

Article

The Multistate Reliability Assessment of Ship Hull Girders

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Abstract: Ships are designed to withstand various types of hull structure damage, including corrosion, fatigue, damage, crack, fouling, etc., throughout their projected life cycle of 25 years. In this study, we used a database of 25 different bulk carriers aged from five to twenty-five years, consisting of a total of 1920 thickness measurements of girder plate damage across 110 fuel tanks. Thickness measurements of longitudinal girder plate were conducted by certified technicians and approved company. Ultrasound thickness gauging equipment was used to collect data in accordance with the developed methodology and gauging scheme. Based on the classification societies' rules, the values of the reduction in steel plate thickness due to corrosion over time fall into three categories: acceptable corrosion, substantial corrosion, and extensive corrosion. While classification societies prescribe permissible thickness reductions between 15 and 30%, in this study, the authors considered the excessive corrosion values to be above 20% reduction in initial thickness. Measurements indicating more than 20% reduction were classified as failures, necessitating the replacement of the corroded surfaces. After applying the multistate approach to the reliability analysis of longitudinal girder plates and improving reliability after reaching the critical state, the results show that usability dropped significantly between ten and fifteen years of service for upper girder plating and between twenty and twenty-five years of service for lower girder plates. These findings highlight the crucial impact of gauging location on reliability analysis.

Keywords: corrosion; failure; girder plate; fuel oil tanks; reliability; risk



Academic Editor: José A. Orosa

Received: 18 April 2025

Revised: 26 May 2025

Accepted: 27 May 2025

Published: 31 May 2025

Citation: Soszyńska-Budny, J.; Ivošević, Š. The Multistate Reliability Assessment of Ship Hull Girders. *Appl. Sci.* **2025**, *15*, 6218. <https://doi.org/10.3390/app15116218>

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1. Introduction

The numerous maritime incidents with vessels in operation have motivated researchers to investigate the causes that lead to these accidents. Research published by Knapp et al. [1] and Heij and Knapp [2] showed that general and dry cargo vessels exhibit the highest likelihood of casualties. Research conducted by EMSE 2019 showed that collision, fire, explosion, flooding, loss of control, hull failure, contact, damage to ship equipment, grounding, and capsizing [3] are the main causes of casualties at sea, while research published by Małyszko [4] presented a model for assessing the impact of various factors on maritime accidents.

Structural damage inside the ship can partially damage the construction and affect the stability of the ship, while cracks in the structure or leaks can cause fuel spillage inside or outside the vessel, which results in environmental pollution and significant economic losses for their repair.

In addition to the aforementioned findings, studies conducted in previous decades support the significance of adopting additional regulations. Research conducted between 1997 and 2001 recorded more than 15% hull failures [5], research between 2002 and 2006

recorded around 4% of hull defects [5], and recent research from 2019 by European Maritime Safety Agency (EMSA) announced that the number of casualties associated with hull failure was around 1% [3].

The most significant contribution to the prevention of corrosion was achieved through the application of Performance Standards for Protective Coatings (PSPCs), implemented in April 2006, which significantly enhance coating quality and efficiency [6].

As corrosion takes place in a monotonous manner, it is very important to identify critical areas and carry out regular visual inspections and measurements regarding the assessment of the condition of corroded surfaces. Visual inspections and thickness measurements aim to assess the condition of the surface protection and the thickness of the metal structure. The rules of classification societies define the state of surface protection as good, fair, and poor, while the state of corrosion is classified as acceptable wear, substantial corrosion, and extensive corrosion, for which immediate reaction should be conducted in order to replace the corroded structure.

Additionally, the most significant contribution to preserving the structural integrity of ships, tankers, and bulk carriers was achieved with the introduction of common structure rules—CSRs [7]. These rules were established to create the minimum classification requirements to mitigate the risks of major hull structural failure that can occur during ship operation, improve safety and environment protection, and ensure the ship's service life of twenty-five years.

Common structure rules were created based on a comprehensive analysis of empirical data on the condition of ship structures in service and actual corrosion values [8]. Through statistical modeling, with a data reliability level of 95%, the corrosion margin was calculated during the designed life cycle of each structural element of the structure depending on the type of ship and the location of the structural element [9]. Thus, the rules define the net thickness of the structural element that meets the strength and resistance criteria of the structure, as well as the gross value that considers allowances for corrosion and safety margins. In addition, these rules ensure greater structural strength and reliability of the hull in relation to corrosion, but increase the weight of the vessel by several percent.

Marine environment and ship-specific factors significantly influence the deterioration of ship structures over time [10]. The corrosion margin is directly influenced by the complex marine environment to which a certain structural element is exposed, and it is under the direct influence of different factors such as atmosphere, cargo, cargo operation, operation procedures, the ship's route, etc. Under the influence of these influential factors, various structural defects develop in the forms of corrosion, crack, deformation, coating breakdown, and biofouling [11]. Different forms of corrosion occur in different ship hull areas and at different times of vessel operation. We can distinguish general corrosion and several other types of corrosion, like pitting, intergranular, galvanic, stress corrosion, corrosion fatigue, fretting, microbiological, crevice, erosion, and cavitation [12].

Various methods are used to inspect ships and detect defects that occur during the ship's life cycle. In this sense, the most important is visual inspection, which is carried out in the traditional way by an inspector at precisely defined time intervals, in accordance with the rules of the classification societies. In addition to the traditional methods of visual inspection, today, significant progress is achieved through the application of autonomous or semi-autonomous robots with missions inside and outside the ship's areas. Recently, unmanned vehicles and robots without a pilot onboard have been extensively used for underwater inspection [11,13]. Furthermore, in addition to pressure, vacuum, roentgen, and magnet tests, various non-destructive inspection methods such as acoustic-based, electromagnetic, and image-based testing are used [11].

Robots are used as an alternative to monitoring the ship's hull in critical ship areas (closed spaces, high altitudes, etc.) in the traditional manner, although the majority of inspections are carried out traditionally even today. Through previous research on ships in operation, and the pursuit of safe and secure vessels, different condition assessment methods have been applied: formal safety assessment, time-variant reliability formulation, failure probability and reliability, inspection and maintenance planning, reliability-centered maintenance, etc. [5]. Recently, Li et al. proposed a dynamic multi-attribute decision-making method for risk-based ship design [14].

In recent years, automation, digitalization, and decarbonization processes have contributed significantly to improving maritime safety. The digitalization of the world is booming, and the evolution of cloud computing, Internet of Things (IoT), Big Data analytics, Virtual Reality (VR), Augmented Reality (AR), Artificial Intelligence (AI), Machine Learning, Deep Learning, Neural Networks, etc., significantly improves maritime security [15]. Additionally, different types of sensors and measurement systems have been developed in marine engineering applications for structural health monitoring and in situ measurements of salinity, vibration, temperature, pressure, etc. [16].

So far, numerous studies have calculated corrosion rates over time in order to assess the degree of corrosion of individual structural areas of bulk carriers and tankers, probability of damage to particular structural areas, type of failure, and the influence of environmental factors and different corrosion classes. In addition, linear and nonlinear models have been well developed for transverse bulkheads and inner bottom plating [17,18], for all structural elements and areas [19,20].

Previous studies on the structural elements of fuel tanks and surrounding areas (sea water, ballast water, cargo, dry space) in bulk carriers in operation have considered corrosion damage in millimeters [17,21] and the percentage of corrosion damage to structural plates [18], resulting in linear and nonlinear models of corrosion development. The reliability of fuel tanks and individual structural elements was the subject of paper [22].

Based on previous research, this study investigated the possibility of applying a multistate approach to reliability analysis on longitudinal girder plating.

This paper is divided into five sections. The Section 2 presents the relevant database of measurement results from different ships and surveys. The Section 3 expounds on the theoretical aspect of the multistate approach for analyzing and improving steel plates' reliability. It defines reliability and risk functions, as well as procedures on how to improve the reliability function after exceeding a critical state. In Section 4, the theory previously described is applied to the case of aging girder plates, using polynomial interpolation. Finally, concluding remarks are presented in Section 5.

2. Materials

This study is based on the analysis of historical data regarding the corrosion damage of structural parts of longitudinal girder plates, i.e., steel plates between the ballast tanks and the fuel tank. The measured data represent general corrosion and are expressed as percentages of damages. They were systematized based on thickness measurement data collected during regular special surveys after five, ten, fifteen, twenty, and twenty-five years, while the corrosion damage data collected during intermediate surveys were included in the results of special surveys.

Previous research on fuel tanks has shown that besides inner bottom plating and longitudinal watertight girder plates, the upper plates of longitudinal girders have significant thickness damages due to corrosion [10,12,17]. Thus, two specific thickness measurement locations on the girder were considered, the upper and lower part of the girder plating, presented as A_5 and A_6 in Figure 1.

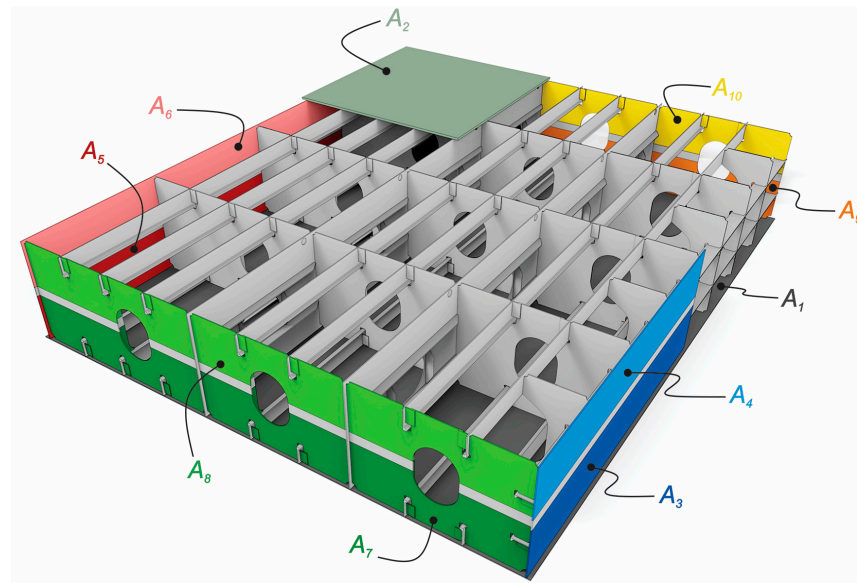


Figure 1. The fuel oil tank structure arrangement areas (A1–A10). Legend: A1—Bottom plate; A2—Inner bottom plate; A3—Inside girder (water tight), lower; A4—Inside girder (water tight), upper; A5—Side (water tight) girder, lower; A6—Side (water tight) girder, upper; A7—Floor after (water tight), lower; A8—Floor after (water tight), upper; A9—Floor fore (water tight), lower; A10—Floor fore (water tight), upper.

This study employed a database gathered over the last two decades, encompassing the following data:

- A total of 25 aging bulk carriers between five and twenty-five years of age;
- The period investigated, between 2005 and 2017;
- The analysis results of 38 varied special surveys.

Table 1 shows a basic database of the considered vessels, number of inspections, number of fuel oil tanks, number of thickness measurements, and specific sections. The methodology for selecting measurement positions was created in such a way as to objectively assess the condition of each structure plate of fuel oil tanks and is described in detail in [10]. Table 1 presents the average diminution (due to corrosion) values of the total number of measured values of the thickness of the girder steel plates at different time intervals, from five to twenty-five years.

Table 1. Dataset used to study longitudinal steel girders.

The Age of Ships (Years)	The Number of Ship Surveys	The Number of Tanks	The Number of Measured Points	The Number of Sections	The Mean Values of Plate Thickness Diminution Caused by Corrosion (%)	
					Lower	Upper
0–5	4	9	180	45	0.3%	0.4%
5–10	4	10	200	55	0.6%	1.4%
10–15	7	19	380	100	1.1%	6.7%
15–20	13	43	800	220	4.8%	9.9%
20–25	10	29	360	150	9.6%	23.0%
Total:	38	110	1920	570		

The measurements were carried out in accordance with the rules of the classification societies, which prescribe a minimum scope of measurements, precisely define the measurement locations, and additionally require measurements in corroded surfaces. This ensures

the consistency of the data in all structural areas of the ships and an objective assessment of the corrosion wear of the steel structure.

The data above enabled the reliability analysis of longitudinal watertight girders' steel plates in upper and lower areas along with further assessment of fuel tank exploitation.

The example of a corroded girder plate after twenty years of exploitation presented in Figure 2b shows corroded surfaces that had to be replaced due to excessive corrosion. The rules and regulations of the classification society under which the ships were classified were used. For the present research, considering different classification societies and their requirements, we adopted their most common rule stating that excessive corrosion occurs when diminution due to corrosion is greater than 20%. Thus, this study identified three characteristic zones of corrosion: acceptable corrosion, significant corrosion, and excessive corrosion, exceeding 20% of diminution. These three categories were used in further research and reliability analysis.

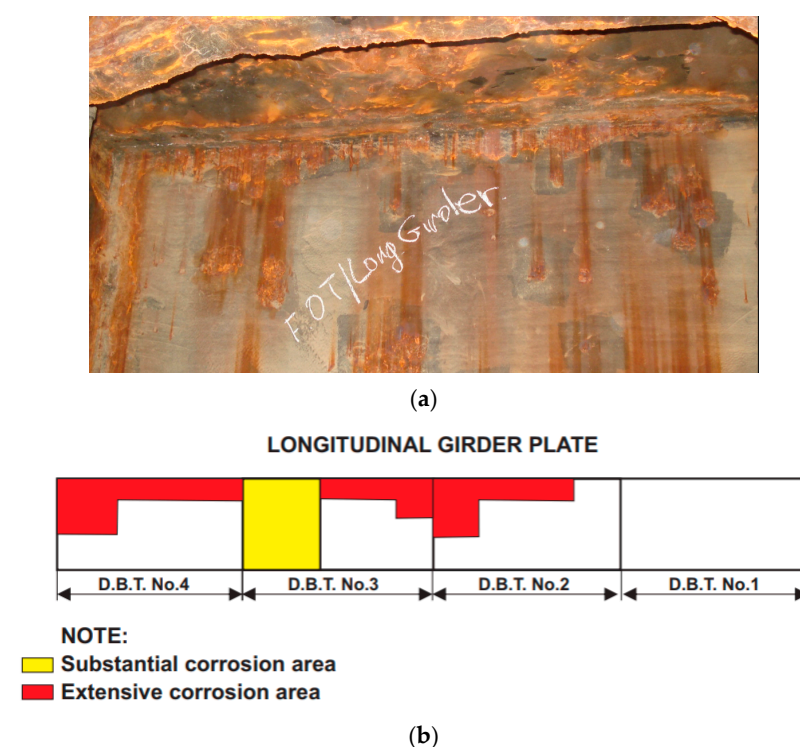


Figure 2. Substantial and extensive corrosion of fuel oil tank longitudinal girder plating: (a) photo of corroded girder plating; (b) sketch of corroded areas of girder plate.

Creating the database in this way enabled us to apply the theory of reliability to the created data, as described in the following section.

3. Methodology

3.1. Multistate Approach to Reliability Analysis of Longitudinal Watertight Girder Plating

During the reliability analysis of the technical systems, we often deviated from the two-state approach, which distinguishes only two states (fit and damaged). Due to the aging of system components, for a more accurate reliability analysis, we distinguish intermediate reliability states. In this study, we use a multistate approach for the reliability assessment [22–24]. For the purposes of this analysis, the steel plates of the longitudinal girder plating between fuel tanks and ballast tanks are considered one system. According

to the multistate approach, the reliability function of the longitudinal girder plating is defined as a vector [22–24]

$$R(t, \cdot) = [R(t, 0), R(t, 1), \dots, R(t, l), \dots, R(t, n)], t \geq 0, \quad (1)$$

with the probability that a steel plate is in a subset of reliability states $\{l, l + 1, \dots, n\}$, $l \in \{0, 1, \dots, n\}$ at moment t , while it was in the best reliability state n at the beginning, i.e., $t = 0$, which defines the coordinate $R(t, l)$ of the reliability function vector.

According to the above assumption,

$$R(t, l) = P(T(l) > t), t \geq 0, l \in \{0, 1, \dots, n\}, \quad (2)$$

where $T(l)$ is a random variable representing the lifetime of a steel plate in reliability state subset $\{l, l + 1, \dots, n\}$, $l \in \{0, 1, \dots, n\}$.

Another significant reliability characteristic used in the reliability analysis is a risk function. The risk function is defined as the probability that steel plates are in a subset of states worse than the critical state (r), where the steel plates were in the best reliability state (n) at time $t = 0$. The risk function can be formulated as follows:

$$r(t) = 1 - R(t, r) = P(T(r) \leq t), t \geq 0. \quad (3)$$

In the above equation, state $r \in \{1, 2, \dots, n\}$ is one of the defined reliability states designated (based on expert opinions) as a critical reliability state, and $R(t, r)$ is a coordinate of the steel plate reliability function.

If an inverse function $r^{-1}(t)$ of risk function (3) exists, then τ can be interpreted as a moment when a steel plate's risk (of damage) exceeds a fixed threshold (δ).

$$\tau = r^{-1}(\delta). \quad (4)$$

3.2. Improving Reliability

Improving the reliability of steel plates' longitudinal watertight girder plating is very important for maintaining the system and for system and environmental safety.

In multistate reliability analysis, it is possible to assume that a system (steel plates) can be repaired when it exceeds its critical reliability state, corresponding to a certain reliability level. Based on our considerations, we chose the state where the acceptable reliability level of 90% is exceeded as a critical state. By replacing some corroded surfaces of steel plates, we improve their reliability, but not necessarily to the best reliability state (the system does not operate like a new one). This is because other non-replaced surfaces of the system may be in a lower reliability state.

In assuming that the reliability of the steel plates is improved, i.e., some girders have been replaced, after crossing a permitted threshold risk level δ , after time τ , τ is then calculated according to Formula (4). Moreover, in assuming that the time of renovation is μ , the coordinates of the improving reliability function in reliability state subsets $\{r, r + 1, \dots, n\}$, $r \in \{1, 2, \dots, n\}$ are presented with the following formulas [22]:

$$R(t, r) = R(t - k \cdot (\tau + \mu), r) \text{ for } k \cdot (\tau + \mu) \leq t < k \cdot (\tau + \mu) + \tau, \quad (5)$$

$$R(t, r) = R(\tau, r) \text{ for } k \cdot (\tau + \mu) + \tau \leq t < (k + 1) \cdot (\tau + \mu), \quad (6)$$

where $k = 0, 1, \dots, N$, $r \in \{1, 2, \dots, n\}$, $t \geq 0$, and N is the number of subsequent reliability improvements.

Figure 3 illustrates the r -th component of the exemplary improving reliability function.

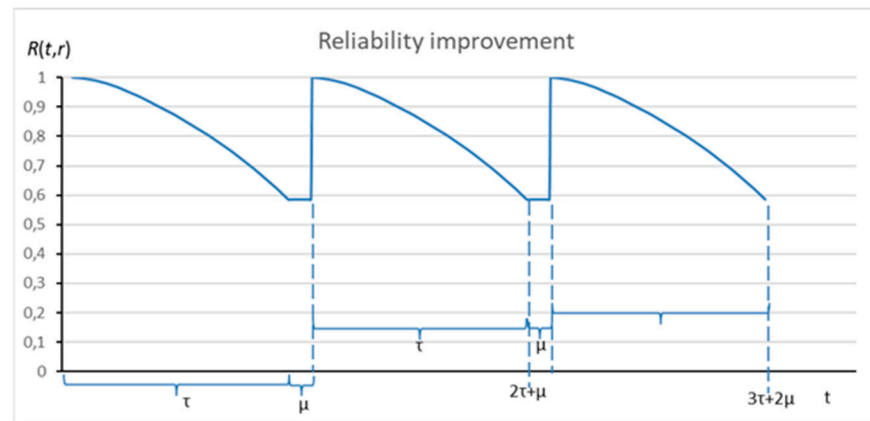


Figure 3. Coordinate of exemplary improvement reliability function.

The diagram in Figure 4, based on the methodology presented above, shows the subsequent steps to improve reliability.

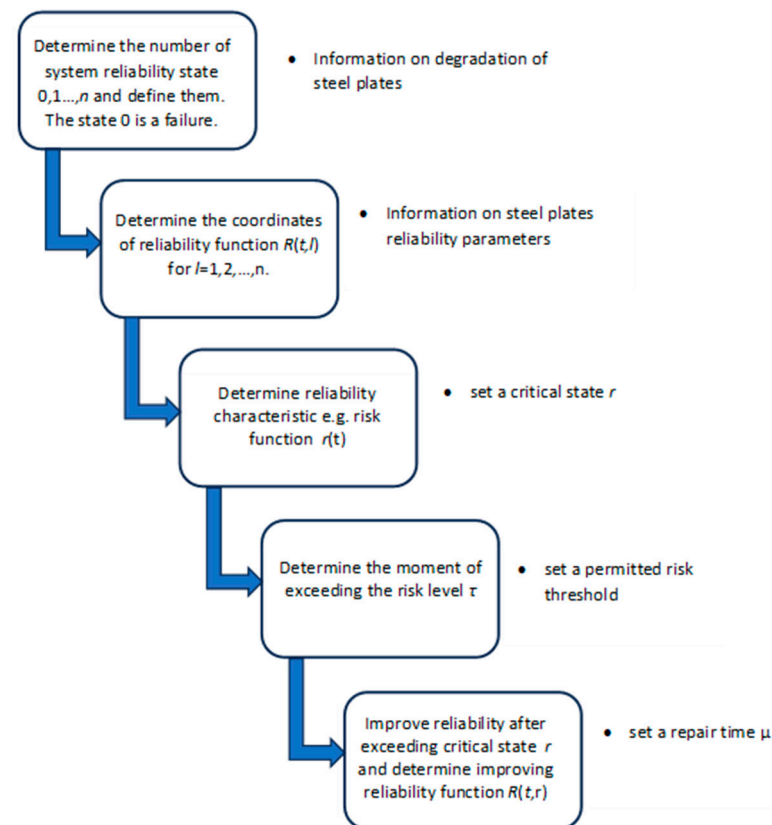


Figure 4. Diagram showing the reliability improvement procedure.

4. Results of Reliability Analysis of Longitudinal Watertight Girder Plating

4.1. Estimation of Reliability and Risk

The reliability analysis of this study is based on the assumption that the permissible wear of steel plates is 20%. This means that the values below 80% of the original thickness are considered unacceptable, i.e., interpreted as failures. Although the remaining thickness ensures the impermeability of steel plates, classification societies still require the replacement of corroded surfaces in these cases.

A multistate approach to reliability was applied, and three reliability states were adopted, defined as follows:

- Second state of reliability—The system works and is totally safe; the corrosion wear of steel plates is in the range of $<0.15\%$).
- First state of reliability—The system works, but its operation is less safe, which may cause environmental pollution, cargo damage, decrease the ship's safety, etc.; the corrosion wear of steel plates is in the range of $<15\%, 20\%$).
- Zeroth state of reliability—The system is damaged (not fulfilling (regulatory) requirements); the corrosion wear of steel plates exceeds 20% .

Since the limit of acceptable reliability is 90% , the first reliability state was chosen as critical state r .

Empirical data of corrosion values were used to estimate reliability characteristics. All data were obtained during regular special surveys after five, ten, fifteen, twenty, and twenty-five years of operation.

In our study, reliability characteristics were determined separately for the upper and lower parts of the girders.

First, a reliability assessment was conducted for the upper side of the girders. Table 2 shows the number of steel plate measurements for the upper sides of girders in particular reliability states $l, l \in \{0, 1, 2\}$.

Table 2. The amount of measurements for steel plates of the upper side of girder in reliability states $l, l \in \{0, 1, 2\}$.

Time (Year)	Number of Measurements That Are $<0.15\%$ of the Original Thickness	Number of Measurements That Are $<15\%, 20\%$ of the Original Thickness	Number of Measurements That Exceed 20% of the Original Thickness
0–5	90	0	0
5–10	99	0	1
10–15	152	16	22
15–20	299	13	88
20–25	65	2	113

Table 3 presents the number of measurements in the reliability state subsets $\{l, l + 1, \dots, 2\}, l \in \{0, 1, 2\}$.

Table 3. Number of measurements of upper sides of girder's steel plates in a subset of reliability states $\{l, l + 1, \dots, 2\}$.

Time (Years)	Number of Measurements	Number of Measurements in Reliability State Subset $\{0, 1, 2\}$	Number of Measurements in Reliability State Subset $\{1, 2\}$	Number of Measurements in Reliability State Subset $\{2\}$
0–5	90	90	90	90
5–10	100	100	99	99
10–15	190	190	168	152
15–20	400	400	312	299
20–25	180	180	67	65

Further, using the data in Table 3, the coordinates of the reliability function values $R(t, l), l \in \{0, 1, 2\}$, after five, ten, fifteen, twenty, and twenty-five years were estimated. These values were estimated as the probability that the upper side of the girder's plates is in reliability state subset $\{0, 1, 2\}, \{1, 2\}$, or $\{2\}$, and they are presented in Table 4.

Table 4. The estimated values of the reliability function in reliability state subsets {0, 1, 2}, {1, 2}, and {2} for the upper side of the girder's plates.

Time (Years)	$R(t, 0)$	$R(t, 1)$	$R(t, 2)$
5	1	1	1
10	1	0.99	0.99
15	1	0.884	0.800
20	1	0.780	0.748
25	1	0.372	0.361

Because the limit of accepted reliability is 90%, reliability state 1 was chosen as a critical state.

From the data presented in Table 4 and from Figure 5, it can be seen that, for ships between ten and fifteen years of age, the reliability function component $R(t, 1)$ has fallen below the acceptable level of 90%.

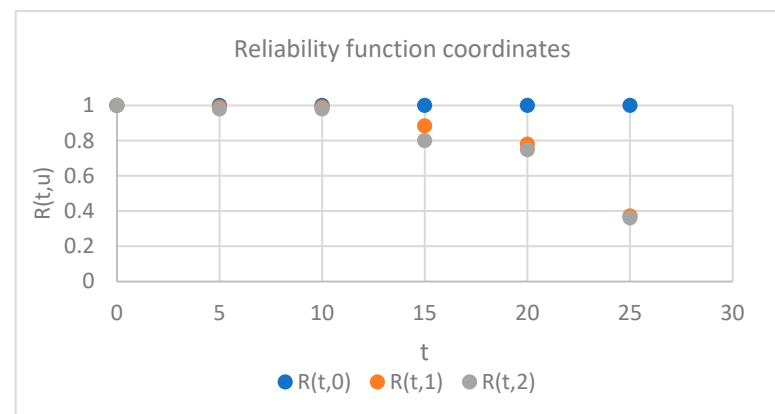


Figure 5. Graphical interpretation of function $R(t, l)$ coordinates.

Now, we have to determine when exactly the risk function exceeds the acceptable level of 0.1. Since we chose state 1 as the reliability critical state, we determine the moment when the steel plates risk exceeds the permitted threshold $\delta = 0.1$. This will help us answer the question when the reliability of the upper side of the girder should be improved (when the steel plates should be replaced).

In the reliability analysis performed, the reliability function coordinates in reliability state subsets {1, 2} and {2} are not at our disposal because the system lifetimes in these subsets were not available. Only the values of the reliability function at five, ten, fifteen, twenty, and twenty-five years were designated based on the thickness reduction measurements conducted during the surveys.

From a mathematical analysis, we know that, if the values of a function at a finite number of points are known, then for any distribution, such a function can be interpolated using Lagrange polynomial interpolation.

Thus, using Lagrange polynomial interpolation, the reliability function coordinate $R(t, 1)$ for the upper side of the girder is given by the following formula:

$$\begin{aligned}
 R(t, 1) &\approx \tilde{R}(t, 1) = R_0(t, 1)p_0(t) + R_1(t, 1)p_1(t) + R_2(t, 1)p_2(t) \\
 &\quad + R_3(t, 1)p_3(t) + R_4(t, 1)p_4(t) \\
 &= p_0(t) + 0.99l_1(t) + 0.884p_2(t) + 0.780p_3(t) + 0.372p_4(t),
 \end{aligned} \tag{7}$$

where

$$p_0(t) = \prod_{\substack{j=0 \\ j \neq 0}}^4 \frac{t-t_j}{t_0-t_j} = \frac{(t-t_1)(t-t_2)(t-t_3)(t-t_4)}{(t_0-t_1)(t_0-t_2)(t_0-t_3)(t_0-t_4)} \\ = \frac{(t-10)(t-15)(t-20)(t-25)}{15,000}, \quad (8)$$

$$p_1(t) = \prod_{\substack{j=0 \\ j \neq 1}}^4 \frac{t-t_j}{t_1-t_j} = \frac{(t-t_0)(t-t_2)(t-t_3)(t-t_4)}{(t_1-t_0)(t_1-t_2)(t_1-t_3)(t_1-t_4)} \\ = -\frac{(t-5)(t-15)(t-20)(t-25)}{3750}, \quad (9)$$

$$p_2(t) = \prod_{\substack{j=0 \\ j \neq 2}}^4 \frac{t-t_j}{t_2-t_j} = \frac{(t-t_0)(t-t_1)(t-t_3)(t-t_4)}{(t_2-t_0)(t_2-t_1)(t_2-t_3)(t_2-t_4)} \\ = \frac{(t-5)(t-10)(t-20)(t-25)}{2500}, \quad (10)$$

$$p_3(t) = \prod_{\substack{j=0 \\ j \neq 3}}^4 \frac{t-t_j}{t_3-t_j} = \frac{(t-t_0)(t-t_1)(t-t_2)(t-t_4)}{(t_3-t_0)(t_3-t_1)(t_3-t_2)(t_3-t_4)} \\ = -\frac{(t-5)(t-10)(t-15)(t-25)}{3750}, \quad (11)$$

$$p_4(t) = \prod_{\substack{j=0 \\ j \neq 4}}^4 \frac{t-t_j}{t_4-t_j} = \frac{(t-t_0)(t-t_1)(t-t_2)(t-t_3)}{(t_4-t_0)(t_4-t_1)(t_4-t_2)(t_4-t_3)} \\ = \frac{(t-5)(t-10)(t-15)(t-20)}{15,000} \quad (12)$$

Through further considerations, using (7)–(12), the moment when the steel plates' risk function $r(t) = 1 - R(t, 1)$ exceeds an acceptable level of 0.1 was found. The approximately risk and reliability functions are given in Figure 6.

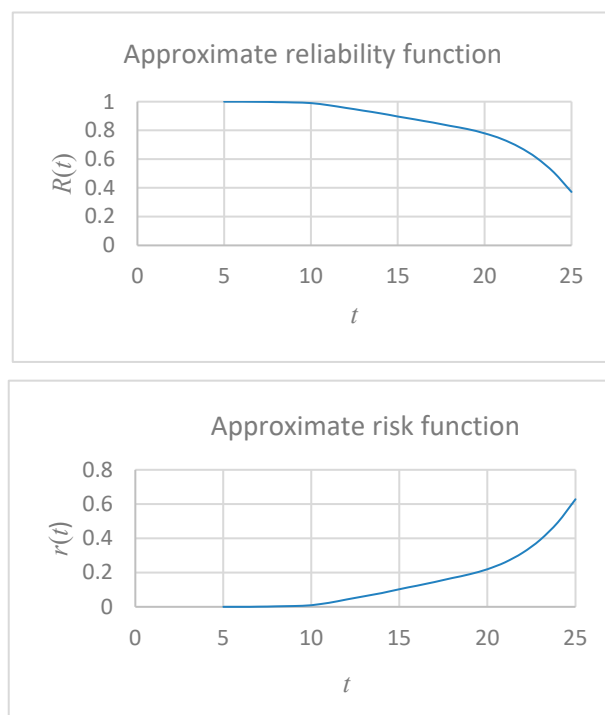


Figure 6. Approximate function obtained from Lagrange interpolation.

According to Formulas (3), (4), and (7)–(12), when the risk function of the steel plate crossed the acceptable threshold $\delta = 0.1$ was at $\tau = 14.2$ years. The above analysis shows that the risk function will exceed the acceptable level of 0.1 after only 14.2 years. This indicates

that the upper side of the girder plates' reliability should be improved by replacing corroded surfaces after the second special survey/inspection, i.e., after ten years of operation. This replacement could minimize hazardous effects of corrosion, such as environmental pollution or structural damage.

A similar reliability analysis was performed for the lower side of girders. Table 5 presents the number of measurements of the lower side of the girders' steel plates in the particular reliability states $l, l \in \{0, 1, 2\}$.

Table 5. The number of measurements for the steel plates of the lower side of the girder in reliability states $l, l \in \{0, 1, 2\}$.

Time (Years)	Number of Measurements That Are <0.15% of the Original Thickness	Number of Measurements That Are <15%, 20% of the Original Thickness	Number of Measurements That Exceed 20% of the Original Thickness
0–5	90	0	0
5–10	100	0	0
10–15	190	0	0
15–20	364	3	33
20–25	130	1	49

Table 6 presents the number of measurements in the reliability state subsets $\{l, l + 1, \dots, 2\}, l \in \{0, 1, 2\}$.

Table 6. Number of measurements of the lower side of the girder's steel plates in a subset of reliability states.

Time (Years)	Number of Measurements	Number of Measurements in Reliability State Subset $\{0, 1, 2\}$	Number of Measurements in Reliability State Subset $\{1, 2\}$	Number of Measurements in Reliability State Subset $\{2\}$
0–5	90	90	90	90
5–10	100	100	100	100
10–15	190	190	190	190
15–20	400	400	367	364
20–25	180	180	131	130

Next, using the data in Table 5, the coordinates of reliability function values $R(t, l), l \in \{0, 1, 2\}$ after five, ten, fifteen, twenty, and twenty-five years were estimated. These values are defined as the probability that the lower side of the girder's plates is in reliability state subset $\{0, 1, 2\}, \{1, 2\}$, or $\{2\}$, and they are presented in Table 7.

Table 7. The estimated values of the reliability function in reliability state subsets $\{0, 1, 2\}, \{1, 2\}$, and $\{2\}$ for the lower side of the girder's plates.

Time (Years)	$R(t, 0)$	$R(t, 1)$	$R(t, 2)$
5	1	1	1
10	1	1	1
15	1	1	1
20	1	0.918	0.910
25	1	0.728	0.722

From the data contained in Table 6, and from Figure 7 it can be concluded that sometime between twenty and twenty-five years of operation, the reliability function's component $R(t, 1)$ fell below the acceptable level of 90%.

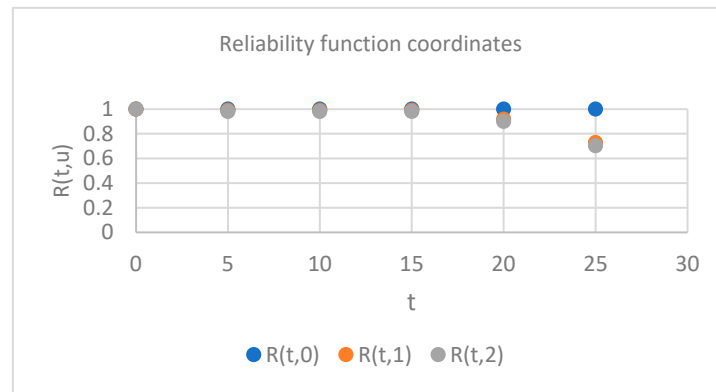


Figure 7. Graphical interpretation of function $R(t, l)$ coordinates.

For the upper side of the girder, we have to determine the time when the risk function exceeds an acceptable level 0.1. This will help us answer the question of when the reliability of the lower side of the girder should be improved (when the steel plates should be replaced).

Using Lagrange polynomial interpolation, the reliability function coordinate $R(t, 1)$ is given by the formula below:

$$\begin{aligned}
 R(t, 1) &\approx \tilde{R}(t, 1) = R_0(t, 1)p_0(t) + R_1(t, 1)p_1(t) + R_2(t, 1)p_2(t) \\
 &\quad + R_3(t, 1)p_3(t) + R_4(t, 1)p_4(t) \\
 &= p_0(t) + p_1(t) + p_2(t) + 0.918p_3(t) + 0.728p_4(t),
 \end{aligned} \tag{13}$$

where $p_0(t)$, $p_1(t)$, $p_2(t)$, $p_3(t)$, and $p_4(t)$ are given by (8)–(12).

The next step is to determine the moments when the steel plates' risk function $r(t) = 1 - R(t, 1)$ exceeds the acceptable level of 0.1. The approximately risk and reliability functions are given in Figure 8.

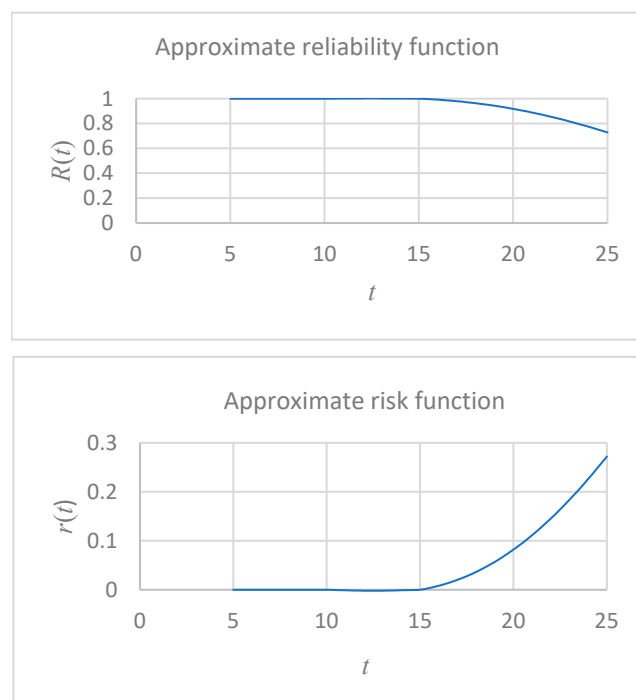


Figure 8. Approximate functions obtained from Lagrange interpolation.

Using Formulas (3), (4), and (13), the moment τ when the risk function of the steel plates exceeds a permitted threshold $\delta = 0.1$ is $\tau = 20.6$ years.

The above analysis indicates that the risk function for the steel plates of the lower side of girder will exceed the acceptable level of 0.1 after 20.6 years. This shows that the reliability of the steel plates of the lower side of the girder should be improved by replacing corroded surfaces after the fourth special survey/inspection, i.e., after twenty years of operation.

The reliability assessment shows that the upper girder plates are corroded significantly earlier than lower girder plates and need to be replaced with new ones to keep the system in good working condition. This study proves that upper girder plates exceed acceptable levels of risk after fourteen years of operation, while lower girder plates exceed these levels after twenty years.

4.2. Reliability Improvement

The time required for the replacement of corroded surfaces on a ship depends on many factors. Based on the opinion of experts, the average time that a ship must spend in a yard (renovation time μ) is about a month. As ships are generally designed for about twenty-five years of operation, the reliability analysis carried out shows that corroded steel plates of both the lower and upper sides of the girders should be replaced once during operation.

Using Formulas (5) and (6), the coordinate of the improved reliability function in reliability state subset $\{1, 2\}$ are given by the following formulas:

- For the upper side of girders, reliability is improved upon the second special survey, i.e., after ten years.

$$R(t, 1) = R(t - k \cdot (10 + 0.083), 1) \quad (14)$$

$$\text{for } k \cdot (10 + 0.083) \leq t < k \cdot (10 + 0.083) + 10,$$

$$R(t, 1) = R(10, 1) \quad (15)$$

$$\text{for } k \cdot (10 + 0.083) + 10 \leq t < (k + 1) \cdot (10 + 0.083),$$

where $k = 0, 1$.

- For the lower side of girders, reliability is improved upon the fourth special survey, i.e., after twenty years.

$$R(t, 1) = R(t - k \cdot (20 + 0.083), 1) \quad (16)$$

$$\text{for } k \cdot (20 + 0.083) \leq t < k \cdot (20 + 0.083) + 20,$$

$$R(t, 1) = R(20, 1) \quad (17)$$

$$\text{for } k \cdot (20 + 0.083) + 20 \leq t < (k + 1) \cdot (20 + 0.083),$$

where $k = 0, 1$.

5. Conclusions

The reliability analysis shows that the upper girder plates corrode significantly earlier than lower girder plates. It is shown that there is a significant difference between the values of the reliability function for the lower and upper girders. The results of this study show that applying a multi-stage approach to reliability analysis and improving reliability after reaching a critical state allows us to identify the time when corrosion reaches a critical value. They also show that there is a significant difference in the intensity of corrosion depending on the measurement location and that the corrosion of steel plates in the upper zone of the

impermeable girder exceeds the critical value after 14.6 years, while it is significantly later, i.e., after 20.6 years, in the lower locations.

The moment when the risk function exceeds the allowable level is also significantly different for the lower and upper sides of the examined girders. Exceeding a critical state by the risk function eventually may cause fuel to mix with water in adjacent ballast tanks and environmental pollution. The girder plates should be replaced, after exceeding a critical state, with new ones in order to keep the system in good working condition. The analysis revealed that ship management and maintenance companies should thoroughly inspect the steel plates on the upper side of girders after ten years of operation and on the lower side after twenty years. Regular monitoring and preventive maintenance can improve reliability, prolong ship lifespan, ensure safety, and significantly reduce the costs of potential major repairs.

In this study, we showed that the multistate reliability approach can be applied to the reliability analysis of steel plates. Conducting Lagrange polynomial interpolation in five-year periods gives us some limitations on the precision of the estimated function. However, regular ship inspections take place every five years, so the proposed method allows us to answer the question of when to replace corroded surfaces. We applied the presented model to one type of vessel, i.e., bulk-carriers; due to significant differences in size, cargo properties (liquid, containerized, etc.), and different kinds of operation, the model should be applied separately to different types of ships in future work.

Further research should also focus on analyzing the cost of replacing corroded surfaces. Since the cost of replacing steel plates on ships is very high, this replacement takes the ship out of service and makes the ship unprofitable, so owners may choose not to replace corroded surfaces. Thus, the problem of cost optimization seems worthy of consideration, and it is an interesting research problem that needs to be studied for human protection (seamen, employees) and for environmental reasons.

Author Contributions: Conceptualization, J.S.-B. and Š.I.; methodology, J.S.-B.; validation, J.S.-B. and Š.I.; formal analysis, J.S.-B. and Š.I.; investigation, Š.I.; resources, J.S.-B. and Š.I.; data curation, Š.I.; writing—original draft preparation, J.S.-B. and Š.I.; writing—review and editing, J.S.-B. and Š.I. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the statutory activities of Gdynia Maritime University, grant number WN/2025/PZ/16.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The original contributions presented in the study are included in the article; further inquiries can be directed to the corresponding author.

Acknowledgments: This research work has been supported by the Invar-Ivošević Company. Some more information about the Company can be found at: <http://www.invar.me>.

Conflicts of Interest: The authors declare no conflicts of interest.

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