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Application of Unsteady Fluid Flow Simulation in the Process of Regulating an Industrial Hydraulic Network

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Abstract: In this article, an analytical solution to a hydraulic network with a wide range of pipe lengths (up to 10 km) is proposed. The Finite-Difference Time-Domain (FDTD) method was applied with the aim of creating a regulation model for controlling both the flow rate of water from one of the two sources and the discharge pressure in the system. The system inertia requires an understanding of boundary conditions in the operation of pipeline networks, which must be known in order to regulate the required parameters with only minor deviations. The proposed model was compared to experimental data, while the mean absolute deviations in the individual system branches ranged from 1 to 5.19%. The created regulation model was subsequently tested by applying linear, sine and stochastic changes in the output load, while the ability to control the discharge pressure and the selected water flow rate was analysed. The effect of coefficient ϵ , which multiplies the effect of the difference between the measured and the predicted value of the discharge pressure on the boundary conditions of the discharge pressure in the system, was analysed in this paper. With the use of the proposed unsteady simulation of the fluid flow in the hydraulic system arranged in parallel and in series, the maximum deviation of the regulated pressure was 1.2% and the maximum deviation of the regulated flow rate was 5.3%.

Keywords: hydraulic network; unsteady fluid flow; regulation; dynamic model; water



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1. Introduction

Implementation of novel technologies in production processes often requires optimising water supply systems, which range from simple open channels for the gravity-fed water supply to complex and sophisticated systems comprising a water treatment station, pumping technology, a reservoir and water distribution pipelines with a length of as many as several kilometres. Newly constructed and complex water distribution systems represent only a small portion of the total number of operated water supply systems. The majority of industrial hydraulic installations are sporadically innovated based on the current production process requirements or changes in the production portfolios of undertakings. A similar situation exists with regard to new installations, which often need adjustments, after only a few years, to the current requirements regarding the demand, operation cost-efficiency and effectivity, as well as the environmental impact [1–6].

Just like any other systems, water distribution networks are exposed to various internal and external factors, associated with their operation and atmospheric phenomena, which may change the characteristics of the network that controls the system. Pressure losses in a pipeline network largely depend on the quality of the surface of inner pipe walls, which

deteriorates over the distribution system utilisation period. The surface degradation is caused by the corrosive effect of transported fluids and incrustation [7,8].

In long hydraulic pipelines, pressure losses are mainly caused by the pipeline length. In order to ensure that a pipeline network exhibits the required pressure or flow rate parameters, it is necessary to correctly identify the losses. Interventions performed in connection with the replacement of certain functional parts, repairs and accidents represent another source of contamination of hydraulic networks. Major changes and extensions of installations, primarily the extensions of the pipeline networks of the operated facilities and the attachment of new systems to the existing networks, may also contaminate water distribution networks. The accumulation of sediments on the walls reduces the flow rate profile of water pipelines, increases the pressure loss, and over time causes a disruption of the characteristics of the distribution networks, which is also facilitated by the corrosive effect. A similar case is the installation of new, remotely controlled regulating components, the replacement of which requires that the distribution system settings comply with the operational requirements of production. At present, it is a priority to regulate a hydraulic network based on the monitoring of its operational parameters in the steady-state fluid flow. This facilitates analysing the changes in the flow-rate characteristics that are caused by the gradual, prolonged incrustation and clogging of pipes [7–9].

Economically and environmentally efficient optimisation of the operation and the regulating components of water distribution systems, or the water management system of a particular enterprise from the global point of view, requires thorough monitoring of operational parameters with the aim of better understanding the behaviour of water or any other fluid flowing in the analysed system. A tool that is beneficial in the process of increasing operation efficiency is an analysis of the related laws of physics while applying mathematical and numerical models. Predicting the behaviour of a water distribution network with the use of mathematical models will increase the efficiency of a search for critical points in the system that are associated with higher risk; in terms of ensuring a faultless operation, it will facilitate identifying the behaviour of the hydraulic network in case of a deviation from the standard conditions and simplifying the process of assessing the risks associated with the network extension, since that usually results in elevated water consumption [10–16]. Historically, it was relatively costly, time-consuming and labour-intensive to design and create mathematical models. Today, the model-building process is much easier, also thanks to the possibility of sharing data with other systems and applications, such as geographic information systems, SCADA (Supervisory Control and Data Acquisition) systems and CAD (Computer-Aided Design) systems [17]. The expertise of 1D fluid flow simulation was applied by Ghostine et al. (2012). In their paper, a comparison of the 1D and the 2D approaches to simulating combined flows at open-channel junctions was presented. Channel junctions are found in nature; for example, in blood or hydrographic networks. Typical examples of such systems include municipal water distribution systems, irrigation and drainage systems, etc. They consist of the main channel and tributaries. In the 1D simulation, the Saint-Venant equations were coupled with the characteristic equations and junction models to find the solution in the network system. In the 2D simulation, the whole system was discretised into triangular cells forming an unstructured computational mesh. Therefore, the 2D Saint-Venant equations were applied without using empirical equations. The results of the numerical simulation indicated that the 1D approach is acceptable for small junction angles and subcritical and transitional flows. The 2D approach to simulating the junction flow is essential especially for a rectangular network, such as the branches in urban areas [12]. Branco et al. (2023) addressed linear stability and numerical analysis with the use of a model for the momentum flux parameters of the 1D two-fluid model in vertical annular flows. The authors claimed that the models improved the prediction of the 1D two-fluid model and proved consistency in the accuracy of the analysed databases [18]. Other examples of mathematical models were applied by Črnjarić-Žic and Mujaković (2016). The authors investigated solutions for a one-dimensional compressible viscous micropolar fluid flow

with different boundary conditions. The mathematical model consisted of four partial differential equations, transformed from the Eulerian to the Lagrangian description. The equations were also associated with various boundary conditions. In their study, they worked with numerical approximations of the one-dimensional viscous heat conducting and micropolar fluid flow problems while using two different numerical schemes—the finite difference scheme and the Faedo–Galerkin method. Both approaches exhibited good concordance between the results. The authors also found out that the finite difference scheme exhibited a lower computational complexity [19].

2D and 3D numerical models facilitate describing multidimensional relationships; however, their computational complexity is high, and the solutions are significantly time-consuming. In order to achieve optimal computation times, some authors use one-dimensional averaged models. The focus of this article is the prediction of pressures and flow rates in an open distribution system that supplies water to a production company with the use of an analytical dynamic model for fluid flow simulation and by applying the finite-difference time-domain method [19–22]. In this paper, a dynamic model of main distribution pipelines is analysed with the aim of stabilising the discharge pressure in the industrial facility while maintaining a constant flow rate of water from the selected sources of industrial water.

2. Mathematical Model

The analysed system is an existing distribution pipeline system in an industrial facility with two different sources of industrial water (Source 1 and Source 2) and two independent outlets (V2 and V4), as shown in Figure 1. Water is supplied to the V4 Branch at a flow rate Q_2 through a pipeline with a constant cross-section, a length L_{2-V4} , a diameter d_{2-V4} and a difference between the pipeline inlet and outlet elevations ΔH_{2-V4} . Source 1 is equipped with pumps arranged in parallel to facilitate supplying water separately to the V2 and V4 Branches. Water is supplied to the V2 Branch through the pipes arranged in series and in parallel from two different sources of industrial water. Water from Source 1 is supplied at a flow rate Q_1 through pipes 1–4 (length L_{1-4} ; diameter d_{1-4} ; elevation difference ΔH_{1-4}). Water from Source 2 is supplied at a flow rate Q_3 through pipes 3–4 (length L_{3-4} ; diameter d_{3-4} ; elevation difference ΔH_{3-4}); water in that source originates in the wastewater treatment plant and is supplied back to the facility. The requirement was to maximise the utilisation of that source while maintaining a preset water flow rate. Subsequently, water is transported to pipe 4–V2 (length L_{4-V2} ; diameter d_{4-V2} ; elevation difference ΔH_{4-V2}) in the facility. The main purpose of creating the model was to produce a hydraulic description of a dynamic system that would facilitate stabilising discharge pressures p_{V2} and p_{V4} while maintaining a constant flow rate of water from Source 2, in particular by regulating pressures p_1 , p_2 and p_3 .

The hydraulic model was based on the following design assumptions:

- The inside diameter of the pipeline is free of any internal incrustation;
- With regard to the pressure differences, the pipe expansion is neglected, and the pipes are regarded as perfectly rigid;
- The values of local losses caused by hydraulic components (valves, check valves, etc.) are known.

The calculations below apply to an unsteady flow of an incompressible fluid. All the pressures below are relative when compared to the atmospheric pressure. For the individual V2 and V4 Branches, the following equations were applied to describe the pressure equilibrium:

$$p_1 - \Delta p_{1-4} - \Delta p_{4-V2} = p_{V2} \text{ (Pa)} \quad (1)$$

$$p_3 - \Delta p_{3-4} - \Delta p_{4-V2} = p_{V2} \text{ (Pa)} \quad (2)$$

$$p_2 - \Delta p_{2-V4} = p_{V4} \text{ (Pa)} \quad (3)$$

wherein p_1 is the relative pressure at the inlet into the V2 Branch from Source 1 (Pa); Δp_{1-4} is the difference between the pressures in the 1–4 Branch (Pa); Δp_{x-y} is the difference between the pressures in the x–y Branch (Pa); p_{V2} is the pressure at the outlet from the V2 Branch into the facility (Pa); p_3 is the pressure at the inlet into the V2 Branch from Source 2 (Pa); and p_2 is the pressure at the inlet into the V4 Branch from Source 1 (Pa). The flow rate balance was described using Equation (4):

$$Q_1 + Q_3 = Q_4 \quad (\text{m}^3 \cdot \text{s}^{-1}) \quad (4)$$

wherein Q_i represents the flow rates in the individual branches and sections, as shown in Figure 1 ($\text{m}^3 \cdot \text{s}^{-1}$).

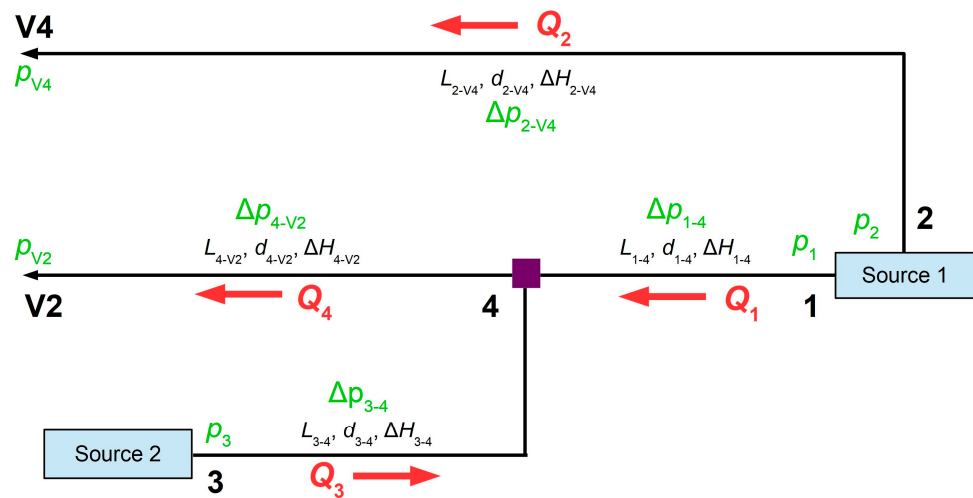


Figure 1. A diagram of flow rates, pressure differences and relative pressures in the pipeline system.

The pressure differences in the individual branches may be calculated by adjusting Bernoulli's equation for an unsteady flow of a viscous fluid [23]:

$$\rho \cdot g \cdot z + p + \rho \cdot \frac{v^2}{2} + \rho \cdot \int_l \frac{\partial v}{\partial \tau} ds = const. \quad (5)$$

$$\Delta p_{1-4} = \rho \cdot g \cdot \frac{0.0827 \cdot \lambda_{1-4}}{d_{1-4}^5} \cdot L_{ekv,1-4} \cdot Q_1^2 + \rho \cdot g \cdot \Delta H_{1-4} + \rho \cdot \frac{4 \cdot L_{1-4}}{\pi \cdot d_{1-4}^2} \cdot \frac{\Delta Q_1}{\Delta \tau} \quad (6)$$

$$\Delta p_{4-V2} = \rho \cdot g \cdot \frac{0.0827 \cdot \lambda_{4-V2}}{d_{4-V2}^5} \cdot L_{ekv,4-V2} \cdot [Q_1 + Q_3]^2 + \rho \cdot g \cdot \Delta H_{4-V2} + \rho \cdot \frac{4 \cdot L_{4-V2}}{\pi \cdot d_{4-V2}^2} \cdot \frac{\Delta Q_4}{\Delta \tau} \quad (7)$$

$$\Delta p_{3-4} = \rho \cdot g \cdot \frac{0.0827 \cdot \lambda_{3-4}}{d_{3-4}^5} \cdot L_{ekv,3-4} \cdot Q_3^2 + \rho \cdot g \cdot \Delta H_{3-4} + \rho \cdot \frac{4 \cdot L_{3-4}}{\pi \cdot d_{3-4}^2} \cdot \frac{\Delta Q_3}{\Delta \tau} \quad (8)$$

$$\Delta p_{2-V4} = \rho \cdot g \cdot \frac{0.0827 \cdot \lambda_{2-V4}}{d_{2-V4}^5} \cdot L_{ekv,2-V4} \cdot Q_2^2 + \rho \cdot g \cdot \Delta H_{2-V4} + \rho \cdot \frac{4 \cdot L_{2-V4}}{\pi \cdot d_{2-V4}^2} \cdot \frac{\Delta Q_2}{\Delta \tau} \quad (9)$$

wherein ρ is the density of the transported water ($\text{kg} \cdot \text{m}^{-3}$); g is the gravitational acceleration ($\text{m} \cdot \text{s}^{-2}$); λ_{x-y} is the coefficient of the pressure loss caused by the pipe length (1); d_{x-y} is the pipe diameter (m); $L_{ekv,x-y}$ is the equivalent length of the pipe in the x–y Branch (m); Q_i is the water flow rate ($\text{m}^3 \cdot \text{s}^{-1}$); ΔH_{x-y} is the difference between the elevations of the x–y Branch inlet and outlet (m); L_{x-y} is the real length of the pipe (m); ΔQ_{x-y} is the change in the water flow rate in the pipe over the time interval $\Delta \tau$ ($\text{m}^3 \cdot \text{s}^{-1}$); and $\Delta \tau$ is the analysed time interval (s).

The coefficients of the pressure loss in the pipes caused by the pipe length may be calculated using Moody's equation for the transition zone of a turbulent flow with a Reynolds number range of $4000 < Re < 10^7$:

$$\lambda = 0.0055 \left[1 + \left(2 \cdot 10^4 \cdot \frac{k}{d} + \frac{10^6}{Re} \right)^{\frac{1}{3}} \right] \quad (10)$$

wherein k is the absolute roughness of the pipe (m) and Re is the Reynolds number (1).

By substituting k_1 with k_8 and k_a with k_d , Equations (6)–(9) may evidently be simplified as follows:

$$\Delta p_{1-4} = k_1 \cdot Q_1^2 + k_2 + k_a \cdot \frac{\Delta Q_1}{\Delta \tau} \quad (11)$$

$$\Delta p_{4-V2} = k_3 \cdot [Q_1 + Q_3]^2 + k_4 + k_b \cdot \frac{\Delta Q_1 + \Delta Q_3}{\Delta \tau} \quad (12)$$

$$\Delta p_{3-4} = k_5 \cdot Q_3^2 + k_6 + k_c \cdot \frac{\Delta Q_3}{\Delta \tau} \quad (13)$$

$$p_{2-V4} = k_7 \cdot Q_2^2 + k_8 + k_d \cdot \frac{\Delta Q_2}{\Delta \tau} \quad (14)$$

In cases where it is necessary to analyse a steady-state system with constant flow rates and pressures, Equations (11)–(14) may be used while neglecting the last term with a time difference in the flow rate. Otherwise, with known boundary conditions of pressures and baseline flow rates, it is possible to identify the changes in flow rates over time. By substituting Equations (11)–(13) in Equations (1) and (2), the following set of equations is obtained for the V2 Branch:

$$p_1 - p_{V2} - k_1 \cdot Q_1^2 - k_2 - k_3 \cdot [Q_1 + Q_3]^2 - k_4 = (k_a + k_b) \cdot \frac{\Delta Q_1}{\Delta \tau} + k_b \cdot \frac{\Delta Q_3}{\Delta \tau} \quad (15)$$

$$p_3 - p_{V2} - k_5 \cdot Q_3^2 - k_6 - k_3 \cdot [Q_1 + Q_3]^2 - k_4 = (k_c + k_b) \cdot \frac{\Delta Q_3}{\Delta \tau} + k_b \cdot \frac{\Delta Q_1}{\Delta \tau} \quad (16)$$

By substituting the constants, the following set of equations that describe a correlation between the changes in flow rates ΔQ_1 and ΔQ_3 is obtained:

$$z_1 = z_2 \cdot \frac{\Delta Q_1}{\Delta \tau} + z_3 \cdot \frac{\Delta Q_3}{\Delta \tau} \quad (17)$$

$$z_4 = z_5 \cdot \frac{\Delta Q_3}{\Delta \tau} + z_6 \cdot \frac{\Delta Q_1}{\Delta \tau} \quad (18)$$

The above equations may be used to identify the change in flow rates over a time unit with the known existing pressures and flow rates:

$$\Delta Q_3 = \frac{z_1 \cdot z_6 - z_2 \cdot z_4}{z_3 \cdot z_6 - z_2 \cdot z_5} \cdot \Delta \tau \quad (19)$$

$$\Delta Q_1 = \frac{z_4}{z_6} \Delta \tau - \frac{z_5}{z_6} \cdot \Delta Q_3 \quad (20)$$

By substituting Equation (14) in Equation (3) and making subsequent adjustments, the following equation for calculating the change in the flow rate in the V4 Branch is obtained:

$$\Delta Q_2 = \frac{p_2 - \Delta p_{V4} - k_7 \cdot Q_2^2 - k_8}{k_d} \Delta \tau \quad (21)$$

The flow rates in the individual branches as a function of time may be identified by applying the Finite-Difference Time-Domain method. In the case that the parameters to be identified are flow rates in the individual branches with known pressure boundary

conditions (p_1 , p_3 and p_{V2}) and initiation flow rates ${}_{(i=0)}Q_1$ and ${}_{(i=0)}Q_3$, the numerical solution for the V2 Branch is defined using the following equations:

$${}_{(i+1)}Q_3 = {}_{(i)}Q_3 + \frac{z_1 \cdot z_6 - z_2 \cdot z_4}{z_3 \cdot z_6 - z_2 \cdot z_5} \cdot {}_{(i)}\Delta\tau \quad (22)$$

$${}_{(i+1)}Q_1 = {}_{(i)}Q_1 + \left(\frac{z_4}{z_6} - \frac{z_5}{z_6} \cdot \frac{z_1 \cdot z_6 - z_2 \cdot z_4}{z_3 \cdot z_6 - z_2 \cdot z_5} \right) {}_{(i)}\Delta\tau \quad (23)$$

For the V4 Branch, the numerical calculation by applying the finite-difference time-domain method was made using the following equation:

$${}_{(i+1)}Q_2 = {}_{(i)}Q_2 + \frac{p_2 - \Delta p_{V4} - k_7 \cdot {}_{(i)}Q_2^2 - k_8}{k_d} {}_{(i)}\Delta\tau \quad (24)$$

The calculations require knowing the pressure boundary conditions. Results of such calculations typically exhibit good concordance with experimental data in the case of a newly constructed pipeline system. In many cases, it is necessary to analyse hydraulic systems with a potential presence of incrustation and sediments along large distribution distances with multiple local hydraulic resistances. For those types of distribution systems, it is more appropriate to use experimental measurements of pipeline coefficients.

2.1. A Model for Calculating the Pipeline Parameters through an Experiment

Due to the fact that the previous calculations were based on the design parameters of the pipeline that no longer exhibits good concordance between the measured data and the experiments due to the effects of incrustation, a different approach was applied. In that approach, coefficients k_1 , k_3 and k_5 for the V2 Branch and coefficients k_7 and k_8 for the V4 Branch were identified by analysing the measured values of flow rates and pressures in a real and steady-state water flow. The steady state may be achieved with a constant water withdrawal and constant pressures, which often occur in a real facility. In this case, mathematical procedures according to Equations (11)–(14) may be used, neglecting the last terms of the equations that describe the dynamic changes in the system.

By deducting Equations (1) and (2), an equation describing the pressure equilibrium at the point where the transported water mixes in point 4 is obtained:

$$p_1 - \Delta p_{1-4} = p_3 - \Delta p_{3-4} \quad (25)$$

By substituting Equations (11) and (13), while neglecting the terms of equations that describe the changes in the flow rate (the steady state), in Equation (25), the following set of equations for the two sets of measured data is obtained:

$$p'_1 - k_1 \cdot Q'^2_1 - k_2 = p'_3 - k_5 \cdot Q'^2_3 - k_6 \quad (26)$$

$$p''_1 - k_1 \cdot Q''^2_1 - k_2 = p''_3 - k_5 \cdot Q''^2_3 - k_6 \quad (27)$$

wherein the pressures and flow rates with one apostrophe represent the data obtained in Measurement 1, while the values with two apostrophes represent the data obtained in Measurement 2. Achieving the required calculation accuracy is conditioned by conducting analyses in two steady states with significantly different flow rates and pressures in the system during the measurement. Coefficients k_2 and k_6 represent the hydrostatic pressure of the water column in the pipeline, and they may be simply calculated using the known differences between the pipe inlet and outlet elevations. The only unknown parameters in the set of equations are k_1 and k_5 , and they may be calculated as follows:

$$f k_5 = \frac{p'_1 \cdot Q''^2_1 - p''_1 \cdot Q'^2_1 + k_2 (Q'^2_1 - Q''^2_1) - p'_3 \cdot Q''^2_3 + p''_3 \cdot Q'^2_3 - k_6 (Q'^2_3 - Q''^2_3)}{Q'^2_1 \cdot Q''^2_3 - Q'^2_3 \cdot Q''^2_1} \quad (28)$$

$$k_1 = \frac{p_1'' - k_2 - p_3'' + k_5 \cdot Q_3''^2 + k_6}{Q_1''^2} \quad (29)$$

Coefficient k_3 is identified from Measurement 1 by adjusting Equation (1):

$$k_3 = \frac{p_1' - k_1 \cdot Q_1'^2 - k_2 - k_4 - p_{V2}}{(Q_1' + Q_3')^2} \quad (30)$$

In order to identify coefficients k_7 and k_8 , Equation (3) was used in the two measurements of the steady-state flow:

$$k_7 = \frac{(p_{V4}' - p_{V4}'') - (p_2' - p_2'')}{Q_2''^2 - Q_2'^2} \quad (31)$$

$$k_8 = p_2' - k_7 \cdot Q_2'^2 - p_{V4}' \quad (32)$$

In order to identify coefficients k_7 and k_8 , only one measurement can be used in a steady water flow in the V4 Branch, while coefficient k_8 is deducted from the known value of the difference between the pipeline inlet and outlet elevations.

The identification of the pipe coefficients facilitated identifying the real parameters of the analysed branches, with an option of the reverse identification of average incrustation (a reduction of the inside diameter due to accumulation of sediments). By adjusting Equations (6) and (11), it was possible to use the measured data in order to calculate the real diameter of the pipes in the 1–4 Branch:

$$d_{1-4} = \sqrt[5]{\rho \cdot g \cdot \frac{0.0827 \cdot \lambda_{1-4}}{k_1} \cdot L_{ekv,1-4}} \quad (33)$$

Similar procedures were applied to calculate the average diameters of pipes in the other branches with the use of coefficients k_3 , k_5 and k_7 .

2.2. Application of the Analytical Approach to the Regulation of the Hydraulic System

When the hydraulic system is sufficiently described by an analytical model and the coefficients of pipes in the individual branches are identified, those correlations may be used to regulate the system. The requirement determined for the regulation of the network in the industrial enterprise was to maximise the use of water from the wastewater treatment plant, supplied from Source 2 (the environmental aspect of the operation), while stabilising the water discharge pressure in the enterprise (p_{V2} and p_{V4}). The required pressures are regulated at the inlets into pipes (p_1 and p_3) from Source 1 and Source 2 by continuous regulation of rotational speeds of the pumps. For the purpose of regulating the V2 Branch, it was proposed to modify Equations (1) and (2) and supplement the term that adjusts the preset pressure to the difference between the real (p_{V2}) and the required value ($p_{V2\text{-required}}$) of pressure at the inlet into the facility:

$$p_1 = p_{V2\text{-required}} + k_1 \cdot Q_1^2 + k_2 + k_3 \cdot [Q_1 + Q_{3\text{-required}}]^2 + k_4 - \varepsilon \cdot (p_{V2} - p_{V2\text{-required}}) \quad (34)$$

$$p_3 = p_{V2\text{-required}} + k_5 \cdot Q_{3\text{-required}}^2 + k_6 + k_3 \cdot [Q_1 + Q_{3\text{-required}}]^2 + k_4 - \varepsilon \cdot (p_{V2} - p_{V2\text{-required}}) \quad (35)$$

The regulation is functional in the continuous measurement of the flow rate Q_1 and the discharge pressure p_{V2} . The flow rate $Q_{3\text{-required}}$ is substituted with the value of the required flow rate of water from Source 2. An increase in the water withdrawal in the industrial enterprise will result in a short-term decrease in the measured pressure p_{V2} and an increase in the flow rate Q_1 . Subsequently, the model is used to calculate new required pressures p_1 and p_3 at which the system is in equilibrium. Based on the identified pressures,

the rotational speeds of the pumps are adjusted to achieve higher values; this facilitates that pressure p_{V2} acquires the required operational value $p_{V2\text{-required}}$. Flow rate Q_1 is also adjusted. A variable parameter in the proposed regulation system is coefficient ε , which facilitates multiplying the deviation between the required and the measured pressure values, thereby significantly stabilising the dynamic changes in the discharge pressure p_{V2} . However, if the ε value is increased too much, the solution may become unstable; therefore, the solution settings depend on the type of the hydraulic system and the response of the controlling elements. Based on the testing of the regulated system, the value of parameter ε that is critical for the system stability was determined; when that value exceeded the critical limit, the regulated system parameters exhibited a divergence. The critical value ε was identified using the ratio of the average time constant for the pipe τ_{pipe} , which was defined as the time required for the regulated parameter, diverted by 63.2% from its amplitude, to stabilise to the reaction time of the pumps τ_{pump} :

$$\varepsilon < \varepsilon_{\text{crit}} = \frac{\tau_{\text{pipe}}}{\tau_{\text{pump}}} \quad (36)$$

The reaction time of the pumps is the average time required for the preset output pressure to reach the required value.

A similar regulation procedure was applied to the V4 Branch, in which the flow rate Q_2 and the discharge pressure p_{V4} were measured:

$$p_2 = p_{V4\text{-required}} + k_7 \cdot Q_2^2 + k_8 - \varepsilon \cdot (p_{V4} - p_{V4\text{-required}}) \quad (37)$$

The regulating person analyses the measured data and makes adequate adjustments to the inlet pressure p_2 at the entry into the V4 Branch.

3. Results and Discussion

For the purpose of verifying the correctness and accuracy of the analytical model, experimental measurements were performed on the hydraulic system in the industrial facility. Its basic dimensional parameters are listed in Table 1.

Table 1. Pipe parameters in the individual system branches.

Branch elevation	$\Delta H_{1-4} = 23.76 \text{ m}$	$\Delta H_{4-V2} = 0.8 \text{ m}$	$\Delta H_{3-4} = 13.3 \text{ m}$	$\Delta H_{2-V4} = 28.46 \text{ m}$
Pipe diameter	$d_{1-4} = 1.1 \text{ m}$	$d_{4-V2} = 1.1 \text{ m}$	$d_{3-4} = 0.9 \text{ m}$	$d_{2-V4} = 1.1 \text{ m}$
Pipe length	$L_{1-4} = 7269 \text{ m}$	$L_{4-V2} = 792 \text{ m}$	$L_{3-4} = 4317 \text{ m}$	$L_{2-V4} = 8700 \text{ m}$

In the measured sections with a steady water flow, the data required for calculating the pipe coefficients were obtained. They were used to calculate the flow rates in the pipeline over a 10 h interval of operation. This facilitated comparing the measured data with the computational model. Subsequently, various simulations of water withdrawal in the industrial enterprise with subsequent regulation of the system at variable values of coefficient ε were analysed.

3.1. A Comparison of the Experimental Data and the Model

The analytical and experimental data were compared while using the data of the hydraulic system that was measured in the industrial enterprise with a 60 s interval over the total time of 10 h. Flow rates Q_1 through Q_4 and pressure boundary conditions of the system were measured. The measured data was used to specify two sets of the measured data at various flow rates and pressures, while the measurement deviation in the defined 5 min interval corresponded to the accuracy of the used sensors.

The aforesaid sections could be regarded as steady, without any dynamic changes. Subsequently, the coefficients defining the parameters of the pipeline sections listed in Table 2 were calculated using Equations (28) through (32).

Table 2. Coefficients k_1 through k_8 identified by calculation (k_2 , k_4 and k_6) and experiments (k_1 , k_3 , k_5 , k_7 , k_8).

k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8
262,251	232,853	87,275	122,502	1,190,691	130,343	292,535	334,896

The key advantage of the approach described above is defining the coefficients that take into account the real condition of the pipeline, including the incrustation and hidden local resistances. For the purpose of dynamic calculations, it was necessary to define coefficients k_a through k_d , and they were identified using the pipeline design parameters. The analytical calculations were made by applying the finite-difference time-domain method with the aim of identifying the changes in flow rates, using Equations (19)–(21). The input data included the previously identified pipeline coefficients and the pressure boundary conditions. The comparisons of the measured data and the resulting flow rate values that were calculated with the use of the mathematical model are presented in Figures 2–4. The time step applied in the calculations was 60 s, which was in concordance with the time step applied in recording the measured data.

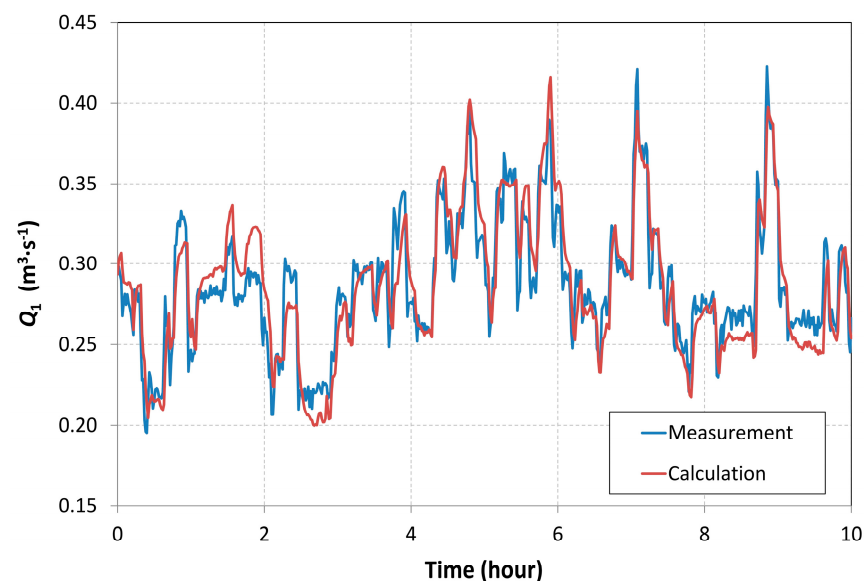


Figure 2. The curves of the measured and the calculated flow rate Q_1 with the known pressure boundary conditions in the V2 Branch.

The results of the calculations were in good concordance with the experimental measurements performed in the real facility, while the average deviation of the absolute value of the difference between the measured and the calculated flow rate values was 5.19% for flow rate Q_1 , 0.99% for flow rate Q_2 , and 2.7% for flow rate Q_3 . Figure 5 shows the instantaneous deviations between the calculated and the measured flow rate values.

Local deviations between the calculated and the measured values were the highest at the flow rate Q_1 , where the proposed model is more complicated due to the fact that water is supplied from two different sources. The lowest deviations were observed in the simple V4 Branch with the flow rate Q_2 . The deviations were also significantly affected by the measurement uncertainty, which was used in the calculation in the form of the pressure boundary conditions, and this increases the probability of inaccuracy. Secondary reasons

for the deviations were potential local withdrawals of water, which might have altered the coefficients of the pipes due to local changes in the local hydrodynamic resistances that may have occurred due to the robust extent of the operation. Also, the dynamic development of the pressures in the pipes caused a slight change in the pipe diameter, which eventually resulted in a significant change in the volume of water in the system over long distances. Moreover, the deviation between the simulation and the measurement was also affected by the time step used in the data scanning, which is often affected by a delay in the collection of certain data as a result of the internet connection between the collection points and the dataloggers in the individual facilities.

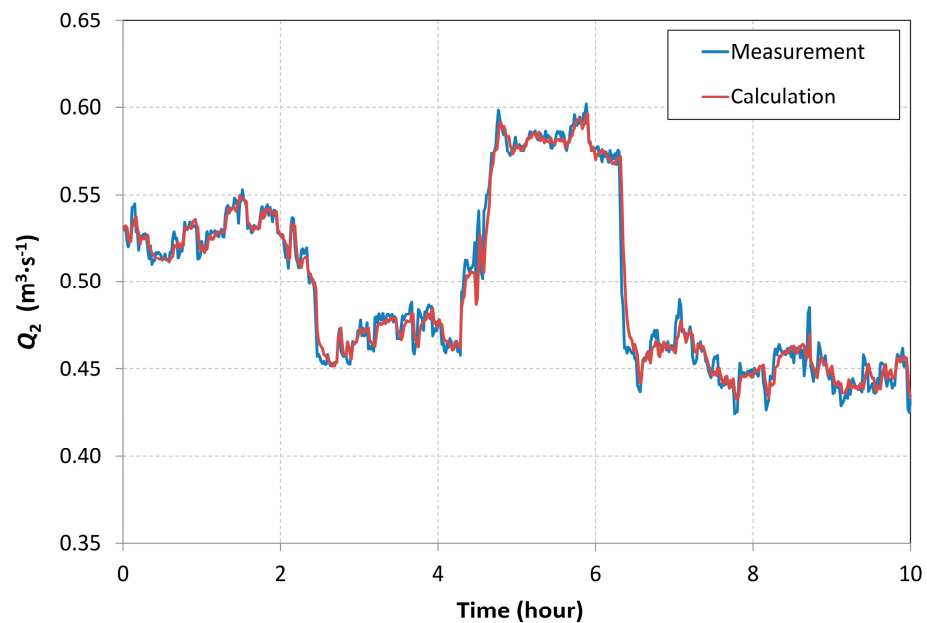


Figure 3. The curves of the measured and the calculated flow rate Q_2 with the known pressure boundary conditions in the V4 Branch.

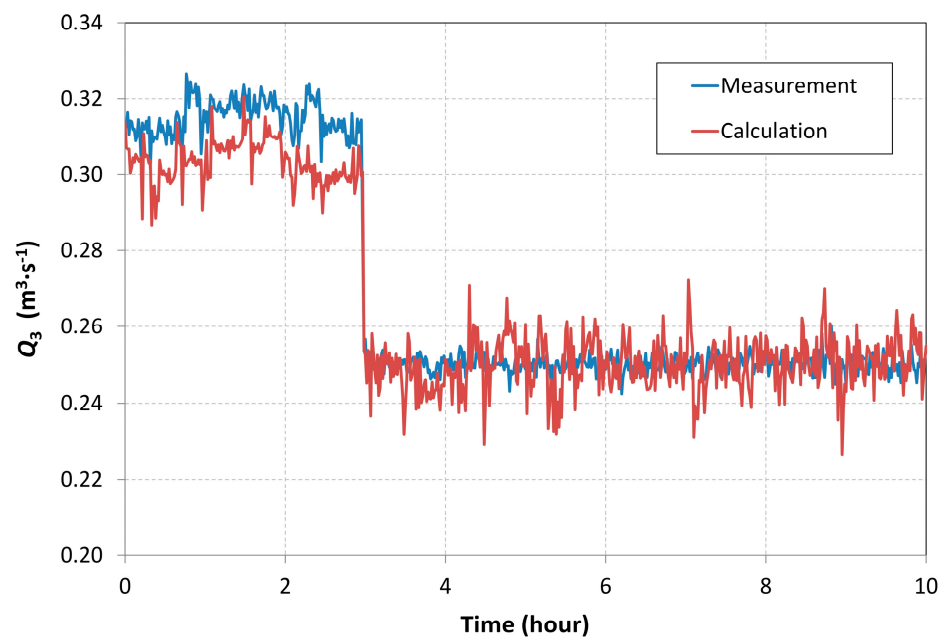


Figure 4. The curves of the measured and the calculated flow rate Q_3 with the known pressure boundary conditions in the V2 Branch.

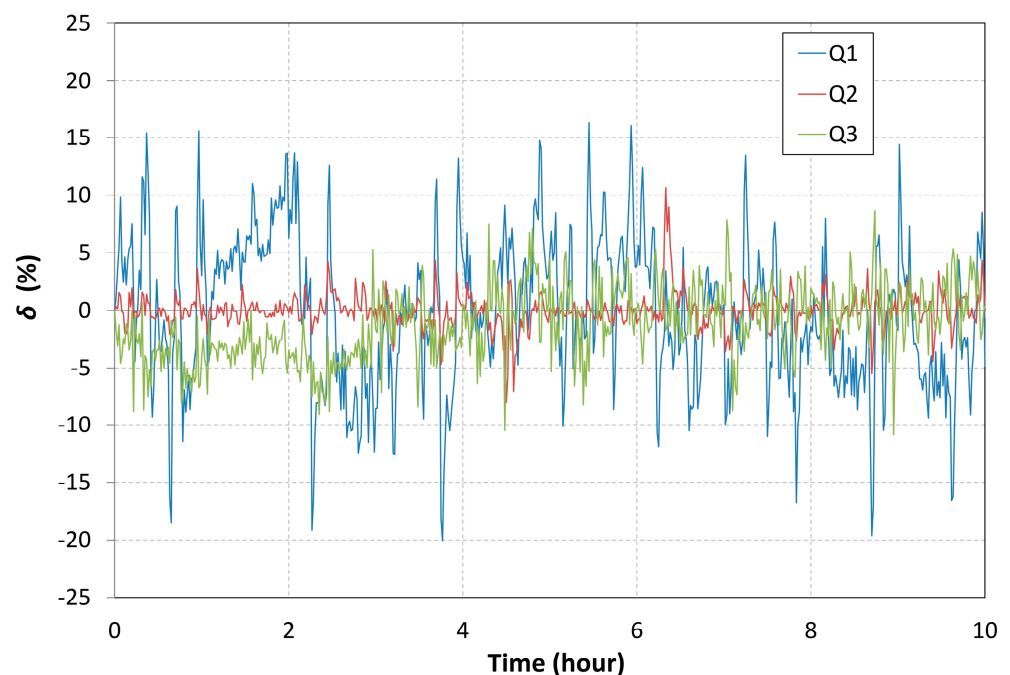


Figure 5. Deviations between the calculated and the measured flow rate values.

3.2. A Regulation of the Hydraulic System through the Defined Mathematical Model

The primary purpose of the proposed model was to implement the system of regulating the hydraulic system in the industrial enterprise with the requirement to stabilise discharge pressures p_{V2} and p_{V4} while maintaining the required flow rate of water from Source 2. The regulation is based on the knowledge described in Equations (34)–(37). In order to demonstrate the regulation model, three model examples were created for the V2 Branch:

- Model Example 1: constant increasing of the flow rate, subsequent stabilising and then reducing the flow rate to the initial value;
- Model Example 2: a sine curve of the output flow rate;
- Model Example 3: a stochastic curve of the output flow rate.

The analysed pipe dimensions are listed in Table 1 and the measured pipe coefficients are listed in Table 2. Table 3 contains the long-term averages that were taken into account when determining the baseline flow rates at the beginning of the simulation.

Table 3. Baseline flow rates at the pipe inlets.

$Q_1 \text{ (m}^3 \cdot \text{s}^{-1}\text{)}$	$Q_2 \text{ (m}^3 \cdot \text{s}^{-1}\text{)}$	$Q_3 \text{ (m}^3 \cdot \text{s}^{-1}\text{)}$
0.294	0.53	0.314

The total time of the numerical simulation of the flow was 2000 s, with a time step of 1 s. The pressure boundary conditions were calculated using Equations (34) through (37). The density and dynamic viscosity of the flowing water was defined at a temperature of 12 °C. The required value of output pressures p_{V2} and p_{V4} was 0.26 MPa. Flow rate Q_3 was regulated with the aim of achieving a constant value of $0.314 \text{ m}^3 \cdot \text{s}^{-1}$.

In order to facilitate the use of numerical simulation of water withdrawal in the industrial enterprise, pressure p_{V2} was calculated using an equation in which the sum of all length-induced and local losses in the enterprise may be substituted with the local loss factor ζ_{eq} :

$$p_{V2} = \zeta_{ekv} \cdot \frac{8 \cdot Q_4^2}{\pi^2 \cdot d_{4-V2}^4} \cdot \rho \quad (38)$$

In all the model situations, the considered baseline flow rate was $0.607 \text{ m}^3 \cdot \text{s}^{-1}$ at the required pressure p_{V2} . This corresponded to the value of the equivalent local loss of 1275. In Model Example 1, the value of the local loss factor was reduced over a 4 min interval by 37.25% down to the value of 800; this simulated an increase in the water flow rate at the outlet from the V2 Branch. After the flow rates and pressures were stabilised, the ζ_{eq} value was increased again to the baseline value of 1275 over a 2 min interval. Subsequently, the flow rates and pressures were left to stabilise again. Figure 6 shows the curves of pressures and flow rates with coefficient $\varepsilon = 0$, at which the regulation was carried out without applying the difference between the measured and the required value of pressure p_{V2} . The calculated pressures p_1 and p_3 in the V2 Branch were used in the dynamic model with a 10 s delay, which is the mean time of change in the rotational speeds of the pumps within the selected range of pressures.

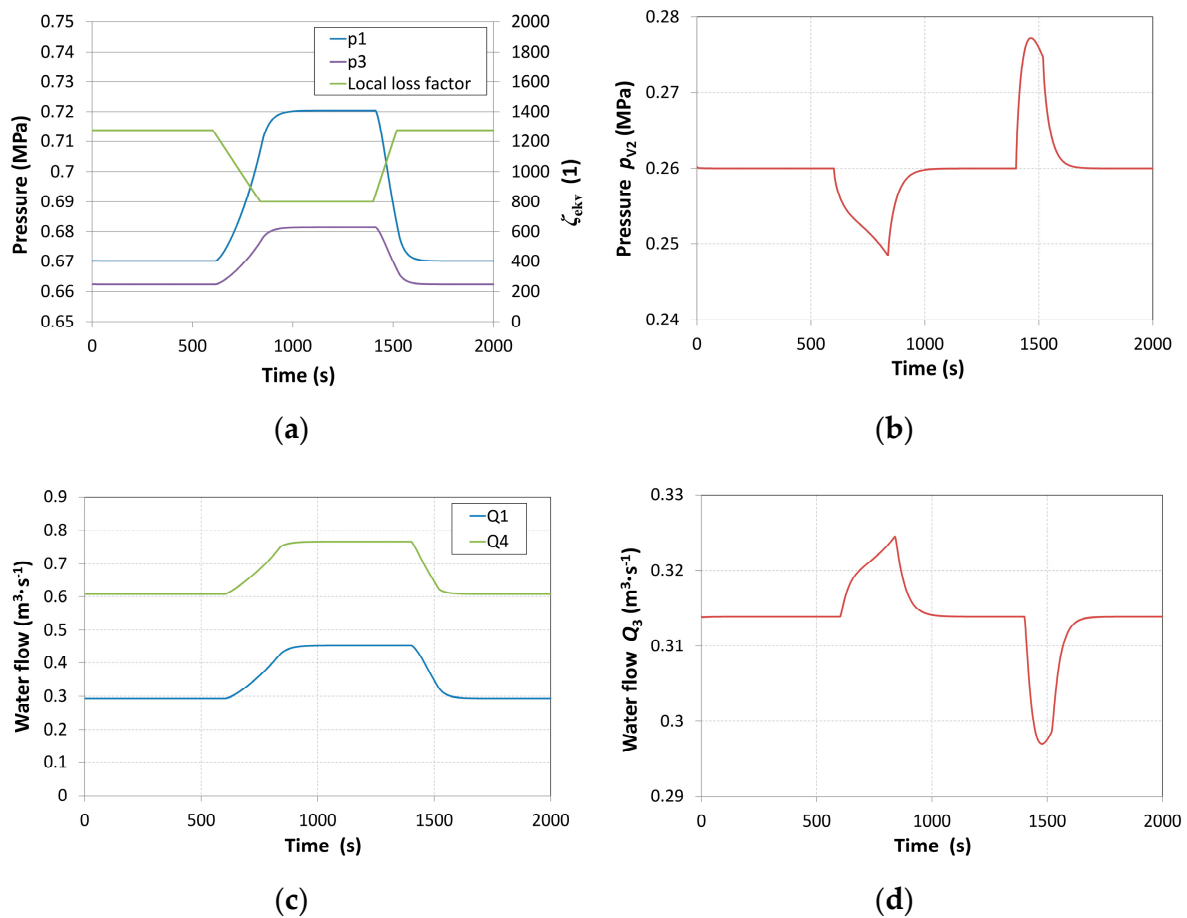


Figure 6. The curves of the operational parameters in Model Example 1 with the value of coefficient $\varepsilon = 0$; (a) the curves of p_1 , p_2 pressures and ζ_{eq} ; (b) the regulated pressure of water at the inlet into the facility in the V2 Branch; (c) flow rates Q_1 and Q_4 ; (d) the regulated flow rate Q_3 .

The required pressure at the outlet from the V2 Branch was set to 0.26 MPa. At the time of 600 s, ζ_{eq} decreased, which resulted in an increase in flow rate Q_1 and a decrease in pressure p_{V2} . Based on the proposed model, pressures p_1 and p_3 gradually increased and, consequently, the water flow rate in the V2 Branch increased. After 1000 s, the flow rates were stabilised with an unchanged regulated pressure p_{V2} and flow rate Q_3 . During the 26.27% increase in flow rate Q_1 , there was a short-term change in the regulated pressure p_{V2} by -4.38% and a 3.3% change in flow rate Q_3 . After the system stabilised (after 1400 s), a repeated increase in ζ_{eq} was simulated at a rate of double the amount of the previous decrease. That change caused a short-term increase in the regulated pressure p_{V2} and a

decrease in pressures p_1 and p_3 and in flow rate Q_1 . After 1700 s, the flow rates were stabilised with an unchanged regulated pressure p_{V2} and flow rate Q_3 . During the 20.81% decrease in flow rate Q_1 , there was a short-term change in the regulated pressure p_{V2} by 6.5% and a -5.3% change in flow rate Q_3 .

For the purpose of comparison, Figure 7 shows the curves of pressures and flow rates with the coefficient $\varepsilon = 5$, at which the regulation was carried out with the use of the difference between the measured and the required values of pressure p_{V2} .

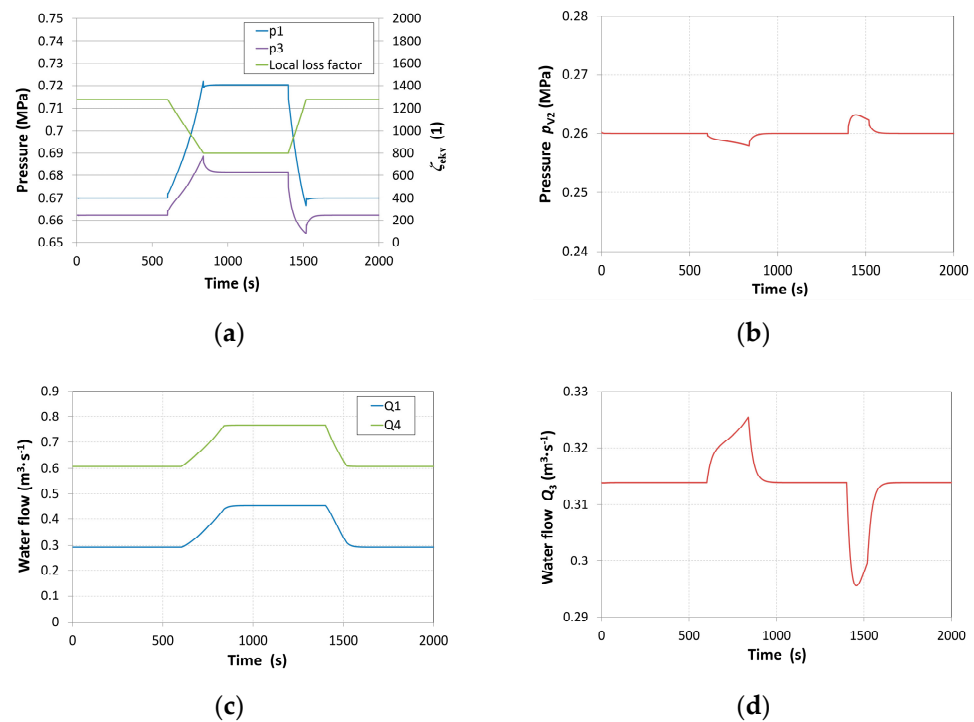


Figure 7. The curves of the operational parameters in Model Example 1 with the value of coefficient $\varepsilon = 5$; (a) the curves of p_1 , p_2 pressures and ζ_{eq} ; (b) the regulated water pressure at the inlet into the facility in the V2 Branch; (c) flow rates Q_1 and Q_4 ; (d) the regulated flow rate Q_3 .

A significant difference that was observed with the coefficient ε being set to 5, compared to the previous example, was the change in the curves of pressures p_1 and p_3 , which in the regulation apply the difference between the measured and the preset value of pressure p_{V2} . This resulted in a significant decrease in the short-term deviation of the regulated pressure p_{V2} , which amounted to -0.77% (after 827 s) and 1.18% (after 1430 s). At the flow rate Q_3 , only a negligible change in the short-term deviation was observed when compared to the previous case.

In Model Example 2, the curve of the value of the equivalent local loss was simulated with a sine curve, representing the alternating changes in the flow rate in the V2 Branch and in time. The ζ_{eq} curve was simulated with an amplitude of 200 and an interval of 630 s. Figure 8 shows the curves of pressures and flow rates with the preset coefficient $\varepsilon = 0$.

The deviation of pressure p_{V2} in the Model Example 2 with coefficient $\varepsilon = 0$ was $\pm 2.3\%$, while the deviation of flow rate Q_3 was $\pm 1.8\%$. When the coefficient ε was increased to 5, the deviation of pressure p_{V2} decreased to $\pm 0.39\%$ (Figure 9).

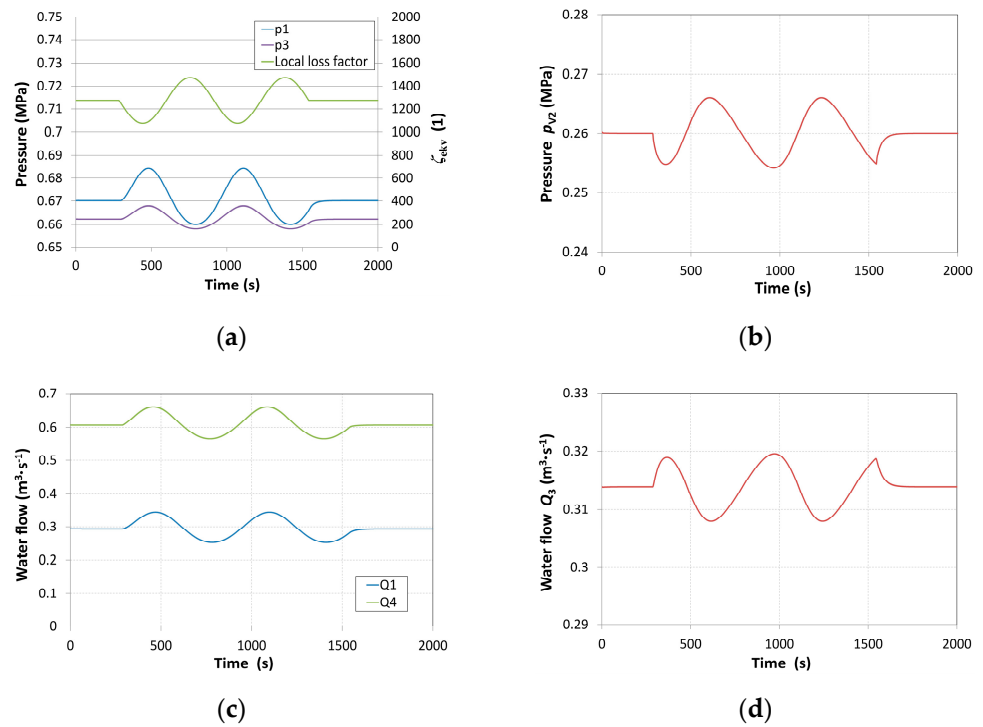


Figure 8. The curves of the operational parameters in Model Example 2 with the value of coefficient $\varepsilon = 0$; (a) the curves of p_1 , p_2 pressures and ζ_{eq} ; (b) the regulated water pressure at the inlet into the facility in the V2 Branch; (c) flow rates Q_1 and Q_4 ; (d) the regulated flow rate Q_3 .

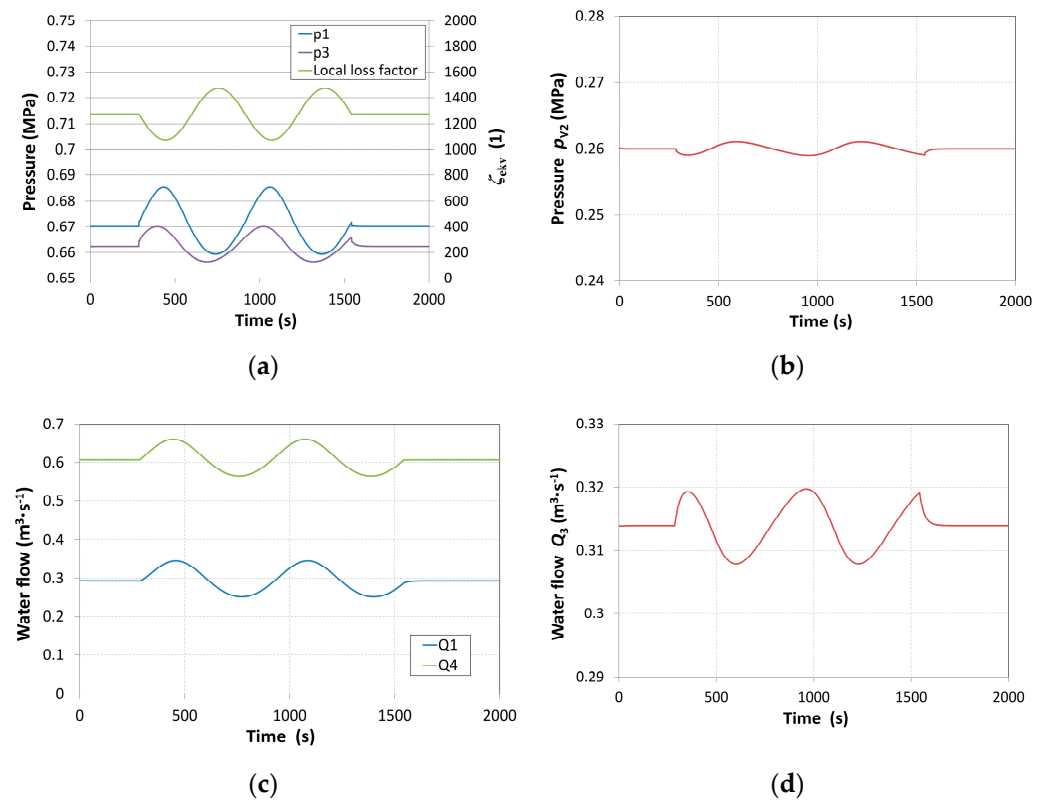


Figure 9. The curves of the operational parameters in Model Example 2 with the value of coefficient $\varepsilon = 5$; (a) the curves of p_1 , p_2 pressures and ζ_{eq} ; (b) the regulated water pressure at the inlet into the facility in the V2 Branch; (c) flow rates Q_1 and Q_4 ; (d) the regulated flow rate Q_3 .

In order to point out the regulation possibility in the standard operation, local loss factor ζ_{eq} was subjected to the stochastic signal with a maximum absolute value of 1343 and a minimal value of 1200 (Figure 10).

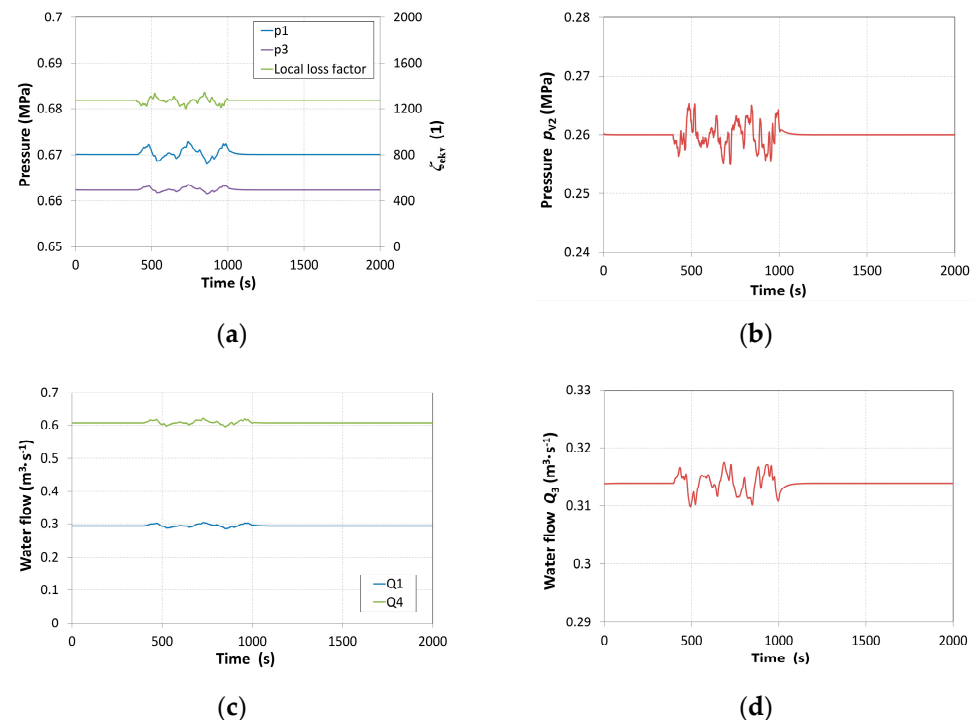


Figure 10. The curves of the operational parameters in Model Example 3 with the value of coefficient $\varepsilon = 0$; (a) the curves of p_1 , p_2 pressures and ζ_{eq} ; (b) the regulated pressure of water at the inlet into the facility in the V2 Branch; (c) flow rates Q_1 and Q_4 ; (d) the regulated flow rate Q_3 .

The local loss factor ranged from +5.3% to −5.9%. The deviation of pressure p_{v2} in Model Example 3 with the preset coefficient $\varepsilon = 0$ ranged from −1.9% to 2.04%, while the deviation of flow rate Q_3 ranged from −1.23% to 1.15%. Increasing the value of coefficient ε to 5 (Figure 11) resulted in a decrease in the maximum deviations of the regulated pressure p_{v2} to $\pm 0.6\%$.

The above simulations clearly indicate that the regulation system is capable of stabilising the required pressures at the end of the V2 and V4 Branches, as well as the regulated flow rate Q_3 . Increasing the coefficient ε reduced the oscillation amplitudes of the regulated parameters and shortened the time constant of the pipe, i.e., the time required for the regulated parameter that had deviated by 63.2% from its amplitude to stabilise. In Model Example 1, the time constant of the regulated parameters was reduced by 71% while the coefficient ε increased from 0 to 5.

In all three regulation examples, increasing the coefficient ε resulted in a decrease in the deviation of regulated pressure p_{v2} . However, more complex curves of pressures p_1 and p_3 at the outlet from the pumps are required, which may lead to sudden and frequent changes in the rotational speed. This is most apparent in the stochastic curve of the local loss factor. As a result, it may affect the service life of the pumps and subsequently also the economy of the transport of industrial water.

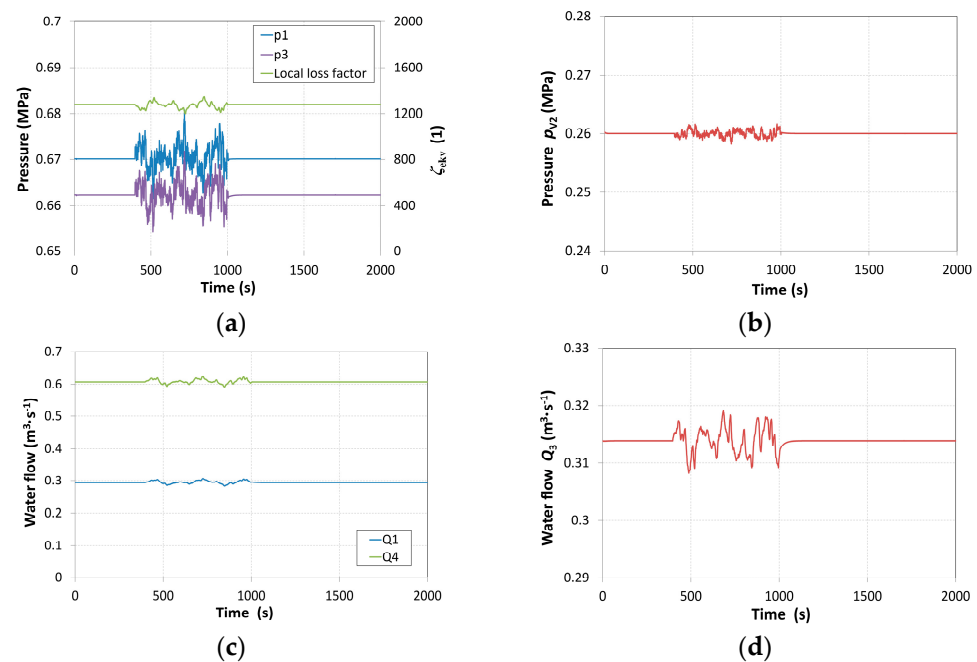


Figure 11. The curves of the operational parameters in Model Example 3 with the value of coefficient $\varepsilon = 5$; (a) the curves of p_1 , p_2 pressures and ζ_{eq} ; (b) the regulated water pressure at the inlet into the facility in the V2 Branch; (c) flow rates Q_1 and Q_4 ; (d) the regulated flow rate Q_3 .

4. Conclusions

In this article, the application of an analytical calculation of a hydraulic system, coupled with the finite-difference time-domain method, which is used to calculate a dynamic curve of the flow rate of water supplied to the consumption site from multiple sources, is described. The focus of the article is the identification of the pipeline parameters based on the measurements of the pressure boundary conditions and flow rates in a steady state, which facilitates taking into account the pipeline incrustation, as well as the absence of data on the local hydraulic resistances in the individual branches of the fluid distribution system. The created model exhibited good concordance with experimental data, and the pipeline coefficients may be continuously analysed during the operation of the distribution system. Such data are crucial especially in the regulation of flow rates and pressures in obsolete massive water distribution systems, for which it is impossible to clearly identify the pipeline parameters and which require simultaneous pipe repairs that involve implementing the action components that alter the pipeline parameters over time. The proposed regulation model facilitates controlling the discharge pressures of the pumps so that two different parameters may be concurrently regulated—a flow rate of water from one of the sources, and a discharge pressure of water flowing from the analysed hydraulic branch. In real-life facilities, the regulation of input pressures p_1 , p_2 and p_3 is carried out through the automatic regulation of rotational speeds of the pumps while respecting their operational characteristics. A time delay in the stabilisation of the output pressures of the pumps (reaction time of pumps) also affects the identification of the critical value of parameter ε_{crit} . The regulation model was tested in three simulation examples, in which gradual, sine and stochastic changes in the output load from the pipeline were analysed, and the system's ability to sufficiently stabilise the analysed operation was tested. It was observed that by applying the difference between the measured and the preset values of discharge pressure, or the product of that value and coefficient ε , it is possible to significantly reduce the deviation of the discharge pressure from the preset value while concurrently regulating the flow rate. The stability of the regulation system depends on the defined value of coefficient ε . Increasing that value complicates the curves of pressures at the pump outlets, and that may have a negative effect on their service life. Coefficient ε should therefore be preset

to different values in different systems, and its effect on the operation stability should be tested by simulation before it is actually applied. Following the adjustments of the model, it may also be applied to any type of pipeline system, and after its material properties are changed, it may be applied to any type of fluid. As the complexity of the system and the number of branches increase, the complexity of the analytical identification of the resulting equations for the calculation of the output pressure boundary conditions also increases. The equations presented in this paper that were used to calculate the pressure boundary conditions are not universal and must be modified for a particular pipeline network.

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