

Article

Preliminary Design and Parametric Study of Minimum-Weight Steel Tied-Arch Bridges Obtained According to a Maximum Allowable Deflection Criterion

Juan José Jorquera-Lucerga * and Juan Manuel García-Guerrero 

Mining and Civil Engineering Department, Universidad Politécnica de Cartagena (UPCT), Paseo Alfonso XIII, 50, 30203 Cartagena, Spain; jm.guerrero@upct.es

* Correspondence: juanjo.jorquera@upct.es

Featured Application

The methods, formulations, and parametric studies described in this paper, as well as their conclusions and design recommendations, were developed to assist designers in the conceptual design of all-steel arch bridges, particularly in cases where minimizing the bridge weight is a priority.

Abstract

In this paper, we present a novel iterative method that minimizes the weight of an all-steel arch bridge during the in-plane preliminary design stage. The behavior of the bridge is assumed to be contained within the plane of the arch. The preliminary design is assumed to be governed by the maximum allowable static deflection at a given checkpoint at the deck under a simplified load combination selected by the designer. The designer can select variables commonly used in preliminary design, such as the web slenderness of the cross-sections of both the arch and the deck and their relative flexural stiffness levels. Moreover, the general method is particularized for tied-arch bridges with vertical hangers: its iterative flowchart is adapted, an approximate analytical formulation that allows manual calculations is provided, and a parametric study that illustrates the effect of the main variables on the weight of the bridge is carried out. The main design recommendations drawn from this research for minimizing the weight of a bridge are as follows: a rise/span ratio between 1/5 and 1/7; cross-sections with significantly different stiffnesses in the arch and deck, ideally with highly flexible arches; and cross-sections with the thinnest possible webs.

Keywords: arch bridge; preliminary design; in-plane behavior; weight optimization; serviceability limit state; conceptual design; parametric study



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1. Introduction

1.1. Preliminary In-Plane Design of Arch Bridges

Arch bridges are usually large and economically important structures, and they require very careful construction processes. Moreover, for arch bridges, there is a very close relationship between geometry, proportions, and structural behavior. Thus, the distribution of material between the arch and the deck or the shape of the arch are crucial for ensuring the quality of the design, perhaps more so than in other typologies. Thus, the conceptual design process of an arch bridge, which includes its preliminary design, is particularly

important, and many research studies have been devoted to it. For example, Stavridis [1] has studied the structural behavior of arch bridges and Granata et al. describe (in [2]) the conceptual design of steel and composite concrete–steel tied-arch bridges, highlighting the influence of the geometry of the arch shape, the design of the arch–deck joint, and the construction sequence on its global behavior.

During the preliminary design stage of an arch bridge, it is usually assumed that its structural response can be divided into two partial behaviors: in-plane and out-of-plane arch behaviors; it is assumed that these can be analyzed separately, since, frequently, they are uncoupled structural systems. The preliminary in-plane design stage (i.e., under actions contained within the plane of the arch) is generally the most time-consuming step in this process, and it is usually carried out by experienced designers, because simplified yet accurate preliminary dimensioning can significantly speed up the entire design process. Preliminary dimensioning is usually based on the designer's experience and studying examples of similar bridge typologies.

An example that illustrates how new arch bridges can be inspired by existing ones is provided by Mermigas and Wang [3], who collected the geometric information of 20 steel arch bridges to investigate the correlation between proportions and behavior. This information, presented in graphs, was used by the Ministry of Transportation in Ontario to evaluate a proposed design for a 75 m steel half-through arch bridge. Similarly, Manterola-Armisén [4] compiles the geometries, proportions, and construction processes (a very relevant aspect in bridge design) of several sets of real arch bridges.

The main dimensions of a bridge are often defined by rules of thumb. For example, Lebet and Hirt [5] recommend a range for the slenderness of steel arch bridges (given by the ratio f/L , where f is the rise of the arch and L is its span) and for the ratio $(z_A + z_D)/L$, where $z_A + z_D$ are, respectively, the depths of the cross-sections of the arch and the deck. The geometry and proportions of concrete arch bridges have been studied, for example, by Leonhardt [6], Menn [7] or Salonga and Gavreau [8].

Parametric or sensitivity studies are also often carried out, mainly for long-span bridges. Modern studies make the most of the possibilities of computer-aided parametric design and optimization algorithms, as shown in Korus et al. [9], who combine FEM and genetic algorithms. In addition, they include studies on the span and span–rise ratios of a set of arch bridges constructed in Poland in the past 20 years.

An important aspect of arch bridge design is the geometry of the arch shape, especially in long-span bridges. Ideally, the arch should have an anti-funicular geometry, i.e., the geometric axis of the arch should be located as close as possible to the pressure line (thus being free of bending moments) for a given load case, usually permanent loads. However, due to the variability in traffic loads, it will not always be possible to completely avoid bending moments. Due to the structural and economic advantages of reducing bending in the arch and deck cross-sections, form-finding methods have received considerable attention from researchers. Recent examples include the studies of Jorquera-Lucerga [10] and Zhao [11].

The distribution of material within the arch and deck cross-sections is also very relevant. In general, cross-sections will have a high radius of gyration, $\sqrt{I/\Omega}$, where I is the second moment of area and Ω is the cross-section area. This means that the material is distributed in such a way that it is separated from the section centroid. This is true for hollow-box or symmetrical H sections, which are widely used in arch bridges. Doubly symmetric cross-sections are used in arches since live loads induce both positive and negative bending moments. In general, these types of sections maximize their second moments of area (which provide flexural stiffness and reduce deflections) and section modulus (which increases bending resistance) and reduce the arch's buckling sensitivity.

The problem of finding the most appropriate area distribution (i.e., minimizing the amount of material) has also been the subject of intensive research. As shown later in this paper, the process for the minimization of cross-section weight will follow Haaijer's work [12] due to the simplicity of its formulation and the use of variables commonly employed in preliminary design.

For preliminary dimensioning purposes, it is common practice to consider that a solution is valid if certain simplified criteria regarding the serviceability limit state and ultimate limit state (SLS and ULS), such as deflections or stress limits, are fulfilled under simplified load combinations (SLCs). SLCs, which are defined by the designer, consist of simplified versions of the load combinations defined in codes and regulations (such as the Eurocodes [13,14]), in which only the most relevant actions are considered. Figure 1 shows an example of this approach, where SLCs only include the self-weight (Ω_A and Ω_D are the areas of the arch and deck cross-sections and γ is the density of steel), the most relevant superimposed dead loads g_{DL} (which consider other permanent loads such as road surfacing, parapets, services, etc.), and the live load q due to traffic. Deflections and stresses under SLCs are evaluated at checkpoints (i.e., at specific locations chosen by the designer) and compared with allowable limits, i.e., δ_{lim} and σ_{lim} .

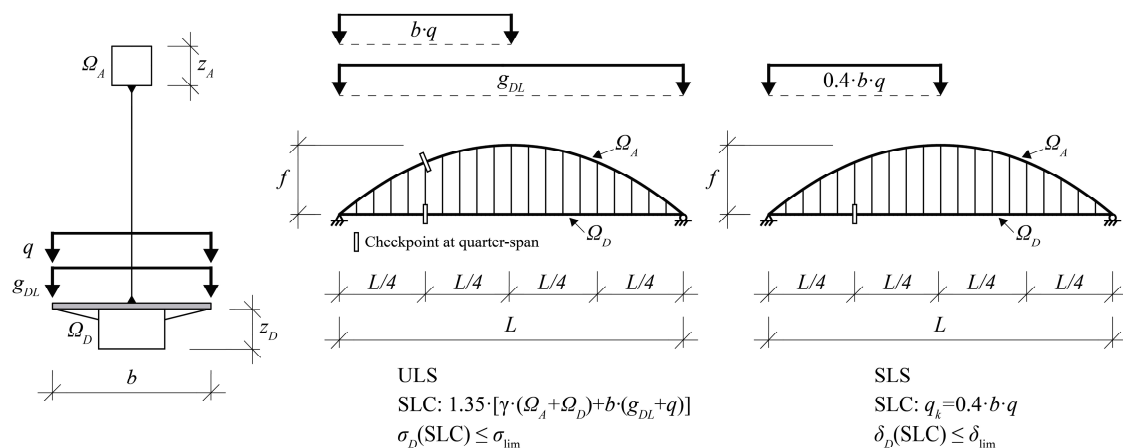


Figure 1. An example of preliminary design based on stress and deflection limits at checkpoints under SLCs defined by the designer.

The preliminary design stage is an intrinsically iterative process: according to the results of the simplified calculations, the dimensions of the arch and deck cross-sections are refined until the criteria are met. This is by no means straightforward, since relatively simple steel cross-sections, such as hollow-box or symmetrical H-sections, are defined by four magnitudes (depth, width and thickness of web(s), and flanges, Figure 1). This process becomes even more complicated in arch bridges due to arch–deck interaction, since a change in the arch stiffness can significantly modify the internal forces of the deck, and vice versa.

1.2. Research Objectives

Therefore, in this context, there is a need for a method that helps designers simplify this step of the preliminary design process. The research presented in this paper aims to achieve the following research objectives:

1. Developing a method for the preliminary design of an arch bridge that verifies that, at a selected checkpoint at the deck, the maximum deflection δ_D is smaller than the maximum allowable deflection at the same point, δ_{lim} , under a given SLC.

2. Simplifying the design process by reducing the number of geometrical variables used to define the cross-sections.
3. Minimizing the weight of the bridge since, in general, the cost will be lower for a smaller amount of material.
4. Ensuring the usefulness of the method for designers. To achieve this goal, the input data must consist of variables commonly used in preliminary design. In particular, it should enable designers to select the relative flexural stiffness of the arch and deck, since this has a significant impact on arch bridge design (see Stavridis [1], Manterola-Armisen [4], or Menn [7]).
5. Realizing parametric studies to illustrate the effect of the main variables on preliminary design.
6. Developing an approximate analytical formulation to allow manual calculations of tied-arch bridges with vertical hangers, aiming to speed up the conceptual design step.

1.3. The δ -Method

The δ -method, which is presented in this paper, assumes that the preliminary design is governed by deflection limitations under a given SLC. The method has the same iterative structure as the preliminary design process. Firstly, a Finite Element Model (FEM) analysis (or, alternatively, accurate manual calculations) is carried out. Secondly, depending on the results, the method guides the designer on how to modify the cross-sections of the structure until the preliminary design goal is achieved. As a result, the cross-sections of both the deck and the arch are dimensioned to verify $\delta_D = \delta_{lim}$. The above-mentioned research objectives 2 to 4 are fulfilled as follows:

Research objective 2: It is assumed that the cross-sections of both the arch and the deck are doubly symmetric, a configuration often used for arch bridges. Additionally, it is assumed that the thicknesses of the flanges of both the arch and deck cross-sections are small compared to their depths. Thus, Section 2.5 will demonstrate how a cross-section and its mechanical properties can be fully defined by its area Ω , its depth z , and the slenderness of its web β .

Research objective 3: Section 4 shows that the vertical deflection δ_D depends not only on the area of the arch and the deck, Ω_A and Ω_D , respectively, but also on their second moments of area, I_A and I_D . Based on Argüelles [15] and Haaijer [12], Section 2.5 demonstrates how, for a given β , the material of the cross-section can be distributed to achieve a target value of I with a minimum Ω , i.e., a minimum weight.

Research objective 4: As intended, Ω and β are variables widely used for preliminary design. Moreover, Section 2.3 shows how arch–deck interaction is defined by a single parameter, C_A , which varies between 0 and 1, and represents the contribution of the arch to the total flexural stiffness of the structure.

This method intends to help designers, since deflection limitations are often the governing criteria for dimensioning some types of arch bridges, such as tied-arch bridges with vertical hangers (Manterola-Armisen [4]). Therefore, other verifications related to the ULS, buckling, etc., are outside the scope of this method and should be carried out separately. Ongoing research conducted by the authors should provide a method for preliminary design that also considers cross-section failure.

1.4. Structure of the Paper

Section 2 describes the general in-plane preliminary design strategy, detailing the assumptions of the method, as well as its scope. Section 3 describes the general δ -method, which can be used for many different types of arch bridge typologies. Section 4 adapts the

δ -method specifically to tied-arch bridges with vertical hangers, based on an approximate theoretical formulation, which is also described in detail. Section 5 shows three case studies, with examples of applications of the δ -method. In the second example, the method results are compared to the dimensions of real bridges, while in the third example, the effect of an alternative SLC is shown. In Section 6, a set of preliminary parametric studies are carried out. Thanks to these studies, the number of case studies needed in the parametric studies (Research objective 5) of Section 7 is reduced. In Section 8, the theoretical formulation developed in Section 4 is adjusted thanks to the results of the parametric studies carried out in Section 7 (Research objective 6). Finally, Section 9 provides the conclusions drawn from this research and some design recommendations.

2. Preliminary In-Plane Design Approach

This paper considers a typical bridge composed of a vertical planar arch and a straight deck of span L , supported by a second-degree parabolic arch with rise f (Figure 2). It is assumed that its behavior is contained within the vertical plane. The method can be easily applied to twin-arch bridges simply by considering the tributary load acting on each arch and deck.

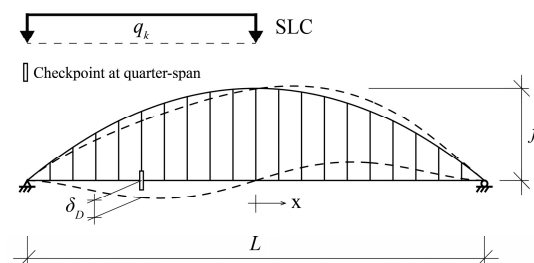


Figure 2. Bridge definition, SLC, and checkpoint location.

2.1. Maximum Deflection Criterion

The method described in this paper, namely, the δ -method, provides the in-plane dimensions of the cross-sections of both the arch and the deck, which correspond to a minimum-weight bridge that strictly verifies the SLS criterion, i.e., that, at a selected checkpoint, the difference between the deflection of the deck under an SLC, δ_D , and the maximum allowable deflection, δ_{lim} , is smaller than a previously defined tolerance: $|\delta_D(SLC) - \delta_{lim}| < tol$.

Although it optimizes the weight of the structure, the method does not make use of any optimization methodologies, such as genetic (e.g., Prendes Gero et al. [16] and Liu et al. [17]) or metaheuristic algorithms (e.g., Saka et al. [18]). Instead, the method uses a classical structural approach, allowing for an intuitive interaction with the designer and making the process extremely adaptable.

2.2. Definition of SLC and Checkpoints

The deflections under the SLC are obtained through a FEM linear elastic analysis of the structure. Non-linearity can also be considered when needed. As in every preliminary design process, the designer's judgement plays a crucial role, since they define the locations of the checkpoints, δ_{lim} , and the factored load cases included in the SLCs. The selected SLC and checkpoint location depend on the bridge typology. This paper, which focuses on tied-arch bridges with vertical hangers, considers an SLC that corresponds to a partial live load arrangement over the left half of the span, as described in Stavridis [1], Lebet [5], or Menn [7]. The checkpoint is located at the quarter-span of the deck.

2.3. Arch–Deck Interaction

The arch–deck interaction is crucial in determining the structural response of an arch bridge. In the δ -method, it is implemented by defining a single parameter, C_A , which represents the contribution of the arch to the global bending stiffness of the bridge:

$$C_A \equiv \frac{E_A \cdot I_A}{E_A \cdot I_A + E_D \cdot I_D} \quad (1)$$

where E_A and E_D are the Young moduli for the materials of the arch and the deck, respectively. Since both the arch and the deck are made of steel, $E_A = E_D$, and Equation (1) can be written as

$$C_A \equiv \frac{I_A}{I_A + I_D} \quad (2)$$

Figure 3 illustrates how C_A ranges from very low values, close to 0, for deck-stiffened arches, such as those pioneered by Maillart (see Billington [19]), to values close to 1 when the flexural stiffness of the deck is very small with respect to that of the arch. C_A can be exactly 1 when the contribution of the deck is negligible, as occurs when the deck is composed of simply supported (i.e., non-continuous) spans.

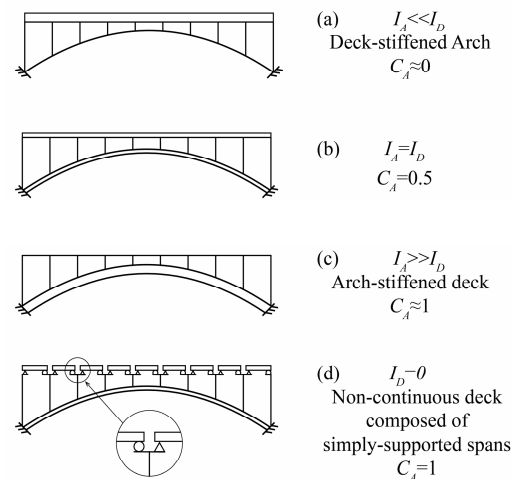


Figure 3. Values of C_A for different arch–deck relative stiffness levels.

2.4. Doubly Symmetric Cross-Sections

This paper considers doubly symmetrical (i.e., symmetrical with respect to the horizontal neutral axis) all-steel hollow boxes and H-sections (Figure 4). This approach was chosen because, usually, arches have a doubly symmetrical cross-section and the deck cross-section can often be approximated as a doubly symmetrical section for preliminary sizing purposes. When this assumption is inaccurate for a preliminary design, as is the case for composite steel–concrete decks, the formulation shown in this paper should be adapted accordingly. Using symmetric cross-sections also has the advantage that the coordinates of the centroidal axes of the arch and the deck remain unchanged when refining the FEM model at each iteration.

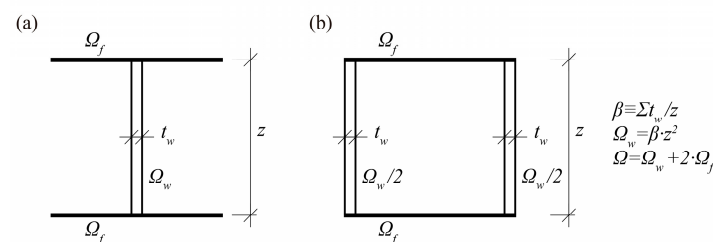


Figure 4. Definitions of cross-sections: (a) H-sections; (b) hollow-box sections.

2.5. Mechanical Properties of Cross-Sections

Since the structural behavior of the bridge is assumed to be contained within the vertical plane, only the in-plane mechanical properties of the cross-sections (such as the area Ω , its second moment of area I , and its elastic section modulus W) are relevant for its in-plane preliminary design. These properties can be accurately estimated (Argüelles [15], Haaijer [12]) by using only three intuitive and commonly used variables (Figure 4):

1. The cross-section area, Ω .
2. The distance between the center lines of the flanges, z . The flange thickness is assumed to be small compared to z , which means that, in practical terms, z coincides with the cross-section depth, and, thus, the area of each flange Ω_f , is concentrated at each end of the web.
3. The geometrical slenderness of the web (or webs) of the cross-section, β , which is very relevant in the classification and behavior of steel cross-sections (see Eurocode 1 [14]). Equation (3) defines β for both the box and H-section cross-sections shown in Figure 4, regardless of the number of webs:

$$\beta \equiv \frac{\sum_i t_{w,i}}{z} \quad (3)$$

For both sections shown in Figure 4, and for the given values of Ω and β , I and W can be expressed as follows (Jorquera-Lucerga and García-Guerrero [20]):

$$I \approx \frac{1}{12} \cdot \beta \cdot z^4 + 2 \cdot \Omega_f \cdot \left(\frac{z}{2}\right)^2 = \frac{z^2}{2} \cdot \left(\frac{\beta \cdot z^2}{6} + \Omega_f\right) = \Omega \cdot \frac{z^2}{4} - \beta \frac{z^4}{6} \quad (4)$$

$$W = \frac{I}{\frac{z}{2}} = z \cdot \left(\beta \cdot \frac{z^2}{6} + \frac{\Omega - \beta \cdot z^2}{2}\right) = \Omega \cdot \frac{z}{2} - \beta \frac{z^3}{3} \quad (5)$$

The maximum possible value of z corresponds to $\Omega_f = 0$ (i.e., when the whole area is in the web and the cross-section becomes rectangular), which leads to

$$0 < z \leq \sqrt{\frac{\Omega}{\beta}} \quad (6)$$

The areas of the cross-sections shown in Figure 4 can be distributed in such a way that, for the given Ω and β , their second moment of area, or inertia, is the maximum, I_{max} . The depth of this section is z_I . These values are obtained as follows:

$$\frac{d(I(z))}{dz} = 0 \quad (7)$$

$$z_I = \sqrt{\frac{3 \cdot \Omega}{4 \cdot \beta}} \quad (8)$$

$$I_{max} = I(\Omega, \beta, z_I) = \frac{3 \cdot \Omega^2}{32 \cdot \beta} \quad (9)$$

which is the maximum possible value of I for a given area Ω and web slenderness β . Since the web area $\Omega_w = \beta \cdot z^2$, a very practical consequence of Equation (8) is that $I = I_{max}$ for a given β when $\Omega_w = 0.75 \cdot \Omega$.

The reader should bear in mind that Equation (9) cannot be applied to Class 4 sections (classified according to Section 5.5 of Eurocode 3.1 [14]), since the cross-section is assumed to be doubly symmetric and its whole area is considered in the estimation of I (i.e., it is

effective). Therefore, the δ -method, as presented in this paper, is limited to Class 1, 2, and 3 sections.

As Equation (9) shows, a low β value (i.e., a thin web) means that a smaller area is needed to achieve a target value of I . In practical terms, $\beta = 1/100$, which corresponds to the web slenderness for pure bending of a Class 3 section for S355 steel (according to Table 5.2 of Eurocode 3.1 [14], $c/t \leq 124 \cdot 0.81 = 100.44$), can be considered, approximately, to be a lower bound for web slenderness, especially in arch cross-sections.

2.6. Scope of the Method

The authors have previously studied different typologies of arch bridges (see, for example, García-Guerrero [21]) prior to this study. Since the authors were already familiar with these typologies, the method was tested and successfully applied to some of them, as shown in Figure 5, such as arches with or without hinges, tied-arch bridges, open spandrel arch bridges, Nielsen–Löhse bridges (see Manterola-Armisen [4], Menn [7], and Park and Hewson [22]), or network-type arches (see Tveit [23]). The SLC and checkpoint location depend on the bridge typology. The results of these studies are expected to be published soon.

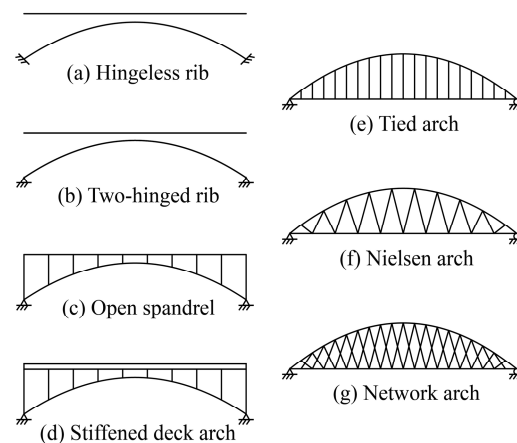


Figure 5. Examples of bridge typologies where the δ -method has been successfully applied by the authors.

The δ -method can be also applied, with no modification, to arches or decks with variable cross-sections, provided the whole arch or the deck are fully defined by knowing the size of one given cross-section. Figure 6 shows examples of area distributions to which the method can be applied.

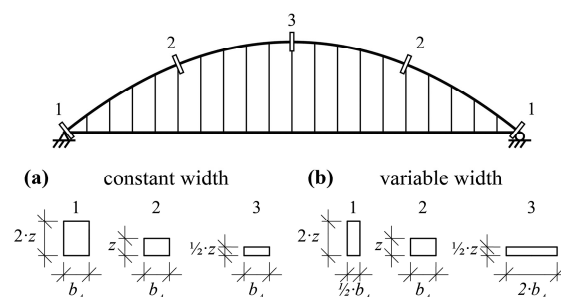


Figure 6. Examples of the distributions of arch area within the scope of the method: (a) constant width; (b) variable width.

One feature of the method with great potential is that the designer can update the typology of the bridge at will during any step of the process. This is as simple as updating

the FEM model according to any desired design criteria at any iteration. For example, the arch can be forked if it is wider than a given threshold value. Although the effects of non-linearity under SLS combinations are rarely significant for these types of bridges, both geometric and material non-linearity can be considered just by including them in the FEM analysis at each iteration.

This paper assumes that design is governed exclusively by deflection limitations under a given SLC. Therefore, readers should bear in mind that this method does not consider all possible in-plane failure modes, such as section failure or in-plane buckling. Although these bridges rarely fail due to in-plane buckling (see, for example, Lebet and Hirt [5], Romeijn and Bouras [24], Chengyi et al. [25], and Tang et al. [26]), this issue must be checked at a later stage of the design process if necessary.

3. The δ -Method: General Case

As will be demonstrated, the essence of the δ -method is that although δ_D , under the SLC, depends on Ω_A , Ω_D , I_A , and I_D , the assumptions in Section 2 allow us to express δ_D as a function of Ω_A exclusively, which becomes the only unknown variable. The auxiliary coefficient μ makes this easier:

$$\mu \equiv \frac{E_D \cdot I_D}{E_A \cdot I_A} = \frac{1}{C_A} - 1 \quad (10)$$

As is well known, the greater the second moment of area, the smaller the deflections. Therefore, Ω_A and Ω_D must be distributed in such a way that their inertias, I_A and I_D , are maximized. The maximum inertia for the cross-section of the arch and the deck, $I_{max,A}$ and $I_{max,D}$, respectively, are given by Equations (11) and (12), where Equation (9) has been particularized for the arch and deck cross-sections, as follows:

$$I_A = I_{max,A} = \frac{3\Omega_A^2}{32 \cdot \beta_A} \quad (11)$$

$$I_D = I_{max,D} = \frac{3\Omega_D^2}{32 \cdot \beta_D} \quad (12)$$

If Equations (11) and (12) are substituted into Equation (10), then

$$\mu = \frac{E_D \frac{3\Omega_D^2}{32 \cdot \beta_D}}{E_A \frac{3\Omega_A^2}{32 \cdot \beta_A}} \quad (13)$$

$$\Omega_D = \Omega_A \sqrt{\mu \frac{E_A}{E_D} \frac{\beta_D}{\beta_A}} \quad (14)$$

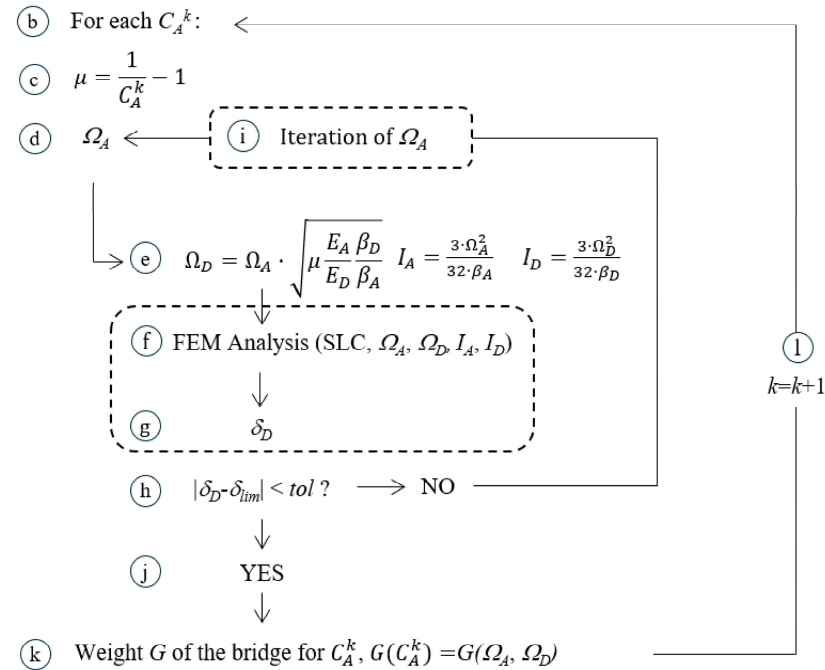
So, Equations (11), (12), and (14) demonstrate that Ω_D , I_A , and I_D , respectively, can be expressed as a function of Ω_A . Figure 7 shows the flowchart for the δ -method. The process begins with defining the data (step a): the geometry, topology, and cable arrangement of the bridge; the SLC; the maximum allowable deflection δ_{lim} ; and the location of the checkpoint at which the vertical deck deflection under the SLC, δ_D , will be evaluated and compared to δ_{lim} . Then, the designer selects the slenderness of the cross-sections of the arch and the deck, β_A and β_D , as well as the range of C_A values ($0 \leq C_A \leq 1$) for which the method will be applied. One advantage of this method is that the designer can restrict the range of C_A values for which the method is applied, although it is always possible to conduct a systematic search, i.e., without a predefined range of C_A . This is the reason why C_A has been defined in a way that varies between 0 and 1.

δ-METHOD (GENERAL METHOD)

DATA:

- Bridge definition: L, f, E_A, E_D , cable arrangement, etc.
- SLC
- (a) • Location of checkpoint, δ_{lim}
- β_A, β_D
- Selected set of values C_A^k ($0 \leq C_A^k \leq 1$)

ITERATIVE PROCESS:



RESULTS:

- (m) $G_{min} = \min_k G(C_A^k)$

Figure 7. A flowchart of the general δ -method.

The method is applied, sequentially, for all the chosen values of C_A (step b). For each one, the auxiliary coefficient μ is evaluated (step c). The designer (step d) only needs to provide Ω_A , since Ω_D , I_A , and I_D are determined by Equations (14), (11), and (12), respectively (step e). With these mechanical properties, a FEM analysis of the bridge is carried out (step g), and the deflection δ_D (SLC) is obtained. The FEM analysis can either be linear or non-linear. The process is repeated until the difference between δ_D and δ_{lim} is smaller than the predefined tolerance tol (step h). Bisection or Newton methods usually provide a reduced number of iterations for Ω_A . As an alternative, although Equation (29) was obtained for tied-arch bridges, it often achieves very fast convergence for other types of bridge. The designer should bear in mind that preliminary designs are carried out for one or several SLCs and that refining the solution is likely to be necessary. Therefore, setting tol too low does not lead to better designs, but rather an excessive calculation time.

The weight of the bridge (G , step k) can then be obtained directly from the output of the FEM model as the sum of the vertical reactions for the self-weight of the arch or the deck. Alternatively, for constant cross-sections, it can be estimated using

$$G = G_A + G_D = \gamma \cdot (L_A \cdot \Omega_A + L \cdot \Omega_D) \quad (15)$$

where γ corresponds to the density of steel (78.5 kN/m^3). The length of the 2nd-degree parabolic arch, L_A , can be estimated as the sum of the lengths of the arch segments or, alternatively, according to Equation (16) (from Strasky [27]):

$$L_A \approx L + \frac{8f^2}{3L} \quad (16)$$

The process is repeated for the next value of C_A (step l). The minimum-weight bridge can be easily found simply by comparing the weights of a set of bridges obtained for all the values of C_A (step m). In the following section, the steps inside the dashed rectangles in the flowchart will be tailored for tied-arch bridges.

4. Method for Tied-Arch Bridges with Vertical Hangers

4.1. SLC and Location of Checkpoints

According to Manterola-Armisen [4] and Menn [7], the maximum deflections for tied-arch bridges with vertical hangers are supposedly due to an SLC corresponding to a uniformly distributed load (UDL) q_k acting on half the deck (Figure 8). This SLC can be decomposed into two other load combinations: SLC-S (symmetric), with a UDL load of $q_k/2$ acting downwards on the whole deck, and SLC-A (antisymmetric), where two opposing UDLs, upwards and downwards, equal to $-q_k/2$ and $q_k/2$, respectively, act on each half of the deck. The checkpoint is located at the quarter-span ($x = -L/4$), where the deflections are expected to be maximum under SLC-A. Obviously, depending on the designer's experience, alternative SLCs can be defined.

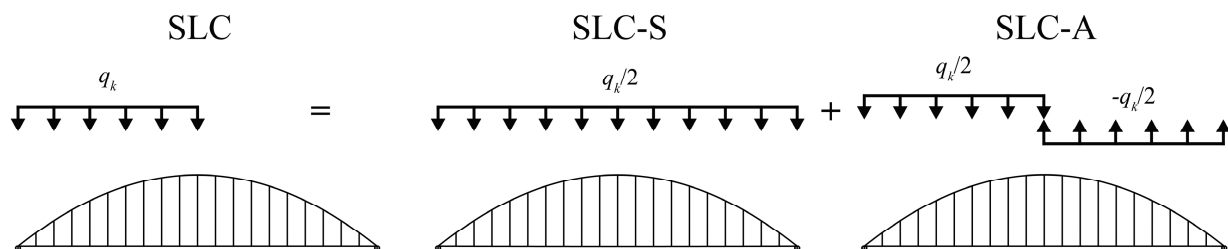


Figure 8. SLC decomposed into symmetric (SLC-S) and antisymmetric (SLC-A) load cases.

The designer also defines the maximum allowable deflection, δ_{lim} , which usually depends on how close the SLCs are to the exact combination defined by regulations. As an example, the Spanish IAP [28], inspired by Eurocode 1 [13], and the Spanish National Annex to Eurocode 0 [29] consider $\delta_{lim} = L/1000$ for bridges and $\delta_{lim} = L/1200$ for footbridges under the frequent part (i.e., factored by $\Psi = 0.4$) of the vertical live load. For arch bridges, L should be the distance between the inflection points of the deformed shape. Therefore, since most of the deflection is due to SLC-A, a value of $\delta_{lim} = L/2000$ can be considered.

A value of 5 kN/m^2 has been considered for the SLC. This coincides with the UDL defined for Load Model 4 of Eurocode 1 [13] and is an intermediate value of the UDLs defined in Load Model 1 for the different notional lanes. Therefore, $q_k = 0.4 \cdot 5 = 2.0 \text{ kN/m}^2$.

4.2. Theoretical Formulation of Deflections for Tied-Arch Bridges with Vertical Hangers

For preliminary design purposes, a theoretical formulation is presented for estimating the deflections of tied-arch bridges under symmetrical and non-symmetrical UDLs. Although it can be used in other contexts, the most relevant consequence is that it provides the basis for adapting the δ -method for tied-arch bridges composed of an arch and a deck, linked by a set of vertical hangers pinned at both ends. Moreover, the arch–deck joint is supposed to be hinged for the sake of simplicity. The proposed formulae decompose the

total deflection δ into three addends: δ_{T1} , δ_{T2} , and δ_{T3} . The sub-index T indicates that these deflection values correspond to theoretical formulae rather than to the “actual” values (without the sub-index) that can be obtained, for example, from a FEM analysis. As shown in Figure 9, they correspond, respectively, to the vertical deflections at the deck checkpoint (i.e., at $x = -L/4$) due to the axial shortening of the arch, the elongation of the deck acting as a tie between the springings, and, finally, the effect of the bending moments at both the arch and the deck.

$$\delta_{T,x=-L/4} = \delta_{T1,x=-L/4} + \delta_{T2,x=-L/4} + \delta_{T3,x=-L/4} \quad (17)$$

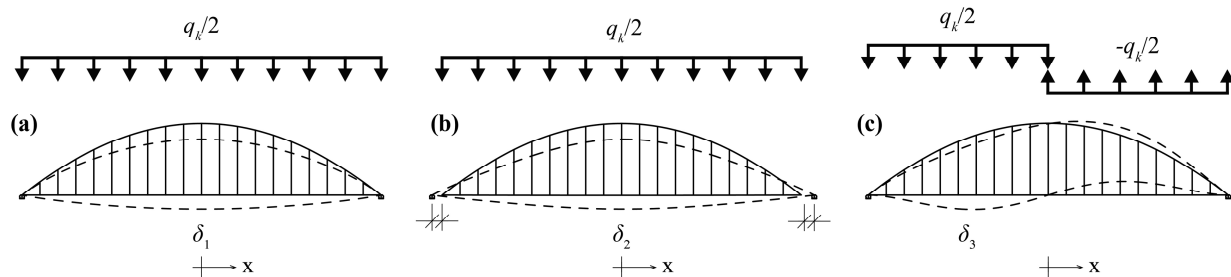


Figure 9. Deflections due to (a) axial shortening, (b) tie elongation, and (c) flexural deformation.

According to Nettleton [30], the following expression can be used to estimate δ_{T1} at the crown ($x = 0$), where N_A is the axial force at the arch under SLC-S and f is its rise:

$$\delta_{T1,x=0} = \frac{N_{A,x=0}}{E_A \cdot \Omega_A} \cdot \left(\frac{L^2}{5 \cdot f} + f \right) \quad (18)$$

For estimating δ_{T2} , Equation (18), where N_D is the axial force at the deck under SLC-S, adapted from Buchholdt [31], originally developed at $x = 0$ for a funicular cable with flexible supports, provides good results:

$$\delta_{T2,x=0} = \frac{120 \cdot L^4 - 320 \cdot f^2 \cdot L^2 + 2304 \cdot f^4}{640 \cdot f \cdot L^3 - 3072 \cdot f^3 \cdot L} \cdot \frac{N_D \cdot L}{E_D \cdot \Omega_D} \quad (19)$$

The values of N_A at $x = 0$ (which is a compressive axial force) and N_D (which is a constant tension axial force, since the deck is only horizontally loaded at the springings) can be obtained from

$$N_{A,x=0} = N_D = \frac{q_k \cdot L^2}{16f} \quad (20)$$

The deflection δ_{T3} at the quarter-span under SLS-A can be estimated using the classical explicit expression

$$\delta_{T3,x=L/4} = \frac{\frac{q_k}{2} \cdot \left(\frac{L}{2} \right)^4}{384 \cdot (E_A I_A + E_D I_D)} = \frac{q_k \cdot L^4}{12,288 \cdot (E_A I_A + E_D I_D)} \quad (21)$$

In the studied bridges, $\delta_{T1} + \delta_{T2}$ is usually several times smaller than δ_{T3} , which means that the deflections are mainly due to the bending moment of the structure, which are due to SLC-A, and not to its axial forces, which are due to SLC-S.

To apply the method, the values of δ_{T1} and δ_{T2} must be calculated at the location of the checkpoint ($x = -L/4$, Figure 9), where δ_D is compared to δ_{lim} . To obtain the

deflection at $x = -L/4$, the values of $\delta_{T1,x=0}$ and $\delta_{T2,x=0}$ must be factored by two coefficients, $k_1 \equiv \delta_{T1,x=-L/4}/\delta_{T1,x=0}$ and $k_2 \equiv \delta_{T2,x=-L/4}/\delta_{T2,x=0}$, as follows:

$$\delta_{Tx=-L/4} = k_1 \cdot \delta_{T1,x=0} + k_2 \cdot \delta_{T2,x=0} + \delta_{T3,x=-L/4} \quad (22)$$

As a first approximation, k_1 and k_2 can be considered 0.75, which corresponds to the assumption that the deformed shape of the deck under SLC-S is a 2nd-degree-parabola:

$$\delta_{Tx=-L/4} \approx 0.75 (\delta_{T1,x=0} + \delta_{T2,x=0}) + \delta_{T3,x=-L/4} \quad (23)$$

More precise values of k_1 and k_2 will be obtained in Section 6.

4.3. Correction of Area Between Iterations

As just shown in Equations (18) and (19), $\delta_1 + \delta_2$, due to SLC-S, depend on Ω_A and Ω_D , and δ_3 , due to SLC-A, depends on $I_A + I_D$ according to Equation (21). Therefore, the three deflections can be rewritten in the following way, in which the coefficients a_i gather in one constant the remaining terms of each equation:

$$\delta_D = \delta_{T1} + \delta_{T2} + \delta_{T3} = \frac{a_1}{\Omega_A} + \frac{a_2}{\Omega_D} + \frac{a_3}{E_A I_A + E_D I_D} \quad (24)$$

As shown in Equation (14), Ω_D can be expressed as a function of Ω_A , and it is determined to be the same for $E_A \cdot I_A + E_D \cdot I_D$ using the auxiliary coefficient μ :

$$E_A I_A + E_D I_D = E_A I_A (1 + \mu) = E_A \cdot \frac{3\Omega_A^2}{32 \cdot \beta_A} (1 + \mu) \quad (25)$$

Thus, Equation (24) can be rewritten in such a way that Ω_A is the only unknown variable and a_i is just new auxiliary coefficients (i.e., different from those shown in the Equation (24)) that gather the remaining terms of the expression:

$$\delta_D = \delta_{T1} + \delta_{T2} + \delta_{T3} = \frac{a_1}{\Omega_A} + \frac{a_2}{\Omega_A} + \frac{a_3}{\Omega_A^2} \quad (26)$$

Gathering δ_{T1} and δ_{T2} in the same term, Equation (26) can be rewritten as

$$\delta_D = \delta_{T1} + \delta_{T2} + \delta_{T3} = \frac{a_{12}}{\Omega_A} + \frac{a_3}{\Omega_A^2} \quad (27)$$

Therefore, for tied-arch bridges with vertical hangers, once δ_D is calculated for a given area Ω_A (for example, as a result of a FEM calculation), the value of δ_D for a different value of the area of the arch, Ω_A' , can be expressed as

$$\delta_D(\Omega_A') = \Omega_A' \cdot \frac{\delta_1 + \delta_2}{\Omega_A} + (\Omega_A')^2 \cdot \frac{\delta_3}{\Omega_A^2} \quad (28)$$

This equation can be used (step i) to iteratively refine the value of Ω_A in the δ -method.

4.4. Flowchart

Figure 10 shows the flowchart of the δ -method adapted to tied-arch bridges with vertical hangers, where steps *i*, *f*, and *g*, inside the dashed rectangles, have been rewritten, according to Equations (23) and (28), as detailed in this Section.

δ-METHOD FOR TIED-ARCH BRIDGES (W/VERTICAL HANGERS)

DATA:

- Bridge definition: L, f, E_A, E_D , cable arrangement, etc.
- SLC
- Location of checkpoint, $\mathcal{S}_{lim}$
- β_A, β_D
- Selected set of values C_A^K ($0 \leq C_A^K \leq 1$)

ITERATIVE PROCESS:

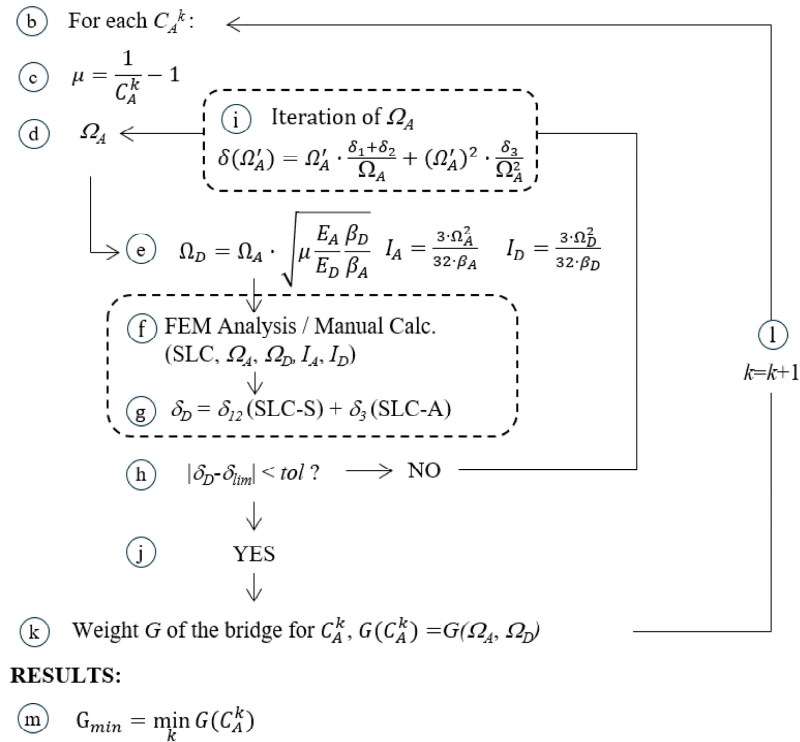


Figure 10. A flowchart of the δ -method adapted to tied-arch bridges with vertical hangers.

5. Example of Application

5.1. Tied Arch with Vertical Hangers

To illustrate the capabilities of the adapted δ -method detailed in Section 4, the weight minimization of the arch and the deck of the tied-arch bridge defined in Figure 11 is now fully developed. The main data are $\psi = 0.4$, $b = 10$ m, $q = 5$ kN/m² (therefore, $q_k = b \cdot \psi \cdot q = 20$ kN/m), $L = 100$ m, $f/L = 0.2$, $\delta_{lim} = L/2000 = 50$ mm, and $\beta_A = \beta_D = 1/100$. The results for a set of 25 values for C_A between 0.02 and 0.98 are shown in Figures 11–13.

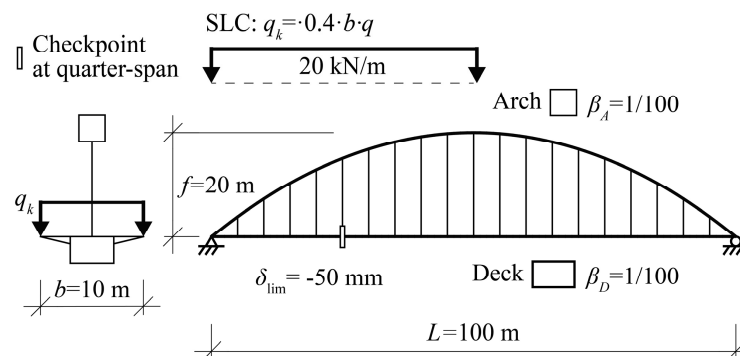


Figure 11. Bridge definition.

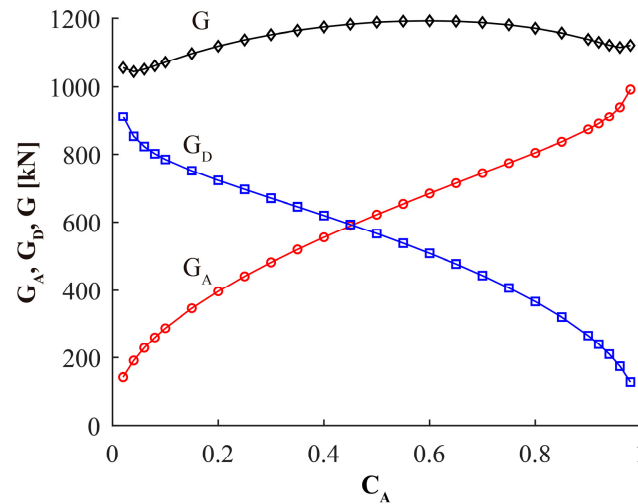


Figure 12. Arch weight, G_A ; deck weight, G_D ; and combined weight, $G = G_A + G_D$ vs. C_A , for the bridge defined in Figure 11.

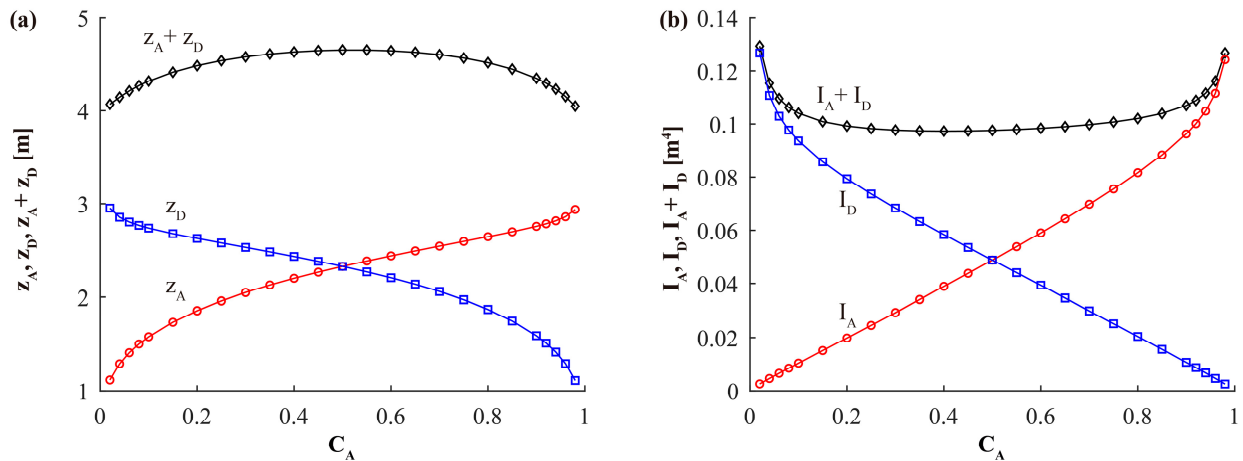


Figure 13. The results for the bridge defined in Figure 11: (a) z_A , z_D , and $z_A + z_D$ vs. C_A ; (b) I_A , I_D , and $I_A + I_D$ vs. C_A .

The initial value of Ω_A when first implementing the method is obtained by assuming that $\delta_3/(\delta_1 + \delta_2) = 1.2$, which is a frequent value in the bridges studied, and leads to

$$\delta_{lim} = \frac{L}{2000} \geq 1.2 \cdot \delta_{T3} = 1.2 \cdot \frac{5 \cdot q_k \cdot \left(\frac{L}{2}\right)^4}{384 \cdot E_A \cdot I_A \cdot (1 + \mu)} \quad (29)$$

Once I_A is obtained, the initial value of Ω_A can be obtained from Equation (11).

As observed in Figure 12, the combined weight of the arch and the deck, $G = G_A + G_D$, is maximum in the central zone. The minimum weight is obtained for very low values of C_A , i.e., for arches with very low bending capacity, practically deck-stiffened arches. Slightly heavier bridges with C_A values close to 1 (i.e., with very stiff arches and very flexible decks) are also lighter than those in the central zone. The lightest bridges are obtained for the lowest values of C_A as, since G depends both on the length of the arch L_A and the span of the deck L (Equation (15)), and the algorithm tends to penalize heavy arch cross-sections rather than heavy deck cross-sections, because L_A is always greater than L .

Figure 13a represents the depth of the arch cross-section, z_A ; the deck cross-section, z_D ; and $z_A + z_D$ vs. C_A for the bridge defined in Figure 11. Figure 13b shows the second moment of area both of the arch and deck cross-sections, I_A and I_D , and $I_A + I_D$ vs. C_A .

5.2. Comparison with Real Bridges

To compare the results of the δ -method with the sizes of cross-sections of real bridges, the δ -method was applied a second time to the bridge defined in Figure 11, where all the data, except β_A and β_D , are repeated. The method has been applied to two bridges: the first bridge, where $\beta_A = \beta_D = 0.04$, and the second bridge, where $\beta_A = \beta_D = 0.08$. The weights of the arch, G_A ; the deck, G_D ; and $G_A + G_D$ are shown in Figure 14a for both values of β . In Figure 14b, the depths of the cross-sections of both the arch and the deck, z_A and z_D , as well as the total depth of the arch and deck, $z_A + z_D$, are plotted for the two values of β . As shown, the values of $z_A + z_D$ lie within the range recommended by Lebet and Hirt [5], i.e., between $L/30$ and $L/45$, and the range defined by the set of values compiled from real bridges by Mermigas and Wang [3], i.e., between $L/25$ and $L/50$.

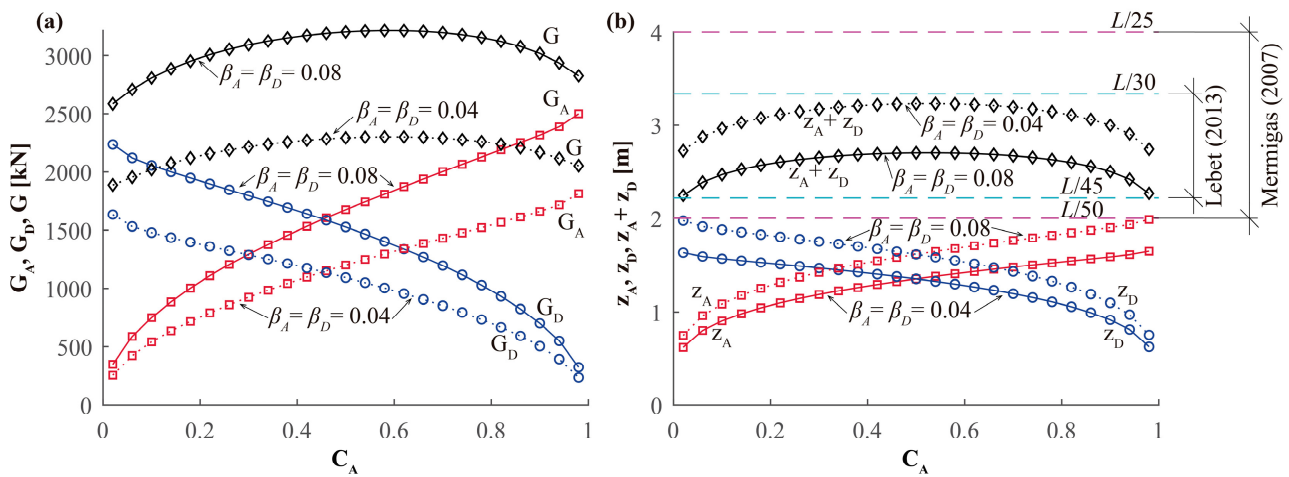


Figure 14. The results for the bridge defined in Figure 11 for $\beta_A = \beta_D = 0.04$ and $\beta_A = \beta_D = 0.08$: (a) G_A , G_D , and $G = G_A + G_D$ vs. C_A ; (b) z_A , z_D , and $z_A + z_D$ vs. C_A and comparison with the ranges defined in references [3,5].

Both case studies illustrate the effect of β on cross-section weight and depth. Lower values of β (i.e., thin webs) result in deeper and lighter cross-sections, and vice versa. Thus, if the designer decides to reduce the cross-section depth by increasing β (for example, for esthetic reasons), the cross-section weight will be increased, as can be deduced from Equation (9).

5.3. Influence of the SLC

As explained in Section 4.1, the SLC used in this paper corresponds to a UDL acting in the left half of the span. Other SLCs correspond, for example, to a UDL over the middle third of the span, as described by Menn in [7]. Figure 15 compares the weights obtained for both SLCs for the bridge defined in Figure 11 for $\beta_A = \beta_D = 0.05$. The same value of $\delta_{lim} = L/2000 = 50$ mm is considered for both SLCs, although it is on the safe side for the SLC over the middle third. As shown in the figure, the UDL acting in the left half of the span is the most adverse SLC.

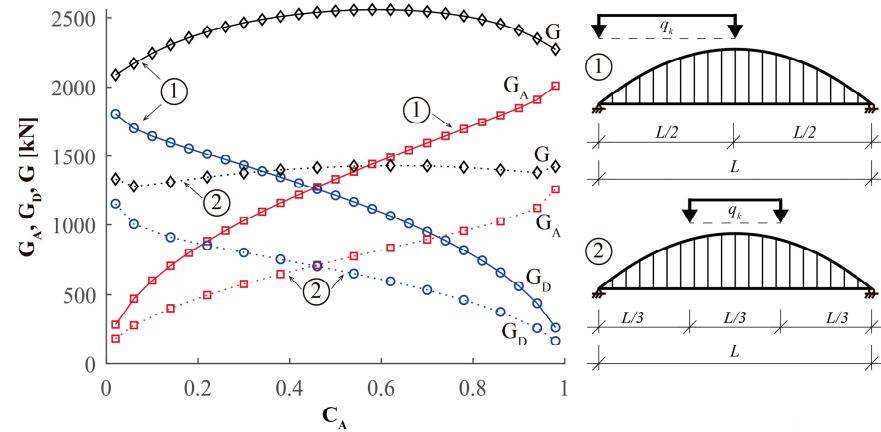


Figure 15. A comparison of results for the bridge defined in Figure 11 for $\beta_A = \beta_D = 0.05$ for different SLCs.

6. Preliminary Parametric Studies

To understand the influence of the parameters that govern the design of the bridges shown in this paper, a parametric study is carried out in Section 7. To reduce the number of models in that section, two preliminary minor studies, corresponding to the load q_k and the allowable deflection, δ_{lim} , are shown in this section. As a result, the number of bridges studied in Section 7 has been reduced to 5544.

6.1. Effect of the Load q_k

One of the most relevant factors for preliminary sizing is the load q_k . Figure 16a shows, for $C_A = 0.3$, the results of Ω_A , Ω_D , and $\Omega_A + \Omega_D$ for values of bridge width b varying between 2.5 and 20 m (and q_k between 5 and 40 kN/m).

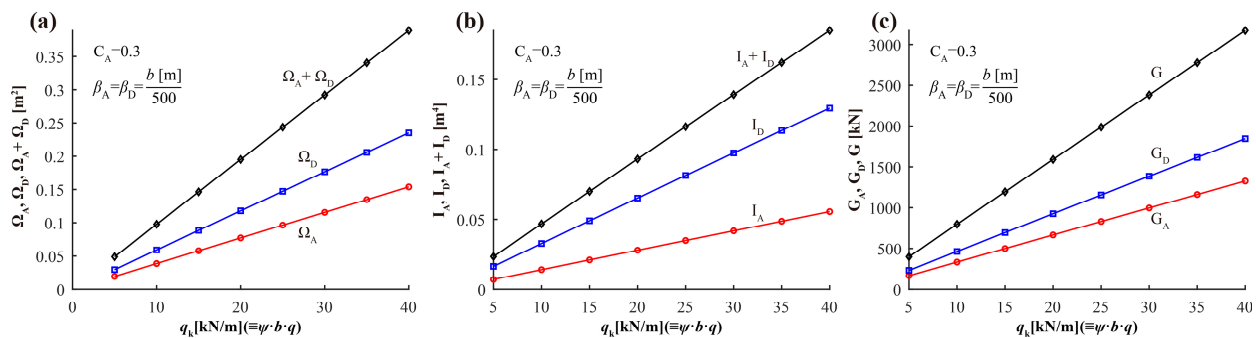


Figure 16. (a) q_k vs. Ω_A , Ω_D , and $\Omega_A + \Omega_D$; (b) q_k vs. I_A , I_D , and $I_A + I_D$; (c) q_k vs. G_A , G_D , and $G = G_A + G_D$.

For each value of b , a value of $\beta_A = \beta_D = b/500$ has been considered, i.e., for example, for $b = 5$ m, $\beta_A = \beta_D = 0.01$. As shown, all curves are linear. The same is true for the second moments of area, shown in Figure 16b, and the weights, shown in Figure 16c.

If the weight per unit load, G/q_k , is plotted (Figure 17a), the lines are horizontal. The same is true in Figure 17b, which shows the depths of the cross-sections. Therefore, the main conclusion here is that the results per unit width of the bridge coincide when the web thickness ($t_w = \beta \cdot z$) is proportional to the value of b (i.e., of q_k).

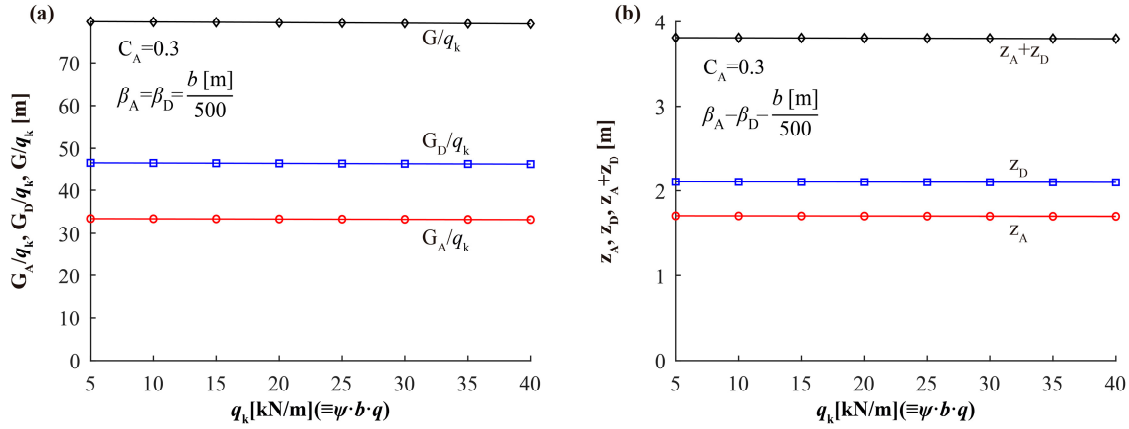


Figure 17. (a) q_k vs. G_A/q_k , G_D/q_k , and $G_A/q_k + G_D/q_k$; (b) q_k vs. z_A , z_D , and $z_A + z_D$.

6.2. Effect of δ_{lim}

Figure 18a shows, for $C_A = 0.3$, the results of I_A , I_D , and $I_A + I_D$ for values of L/δ_{lim} varying between 500 and 2500. As shown, all curves can be assumed to be linear with sufficient accuracy. The same applies to Figure 18b, which represents Ω_A^2 and Ω_D^2 , and Figure 18c, where the weights G_A^2 and G_D^2 are drawn. The main conclusion here is that the relationship between L/δ_{lim} and I_A and I_D can be considered linear. The same is true for Ω_A^2 and Ω_D^2 or G_A^2 and G_D^2 .

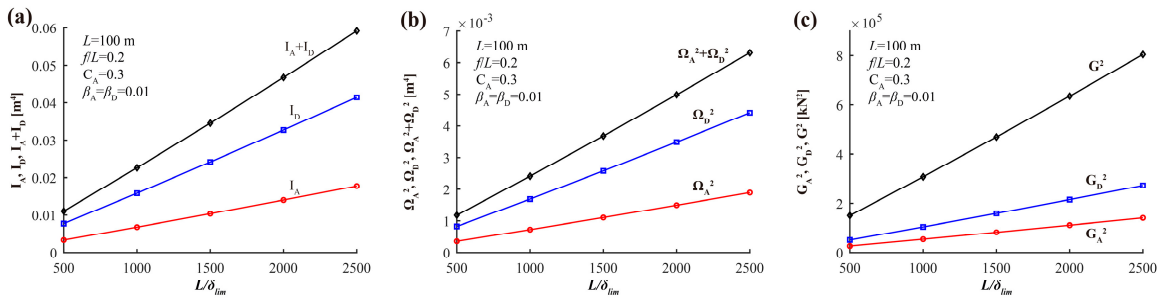


Figure 18. (a) I_A , I_D , and $I_A + I_D$ vs. L/δ_{lim} ; (b) Ω_A^2 , Ω_D^2 , and $\Omega_A^2 + \Omega_D^2$ vs. L/δ_{lim} ; (c) G_A^2 , G_D^2 , and $G_A^2 + G_D^2$ vs. L/δ_{lim} .

7. Parametric Studies

To estimate the effect of the main parameters on the weight of the bridge, G , the models detailed in Table 1 are generated using nested loops. The δ -method is applied for n_i variations in the i most relevant parameters, resulting in $n = \Pi(n_i) = 5544$ FEM models. Data and results are available on request. In every model, the δ -method follows the flowchart detailed in Figure 10. Computational flow control is performed using MatlabR2017b [32], and the FEM analyses are carried out using SAP2000 v26 [33]. The typical elevation of the models is shown in Figure 19.

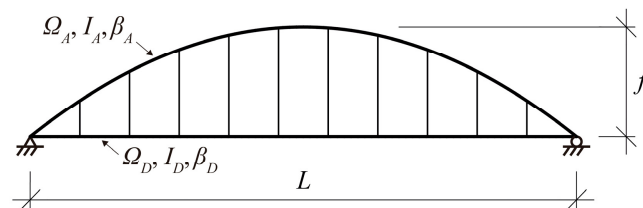


Figure 19. The elevation of one of the analyzed models.

Table 1. Parametric analysis: case studies.

Symbol	Units	Values	n_i
L	m	50 75 100 150	4
f/L	-	0.25 0.2 0.167 0.143 0.125 0.111 0.10	7
C_A	-	0.05 0.10 0.20 0.30 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95	11
β_A	-	1/25 1/50 1/100	3
β_D	-	1/25 1/50 1/100	3
q_k	kN/m	10	1
δ_{lim}	m	$L/500$ $L/2000$	2
Total n models = $P(n_i)$			5544

Other data values used for all the models are $b = 5$ m, $\psi = 0.4$, $q = 5$ kN/m², $E_A = E_D = 200$ GPa, and $\gamma = 78.5$ kN/m³.

7.1. Relationship Between Inertia, Area, and Weight and Span

Since δ_{lim} is defined as a factor of $1/L$, it follows, from Equation (29), that the inertia I_A (or I_D) is proportional to L^3 , as shown in Figure 20a for different values of the ratio f/L . As, based on Equation (11), the cross-sectional area of the arch is proportional to $I_A^{1/2}$, Ω_A is also proportional to $L^{3/2}$, as shown in Figure 20b.

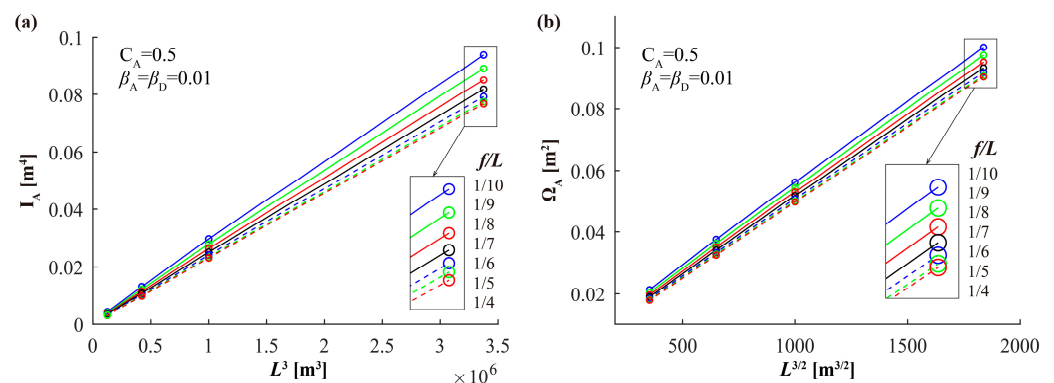


Figure 20. $C_A = 0.5$ and $\beta_A = \beta_D = 0.01$ for different values of f/L : (a) I_A vs. L^3 ; (b) Ω_A vs. $L^{3/2}$.

According to the results, since the weight of the bridge (see Equation (15)) is obtained by multiplying the area by the span of the bridge, it is proportional to $L^{5/2}$, as shown in Figure 21a. It is also interesting to observe the influence of the f/L ratio on G . Figure 21b shows that, in these examples, the value of $G/L^{5/2}$ is lower for f/L values between 0.15 and 0.2 and the weight of the bridge increases for both shallower and deeper arches. In the same example, the smaller the L value, the higher the ratio $G/L^{5/2}$.

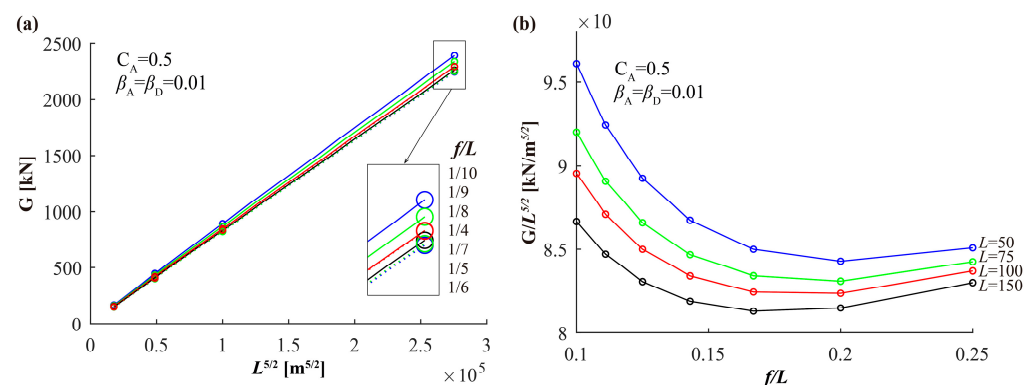


Figure 21. $C_A = 0.5$ and $\beta_A = \beta_D = 0.01$: (a) G vs. $L^{5/2}$ for different values of f/L ; (b) $G/L^{5/2}$ vs. f/L for different values of L .

To illustrate the variability in the values of C_A for which minimum-weight bridges are obtained, Figure 22 shows the results for various values of L ($L = 50, 75, 100$, and 150 m) and f/L ($f/L = 1/10, 1/6$, and $1/4$). In all cases, minimum-weight bridges show a clear tendency to appear at points with low C_A values.

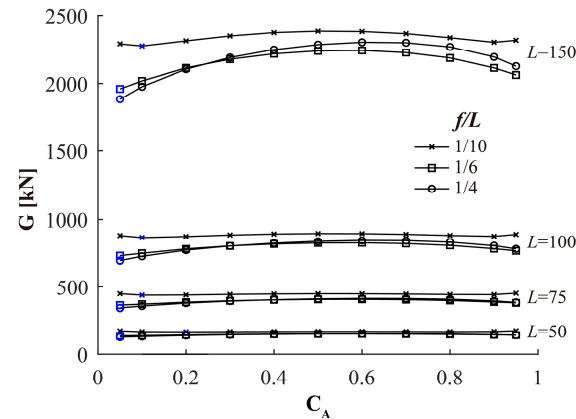


Figure 22. $\beta_A = \beta_D = 0.01$: G vs. C_A for different values of L and f/L .

7.2. Effect of the Web Slenderness

As shown in Figure 23a, the effects of β_A and β_D on the weight of the bridge are quite significant. β_A remains constant, being equal to 0.01, while β_D varies between 0.01 and 0.04. As shown, the more slender the web, the lighter the cross-section, because the required inertia can be achieved with less material. In the results, G_A remains virtually unchanged for different values of β_D . However, G_D increases considerably, since, for higher values of β_D (i.e., thicker webs), the deck needs a larger area to achieve the target value.

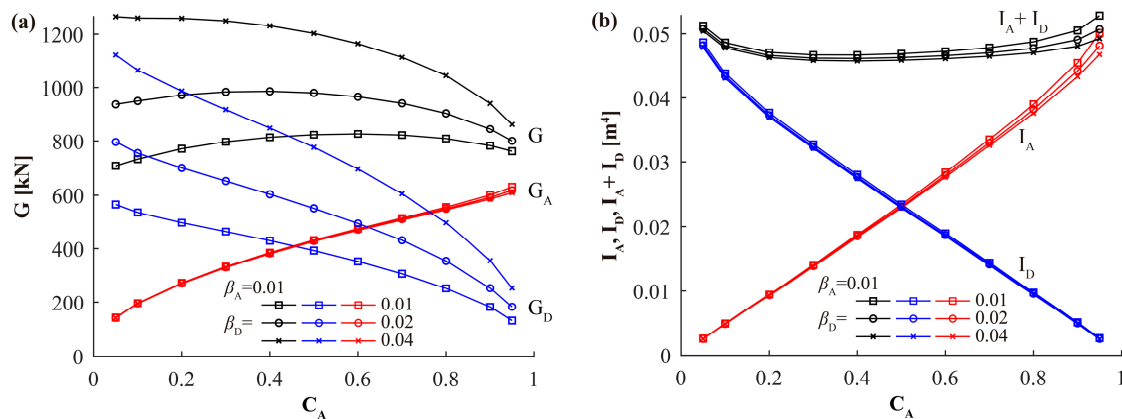


Figure 23. $L = 100, f/L = 0.20, \beta_A = 0.01$: (a) G vs. C_A ; (b) I_A, I_D , and $I_A + I_D$ vs. C_A for different β_D .

Nevertheless, the increase in G_D with β_D shown in Figure 23a is not echoed in the variation in I_D shown in Figure 23b, which is almost independent of β_D . This represents the fact that I_A and I_D are the main parameters governing deflections. Therefore, using sections with very slender webs reduces the area required to achieve the required level of stiffness. In conclusion, cross-sections with the thinnest possible webs (with low β_A and β_D values) should be used to minimize the bridge weight.

8. Manual Calculation of Tied-Arch Bridges

The FEM analyses required for the applications of the δ -method presented in this paper are not especially complex: once the bridge has been modeled with its cross-sections, only two load combinations are needed: the SLC-S and SLC-A. Depending on the results,

the cross-sections are corrected at each iteration until convergence is achieved. This process, which is already very fast, can be accelerated even further through automation. Using a very simple code to implement the flowchart shown in Figure 10, each case study, including all necessary iterations, takes approximately 2 s, with most of the time spent on the FEM analyses in SAP2000 [33]. However, even with the availability of automated tools for pre- and post-processing, the iterative process can be very time-consuming, particularly when performing a parametric study that requires many different parameters. Therefore, it would be very helpful to the designer if the calculations could be performed using an explicit manual yet accurate formulation able to distinguish the effects of each parameter and inform design decisions.

For this purpose, the coefficients k_1 and k_2 that multiply the “theoretical” approximate values $\delta_{T1,x=0}$ and $\delta_{T2,x=0}$ in Equation (22) have been adjusted to ensure sufficient accuracy for manual calculations, and a new third coefficient k_3 that multiplies $\delta_{T3,x=-L/4}$ has been introduced with the same purpose. The three coefficients have been obtained with the help of the “exact” values of $\delta_{12} = \delta_1 + \delta_2$ (under SLC-S) and δ_3 (under SLC-A) obtained from the 5544 FEM analyses carried out in Section 7. In our case, since δ_{12} was calculated all at once in the FEM analysis, it was impossible to distinguish which fraction of δ_{12} corresponds to δ_1 and which one corresponds to δ_2 . Therefore, we took $k_1 = k_2 = k_{12}$, a condition represented in Figure 24 with the diagonal $k_1 = k_2$. The coefficient k_{12} was obtained so that the mean of δ_{12} for the n FEM models coincided with the mean of $k_1 \cdot \delta_{T1} + k_2 \cdot \delta_{T2}$. The solution was $k_1 = k_2 \approx 0.71$. The process is shown in Figure 24, where each point of the Figure 24a belongs to an isoline of Equation (30). For these values, a mean of 0.9993 and a standard deviation (Figure 24b) of 0.0354 were obtained. Equation (30) is defined as follows:

$$\frac{1}{n} \sum_1^n \left(\frac{\delta_{12,x=L/4}}{k_1 \cdot \delta_{T1,x=0} + k_2 \cdot \delta_{T2,x=0}} \right)_i \quad (30)$$

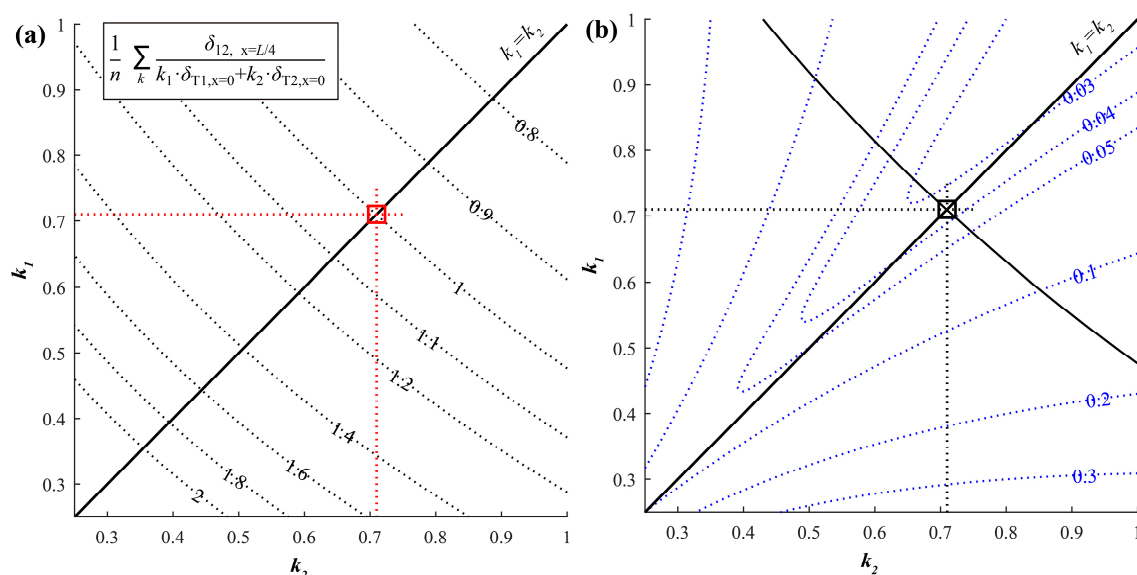


Figure 24. The numerical determination of the coefficients k_1 and k_2 : (a) mean Equation (30) and (b) standard deviation.

The value of the coefficient of k_3 for the n bridges has been obtained as follows by simply comparing “theoretical” and “real” deflections:

$$k_3 = \frac{1}{n} \sum_1^n \left(\frac{\delta_{3,x=L/4}}{\delta_{T3,x=L/4}} \right)_i \quad (31)$$

resulting in $k_3 = 1.03$. For this value, a mean of 0.9994 and a standard deviation of 0.0254 are obtained.

Therefore, following these analyses, the δ -method can be carried out in the same way as shown in Section 4, where the values of δ_{12} and δ_3 obtained from the FEM output can be substituted by the following theoretical formulas ($\delta_{T1,x=0}$, $\delta_{T2,x=0}$, and $\delta_{T3,x=L/4}$; Equations (18), (19) and (21)) factored by the just obtained coefficients:

$$\delta_{12,x=L/4} \approx 0.71 \cdot (\delta_{T1,x=0} + \delta_{T2,x=0}) \quad (32)$$

$$\delta_{3,x=L/4} \approx 1.03 \cdot \delta_{T3,x=L/4} \quad (33)$$

which leads to the following approximate formulation:

$$\delta_{D,x=-\frac{L}{4}} \approx 0.71 \cdot \left[\frac{N_{A,x=0}}{E_A \cdot \Omega_A} \cdot \left(\frac{L^2}{5 \cdot f} + f \right) + \frac{120 \cdot L^4 - 320 \cdot f^2 \cdot L^2 + 2304 \cdot f^4}{640 \cdot f \cdot L^3 - 3072 \cdot f^3 \cdot L} \cdot \frac{N_D \cdot L}{E_D \cdot \Omega_D} \right] + 1.03 \cdot \frac{q_k \cdot L^4}{12,288 \cdot (E_A I_A + E_D I_D)} \quad (34)$$

To assess the validity of Equation (34), the histogram in Figure 25 shows the ratio between the “real” results obtained via FEM analysis and the “theoretical” results for the n bridges obtained via manual calculations.

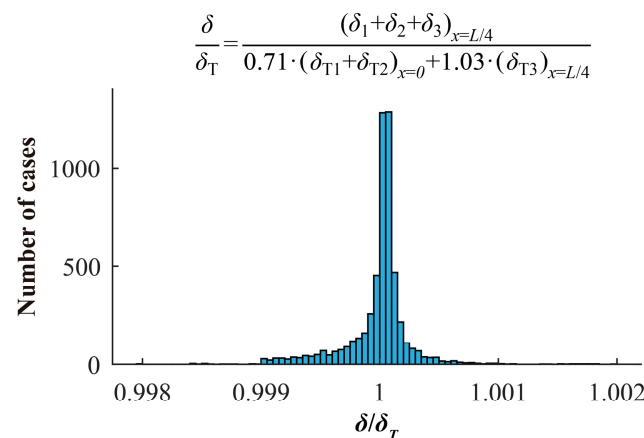


Figure 25. $(\delta/\delta_T)_i$ histogram.

Similarly, to verify the validity of the above expressions in evaluating the bridge weight, the n results of the ratio $(G/G_T)_i$ are shown (Figure 26a), where G has been calculated with FEM models and G_T has been calculated with the approximated theoretical formulation.

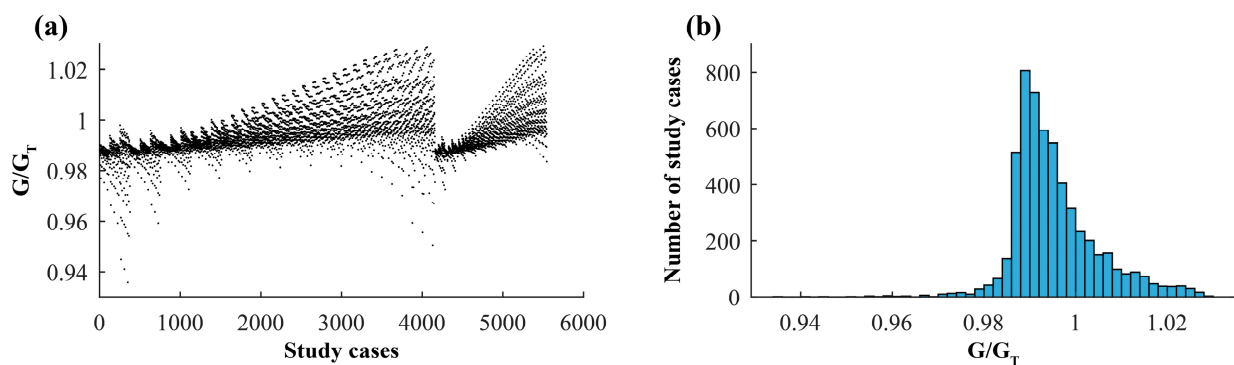


Figure 26. $(G/G_T)_i$: (a) graph; (b) histogram.

The mean of $(G/G_T)_i$ is 0.9953, and its standard deviation is 0.0093. Regarding the error, 227 models out of 5544 contain an error that is greater than $\pm 2\%$, while only 24 contain an error greater than $\pm 3\%$. These results are considered sufficiently accurate for a manual analysis, especially as the advantage in time consumption is immense: while SAP2000 calculations take about 2 s per case study (about 11,000 s in total), the same 5544 calculations with the theoretical formulation take between 2.5 and 2.8 s for all case studies. In other words, the approximate calculations are about 4000 times faster. The process, therefore, seems clear: if many models are to be developed, the first studies of the solution should be carried out using the manual approximate formulas (Equations (20) and (32)–(34)), whereas computer calculations should be used in the final stages of the process, when more accurate results are required.

9. Conclusions

The methods, formulations, and parametric studies described in this paper, as well as their conclusions, are designed to assist designers in the conceptual design of all-steel arch bridges, particularly when minimizing the bridge weight is a priority.

This paper presents a novel iterative method, the δ -method, that minimizes the weight of an all-steel arch bridge during the in-plane preliminary design stage. This method reproduces the iterative structure of the preliminary design process. Its input data are variables commonly used in preliminary design, such as the slenderness of the webs of the cross-sections and their relative flexural stiffness. It can be successfully applied to different typologies, such as deck-stiffened, tied-arch, Nielsen–Löhse, or network-type arch bridges. This paper emphasizes how relevant good engineering judgement may become in the early stages of conceptual design.

The δ -method, as presented in this paper, is limited to Class 1, 2, and 3 (classified according to Section 5.5 of Eurocode 3 [14]) sections, since the cross-sections are assumed to be doubly symmetric and their whole areas are considered when estimating I_A and I_D . In practical terms, $\beta = 1/100$, which corresponds to the web slenderness for pure bending of a Class 3 section for S355 steel (according to Table 5.2. of Eurocode 3 [14]) can be considered, approximately, to be a lower bound for web slenderness.

The method can be applied without restricting the span, L , or stiffness ratio, C_A . However, as is usually mentioned regarding arch bridge design (as it happens in references [1,2] or [3]), very low rise–span ratio values should be avoided, as internal forces and sensitivity to geometrical non-linearity increase significantly, especially for $f/L < 1/10$.

Moreover, in this paper, the general method has been particularized for tied-arch bridges with vertical hangers: Firstly, its flowchart has been adapted to this type of bridge. Secondly, an approximate analytical formulation that allows manual calculations is provided. This formulation can be considered sufficiently accurate for a manual analysis. Finally, a parametric study with 5544 FEM models, which illustrates the effect of the main variables on the weight of the bridge, is carried out.

9.1. Conclusions from Parametric Studies

The main conclusions of the parametric studies can be summarized as follows:

1. Minimum-weight bridges usually are obtained for very low C_A values, i.e., for arches with very low bending stiffness, which are practically deck-stiffened arches. The reason for this is that G depends both on the length of the arch, L_A , and the span of the deck, L , and, consequently, the algorithm tends to penalize heavy arch cross-sections rather than heavy deck cross-sections, as $L_A > L$. This trend is confirmed in the remaining parametric studies carried out.

2. When the web thickness ($t_w = \beta \cdot z$) is proportional to the value of q_k (which is the value of the UDL used for the SLC), the area of the cross-sections, Ω ; their inertias, I ; and the weight of the bridge, G , are directly proportional to q_k . In this case, the ratios G/q_k and z/q_k for the arch and the deck are constant for any value of q_k .
3. Regarding δ_{lim} , the relationship between L/δ_{lim} and I_A and I_D can be considered linear. The same thing happens for Ω_A^2 and Ω_D^2 or G_A^2 and G_D^2 .
4. The parametric study has shown that I_A or I_D , Ω_A or Ω_D , and G are directly proportional to L^3 , $L^{3/2}$, and $L^{5/2}$, respectively, for all the studied values of f/L .
5. Regarding the influence of the f/L ratio on G , in the studied bridges, the value of $G/L^{5/2}$ is lower for f/L values between 0.15 and 0.2, and G increases for both shallower and deeper arches.
6. In the same example, the smaller the L value, the higher the ratio $G/L^{5/2}$.
7. Cross-sections with the thinnest possible web (with low values of β_A and β_D) should be used to minimize the bridge weight.
8. Thicker webs (with higher values of β_A and β_D) should be used to reduce the depths of the cross-sections.

9.2. Design Recommendations

According to the developed formulations and the results of the parametric studies, the following design recommendations should be adopted to minimize the weight of the bridge:

1. Use rise–span ratios f/L between 1/5 and 1/7, with f/L being more than 1/10;
2. Use sections with very different stiffness levels in the arch and deck, preferably with very flexible arches;
3. Use the thinnest possible web thickness, β , to achieve higher stiffness with less material, with β being more than 1/100.

Readers should bear in mind that these conclusions and design recommendations are valid for tied-arch bridges with vertical hangers, where the connection between the arch and the deck is hinged. The design is assumed to be governed by the maximum allowable in-plane deflection. Therefore, different design recommendations could be obtained for other criteria, such as in-plane and out-of-plane buckling, fatigue, ultimate strength, and constructability issues, or for different arch bridge typologies.

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Abbreviations

The following symbols and abbreviations are used in this paper:

b	Loaded deck width (Figure 1)
C_A	Contribution of the arch to the global flexural stiffness of the bridge (Section 2.3, Equation (1))
E_A, E_D	Young's modulus for the materials of the arch and the deck
f	Rise of the arch (Figure 1)
FEM	Finite elements method
g_{DL}	Superimposed dead load (Figure 1)
G_A, G_D, G	Arch, deck, and bridge weight.
I	Inertia (second moment of area) of a cross-section (Section 2.5, Equation (4))
I_A, I_D	Second moment of area (inertia) of the arch and deck cross-sections (Section 3, Equations (11) and (12))
I_{max}	Maximum second moment for a given area and web slenderness (Section 2.5, Equation (9))
k_1, k_2, k_3	Correction coefficients of theoretical formulation (Section 8).
L	Bridge span (Figure 1)
L_A	Length of the arch
N_A, N_D	Axial force at crown of the arch and at the deck (Section 4.2, Equation (20))
q	Live load
q_k	Live load used for the SLC in the δ -method
SLC	Simplified load combination used for the preliminary design stage (Sections 1.1 and 2.2)
SLC-A, SLC-S	Symmetric and antisymmetric load combinations in which the SLC is decomposed (Section 4, Figure 8)
t_w	Web thickness (Figure 4)
tol	Tolerance of iterative process (Section 2.1, Figure 7)
UDL	Uniformly distributed load
z	Web depth, approximately equal to cross-section depth (Figure 4)
z_A, z_D	Arch and deck web depth
z_I	For a given area, web depth that maximizes the second moment of area (Section 2.5, Equation (8))
β	Geometrical slenderness of the web(s) (Section 2.5, Equation (3))
β_A, β_D	Geometrical slenderness of the web of the arch and deck
γ	Density of the arch and deck materials
δ	Vertical deflection at the arch
$\delta_1, \delta_2, \delta_3$	Deflections, calculated with FEM, due to axial shortening of the arch, axial elongation of the deck, and a UDL q acting upon half of the deck
δ_D	Total deflection ($\delta_1 + \delta_2 + \delta_3$) at the deck checkpoint
δ_{lim}	Maximum allowable deflection at the deck checkpoint
$\delta_{T1}, \delta_{T2}, \delta_{T3}$	Values of $\delta_1, \delta_2, \delta_3$, calculated with theoretical formulae (Figure 9, Equations (18), (19), and (21))
μ	Auxiliary coefficient (Equation (10))
Ω	Area of a cross-section (Figure 4)
Ω_A, Ω_D	Area of the arch and deck cross-sections
Ω_f, Ω_w	Areas of one flange and the web of the cross-section (Figure 4)

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