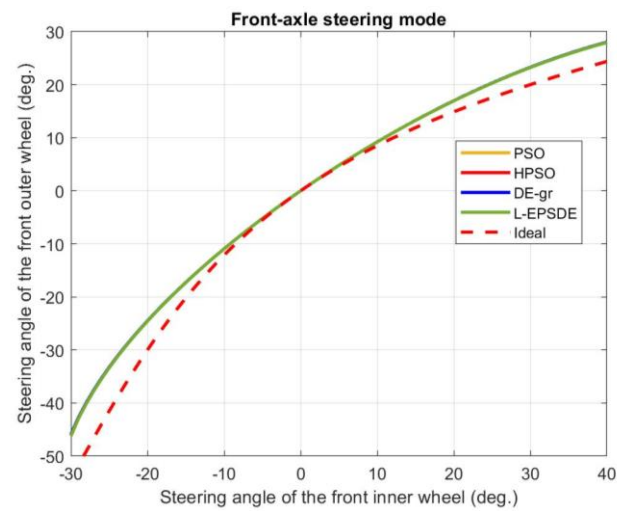


Figure 10. Steering angle relation of optimized four-bar steering mechanisms (Front-axle steering mode).

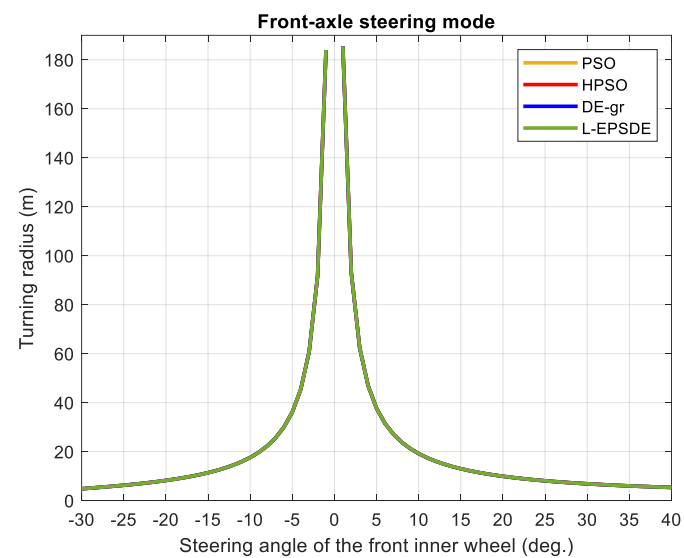


Figure 11. Turning radius of optimized four-bar steering mechanisms (Front-axle steering mode).

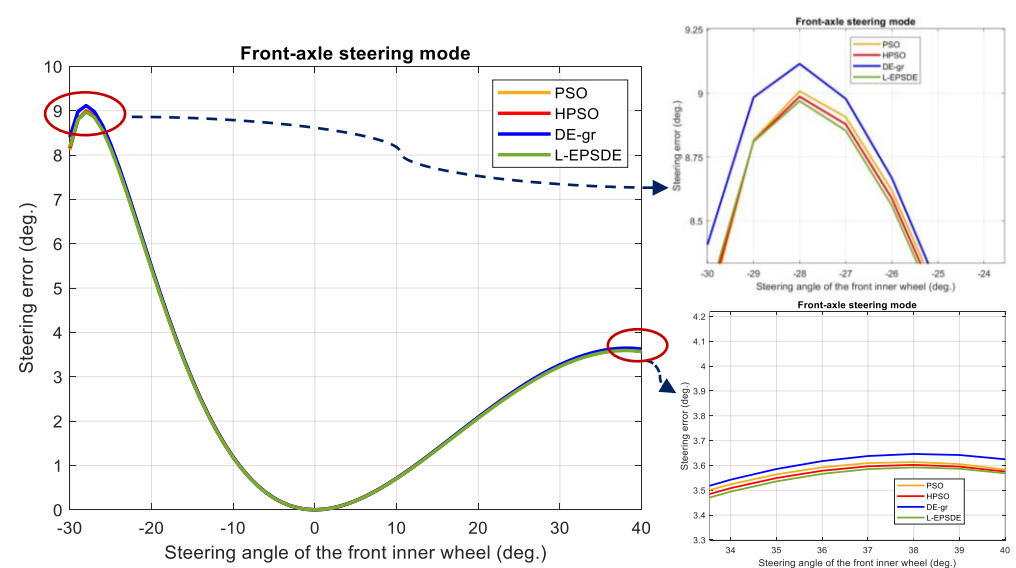


Figure 12. Steering error of optimized four-bar steering mechanisms (Front-axle steering mode).

4.3.2. Optimal Design of the Reverse-Phase ($K = 1$) Four-Bar Steering Mechanism

In reverse-phase ($K = 1$) steering mode, with the front and rear steerable axles, and a non-steerable middle axle. The design parameters of this steering mode are θ_{20f} , r_{2f} and θ_{20r} , r_{2r} . Table 6 summarizes the statistical results of two hundred optimization experiments for the reverse-phase ($K = 1$) four-bar steering mechanism using four optimization methods. L-EPSDE achieves the smallest standard deviation (2.53×10^{-4}), worst error value (1.51×10^{-2}), mean value (1.022×10^{-2}), and minimum error (9.9997×10^{-3}). DE-gr and PSO also achieve the same minimum error, with DE-gr ranking second in mean value and standard deviation, and PSO has a maximum mean value (1.062×10^{-2}) and standard deviation (3.85×10^{-3}). Figure 13 illustrates the convergence speed of the four optimization methods. The DE-gr method is the fastest to converge to the optimal solution, followed by L-EPSDE, while HPSO and PSO converge slightly slowly but still achieve the optimal solution within 200 iterations. Overall, DE-gr and L-EPSDE exhibit the fastest convergence, highest accuracy, and precision, while HPSO and PSO perform slightly worse.

Table 6. Statistical results of the four optimization methods for the reverse-phase steering mode ($K = 1$) (unit: rad^2).

Method	Best Error	Worst Error	Median	Mean	SD
PSO	9.997×10^{-3}	5.66×10^{-2}	1.007×10^{-2}	1.062×10^{-2}	3.85×10^{-3}
HPSO	1.000×10^{-2}	8.22×10^{-2}	1.052×10^{-2}	1.035×10^{-2}	3.09×10^{-4}
DE-gr	9.997×10^{-3}	1.52×10^{-2}	1.002×10^{-2}	1.025×10^{-2}	2.57×10^{-4}
L-EPSDE	9.997×10^{-3}	1.51×10^{-2}	1.026×10^{-2}	1.022×10^{-2}	2.53×10^{-4}

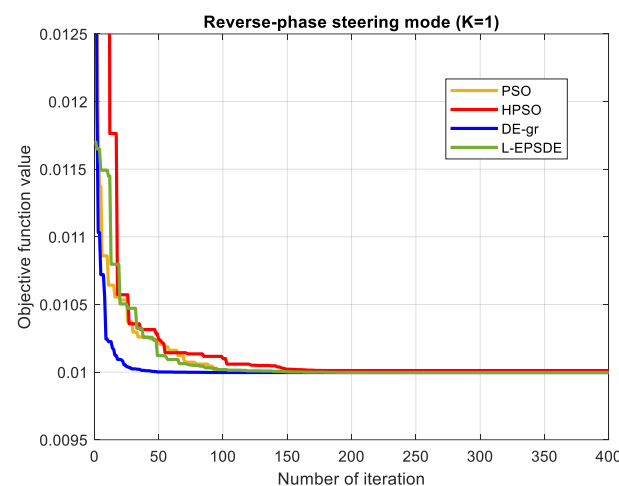


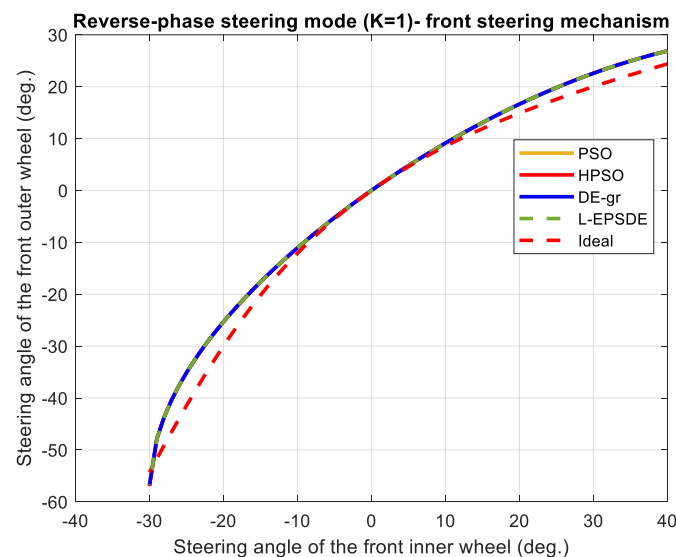
Figure 13. Convergence speed of the four optimization methods (Reverse-phase, $K = 1$).

Table 7 shows the optimal objective function values, dimensions, and maximum steering errors for the reverse-phase ($K = 1$) four-bar steering mechanisms synthesized from two hundred optimization experiments using four optimization methods. Among them, Δ_f and Δ_r mean maximum steering errors of the front and rear steering mechanisms. It is observed that the dimensions of the optimized four-bar steering mechanisms are different between the front and rear sets, the maximum steering error of the front axle is the smallest (6.3850°) by the DE-gr, L-EPSDE, and PSO methods, and the largest (6.4167°) by the HPSO method. The maximum steering error of the rear axle is also minimized by the DE-gr, L-EPSDE, and PSO methods (17.4121°) and maximized by the HPSO method (17.4145°).

Table 7. Optimized four-bar steering mechanisms using four optimization methods for reverse-phase mode ($K = 1$).

	Lima et al. [32]	PSO	HPSO	DE-gr	L-EPSDE
Iteration number	--	1000	1000	1000	1000
Objective fun. (rad. ²)	0.07076	0.009997	0.010012	0.009997	0.009997
W_f (mm)	2450	2180	2180	2180	2180
W_r (mm)	2450	2170	2170	2170	2170
L (mm)	5000	4300	4300	4300	4300
r_{2f} (mm)	124.40	218.0000	227.9441	218.0000	218.0000
r_{3f} (mm)	1971.1	1986.9669	1979.2280	1986.9676	1986.9675
r_{2r} (mm)	100.00	300.0000	299.5763	300.0000	300.0000
r_{3r} (mm)	2025.5	2399.1092	2398.7244	2399.1092	2399.1092
θ_{2of} (°)	57.0833	63.7214	63.8708	63.7215	63.7215
θ_{2or} (°)	50.5726	112.4482	112.4419	112.4482	112.4482
Δ_f (°)	--	6.3850	6.4167	6.3850	6.3850
Δ_r (°)	--	17.4121	17.4145	17.4121	17.4121

Figures 14 and 15 show the relationship of steering angle and steering error curves, respectively, of each optimized steering mechanism of the front axle, respectively, the maximum steering error is about 6.4 degrees. Figure 16 shows the curve of the transmission angle of each optimized four-bar steering mechanism of the front axle, and the transmission efficiency within the steering angle range $[-30^\circ, -20^\circ]$ is not satisfactory. Figure 17 shows the curve of the turning radius at the center of gravity of the front part of the vehicle. In Figures 14–17, the curves of the four methods almost overlap and are difficult to distinguish due to the similarity of their optimal results.

**Figure 14.** Steering angle relation of the optimized front four-bar steering mechanisms (Reverse-phase, $K = 1$).

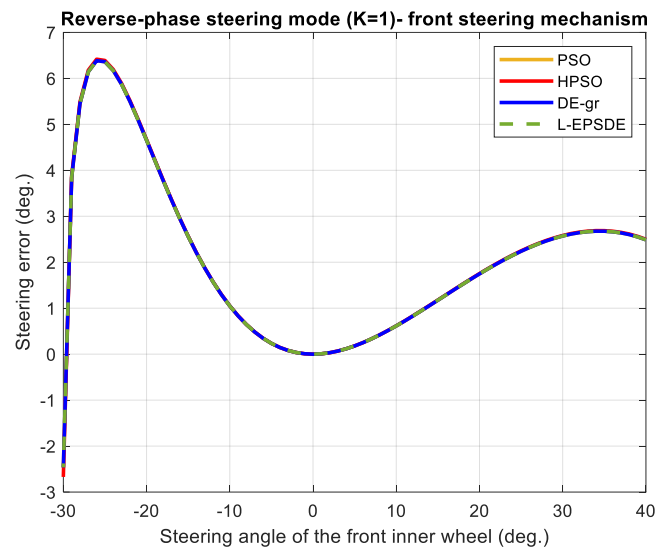


Figure 15. Steering error of the optimized front four-bar steering mechanisms (Reverse-phase, $K = 1$).

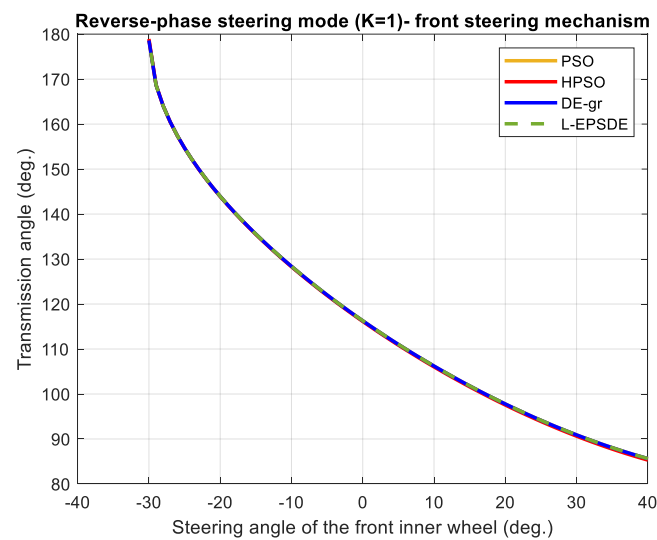


Figure 16. Transmission angle of the optimized front four-bar steering mechanisms (Reverse-phase, $K = 1$).

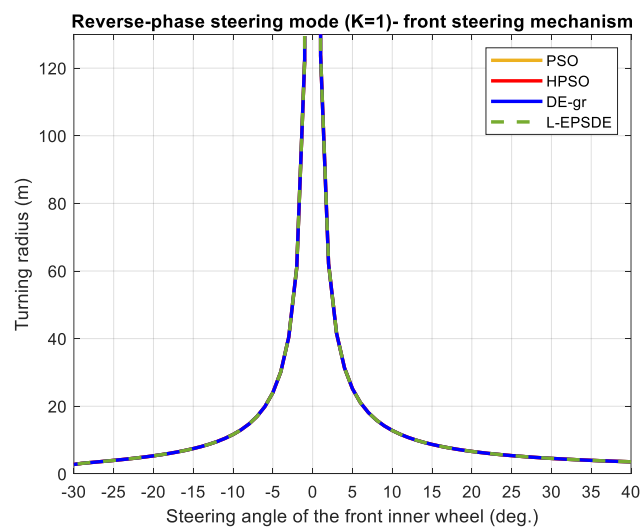


Figure 17. Turning radii of the gravity center of the vehicle's front part (Reverse-phase, $K = 1$).

Figures 18 and 19 show the curves of steering angles and steering errors of each optimized four-bar steering mechanism of the rear axle, respectively; the maximum steering error is about 17.4 degrees. Figure 20 shows the curve of the transmission angle of each optimized four-bar steering mechanism of the rear axle, and the transmission efficiency within the steering angle range $[-40^\circ, -20^\circ]$ is not satisfactory. Figure 21 shows the curve of turning radius at the center of gravity of the rear part of the vehicle. For comparison, the part data of the turning radius (Figures 17 and 21) at the center of gravity of the front and rear parts of the vehicle are shown in the third and fourth columns of Table 1. In Figures 18–21, the curves of the four methods almost overlap and are difficult to distinguish due to the similarity of their optimal results.

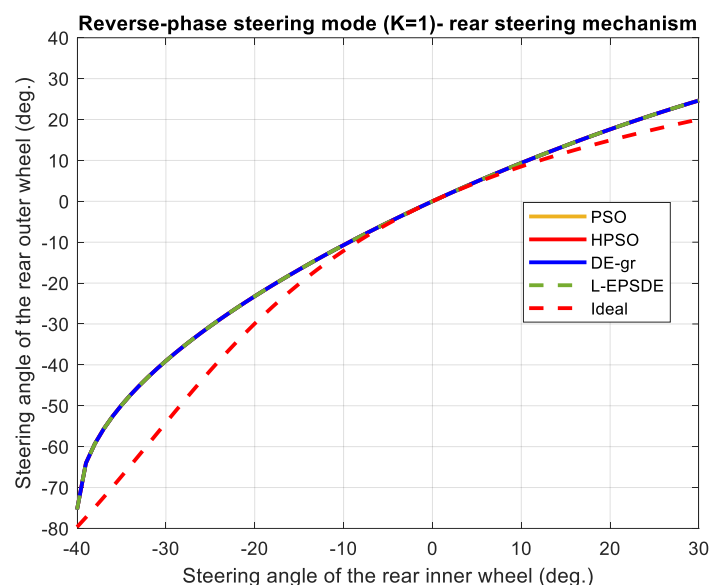


Figure 18. Steering angle of the optimized rear four-bar steering mechanisms (Reverse-phase, $K = 1$).

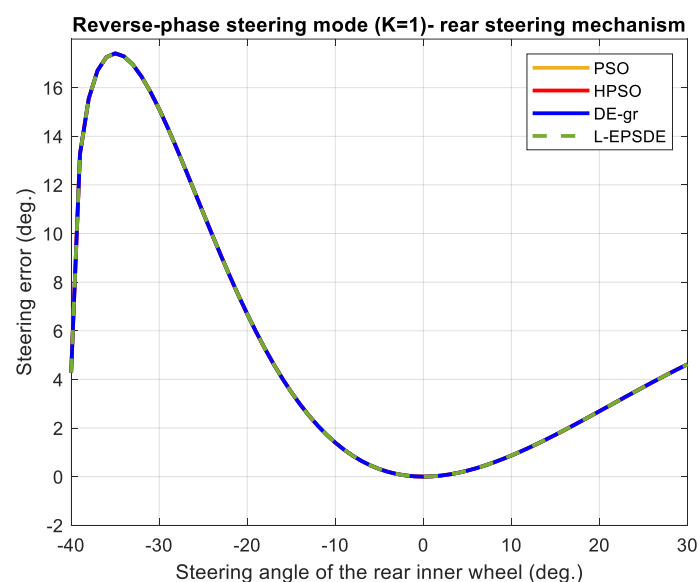


Figure 19. Steering error of the optimized rear four-bar steering mechanisms (Reverse-phase, $K = 1$).

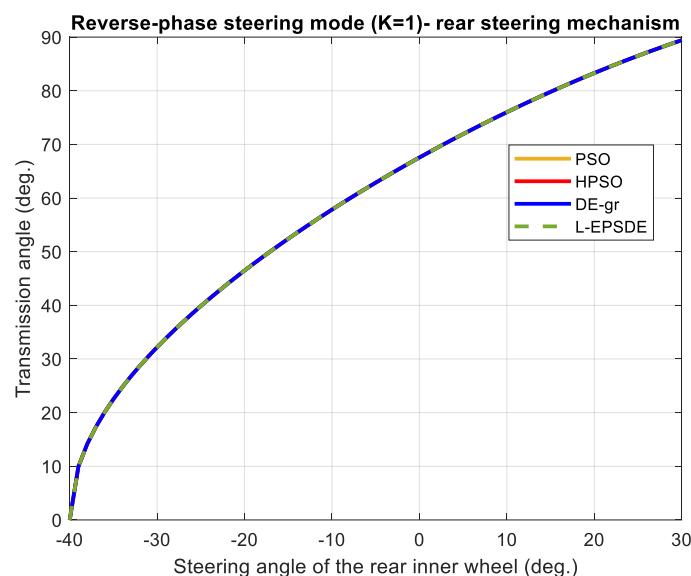


Figure 20. Transmission angle of the optimized rear four-bar steering mechanisms (Reverse-phase, $K = 1$).

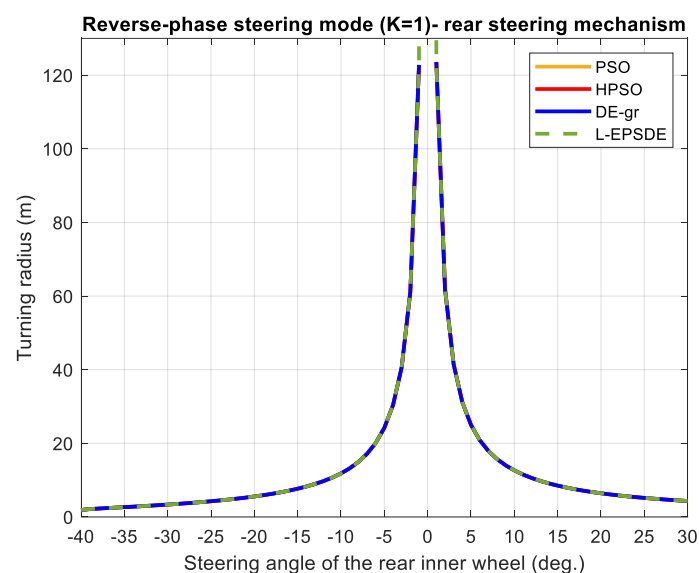


Figure 21. Turning radii of the gravity center of the vehicle's rear part (Reverse-phase, $K = 1$).

4.3.3. Optimal Design of the Reverse-Phase ($K \neq 1$) Four-Bar Steering Mechanism

The design parameters of this steering mode are θ_{20f} , r_{2f} , θ_{20r} , r_{2r} and K . Table 8 presents the statistical results of two hundred optimization experiments for the reverse-steering ($K \neq 1$) four-bar mechanism of a three-axle, six-wheeled vehicle. The DE-gr method achieves the best performance, with the lowest mean value (1.41×10^{-4}), standard deviation (1.61×10^{-5}), and worst error value (1.58×10^{-4}). Its minimum error (1.256×10^{-4}) is nearly identical to HPSO and L-EPSDE's (1.254×10^{-4}). The HPSO method shows slightly better mean and standard deviation values than the L-EPSDE method, but its worst value is slightly inferior. PSO method performs the poorest on best error (1.298×10^{-4}) and standard deviation (2.15×10^{-3}). Figure 22 shows the convergence speed of the four optimization methods, where the DE-gr method converges fastest, followed by L-EPSDE and PSO, and HPSO converges more slowly. Overall, the DE-gr method excels in accuracy, converge speed, and stability, with L-EPSDE and HPSO performing comparable performance and PSO slightly lagging.

Table 8. Statistical results of the four optimization methods for reverse-phase steering mode ($K \neq 1$).

Method	Best Error	Worst Error	Median	Mean	SD
PSO	1.298×10^{-4}	5.44×10^{-3}	1.71×10^{-3}	2.39×10^{-3}	2.15×10^{-3}
HPSO	1.256×10^{-4}	1.06×10^{-2}	9.64×10^{-3}	7.83×10^{-4}	1.28×10^{-3}
DE-gr	1.256×10^{-4}	1.58×10^{-4}	1.26×10^{-4}	1.41×10^{-4}	1.61×10^{-5}
L-EPSDE	1.254×10^{-4}	2.63×10^{-3}	2.39×10^{-3}	2.69×10^{-3}	1.92×10^{-3}

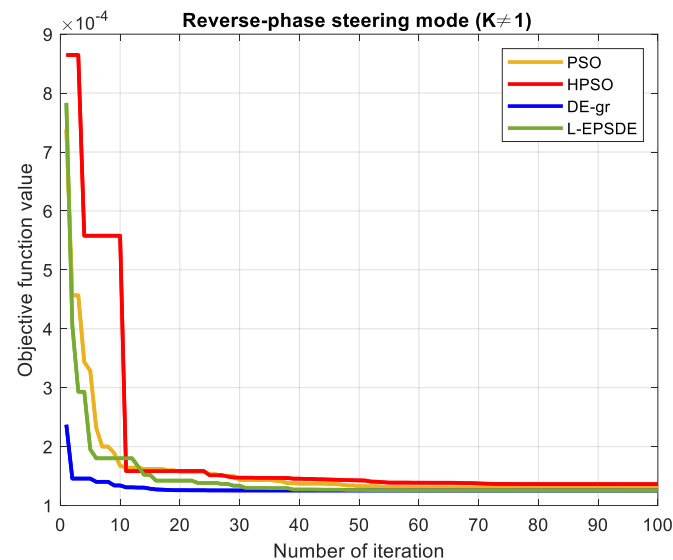
**Figure 22.** Convergence speed of the four optimization methods for reverse-phase steering mode ($K \neq 1$).

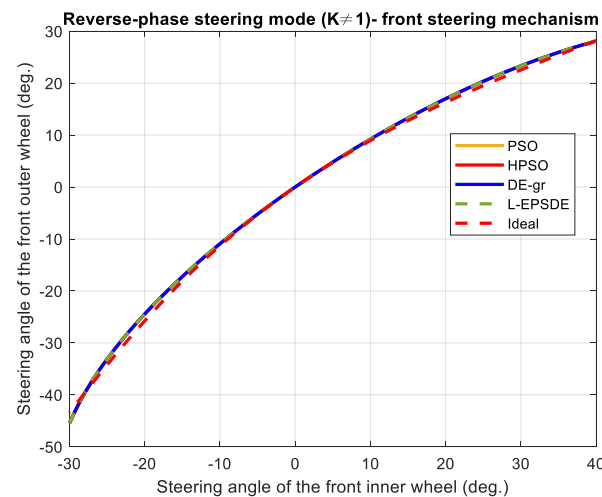
Table 9 shows the objective function values, optimal dimensions, and maximum steering errors for the three-axle, six-wheeled reverse-phase four-bar steering mechanisms synthesized using four optimization methods. The steering angle range of the front steering wheel is a little different from the literature [33]. The result revealed that the front and rear-optimized four-bar mechanisms have distinct dimensions. The red marks in Table 9 highlight the maximum steering errors (Δ_f, Δ_r) for the front and rear axles. For the front axle, PSO and DE-gr methods performed best, with maximum errors of 2.0286° and 2.0646° , respectively, while the HPSO method was slightly worse at 2.1978° . For the rear axle, DE-gr and L-EPSDE methods achieved the best results, with maximum errors of 3.3272° and 3.3301° , respectively, whereas HPSO and PSO methods performed slightly poor at 3.8740° and 3.8985° . Overall, the maximum steering errors of front and rear axles in this study are improved compared to the literature [33].

Comparing the convergence trends in Figure 13 ($K = 1$) and Figure 22 ($K \neq 1$), it is evident that all four methods converge more rapidly to the optimal solution in the $K \neq 1$ case. For example, the DE-gr method reaches the optimal solution after 50 iterations for $K = 1$, but only 25 iterations for $K \neq 1$. Additionally, referring to Table 6 ($K = 1$) and Table 8 ($K \neq 1$), the optimal objective function values for all four methods (e.g., 1.254×10^{-4} for L-EPSDE) are lower in the $K \neq 1$ case compared to $K = 1$ (9.997×10^{-3} for L-EPSDE). In the optimal synthesis of the reverse-phase steering mechanism for a three-axle, six-wheeled vehicle, considering K as a design parameter not only enhances design flexibility for intermediate axle placement but also improves convergence efficiency and solution accuracy.

Table 9. Optimized four-bar steering mechanisms using four optimization methods for reverse-phase mode ($K \neq 1$).

	Buzzi et al. [33]	PSO	HPSO	DE-gr	L-EPSDE
Iteration number	--	1000	1000	1000	1000
Obj. fun. (rad. ²)	0.000848	0.0001298	0.0001256	0.0001256	0.0001254
r_{2f} (mm)	229.3	263.0771	220.3491	219.5021	218.0000
r_{3f} (mm)	2322.0	1974.3805	2003.8093	2004.4058	2005.4882
r_{2r} (mm)	322.4	220.0000	217.0070	218.7651	217.0000
r_{3r} (mm)	2648.5	2544.6052	2539.3952	2542.4568	2539.3148
θ_{2of} (°)	72.0	66.9959	66.4344	66.4229	66.4058
θ_{2or} (°)	42.4	148.3616	148.3330	148.3502	148.3158
K	0.333	0.323	0.323	0.333	0.333
w_1	0.752	0.758	0.758	0.752	0.752
w_3	0.248	0.242	0.242	0.248	0.248
Δ_f (°)	6.8	2.0286	2.1978	2.0646	2.0648
Δ_r (°)	15	3.8985	3.8740	3.3272	3.3301

Figures 23 and 24 illustrate the curves of steering angles and steering errors, respectively, for each optimized front four-bar steering mechanism. The maximum steering errors for the PSO, DE-gr, and L-EPSDE methods occur near the end of the steering stroke but remain around 2.1° , significantly lower than in the $K = 1$ case. Figure 25 shows the transmission angle curves for each optimized front four-bar steering mechanism. In the larger range of $[-20^\circ, 40^\circ]$, the deviation from 90° is less than 50° , indicating better transmission efficiency. However, in the smaller range of $[-30^\circ, -20^\circ]$, the deviation from 90° is more than 50° , resulting in reduced transmission efficiency. Figure 26 shows the turning radius curve at the center of gravity for the front part of the vehicle under the reverse-phase steering mode. In Figures 23–26, the curves of the four methods almost overlap and are difficult to distinguish due to the similarity of their optimal results.

**Figure 23.** Steering angle relation of the optimized front four-bar steering mechanisms (Reverse-phase, $K \neq 1$).

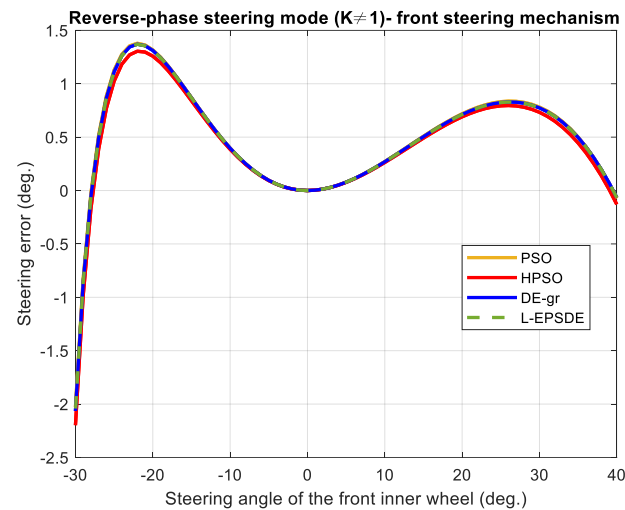


Figure 24. Steering error of the optimized front four-bar steering mechanisms (Reverse-phase, $K \neq 1$).

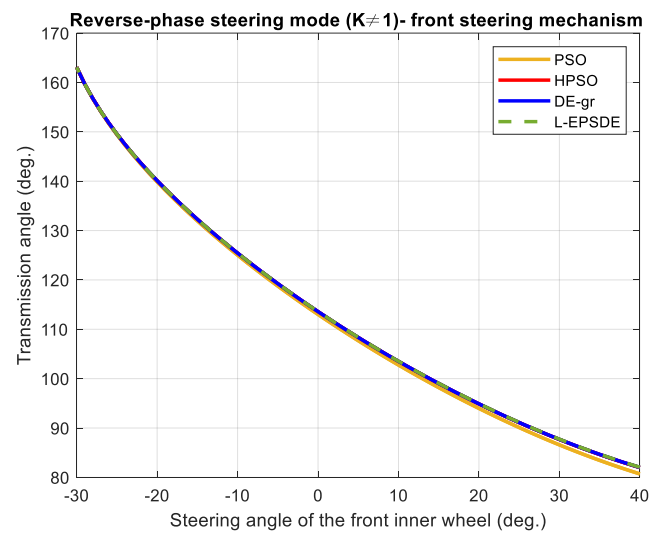


Figure 25. Transmission angle of the optimized front four-bar steering mechanisms (Reverse-phase, $K \neq 1$).

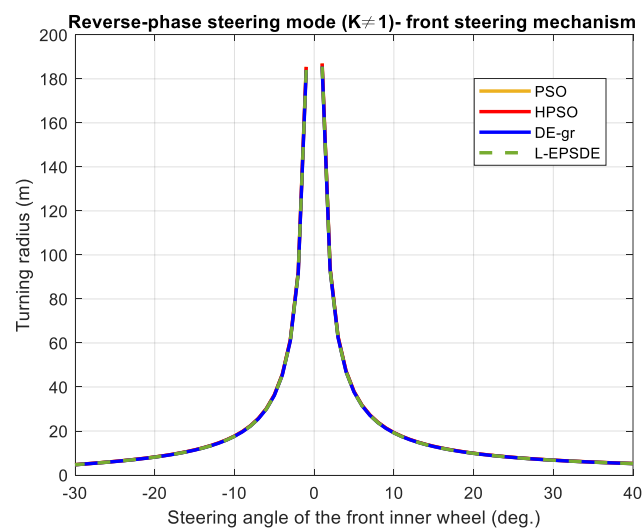


Figure 26. Turning radii of the gravity center of the vehicle's front part (Reverse-phase, $K \neq 1$).

Figures 27 and 28 depict the relation of steering angles and steering error curves, respectively, for each optimized rear four-bar steering mechanism. The DE-gr method achieves the smallest maximum steering error (3.3272°), followed closely by the L-EPSDE method (3.3301°). In contrast, the PSO and HPSO methods result in slightly larger steering errors (3.8985° and 3.874° , respectively), as shown in the enlarged figure on the right side. Figure 29 shows the transmission angle curves for the optimized rear four-bar steering mechanisms, with deviations exceeding 50° from 90° , indicating lower transmission efficiency compared to the front axle steering mechanism. This implies that the transmission effectiveness of this reverse-phase steering mode relies primarily on the front steering mechanism. Figure 30 illustrates the turning radius curves at the rear part's center of gravity under the reverse-phase steering mode. For comparison, the part data of the turning radius (Figures 26 and 30) at the center of gravity of the front and rear parts of the vehicle are shown in the fifth and sixth columns of Table 1. In Figures 27, 29 and 30, the curves of the four methods almost overlap and are difficult to distinguish due to the similarity of their optimal results.

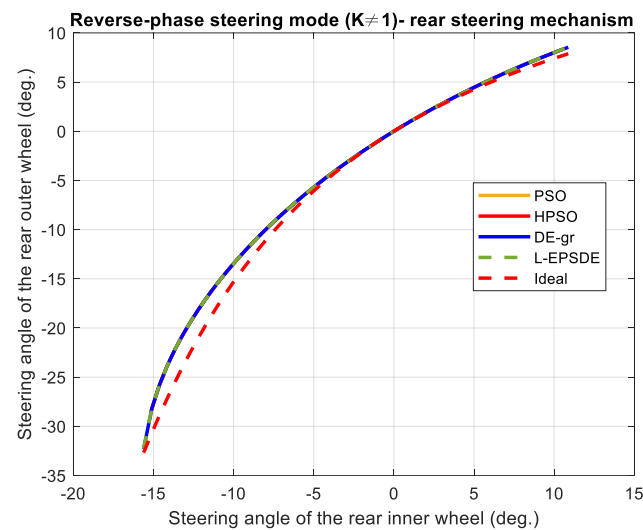


Figure 27. Steering angle relation of the optimized rear four-bar steering mechanisms (Reverse-phase, $K \neq 1$).

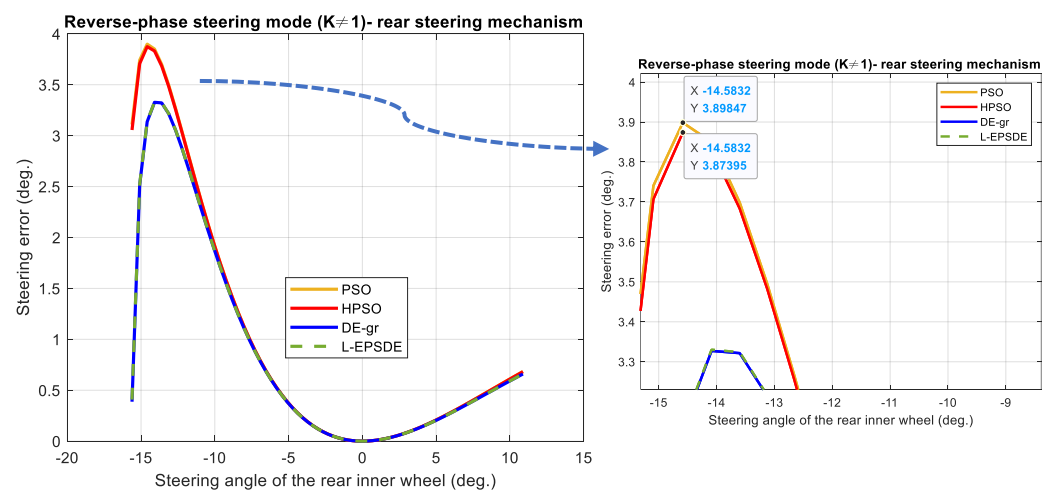


Figure 28. Steering error of the optimized rear four-bar steering mechanisms (Reverse-phase, $K \neq 1$).

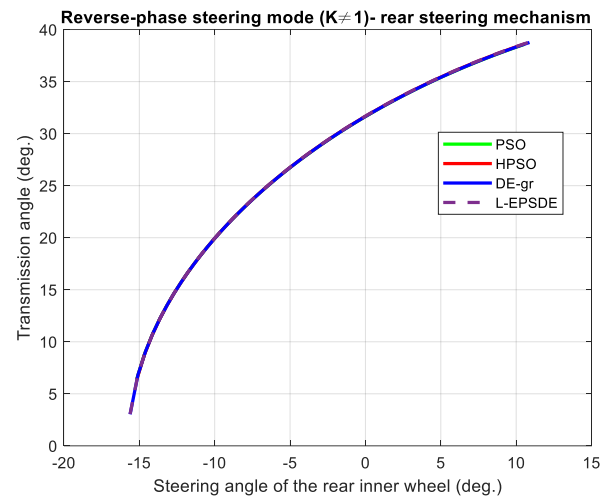


Figure 29. Transmission angle of the optimized rear four-bar steering mechanisms (Reverse-phase, $K \neq 1$).

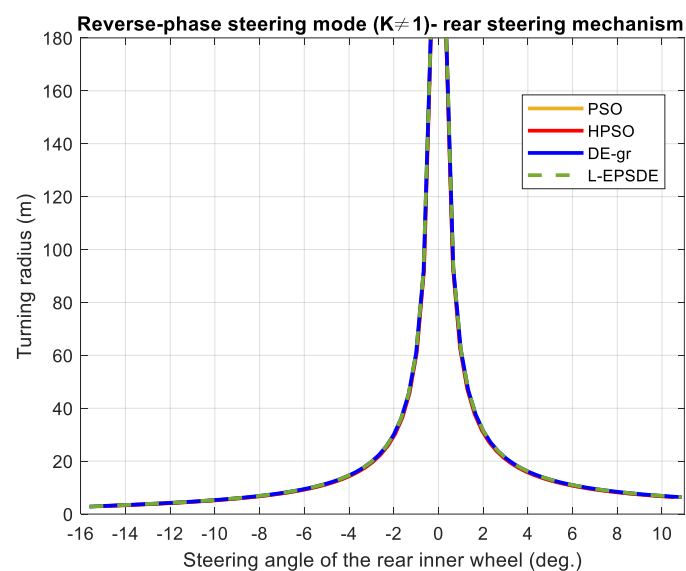


Figure 30. Turning radii of the gravity center of the vehicle's rear part (Reverse-phase, $K \neq 1$).

This study employs the multi-body dynamic analysis software MSC-ADAMS v2015 to perform the kinematic simulation of the optimized Ackermann four-bar steering mechanisms in the three-axle, six-wheeled, reverse-phase steering mode obtained using the DE-gr method and validates the results with MATLAB analysis. Figure 31 shows the solid model of the optimized reverse-phase four-bar steering mechanisms of the three-axle, six-wheeled vehicle. Figures 32 and 33 depict the kinematic simulations of the optimized steering mechanisms, which are the trailing type and leading type Ackermann steering mechanisms [13], respectively, for the front and rear axles. Figures 34 and 35 compare the analysis errors of output steering angle between MATLAB and MSC-ADAMS for the front and rear axles, respectively. The steering error results derived from ADAMS and MATLAB are very consistent, even though there exist a few analysis errors with maximum errors of only 0.222° (front axle) and 0.234° (rear axle), confirming the accuracy of the MATLAB analysis.

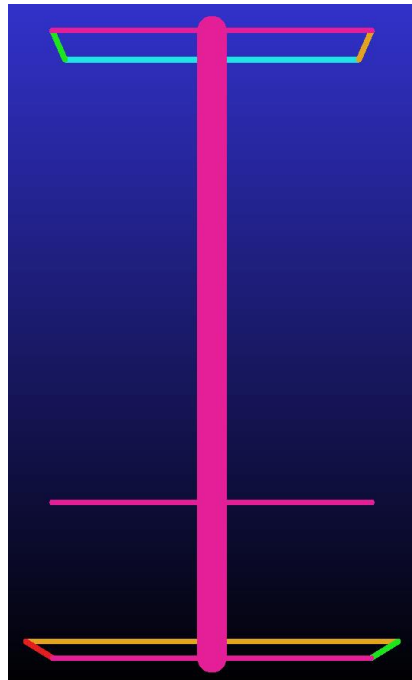


Figure 31. Solid model of the optimized reverse-phase four-bar steering mechanisms by ADAMS.

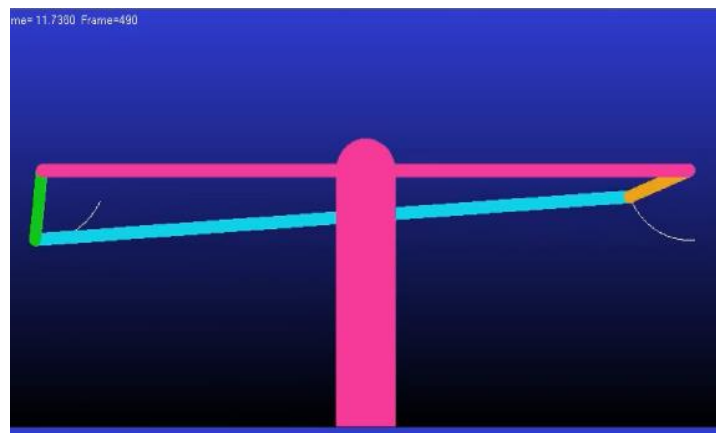


Figure 32. Kinematic simulation of the optimized front four-bar steering mechanism by ADAMS.

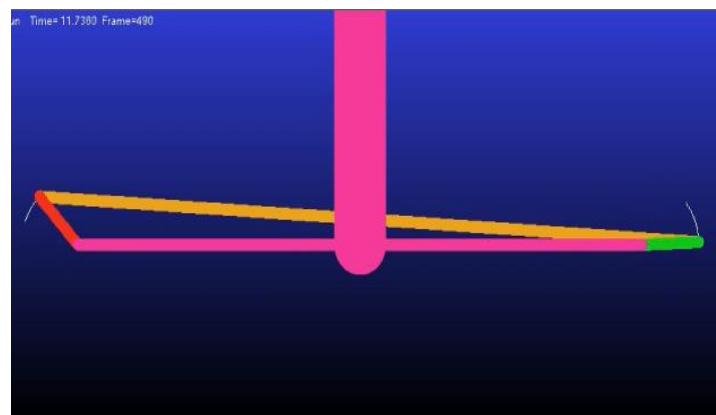


Figure 33. Kinematic simulation of the optimized rear four-bar steering mechanism by ADAMS.

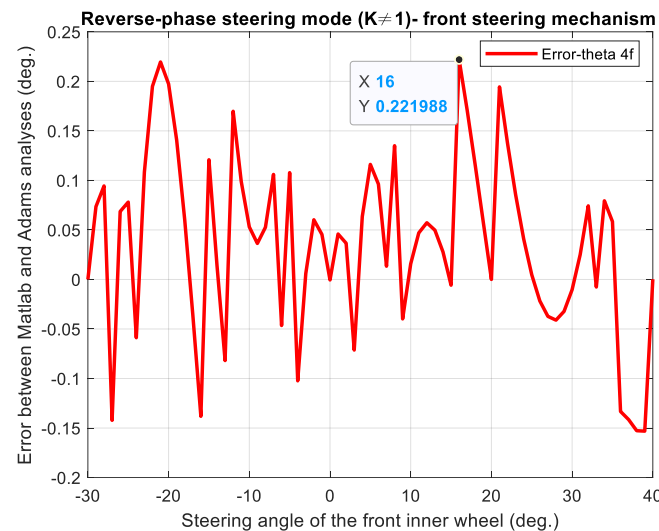


Figure 34. Analyses error between ADAMS and MATLAB for the front steering mechanism ($K \neq 1$).

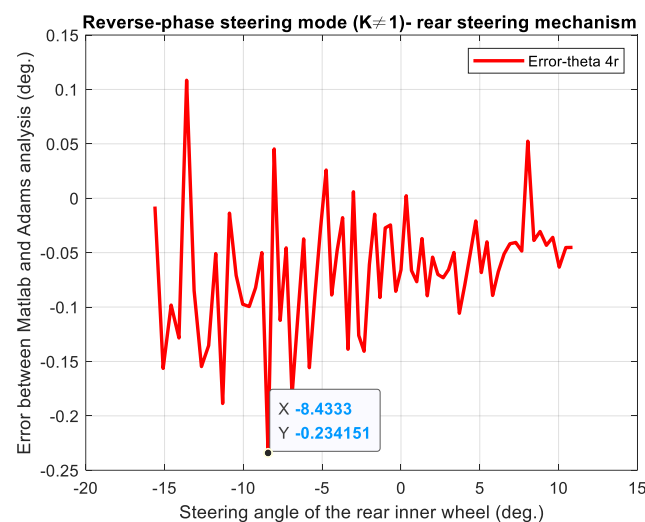


Figure 35. Analyses error between ADAMS and MATLAB for the rear steering mechanism ($K \neq 1$).

5. Conclusions

This paper investigates the optimal dimensional synthesis of Ackerman trapezoidal steering mechanisms for a three-axle, six-wheeled vehicle in front-axle and reverse-phase steering modes. The optimization objective minimizes the mean squared steering errors within the steering range with a penalty for constraint violations of Grashof's criteria. In reverse-phase ($K \neq 1$) steering mode, the transmission ratio of the front to rear axle (K) is considered as a design parameter. Four metaheuristic optimization algorithms—PSO, HPSO, DE-gr, and L-EPSDE—were employed through two hundred optimization numerical experiments. For front-axle steering mode, the L-EPSDE method achieved the smallest maximum steering error (8.96986°). In reverse-phase ($K = 1$) steering mode, DE-gr, L-EPSDE, and PSO methods minimized front axle steering errors (6.3850°) and rear-axle steering errors (17.4121°), respectively, while the HPSO method had slightly higher errors, 6.4167° and 17.4145° , in the front and rear axles, respectively. Additionally, the reverse-phase ($K = 1$) steering mode yielded a significantly smaller turning radius compared to the front-axle steering mode. In reverse-phase ($K \neq 1$) steering mode, the DE-gr method performed best overall, reducing the steering errors of front and rear axles to 2.0646° and 3.3272° , respectively. And the L-EPSDE method has comparable performance with steering error of front and rear axles of 2.0648° and 3.3301° , respectively. However, for reverse-phase

($K \neq 1$) steering mode, the turning radii are slightly larger than those of the case $K = 1$. Statistical analysis of the two hundred optimization numerical experiments revealed that DE-gr and L-EPSDE methods had the fastest convergence, highest accuracy, and precision, with PSO and HPSO methods slightly behind but still better than previous studies [33]. For the optimal synthesis of the reverse-phase Ackermann four-bar steering mechanism of a three-axle, six-wheeled vehicle, considering K as a design parameter not only enhances design flexibility for intermediate axle placement but also significantly reduces the steering error and improves the accuracy and convergence efficiency.

Overall, the study concludes that the Ackermann four-bar steering mechanism in front-axle and reverse-phase steering modes is effectively modeled and efficiently solved as a low-dimension optimization problem with four and five design parameters. Among the four utilized metaheuristic algorithms, DE-gr and L-EPSDE performed the best. Furthermore, the DE-gr-optimized reverse-phase Ackermann four-bar steering mechanisms were validated using multi-dynamic analysis software MSC-ADAMS v2015 and the MATLAB program, showing the analysis errors of the output steering angle of 0.222° (front axle) and 0.234° (rear axle), respectively, confirming the accuracy and feasibility of the proposed dimensional synthesis approach of the Ackermann trapezoidal steering mechanism for a three-axle, six-wheeled vehicle. The result of this study indicates that two metaheuristic optimization methods, DE-gr and L-EPSDE, with their high efficiency, accuracy, and fast convergence speed, are well-suited for further investigation on the optimal dimensional synthesis of a front-wheel drive two-axle four-wheeled vehicle's six-bar and eight-bar steering mechanisms and a three-axle, six-wheeled vehicle's four-bar and six-bar steering mechanisms (with two front steerable axles and a rear non-steerable axle) and a six-bar steering mechanism in reverse-phase steering mode.

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