

Article

Numerical Investigation of the Influence of the Stress Multiaxiality on the Propagation Behavior of Rayleigh Waves

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Abstract

The influence of stress state multiaxiality on the propagation velocity of Rayleigh waves is explored through a detailed numerical study. The study uses the Murnaghan model to capture nonlinear elastodynamics in the material behavior, necessitating consideration of third-order elastic constants. Various invariant stress variables are compared for their suitability to describe the relationship between multiaxiality of the stress state and change in propagation velocity. The results are interpreted physically and provide information about the interaction between stress state multiaxiality and wave propagation. Finite element simulations are conducted using Abaqus/Explicit, with the material behavior implemented via a VUMAT user subroutine. Transformation relations for rotated axes are used to understand how the stress state affects the directional dependence of wave velocity. This study offers valuable insights into the complex relationship between stress state and Rayleigh wave propagation, essential for applications in reconstruction of residual stress fields. The results show that the change in propagation velocity is best described by models that include the principal stresses. Different stress states lead to different distortion of the propagation front. The numerical results are compared and validated with the semianalytical solution. The results show good agreement.

Keywords: acoustoelastic effect; Rayleigh waves; stress triaxiality; wave propagation; FEA; Murnaghan model; wave front; residual stress



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1. Introduction

Ultrasonic measurements have long served as a tool for defect detection, notably for identifying cracks or voids within engineering components. Such measurements exploit the fundamental principle that sound waves exhibit variable propagation velocities in different media. The surface roughness of engineering surfaces can be determined using ultrasonic waves. Yuan et al. [1] dealt with this in their work on the scattering of ultrasonic waves by rough surfaces. Liang et al. [2], in contrast, focused on the dispersion and attenuation of surface acoustic waves. Liang analyzed this effect using numerical simulations and then validated it with experimental data. Ultrasonic technology is also suitable for detecting microstructural changes (represented by the dislocation density) in metal physics.

Ultrasonic measurements are additionally useful for assessing residual stresses within engineering components. The propagation velocity of both solid and surface waves shows a weak, but nevertheless detectable, dependence on the stress state, a phenomenon known as the acoustoelastic effect and extensively documented in the literature. More precisely, the acoustoelastic effect does not result from the stress state but, rather, from the strain

state. Biot [3] was the first to study the influence of initial stresses on elastic waves. In his theoretical work, Biot showed that the propagation of elastic waves under initial stresses follows laws that cannot be explained by elastic anisotropy or the change in second-order elastic constants.

In their publication, Hughes and Kelly [4] describe the stress-dependent change in longitudinal and transverse waves for the first time mathematically and validate the results with experimental data. With their scientific work, Hughes and Kelly established the theory of acoustoelasticity and, thus, provided the foundation for the nondestructive detection of residual stresses using ultrasonic waves. Their theory is based on Murnaghan's theory of nonlinear elasticity. Murnaghan [5,6] extended the classical theory of linear elasticity by expanding the elastic potential or the strain energy density function. The influence of a uniaxial stress state on all longitudinal and transverse waves was described by Egle and Bray [7] in their publication and validated using experimental data. Bland [8] published a paper in the field of nonlinear elastodynamics. Toupin and Bernstein [9] described the acoustoelastic effect for perfectly elastic materials. Hayes and Rivlin [10] describe in their publication the propagation of plane volume waves for a homogeneous deformation field analytically. In their work, Pao et al. [11] describe the fundamentals of acoustoelasticity and the measurement of residual stresses.

Crecraft [12] was among the first to show the applicability of the acoustoelastic effect for estimating the stresses in engineering components. Salamanca [13] measured the stresses caused by welding processes in steel using critically refracted longitudinal waves. The same idea was used by Srinivasan et al. [14] for analyzing cast components. Tang and Bray [15] investigated the influence of stress and plastic deformation on the propagation velocity of critically refracted longitudinal waves. The dependence on plastic strain was described using regression functions but was not physically interpreted. The influence of the stress gradient was investigated by several scientists on the basis of experiments; see the works of Bescond et al. [16] and Si-Chaib et al. [17,18], where Si-Chaib investigated the influence of the stress gradient on the velocity of longitudinal waves based on the three-point bending test. Ivanova et al. [19] experimentally investigated the influence of stress and stress gradient on the propagation velocity of Rayleigh waves using a beam with constant bending moment and constant stress gradient along the length of the beam. This work focuses on the propagation behavior of Rayleigh waves. Rayleigh waves combine both longitudinal (compression and dilation motions) and transverse (shear motions) oscillations. The particles in a medium move in an elliptical path. Understanding the effect of stress multiaxiality on Rayleigh wave propagation velocity is crucial for reconstructing residual stress states in engineering components.

The aim of this work is to describe the dependence of the Rayleigh wave propagation velocity on the existing stress state with as few parameters as possible. In addition, these parameters should be easy to interpret in a physically meaningful way. This paper evaluates different stress descriptors, such as principal stresses and Mises equivalent stress, aiming at finding a universally applicable method for assessing the influence of multiaxiality. The relationship is generalized in order to retain the vector character of the velocity. Additionally, this study seeks to mathematically describe the directional dependence of propagation velocity, which allows statements about the existing stress triaxiality. Moreover, understanding the distortion of the wave propagation front as a function of the existing stress state allows us to determine the principal stresses.

2. Materials and Methods

2.1. Numerical Simulation

The finite element software Abaqus/Explicit 2023 (Dassault Systèmes, Vélizy-Villacoublay, France). is employed for conducting numerical simulations, where the material behavior represented by the Murnaghan model is implemented via a VUMAT user subroutine. The excitation signal is applied as a pressure within a designated region of the model, with the amplitude of the signal defined by means of a VDLOAD subroutine. Figure 1 illustrates the excitation signal.

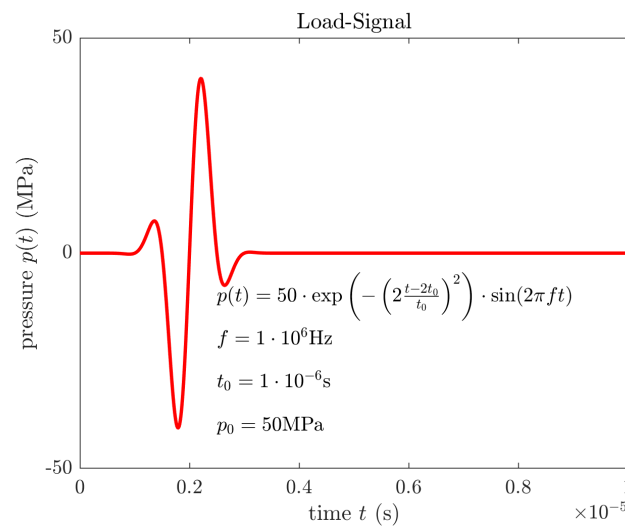


Figure 1. Input signal (pressure–time curve) used for introducing the acoustic surface wave, along with the corresponding equation integrated into the VDLOAD subroutine.

The excitation signal is mathematically expressed as

$$p(t) = p_0 \exp\left(-\left(2\frac{t-2t_0}{t_0}\right)^2\right) \sin(2\pi ft) \quad (1)$$

where $p(t)$ denotes the pressure as a function of time t , f represents the frequency of the wave, and t_0 stands for a reference time. The pre-existing stress field, emulating residual stresses in a real world scenario, is applied in a quasi-static load step. The load amplitude is slowly increased during a sufficiently long step time to mitigate dynamic artifacts due to inertia effects. For the subsequent wave propagation analysis, in order to accurately resolve wave propagation dynamics, the time increments are set to a sufficiently small value of 0.05 ns. Figure 2 shows the wave propagation in the material in terms of the magnitude of the velocity vector.

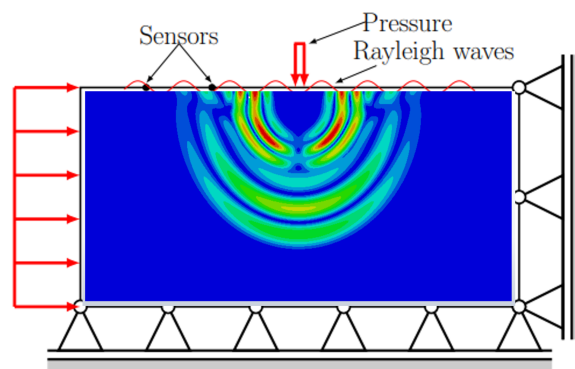


Figure 2. Wave propagation analysis in the material; the magnitude of the velocity vector is shown.

The second- and third-order elastic constants of the material used in the numerical simulations are shown in Table 1. The material data is sourced from [20].

Table 1. Elastic constants of 2nd and 3rd order of the material used for the numerical simulations.

λ (MPa)	μ (MPa)	ν_1 (MPa)	ν_2 (MPa)	ν_3 (MPa)
90,900	78,000	−728,000	−265,000	179,000

The geometric configuration of the finite element model is designed to mimic a cuboid with a rectangular cross-section. The model is discretized using continuum elements with linear shape functions and reduced integration (ABAQUS nomenclature C3D8R). To ensure high-fidelity representation of acoustic wave propagation, the element size is set to $1/10\lambda$, where λ denotes the wavelength.

2.2. Methodology

As depicted in Figure 3, mechanical stress is uniformly applied in the form of a homogeneously distributed load acting in x-direction, either in the compressive (+) or tensile (−) direction. Additionally, uniform stress is exerted in the y-direction to address multiaxial stress conditions.

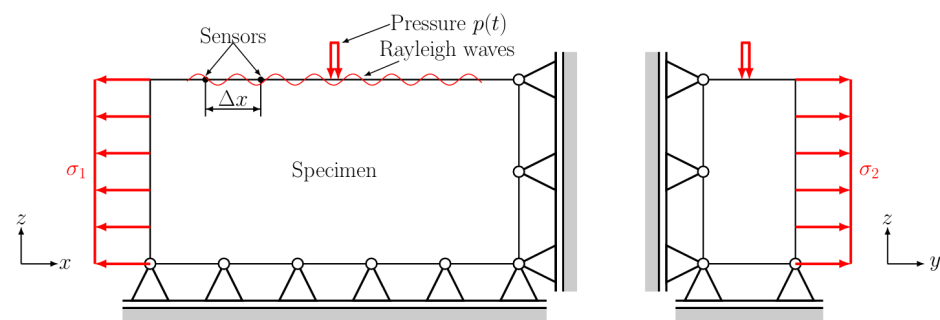


Figure 3. Finite element model for simulating the impact of stress multiaxiality on the propagation velocity of Rayleigh waves. The virtual sensor placement is essential for the determination of the velocity change.

Virtual sensors (see Figure 3) are attached to the finite element (FE) model to ascertain the propagation velocity of Rayleigh waves. These virtual sensors are positioned at equal distances along the model's bottom edge. They serve the purpose of detecting the displacements or velocities of material points of the structure under investigation. The propagation time is measured from the displacement history observed in the detected signals recorded by the two sensors. Knowing the time difference of the arriving signals and the distance between the sensors allows us to compute the wave propagation velocity.

$$v_R = \frac{\Delta x}{\Delta t} \quad (2)$$

In this equation, Δx represents the distance between the two sensors, while Δt denotes the time shift. However, it is imperative to correct the distance between the sensors due to the alterations induced by elastic strain. For the sake of simplicity, Hooke's law is used here for the displacement correction. This simplification is admissible as the differences to the displacement correction as predicted by the Murnaghan model are negligible and the expressions become much more manageable. The corrected distance, denoted as Δx , is calculated as follows:

$$\Delta x = \Delta x_{\text{init}} \left[1 + \frac{1}{E} (\sigma_x - \nu \sigma_y) \right] \quad (3)$$

Here, Δx_{init} stands for the distance in the unloaded state, while E and ν represent the second-order elastic constants. Additionally, σ_x and σ_y denote the prevalent stresses.

2.3. Signal Processing

Cross-correlation between the original and the distorted signal is a common method for assessing similarity and detecting phase shifts. A substantial drop in correlation suggests that the distorted signal deviates significantly from a linear response. This mathematical technique evaluates the similarity between two signals by measuring how well one signal aligns with the other at various time shifts (lags). In signal processing, cross-correlation is frequently employed to determine the time shift between two signals.

$$R_{xy}(\tau) = (x \star y) = \int_{-\infty}^{\infty} x(t) \cdot y(t + \tau) dt \quad (4)$$

$R_{xy}(\tau)$ is the cross-correlation of two continuous signals $x(t)$ and $y(t)$ for a certain lag τ .

2.4. Analyzed Stress States

The stress states multiaxiality is characterized by the triaxiality parameter, denoted as ξ , which is defined as the ratio of hydrostatic stress p and the von Mises equivalent stress σ_{eq} (see Equation (5)).

$$\xi = \frac{p}{\sigma_{eq}} = \frac{\sigma_{ii}}{3 \left(\frac{3}{2} \sigma_{ij}^{dev} \sigma_{ij}^{dev} \right)^{1/2}} \quad (5)$$

Here, $\sigma_{ii} = \text{trace}(\sigma_{ij})$ denotes the trace of the stress tensor, and $\sigma_{ij}^{dev} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij}$ represents the deviatoric stress tensor, with δ_{ij} being the Kronecker Delta representing the unit tensor. The stress states selected to elucidate the influence of stress triaxiality are shown in Figure 4. Each stress state is represented by its corresponding Mohr's stress circle. Case (a) shows a uniform uniaxial tensile stress state, case (b) a pure shear stress state, case (c) a biaxial tensile stress state, and case (d) a generalized biaxial stress state. The corresponding equivalent von Mises stresses are denoted as σ_{eq} , and the hydrostatic stress component is p . The principal stresses are termed σ_1 , σ_2 , and σ_3 , and the maximum principal shear stress is τ_{max} .

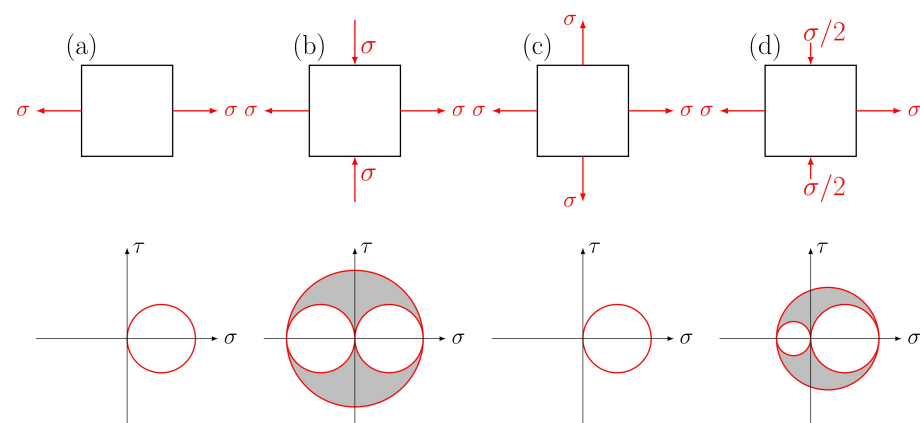


Figure 4. Visualization of stress states on the solid element and their corresponding representation in Mohr's stress circle for various scenarios: (a) Pure tension ($\xi = 0.333$), (b) pure shear ($\xi = 0$), (c) biaxial tension ($\xi = 0.667$), (d) general tension-compression (where $\xi = 0.125$).

In the numerical simulations, the stress value σ is incrementally increased. The triaxiality parameter is kept constant for each case (a) to (d). The findings from these simulations are discussed in the subsequent section. Different stress metrics are employed

to ascertain the most effective characterization of stress state effects on Rayleigh wave propagation velocity.

2.5. Murnaghan Model

To model the acoustoelastic effect, it is essential to capture the stress/strain-dependency of the elastic constants. The nonlinearity of the elastic constants follows from the atomic potential, which means that linear elasticity is merely a simplification for infinitesimal deformations. By extending the elastic potential (see Equation (6)) to include the third-order elastic constants, the Murnaghan model is able to describe the strain dependency of the entries of elasticity tensor. Furthermore, the Murnaghan model also captures the influence of nonlinear deformations by higher-order stress–strain relationship. The acoustoelastic effect often occurs at small deformations while the material is under significant stress in engineering components. The Murnaghan model is designed to model this situation as it considers higher order stresses that correlate with minimal deformations. Other hyperelastic material models, such as the neo-Hooke or Mooney–Rivlin models, are well suited for large deformations, but often only consider first-order nonlinear effects and are therefore less accurate for small deformations under stress and the acoustoelastic effect. Murnaghan’s model extends the strain energy density function through a Taylor series expansion, see Equation (6). Here and henceforth in this paper we adopt the common summation convention over repeated indices $i, j = 1 \dots 3$. This extension requires the introduction of third-order elastic constants, as also described in detail by Landau [21]. The extension leads to a slightly nonlinear component in the stress–strain relationship. Murnaghan’s model is often employed for modeling stress-dependent propagation velocities of ultrasonic waves in solid media, i.e., for describing the acoustoelastic effect.

$$U = \frac{\lambda}{2} \text{tr}(E_{ij})^2 + \mu \text{tr}(E_{ij}^2) + \frac{\nu_1}{6} \text{tr}(E_{ij})^3 + \nu_2 \text{tr}(E_{ij}) \text{tr}(E_{ij}^2) + \frac{4}{3} \nu_3 \text{tr}(E_{ij}^3) \quad (6)$$

Within the framework of elasticity, the elastic strain energy density, denoted by U , is governed by second-order elastic constants, λ and μ , commonly referred to as Lamé constants, alongside third-order elastic constants, ν_1 , ν_2 , and ν_3 , referred to as Murnaghan constants. The strain energy density in the Murnaghan model is described as a third-order function of the strain tensor.

$$U = \frac{1}{2} C_{ijkl} E_{ij} E_{kl} + \frac{1}{6} C_{ijklmn} E_{ij} E_{kl} E_{mn} \quad (7)$$

Here, U is the strain energy density (elastic potential), C_{ijkl} is the fourth-order elasticity tensor, C_{ijklmn} is the sixth-order elasticity tensor, and E_{ij} is the Green–Lagrange strain tensor. The fourth-order elasticity tensor contains the second-order elastic constants (Lame constants) λ , μ .

$$C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \quad (8)$$

The sixth-order elasticity tensor contains the third-order elastic constants (Murnaghan constants).

$$\begin{aligned} C_{ijklmn} = & 2\nu_3 \delta_{ij} \delta_{kl} \delta_{mn} + \nu_2 (\delta_{ij} \delta_{km} \delta_{ln} + \delta_{ij} \delta_{kn} \delta_{lm} + \delta_{kl} \delta_{mi} \delta_{nj} + \delta_{kl} \delta_{mj} \delta_{ni} \\ & + \delta_{mn} \delta_{ik} \delta_{jl} + \delta_{mn} \delta_{il} \delta_{jk}) + \frac{1}{4} \nu_1 (\delta_{ik} \delta_{jm} \delta_{kn} + \delta_{ik} \delta_{jn} \delta_{km} + \delta_{il} \delta_{jm} \delta_{kn} \\ & + \delta_{il} \delta_{jn} \delta_{km} + \delta_{jk} \delta_{im} \delta_{ln} + \delta_{jk} \delta_{in} \delta_{lm} + \delta_{jk} \delta_{im} \delta_{kn} + \delta_{jk} \delta_{in} \delta_{km}) \end{aligned} \quad (9)$$

The components of the first Piola–Kirchhoff stress tensor follows from the tensorial product of the deformation gradient F_{ij} with the partial derivative of the elastic potential U according to the Green–Lagrange strain tensor E_{ij} .

$$P_{ij} = F_{ik} \frac{\partial U}{\partial E_{kj}} \quad (10)$$

During nonlinear wave propagation, the total strain E_{ij} is decomposed into static prestrain E_{ij}^s and the strain through dynamic disturbance E_{ij}^d due to wave propagation. These strains do not interact with each other and cannot easily be superimposed through the nonlinearity.

$$E_{ij} \neq E_{ij}^s + E_{ij}^d \quad (11)$$

Figure 5 shows the kinematics of a material point in three different states. (I) shows the reference configuration, (II) shows the prestressed configuration. (III) shows the current configuration following a dynamic disturbance while passing through the elastic wave.

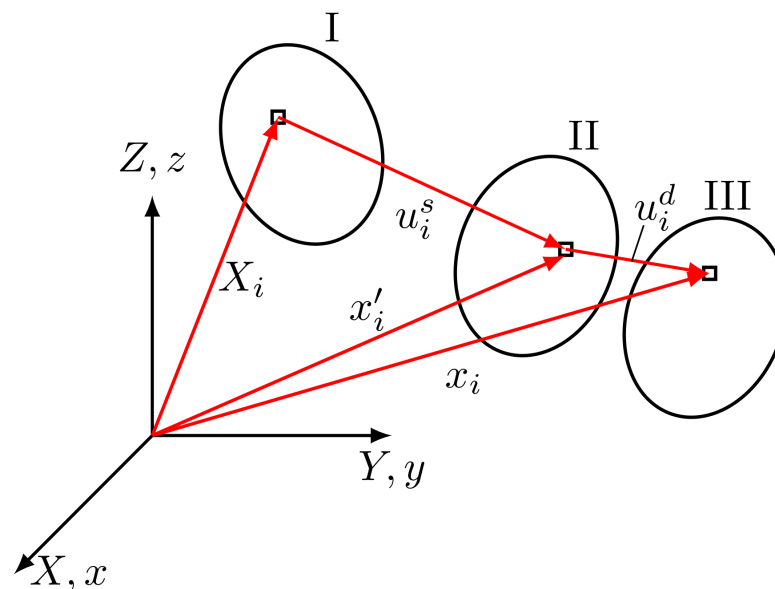


Figure 5. Kinematics of a material point in three different states. I shows the (initial) reference configuration. II shows the prestressed configuration. III shows the current configuration according to an additional dynamic disturbance (elastic wave).

The displacement vector u_i^s for the prestressed configuration (II) is given by

$$u_i^s = x'_i - X_i \quad (12)$$

The displacement vector for the dynamic disturbance u_i^d in the current configuration (III) is given by

$$u_i^d = x_i - x'_i \quad (13)$$

The first Cauchy's law of motion related to the reference state is given by

$$\rho_0 \frac{\partial^2 x_i}{\partial t^2} = \rho_0 \frac{\partial^2}{\partial t^2} (X_i + u_i^s + u_i^d) = \rho_0 \frac{\partial^2 u_i^d}{\partial t^2} = \nabla_j P_{ij} \quad (14)$$

The transformation from the reference configuration (Lagrange's description) to the prestressed configuration (Euler's description) follows according to the chain rule of

differential calculus and the use of the relationships between the displacement vectors. Furthermore, it is assumed that the dynamic disturbance is negligible compared to the strain in the prestressed state.

$$\nabla_j P_{ij} = \frac{\partial P_{ij}}{\partial X_j} \approx \frac{\partial P_{ij}}{\partial x_j} + \frac{\partial u_k^s}{\partial x_j} \frac{P_{ij}}{\partial x_k} \quad (15)$$

A second-order Taylor expansion of the constitutive law (Equation (10)) about the prestrained state and the equilibrium of the prestress yield the wave equation for the dynamic disturbance:

$$\rho_0 \frac{\partial^2 u_i^d}{\partial t^2} = B_{ijkl} \frac{\partial^2 u_j^d}{\partial x_k \partial x_l} \quad (16)$$

where B_{ijkl} considers the reduction of the effective stiffness through static prestrain. The derivative can be found in Appendix A.

$$B_{ijkl} = C_{ijkl} + \delta_{ik} C_{ijqr} u_{q,r}^s + C_{rjkl} u_{i,r}^s + C_{irkl} u_{j,r}^s + C_{ijrl} u_{k,r}^s + C_{ijkr} u_{l,r}^s + C_{ijklmn} u_{m,n}^s \quad (17)$$

3. Results

3.1. Fundamental Consideration

The aim of this work is to describe the stress state dependent change in velocity of Rayleigh waves. The correlation should be described with as few parameters as possible. The parameters should be physically motivated. The best invariant stress quantity to describe the influence of multiaxial stress states on the propagation velocity of Rayleigh waves has to be found first.

The multiaxiality of stress states is typically characterized with the triaxiality parameter (see Equation (5)). It therefore seems obvious to use the hydrostatic stress as well as the equivalent stress to describe the influence of the stress state on the change of the propagation velocity of Rayleigh waves. The definition of the Mises stress makes it an inappropriate measure to explain tension–compression asymmetry as it is, by definition, always positive and does not differentiate between tension and compression. Thus, better descriptions for the velocity function should be used. Both the hydrostatic stress and the equivalent stress according to von Mises can equally be expressed in terms of principal stresses, which also allows consideration of the tension–compression asymmetry of the propagation velocity.

From a physical standpoint, the acoustoelastic effect results from strain rather than stress, i.e., from the strain-dependent stiffness of the material, which is a consequence of the nonlinearity of interatomic potentials. Evidently, uniaxial tensile stress also causes changes in propagation velocity in the transverse direction. Consequently, the change in wave velocity should better be defined as a function of the principal strain vector. To this end, it is advisable to switch to Voigt notation of tensorial quantities, expressed in index notation, aligned with principal axes, $i = 1, \dots, 3$, in the principal strain space. The velocity vector v_i is expressed as a function of the tensor of acoustoelastic constants L_{ij} and the principal strain vector ε_j ,

$$\frac{\Delta v_i}{v_0} = L_{ij} \varepsilon_j \quad (18)$$

The wave propagation velocity in any given direction is directly linked to the atom spacing and, thus, the strain in that direction. Thus, there will be no coupling between a

velocity component in one direction and strain in a direction perpendicular to it, making the matrix of the acoustoelastic constants for the strain formulation L_{ij} a diagonal matrix,

$$L_{ij} = \begin{bmatrix} L & 0 & 0 \\ 0 & L & 0 \\ 0 & 0 & L \end{bmatrix} \quad (19)$$

For elasticity, stress and strain state are linked via Hooke's law $\sigma_i = C_{ij}\varepsilon_j$, with C_{ij} being the elasticity tensor in Voigt notation, aligned with principal axes, $i = 1, \dots, 3$ in the principal stress space. Therefore, we obtain

$$\frac{\Delta v_i}{v_0} = L_{ij}C_{jk}^{-1}\sigma_k \quad (20)$$

The result of the product from the tensor of the acoustoelastic constants in the strain formulation L_{ik} and the compliance tensor C_{ik}^{-1} is the tensor of the acoustoelastic constants in the stress formulation K_{ij} .

$$\frac{\Delta v_i}{v_0} = \frac{v_i - v_0}{v_0} = K_{ij}\sigma_j \quad (21)$$

where v_0 is the velocity of the Rayleigh waves in the unloaded case. The tensor of acoustoelastic constants for Rayleigh waves assumes the following form:

$$K_{ij} = \begin{bmatrix} K_1 & K_2 & K_2 \\ K_2 & K_1 & K_2 \\ K_2 & K_2 & K_1 \end{bmatrix} \quad (22)$$

3.2. Description Using Principal Stresses

Figure 6 shows the impact of the two in-plane principal stresses on the propagation velocity of Rayleigh waves. As expected, an increase in tensile loading entails a decrement in the propagation velocity of Rayleigh waves along the loading direction. On the other hand, as compressive stress grows in the transverse direction, a pronounced reduction of the propagation velocity is detected. In contrast, due to the Poisson effect, an increase in tensile stress along the transverse direction yields a less significant decrease of the velocity.

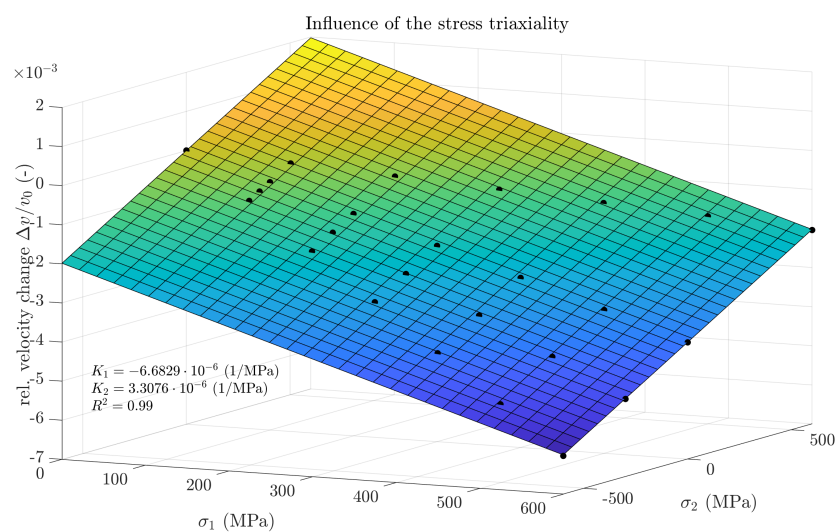


Figure 6. Influence of stress multi-axiality, characterized by the in-plane principal stresses, on the propagation velocity of Rayleigh waves, where the points represent the simulation data and the plane represents the mathematical description.

Equation (23) represents a plane in principal stress space that, if one of the principal stress components can be omitted—as this is the case for a plane stress state on the surface of a body—can be displayed graphically. In this representation, the relative change in propagation velocity, denoted as $\Delta v/v_0$, is expressed as a linear function of the principal stresses σ_1 and σ_2 .

$$\frac{\Delta v}{v_0} = K_1\sigma_1 + K_2\sigma_2 \quad (23)$$

In this equation, K_1 and K_2 signify the acoustoelastic constants of Rayleigh waves. This model exhibits good agreement with simulation data across all stress states ($R^2 = 0.99$). In reality, not only do the principal stresses within the plane of wave propagation impact the propagation velocity of Rayleigh waves, but also the principal stress normal to the plane of wave propagation and some distance underneath the surface, which calls for an additional acoustoelastic constant K_3 associated with the third principal stress σ_3 .

$$\frac{\Delta v}{v_0} = K_1\sigma_1 + K_2\sigma_2 + K_3\sigma_3 \quad (24)$$

This additional modification improves the already good agreement with simulation data even further to a correlation coefficient of $R^2 = 0.986$. These results show that while it is very easy to determine the influence of general stress states on the change in propagation velocity using numerical methods, it is difficult or impossible to realize it experimentally.

3.3. Separation of the Effects of Stress Multiaxiality and Other Nonlinearities

Nonlinear wave propagation describes the changes in wave propagation that are influenced by effects outside the linear range. With linear wave propagation, the frequency, wave shape, and wave velocity remain constant, provided that the material is homogeneous and isotropic. Nonlinearities change the propagation velocity and shape of the wave as it propagates through the material.

A nonlinear influencing factor is the nonlinearity of the material behavior. The acoustoelastic effect results from this nonlinearity. In addition, this nonlinearity leads to an increase or reduction in the displacement of a material point compared to the linear elastic solution. In the case of nonlinear elastic material behavior, the stress amplitude as the wave propagates also affects the propagation velocity. At low amplitudes, wave propagation is approximately linear in many unloaded materials. However, as the amplitude increases, the nonlinear effects increase because the stress–strain relationship becomes increasingly nonlinear. This leads to effects such as the distortion of the wave shape, the generation of harmonics, and a change in wave velocity. For waves that propagate in curved or highly deformed materials, the geometry of the medium itself can have a nonlinear effect on the wave.

To analyze the distortion due to the nonlinearity of a signal, various techniques and metrics can be used to quantify the deviations of the signal from its ideal, linear form.

Influence of the Amplitude

Firstly, the influence of the stress amplitude of the propagating surface wave on the velocity is analyzed for the unloaded material. During the propagation of the Rayleigh wave, normal stresses occur parallel and normal to the surface as well as shear stresses. The hydrostatic stress is used to characterize the stress state. Figure 7 shows the influence of the hydrostatic stress on the propagation velocity of the wave. The results show that the related velocity changes are very small compared with the residual stresses to be analyzed. For the given hydrostatic stress of $p = 2.319$ MPa, the maximum correlation also barely

deviates from 1. This means that the influence of the amplitude on the wave propagation velocity can be neglected.

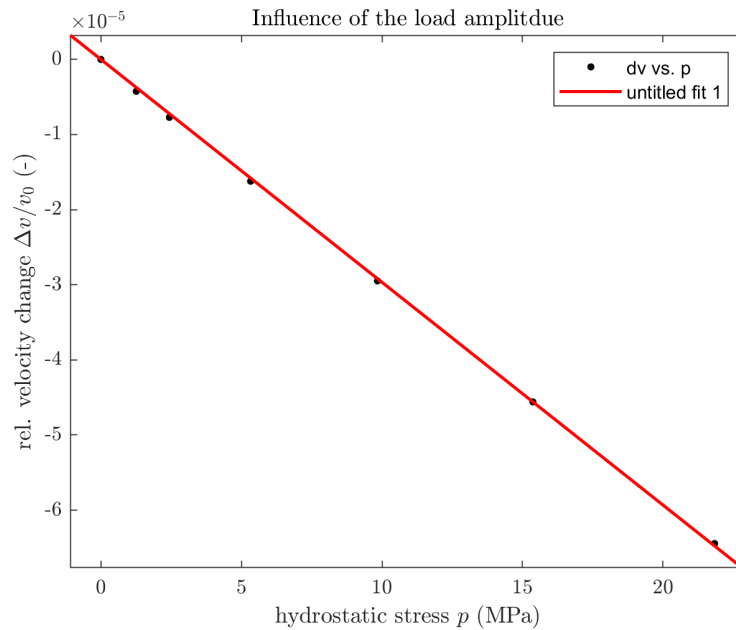


Figure 7. Influence of the hydrostatic stress of the wave propagation on the propagation velocity of the Rayleigh wave.

As the analyzed structure is neither curved nor strongly nonlinearly deformed, these sources of nonlinearities can also be neglected. The changes in velocity can, therefore, be attributed to the existing stress state.

3.4. Distortion of the Wave Propagation Front

In the absence of external stress, Rayleigh waves propagate uniformly in all directions within the plane, forming a circular wave front under the condition of isotropic material behavior. However, the introduction of mechanical stresses distorts this wave front into an elliptical shape. When subjected to uniaxial tensile stress along the 1-direction, the propagation velocity diminishes in the loading direction while increasing in the transverse direction due to the Poisson effect.

The velocity changes along the two principal axes directions are mathematically described for general principal stress states by the following equations:

$$\frac{\Delta v_1}{v_0} = \frac{v_1 - v_0}{v_0} = K_1 \sigma_1 + K_2 \sigma_2 + K_2 \sigma_3 \quad (25)$$

$$\frac{\Delta v_2}{v_0} = \frac{v_2 - v_0}{v_0} = K_1 \sigma_2 + K_2 \sigma_1 + K_2 \sigma_3 \quad (26)$$

In this context, Δv_1 represents the velocity change along the 1-direction, while Δv_2 denotes the change in propagation velocity along the 2-direction. Equations (25) and (26) define the primary axes of the elliptical distortion. The entire ellipse is then described by the following equation:

$$\frac{\Delta v_1^2}{v_0^2 (K_1 \sigma_1 + K_2 \sigma_2 + K_2 \sigma_3)^2} + \frac{\Delta v_2^2}{v_0^2 (K_1 \sigma_2 + K_2 \sigma_1 + K_2 \sigma_3)^2} = 1 \quad (27)$$

In polar coordinates, the variation in velocity can be expressed as follows:

$$\Delta v(\varphi) = v_0 \frac{(K_1\sigma_1 + K_2\sigma_2 + K_2\sigma_3)(K_1\sigma_2 + K_2\sigma_1 + K_2\sigma_3)}{\sqrt{(K_1\sigma_1 + K_2\sigma_2 + K_2\sigma_3)^2 \sin^2 \varphi + (K_1\sigma_2 + K_2\sigma_1 + K_2\sigma_3)^2 \cos^2 \varphi}} \quad (28)$$

Figure 8 illustrates the directional dependence of Rayleigh wave propagation velocity in polar representation across various stress states for the same maximum principal stress σ_1 . It effectively visualizes the polar form shown in Equation (28). The dotted circles indicate the change in velocity in meters per second. The radial lines indicate the angles in radians. Uniaxial stresses distort the propagation front, while, as expected, biaxial stress states induce velocity changes without altering the wave front's shape, which remains circular. Biaxial tension uniformly reduces propagation velocity in all directions, whereas biaxial compression uniformly increases it.

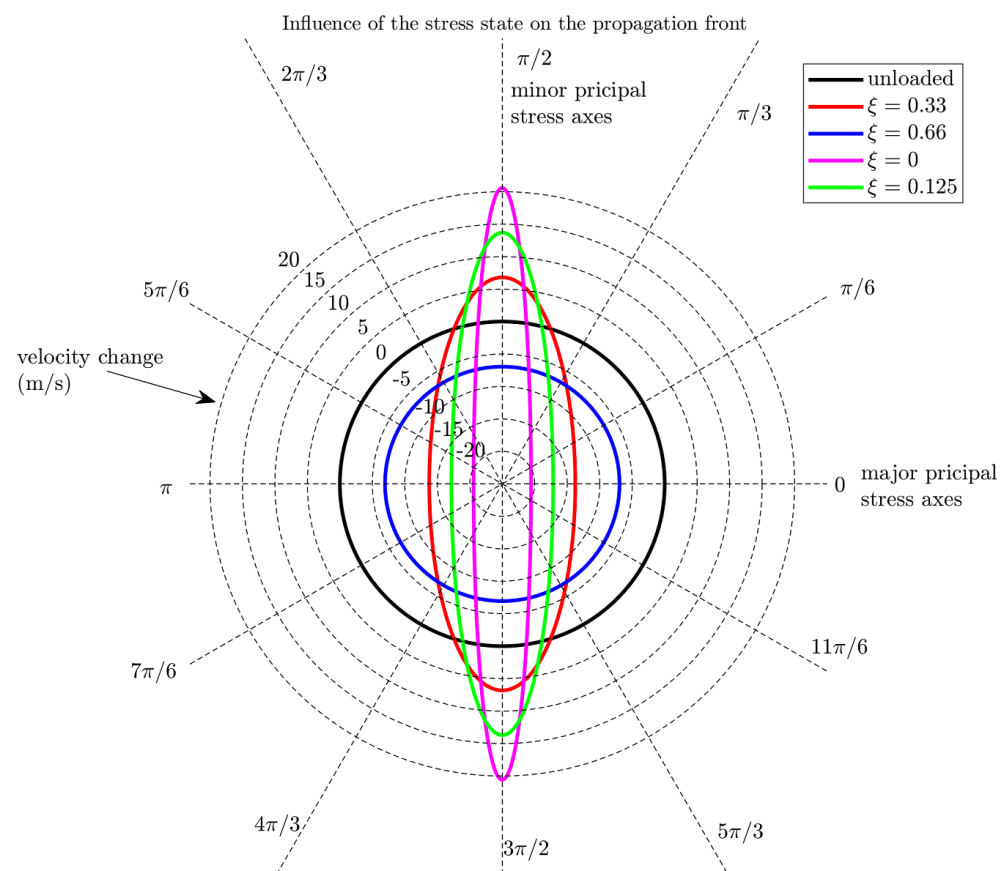


Figure 8. Directional dependence of the change in Rayleigh wave propagation velocity, depicted in polar coordinates, for various stress states: Uniaxial tension ($\xi = 0.33$), pure shear ($\xi = 0$), biaxial tension ($\xi = 0.66$), and a general uniaxial stress state ($\xi = 0.125$).

By incorporating the rotation tensor R_{ij} , the velocity changes can be expressed as follows:

$$\frac{\Delta v'_i}{v_0} = R_{ij} K_{jk} \sigma_k = R_{ij} \frac{\Delta v_j}{v_0} \quad (29)$$

The rotation tensor is defined as follows:

$$R_{ij} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (30)$$

Understanding the distortion of the wave front allows us to make conclusions about the existing stress state (triaxiality), as well as detect the principal stress axes and recalculate the principal stresses themselves. If the directions of the principal axes are known, two velocity measurements in precisely these directions are necessary to determine the principal stresses. If the principal axes are unknown, the wave front has to be reconstructed. This is achieved by measuring the velocities in three different directions. The directions of the principal axes and the changes in velocity in these directions then have to be determined.

3.5. Validation of the Results

Rayleigh waves are a combination of longitudinal and transverse waves. The implicit function, that describes the velocity of the wave propagation as a function of the entries of the elasticity tensor, the determinant of the deformation gradient and the density, is given below in Equation (31). For its derivation, see [22].

$$0 = \left(\frac{(JB_{1111} - \rho_0 c_R^2)(JB_{1212} - \rho_0 c_R^2)}{(JB_{2222})(JB_{2121})} \right)^{1/2} [JB_{2222}(JB_{1111} - \rho_0 c_R^2) - (JB_{1122})^2] + (JB_{1111} - \rho_0 c_R^2)(JB_{1212} - \rho_0 c_R^2 - JB_{2121}) \quad (31)$$

Due to the strong nonlinearity, it is not possible to give a closed analytical solution. Nevertheless, a semianalytical solution can be found by solving the analytical Equation (31) numerically. Nonlinear effects such as the influence of amplitude or the influence of dispersion on the propagation velocity cannot be covered by this equation because only homogeneous stress states are taken into account. Because in the Murnaghan model the components of the elasticity tensor depend on strain, the Green–Lagrange strain tensor has to be determined iteratively. The starting value is guessed on the basis of linear elasticity. The Frobenius norm of the strain tensors is then calculated. If the residual is too large, the calculation steps are repeated. If the residual is sufficiently small, the nonlinear equation can be solved numerically. As the nonlinear equation may have multiple solutions, the range has to be chosen in a meaningful way. The velocity difference to the velocity in the unloaded state is then calculated. Table 2 shows a comparison of the related change in velocity between the FE solution and the analytical solution for a first principle stress $\sigma_1 = 600$ MPa in the axial direction and the different triaxiality parameters. The relative error between simulation and semianalytical solution increases as the change in velocity decreases. As the table shows, the simulation results describe the trends quite accurately. Deviations can be attributed, for example, to the discretization of the system and the explicit solution of the time integration.

Table 2. Comparison of the relative error between semianalytical solution and FE solution for the related propagation velocity at a first principal stress $\sigma_1 = 600$ MPa and different stress states.

Stress Multiaxiality ζ	$\Delta v/v_0$ (Semianalytic)	$\Delta v/v_0$ (FE)	Rel. Error
0.33	-3.35×10^{-3}	-3.41×10^{-3}	1.76%
0	-6.018×10^{-3}	-6.094×10^{-3}	1.25%
0.66	-2.109×10^{-3}	-2.158×10^{-3}	2.27%
0.125	-5.043×10^{-3}	-5.118×10^{-3}	1.47%

4. Discussion

The numerical study highlights the substantial impact of stress state triaxiality on Rayleigh wave propagation velocity. Optimal representation of the influence of multiaxial stress state is achieved through principal stresses. Following the initial goal of this work, the dependence of the propagation velocity of Rayleigh waves on the prevalent stress state is described with a minimal set of physically interpretable material parameters, making its use for practical applications as convenient as possible. The presented linear model demonstrates a strong agreement with the data obtained from numerical simulations. The high coefficient of determination ($R^2 = 0.99$) indicates that the model effectively captures the relationship between stress and wave velocity for a variety of stress states. The resulting linear Equation (23) derived in this work closely resembles the results reported by Bach [23] for transverse waves. According to [23], a proportional relationship exists between the relative velocity difference and the principal stress in solids. Bach found his results experimentally using biaxial stress states. Our results exhibit the same general trends as the experimental data reported by Bach [23], showing a linear dependence on the applied principal normal stresses. However, a direct comparison of absolute values is not possible, as Bach investigated a different material, and the third-order elastic constants, which strongly affect the velocity variations, are material-specific. Furthermore, it should be noted that Bach did not examine surface waves but focused on shear wave propagation.

In Section 3, a system of equations was introduced to describe the change in propagation velocity along the principal axes (see Equation (21)). This formulation is based on the tensor of acoustoelastic constants and the stress vector. In the inverse problem, the propagation velocities and the acoustoelastic tensor are assumed to be known, while the principal normal stresses are unknown. These stresses can be determined by solving the system of equations. If the directions of the principal stress axes are known, the problem is significantly simplified, as only the velocity changes along those specific directions are required. However, if the principal axes are unknown, velocity measurements must be conducted in three distinct directions. From these, the distortion of the wave front can be inferred. The shape and orientation of the resulting ellipse provide information on the directions of the principal stresses. By applying the rotation tensor, the velocity changes can be transformed into the principal stress coordinate system (see Equation (29)). The principal normal stresses can then be calculated by solving the system of equations given in Equation (21). Finally, using standard transformation laws for stress tensors, the full set of stress components can be reconstructed.

Evidently, biaxial and hydrostatic stress conditions do not distort the shape of the wave fronts but alter propagation velocity. Thus, an additional reference measurement of an unstressed portion of the component is necessary to calibrate the procedure. It should also be mentioned that, clearly, the shape of the elasticity tensor influences the distortion of the wave front. For orthotropic or anisotropic materials, the wave front deviates from the circular shape in the case of point-shaped excitation, even in the stress-free state. In the case of composite materials, significantly more complex propagation fronts can result depending on the layer structure and layer stiffness.

With regard to the metallic materials investigated in this work, the magnitude of the velocity change and the calculated acoustoelastic constants of the Rayleigh waves are in the range to be expected for steels [24–27]. At this point, it should be noted that velocities will vary between different steel qualities because even though the second-order elastic constants are essentially equal across a wide range of alloys, the third-order elastic constants are not. The latter are highly sensitive to influencing factors such as the aforementioned chemical composition and microstructural features as a result of the processing route of the material. Evidently, inhomogeneities such as grain boundaries, defects, and dislocations

will serve as obstacles for wave propagation and lead to additional dispersion of the elastic wave and, thus, of the detector signal, further exacerbating experimental validation. The change of the propagation velocity depending on the dislocation density above the initial yield strength can be analyzed using a model described by Mujica et al. [28], based on a generalization of the Granato–Lücke theory by Maurel et al. [29]. The influence of plastic deformation on the acoustoelastic effect was comprehensively investigated by the authors of [30]. In their work, a coupling between micromechanical processes, in particular dislocation accumulation, and macroscopic deformation variables, such as plastic strain, is established. It is shown that the propagation velocity of ultrasonic waves decreases quadratically with increasing plastic strain.

As long as the wavelength of the ultrasonic wave—typically in the millimeter range for the frequencies considered here—is significantly larger than the grain size, the phase velocity remains largely unaffected. In cases where the grain size increases substantially, enhanced scattering and microstructural inhomogeneities can lead to a reduction in the effective measured wave velocity.

Each phase within a material exhibits distinct mechanical properties, which results in local variations in elastic wave propagation speed. On average, the presence of stiffer or more compliant phases can slightly shift the overall propagation velocity. If a phase transformation occurs (e.g., austenite transforming into martensite), this can cause a sudden change in the local phase velocity.

Textures develop when grains become preferentially oriented due to manufacturing processes. This crystallographic alignment introduces anisotropy in the elastic properties of the material, resulting in direction-dependent wave propagation. Depending on the angle between the propagation direction and the texture orientation, the wave velocity may vary. As plastic deformation increases, the texture typically intensifies along a specific direction, leading to a further decrease in propagation velocity in that orientation.

For nonmetallic materials such as polymers, the presented concepts will, in principle, work as well. However, it has to be mentioned that viscoelastic effects may have to be taken into account, leading to a gradual decrease of the propagation velocity.

Experimental validation of the presented results poses a number of challenges: First it has to be noted that the excitation applied in the simulations is significantly larger than the excitation exerted by transducer in real experiments. This is due to the fact that the actual excitation amplitude is of a magnitude of a few decimals in the chosen N, mm, s unit system, i.e., numerical values where truncation errors become relevant. To forestall their influence, the excitation amplitude has deliberately been chosen to be large. In reality, the measured signal is subject to measurement noise which is sometimes difficult to distinguish from the actual signal. This noise originates from many different sources, such as surface roughness, grain boundaries, the electronic device, etc. Surface roughness additionally leads to a change in velocity and dispersion of the wave, which cannot be modeled with a semianalytical solution. The influence of surface roughness on the propagation velocity and dispersion of surface waves is caused by energy loss. The main causes of this are for wavelengths $\lambda > R_a$ conversion to other wave modes, reflections, interference, and increased attenuation. If the wavelength λ is smaller than the mean surface roughness R_a , the path also becomes longer because the wave follows the surface. Taking these dispersion effects into account would require a full FE analysis of a rough surface. However, it can be considered with the FE solution if the surface roughness is modeled.

Despite the aforementioned issues having an impact on the wave propagation velocity, the predictive quality of the presented approach based on homogenized material parameters is reasonably high and reproducible within a given steel grade. This is also corroborated by the fact that the FE solution shows a good agreement with the semianalytical

solution. It should be noted that the semianalytical solution is limited to homogeneous stress states only, whereas the FE solution is applicable to arbitrarily complex stress states, albeit at significantly higher computational cost.

In contrast, real-world measurements are inherently affected by various sources of measurement noise, which are absent in the idealized simulation environment. This includes electronic noise from sensors and amplifiers, quantization noise from analog-to-digital conversion, and external disturbances such as electromagnetic interference and mechanical vibrations. As a result, the measured signal in practical experiments is often superimposed with noise, which may be difficult to distinguish from the actual system response, especially at low excitation levels. Moreover, tolerances in physical components and environmental fluctuations contribute additional uncertainty.

For industrial adoption, several practical aspects must be addressed to enable the reliable application of Rayleigh-wave-based stress evaluation. A key factor is the precise placement and orientation of ultrasonic sensors. To accurately capture the directional dependence of wave velocity, the sensor configuration must ensure that wave propagation occurs along well-defined paths with known angular alignment relative to potential principal stress directions. In particular, measurements in at least three noncollinear directions are required to resolve multiaxial stress states when the orientation of principal axes is unknown. Another critical consideration is the mitigation of measurement noise, which may obscure the relatively small velocity variations associated with stress-induced acoustoelastic effects. To enhance signal stability, high-frequency sensors with broad bandwidth and high signal-to-noise ratio are recommended. Additionally, surface preparation—e.g., through polishing or light grinding—can significantly reduce scattering due to surface roughness and improve wave coupling.

5. Conclusions

The presented results are of significant importance for the reconstruction of residual stress states in engineering components. It has been shown that the change in the propagation velocity of Rayleigh waves is best described using the principal stresses for any given stress state. The results of the present work also show that the acoustoelastic constants for Rayleigh waves can easily be determined using numerical methods. Furthermore, the results show that the distortion of the wave propagation front allows conclusions about the present stress state and the triaxiality parameter.

For the future, investigating the influence of additional factors like stress gradient and other factors on the acoustoelastic effect could lead to a more holistic understanding of wave propagation in stressed materials. The residual stresses of engineering components always exhibit a stress gradient. These factors have to be taken into account for precise verification. Furthermore, the development of more sophisticated algorithms for the real-time reconstruction of stress states from Rayleigh wave measurements could further improve the practical applicability of this technique.

The presented model should be experimentally validated under defined multiaxial stress states using laser ultrasonics. Noncontact, laser-based excitation and detection enable high-resolution measurements of Rayleigh-wave propagation. By applying controlled uniaxial tension and biaxial loading and monitoring directional velocity changes, the stress dependence predicted by the model can be quantitatively assessed.

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Appendix A. Additional Mathematical Derivation

Appendix A.1. Chain-Rule Transformation of the Derivative Operator

We briefly derive how derivatives with respect to the material coordinate X_j map to spatial derivatives in x'_k with the kinematics $x'_k = X_k + u_k^s$ and the chain rule.

$$\begin{aligned}\frac{\partial}{\partial X_j} &= \frac{\partial}{\partial x_k} \frac{\partial x_k}{\partial X_j} = \frac{\partial}{\partial x_k} \frac{\partial}{\partial X_j} (X_k + u_k^s) \\ &= \frac{\partial}{\partial x_j} + \frac{\partial u_k^s}{\partial X_j} \frac{\partial}{\partial x_k} \\ &= \frac{\partial}{\partial x_j} + u_{k,j}^s \frac{\partial}{\partial x_k} + \dots\end{aligned}\quad (A1)$$

Appendix A.2. Effective Elasticity Tensor in the Prestressed State

$$\begin{aligned}\frac{\partial P_{ij}}{\partial X_j} &\approx \frac{\partial P_{ij}}{\partial x_j} + u_{p,j}^s \frac{\partial P_{ij}}{\partial x_p} \\ &\approx C_{ijkl} \left(\frac{\partial^2 u_k^d}{\partial x_j \partial x_l} + u_{q,l}^s \frac{\partial^2 u_k^d}{\partial x_j \partial x_q} \right) + u_{p,j}^s C_{ijkl} \left(\frac{\partial^2 u_k^d}{\partial x_p \partial x_l} \right) \\ &\quad + \frac{1}{2} C_{ijklmn} \left(\frac{\partial^2 u_k^d}{\partial x_j \partial x_l} u_{m,n}^s + u_{k,l}^s \frac{\partial^2 u_m^d}{\partial x_j \partial x_n} \right) \\ &= \left(C_{ijkl} + C_{ijkq} u_{l,q}^s + C_{ipkl} u_{j,p}^s \right) \frac{\partial^2 u_k^d}{\partial x_j \partial x_l} \\ &\quad + \frac{1}{2} \left(C_{ijklmn} + C_{ijln} \delta_{km} + C_{jnkl} \delta_{im} + C_{jlmn} \delta_{ik} \right) \left(\frac{\partial^2 u_k^d}{\partial x_j \partial x_l} u_{m,n}^s + u_{k,l}^s \frac{\partial^2 u_m^d}{\partial x_j \partial x_n} \right) \\ &= \left(C_{ijkl} + \delta_{ik} C_{ijqr} u_{q,r}^s + C_{rjkl} u_{i,r}^s + C_{irkl} u_{j,r}^s + C_{ijrl} u_{k,r}^s + C_{ijkr} u_{l,r}^s + C_{ijklmn} u_{m,n}^s \right) \frac{\partial^2 u_k^d}{\partial x_j \partial x_l} \\ &= B_{ijkl} \frac{\partial^2 u_k^d}{\partial x_j \partial x_l}\end{aligned}\quad (A2)$$

whereby

$$B_{ijkl} = C_{ijkl} + \delta_{ik} C_{ijqr} u_{q,r}^s + C_{rjkl} u_{i,r}^s + C_{irkl} u_{j,r}^s + C_{ijrl} u_{k,r}^s + C_{ijkr} u_{l,r}^s + C_{ijklmn} u_{m,n}^s.$$

For small deformations ($u_{i,j}^s \ll 1$) and negligible rigid rotations, the Green–Lagrange strain reduces to the symmetric part of the displacement gradient,

$$E_{ij}^s \approx \frac{1}{2} (u_{i,j}^s + u_{j,i}^s).$$

In particular, in the absence of shear (pure normal strains), this implies that

$$u_{i,j}^s \approx E_{ij}^s.$$

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